9.4 Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

1. Find the 100th term in the following sequence of numbers.

a) 2,5,8,11, ... **b)** 2,4,8,16, ...

2. Find the sum of the 1st 100 positive integers.

More Practice

Arithmetic and Geometric Sequences

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html

http://www.mathsisfun.com/algebra/sequences-sums-geometric.html

SAT Connection

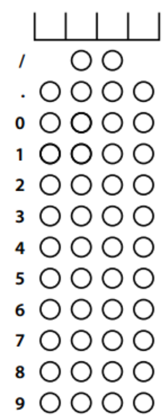
Heart of Algebra

8. Interpret the variables and constants in expressions for linear functions within the context presented.

Example:

$$a = 18t + 15$$

Jane made an initial deposit to a savings account. Each week thereafter she deposited a fixed amount to the account. The equation above models the amount a, in dollars, that Jane has deposited after t weekly deposits. According to the model, how many dollars was Jane's initial deposit? (Disregard the $ sign when gridding your answer.)



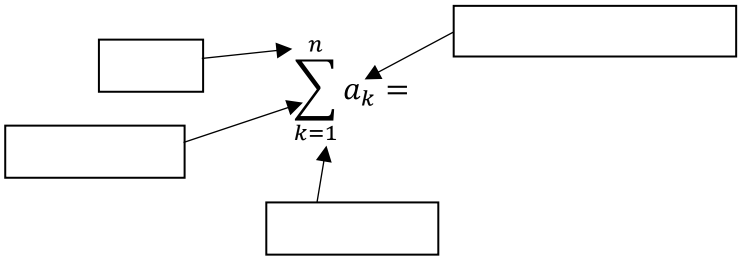
NOTE: You may start your answers in any column, space permitting. Columns you don't need to use should be left blank.

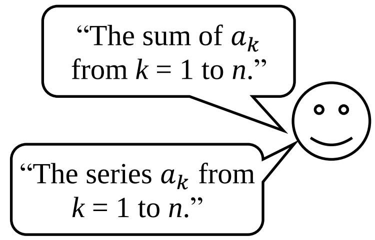
Solution

Summation/Series

Summation (or Series) - sum up the terms of a sequence

Summation Notation





Example 1: Find the value of:

$$\sum_{k=2}^{5} 3k$$

Example 2: Write the summation $2 + 5 + 8 + 11 + \cdots + 29$ in sigma notation.

Example 3: Write the series $5 - 15 + 45 - 135 + \cdots$ in sigma notation.

Sum of the Terms in an Arithmetic Sequence

Proof

Start with the sum of an arithmetic sequence

$$\sum_{k=1}^{n} a_k = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-1)d)$$

Write the terms backwards, starting with a_n,

$$\sum_{k=1}^{n} a_k =$$

Add the two equations together,

Simplify,

Solve for sigma,

∴, Formula for Sum of the Terms in an Arithmetic Sequence is:

$$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n) \qquad OR \qquad \sum_{k=1}^{n} a_k = \frac{n}{2}(2a_1 + (n-1)d)$$

Example 1:
Find the sum of: $2 + 5 + 8 + 11 + \cdots + 29$

Example 2:
Find the sum of the sequence: $-3, 1, 5, 9, \ldots, 133$

More Practice

Arithmetic Series
https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html
http://www.purplemath.com/modules/series4.htm
https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-seq-and-series/e/arithmetic_series
https://youtu.be/cYw4MFWsB6c
https://youtu.be/xWHfQGBzgbc
https://youtu.be/UHkueFmPC6s

Homework Assignment

p.657 #43-45all; p.664 #1–11odd

SAT Connection

Solution

The correct answer is 15. The amount, a, that Jane has deposited after t fixed weekly deposits is equal to the initial deposit plus the total amount of money Jane has deposited in the t fixed weekly deposits. This amount a is given to be $a = 18t + 15$. The amount she deposited in the t fixed weekly deposits is the amount of the weekly deposit times t; hence, this amount must be given by the term $18t$ in $a = 18t + 15$ (and so Jane must have deposited 18 dollars each week after the initial deposit). Therefore, the amount of Jane's original deposit, in dollars, is $a - 18t = 15$.