9.3 Sequences

Target 7C: Generate and identify the explicit rule for geometric sequences and series

Review of Prior Concepts

Is the sequence arithmetic? If yes, find the common difference.

a) 1,5,9,13,17, … yes, $d = 4$

b) 1,4,9,16,25, … no

c) $4x, x, -2x, -5x, -8x, \ldots$ yes, $d = -3x$

More Practice

Arithmetic Sequences

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html
https://www.khanacademy.org/math/algebra/sequences/constructing-arithmetic-sequences/a/writing-recursive-formulas-for-arithmetic-sequences
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_arithseq.xml
http://www.coolmath.com/algebra/19-sequences-series/05-arithmetic-sequences-01
https://youtu.be/_cooC3yG_p0
https://youtu.be/lj_X9JVSF8k

SAT Connection

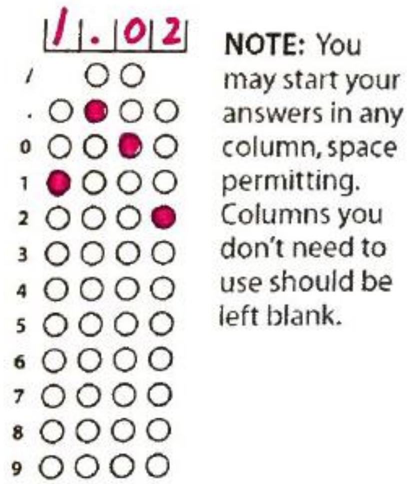
Passport to Advanced Math

10. Interpret parts of nonlinear expressions in terms of their context

Example:

Jessica opened a bank account that earns 2 percent interest compounded annually. Her initial deposit was \$100, and she uses the expression $\$100(x)^t$ to find the value of the account after t years.

What is the value of x in the expression?



year 1 → $\$100 + \$100(.02) = \$100(1.02)^1$

year 2 . . . . $= \$100(1.02)^2$

⋮

year t $= \$100\underbrace{(1.02)}_{x = 1.02}{}^t$

Solution

Geometric Sequence

Geometric Sequence – sequence written as $\{a, ar, ar^2, ar^3, \dots, ar^{n-1}, \dots\}$

Explicit Rule

$$a_n = a_1 r^{n-1}$$

nth term   1st term   common ratio

Recursive Rule

$$a_n = a_{n-1} \cdot r, \quad \text{for all } n \geq 2$$

nth term   (n-1)th term   common ratio

Example 1: Find the common ratio and 10th term, and write a recursive rule and explicit rule for the sequence: 9,18,36,72, ...

$\boxed{r = 2}$ $\frac{18}{9} = 2$ $\frac{36}{18} = 2$ ☺

$a_n = a_1 r^{n-1}$
$a_{10} = 9(2)^{10-1}$
$= 9(2^9)$
$\boxed{a_{10} = 4608}$

Recursive Rule:
$a_n = a_{n-1} \cdot 2$
$\boxed{a_n = 2a_{n-1}}$

Explicit Rule
$a_n = a_1 r^{n-1}$
$\boxed{a_n = 9(2)^{n-1}}$ OR
OR $= 9(2^n) \cdot 2^{-1}$
$= \frac{9(2^n)}{2} = \boxed{\frac{9}{2} \cdot 2^n}$

Example 2: Find the common ratio and 10th term, and write a recursive rule and explicit rule for the sequence: 7,0.7,0.07,0.007, ...

$\boxed{r = .1}$ or $\boxed{\frac{1}{10}}$ $\frac{.7}{7} = .1$ $\frac{.07}{.7} = .1$ ☺

$a_n = a_1 r^{n-1}$
$a_{10} = 7(.1)^{10-1}$
$\boxed{a_{10} = 7 \times 10^{-9}}$
or $\boxed{a_{10} = .000000007}$
OR $\boxed{a_{10} = \frac{7}{1000000000}}$

Recursive Rule
$a_n = a_{n-1} \cdot (.1)$
$\boxed{a_n = .1 a_{n-1}}$
OR
$\boxed{a_n = \frac{1}{10} a_{n-1}}$

Explicit Rule
$a_n = a_1 r^{n-1}$
$\boxed{a_n = 7(.1)^{n-1}}$ OR
OR $= \boxed{7\left(\frac{1}{10}\right)^{n-1}}$ OR
OR $= 7\left(\frac{1}{10}\right)^n\left(\frac{1}{10}\right)^{-1}$
$= 7\left(\frac{1}{10}\right)^n \cdot 10$
$= \boxed{70\left(\frac{1}{10}\right)^n}$

Example 3: Given the geometric sequence terms $a_3 = \frac{1}{2}$ and $a_5 = \frac{9}{2}$, find a_1.

$a_n = a_1 r^{n-1}$
$a_3 = a_1 r^{3-1}$
$\frac{1}{2} = a_1 r^2$

$a_5 = a_1 r^{5-1}$
$\frac{9}{2} = a_1 r^4$
$\frac{9}{2} = a_1 r^2 \cdot r^2$
$\frac{9}{2} = \frac{1}{2} \cdot r^2$
$9 = r^2$
$\pm 3 = r$

$\frac{1}{2} = a_1 r^2$

$\frac{1}{2} = a_1 (\pm 3)^2$
$\frac{1}{9} \cdot \frac{1}{2} = 9a_1 \cdot \frac{1}{9}$
$\boxed{\frac{1}{18} = a_1}$

Example 4: The fifth and eighth terms of a geometric sequence are 1920 and 30, respectively. Find a_1.

$a_n = a_1 r^{n-1}$

$a_5 = a_1 r^{5-1}$ $a_8 = a_1 r^{8-1}$

$1920 = a_1 r^4$ $30 = a_1 r^7$

$30 = a_1 r^4 \cdot r^3$

$30 = 1920 \cdot r^3$

$\frac{1}{64} = r^3$

$\frac{1}{4} = r$

$1920 = a_1 r^4$

$1920 = a_1 (\frac{1}{4})^4$

$256 \cdot 1920 = a_1 (\frac{1}{256}) \cdot 256$

$\boxed{491520 = a_1}$

Example 5: A population of ants is growing at a rate of 8% a year. If there are 160 ants in the initial population, find the number of ants after 6 years.

$r = 1 + 8\%$

$r = 1.08$

$a_1 = 160$

Sequence method:

$$a_n = a_1 r^{n-1}$$
$$a_7 = 160(1.08)^{7-1}$$
$$= 160(1.08)^6$$
$$= 253$$

Exponential growth method:

$$P = P_o(1 + r)^t$$
$$P = 160(1 + 0.08)^t$$
$$= 160(1.08)^6$$
$$= 253$$

Therefore, there are approximately 253 ants after 6 years

Example 6:
Find which term in the geometric sequence 1, 3, 9, 27, ... is the first to exceed 7,000.

a_1

$n = ?$

$r = 3$ $\frac{3}{1} = 3$ $\frac{9}{3} = 3$

☺

$a_n = a_1 r^{n-1}$

$1(3)^{n-1} > 7000$

$3^{n-1} > 7000$

$\ln 3^{n-1} > \ln 7000$

$(n-1)\ln 3 > \ln 7000$

$n - 1 > \frac{\ln 7000}{\ln 3}$

$n > 1 + \frac{\ln 7000}{\ln 3}$

$n > 9.059$

$\therefore$, $\boxed{10^{th} \text{ term}}$

More Practice

Geometric Sequences

http://www.mathsisfun.com/algebra/sequences-sums-geometric.html
http://www.algebralab.org/lessons/lesson.aspx?file=algebra_geoseq.xml
http://www.mathguide.com/lessons/SequenceGeometric.html
https://youtu.be/EJjCXIhP7X0
https://youtu.be/h1HJEOD6u8E
https://youtu.be/C7tE26CDI2M
https://youtu.be/cXy_LJK0Ui8
https://youtu.be/lj_X9JVSF8k

Homework Assignment

p.656 #2-10even,25,27,31

SAT Connection
Solution

The correct answer is 1.02. The initial deposit earns 2 percent interest compounded annually. Thus at the end of 1 year, the new value of the account is the initial deposit of \$100 plus 2 percent of the initial deposit: $\$100 + \frac{2}{100}(\$100) = \$100(1.02)$. Since the interest is compounded annually, the value at the end of each succeeding year is the sum of the previous year's value plus 2 percent of the previous year's value. This is again equivalent to multiplying the previous year's value by 1.02. Thus, after 2 years, the value will be $\$100(1.02)(1.02) = \$100(1.02)^2$; after 3 years, the value will be $\$100(1.02)^3$; and after t years, the value will be $\$100(1.02)^t$. Therefore, in the formula for the value for Jessica's account after t years, $\$100(x)^t$, the value of x must be 1.02.