DATE: _______________

Unit 7 (Chapter 9): Discrete Mathematics

Pre-Calculus

9.2 The Binomial Theorem

Target 7A: Expand the power of a binomial using the Binomial Theorem

Review of Prior Concepts

1. Predict the next 2 rows:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

2. How does the following compare with problem #1?

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

More Practice

Pascal's Triangle

http://www.mathsisfun.com/pascals-triangle.html

https://youtu.be/XMriWTvPXHI

SAT Connection

Heart of Algebra

4. Create an equivalent form of an algebraic expression

Example:

$$9a^4 + 12a^2b^2 + 4b^4$$

Which of the following is equivalent to the expression shown above?

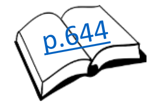
A) $(3a^2 + 2b^2)^2$

B) $(3a + 2b)^4$

C) $(9a^2 + 4b^2)^2$

D) $(9a + 4b)^4$

Solution



Find the terms in:

$(a+b)^0$

$(a+b)^1$

$(a+b)^2$

$(a+b)^3$

$(a+b)^4$

$(a+b)^5$

Binomial Coefficient: ___________________________

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Example 1: Expand $(x+2)^6$

Example 2: Expand $(x-3)^4$

Finding one Term in a Binomial Expansion

$$(a+b)^n = \underbrace{\binom{n}{0}a^n b^0}_{1^{st}\text{ term}} + \underbrace{\binom{n}{1}a^{n-1}b^1}_{2^{nd}\text{ term}} + \underbrace{\binom{n}{2}a^{n-2}b^2}_{3^{rd}\text{ term}} + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \underbrace{\binom{n}{n}a^0 b^n}_{n^{th}\text{ term}}$$

In general, the r^{th} term is: $\binom{n}{\quad}\,a^{\quad}\;b^{\quad}$

Example 3: Find the coefficient of the 8^{th} term of $(x+2)^{11}$

Example 4: Find the coefficient of the 3rd term of $(x-3)^6$

Example 5: Find the 4th term of $(x^2+y)^5$

More Practice

Binomial Theorem

http://www.purplemath.com/modules/binomial.htm
https://www.mathsisfun.com/algebra/binomial-theorem.html
https://people.richland.edu/james/lecture/m116/sequences/binomial.html
https://youtu.be/ojFuf9RYmzI

Homework Assignment

p.648 #1–15odd,27,28

SAT Connection

Solution

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3\,(2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$. Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$. Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b)\,(9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6{,}561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6{,}561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.