

7.3 Solve Systems of Equations Using Matrices

Target 8F: Find the inverse of a matrix, if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Review of Prior Concepts

Find the equation of a parabola that passes through the points $(-1,9)$, $(1,5)$, and $(2,12)$ using Inverse Matrices.

$$\hookrightarrow y = ax^2 + bx + c$$

$$9 = a(-1)^2 + b(-1) + c \rightarrow a - b + c = 9$$

$$5 = a(1)^2 + b(1) + c \rightarrow a + b + c = 5$$

$$12 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + c = 12$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 5 \\ 12 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$$

$$y = 3x^2 - 2x + 4$$

More Practice**Solving Systems Using Inverse Matrices**

<http://www.mathplanet.com/education/algebra-2/matrices/using-matrices-when-solving-system-of-equations>

<http://math.uww.edu/~mcfarlat/matrix.htm>

<https://www.mathsisfun.com/algebra/systems-linear-equations-matrices.html>

<https://youtu.be/Re1F4d24Fxc>

https://youtu.be/0_DYEFtCiM

<https://youtu.be/FILsx1WD6a8>

Augmented Matrices

Augmented Matrix – one matrix with coefficients and answers

Example:

Write the system of equations as an augmented matrix:
$$\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{array} \right]$$

optional: represents "=" sign

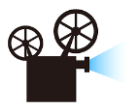
Solving Augmented Matrices

To solve an augmented matrix,

use Gaussian elimination to have the matrix in **Reduced Row Echelon Form**.

Reduced Row Echelon Form:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \cdots & 0 & a_1 \\ 0 & 1 & 0 & \cdots & 0 & a_2 \\ 0 & 0 & 1 & & & \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & a_n \end{array} \right]$$



WATCH THIS VIDEO: https://youtu.be/0-feBnP7q_k

Examples:

1. Solve the system of equations using **Reduced Row Echelon Form**:

$$\begin{cases} x - 2y + z = 7 \\ 3x - 5y + z = 14 \\ 2x - 2y - z = 3 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 1 & -2 & 1 & 7 \\ 3 & -5 & 1 & 14 \\ 2 & -2 & -1 & 3 \end{bmatrix} \right) = \begin{array}{c|ccc|c} x & y & z & = & \\ \hline 1 & 0 & 0 & 2 & \rightarrow x=2 \\ 0 & 1 & 0 & -1 & \rightarrow y=-1 \\ 0 & 0 & 1 & 3 & \rightarrow z=3 \end{array}$$

$$\boxed{(2, -1, 3)}$$

2. Solve the system of equations using **Reduced Row Echelon Form**:

$$\begin{cases} x + y + 3z = 2 \\ 3x + 4y + 10z = 5 \\ x + 2y + 4z = 3 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 1 & 1 & 3 & 2 \\ 3 & 4 & 10 & 5 \\ 1 & 2 & 4 & 3 \end{bmatrix} \right) = \begin{array}{c|ccc|c} x & y & z & = & \\ \hline 1 & 0 & 2 & 0 & \rightarrow x+2z=0 \\ 0 & 1 & 1 & 0 & \rightarrow y+z=0 \\ 0 & 0 & 0 & 1 & \rightarrow 0=1 \end{array}$$

↓
not true $0 \neq 1$,
 \therefore **no solution**

3. Solve the system of equations using **Reduced Row Echelon Form**:

$$\begin{cases} x - z = 2 \quad \text{notice, no } y \\ -2x + y + 3z = -5 \\ 2x + y - z = 3 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 1 & 0 & -1 & 2 \\ -2 & 1 & 3 & -5 \\ 2 & 1 & -1 & 3 \end{bmatrix} \right) = \begin{array}{c|ccc|c} x & y & z & = & \\ \hline 1 & 0 & -1 & 2 & \rightarrow x-z=2 \\ 0 & 1 & 1 & -1 & \rightarrow y+z=-1 \\ 0 & 0 & 0 & 0 & \rightarrow 0=0 \end{array}$$

↓
always true, zero always equals zero
 \therefore infinitely many solutions
where $x-z=2$ and $y+z=-1$
 $x=z+2$ $y=-z-1$

$$\boxed{(z+2, -z-1, z)}$$

Now you try...

Solve each system of equations using **Reduced Row Echelon Form**:

$$1. \begin{cases} x + 2y + 3z = 9 \\ 2x - y + z = 8 \\ 3x - z = 3 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 1 & 2 & 3 & 9 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{bmatrix} \rightarrow \begin{cases} x=2 \\ y=-1 \\ z=3 \end{cases}$$

$$(2, -1, 3)$$

not every solution will always be $(2, -1, 3)$... that was a coincidence

$$2. \begin{cases} x + z = 1 \\ x + y + z = 2 \\ x - y + z = 1 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{cases} x+z=0 \\ y=0 \\ 0=1 \end{cases}$$

$0 \neq 1$
 \therefore , no solution

$$3. \begin{cases} 3x + y - 6z = -10 \\ 2x + y - 5z = -8 \\ 6x - 3y + 3z = 0 \end{cases}$$

$$\text{rref} \left(\begin{bmatrix} 3 & 1 & -6 & -10 \\ 2 & 1 & -5 & -8 \\ 6 & -3 & 3 & 0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & -3 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{cases} x+z=-2 \\ y-3z=-4 \\ 0=0 \end{cases}$$

\downarrow
 always true.
 \therefore , infinite # solutions
 where $x+z=-2$ $y-3z=-4$
 $x=+z-2$ $y=3z-4$

$$(+z-2, 3z-4, z)$$

More Practice

Augmented Matrices

<http://www.purplemath.com/modules/matrices.htm>

<http://www.mathbootcamps.com/augmented-matrices-and-systems-of-linear-equations/>

<https://braingenie.ck12.org/skills/106514>

https://youtu.be/A_fIRE0NJ8Y

<https://youtu.be/dHmqVGXyVG4>

<https://youtu.be/N9tFrUK83Uk>

Gaussian Elimination for Augmented Matrices

<https://www.mathway.com/examples/linear-algebra/systems-of-linear-equations/solving-using-an-augmented-matrix?id=227>

<http://tutorial.math.lamar.edu/Classes/Alg/AugmentedMatrix.aspx>

https://youtu.be/WiVeiVIu_SM

<https://youtu.be/2j5Ic2V7wq4>

Homework Assignment

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