

Target 8A: Experimental & Theoretical Probability & Probability Distributions and Frequency Tables

The probability of an event A of occurring is a number from 0 to 1, $0 \leq P(A) \leq 1$, that describes how likely it is that an event will occur. Probabilities closer to 1 are more likely to occur, and probabilities closer to 0 are less likely to occur.

Outcome: the possible result of a situation or experiment.

Event: a single or a group outcome.

Sample space: the set of all possible outcomes.

Experimental probability: measures the likelihood that the event occurs based on the actual results of an experiment.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of times the experiment is done}}$$

Theoretical probability: describes the likelihood of an event based on mathematical reasoning.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Complement of an event: all the possible outcomes in the sample space that are not part of the event.

$$P(\text{event}) + P(\text{not event}) = 1 \quad P(\text{not event}) = 1 - P(\text{event})$$

Calculating Experimental Probability

Quality Control A quality control inspector samples 500 LCD monitors and finds defects in three of them.

A What is the experimental probability that a monitor selected at random will have a defect?

$$\begin{aligned} P(\text{defect}) &= \frac{\text{number of monitors with a defect}}{\text{number of monitors inspected}} \\ &= \frac{3}{500} \\ &= 0.006 \text{ or } 0.6\% \end{aligned}$$

The experimental probability that a monitor selected at random is defective is 0.6%.

B If the company manufactures 15,240 monitors in a month, how many are likely to have a defect based on the quality inspector's results?

$$\begin{aligned} \text{number of defective monitors} &= P(\text{defect}) \cdot \text{total number of monitors} \\ &= 0.006 \cdot 15,240 \\ &= 91.44 \end{aligned}$$

It is likely that approximately 91 monitors are defective.

Using Probabilities of Events and Their Complements

A jar contains 10 red marbles, 8 green marbles, 5 blue marbles, and 6 white marbles. What is the probability that a randomly selected marble is not green?

$$\begin{aligned} P(\text{not green}) &= 1 - P(\text{green}) \quad \text{Probability of the complement} \\ &= 1 - \frac{8}{29} \quad \text{Find } P(\text{green}). \\ &= \frac{21}{29} \quad \text{Simplify.} \end{aligned}$$

The probability that the chosen marble is not green is $\frac{21}{29}$.

Practice

1) You roll a standard number cube 10 times. The results are 6, 4, 6, 1, 5, 2, 4, 2, 4, 3. Find the experimental probability of each outcome:

$$\begin{aligned} \text{a. } P(\text{rolling a 5}) &= \frac{1}{10} & \text{b. } P(\text{rolling a 6}) &= \frac{2}{10} = \frac{1}{5} & \text{c. } P(\text{rolling an even number}) &= \frac{7}{10} & \text{d. } P(\text{rolling a 1}) &= \frac{1}{10} \end{aligned}$$

2) What is the experimental probability of rolling an odd number on a standard number cube? For 50 rolls of the number cube, predict the number of rolls that will result in an odd number.

3) Find the theoretical probability of each outcome:

$$\begin{aligned} \text{a. } P(\text{rolling a 5}) &= \frac{1}{6} & \text{b. } P(\text{rolling a 6}) &= \frac{1}{6} & \text{c. } P(\text{rolling an even number}) &= \frac{3}{6} = \frac{1}{2} & \text{d. } P(\text{rolling a 1}) &= \frac{1}{6} \\ \text{e. } P(\text{rolling an odd \#}) &= \frac{3}{6} = \frac{1}{2} & \text{f. } P(\text{rolling a multiple of 3}) &= \frac{2}{6} = \frac{1}{3} \end{aligned}$$

4) A bag contains 2 red ping pong balls, 3 green ping pong balls, 3 blue ping pong balls, and 1 yellow ping pong ball. Find the probability of randomly selecting each outcome:

$$\begin{aligned} \text{a. } P(\text{not red}) &= 1 - P(\text{red}) = 1 - \frac{2}{9} = \frac{7}{9} & \text{b. } P(\text{not green}) &= 1 - P(\text{green}) = 1 - \frac{3}{9} = \frac{6}{9} = \frac{2}{3} & \text{c. } P(\text{not blue}) &= \frac{3}{9} & \text{d. } P(\text{not yellow}) &= 1 - P(\text{yellow}) = 1 - \frac{1}{9} = \frac{8}{9} \end{aligned}$$

Finding Relative Frequencies

Surveys The results of a survey of students' music preferences are organized in this frequency table. What is the relative frequency of preference for rock music?

Use the frequency table to find the number of times rock music is chosen as the preference, and the total number of survey results.

$$\begin{aligned} \text{relative frequency} &= \frac{\text{frequency of rock music preference}}{\text{total frequency}} \\ &= \frac{10}{10 + 7 + 8 + 5 + 6 + 4} = \frac{10}{40} = \frac{1}{4} \end{aligned}$$

The relative frequency of preference for rock music is $\frac{1}{4}$.

Type of Music Preferred	Frequency
Rock	10
Hip Hop	7
Country	8
Classical	5
Alternative	6
Other	4

Calculating Probability by Using Relative Frequencies

A student conducts a probability experiment by tossing 3 coins one after the other. Using the results below, what is the probability that exactly two heads will occur in the next three tosses?

Coin Toss Result	HHH	HHT	HTT	HTH	THH	THT	TTT	TTH
Frequency	5	7	9	6	2	9	10	2

Step 1 Find the number of times a trial results in exactly two heads.

The possible results that show exactly two heads are HHT, HTH, and THH.

The frequency of these results is $7 + 6 + 2 = 15$.

Step 2 Find the total of all the frequencies.

$$5 + 7 + 9 + 6 + 2 + 9 + 10 + 2 = 50$$

Step 3 Find the relative frequency of a trial with exactly two heads.

$$\text{relative frequency} = \frac{\text{frequency of exactly two heads}}{\text{total of the frequencies}} = \frac{15}{50} = \frac{3}{10}$$

Based on the data collected, the probability that the next toss will be exactly two heads is $\frac{3}{10}$.

Practice

1) A camp counselor records the number of camp attendees who participate in the daily activities. The results are shown in the table. Find the relative frequency of each activity:

Total $12 + 18 + 13 = 43$

- a. Waterskiing = $\frac{12}{43}$ b. Hiking = $\frac{18}{43}$ c. Canoeing = $\frac{13}{43}$

Camp Activities

Activity	Number of People
Waterskiing	12
Hiking	18
Canoeing	13

2) A spinner has 3 equal sections colored red, blue, and green. A student conducts an experiment where she spins the spinner twice and records the results. The results are shown in the frequency table below:

- a. What is the probability of spinning red exactly once on the next two spins? ** Red exactly once* $\frac{5+2+4+2}{23} = \frac{13}{23}$
- b. What is the probability of spinning blue twice on the next two spins? ** Blue twice* $\frac{1}{23}$

Colors	RR	*RB	*RG	*BB	*BR	BG	GG	*GR	GB
Frequency	3	5	2	1	4	2	3	2	1

Total = 23

Favorite Season

Season	Number of Responses
Winter	7
Spring	13
Summer	19
Fall	11

Total 50

3) A student records the favorite season for 50 students. The results are shown in the table.

- a. What is the relative frequency of spring? $\frac{13}{50}$
- b. What is the relative frequency of winter? $\frac{7}{50}$
- c. If the table included the number of responses for only three of the seasons, could you determine the relative frequency of the remaining season? Explain.

Of course! Number of responses must add up to 50!

There is a total # of 50 students...