DATE: ________________

6.2 Dot Product of Vectors (*Target 8C & 8D*)

RALLY COACH

Find the component form of $\overrightarrow{AB}$ and find the magnitude of $\overrightarrow{AB}$.

1. $A\ (2,4)$, $B\ (-1,3)$

$\overrightarrow{AB} = \langle -1-2, 3-4 \rangle$

$\boxed{\overrightarrow{AB} = \langle -3, -1 \rangle}$

$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (-1)^2}$

$\boxed{|\overrightarrow{AB}| = \sqrt{10}}$

1. $A\ (4,-2)$, $B\ (5,-5)$

$\overrightarrow{AB} = \langle 5-4, -5--2 \rangle$

$\boxed{\overrightarrow{AB} = \langle 1, -3 \rangle}$

$|\overrightarrow{AB}| = \sqrt{1^2 + (-3)^2}$

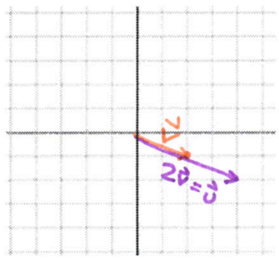
$\boxed{|\overrightarrow{AB}| = \sqrt{10}}$

Let $\vec{v} = \langle 2, -1 \rangle$ and $\vec{w} = \langle -3, 1 \rangle$. Find $\vec{u}$ and sketch the vector operation.

2. $u = 2v$

$u = 2\langle 2, -1 \rangle$

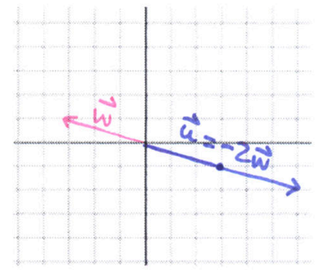
$\vec{u} = \langle 4, -2 \rangle$



2. $u = -2w$

$u = -2\langle -3, 1 \rangle$

$\vec{u} = \langle 6, -2 \rangle$



Find the unit vector.

3. $v = \langle -2, 5 \rangle$

unit vector $= \dfrac{\langle -2, 5 \rangle}{\sqrt{(-2)^2 + 5^2}}$

$= \dfrac{\langle -2, 5 \rangle}{\sqrt{29}}$

$\boxed{\text{unit vector} = \left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle}$

3. $v = \langle 3, -2 \rangle$

unit vector $= \dfrac{\langle 3, -2 \rangle}{\sqrt{3^2 + (-2)^2}}$

$= \dfrac{\langle 3, -2 \rangle}{\sqrt{13}}$

$\boxed{\text{unit vector} = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle}$

Find the direction angle of the vector.

4. $v = -2i + 5j$

$\cos\theta = \dfrac{-2}{\sqrt{(-2)^2 + 5^2}}$

$\cos\theta = \dfrac{-2}{\sqrt{29}}$

$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{29}}\right)$ $\boxed{\theta = 111.801^\circ}$

4. $v = 3i - 2j$

$\cos\theta = \dfrac{3}{\sqrt{3^2 + (-2)^2}}$

$\cos\theta = \dfrac{3}{\sqrt{13}}$

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right) \Rightarrow$ $\theta = 33.690^\circ$

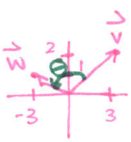
QUAD IV, ∴

$\theta = 360 - 33.690$

$\boxed{\theta = 326.310^\circ}$

Sketch the two vectors. Find the angle between the two vectors

5. $v = 3i + 2j$, $w = -3i + j$



$\cos\theta = \dfrac{v \cdot w}{|v||w|}$

$\cos\theta = \dfrac{3(-3) + 2(1)}{\sqrt{13}\ \sqrt{10}}$

$\cos\theta = \dfrac{-7}{\sqrt{13}\sqrt{10}}$

$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{13}\sqrt{10}}\right)$

$\boxed{\theta = 127.875^\circ}$

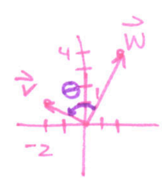
5. $v = -2i + j$, $w = 2i + 4j$

$\cos\theta = \dfrac{-2(2) + 1(4)}{\sqrt{5}\ \sqrt{20}}$

$\cos\theta = \dfrac{0}{\sqrt{5}\sqrt{20}}$ $\vec{u} \cdot \vec{v} = 0$

∴, u & v orthogonal

$\boxed{\theta = 90^\circ}$



Find $u \cdot v$.

6. $|\vec{u}| = 8$, $|\vec{v}| = 12$, and angle between $\vec{u}$ and $\vec{v}$ is 60°.

$\cos\theta = \dfrac{u \cdot v}{|u||v|}$

$\cos 60^\circ = \dfrac{u \cdot v}{8 \cdot 12}$

$8 \cdot 12 \cdot \cos 60^\circ = u \cdot v$

$\boxed{u \cdot v = 48}$

6. $|\vec{u}| = 4$, $|\vec{v}| = 5$, and angle between $\vec{u}$ and $\vec{v}$ is 120°.

$\cos\theta = \dfrac{u \cdot v}{|u||v|}$

$\cos 120^\circ = \dfrac{u \cdot v}{4 \cdot 5}$

$4 \cdot 5 \cdot \cos 120^\circ = u \cdot v$

$\boxed{u \cdot v = -10}$

Now, WORK TOGETHER.

7. Which pairs of vectors are orthogonal?

(A) $\vec{v} = \langle 3,-2\rangle, \vec{w} = \langle -1,2\rangle$

$v \cdot w = 3(-1)+(-2)(2) = -7 \neq 0$ ✗ not orthogonal

$\boxed{\text{(B)}}$ $\vec{v} = \langle -2,0\rangle, \vec{w} = \langle 0,5\rangle$

$v \cdot w = -2(0)+0(5) = 0$ ✓ $\therefore \vec{v}$ & $\vec{w}$ are orthogonal

$\boxed{\text{(C)}}$ $\vec{v} = \langle 3,-6\rangle, \vec{w} = \langle 2,1\rangle$

$v \cdot w = 3(2)+(-6)(1) = 0$ ✓ $\therefore \vec{v}$ & $\vec{w}$ are orthogonal

(D) $\vec{v} = \langle 2,-3\rangle, \vec{w} = \langle -2,3\rangle$

$v \cdot w = 2(-2)+(-3)(3) = -13 \neq 0$ ✗ not orthogonal

8. Find k so that $\vec{u}$ and $\vec{v}$ are orthogonal.

$u = -4ki + 5j, v = 2i - 6j$ → $\vec{u}\cdot\vec{v} = 0$

$\vec{u}\cdot\vec{v} = 0$

$-4k(2)+5(-6) = 0$

$-8k - 30 = 0$ → $-8k = 30$

$k = \frac{30}{-8}$ ⇒ $\boxed{k = -\frac{15}{4} \text{ or } -3.75}$

WORK Problems from **http://www.physicsclassroom.com/calcpad/energy/problem**

9. Renatta Gass is out with her friends. Misfortune occurs and Renatta and her friends find themselves getting a *work*out. They apply a cumulative force of 1080 N to push the car 218 m to the nearest fuel station. Determine the work done on the car.

↳ horizontal force

distance horizontal

$W = F \cdot d$

$= 1080\,N \cdot 218\,m$

$= 235440\ N\cdot M$ → $\boxed{235440\ J}$

J = Joule ... ☺

10. Hans Full is pulling on a rope to drag his backpack to school across the ice. He pulls upwards and rightwards with a force of 22.9 Newtons at an angle of 35 degrees above the horizontal to drag his backpack a horizontal distance of 129 meters to the right. Determine the work (in Joules) done upon the backpack.

Force

distance horizontal

$|\vec{F}_H| = 22.9\cos 35°$

$= 18.759$ N

$W = F \cdot d$

$= (18.759)(129)$

$= 2419.857\ N\cdot m$ ⇒ $\boxed{2419.857\ J}$

11. Lamar Gant, U.S. powerlifting star, became the first man to deadlift five times his own body weight in 1985. Deadlifting involves raising a loaded barbell from the floor to a position above the head with outstretched arms. Determine the work done by Lamar in deadlifting 300 kg to a height of 0.90 m above the ground.

↳ Force vertical

↳ vertical distance

$|\vec{F}_V| = 300\sin 90°$

$= 300$

$W = F \cdot d$

$= (300\ kg)(.9\ m)$

$= 270\ kg \cdot m$

1 kg ≈ 10 N ... ☺

$W = 270\ \cancel{kg}\cdot m \cdot \frac{10\,N}{1\,\cancel{kg}}$

$= 2700\ N\cdot m$ ⇒ $\boxed{2700\ J}$

12. Sheila has just arrived at the airport and is dragging her suitcase to the luggage check-in desk. She pulls on the strap with a force of 190 N at an angle of 35° to the horizontal to displace it 45 m to the desk. Determine the work done by Sheila on the suitcase.

Force

↳ distance horizontal

$|\vec{F}_H| = 190\cos 35°$

$= 155.639$

$W = F \cdot d$

$= (155.639\ N)(45\,m)$

$= 7003.750\ N\cdot m$ ⇒ $\boxed{7003.750\ J}$

13. While training for breeding season, a 380 gram male squirrel does 32 pushups in a minute, displacing its center of mass by a distance of 8.5 cm for each pushup. Determine the total work done on the squirrel while moving upward (32 times).

Force

vertical distance

1 kg = 1000g
1 m = 100 cm
1 kg = 10 N ☺

squirrel, not scorpion

$|\vec{F}_V| = 380\sin 90°$

$= 380$

one pushup

$W = F \cdot d$

$= (380\,g)(8.5\,cm)$

$= 3230\ g\cdot cm$

$3230\ \cancel{g}\cdot \cancel{cm}\left(\frac{1\,kg}{1000\,\cancel{g}}\right)\left(\frac{1\,m}{100\,\cancel{cm}}\right)\left(\frac{10\,N}{1\,\cancel{kg}}\right)$

$= .323\ N\cdot m$

32 pushups → $32(.323) = \boxed{10.336\ J}$