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Unit 8 (Chapter 6 & 7): Matrices & Vectors **Pre-Calculus**

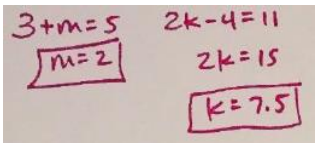
7.2 Matrix Algebra

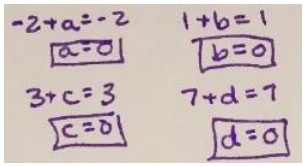
Target 8E: Represent a system of linear equations as a single matrix equation in a vector variable
Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

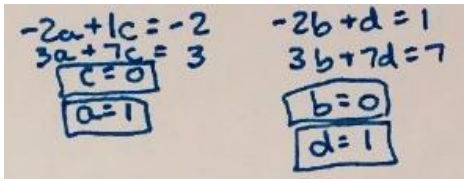
Review of Prior Concepts

Find the value of the variables.

1. $\begin{bmatrix} 3 \\ 2k \end{bmatrix} + \begin{bmatrix} m \\ -4 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$ **2.** $\begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}$ **3.** $\begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 7 \end{bmatrix}$







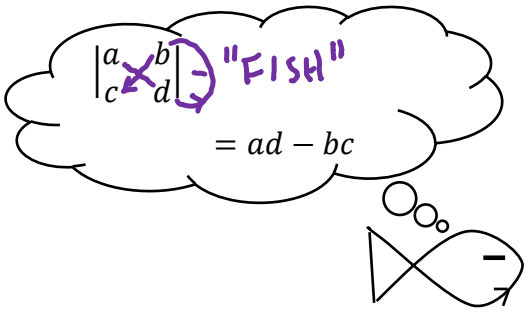
More Practice

Solving Equal Matrices
http://www.purplemath.com/modules/matrices3.htm
http://www.onlinemathlearning.com/equal-matrices.html
http://math.tutorvista.com/algebra/equal-matrices.html
https://www.youtube.com/watch?v=nOlRFQsHGOg
https://www.youtube.com/watch?v=LBfv7LtZHWk

Determinant of a 2 × 2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of A is:

$$\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$
$$= ad - bc$$



Examples:

1. Find $\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix}$.

$\begin{vmatrix} 3 & 1 \\ 2 & -1 \end{vmatrix} = 3(-1) - 2(1)$
$= -3 - 2$
$= -5$

2. Find $\det A$, where $A = \begin{bmatrix} 6 & 3 \\ 1 & -2 \end{bmatrix}$.

$\det A = \begin{vmatrix} 6 & 3 \\ 1 & -2 \end{vmatrix}$
$= 6(-2) - 3(1)$
$= -12 - 3$
$= -15$

3. Using your graphing calculator, find $\begin{vmatrix} 3 & -1 & 7 \\ 2 & 0 & 1 \\ 1 & 4 & 2 \end{vmatrix}$.

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$\begin{vmatrix} 3 & -1 & 7 \\ 2 & 0 & 1 \\ 1 & 4 & 2 \end{vmatrix} = 47$

Watch how to find the determinant of a 3 × 3 matrix by hand:
https://youtu.be/21LWuY8i6Hw

Identity Matrix (for a square matrix)

$A \cdot I_n = I_n \cdot A = A$

where $I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$ → ones along diagonal

Additive Identity: $x + 0 = x$ and $0 + x = x$

$2+0=2 \quad 0+2=2$

Multiplicative Identity: $x \cdot 1 = x$ and $1 \cdot x = x$

$2 \cdot 1 = 2 \quad 1 \cdot 2 = 2$

Examples:

1. Write I_2

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. Write I_4

$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Inverse of a Square Matrix

If A and B are both $n \times n$ matrices and $= BA = I_n$,

then B is the inverse of A (B *is* A^{-1}).

$AB = I_n \qquad BA = I_n$

$A(A^{-1}) = I_n \qquad (A^{-1})A = I_n$

✸Matrix A has an inverse if $\det A \neq 0$

Find the Inverse of a 2 × 2 Matrix

If $\det A \neq 0$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

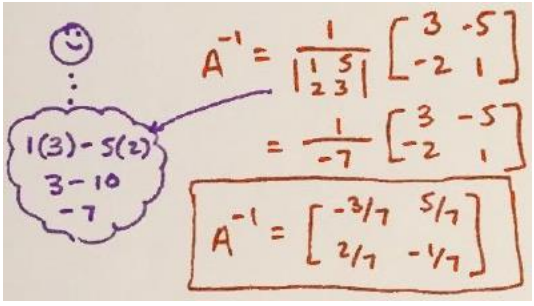
$ad - bc \neq 0$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

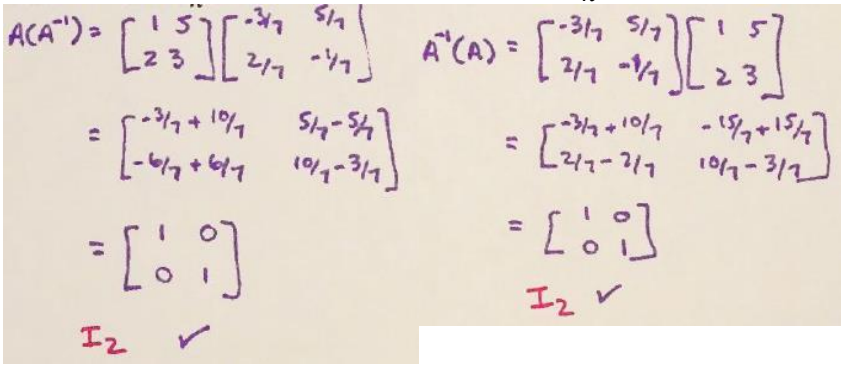
$$= \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Examples:

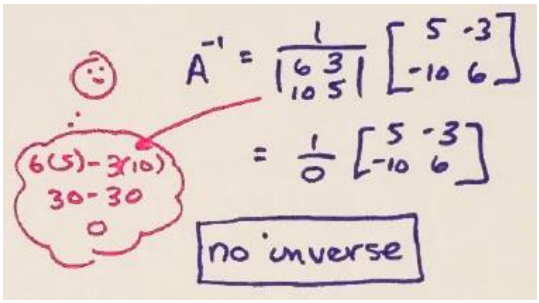
1. Find the inverse of $A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$ and check your answer.



Check: $AB = BA = I_n$?



2. $A = \begin{bmatrix} 6 & 3 \\ 10 & 5 \end{bmatrix}$. Find A^{-1}.



3. Find inverse of: $\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & -2 \\ 0 & 1 & 1 \end{bmatrix} = A$

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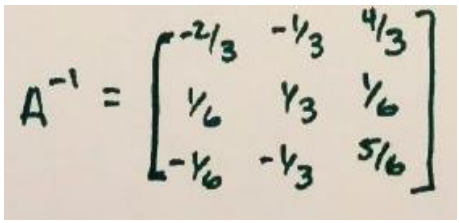

Watch how to find the inverse of a 3 × 3 matrix by hand: https://youtu.be/YvjkPF6C_LI

More Practice

Inverse Matrices

https://www.mathsisfun.com/algebra/matrix-inverse.html
https://www.khanacademy.org/math/precalculus/precalc-matrices/practice-finding-inverses-of-2x2-matrices/e/matrix_inverse_2x2
http://www.intmath.com/matrices-determinants/5-inverse-matrix.php
https://youtu.be/OU9sWHk_dlw
https://youtu.be/y4B_EC5MNS8

Determinants

http://www.mathsisfun.com/algebra/matrix-determinant.html
http://www.virtualnerd.com/algebra-2/matrices/determinants-cramers-rule/determinants/determinant-2-by-2
https://www.khanacademy.org/math/precalculus/precalc-matrices/determinant-of-2x2-matrix/e/matrix_determinant
http://www.purplemath.com/modules/determs.htm
http://www.coolmath.com/algebra/14-determinants-cramers-rule/01-determinants-cramers-rule-2x2-02
https://youtu.be/OU9sWHk_dlw
https://youtu.be/0S5VfMNqgaM
https://youtu.be/Ympm-AxmJ14

Homework Assignment

p.539 #33,35,37,41,44,45,62,66