DATE: _______________

Unit 7 (Chapter 9): Discrete Mathematics **Pre-Calculus**

9.4 Sequences & Series

Target 7B: Generate and identify the explicit rule for arithmetic sequences and series

Review of Prior Concepts

1. Find the 17th term of:

$$(2x - y)^{24}$$

$\binom{24}{16}(2x)^{8}(-y)^{16}$

$= \boxed{188280576x^{8}y^{16}}$

2. Find the sum of the coefficients of:

$$(2x - y)^{3}$$

$\binom{3}{0}(2x)^{3}(-y)^{0} + \binom{3}{1}(2x)^{2}(-y)^{1} + \binom{3}{2}(2x)(-y)^{2} + \binom{3}{3}(2x)^{0}(-y)^{3}$

$8x^3 - 12x^2y + 6y^2 - y^3$

$8 - 12 + 6 - 1 = \boxed{1}$

or $(2-1)^3 = \boxed{1}$ … ☺

More Practice

Binomial Expansion

http://www.purplemath.com/modules/binomial2.htm
https://www.mathsisfun.com/algebra/binomial-theorem.html
https://braingenie.ck12.org/skills/106589
https://www.khanacademy.org/math/algebra2/polynomial-functions/binomial-theorem/v/coefficient-in-binomial-expansion
https://youtu.be/--3KJdcbJ0g
https://youtu.be/fndaGW7Tcz0

SAT Connection

Heart of Algebra

8. Interpret the variables and constants in expressions for linear functions within the context presented.

Example:

Kathy is a repair technician for a phone company. Each week, she receives a batch of phones that need repairs. The number of phones that she has left to fix at the end of each day can be estimated with the equation $P = 108 - 23d$, where P is the number of phones left and d is the number of days she has worked that week. What is the meaning of the value 108 in this equation?

A) Kathy will complete the repairs within 108 days.
B) Kathy starts each week with 108 phones to fix.
C) Kathy repairs phones at a rate of 108 per hour.
D) Kathy repairs phones at a rate of 108 per day.

$P = 108 - 23d$

arithmetic explicit rule.

$a_1 + (n-1)d$

$a_1 = 108$

initial value, so

(B) Kathy starts w/ 108 phones to fix.

Solution

Sequences

p. 650

Sequence – ordered progression of numbers

- Infinite – unmeasurable or infinite # of terms
  Example: $2,4,8,16,\ldots,2^k,\ldots$ ← #terms : ∞
- Finite – countable or a set # of terms
  Example: 2,4,8,16,32 ← only 5 terms…finite

Terms of sequences: k^{th} term = a_k n^{th} term = a_n

Example: Find the 3rd term in the sequence: 2,4,8,16,32

$\boxed{a_3 = 8}$

Defining Sequences

Recursively-Defined Sequence – Each term depends on previous term(s)

Example 1: Given $a_n = 3a_{n-1}$ where $a_1 = 2$ and $n \geq 2$, find a_4

start to find a_4 →

$a_4 = 3a_{4-1}$
$= 3a_3$ ← need a_3
$= 3(18)$
$\boxed{a_4 = 54}$

$a_3 = 3a_{3-1}$
$= 3a_2$ ← need a_2
$= 3(6)$
$a_3 = 18$

$a_2 = 3a_{2-1}$
$= 3a_1$ ← need a_1, $a_1 = 2$ (given)
$= 3(2)$
$a_2 = 6$

Example 2: Given $a_n = a_{n-1} - 5$ where $a_1 = 11$ and $n \geq 2$, find the 4th and 8th terms.

↳ to get 4th + 8th terms, need $a_2, a_3, a_5, a_6, a_7 \ldots$

start w/

$a_2 = a_{2-1} - 5$
$= a_1 - 5$
$= 11 - 5$
$a_2 = 6$

$a_3 = a_{3-1} - 5$
$= a_2 - 5$
$= 6 - 5$
$a_3 = 1$

$a_4 = a_{4-1} - 5$
$= a_3 - 5$
$= 1 - 5$
$\boxed{a_4 = -4}$

now, write the terms… look for pattern…predict for other terms

$11, 6, 1, -4, -9, -14, -19, -24$

subtracting 5

↳ $\boxed{a_8 = -24}$

Explicitly-Defined Sequence – Terms based on n

Example: Given $a_n = -5n + 16$, find the 4th and 8th terms.

$a_4 = -5(4) + 16$

$\boxed{a_4 = -4}$

$a_8 = -5(8) + 16$

$\boxed{a_8 = -24}$

Arithmetic Sequence

Arithmetic Sequence – sequence written as $\{\underbrace{a}_{1^{st}\text{ term}}, \underbrace{a+d}_{2^{nd}\text{ term}}, a+2d, a+3d, \ldots, \underbrace{a+(n-1)d}_{n^{th}\text{ term}}, \ldots\}$

Explicit Rule

$$a_n = a_1 + (n-1)d$$

n^{th} term

1^{ST} term

common difference

Recursive Rule

$$a_n = a_{n-1} + d, \qquad \text{for all } n \geq 2$$

n^{th} term

$(n-1)^{th}$ term

common difference

Example 1: Find the common difference and 10th term, and write a recursive rule and explicit rule for the sequence: 7,20,33, ...

$\boxed{d = 13}$

$20 - 7 = 13$
$33 - 20 = 13$ ... ☺

$a_n = a_1 + (n-1)d$

$a_{10} = 7 + (10-1)13$

$a_{10} = 7 + 9(13)$

$\boxed{a_{10} = 124}$

Recursive Rule

to get next term, add 13.

$\boxed{a_n = a_{n-1} + 13}$

Explicit Rule

$a_n = a_1 + (n-1)d$

$= 7 + (n-1)13$

$= 7 + 13n - 13$

$\boxed{a_n = 13n - 6}$

Example 2: Find the common difference and 10th term, and write a recursive rule and explicit rule for the sequence: $\ln 3, \ln 6, \ln 12, \ln 24, \ldots$

$d = \ln 6 - \ln 3$

$= \ln\left(\frac{6}{3}\right)$

$\boxed{d = \ln 2}$

$d = \ln 12 - \ln 6$
$= \ln\left(\frac{12}{6}\right)$
$= \ln 2$ ✓ ... ☺

$a_n = a_1 + (n-1)d$

$a_{10} = \ln 3 + (10-1)\ln 2$

$a_{10} = \ln 3 + 9\ln 2$

$= \ln 3 + \ln 2^9$

$\boxed{a_{10} = \ln(3 \cdot 2^9)}$

Recursive Rule

to get next term, add $\ln 2$

$\boxed{a_n = a_{n-1} + \ln 2}$

Explicit Rule

$a_n = a_1 + (n-1)d$

$a_n = \ln 3 + (n-1)\ln 2$

$= \ln 3 + (\ln 2)n - \ln 2$

$= (\ln 2)n + \ln 3 - \ln 2$

$\boxed{a_n = n\ln 2 + \ln\left(\frac{3}{2}\right)}$

OR

$$a_n = \ln 2^n + \ln\left(\frac{3}{2}\right)$$
$$= \ln 2^n + \ln\left(\frac{3}{2}\right)$$
$$= \ln\left(\frac{3 \cdot 2^n}{2^1}\right)$$
$$= \ln(3 \cdot 2^{n-1})$$

More Practice

Arithmetic Sequences

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html
https://www.khanacademy.org/math/algebra/sequences/constructing-arithmetic-sequences/a/writing-recursive-formulas-for-arithmetic-sequences
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_arithseq.xml
http://www.coolmath.com/algebra/19-sequences-series/05-arithmetic-sequences-01
https://youtu.be/_cooC3yG_p0
https://youtu.be/lj_X9JVSF8k

Homework Assignment

p.656 #1–9odd,21,23,29

SAT Connection

Solution

Choice B is correct. The value 108 in the equation is the value of P in $P = 108 - 23\,d$ when $d = 0$. When $d = 0$, Kathy has worked 0 days that week. In other words, 108 is the number of phones left before Kathy has started work for the week. Therefore, the meaning of the value 108 in the equation is that Kathy starts each week with 108 phones to fix because she has worked 0 days and has 108 phones left to fix.

Choice A is incorrect because Kathy will complete the repairs when $P = 0$. Since $P = 108 - 23d$, this will occur when $0 = 108 - 23d$ or when $d = \frac{108}{23}$, not when $d = 108$. Therefore, the value 108 in the equation does not represent the number of days it will take Kathy to complete the repairs. Choices C and D are incorrect because the number 23 in $P = 108 - 23P = 108$ indicates that the number of phones left will decrease by 23 for each increase in the value of d by 1; in other words, that Kathy is repairing phones at a rate of 23 per day, not 108 per hour (choice C) or 108 per day (choice D).