

Warm up: Simplify.

① $\frac{3x^4}{6} \div \frac{x^3y}{18}$

$\frac{3x^4}{6} \cdot \frac{18}{x^3y} = \frac{\cancel{3} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{2} \cdot 3 \cdot 3}{\cancel{2} \cdot 3 \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot y} = \frac{9x}{y}$

② $\frac{x^2-9}{3} \cdot \frac{3x-21}{x^2+6x+9}$

$\frac{(x-3)(x+3)}{3} \cdot \frac{3(x-7)}{(x+3)(x+3)} \parallel$

$\frac{(x-3)(x-7)}{x+3}$

③ $\frac{1}{10m} - \frac{3}{12m}$

$\frac{\cancel{3} \cdot 2 \cdot 1}{\cancel{3} \cdot 2 \cdot 2 \cdot 5 \cdot m} - \frac{5 \cdot \cancel{3}}{5 \cdot 2 \cdot 2 \cdot 3 \cdot m}$

COMMON DENOMINATOR

$\frac{6-15}{60m} = \frac{-9}{60m} = \frac{-3}{20m}$

④ $3 \cdot \frac{1}{2x+4} + \frac{-5(2x+4)}{3(2x+4)}$

$\frac{3-10x-20}{3(2x+4)}$

$\frac{2x+3 \cdot 2}{2(x+2)}$

$= \frac{-17-10x}{6(x+2)}$

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9.3. Advanced Algebra

Graphing Rational Functions

DATE: 4/7

Target 8D. Understand the relationship between a rational function and its graph.



Vertical Asymptotes and Point Discontinuity

Examples of rational functions:

$$f(x) = \frac{x}{x+3}$$

$-3+3=0$

$$g(x) = \frac{5}{x-6}$$

$6-6=0$

$$h(x) = \frac{x+4}{(x-1)(x+4)}$$

$1-1 \quad -4+4$

No denominator in a rational function can be zero because division by zero is not defined. In the examples above, the functions are not defined at $x = -3$, $x = 6$, and $x = 1$ and $x = -4$, respectively.

The graphs of rational functions may have breaks in *continuity*. This means that, unlike polynomial functions, which can be traced with a pencil never leaving the paper, not all rational functions are traceable. Breaks in continuity can appear as a *vertical asymptote* or as *point discontinuity (hole)*.

9-3 Graphing Rational Functions - Microsoft Word

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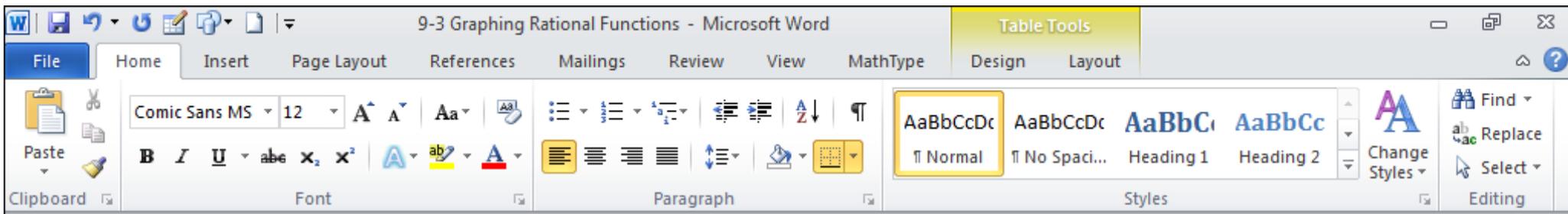
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Discontinuity

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.	For $f(x) = \frac{x}{x-3}$, $x = 3$ is a vertical asymptote. $\begin{array}{r} x-3=0 \\ +3+3 \\ \hline x=3 \checkmark \end{array}$	
Point Discontinuity	If the original function is undefined for $x = a$ but the rational expression in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.	$f(x) = \frac{(x+2)(x-1)}{x+2}$ $-2+2=0$ Can be simplified to $f(x) = x-1$. So, $x = -2$ represents a hole in the graph.	



Determine the equations of any **vertical asymptotes** and the values of x for any **holes** for each graph.

$$1. f(x) = \frac{x^2 - 1}{x^2 - 6x + 5} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x-5)} = \frac{x+1}{x-5}$$

$\begin{matrix} -1 \\ \wedge \\ -1 \cdot 1 \end{matrix}$ $\begin{matrix} 5 \\ \wedge \\ -1 \cdot -5 \end{matrix}$ $\textcircled{1}$

simplest form
↓

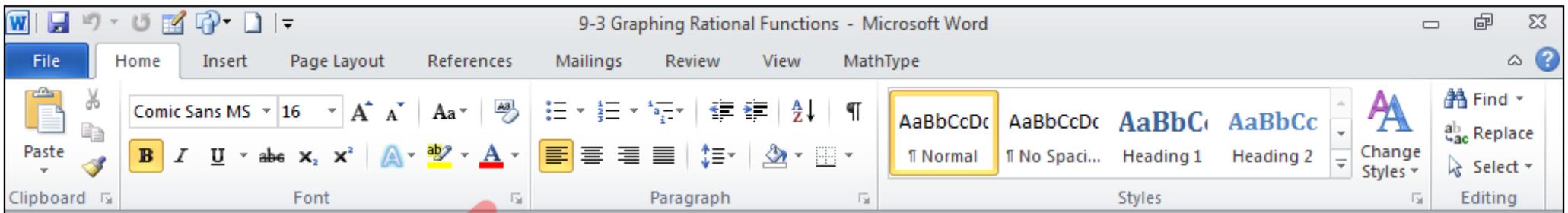
$x-5=0$
 $+5+5$
 $\textcircled{x=5}$

$$2. f(x) = \frac{x^2 - 4}{x^2 + 5x + 6} = \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}(x+3)} = \frac{x-2}{x+3}$$

$\begin{matrix} -4 \\ \wedge \\ -2 \cdot 2 \end{matrix}$ $\begin{matrix} 6 \\ \wedge \\ 2 \cdot 3 \end{matrix}$ $\begin{matrix} -2+2=0 \\ \textcircled{2} \end{matrix}$

$x+3=0$
 $-3-3$
 $\textcircled{x=-3}$

$$3. f(x) = \frac{x^2 - 9}{x + 3}$$



$$3. f(x) = \frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3} = x-3$$

$\begin{matrix} -9 \\ +3 \cdot -3 \end{matrix}$

(-3)

No vertical asymptotes because original $f(x)$ is not in simplest form.

$$4. f(x) = \frac{x-1}{x^2+4x-5} = \frac{x-1}{(x+5)(x-1)} = \frac{1}{x+5}$$

$\begin{matrix} -5 \\ +3 \cdot -1 \end{matrix}$

(1)

$$\begin{matrix} x+5=0 \\ -5 \quad -5 \\ \hline x=-5 \end{matrix}$$

$$5. f(x) = \frac{2}{x^2-5x+6} = \frac{2}{(x-2)(x-3)} \rightarrow \text{in simplest form}$$

$\begin{matrix} 6 \\ -2 \cdot -3 \end{matrix}$

No holes

$$\begin{matrix} x-2=0 \\ +2 \quad +2 \\ \hline x=2 \end{matrix}$$

$$\begin{matrix} x-3=0 \\ +3 \quad +3 \\ \hline x=3 \end{matrix}$$

Notice two vertical asymptotes