2nd Semester Final Review
Advanced Algebra

Name: Key
Date: _____ Period: _____

Target 6.A. Simplify expressions using properties of exponents.

Simplify the expressions. $3+7=10$, $4+(-5)=-1$

1. $\left(\frac{y^5}{x^3}\right)^8$

$\frac{y^{5\cdot 8}}{x^{3\cdot 8}} = \frac{y^{40}}{x^{24}}$

2. $w^3w^7x^4x^{-5}$

$w^{10}x^{-1}$

$= \frac{w^{10}}{x}$

3. $\left(\frac{y^9}{x^2}\right)^7$

$\frac{y^{9\cdot 7}}{x^{2\cdot 7}} = \frac{y^{63}}{x^{14}}$

4. $w^5w^{11}x^6x^{-7}$

$w^{5+11}x^{6+(-7)}$

$= w^{16}x^{-1}$

$= \frac{w^{16}}{x}$

Target 6.B. Perform operations on polynomial functions.

Simplify the expression.

5. $(x+2)(x^2-x+3)$ Dist. property

$x^3 - 1x^2 + 3x + 2x^2 - 2x + 6$

$x^3 + x^2 + x + 6$

6. $(x+3)(x^2-2x+7)$ Box method

	x^2	$-2x$	$+7$
x	x^3	$-2x^2$	$7x$
$+3$	$3x^2$	$-6x$	$+21$

$x^3 + x^2 + x + 21$

7. Use the functions $f(x) = 7x - 18$ and $g(x) = 4x - 6$ to find $f(x) - g(x)$.

$(f-g)(x) = f(x) - g(x) = 7x - 18 - (4x-6)$

$= 7x - 18 - 4x + 6 = 3x - 12$

8. Use the functions $f(x) = x^3 - 4x - 6$ and $g(x) = x - 2$ to find $\frac{f(x)}{g(x)}$. Use synthetic division.

$1x^3 + 0x^2 - 4x - 6$ $+2$

$\frac{f(x)}{g(x)} = ?$

2	1	0	-4	-6
	↓	2	4	0
	1	2	0	-6

1 power less than 3 ← REMAINDER

$2\cdot 1 = 2$; $0+2=2$
$2\cdot 2 = 4$; $-4+4=0$
$2\cdot 0 = 0$; $-6+0=-6$

$x^2 + 2x + \frac{-6}{x-2}$

Target 6.C. Create compositions of polynomial functions.

9. Use the functions $f(x) = 7x - 18$ and $g(x) = 4x - 6$ to find $f(g(x))$.

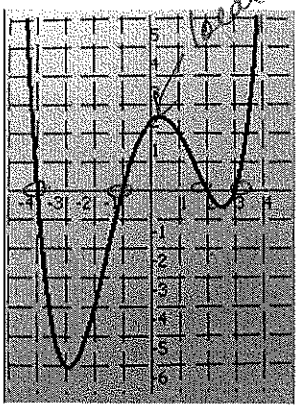
$f(g(x)) = f(4x-6) = 7(4x-6) - 18 = 28x - 42 - 18$

$= 28x - 60$

Target 7.A. Analyze the graph of a polynomial function by: identify its degree, number and location of its real zeros, determine its end behavior, and determining the maxima and minima.

Use the graph to the right for #10-13.

local max



10. Determine whether the degree of the function is odd or even.

Even

11. State the number of real zeros.

4

12. Determine the real zeros of the polynomial function.

$-4, -1, 2, 3$

13. Determine the ordered pair(s) of the local maximum.

$\approx \left(\frac{1}{3}, 2.5\right)$

Target 7.B. Write a polynomial function, in standard form, given its roots.

14. Write a polynomial function with the roots $x = -4, x = 5$, and $x = 7$.

$f(x) = (x-(-4))(x-(5))(x-(7)) = (x+4)(x-5)(x-7)$

Target 7.C. Apply the Remainder Theorem and the Factor Theorem to determine the factors and roots of a polynomial.

15. Given $f(x) = 6x^3 - 5x^2 + 7x + 3$, find $f(2)$.

$f(2) = 6(2)^3 - 5(2)^2 + 7(2) + 3$
$= 6(8) - 5(4) + 14 + 3 = 48 - 20 + 14 + 3 = 45$

16. Given the polynomial and one of its factors, find the remaining factors: $4x^3 - 4x^2 - 9x + 9$; $(x - 1)$ $+1$

1	4	-4	-9	9
		4	0	-9
	4	0	-9	0

So $4x^2 - 9 \leftarrow$ Factor

$(2x+3)(2x-3)$

Target 8.A. Classify an equation as direct, inverse, or joint variation.

17. Determine which of the following equations represents an inverse variation equation.

a. $y = 5x$ $(y = kx)$ Direct

~~b.~~ $y = \sqrt{7x}$

c. $y = \frac{11}{x}$ $xy = k$ or $y = \frac{k}{x}$ Inverse

d. $y = -9xz$ Joint $(y = kxz)$

~~e.~~ $y = |-10x|$

Target 8.B. Create equations to solve direct, inverse, or joint variation problems.

18. Suppose y varies jointly with x and z. If $y = 2$ when $x = 5$ and $z = 4$, find the value of y when $x = -5$ and $z = 12$.

METHOD 1:

$\frac{y}{xz} = \frac{\cancel{xz}k}{\cancel{xz}}$

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2} \Rightarrow \frac{2}{(5)(4)} = \frac{y}{(-5)(12)}$

$\Rightarrow \frac{2}{20} = \frac{y}{-60} \Rightarrow \frac{-120}{20} = \frac{20y}{20}$

$-6 = y$

METHOD 2:

$y = kxz$
$2 = k(5)(4)$
$\frac{2}{20} = \frac{k \cdot 20}{20}$
$\frac{1}{10} = k$

$y = \frac{1}{10}(-5)(12)$
$= \frac{-60}{10}$
$y = -6$

Target 8.C. Perform operations and simplify rational expressions.
Simplify the expression for problems #19-20.

19. $\frac{25(x+3)}{x^2} \cdot \frac{x}{5(x+3)}$

$\frac{25 \cdot \cancel{x}}{x \cdot \cancel{x}} = \frac{25}{x}$

20. $\frac{7}{6x} + \frac{x}{6x(x-5)}$

$= \frac{7}{6x} \cdot \frac{(x-5)}{(x-5)} + \frac{x}{6x(x-5)}$

$= \frac{7x - 35 + x}{6x(x-5)} = \frac{8x - 35}{6x(x-5)}$

Target 8.D. Understand the relationship between a rational function and its graph.

21. Write the equation of the vertical asymptote of the graph of $y = \frac{x^2+3x-4}{x-5}$. $\rightarrow$ Factor

-4
$4 \cdot -1$
$4 + (-1) = 3$

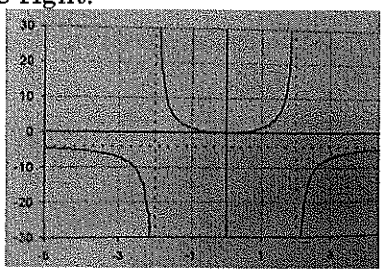
$y = \frac{(x+4)(x-1)}{(x-5)}$ $\rightarrow$ vertical asymptote

$x - 5 = 0$
$+5 \quad +5$
$\Rightarrow x = 5$

NO HOLES

22. Determine the equation(s) of the vertical asymptote(s) from the graph to the right.

$x = -2$ and $x = 2$



-2

$(x+4)\cdot\frac{6}{x+4} = 2\cdot(x+4) - \frac{12}{x+4}\cdot(x+4)$

Target 8.E. Solve rational equations.
Solve each equation for problems #23-24.

23. $\frac{-8}{(x+5)} = \frac{4}{3}$

$-8\cdot 3 = 4(x+5)$

$-24 = 4x + 20$

$-20 \qquad -20$

$\frac{-44}{4} = \frac{4x}{4}$

$-11 = x$

24. $\frac{6}{x+4} = 2 - \frac{12}{x+4}$

$6 = 2x + 8 - 12$

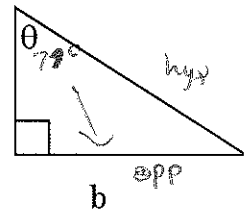
$6 = 2x - 4$

$+4 \qquad +4$

$\frac{10}{2} = \frac{2x}{2} \Rightarrow 5 = x$

Target 9.A. Solve right triangles and extend knowledge of sine, cosine and tangent ratios to their respective reciprocals.

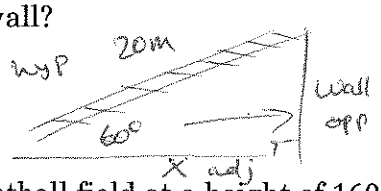
25. The measure of $\theta = 74°$, find the value of b. Round your answer to the nearest tenth.



$\tan 74° = \frac{opp}{adj} = \frac{b}{8} \Rightarrow \frac{\tan 74°}{1} = \frac{b}{8}$

$\Rightarrow b = 8\cdot\tan 74° = 27.9$

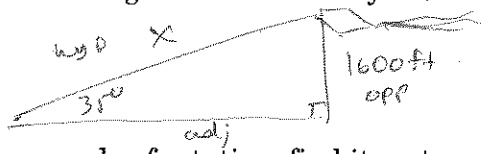
26. A 20 m long ladder rests against a wall at an angle of 60° with the ground. How far is the foot of the ladder from the bottom of the wall?



$\frac{\cos 60°}{1} = \frac{x}{20}$

$x = 20\cdot\cos 60° = 10\,m$

27. A kite is flying over the football field at a height of 1600 ft with an angle of elevation of 35°. Calculate the length of string that is used to fly the kite. Round your answer to the nearest foot.



$\frac{\sin 35°}{1} = \frac{1600}{x} \Rightarrow x = \frac{\sin 35°}{\sin 35°} = \frac{1600}{\sin 35°}$

$x = 2790\,ft$

Target 9.B. Draw an angle of rotation, find its coterminal angles and determine the quadrant in which it lands.

28. Find two angles that are coterminal with a 125° angle.

Many ans. possible: $125 + 360 = 485°$ and $125 - 360 = -235°$

29. Sketch the graph of an angle measuring −65°.

↙ negative ∠ , ↶ positive ∠ meas.

Target 9.C. Understand how to move between radian measure and degree measure.

30. Convert $\frac{3\pi}{4}$ *radians* to degrees.

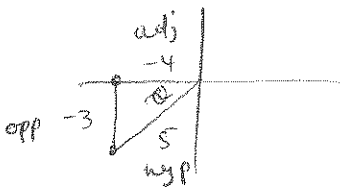
$\frac{3\pi}{4}\cdot\frac{180}{\pi} = \frac{540}{4} = 135°$

31. Convert 180° to radians.

$\frac{180}{1}\cdot\frac{\pi}{180} = \frac{180\pi}{180} = \pi$ radians

Target 9.D. Determine the exact values of the six trigonometric functions given the terminal side of θ passing through a given point P(x,y) or using reference triangles.

32. Given the point (−4, −3) on the terminal side of an angle θ, evaluate tan θ.



$r = \sqrt{(x)^2 + (y)^2}$

$= \sqrt{(-4)^2 + (-3)^2}$

$= \sqrt{16+9} = \sqrt{25} = 25$

$\tan\theta = \frac{-3}{-4} = \frac{3}{4}$

Remember ① MENU ② NUMBER ③ FACTOR

Target 10.A. Simplify radical expressions with various indices.

Simplify the expression; assume all variables are positive for problems #33-36.

33. $\sqrt[4]{81x^8}$

$4 \div 4 = 1$
$8 \div 4 = 2$

$\sqrt[4]{3^4 x^8}$

$3x^2$

34. $\sqrt[4]{75x^6y^{13}}$

$2 \div 2 = 1$
$6 \div 2 = 3$
$12 \div 2 = 6$

$\sqrt[2]{5^2 \cdot 3 \cdot x^6 \cdot y^{12} \cdot y}$

$5x^3y^6\sqrt{3y}$

35. $\sqrt[4]{256x^4}$

$8 \div 4 = 2$
$4 \div 4 = 1$

$\sqrt[4]{2^8 x^4}$

$2^2 x$

$4x$

36. $\sqrt[2]{72x^{12}y^7}$

$2 \div 2 = 1$
$2 \div 2 = 1$
$12 \div 2 = 6$
$6 \div 2 = 3$

$\sqrt[2]{2^3 \cdot 3^2 \cdot x^{12} y^6 \cdot y}$

$\sqrt[2]{2^2 \cdot 2 \cdot 3^2 \cdot x^{12} \cdot y^6 \cdot y}$

$2 \cdot 3 \cdot x^6 \cdot y^3 \sqrt{2y}$

$6x^6y^3\sqrt{2y}$

Target 10.B. Perform operations on radical expressions with various indices.

Simplify the expression for problems #37-40.

37. $x\sqrt[4]{x} + 3x\sqrt[4]{x}$

$4x\sqrt[4]{x}$

38. $\sqrt[5]{2} \cdot \sqrt[5]{16}$

$\sqrt[5]{2} \cdot \sqrt[5]{2^4}$

$\sqrt[5]{2^5}$ $\quad 5 \div 5 = 1$

2

39. $x\sqrt[3]{2x} + 5x\sqrt[3]{2x}$

$6x\sqrt[3]{2x}$

40. $\sqrt[4]{3} \cdot \sqrt[4]{27} = \sqrt[4]{3^3}$

$\sqrt[4]{3 \cdot 3^3}$

$\sqrt[4]{3^4}$ $\quad 4 \div 4 = 1$

3

Target 10.C. Solve equations containing radicals.

Solve the equation for problems #41-44. No solution

41. $5 \cdot \sqrt[3]{x+7} = 15$

$\frac{5 \cdot \sqrt[3]{x+7}}{5} = \frac{15}{5}$

$(\sqrt[3]{x+7})^3 = (3)^3$

$x + 7 = 27$
$-7 \quad -7$
$x = 20$

CHECK:
$5 \cdot \sqrt[3]{20+7} = 15$
$5 \cdot \sqrt[3]{27} = 15$
$5 \cdot 3 = 15$ ✓

42. $\sqrt{5x} + 23 = 18$

$-23 \quad -23$

$(\sqrt{5x})^2 = (-5)^2$

$5x = 25$
$\frac{5x}{5} = \frac{25}{5}$
$x = 5$

CHECK:
$\sqrt{5 \cdot 5} + 23$
$\sqrt{25} + 23$
$5 + 23$
$28 \neq 18$

43. $8 \cdot \sqrt[3]{x+9} = 32$

$\frac{8 \cdot \sqrt[3]{x+9}}{8} = \frac{32}{8}$

$(\sqrt[3]{x+9})^3 = (4)^3$

$x + 9 = 64$
$-9 \quad -9$
$x = 55$

CHECK IS GOOD ✓

44. $\sqrt{7x} + 31 = 22$

$-31 \quad -31$

$(\sqrt{7x})^2 = (-9)^2$

$\frac{7x}{7} = \frac{81}{7}$
$x = \frac{81}{7}$

CHECK NO GOOD
NO SOL

Target 10.D. Graph and state the domain and range of radical functions in $f(x) = a\sqrt{x-h} + k$ form.

45. State the domain of $y = \sqrt{x+11}$.

$x + 11 = 0$
$-11 \quad -11$
$x = -11$

Domain $= \{x \mid x \geq -11\}$

"The set of all x values greater than or equal to 11"

Target 11.A. Write an equation in vertex form for a parabola and in standard form for a circle by completing the square.

46. Write the equation of the circle in standard form: $x^2 + 6x + y^2 - 12y + 17 = 0$

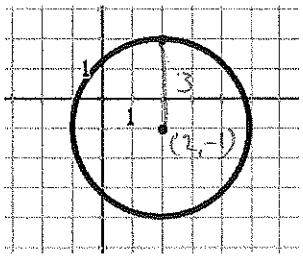
Skip

47. Write the equation of the parabola in vertex form: $x = y^2 - 8y + 5$

Skip

Target 11.B. Write an equation in vertex form for a parabola and in standard form for a circle given its graph.

48. Write the equation for the circle graphed to the right.



$(x-h)^2+(y-k)^2=r^2$ $(h,k)=(-2,1)$

$(x-(-2))^2+(y-1)^2=(3)^2$

$(x+2)^2+(y-1)^2=9$

Target 11.D. Identify a conic section and its characteristics given its equation

49. Identify this equation (circle, parabola, straight line): $6x^2+6y^2+8x-12y-34=0$

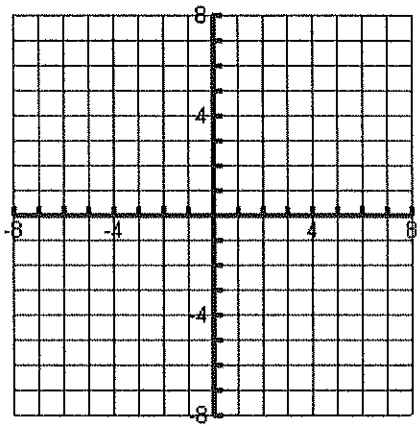
SKIP

50. Determine which direction the parabola opens given its equation: $x=5(y-4)^2+7$

5 is positive so opens up (If 5 was negative, parabola open down,)

Target 11.E. Solve a system involving a quadratic equation with a linear equation graphically and algebraically.

51. Solve the system of equations (find all points of intersection): $\begin{matrix} y=x+1 \\ y=x^2 \end{matrix}$



SKIP

Target 11.C. Graph parabolas and circles and label their parts.

52. Find the center & radius of a circle, then graph the circle: $(x-2)^2+(y+1)^2=9$

Center $(2,-1)$

Radius $\sqrt{9}=3$