6.3 Parametric Equations
Target 10A: Define and graph parametric equations
Target 10B: Perform parametric/rectangular conversions

Think about this: Imagine a baseball is thrown across a field. What are the variables involved?

Parametric Equations

- $x(t) \rightarrow$ horizontal distance in terms of time
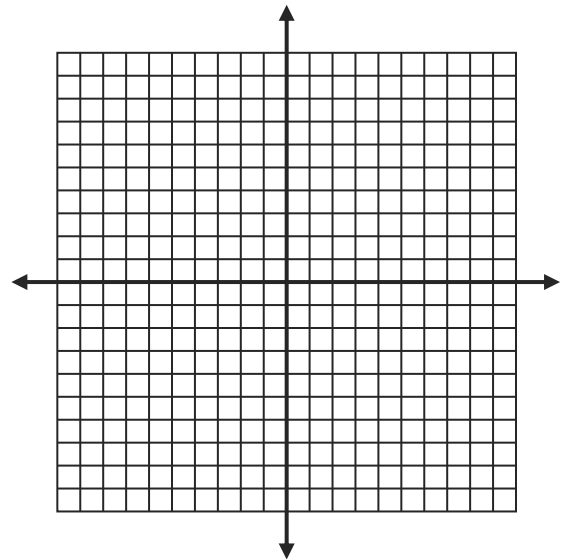- $y(t) \rightarrow$ vertical distance in terms of time

Graphing Parametric Equations

Construct a table choosing values of t and then determine the values of x and y.

Example 1: Make a table of values and sketch the curve, indicating the direction of your graph.
$$x(t) = t^2 - 2, \; y(t) = 3t$$

t	x	y
-3		
-2		
-1		
0		
1		
2		
3		

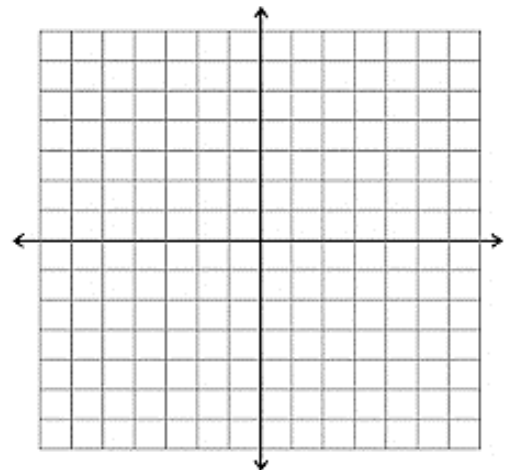*Example 2:* Make a table of values and sketch the curve, indicating the direction of your graph.
$$x(t) = 5 - 3t,$$
$$y(t) = 2 + t, \qquad -1 \le t \le 3$$

t	x	y

Example 3: ⊞ Using your graphing calculator, graph the parametric equations:
$$x(t) = 4t^3 - 16t^2 + 15t, \quad y(t) = 2$$
$0 \le t \le 5,\; tstep = 0.05$
Window: $X[-4,6], Y[-3,6], Xscale: 1, Yscale: 1$
When and where does the particle reverse direction?

Converting from Parametric to Rectangular

Determining a Rectangular Equation for Given Parametric Equations
- Solve either equation for t.
- Substitute that value of t into the other equation.
- Calculate the restrictions on the variables x and y based on the restrictions on t.

Example: Write the parametric equations in rectangular form.
$$x(t) = \sqrt{t - 2}, \ y(t) = 3t$$

Solution:	*Steps:*
$x = \sqrt{t - 2}$	Write one of the parametric equations and then solve for t.
$x^2 = t - 2$	
$x^2 + 2 = t$	
$y = 3t$	Write the other parametric equation.
$y = 3(x^2 + 2)$	Substitute in expression for t.
$y = 3x^2 + 6$	Simplify.

Restriction on t:

Since $x(t) = \sqrt{t - 2}$ & can't $\sqrt{\ }$ a negative #, $t - 2$ needs to be $\geq$ zero.

$\therefore, t - 2 \geq 0$, so $t \geq 2$

If $t \geq 2$, then $x \geq 0$ and $y \geq 6$

ANSWER:

$y = 3x^2 + 6$, where $x \geq 0$ and $y \geq 6$

Now you try, write the parametric equations in rectangular form.

1. $x(t) = \frac{1}{2}t + 4, \ y(t) = t^3$

2. $x(t) = 5 - 3t, \ y(t) = 2 + t, \ -1 \leq t \leq 3$

3. $x(t) = \frac{1}{t-3}, \ y(t) = 4t + 5$

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More Practice

Parametric Equations

http://tutorial.math.lamar.edu/Classes/CalcII/ParametricEqn.aspx

https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1

https://people.richland.edu/james/lecture/m116/conics/parametric.html

https://www.varsitytutors.com/hotmath/hotmath_help/topics/parametric-equations

https://youtu.e/9kKZHQtYP7g

https://youtu.be/tsnHL1Lb5MU

https://youtu.be/lTOSslFAl18

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Homework Assignment
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