DATE: ______________

Unit 7 (Chapter 9): Discrete Mathematics **Pre-Calculus**

9.4 Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

1. Find the 100th term in the following sequence of numbers.

a) 2,5,8,11, ... $a_n = 2+(n-1)3$
$= 3n-1$
$a_{100} = 3(100)-1 = \boxed{299}$

b) 2,4,8,16, ... $a_n = 2(2)^{n-1}$
$a_{100} = 2(2)^{100-1}$
$= \boxed{2^{100}}$

2. Find the sum of the 1st 100 positive integers.

$1+2+\dots+100 = \boxed{5050}$

More Practice

Arithmetic and Geometric Sequences

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html

http://www.mathsisfun.com/algebra/sequences-sums-geometric.html

SAT Connection

Heart of Algebra

8. Interpret the variables and constants in expressions for linear functions within the context presented.

Example:

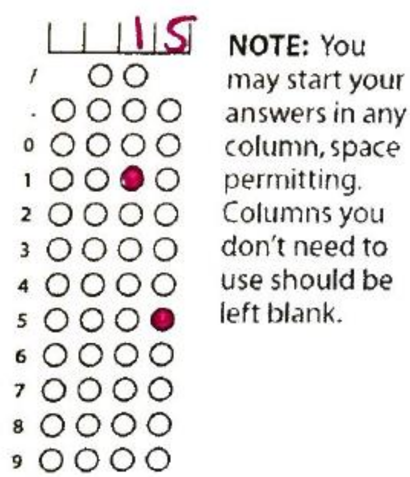
$$a = 18t + 15$$

Jane made an initial deposit to a savings account. Each week thereafter she deposited a fixed amount to the account. The equation above models the amount a, in dollars, that Jane has deposited after t weekly deposits. According to the model, how many dollars was Jane's initial deposit? (Disregard the $ sign when gridding your answer.)

$a = 18t + 15$

$a = 15 + 18t$ arithmetic explicit rule

$a_1 = 15$ initial deposit



Solution

Summation/Series

Summation (or Series) - sum up the terms of a sequence

Summation Notation

stop, sigma, start, explicit rule

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

"The sum of a_k from $k = 1$ to n."

"The series a_k from $k = 1$ to n."

Example 1: Find the value of:

$$\sum_{k=2}^{5} 3k = 3(2) + 3(3) + 3(4) + 3(5)$$
$$= \boxed{42}$$

MENU, CALCULUS, SUM

Example 2 Write the summation $\underbrace{2}_{a_1} + 5 + 8 + 11 + \cdots + \underbrace{29}_{a_n}$ in sigma notation.

start $k=1$

$a_n \leftarrow$ how many terms?

$a_n = a_1 + (n-1)d$
$29 = 2 + (n-1)3$
$29 = 2 + 3n - 3$
$29 = 3n - 1$
$30 = 3n$
$10 = n$

Arithmetic explicit rule

$a_n = a_1 + (n-1)d$
$a_n = 2 + (n-1)3$
$= 2 + 3n - 3$
$a_n = 3n - 1$

$$\boxed{\sum_{k=1}^{10} (3k-1)}$$

Example 3: Write the summation $\underbrace{5}_{a_1} - 15 + 45 - 135 + \cdots$ in sigma notation.

$k=1$

$\leftarrow$ how many terms? ∞

Geometric explicit rule

$a_n = a_1 r^{n-1}$
$a_n = 5(-3)^{n-1}$
or $a_n = 5(-3)^n(-3)^{-1}$
$= \frac{5(-3)^n}{-3}$
$a_n = -\frac{5}{3}(-3)^n$

$$\boxed{\sum_{k=1}^{\infty} 5(-3)^{k-1} \quad \text{or} \quad \sum_{k=1}^{\infty} \frac{-5}{3}(-3)^k}$$

Sum of the Terms in an Arithmetic Sequence

Proof

Start with the sum of an arithmetic sequence

$$\sum_{k=1}^{n} a_k = a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + (a_1 + (n-1)d)$$

Write the terms backwards, starting with a_n,

$$\sum_{k=1}^{n} a_k = a_n + (a_n - d) + (a_n - 2d) + \cdots + (a_n - (n-1)d)$$

Add the two equations together,

$$2\sum_{k=1}^{n} a_k = a_1 + a_n + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)$$

Simplify,

$$2\sum_{k=1}^{n} a_k = n(a_1 + a_n)$$

Solve for sigma,

$$\sum_{k=1}^{n} a_k = \frac{n(a_1 + a_n)}{2}$$

∴, Formula for Sum of the Terms in an Arithmetic Sequence is:

$$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n) \qquad OR \qquad \sum_{k=1}^{n} a_k = \frac{n}{2}(2a_1 + (n-1)d)$$

$$\frac{n}{2}(a_1 + \underbrace{a_1 + (n-1)d})$$

Example 1:

Find the sum of: $\underbrace{2}_{a_1} + 5 + 8 + 11 + \cdots + \underbrace{29}_{a_n}$

$$\sum_{k=1}^{10} a_k = \frac{10}{2}(2+29)$$
$$= \boxed{155}$$

arithmetic w/ d = 3

$a_n = a_1 + (n-1)d$

$29 = 2 + (n-1)3$

$29 = 2 + 3n - 3$

$29 = 3n - 1$

$30 = 3n$

$10 = n$

Example 2:

Find the sum of the sequence: $\underbrace{-3}_{a_1}$, 1, 5, 9, ... , $\underbrace{133}_{a_n}$

$$\sum_{k=1}^{35} a_k = \frac{35}{2}(-3+133)$$
$$= \boxed{2275}$$

arithmetic w/ d = 4

$a_n = a_1 + (n-1)d$

$133 = -3 + (n-1)4$

$133 = -3 + 4n - 4$

$133 = 4n - 7$

$140 = 4n$

$35 = n$

More Practice

Arithmetic Series

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html
http://www.purplemath.com/modules/series4.htm
https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-seq-and-series/e/arithmetic_series
https://youtu.be/cYw4MFWsB6c
https://youtu.be/xWHfQGBzgbc
https://youtu.be/UHkueFmPC6s

Homework Assignment

p.657 #43–45all; p.664 #1-11odd

SAT Connection

Solution

The correct answer is 15. The amount, a, that Jane has deposited after t fixed weekly deposits is equal to the initial deposit plus the total amount of money Jane has deposited in the t fixed weekly deposits. This amount a is given to be $a = 18t + 15$. The amount she deposited in the t fixed weekly deposits is the amount of the weekly deposit times t; hence, this amount must be given by the term $18t$ in $a = 18t + 15$ (and so Jane must have deposited 18 dollars each week after the initial deposit). Therefore, the amount of Jane's original deposit, in dollars, is $a - 18t = 15$.