DATE: _____

Unit 8 (Chapter 6 & 7): Matrices & Vectors **Pre-Calculus**

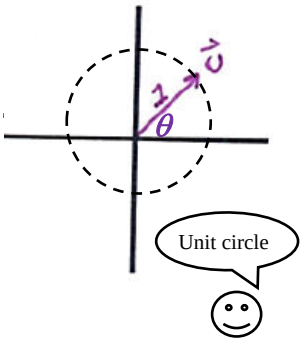
6.1 Vectors in the Plane (continued)

Target 8A: Perform vector operations: scalar multiple and sums and represent them graphically
Target 8B: Perform vector operations: magnitude, direction angle, and unit vector

Unit Vector

Unit Vector – A vector, **u**, with a length of _ one unit

$|\mathbf{u}| =$ _ 1 __



If **v** is not the zero vector, then the unit vector in the direction of **v** is:

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\langle v_1, v_2 \rangle}{\text{magnitude of } v}$$

Note: $\mathbf{u} = \langle \cos\theta, \sin\theta \rangle$

The **Standard Unit Vectors** are $\mathbf{i} = \langle 1,0 \rangle$ and $\mathbf{j} = \langle 0,1 \rangle$.

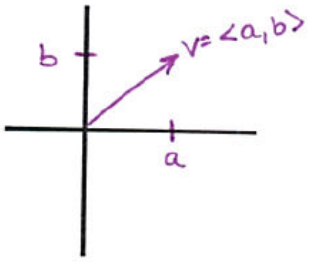
So, any vector $\mathbf{v} = \langle a, b \rangle$ can be expressed as

the linear combination $ai + bj$ of **i** and **j**,

where the scalar *a* is the horizontal component

and *b* is the vertical component

of vector **v**.



Example

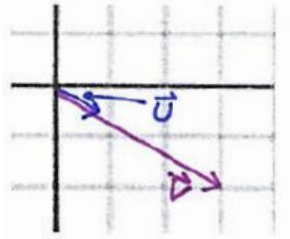
Find a unit vector in the direction of $\mathbf{v} = \langle 3, -2 \rangle$. Write the unit vector as a linear combination of the standard unit vector.

$|v| = \sqrt{3^2 + (-2)^2}$
$= \sqrt{9+4}$
$= \sqrt{13}$

$u = \frac{v}{|v|} = \frac{\langle 3, -2 \rangle}{\sqrt{13}}$

$\vec{u} = \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$\vec{u} = \frac{3}{\sqrt{13}} i - \frac{2}{\sqrt{13}} j$



A vector's **Direction Angle** is the angle θ that **v** makes with the positive *x*-axis. *(This is precise way to describe direction of a vector.)*

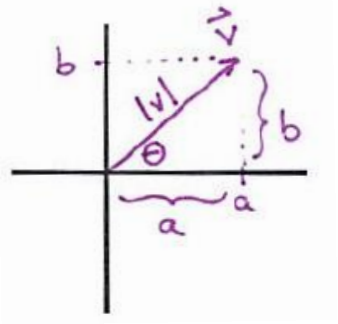
$\sin\theta = \frac{b}{|v|}$ $\cos\theta = \frac{a}{|v|}$

So, $b = |v| \sin\theta$ So, $a = |v| \cos\theta$

The horizontal component is $|v| \cos\theta$.

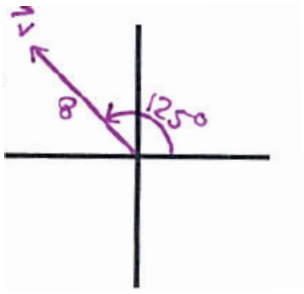
The vertical component is $|v| \sin\theta$.

$\therefore, \mathbf{v} = |v|\cos\theta\, i + |v| \sin\theta\, j = \langle |\mathbf{v}| \cdot \cos\theta, |\mathbf{v}| \cdot \sin\theta \rangle$



Example

Find the components of vector **v** with direction angle of 125° and magnitude of 8.



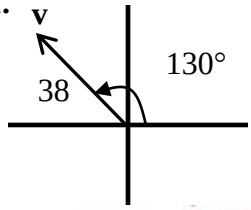
$v = \langle 8\cos 125°, 8\sin 125° \rangle$

$v \approx \langle -4.589, 6.553 \rangle$

Practice

For #1 & 2, find the component form of the vector.

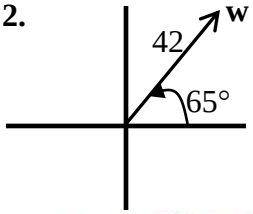
1.



$v = \langle 38\cos 130°, 38\sin 130° \rangle$

$v = \langle -24.426, 29.110 \rangle$

2.



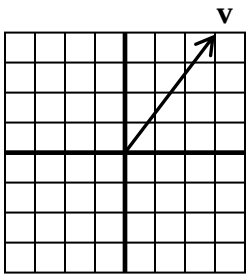
$w = \langle 42\cos 65°, 42\sin 65° \rangle$

$w = \langle 17.75, 38.065 \rangle$

For #3-6, find:

a) the component form of the vector,
b) the unit vector in the direction of the given vector,
c) and the direction angle of the vector.

3.



component form

$v = \langle 3, 4 \rangle$

$|v| = \sqrt{3^2+4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$= 5$

$u = \frac{v}{|v|} = \frac{\langle 3,4 \rangle}{5}$

unit vector $u = \langle \frac{3}{5}, \frac{4}{5} \rangle$

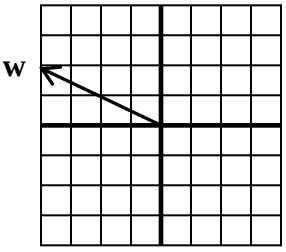
$\sin\theta = \frac{4}{5}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$

$\theta = 53.130°$

direction $\theta \approx 53°$

4.



component form

$w = \langle -4, 2 \rangle$

$|w| = \sqrt{(-4)^2+(2)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$u = \frac{w}{|w|} = \frac{\langle -4,2 \rangle}{2\sqrt{5}}$

unit vector $u = \langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$

$\cos\theta = \frac{-2}{\sqrt{5}}$

$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{5}}\right)$

$\theta = 153.435$

direction $\theta \approx 153°$

5. $\mathbf{v} = \langle -2, 3 \rangle$

$v = \langle -2, 3 \rangle$ component form

$|v| = \sqrt{(-2)^2 + 3^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$

$u = \frac{v}{|v|} = \frac{\langle -2, 3 \rangle}{\sqrt{13}}$

unit vector $u = \left\langle \frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

$\cos\theta = \frac{-2}{\sqrt{13}}$

$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{13}}\right)$

$\theta = 123.690$

$\theta \approx 124^\circ$ direction $\angle$

6. $\mathbf{w} = 5\mathbf{i} + 2\mathbf{j}$

$w = \langle 5, 2 \rangle$ component form

$|w| = \sqrt{5^2 + 2^2}$

$= \sqrt{25+4}$

$= \sqrt{29}$

$u = \frac{\langle 5, 2 \rangle}{\sqrt{29}}$

unit vector $u = \left\langle \frac{5}{\sqrt{29}}, \frac{2}{\sqrt{29}} \right\rangle$

$\cos\theta = \frac{5}{\sqrt{29}}$

$\theta = \cos^{-1}\left(\frac{5}{\sqrt{29}}\right)$

$\theta = 21.801$

$\theta \approx 22^\circ$ direction angle

More Practice

Unit Vectors

https://www.khanacademy.org/math/precalculus/vectors-precalc#unit-vectors
https://www.mathsisfun.com/algebra/vector-unit.html
http://www.algebralab.org/lessons/lesson.aspx?file=Trigonometry_TrigVectorUnits.xml
http://www.softschools.com/formulas/physics/unit_vector_formula/83/
https://youtu.be/6o_S7u7Ddx4
https://youtu.be/iAeKl5h2SJM

Magnitude and Direction Angle of Vectors

https://braingenie.ck12.org/skills/108146
https://www.khanacademy.org/math/precalculus/vectors-precalc#magnitude-direction
https://www.varsitytutors.com/hotmath/hotmath_help/topics/magnitude-and-direction-of-vectors
http://www.softschools.com/math/pre_calculus/direction_angles_of_vectors/
https://youtu.be/WxWJorOVIj8
https://youtu.be/8Eur16foTMw
https://youtu.be/82nu_sAPmmo

Component Form of Vectors

https://www.khanacademy.org/math/precalculus/vectors-precalc#component-form-of-vectors
https://www.math10.com/en/geometry/vectors-operations/vectors-operations.html
https://www.varsitytutors.com/precalculus-help/express-a-vector-in-component-form
https://youtu.be/GxBUbiNL1eE
https://youtu.be/WZ3xzVHT0mc

Homework Assignment

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