

7.3 Partial Fractions

Target 8G: Decompose rational expressions into partial fractions

Review of Prior Concepts

Simplify.

$$1. \frac{3}{5} + \frac{21}{4}$$

$$= \frac{(4)3}{(4)5} + \frac{21(5)}{4(5)}$$

$$= \frac{12}{20} + \frac{105}{20}$$

$$= \boxed{\frac{117}{20}}$$

$$2. \frac{(x-2)^2}{(x-2)^x} + \frac{1}{(x-2)(x)}$$

$$= \frac{2x-4}{x(x-2)} + \frac{x}{x(x-2)}$$

$$= \boxed{\frac{3x-4}{x(x-2)}} \text{ or } \boxed{\frac{3x-4}{x^2-2x}}$$

More Practice

Simplifying Rational Expressions

<http://www.mathplanet.com/education/algebra-1/rational-expressions/add-and-subtract-rational-expressions>

<http://www.purplemath.com/modules/rtnladd.htm>

<https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions#adding-and-subtracting-rational-expressions>

<https://youtu.be/XTZl7Kn6u4Y>

https://youtu.be/y_DweTAEYWk

Partial Fraction Decomposition

To **decompose a fraction** – write one fraction as the sum/difference of 2 or more fractions.

Example:

Write the terms for partial fraction

decomposition of: $\frac{x-3}{x^2+3x}$

① Factor the denominator.

② Write each factor in its own fraction w/numerator as function that is one degree less than denominator.

A, B, C, etc... are constants (#s)

$$① \frac{x-3}{x(x+3)}$$

$$② \frac{x-3}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$$

constants have degree of 0.
degree is 1, ∴, numerator degree is 0.

More Examples:

Write the terms for partial fraction decomposition of each:

1. $\frac{x-14}{x^2-4}$

$$① \frac{x-14}{(x-2)(x+2)}$$

$$② \frac{x-14}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$$

2. $\frac{4x+4}{x^3+2x^2}$

$$① \frac{4x+4}{x^2(x+2)}$$

$$② \frac{4x+4}{x^2(x+2)} = \frac{A+B}{x^2} + \frac{C}{x+2}$$

∴, degree 1
degree 2
where A, B, C are #s

3. $\frac{3x^2+4}{(x-3)(x+5)(x^2+1)}$

already factored

$$\frac{3x^2+4}{(x-3)(x+5)(x^2+1)} = \frac{A}{x-3} + \frac{B}{x+5} + \frac{Cx+D}{x^2+1}$$
 degree 0, degree 1, degree 2

4. $\frac{5x^3-10x^2-5x-5}{(x^2+9)(x^2+4)}$

← already factored i

$$\frac{5x^3-10x^2-5x-5}{(x^2+9)(x^2+4)} = \frac{Ax+B}{x^2+9} + \frac{Cx+D}{x^2+4}$$
 degree 1, degree 2

Decompose the Fraction

Example:

Decompose: $\frac{x-3}{x^2+3x}$

- ① Factor the denominator.
- ② Write each factor in its own fraction w/numerator as function that is one degree less than denominator.
- ③ Solve for the constants: A,B,C, etc.
- ④ Write original fraction as decomposed fractions (partial fractions)

① $\frac{x-3}{x(x+3)}$

② $\frac{x-3}{x(x+3)} = \frac{A}{x} + \frac{B}{x+3}$

③ $\frac{x(x+3)}{x(x+3)} \cdot \frac{x-3}{x(x+3)} = \frac{A(x)(x+3)}{x} + \frac{B(x)(x+3)}{x+3}$

$x-3 = A(x+3) + B(x)$

$x-3 = Ax + 3A + Bx$

$x-3 = (A+B)x + 3A$

$A+B=1 \quad -3=3A$

$-1+B=1 \quad -1=A$

$B=2$

constants = constants
 linear coeff = linear coeff

④ $\frac{x-3}{x^2+3x} = \frac{-1}{x} + \frac{2}{x+3}$

More Examples:

Decompose each fraction.

1. $\frac{x-14}{x^2-4}$

① $\frac{x-14}{(x-2)(x+2)}$

② $\frac{x-14}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

③ $\frac{x-14}{(x-2)(x+2)} = \frac{A(x+2)}{x-2} + \frac{B(x-2)}{x+2}$

$$x-14 = A(x+2) + B(x-2)$$

$$x-14 = Ax+2A+Bx-2B$$

$$x-14 = (A+B)x + 2A-2B$$

$$A+B=1 \quad 2A-2B=-14$$

$$A+B=1 \quad A-B=-7$$

$$\begin{array}{r} A+B=1 \\ A-B=-7 \\ \hline 2A=-6 \end{array} \quad \begin{array}{r} A+B=1 \\ -3+B=1 \\ \hline B=4 \end{array}$$

$$A=-3 \quad B=4$$

$$\frac{x-14}{x^2-4} = \frac{-3}{x-2} + \frac{4}{x+2}$$

2. $\frac{7x-7}{x^2-3x-10}$

① $\frac{7x-7}{(x-5)(x+2)}$

② $\frac{7x-7}{(x-5)(x+2)} = \frac{A}{x-5} + \frac{B}{x+2}$

③ $\frac{7x-7}{(x-5)(x+2)} = \frac{A(x+2)}{x-5} + \frac{B(x-5)}{x+2}$

$$7x-7 = A(x+2) + B(x-5)$$

$$7x-7 = Ax+2A+Bx-5B$$

$$7x-7 = (A+B)x + 2A-5B$$

$$A+B=7 \quad 2A-5B=-7$$

$$A=7-B \quad 2(7-B)-5B=-7$$

$$14-2B-5B=-7$$

$$-7B=-21$$

$$B=3$$

$$A=7-3$$

$$A=4$$

$$\frac{7x-7}{x^2-3x-10} = \frac{4}{x-5} + \frac{3}{x+2}$$

More Practice

Partial Fraction Decomposition

<http://www.purplemath.com/modules/partfrac.htm>

<https://www.mathsisfun.com/algebra/partial-fractions.html>

<https://youtu.be/O14kJpRMY08>

<https://youtu.be/bqf42x6nZoo>

Homework Assignment

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