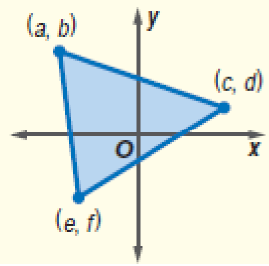
7.3 Solve Systems of Equations Using Matrices

Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

Review of Prior Concepts

The area of a triangle having vertices at (a, b), (c, d), and (e, f) is $|A|$, where ← absolute value

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$ ← determinant



Find the area of a triangle whose vertices are: (−2,1), (3,7) and (8,0).
(a,b) (c,d) (e,f)

$$A = \frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 3 & 7 & 1 \\ 8 & 0 & 1 \end{vmatrix} = \boxed{32.5}$$

More Practice

Area of a Triangle given Vertices

http://www.mathplanet.com/education/algebra-2/matrices/determinants
http://www.purplemath.com/modules/detprobs.htm
https://www.youtube.com/watch?v=bkJX3q7wvJc

SAT Connection
Heart of Algebra
6. Algebraically solve systems of two linear equations in two variables

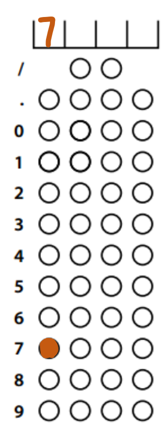
Example:

$$x + y = -9$$
$$x + 2y = -25$$

According to the system of equations above, what is the value of x ?

By substitution,
$x+y=-9$
$y = -9-x$
$x+2y=-25$
$x+2(-9-x)=-25$
$x-18-2x=-25$
$-x-18=-25$
$-x=-7$
$x=7$

By linear combination,
$x+y=-9 \xrightarrow{(-2)} -2x-2y=18$
$x+2y=-25 \rightarrow x+2y=-25$
$-x=-7$
$x=7$



NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

Solution

Solving System of Equations Using Inverse Matrices

If $AX = B$, where A, B, and X are matrices, then

get X matrix by itself…

$$AX = B$$

$$\underbrace{(A^{-1})A}_{I\text{, identity matrix}}X = A^{-1}B$$

$$X = A^{-1}B$$

(if A^{-1} exists)

Examples:

1. Solve the system of equations: $\begin{cases} 3x - 2y = 0 \\ -x + y = 5 \end{cases}$

$$\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1}}\cancel{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{3(1)-(-2)(-1)}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

…Remember $(2\times 2)(2\times 1)$ 2×1 …

$$= \begin{bmatrix} 1(0) + 2(5) \\ 1(0) + 3(5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$\boxed{x = 10,\ y = 15}$ or $\boxed{(10, 15)}$

2. Solve the system of equations: $\begin{cases} x - y + 2z = -3 \\ 2x + y - z = 0 \\ -x + 2y - 3z = 7 \end{cases}$

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ -1 & 2 & -3 \end{bmatrix}^{-1}\begin{bmatrix} -3 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 3 \end{bmatrix}$$

$\boxed{x = -2,\ y = 7,\ z = 3}$ OR $\boxed{(-2, 7, 3)}$

3. Find x and y if $BX = A$, where $A = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}$, and $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

$$\begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix}^{-1}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2(-2)-5(1)}\begin{bmatrix} -2 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{-9}\begin{bmatrix} -2 & -5 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$= -\frac{1}{9}\begin{bmatrix} -2(-3)+-5(1) \\ -1(-3)+2(1) \end{bmatrix}$$

$$= -\frac{1}{9}\begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/9 \\ -5/9 \end{bmatrix}$$

$$\boxed{x = -\frac{1}{9},\ y = -\frac{5}{9}} \quad \text{or} \quad \boxed{\left(-\frac{1}{9}, \frac{-5}{9}\right)}$$

More Practice

Solving Systems Using Inverse Matrices

http://www.mathplanet.com/education/algebra-2/matrices/using-matrices-when-solving-system-of-equations

http://math.uww.edu/~mcfarlat/matrix.htm

https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html

https://youtu.be/Re1F4d24Fxc

https://youtu.be/0_DYEFtlCiM

Homework Assignment

p.553 #25,49,51,53,55,83,85 (answer all questions using inverse Matrices methods)

SAT Connection

Solution

The correct answer is 7. Subtracting the left and right sides of $x + y = -9$ from the corresponding sides of $x + 2y = -25$ gives $(x + 2y) - (x + y) = -25 - (-9)$, which is equivalent to $y = -16$. Substituting -16 for y in $x + y = -9$ gives $x + (-16) = -9$, which is equivalent to $x = -9 - (-16) = 7$.