## 9.4 Series

Target 7D: Calculate the sums of finite and infinite series

Review of Prior Concepts

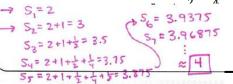
**1.** Find the value of (without calculator):

$$\sum_{k=5}^{9} (11-3k) = \frac{1(-3(5)+1(-3(6)+1(-3(7))+1(-3(7))+1(-3(8)+1(-3(8)$$

**2.** Find the value of (with calculator):

$$\sum_{k=0}^{12} \left(\frac{1}{2}\right)^k = \boxed{\frac{8191}{4096}}$$

3. Find the sum of:  $2 + 1 + \frac{1}{2} + \frac{1}{4} + \cdots$ 



**4.** Find the sum of:  $1 + 2 + 4 + 8 + 16 + \cdots$ 

$$S_1 = 1$$
  
 $S_2 = 1 + 2 = 3$   
 $S_3 = 1 + 2 + 4 = 7$   
 $S_4 = 1 + 2 + 4 + 8 = 15$ 
 $S_5 = 31$   
 $S_6 = 63$   
 $S_6 = 63$ 

### **More Practice**

## Arithmetic & Geometric Sequences and Series

https://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html http://www.mathsisfun.com/algebra/sequences-sums-geometric.html http://www.purplemath.com/modules/series4.htm



#### **SAT Connection**

### **Passport to Advanced Math**

10. Interpret parts of nonlinear expressions in terms of their context. .

#### Example:

Of the following four types of savings account plans, which option would yield exponential growth of the money in the account?

- Each successive year, 2% of the initial savings is added to the value of the account.
- B) Each successive year, 1.5% of the initial savings and \$100 is added to the value of the account.
- Each successive year, 1% of the current value is added to the value of the account.
- Each successive year, \$100 is added to the value of the account.

(c) 
$$a_n = 0.1(a_{n-1}) + a_{n-1}$$
  

$$= a_{n-1}(0.1+1)$$

$$= 1.1 a_{n-1} \longrightarrow \text{matches recurryive rule}$$

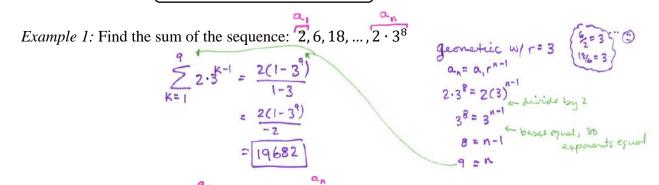
**Solution** 

### **Finite Geometric Series**

$$\sum_{k=1}^{n} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{n-1}$$

Formula for Sum of the Terms in a Finite Geometric Sequence is:

$$\sum_{k=1}^{n} a_1 r^{k-1} = \frac{a_1 (1 - r^n)}{1 - r}$$



Example 2: Find the sum:  $\frac{4}{4} - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \frac{4}{81}$ 

$$\sum_{k=1}^{5} 4(-\frac{1}{5})^{k-1} = \frac{4(1-(-\frac{1}{3})^{5})}{1-\frac{1}{3}}$$

$$= \frac{4(1-(-\frac{1}{2}\frac{1}{4})}{1+\frac{1}{3}}$$

$$= \frac{4(1+\frac{1}{2}\frac{1}{4})}{4|_{3}} = \frac{244}{81} \text{ or } 3.01235$$

#### **Infinite Geometric Series**

$$\sum_{k=1}^{\infty} a_1 r^{k-1} = a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{n-1} + \dots$$

Formula for Sum of the Terms of an Infinite Geometric Sequence is:

$$\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r} \qquad \text{when } |r| < 1$$

Example 1: Find the sum of the sequence: 
$$2+1+\frac{1}{2}+\frac{1}{4}+\cdots$$

$$\sum_{k=1}^{\infty} 2\left(\frac{1}{2}\right)^{k-1} = \frac{2}{1-\sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = 2 \cdot \frac{1}{2} = \boxed{4}$$
The series

"converges to 4"

## **Unit 7 (Chapter 9): Discrete Mathematics**

## **Pre-Calculus**

Example 2: Find the sum: 
$$\frac{1}{48} + \frac{1}{16} + \frac{3}{16} + \frac{9}{16} + \cdots$$

"The series diverges"

geometrice 
$$w/r=3$$

Is  $|3| < 1$ ?

 $3 < 1 \times$ 
 $|3| < 1 \times$ 

## Now, you try...

1. Find the sum of the sequence:  $3, \frac{3}{4}, \frac{3}{16}, \frac{3}{64}, \dots$ 

$$\sum_{k=1}^{\infty} 3(\frac{1}{4})^{k-1} = \frac{3}{1-1/4}$$

$$= \frac{3}{3/4}$$

$$= 3 \cdot \frac{1}{3} = \boxed{4}$$
The seconds

geometric W/r= 4 Is 14/41?

**2.** Find:

$$\sum_{k=1}^{\infty} (-2)^k = (-2)^k + (-2)^k + (-2)^k + (-2)^k + \cdots$$

Since 1-2/ \$1, the seaso diverges.

3. Find the sum:  $\frac{1}{16} - \frac{1}{48} + \frac{1}{144} - \cdots$ 

$$\sum_{k=1}^{\infty} \frac{1}{16} (\frac{1}{3})^{k-1} = \frac{1}{1-\frac{1}{3}}$$

$$= \frac{1}{16} \cdot \frac{3}{4} = \frac{3}{64}$$
The series converges to  $\frac{3}{64}$ 

geometrie  $w/c=\frac{1}{3}$   $\frac{48}{y_{11}}=\frac{1}{48}\cdot\frac{16}{3}=\frac{1}{3}$   $\frac{y_{14}}{y_{12}}=\frac{1}{144}\cdot\frac{16}{3}=\frac{1}{3}$ The series  $\frac{3}{3}$ 

## **More Practice**

### **Geometric Series**

http://www.purplemath.com/modules/series5.htm

https://www.khanacademy.org/math/algebra2/sequences-and-series/copy-of-geometric-sequence-

series/v/geometric-series-introduction

http://www.mathsisfun.com/algebra/sequences-sums-geometric.html

https://youtu.be/yYxzq\_O18Mg

https://youtu.be/-JH5XSvJFTA

https://youtu.be/DO1bIuqFIDQ

https://youtu.be/haK3oC0L\_a8

# **Homework Assignment**

p.656 #11,13; p.664 #13-29odd

### **SAT Connection**

#### Solution

**Choice C is correct**. Let I be the initial savings. If each successive year, 1% of the current value is added to the value of the account, then after 1 year, the amount in the account will be I + 0.01I = I(1 + 0.01); after 2 years, the amount in the account will be  $I(1 + 0.01) + 0.01I(1 + 0.01) = (1 + 0.01)I(1 + 0.01) = I(1 + 0.01)^2$ ; and after t years, the amount in the account will be  $I(1 + 0.01)^t$ . This is exponential growth of the money in the account.

Choice A is incorrect. If each successive year, 2% of the initial savings, I, is added to the value of the account, then after t years, the amount in the account will be I + 0.02It, which is linear growth. Choice B is incorrect. If each successive year, 1.5% of the initial savings, I, and \$100 is added to the value of the the account, then after t years the amount in the account will be I + (0.015I + 100)t, which is linear growth. Choice D is incorrect. If each successive year, \$100 is added to the value of the account, then after t years the amount in the account will be I + 100t, which is linear growth.