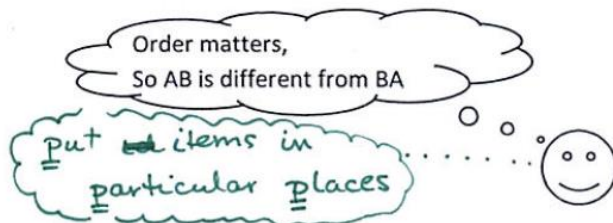


9.1 Basic Combinatorics

Target 7A: Expand the power of a binomial using the Binomial Theorem

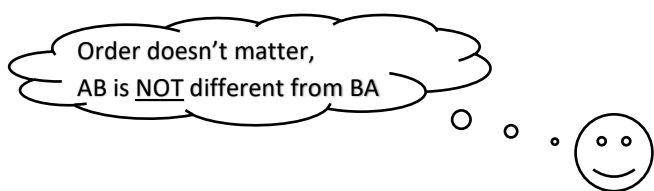
Permutations – # of ways to permute (arrange) n objects taken r at a time

$${}_n P_r = P(n, r) = \frac{n!}{(n-r)!}$$



Combinations – # of combinations of n objects taken r at a time

$${}_n C_r = C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$



Examples

1. Evaluate each expression without using a calculator. Check your answer with a calculator.

a) ${}_9 P_2$

$$\begin{aligned} &= \frac{9!}{(9-2)!} \\ &= \frac{9!}{7!} \\ &= \frac{9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} \\ &= 9 \cdot 8 \\ &= \boxed{72} \end{aligned}$$

b) ${}_{10} C_3$

$$\begin{aligned} &= \frac{10!}{3!(10-3)!} \\ &= \frac{10!}{3! \cdot 7!} \\ &= \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{3 \cdot 2 \cdot \cancel{7!}} \\ &= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2} = 10 \cdot 3 \cdot 4 = \boxed{120} \end{aligned}$$

2. How many distinguishable 11-letter words (not necessarily in the dictionary) can be formed using the letters in...

a) PROBABILITY

$\frac{(\text{all letters})!}{(\text{repeats})!}$

2 B's
2 I's

$$= \frac{11!}{2! \cdot 2!}$$

$$= 9,979,200$$

b) VICISSITUDE

3 I's
2 S's

$$= \frac{11!}{3! \cdot 2!}$$

$$= 3,326,400$$

3. Count the number of ways that each procedure can be done.

a) Arrange 4 books from left to right on a bookshelf.

☺ ... put items in particular places PERMUTATION

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = \boxed{24}$$

b) Elect a President, Vice-President, Treasurer, and Secretary from a Student Council, which consists of 17 members.

☺ ... put items in particular places PERMUTATION

$${}_{17}P_4 = \frac{17!}{(17-4)!} = \frac{17!}{13!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot \cancel{13!}}{\cancel{13!}} = 57,120$$

c) Select a 5-person fundraising committee from the Book Club, which consists of 21 members.

any order of 5 people ☺

$$\binom{21}{5} = \frac{21!}{5!(21-5)!} = \frac{21!}{5!16!} = 20,349$$

d) Unique tickets that can be printed for the Illinois Pick-5 lottery drawing, where players must match all five numbers on their ticket, out of 48 possible numbers.

any order of 5 #s ☺

$$\binom{48}{5} = \frac{48!}{5!(48-5)!} = \frac{48!}{5!(43)!} = 1,712,304$$

More Practice

Permutations & Combinations

<https://www.khanacademy.org/math/precalculus/prob-comb>

<https://www.khanacademy.org/math/precalculus/prob-comb/combinatorics-precalc/v/permutation-formula>

<http://www.coolmath.com/algebra/20-combinatorics/03-Permutations-01>

<http://www.coolmath.com/algebra/20-combinatorics/04-permutations-repeats-reruns-01>

<https://betterexplained.com/articles/easy-permutations-and-combinations/>

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>

<https://www.khanacademy.org/math/precalculus/prob-comb/combinations/v/introduction-to-combinations>

<http://www.coolmath.com/algebra/20-combinatorics/05-combinations-01>

https://www.youtube.com/watch?annotation_id=annotation_50580&feature=iv&src_vid=hJRXXKq2GEO8&v=H9VYDGuxqGg

https://www.youtube.com/watch?v=s_LfN4ItCs4

<https://www.youtube.com/watch?v=0NAASclUm4k>

<https://www.youtube.com/watch?v=3S8hs6aEts0>

Homework Assignment

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