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Unit 7 (Chapter 9): Discrete Mathematics **Pre-Calculus**

9.2 The Binomial Theorem

Target 7A: Expand the power of a binomial using the Binomial Theorem

Review of Prior Concepts

1. Predict the next 2 rows:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

2. How does the following compare with problem #1?

$\binom{0}{0}=1$

$\binom{1}{0}=1$ $\binom{1}{1}=1$

$\binom{2}{0}=1$ $\binom{2}{1}=2$ $\binom{2}{2}=1$

$\binom{3}{0}=1$ $\binom{3}{1}=3$ $\binom{3}{2}=3$ $\binom{3}{3}=1$

$\binom{4}{0}=1$ $\binom{4}{1}=4$ $\binom{4}{2}=6$ $\binom{4}{3}=4$ $\binom{4}{4}=1$

They are the same.

${}_nC_r = \binom{n}{r}$

More Practice

Pascal's Triangle

http://www.mathsisfun.com/pascals-triangle.html

https://youtu.be/XMriWTvPXHI

SAT Connection

Heart of Algebra

4. Create an equivalent form of an algebraic expression

Example:

$$9a^4 + 12a^2b^2 + 4b^4$$

Which of the following is equivalent to the expression shown above?

A) $(3a^2 + 2b^2)^2$

B) $(3a + 2b)^4$

C) $(9a^2 + 4b^2)^2$

D) $(9a + 4b)^4$

$9a^4 + 12a^2b^2 + 4b^4$

$(3a^2)^2 + 12a^2b^2 + (2b^2)^2$

$(3a^2)^2 + (3a^2)(2b^2)\cdot 2 + (2b^2)^2$

$x^2 + 2xy + y^2$

Let $x = 3a^2$, $y = 2b^2$

$(x+y)(x+y)$

$(3a^2+2b^2)(3a^2+2b^2)$

$(3a^2+2b^2)^2$

Solution

Find the terms in:

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$(a+b)^0 = 1$

$(a+b)^1 = a + b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$\rightarrow \binom{5}{0}a^5 + \binom{5}{1}a^4b + \binom{5}{2}a^3b^2 + \binom{5}{3}a^2b^3 + \binom{5}{4}ab^4 + \binom{5}{5}b^5$

coefficient

Binomial Coefficient: $\binom{n}{r}$ or ${}_nC_r$ … ☺

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

always add to $= n$

Example 1: Expand $(\underbrace{x}_{a} + \underbrace{2}_{b})^{6\,n}$ $= \binom{6}{0}x^6(2)^0 + \binom{6}{1}x^5(2)^1 + \binom{6}{2}x^4(2)^2 + \binom{6}{3}x^3(2)^3 + \binom{6}{4}x^2(2)^4 + \binom{6}{5}x^1(2)^5 + \binom{6}{6}x^0(2)^6$

always same #

$= \boxed{x^6 + 12x^5 + 60x^4 + 160x^3 + 240x^2 + 192x + 64}$

Example 2: Expand $(\underbrace{x}_{a} - \underbrace{3}_{b})^{4\,n}$ $= \binom{4}{0}x^4(-3)^0 + \binom{4}{1}x^3(-3)^1 + \binom{4}{2}x^2(-3)^2 + \binom{4}{3}x^1(-3)^3 + \binom{4}{4}x^0(-3)^4$

$= \boxed{x^4 - 12x^3 + 54x^2 - 108x + 81}$

Finding one Term in a Binomial Expansion

$$(a+b)^n = \underbrace{\binom{n}{0}a^n b^0}_{1^{\text{st}}\text{ term}} + \underbrace{\binom{n}{1}a^{n-1}b^1}_{2^{\text{nd}}\text{ term}} + \underbrace{\binom{n}{2}a^{n-2}b^2}_{3^{\text{rd}}\text{ term}} + \cdots + \binom{n}{n-1}a^1 b^{n-1} + \underbrace{\binom{n}{n}a^0 b^n}_{n^{\text{th}}\text{ term}}$$

In general, the r^{th} term is: $\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$

or $\binom{n}{r-1} a^{n-r+1} b^{r-1}$ … ☺

Example 3: Find the coefficient of the 8^{th} term of $(\overset{a}{x} + \overset{b}{2})^{11 \leftarrow n}$

$r-1$
$8-1$

$\binom{11}{7} x^{11-7} (2)^7$

$= 330x^4 \cdot 128 = 42240x^4$

coefficient is: $\boxed{42240}$

Example 4: Find the coefficient of the 3rd term of $(x-3)^6$

r

$r-1$

$3-1$

$$\binom{6}{2} x^{6-2} (-3)^2$$

$$= 15x^4 \cdot 9 = 135x^4$$

coefficient is: $\boxed{135}$

Example 5: Find the 4th term of $(x^2+y)^5$

r

$r-1$

$4-1$

$$\binom{5}{3} (x^2)^{5-3} (y)^3$$

$$= 10(x^2)^2 y^3$$

$$= \boxed{10x^4y^3}$$

More Practice

Binomial Theorem

http://www.purplemath.com/modules/binomial.htm
https://www.mathsisfun.com/algebra/binomial-theorem.html
https://people.richland.edu/james/lecture/m116/sequences/binomial.html
https://youtu.be/ojFuf9RYmzI

Homework Assignment

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SAT Connection

Solution

Choice A is correct. If a polynomial expression is in the form $(x)^2 + 2(x)(y) + (y)^2$, then it is equivalent to $(x + y)^2$. Because $9a^4 + 12a^2b^2 + 4b^4 = (3a^2)^2 + 2(3a^2)(2b^2) + (2b^2)^2$, it can be rewritten as $(3a^2 + 2b^2)^2$.

Choice B is incorrect. The expression $(3a + 2b)^4$ is equivalent to the product $(3a + 2b)(3a + 2b)(3a + 2b)(3a + 2b)$. This product will contain the term $4(3a)^3\,(2b) = 216a^3b$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $216a^3b$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (3a + 2b)^4$. Choice C is incorrect. The expression $(9a^2 + 4b^2)^2$ is equivalent to the product $(9a^2 + 4b^2)(9a^2 + 4b^2)$. This product will contain the term $(9a^2)(9a^2) = 81a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $81a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a^2 + 4b^2)^2$. Choice D is incorrect. The expression $(9a + 4b)^4$ is equivalent to the product $(9a + 4b)(9a + 4b)(9a + 4b)\,(9a + 4b)$. This product will contain the term $(9a)(9a)(9a)(9a) = 6{,}561a^4$. However, the given polynomial, $9a^4 + 12a^2b^2 + 4b^4$, does not contain the term $6{,}561a^4$. Therefore, $9a^4 + 12a^2b^2 + 4b^4 \neq (9a + 4b)^4$.