

5.4 Multiple Angle Identities

Target 6B: Prove trigonometric identities

Target 6C: Solve equations using trigonometric identities

Review Prior Concepts

Solve the trigonometric equation for x on $[0, 2\pi)$.

1) $\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \pi$

2) $\tan x - \sec x \cos x = 0$

$\tan x - \frac{1}{\cos x} \cdot \cos x = 0$

$\tan x - 1 = 0$

$\tan x = 1$
 $x = \frac{\pi}{4}, \frac{5\pi}{4}$

Double-Angle Identities

$\sin 2A = \sin(A + A)$

$= \sin A \cos A + \cos A \sin A$

$= \sin A \cos A + \sin A \cos A$

$\boxed{\sin 2A = 2 \sin A \cos A}$

$\tan 2A = \tan(A + A)$

$= \frac{\tan A + \tan A}{1 - \tan A \tan A}$

$\boxed{\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}}$

$\cos 2A = \cos(A + A)$

$= \cos A \cos A - \sin A \sin A$

$\boxed{\cos 2A = \cos^2 A - \sin^2 A}$

\downarrow
 $= 1 - \sin^2 A - \sin^2 A$

$\boxed{\cos 2A = 1 - 2 \sin^2 A}$

$\cos 2A = \cos^2 A - \sin^2 A$

\downarrow
 $= \cos^2 A - (1 - \cos^2 A)$

$= \cos^2 A - 1 + \cos^2 A$

$\boxed{\cos 2A = 2 \cos^2 A - 1}$

Examples**Solve for x on $[0, 2\pi)$.**

1) $\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x (2 \cos x - 1) = 0$

$\sin x = 0$

$2 \cos x - 1 = 0$

$x = 0, \pi$

$2 \cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

* Don't forget to
check for
extraneous
solutions

$\boxed{x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}}$

2) $\cos 2x = \cos x$

$$\cos 2x - \cos x = 0$$

$$2\cos^2 x - 1 - \cos x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$2u^2 - u - 1 = 0$$

$$(2u+1)(u-1) = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

let $u = \cos x$

$$2\cos x + 1 = 0$$

$$2\cos x = -1$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\cos x - 1 = 0$$

$$\cos x = 1$$

$$x = 0$$

$$x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$$

Prove the identity.

1) $\cos 6x = 2\cos^2 3x - 1$

$$\begin{aligned} \cos(2(3x)) &= \\ \cos(2A) &= \\ 2\cos^2 A - 1 &= \\ 2\cos^2(3x) - 1 &= \end{aligned}$$

$3x = A \dots \textcircled{1}$

2) $2\cot 2x = \cot x - \tan x$

$$\begin{aligned} 2 \cdot \frac{1}{\tan 2x} &= \\ 2 \cdot \frac{1}{2\tan x} &= \\ 2 \cdot \frac{1 - \tan^2 x}{2\tan x} &= \\ \frac{1 - \tan^2 x}{\tan x} &= \\ \frac{1}{\tan x} - \frac{\tan^2 x}{\tan x} &= \\ \cot x - \tan x &= \end{aligned}$$

3) $\sin 3x = \sin x (3 - 4\sin^2 x)$

$$\begin{aligned} \sin(2x+x) &= \\ \sin 2x \cos x + \cos 2x \sin x &= \\ 2\sin x \cos x \cdot \cos x + (1 - 2\sin^2 x) \sin x &= \\ 2\sin x \cos^2 x + \sin x - 2\sin^3 x &= \\ \sin x (2\cos^2 x + 1 - 2\sin^2 x) &= \\ \sin x [2(1 - \sin^2 x) + 1 - 2\sin^2 x] &= \\ \sin x (2 - 2\sin^2 x + 1 - 2\sin^2 x) &= \\ \sin x (3 - 4\sin^2 x) &= \end{aligned}$$

More Practice**Using Double Angle Identities**

<http://www.intmath.com/analytic-trigonometry/3-double-angle-formulas.php>

<http://www.ck12.org/trigonometry/Solving-Equations-with-Double-Angle-Identities/lesson/Solving-Trig-Equations-using-Double-and-Half-Angle-Formulas-ALG-II/>

<https://www.sophia.org/concepts/solving-an-equation-by-applying-a-double-angle-identity>

https://www.youtube.com/watch?v=LSh_Ol_XsaE

https://www.youtube.com/watch?v=rF36a8K_3QM

<https://www.youtube.com/watch?v=9mfvng-9cr0>

Homework Assignment

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