DATE: _____

Unit 10 (Chapter 6): Parametric & Polar **Pre-Calculus 2018-2019**

6.3 Parametric Equations

Target 10C: Solve real-world problems using parametric models

Review of Prior Concepts

Write the parametric equations in rectangular form.

1. $x(t) = 6 - t$
$y(t) = 3t - 4$

$x = 6 - t$
$x - 6 = -t$
$-x + 6 = t$

$y = 3t - 4$
$y = 3(-x + 6) - 4$
$y = -3x + 18 - 4$
$y = -3x + 14$

no restrictions on t, so no restrictions on x or y.

2. $x(t) = 3\cos t$
$y(t) = 3\sin t$

$x = 3\cos t$ $\quad y = 3\sin t$
$\frac{x}{3} = \cos t$ $\quad \frac{y}{3} = \sin t$

$\sin^2 t + \cos^2 t = 1$
$\left(\frac{y}{3}\right)^2 + \left(\frac{x}{3}\right)^2 = 1$
$\frac{y^2}{9} + \frac{x^2}{9} = 1$
$x^2 + y^2 = 9$

circle

Recall:

$$\text{position} = \frac{1}{2}gt^2 + v_0 t + \text{initial position}$$

gravity $-32\ ft/sec^2$ or $-9.8\ m/s^2$

initial velocity

Velocity Vector = $\langle v_0 \cos\theta, v_0 \sin\theta \rangle$

Horizontal position → $x(t) = v_0 \cos\theta\, t + x_0$

Vertical position → $y(t) = \frac{1}{2}gt^2 + v_0 \sin\theta\, t + y_0$

Examples

1a) Maria was a star outfielder for the Oakland Athletics. It was game 7 of the World Series. She was up to bat with 2 outs and the bases were loaded in the bottom of the 9th inning. The count was 3 and 2 (3 balls, 2 strikes) and the score was 3-6.

She was their only hope…The next pitch was a fastball, Maria's favorite pitch and it was in her wheelhouse. She closed her eyes and swung the bat as hard as she could.

She hit the ball at 3 feet above the ground with an initial speed of 150 ft/sec at an angle of 18 degrees with the horizontal. Did the A's win the World Series if the wall is 400 ft away and 20 ft high? Did the ball go over the fence? Did the outfielder catch the ball?

$\theta = 18°$
$y_0 = 3\ ft$
$v_0 = 150\ ft/sec$

$x(t) = v_0 \cos\theta\, t + x_0$
$x(t) = (150\cos 18°)t$

$y(t) = \frac{1}{2}gt^2 + v_0 \sin\theta\, t + y_0$
$y(t) = \frac{1}{2}(-32)t^2 + (150\sin 18°)t + 3$
$y(t) = -16t^2 + (150\sin 18°)t + 3$

$400 = (150\cos 18°)t$
$2.804_{sec} = t$

$y(2.804) = -16(2.804)^2 + (150\sin 18°)(2.804) + 3$
$= 7.178\ ft$ ← less than 20ft, ∴, the ball did not go over the wall.

1b) Suppose there was a 25 ft/sec horizontal wind gust as Maria hit the ball. Does the ball clear the fence? If so, by how much?

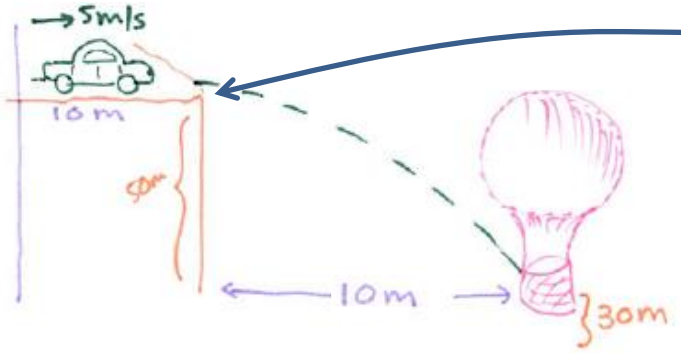
$$x(t) = (150\cos 18° + 25) \cdot t$$
$$400 = (150\cos 18° + 25) \cdot t$$
$$\frac{400}{150\cos 18° + 25} = t$$
$$2.389 \text{ sec} = t$$

$$y(2.389) = -16(2.389)^2 + (150\sin 18°)(2.389) + 3$$
$$= 22.515 \text{ ft}$$

Therefore, the ball clears the 20 ft fence by 2.515 ft.

2) Real-Life Situation from https://www.khanacademy.org/math/algebra-home/alg-trig-functions/alg-parametric/v/parametric-equations-1

Marzine and Jocelyn are driving in a car at a speed of 5 m/s; Mr. B. is chasing them down because he needs a homework assignment from them. They are afraid of being caught because they didn't complete all of the work that supports their answers. They are approaching a cliff 10 m away that is 50 m above Lake Michigan and have to decide to face the wrath of Mr. B., or see if the car can FLY!!!! They are very scared of Mr. B., so they chose the latter. After 2 seconds, will they be able to jump onto a hot air balloon that is 30 m off of the ground and 10 m from the cliff?



(not drawn to scale)

$x_0 = 10\text{m}$

$y_0 = 50\text{m}$

horizontal velocity = 5m/s

$x(t) = v_0 t + x_0$

$x(t) = 5t + 10$

$y(t) = \frac{1}{2}gt^2 + v_0 t + y_0$

$y(t) = \frac{1}{2}(-9.8)t^2 + 0 + 50$

$y(t) = -4.9t^2 + 50$

make table of values to see what happens

t	x	y
0	10	50
1	15	45.1
2	20	30.4
3	25	5.9

At t = 0, you are about to go off the cliff

→ when t = 2, Marzine + Jocelyn will be 10m from the cliff + 30.4 m off the ground... close enough to land safely in the hot air balloon.

More Practice

Applications of Parametric Equations

http://www.ck12.org/book/CK-12-Precalculus-Concepts/section/10.5/
http://www.shelovesmath.com/precal/introduction-to-parametric-equations/#Applications
https://youtu.be/0Fi9iDDjD64
https://youtu.be/4o6MOaVtz8Y

Homework Assignment

p.482 #39,44,45,49,55