

Given:  $\overline{PE} \cong \overline{PR}$   
 $\overline{BE} \cong \overline{BR}$

Prove:  $\overleftrightarrow{PB}$  is  $\perp$  bisector  
 of  $\overline{ER}$

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Statements	Reasons
① $\overline{PE} \cong \overline{PR}$ $\overline{BE} \cong \overline{BR}$	① Given
② $\overline{PB} \cong \overline{PB}$	② Reflexive Property
③ $\triangle PEB \cong \triangle PRB$	③ SSS
④ $\angle EPS \cong \angle RPS$	④ CPCTC
⑤ $\overline{PS} \cong \overline{PS}$	⑤ Reflexive property
⑥ $\triangle EPS \cong \triangle RPS$	⑥ SAS
⑦ $\overline{ES} \cong \overline{RS}$	⑦ CPCTC
⑧ $\overleftrightarrow{PB}$ bis $\overline{ER}$	⑧ Def. of bisection
⑨ $\angle PSE \cong \angle PSR$	⑨ CPCTC
⑩ $\angle PSE$ supp $\angle PSR$	⑩ If 2 $\angle$ form st. $\angle$ , then supp.
⑪ $\angle PSE, \angle PSR$ rt. $\angle$	⑪ If $\angle$ s supp and $\cong$ , then rt. $\angle$ s.
⑫ $\overleftrightarrow{PB} \perp \overline{ER}$	⑫ If 2 lines int. to form rt. $\angle$ s, then $\perp$ .
⑬ $\overleftrightarrow{PB} \perp$ bis $\overline{ER}$	⑬ step 8 and 12

## 4.4. Honors Geometry

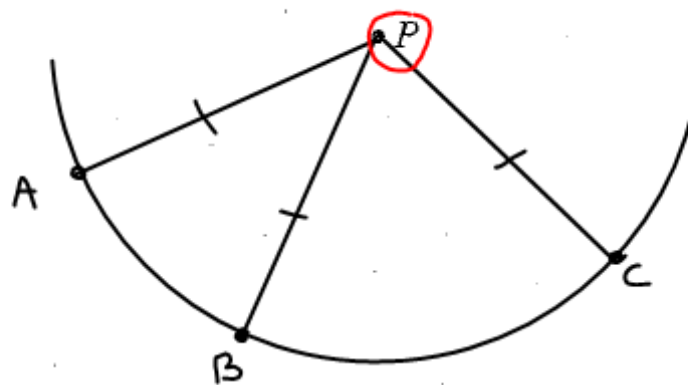
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Target 4E. Recognize and apply the relationship between equidistance and perpendicular bisection

### The Equidistance Theorems

- Equidistant: Two or more points that are the same distance away from a third point.
  - Example: All of the points on a circle are equidistant from the center!

Use a compass to construct points A, B, and C so that they are all equidistant from P.



So,  $\overline{AP} \cong \overline{BP} \cong \overline{CP}$  means that P is equidistant from A, B, and C.

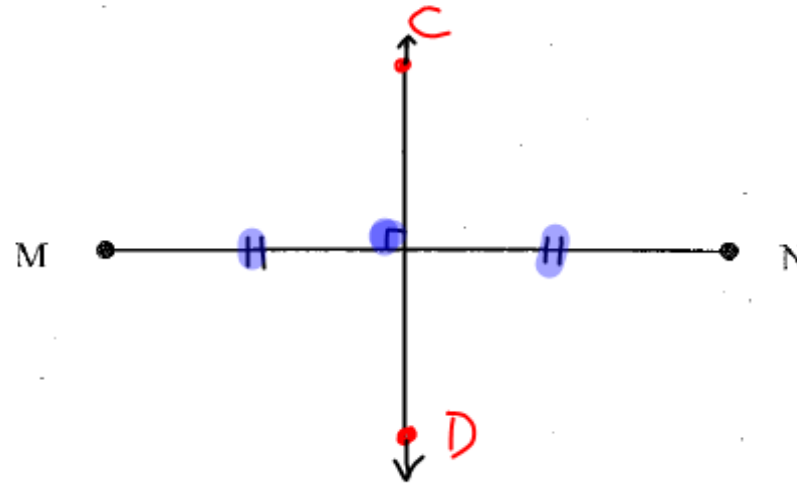
- Perpendicular Bisector: a segment or a line that bisects and is perpendicular to

- Perpendicular Bisector: a segment or a line that bisects and is perpendicular to another segment.



- Example: think back...The median of an isosceles triangle was also a  $\perp$  bisector!

Below, create a sketch of  $\overleftrightarrow{CD}$  so that it is the  $\perp$  bisector of  $\overline{MN}$ . Use the proper tick marks to label all of the important information.



4.4 Equidistance.pdf - Adobe Reader

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Use a compass to construct  $\overline{PQ}$  so that it is the  $\perp$  bisector of  $\overline{AB}$ .

Think about the steps that you just completed in order to construct the  $\perp$  bisector. Why did you want the arcs to intersect? The next theorem proves why our construction worked!

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Think about the steps that you just completed in order to construct the  $\perp$  bisector. Why did you want the arcs to intersect? The next theorem proves why our construction worked!

- Theorem: If **two points** are each equidistant from the endpoints of a segment, then the two points determine the  $\perp$  bisector of that segment.

Given:  $\overline{PA} \cong \overline{PB}$ ,  $\overline{QA} \cong \overline{QB}$  (above)

Prove:  $\overline{PQ}$  is the  $\perp$  bisector of  $\overline{AB}$   
(a detour proof for this theorem is provided on page 181, sample problem #2)

- Theorem: If a point is on a  $\perp$  bisector of a segment, then it is equidistant from the endpoints of that segment.

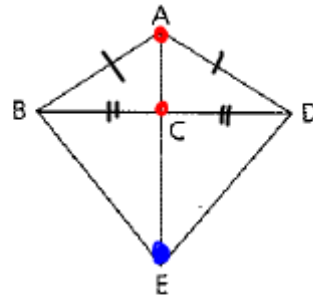
Draw a new point (**name it point C**) on  $\overline{PQ}$  in your construction above. Then draw and measure segments  $\overline{CA}$  and  $\overline{CB}$ . Is our theorem true?

Given:  $\overline{CQ}$  is the  $\perp$  bisector of  $\overline{AB}$

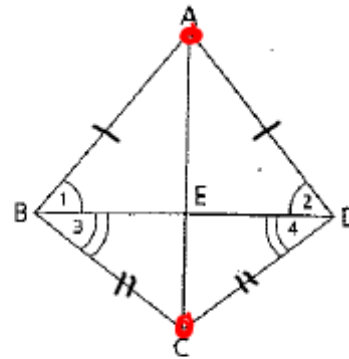
Prove:  $\overline{CA} \cong \overline{CB}$   
(a proof can be created using definition of perpendicular bisector, SAS, and CPCTC)

Given:  $\overline{AB} \cong \overline{AD}$ ,  
 $\overline{BC} \cong \overline{CD}$

Conclusion:  $\overline{BE} \cong \overline{DE}$



statement	Reason
① $\overline{AB} \cong \overline{AD}$ $\overline{BC} \cong \overline{CD}$	① Given
② $\overleftrightarrow{AC} \perp \text{bis } \overline{BD}$	② <u>Two pts.</u> equidistant from the endpoints (B, D) of a segment determine the $\perp$ bisector of the segment
③ $\overline{BE} \cong \overline{DE}$	③ <u>A point</u> on the $\perp$ bisector of a segment is equidistant from the endpoints of the segment.

Given:  $\angle 1 \cong \angle 2$ , $\angle 3 \cong \angle 4$ Prove:  $\overleftrightarrow{AE} \perp \text{bis. } \overline{BD}$ 

S	R
① $\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$	① Given
② $\overline{AB} \cong \overline{AD}$	② $\rightarrow$ $\times$
③ $\overline{CB} \cong \overline{CD}$	③ Same as step 2.
④ $\overleftrightarrow{AE} \perp \text{bis } \overline{BD}$	④ Equidistance Thm. (Step ②, ③)

[Two pts equidistant from the  
endpts of a seg. determine  $\perp$  bis.]

Given:  $\overline{AB} \cong \overline{AD}$ ,  
 $\overline{BC} \cong \overline{CD}$ 