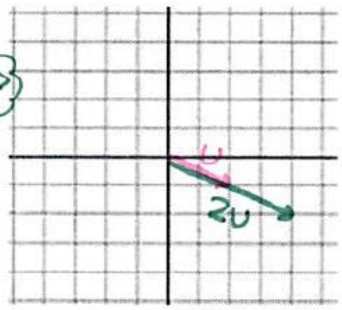
6.2 Dot Product of Vectors

Target 8C: Calculate and use properties of the Dot Product

Review of Prior Concepts

1. Let **u** = $\langle 2, -1 \rangle$. Sketch **u** and 2**u**.

2u= <4,-2>



2. Find AB, if A = $[1 \quad -3]$ and B = $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$.

$AB = [1 \ -3]\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

$= [1(4) + -3(2)]$

$= [-2]$

(1 x 2)(2 x 1)
= 1 x 1
one by one

Dot Product

"u dot v"

If $\vec{u} = \langle a, b \rangle$ and $\vec{v} = \langle c, d \rangle$, then

$\vec{u} \cdot \vec{v} =$ $ac + bd$

constant #
"scalar"

$u = [a \quad b]$, $v = \begin{bmatrix} c \\ d \end{bmatrix}$, then

$uv = [ac + bd]$

Example

Given $\vec{u} = \langle 1,3 \rangle$ and $\vec{v} = \langle 4,5 \rangle$, find $\vec{u} \cdot \vec{v}$.

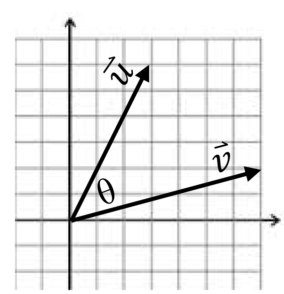
$\vec{u} \cdot \vec{v} = 1(4) + 3(5)$

$= \boxed{19}$

Angle Between Two Vectors

See how this formula comes about at: https://youtu.be/eLMLJkcllBs

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$



Example

Given $\vec{u} = \langle 1,3 \rangle$ and $\vec{v} = \langle 4,5 \rangle$, find the angle between $\vec{u}$ and $\vec{v}$.

$\cos \theta = \dfrac{1(4) + 3(5)}{(\sqrt{10})(\sqrt{41})}$ $\rightarrow \vec{u} \cdot \vec{v}$, $\rightarrow |\vec{u}||\vec{v}|$

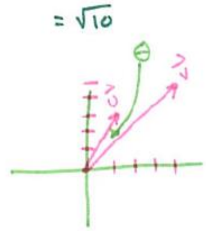
$|\vec{u}| = \sqrt{1^2 + 3^2}$
$= \sqrt{10}$

$|\vec{v}| = \sqrt{4^2 + 5^2}$
$= \sqrt{41}$

$\cos \theta = \dfrac{19}{(\sqrt{10})(\sqrt{41})}$

$\theta = \cos^{-1}\left(\dfrac{19}{\sqrt{10}\sqrt{41}}\right)$

$\boxed{\theta = 20.225°}$



Now you try…

1. Given $\vec{u} = \langle 2, -4\rangle$, and $\vec{v} = \langle -8, 7\rangle$, find $\vec{u} \cdot \vec{v}$.

$\vec{u} \cdot \vec{v} = 2(-8) + (-4)(7)$

$= -16 + {}^{-}28$

$= \boxed{-44}$

2. Given $\vec{u} = 4\mathbf{i} - 11\mathbf{j}$ and $\vec{v} = -3\mathbf{j}$, find $\vec{u} \cdot \vec{v}$.

$\hookrightarrow \vec{u} = \langle 4, -11\rangle$ $\hookrightarrow \vec{v} = \langle 0, -3\rangle$

$\vec{u} \cdot \vec{v} = 4(0) + -11(-3)$

$= \boxed{33}$

3. Given $\vec{u} = \langle -3, 8\rangle$ and $\vec{v} = \langle -1, -9\rangle$, find the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \dfrac{-3(-1) + 8(-9)}{\sqrt{73} \cdot \sqrt{82}}$

$|\vec{u}| = \sqrt{-3^2 + 8^2} = \sqrt{73}$

$|\vec{v}| = \sqrt{(-1)^2 + (-9)^2} = \sqrt{82}$

$\theta = \cos^{-1}\left(\dfrac{-69}{\sqrt{73}\,\sqrt{82}}\right)$

$\boxed{\theta = 153.104°}$

4. Given $\vec{u} = \langle -2, 0\rangle$ and $\vec{v} = \langle 0, 5\rangle$, find the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \dfrac{-2(0) + 0(5)}{2 \cdot 5}$

$|\vec{u}| = \sqrt{0^2 + 2^2} = 2$

$|\vec{v}| = \sqrt{0^2 + 5^2} = 5$

$\cos\theta = \dfrac{0}{10}$

$\cos\theta = 0$

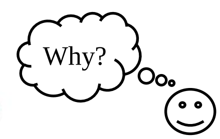
$\theta = 90°$

Orthogonal Vectors

If $\vec{u} \cdot \vec{v} = 0$, then the vectors are orthogonal.

$\hookrightarrow$ "perpendicular"

"the 2 vectors make a 90° ∠"



Example:

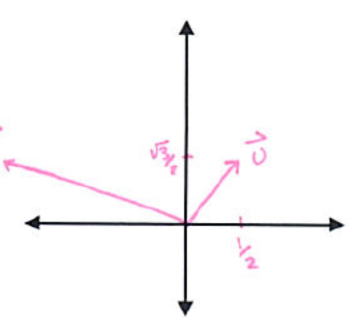
(Non-calculator)

Given $\vec{u} = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3}\rangle$ and $\vec{v} = \langle 3\cos\frac{5\pi}{6}, 3\sin\frac{5\pi}{6}\rangle$, find the angle between $\vec{u}$ and $\vec{v}$.

$\vec{u} = \langle 1/2, \sqrt{3}/2\rangle$

$\vec{v} = \langle 3(\frac{-\sqrt{3}}{2}), 3(\frac{1}{2})\rangle$

$\vec{v} = \langle -\frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$



$\vec{u} \cdot \vec{v} = \frac{1}{2}(\frac{-3\sqrt{3}}{2}) + \frac{\sqrt{3}}{2}(\frac{3}{2})$

$= \frac{-3\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}$

$= 0$

$\therefore$, $\vec{u}$ & $\vec{v}$ are orthogonal, so $\theta = \boxed{90°}$

More Practice

Dot Product

https://www.mathsisfun.com/algebra/vectors-dot-product.html
https://betterexplained.com/articles/vector-calculus-understanding-the-dot-product/
https://youtu.be/KDHuWxy53uM
https://youtu.be/98C7iv8OcnI

Angle Between Vectors

http://onlinemschool.com/math/library/vector/angl/
https://youtu.be/WDdR5s0C4cY
https://youtu.be/4WxniMJYySc

Homework Assignment

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