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Unit 8 (Chapter 6 & 7): Matrices & Vectors **Pre-Calculus**

6.1 Vectors in the Plane

Target 8A: Perform vector operations: scalar multiple and sums and represent them graphically
Target 8B: Perform vector operations: magnitude, direction angle, and unit vector

Review of Prior Concepts

$P = \begin{bmatrix}3\\1\end{bmatrix}$, $Q = \begin{bmatrix}4\\-2\end{bmatrix}$, $R = \begin{bmatrix}-4\\2\end{bmatrix}$, and $S = \begin{bmatrix}-1 & 2\\6 & 1\end{bmatrix}$

Perform the indicated matrix operation.

a) P + Q

$P+Q = \begin{bmatrix}3\\1\end{bmatrix}+\begin{bmatrix}4\\-2\end{bmatrix} = \begin{bmatrix}7\\-1\end{bmatrix}$

b) Q + R

$Q+R = \begin{bmatrix}4\\-2\end{bmatrix}+\begin{bmatrix}-4\\2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

c) Q – R

$Q-R = \begin{bmatrix}4\\-2\end{bmatrix}-\begin{bmatrix}-4\\2\end{bmatrix} = \begin{bmatrix}8\\-4\end{bmatrix}$

d) 3Q

$3Q = 3\begin{bmatrix}4\\-2\end{bmatrix} = \begin{bmatrix}12\\-6\end{bmatrix}$

e) QS

QS cannot be done b/c cannot multiply (2x1)(2x2) matrices

f) SR

$SR = \begin{bmatrix}-1 & 2\\6 & 1\end{bmatrix}\begin{bmatrix}-4\\2\end{bmatrix} = \begin{bmatrix}-1(-4)+2(2)\\6(-4)+1(2)\end{bmatrix} = \begin{bmatrix}8\\-22\end{bmatrix}$

More Practice

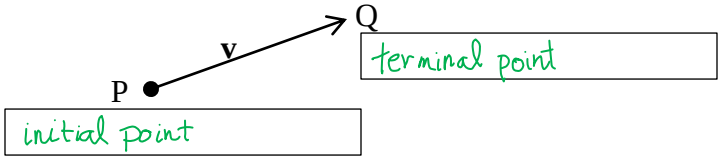
Operations with Matrices

http://www.mathsisfun.com/algebra/matrix-introduction.html
https://www.khanacademy.org/math/precalculus/precalc-matrices#adding-and-subtracting-matrices
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_matrix_operations.xml
https://www.youtube.com/watch?v=xr6qsiEznKU
https://www.youtube.com/watch?v=SPFWVUkxk8E
https://www.youtube.com/watch?v=kuixY2bCc_0
https://www.youtube.com/watch?v=sYlOjyPyX3g

Vectors

Vector -- a directed line segment with an initial point and terminal point.
The vector is represented with a **bold** lowercase letter.

$\overrightarrow{PQ}$ = ✓



Note: The arrow on the vector shows the direction --- a vector is NOT a ray (is a segment)
The vector above stops at Q.

A vector has magnitude (length of the vector) represented by $|\overrightarrow{PQ}|$ or |✓|

and direction (slope of the vector) represented by an angle.

Vectors are equal if both magnitude AND direction are equal.
(pointing in the same direction)

Example

u is a represented by $\overrightarrow{RS}$ and **v** is represented by $\overrightarrow{OP}$, where $R = (7,-3)$, $S = (4,-5)$, $O = (1,1)$, $P = (-2,-1)$. Prove **u** = **v**.

magnitude of $\overrightarrow{RS}$ = $|\overrightarrow{RS}| = \sqrt{(7-4)^2 + (-3--5)^2}$

$|u| = \sqrt{13}$

magnitude of $\overrightarrow{OP}$ = $|\overrightarrow{OP}| = \sqrt{(1--2)^2 + (1--1)^2}$

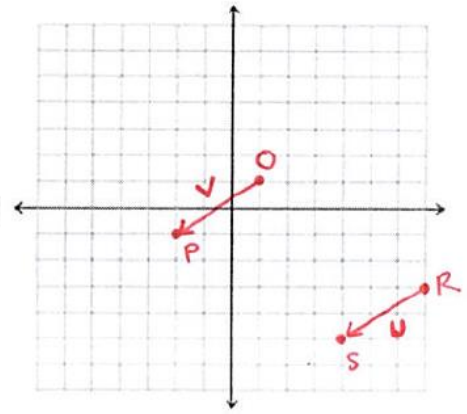
$|v| = \sqrt{13}$

$|u| = |v|$ ✓

Slope of $\overrightarrow{RS}$ = 2/3

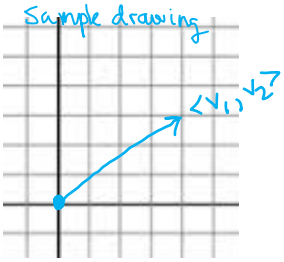
Slope of $\overrightarrow{OP}$ = 2/3 } = ✓

∴, u = v



Component Form and Magnitude of a Vector

If **v** is a vector in a plane equal to the vector with initial point at (0,0) and terminal point at (v_1, v_2), then the **component form** of **v** is



v = ⟨ v_1 , v_2 ⟩

The *components* are v_1 and v_2.

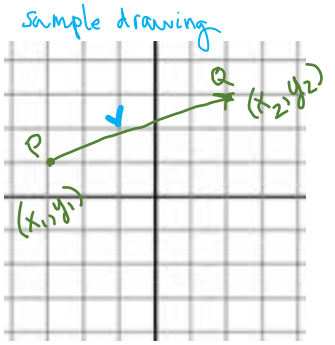
The vector is called the *position vector*, where position = ⟨ v_1 , v_2 ⟩

Writing a Vector in Component Form (Head Minus Tail Rule)

Given $\overrightarrow{PQ}$ = **v**, where $P = (x_1, y_1)$ and $Q = (x_2, y_2)$, then

v = ⟨ $x_2 - x_1$, $y_2 - y_1$ ⟩ (terminal pt − initial pt)

= ⟨ v_1 , v_2 ⟩

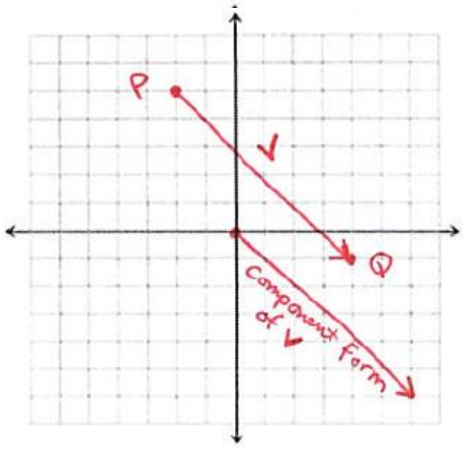


Magnitude of **v** = $|v| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ (distance formula, Pythagorean theorem)

or $\sqrt{(v_1)^2 + (v_2)^2}$

Example

Write the vector, **v** = $\overrightarrow{PQ}$, in component form and find the magnitude of **v**, where $P = (-2,5)$ and $Q = (4,-1)$.



v = ⟨ 4 − −2 , −1 − 5 ⟩

v = ⟨ 6, −6 ⟩

magnitude = $\sqrt{(4--2)^2 + (-1-5)^2}$

$= \sqrt{(6)^2 + (-6)^2}$

$= \sqrt{72}$

or $6\sqrt{2}$

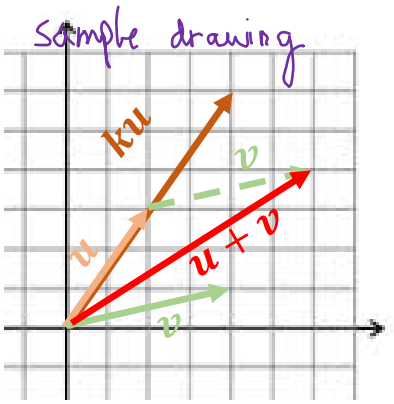
Vector Addition (Tail-to-Head Rule) **and Scalar Multiplication**

Let **u** = $\langle u_1 , u_2 \rangle$ and **v** = $\langle v_1 , v_2 \rangle$ and k is a constant, then

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1 , u_2 + v_2 \rangle$$

and

$$k\mathbf{u} = \langle ku_1 , ku_2 \rangle$$

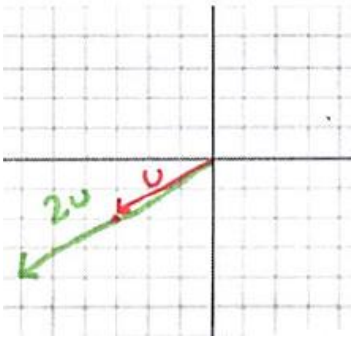


Example
Let **u** = $\langle -3, -2 \rangle$ and **v** = $\langle 1 , 4 \rangle$. Sketch and label a geometric representation of the given vector operations:

a) 2**u**

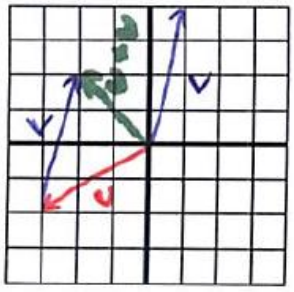
$2\mathbf{u} = 2\langle -3, -2 \rangle$
$= \langle -6, -4 \rangle$

a)



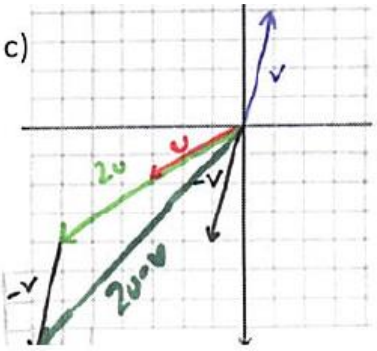
b) **u** + **v** $= \langle -3, -2 \rangle + \langle 1, 4 \rangle$
$= \langle -2, 2 \rangle$

b)



c) 2**u** – **v** $= 2\langle -3, -2 \rangle - \langle 1, 4 \rangle$
$= \langle -6, -4 \rangle - \langle 1, 4 \rangle$
$= \langle -7, -8 \rangle$

c)



Vector Practice

Component Form and Magnitude

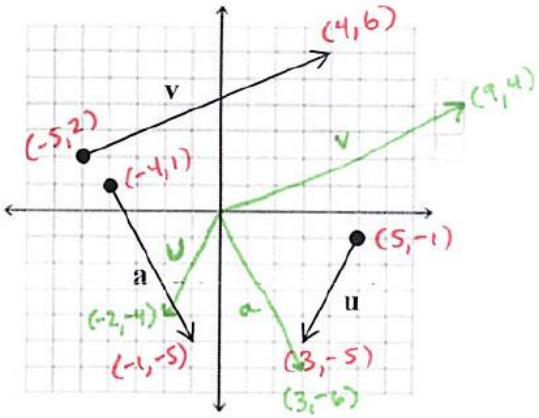
Find the component form and magnitude of each vector.

$v = \langle 4 - -5, 6 - 2 \rangle$
$= \langle 9, 4 \rangle$
$|v| = \sqrt{9^2 + 4^2}$
$= \sqrt{97}$

$u = \langle 3 - 5, -5 - -1 \rangle$
$= \langle -2, -4 \rangle$
$|u| = \sqrt{(-2)^2 + (-4)^2}$
$= \sqrt{20}$ or $2\sqrt{5}$

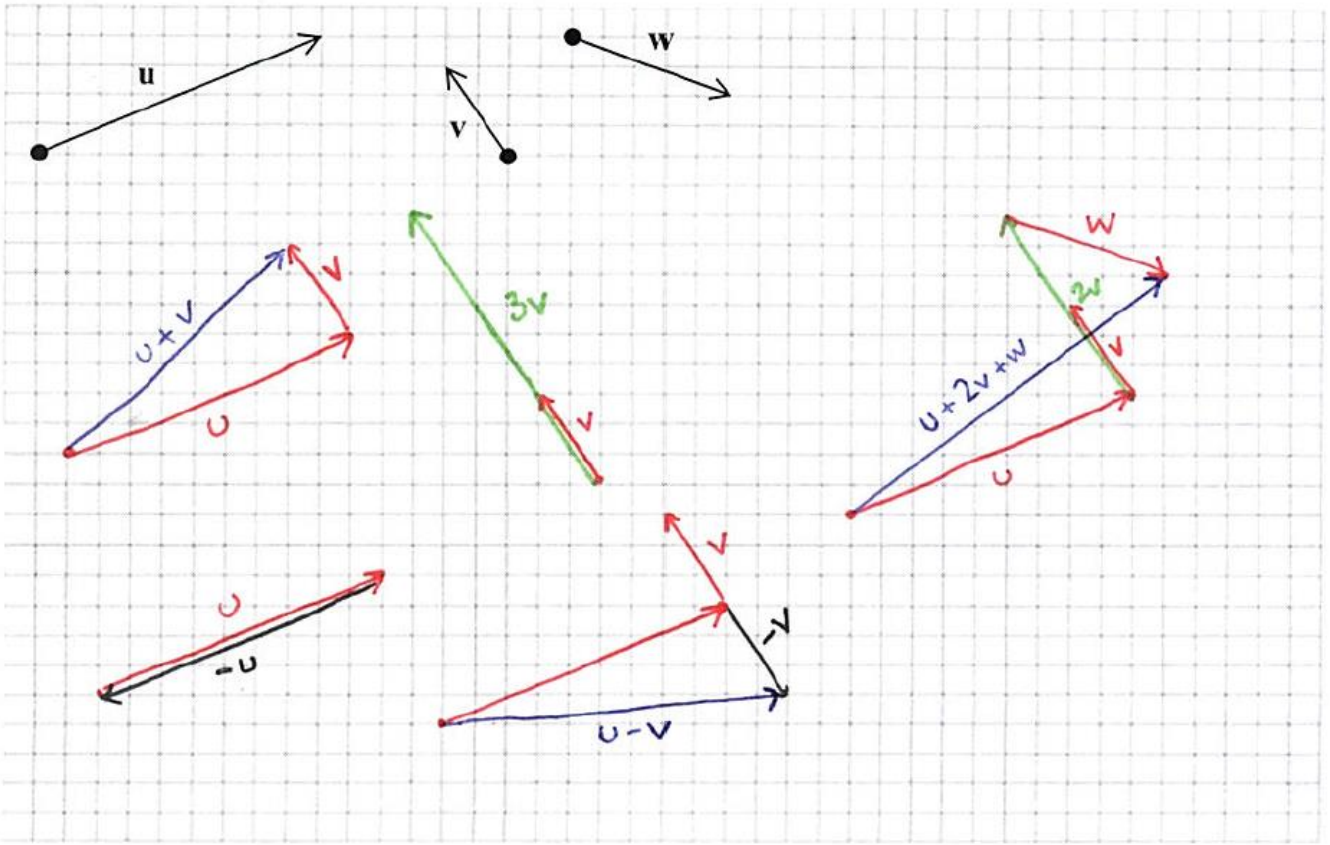
$a = \langle -1 - -4, -5 - 1 \rangle$
$= \langle 3, -6 \rangle$
$|a| = \sqrt{3^2 + (-6)^2} = \sqrt{45}$ or $3\sqrt{5}$

$v = \langle 9, 4 \rangle$
$|v| = \sqrt{97}$
$u = \langle -2, -4 \rangle$
$|u| = 2\sqrt{5}$
$a = \langle 3, -6 \rangle$
$|a| = 3\sqrt{5}$



Addition and Scalar Multiplication

Sketch and label a geometric representation of the given vector operations:
u + **v** , 3**v** , – **u** , **u** – **v**, and **u** + 2**v** + **w**



More Practice

Introduction to Vectors

https://www.mathsisfun.com/algebra/vectors.html
https://www.khanacademy.org/math/precalculus/vectors-precalc
http://philschatz.com/precalculus-book/contents/m49412.html
https://www.varsitytutors.com/precalculus-help/matrices-and-vectors/geometric-vectors
https://youtu.be/_YkIivLaVJs
https://youtu.be/60btq9PN8IM
https://youtu.be/IKzR0Odurm0

Component Form of Vectors

https://braingenie.ck12.org/skills/108146
https://www.khanacademy.org/math/precalculus/vectors-precalc/vector-basic/v/example-finding-components-of-a-vector
https://www.mathway.com/examples/precalculus/vectors/finding-the-position-vector?id=582
https://youtu.be/pimr9I92GZY
https://youtu.be/-0qEDcZZS9E
https://youtu.be/wz5AvyKWjb8

Vector Addition and Scalar Multiplication

http://www.mathplanet.com/education/geometry/transformations/vectors
http://www.softschools.com/math/pre_calculus/scalar_multiplication_and_vector_addition/
https://www.khanacademy.org/math/precalculus/vectors-precalc#scalar-multiplication
https://youtu.be/pNMrYACjHXQ
https://youtu.be/Z1o-DGohM60
https://youtu.be/_YkIivLaVJs
https://youtu.be/lulSApFPw1M
https://youtu.be/0i9Cp-6QuUk

Homework Assignment

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