

8.1, 8.2 & 8.3 Parabolas, Ellipses & Hyperbolas

Target 4A/4C/4E: Investigate the geometric properties of parabolas/ellipses/hyperbolas

Conics in General Form vs. Standard Form

How to write an ellipse equation from general form to standard form (or how to complete a square)

Steps	Example
Start with general form of equation.	$4x^2 + 9y^2 - 48x + 72y + 144 = 0$
Move the constant & group x -terms together and y -terms together.	$4x^2 - 48x + 9y^2 + 72y = -144$
Factor out the coefficient(s) on the square terms.	$4(x^2 - 12x) + 9(y^2 + 8y) = -144$
Leave empty space after the x -terms and the y -terms.	$4(x^2 - 12x \quad) + 9(y^2 + 8y \quad) = -144$
Take $\frac{1}{2}$ the coefficient of the linear terms and square that #.	$\frac{1}{2}(-12)$ $\frac{1}{2}(8)$ $\left(\frac{1}{2}(-12)\right)^2 = 36$ $\left(\frac{1}{2}(8)\right)^2 = 16$
Place the values into the empty spaces. Multiply the values by the coefficients and place on the other side of the equation.	$4(x^2 - 12x + 36) + 9(y^2 + 8y + 16) = -144 + 4(36) + 9(16)$
Write the x -terms and the y -terms in squared form (where the constant is $\frac{1}{2}$ the coefficient of the linear terms).	$4(x - 6)^2 + 9(y + 4)^2 = 144$
Divide by value on right side to get equation into standard form.	$\frac{(x - 6)^2}{36} + \frac{(y + 4)^2}{16} = 1$



SAT Connection

Passport to Advanced Math

12. Understand a nonlinear relationship between two variables

Example: $x^2 + y^2 + 4x - 2y = -1$ The equation of a circle in the xy -plane is shown above. What is the radius of the circle?

- A) 2
B) 3
C) 4
D) 9

[Solution](#)

Now, you try....

Example 1:

Write the equation in standard form and identify the center, vertices, and foci.

$$16x^2 + 4y^2 - 32x + 24y - 12 = 0$$

Example 2:

Write the equation in standard form and identify the center, vertices, and asymptotes.

$$4x^2 - 5y^2 + 40x - 30y - 45 = 0$$

Unit 4 (Chapter 8): Conic Sections**Pre-Calculus**

How to write a parabola equation from general form to standard form (or how to complete a square)

Steps	Example
	$x^2 - 6x - 12y - 3 = 0$

Trade this paper with a classmate.

Your classmate will try to follow your steps for the example below.

Example

Write the equation in standard form and identify the vertex, focus, and directrix.

$$y^2 + 4y + 8x + 12 = 0$$

More Practice

Rewriting Conic Sections

<https://www.algebra.com/algebra/homework/Quadratic-relations-and-conic-sections/Quadratic-relations-and-conic-sections.faq.question.581877.html>

<https://www.mathway.com/examples/algebra/conic-sections/finding-the-vertex-form-of-a-hyperbola?id=818>

<https://www.youtube.com/watch?v=X5rBFTVYCa0>

<https://www.youtube.com/watch?v=qgM37pssnWY>

Homework Assignment

p.579 #49,51; p.591 #45,47; p.600 #47,49

SAT Connection**Solution**

Choice A is correct. The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. To put the equation $x^2 + y^2 + 4x - 2y = -1$ in this form, complete the square as follows:

$$\begin{aligned}x^2 + y^2 + 4x - 2y &= -1 \\(x^2 + 4x) + (y^2 - 2y) &= -1 \\(x^2 + 4x + 4) - 4 + (y^2 - 2y + 1) - 1 &= -1 \\(x + 2)^2 + (y - 1)^2 - 4 - 1 &= -1 \\(x + 2)^2 + (y - 1)^2 &= 4 = 2^2\end{aligned}$$

Therefore, the radius of the circle is 2.

Choice C is incorrect because it is the square of the radius, not the radius. Choices B and D are incorrect and may result from errors in rewriting the given equation in standard form.