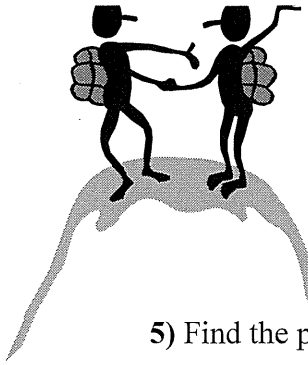

Name: _____

7.3 Mountain Climber

$\therefore \frac{3x+5}{x^3+4x^2+5x+2} = \frac{1}{x+1} + \frac{2}{(x+1)^2} - \frac{1}{x+2}$

5) Find the partial fraction decomposition: $\frac{3x+5}{x^3+4x^2+5x+2}$ (q / p)

$x = \frac{p}{q} = \frac{\pm 1, \pm 2}{\pm 1} = \{\pm 1, \pm 2\}$ Synthetic division:

$-1 \;|\; 1 \;\; 4 \;\; 5 \;\; 2$

$\downarrow \;\; -1 \;\; -3 \;\; -2$

$1 \;\; 3 \;\; 2 \;\; |\; 0$ ∴ Yes ✓

$x = -1$, $x+1 = 0$

x^2+3x+2

$(x+1)(x+2)$ Remaining factors

$\therefore x^3+4x^2+5x+2 = (x+1)(x+1)(x+2)$

$\frac{3x+5}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2}$ LCD: $(x+1)^2(x+2)$

$3x+5 = A[(x+1)(x+2)] + B(x+2) + C(x+1)^2$

$3x+5 = Ax^2 + 3Ax + 2A + Bx + 2B + Cx^2 + 2Cx + C$

$3x+5 = (A+C)x^2 + (3A+B+2C)x + 2A+2B+C$

$0 = A+C \Rightarrow -A = C$

$3 = 3A+B+2C \Rightarrow 3 = 3A+B-2A \Rightarrow \boxed{3 = A+B}$ ✗

$5 = 2A+2B+C \Rightarrow 5 = 2A+2B-A \Rightarrow \boxed{5 = A+2B}$ *

* $3 = A+B$

$5 = A+2B$ (Subtract equation 2 from eq. 1)

$-2 = -B$

$2 = B$

$\therefore A = 1, \; C = -1$

4) Find the partial fraction decomposition: $\frac{3x^3+6x-1}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$ LCD: $(x^2+2)^2$

$3x^3+0x^2+6x-1 = (Ax+B)(x^2+2) + Cx+D$

$3x^3+0x^2+6x-1 = Ax^3+2Ax+Bx^2+2B+Cx+D$ (Rearrange terms)

$3x^3+0x^2+6x-1 = Ax^3+Bx^2+2Ax+Cx+2B+D$

$3x^3+0x^2+6x-1 = Ax^3+Bx^2+(2A+C)x+2B+D$

$3 = A$

$0 = B$

$6 = 2A+C \Rightarrow 6 = 2(3)+C \Rightarrow 6 = 6+C \Rightarrow 0 = C$

$-1 = 2B+D \Rightarrow -1 = 2(0)+D \Rightarrow -1 = D$

$\therefore \frac{3x^3+6x-1}{(x^2+2)^2} = \frac{3x+0}{x^2+2} + \frac{0x+(-1)}{(x^2+2)^2}$

$\boxed{\frac{3x^3+6x-1}{(x^2+2)^2} = \frac{3x}{x^2+2} + \frac{-1}{(x^2+2)^2}}$

3) Find the partial fraction decomposition: $\frac{3x-2}{x^2-3x-4} = \frac{3x-2}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}$

LCD: $(x-4)(x+1)$. Multiply every term in equation by LCD. Then,

$3x-2 = A(x+1) + B(x-4)$

Let $x = -1$.

$3(-1)-2 = A(-1+1) + B(-1-4)$

$-3-2 = A\cdot 0 + B(-5)$

$\frac{-5}{-5} = \frac{-5B}{-5} \Rightarrow \boxed{B = 1}$

Let $x = 4$.

$3(4)-2 = A(4+1) + B(4-4)$

$12-2 = A\cdot 5$

$\frac{10}{5} = \frac{5A}{5} \Rightarrow \boxed{2 = A}$

$\therefore \frac{3x-2}{x^2-3x-4} = \frac{2}{x-4} + \frac{1}{x+1}$

2) Find the partial fraction decomposition: $\frac{-6}{x^2-3x} = \frac{-6}{x(x-3)} = \frac{A}{x} + \frac{B}{x-3}$ LCD: $x(x-3)$. Multiply every term in eq. by the LCD

$\frac{-6}{x(x-3)} \cdot x(x-3) = \frac{A}{x} \cdot x(x-3) + \frac{B}{x-3} \cdot x(x-3)$

$-6 = A(x-3) + Bx$

Let $x = 3$.

$-6 = A(3-3) + B\cdot 3$

$-6 = A\cdot 0 + 3B$

$-6 = 3B \Rightarrow \boxed{B = -2}$

Let $x = 0$.

$-6 = A(0-3) + B\cdot 0$

$-6 = A(-3)$

$\boxed{2 = A}$

$\therefore \frac{-6}{x^2-3x} = \frac{2}{x} + \frac{-2}{x-3}$

or

$\frac{-6}{x^2-3x} = \frac{2}{x} - \frac{2}{x-3}$

1) Write the terms for the partial fraction decomposition of the rational function:

$\frac{x^4-4x^3+x-3}{x^2(x+4)^2(x^2+3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2} + \frac{Ex+F}{x^2+3}$