DATE: _____

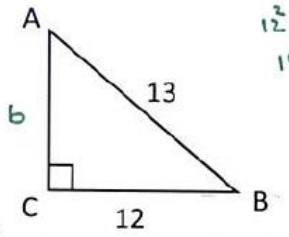
Unit 6 (Chapter 5): Analytic Trigonometry **Pre-Calculus**

5.5 The Law of Sines

Target 6D: Use Law of Sines and Law of Cosines to solve triangles

Review Prior Concepts

Solve the triangle for all missing sides and angles.



$12^2 + b^2 = 13^2$

$144 + b^2 = 169$

$b^2 = 25$

$b = 5$

$\sin A = \frac{12}{13}$

$A = \sin^{-1}\left(\frac{12}{13}\right)$

$\angle A = 67.38°$

$\cos B = \frac{12}{13}$

$B = \cos^{-1}\left(\frac{12}{13}\right)$

$\angle B = 22.62°$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

➲With what given conditions can Law of Sines be used?

* Angle - Side - Angle
* Angle - Angle - Side

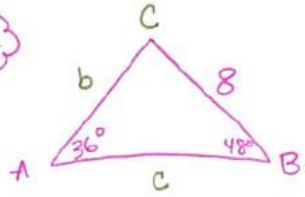
and Side - Side - Angle ... see back

Example

Solve the triangle given $\angle A = 36°$, $\angle B = 48°$, $a = 8$.

Draw picture

AAS



$\angle c = ?$

$b = ?$

$c = ?$

$\angle A + \angle B + \angle C = 180°$

$\angle C = 180 - (\angle A + \angle B)$

$= 180 - (36 + 48)$

$\angle C = 96°$

$\frac{\sin 36°}{8} = \frac{\sin 48°}{b}$

$b \sin 36° = 8 \sin 48°$

$b = \frac{8 \sin 48°}{\sin 36°}$

$b = 10.115$

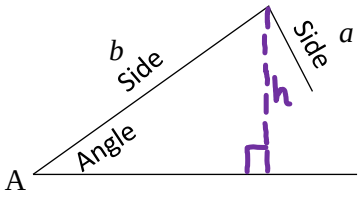
$\frac{\sin 36°}{8} = \frac{\sin 96°}{c}$

$c \sin 36° = 8 \sin 96°$

$c = \frac{8 \sin 96°}{\sin 36°}$

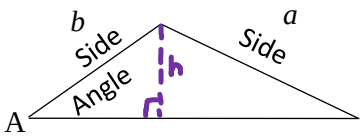
$c = 13.536$

The 3 Situations of Side-Side-Angle (The Ambiguous Case)



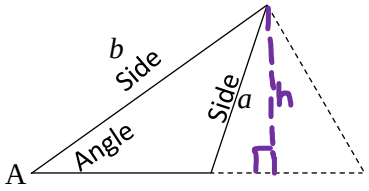
No Triangle

If $a < h$, where $\sin A = \frac{h}{b}$.



One Triangle

If $a > h$, where $\sin A = \frac{h}{b}$, AND $a \geq b$.



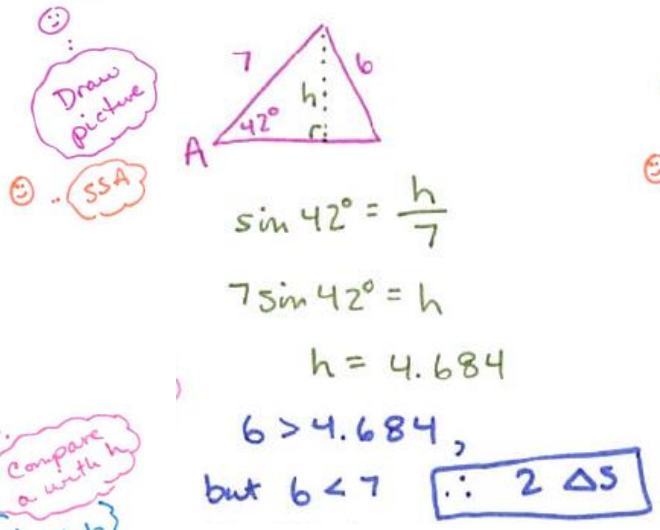
Two Triangles

If $a > h$, where $\sin A = \frac{h}{b}$, AND $a < b$.

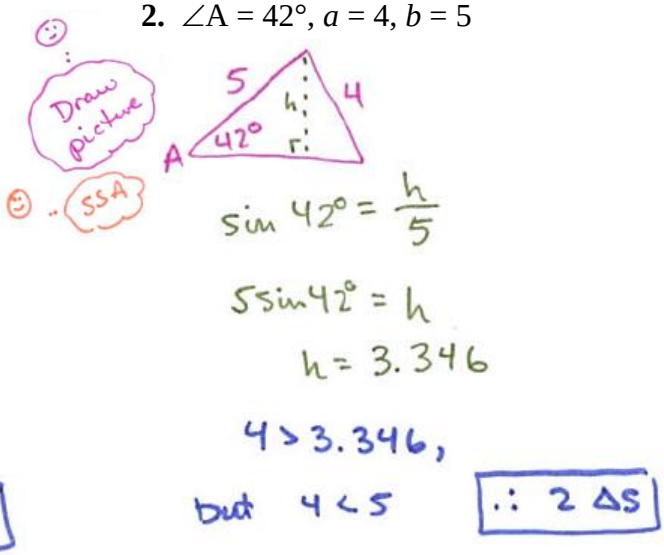
Examples

How many triangles can be made from the given information?

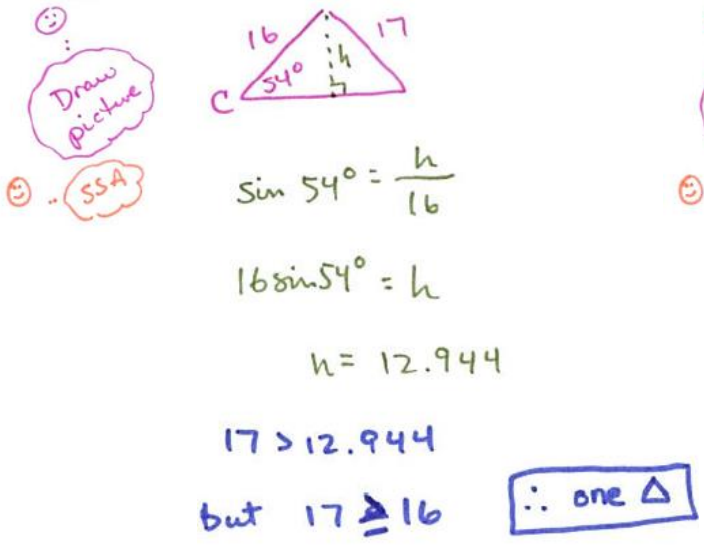
1. $\angle A = 42°$, $a = 6$, $b = 7$



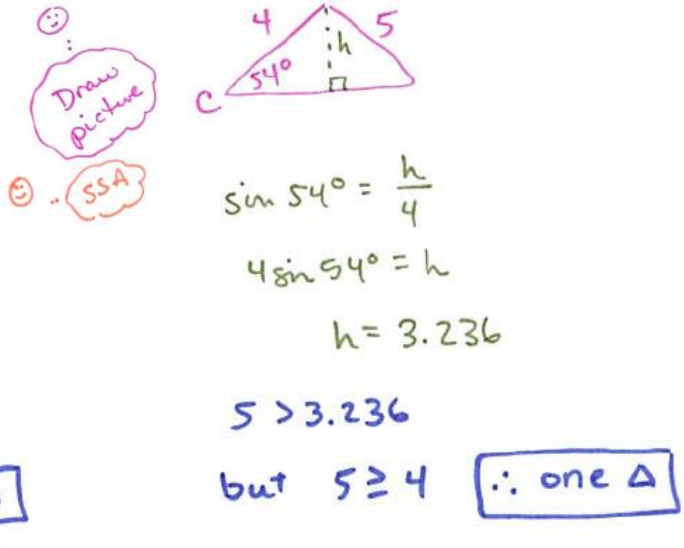
2. $\angle A = 42°$, $a = 4$, $b = 5$



3. $\angle C = 54°$, $b = 16$, $c = 17$

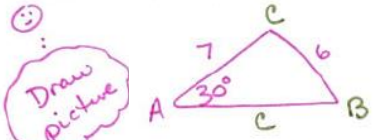


4. $\angle C = 54°$, $b = 4$, $c = 5$



Solve each triangle with the given information or state that a triangle cannot be made.
(there may be one Δ, two Δs, or no Δ)

5) ∠A = 30°, $a = 6$, $b = 7$



Draw picture

SSA

$\sin 30° = \frac{h}{7}$

$7\sin 30° = h$

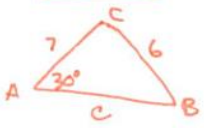
$3.5 = h$

$6 > 3.5$

$6 < 7$ } ∴, 2 Δs

∠B = ?
∠C = ?
c = ?

Triangle 1



$\frac{\sin 30°}{6} = \frac{\sin B}{7}$

$7\sin 30° = 6\sin B$

$\frac{7\sin 30°}{6} = \sin B$

$\sin^{-1}\left(\frac{7\sin 30°}{6}\right) = B$

$\angle B = 35.685°$

$\angle C = 180 - (30 + 35.685)$

$\angle C = 114.315°$

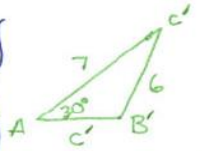
$\frac{\sin 30°}{6} = \frac{\sin 114.315°}{c}$

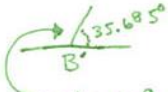
$c\sin 30° = 6\sin 114.315°$

$c = \frac{6\sin 114.315°}{\sin 30°}$

$c = 10.936$

Triangle 2





$\angle B' = 180° - (35.685°)$

$\angle B' = 144.315°$

$\angle C' = 180° - (30° + 144.315°)$

$\angle C' = 5.685°$

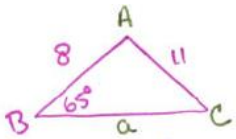
$\frac{\sin 30°}{6} = \frac{\sin 5.685°}{c}$

$c\sin 30° = 6\sin 5.685°$

$c = \frac{6\sin 5.685°}{\sin 30°}$

$c = 1.189$

6) ∠B = 65°, $b = 11$, $c = 8$



Draw picture

SSA

$\sin 65° = \frac{h}{8}$

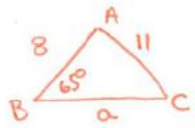
$8\sin 65° = h$

$h = 7.25$

$11 > 7.25$

$11 > 8$ } ∴, one Δ

∠C = ?
∠A = ?
a = ?



$\frac{\sin 65°}{11} = \frac{\sin C}{8}$

$8\sin 65° = 11\sin C$

$\frac{8\sin 65°}{11} = \sin C$

$\sin^{-1}\left(\frac{8\sin 65°}{11}\right) = C$

$\angle C = 41.234°$

$\angle B = 180° - (65° + 41.234°)$

$\angle B = 73.766°$

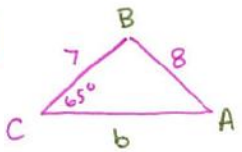
$\frac{\sin 65°}{11} = \frac{\sin 73.766°}{a}$

$a\sin 65° = 11\sin 73.766°$

$a = \frac{11\sin 73.766°}{\sin 65°}$

$a = 11.653$

7) ∠C = 65°, $a = 7$, $c = 8$



Draw picture

SSA

$\sin 65° = \frac{h}{7}$

$h = 7\sin 65°$

$h = 6.344$

$8 > 6.344$

$8 > 7$ } ∴, one Δ

∠A = ?
∠B = ?
b = ?

$\frac{\sin 65°}{8} = \frac{\sin A}{7}$

$7\sin 65° = 8\sin A$

$\frac{7\sin 65°}{8} = \sin A$

$\sin^{-1}\left(\frac{7\sin 65°}{8}\right) = A$

$\angle A = 52.469°$

$\angle B = 180° - (65° + 52.469°)$

$\angle B = 62.531°$

$\frac{\sin 65°}{8} = \frac{\sin 62.531°}{b}$

$b\sin 65° = 8\sin 62.531°$

$b = \frac{8\sin 62.531°}{\sin 65°}$

$b = 7.832$

More Practice

Law of Sines

https://www.khanacademy.org/math/geometry/hs-geo-trig/hs-geo-law-of-sines/v/law-of-sines
https://www.mathsisfun.com/algebra/trig-sine-law.html
http://www.themathpage.com/atrig/law-of-sines.htm
http://www.softschools.com/math/calculus/the_ambiguous_case_of_the_law_of_sines/
https://www.youtube.com/watch?v=yVquId7xJQY
https://www.youtube.com/watch?v=ksBaHrVqhyo
https://www.youtube.com/watch?v=S4xAKewlqA4

Homework Assignment

p.439 #1,3,7,11,13,15,19,25,29