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Unit 4 (Chapter 8): Conic Sections **Pre-Calculus**

8.1 Circles & Parabolas

Target 4A: Investigate the geometric properties of parabolas (vertex, focus, and directrix)
Target 4B: Derive the standard equation of a parabola and graph given two or three criteria

Review of Prior Concepts

https://youtu.be/yaSpxDdKEAs
What do you know about Conic Sections?

answers vary
(circles, parabolas, ellipses, hyperbolas)

More Practice

Introduction to Conics
https://www.khanacademy.org/math/algebra2/intro-to-conics-alg2
http://www.coolmath.com/algebra/25-conic-sections/01-introduction-circles-01
https://www.mathsisfun.com/geometry/conic-sections.html
https://www.youtube.com/watch?v=GDHNoQHQmtQ
https://www.youtube.com/watch?v=SKNybVUuPXA

SAT Connection

Passport to Advanced Math
12. Understand a nonlinear relationship between two variables

Example: Which of the following is an equation of a circle in the xy-plane with center $(0,4)$ and a radius with endpoint $\left(\frac{4}{3}, 5\right)$?

A) $x^2 + (y-4)^2 = \frac{25}{9}$

B) $x^2 + (y+4)^2 = \frac{25}{9}$

C) $x^2 + (y-4)^2 = \frac{5}{3}$

D) $x^2 + (y+4)^2 = \frac{3}{5}$

(h,k) (x,y)

$(x-h)^2+(y-k)^2=r^2$

$(\frac{4}{3}-0)^2+(5-4)^2=r^2$

$(\frac{4}{3})^2+1^2=r^2$

$\frac{25}{9}=r^2$

$(x-0)^2+(y-4)^2=\frac{25}{9}$

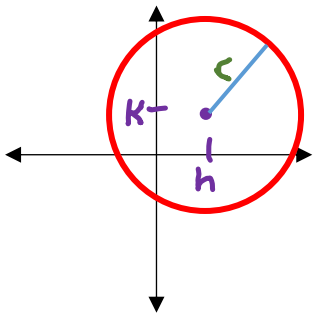
$x^2+(y-4)^2=\frac{25}{9}$

Solution

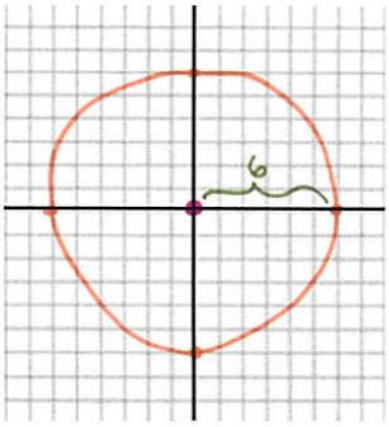
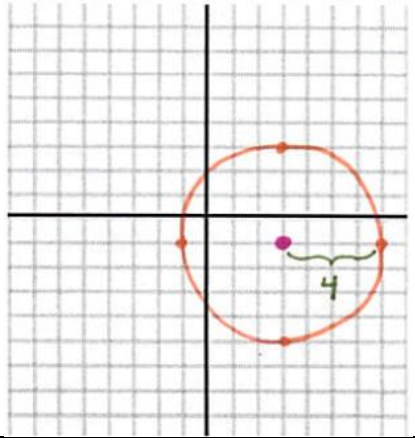
Circle

Definition (in your own words)

Set of all points equidistant from a given point (equal distance)

Standard Form of Equation	$(x-h)^2+(y-k)^2=r^2$	*Sketch* 
Center	(h,k)	
Radius	r	
Eccentricity	$e=0$	

Examples

Standard Form of Equation	$x^2+y^2=36$	$(x-3)^2+(y+1)^2=16$
Center	$(0,0)$	$(3,-1)$
Radius	$r^2=36 \rightarrow r=6$	$r^2=16 \rightarrow r=4$
Eccentricity	$e=0$	$e=0$
Sketch		

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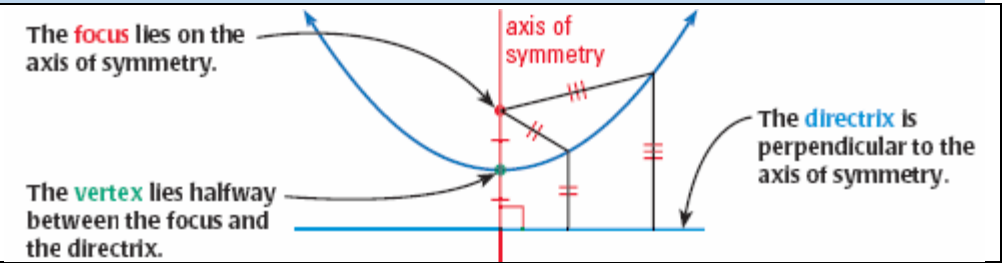
Unit 4 (Chapter 8): Conic Sections **Pre-Calculus**

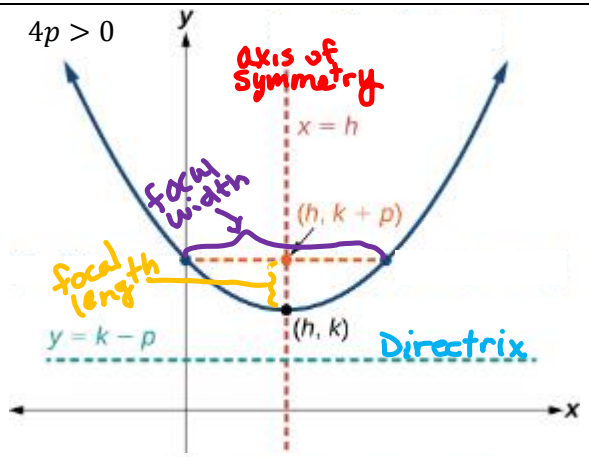
Parabola

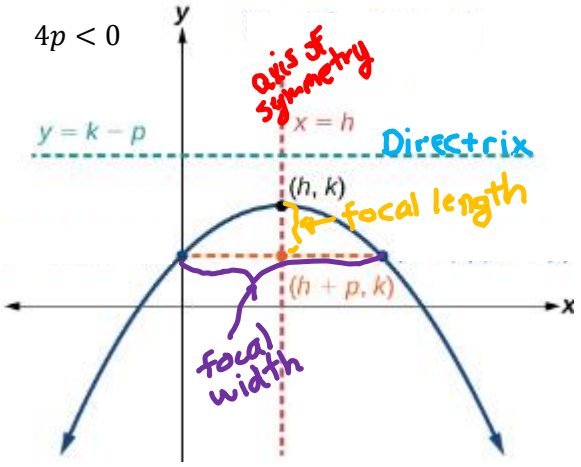
Definition (in your own words)

Set of all points equidistant from a given point (focus) and a given line (directrix) (point not on the line) ... ☺



Vertical Axis (of Symmetry)		*Sample Sketch*	*Example*
Standard Form of Equation	$(x-h)^2 = 4p(y-k)$		
Opening	up when $4p>0$ down when $4p<0$		
Vertex	(h,k)		
Focus	$(h, k+p)$		
Directrix	$y = k-p$		
Axis of Symmetry	$x = h$		
Focal Length	length from focus to vertex or directrix to vertex p		
Focal Width	width of parabola thru the focus $\lvert 4p \rvert$		
Eccentricity	$e = 1$		





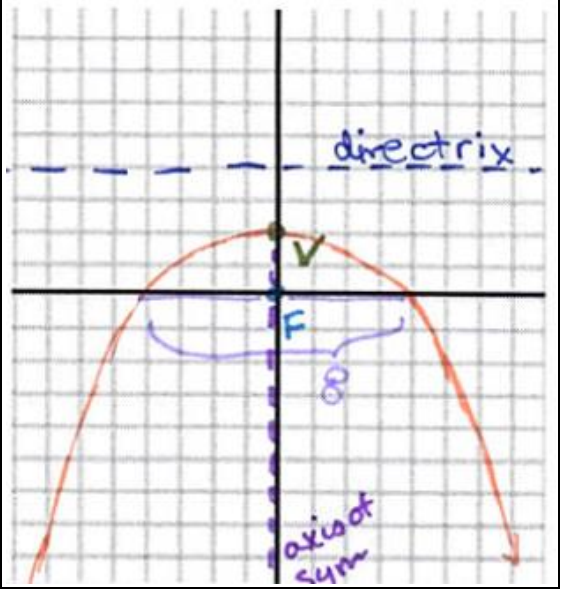
$x^2 = -8(y-2)$

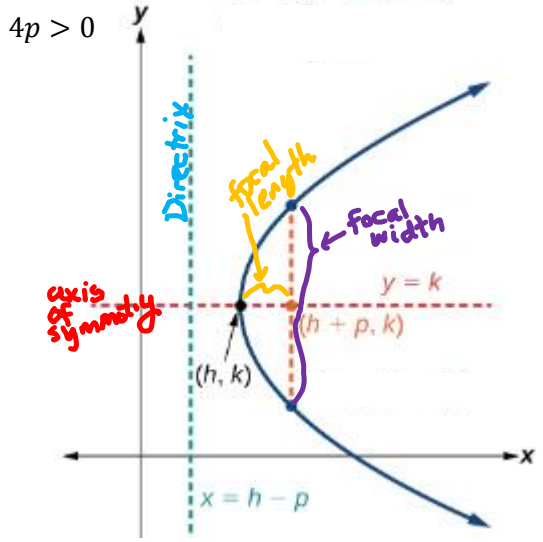
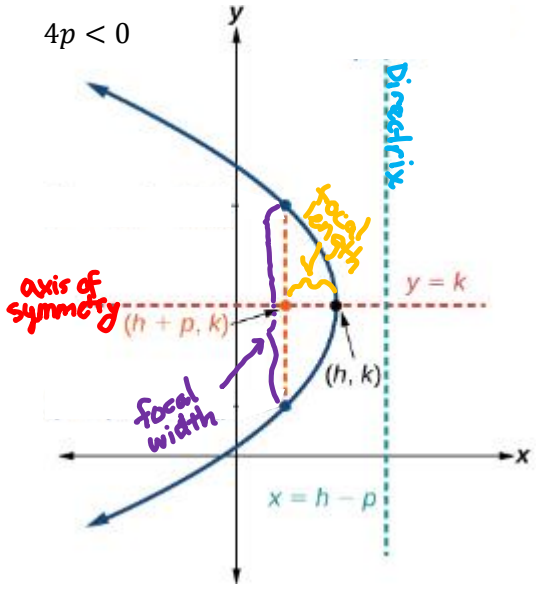
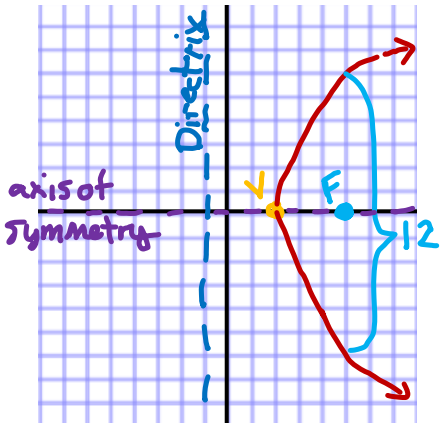
vertex: (0, 2) $-8 = 4p$, $-2 = p$

focus: (0, 0)

directrix: $y = 4$

axis of symmetry: $x = 0$



Horizontal Axis (of Symmetry)		*Sketch*	*Example*
Standard Form of Equation	$(y-k)^2 = 4p(x-h)$	$4p > 0$ 	$y^2 = 12(x-2)$
Opening	right when $4p>0$ Left when $4p<0$		Vertex (2, 0) Focus (5, 0) $4p=12$ $p=3$ directrix $x=-1$ axis of symmetry $y=0$
Vertex	(h,k)		
Focus	$(h+p, k)$		
Directrix	$x = h-p$		
Axis of Symmetry	$y=k$		
Focal Length	length from focus to vertex or directrix to vertex p	$4p < 0$ 	
Focal Width	width of parabola thru the focus $\|4p\|$		
Eccentricity	$e = 1$		

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Unit 4 (Chapter 8): Conic Sections **Pre-Calculus**

Examples:

1. Find the vertex, focus, directrix, and focal width of the parabola: $(y+4)^2 = 6x - 12$

$(y+4)^2 = 6(x-2)$

vertex: $(2, -4)$

focus: $(2+\frac{3}{2}, -4)$ $(\frac{7}{2}, -4)$

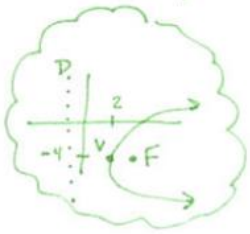
directrix: $x = 2 - \frac{3}{2}$ $x = \frac{1}{2}$

focal width: 6

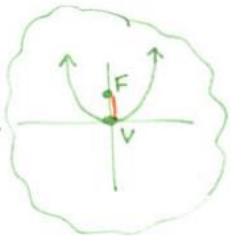
$6 = 4p$

$\frac{6}{4} = p$

$\frac{3}{2} = p$



2. Find an equation in standard form for the parabola that satisfies the given conditions: vertex (0,0), focus (0,2)



$(x-h)^2 = 4p(y-k)$

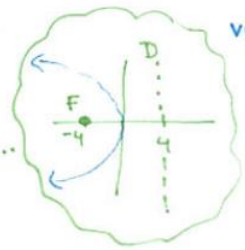
$(x-0)^2 = 4(2)(y-0)$

$x^2 = 8y$

p → distance btn focus + vertex

$p = 2$

3. Find an equation in standard form for the parabola that satisfies the given conditions: focus (–4,0), directrix $x = 4$



vertex is mdpt btn focus & directrix

Vertex: (0,0)

p → distance btn focus + vertex

$p = 4$

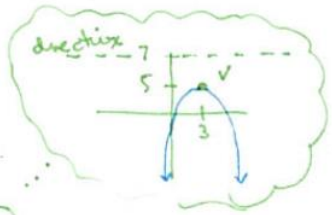
↑ negative since parabola opens left

$(y-k)^2 = 4p(x-h)$

$(y-0)^2 = 4(-4)(x-0)$

$y^2 = -16x$

4. Find an equation in standard form for the parabola that satisfies the given conditions: vertex (3,5), directrix $y = 7$



p → distance btn vertex & directrix

$p = 2$

↑ negative b/c parabola opens down

$(x-h)^2 = 4p(y-k)$

$(x-3)^2 = 4(-2)(y-5)$

$(x-3)^2 = -8(y-5)$

5. What is the relationship between the vertex, focus, and directrix in the parabola?

The vertex is an equal distance from the focus as it is from the directrix.

The distance from vertex to focus is p and from vertex to directrix is p.

More Practice

Circles

http://www.regentsprep.org/regents/math/algtrig/atc1/circlelesson.htm
http://www.mathsisfun.com/algebra/circle-equations.html
http://www.mathwarehouse.com/geometry/circle/equation-of-a-circle.php
https://www.youtube.com/watch?v=FLM3xlqw3WY
https://www.youtube.com/watch?v=6r1GQCxyMKI

Parabolas

http://www.purplemath.com/modules/parabola3.htm
http://www.protutorcompany.com/finding-the-vertex-focus-and-directrix-of-a-parabola-given-in-standard-form/
https://braingenie.ck12.org/skills/108316
https://www.youtube.com/watch?v=qxjs3NHI5T8
https://www.youtube.com/watch?v=a3qUuSqxzBk

Homework Assignment

p.578 #1,3,7-10all,11,13,15

SAT Connection

Solution

Choice A is correct. The equation of a circle can be written as $(x - h)^2 + (y - k)^2 = r^2$ where (h, k) are the coordinates of the center of the circle and r is the radius of the circle. Since the coordinates of the center of the circle are $(0, 4)$, the equation is $x^2 + (y - 4)^2 = r^2$, where r is the radius. The radius of the circle is the distance from the center, $(0, 4)$, to the given endpoint of a radius, $\left(\frac{4}{3}, 5\right)$. By the distance formula, $r^2 = \left(\frac{4}{3} - 0\right)^2 + (5 - 4)^2 = \frac{25}{9}$. Therefore, an equation of the given circle is $x^2 + (y - 4)^2 = \frac{25}{9}$.

Choice B is incorrect; it results from the incorrect equation $(x + h)^2 + (y + k)^2 = r^2$. Choice C is incorrect; it results from using r instead of r^2 in the equation for the circle. Choice D is incorrect; it results from using the incorrect equation $(x + h)^2 + (y + k)^2 = \frac{1}{r}$.