

10.4. Honors Geometry

DATE: 4/29

Target 9A. Know and apply the properties of tangents, secants, chords, and arcs/

Tangents and Secants

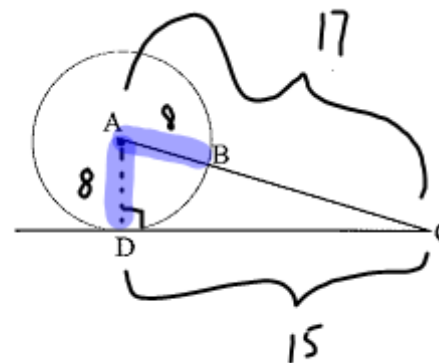
Postulate: A tangent line is perpendicular to the radius at a point of tangency (point of contact).

Example

Given: $DC = 15$ cm
 $AD = 8$ cm

Find BC.

$(8, 15, 17)$
 $17 - 8 = 9$
 $\therefore BC = 9 \checkmark$

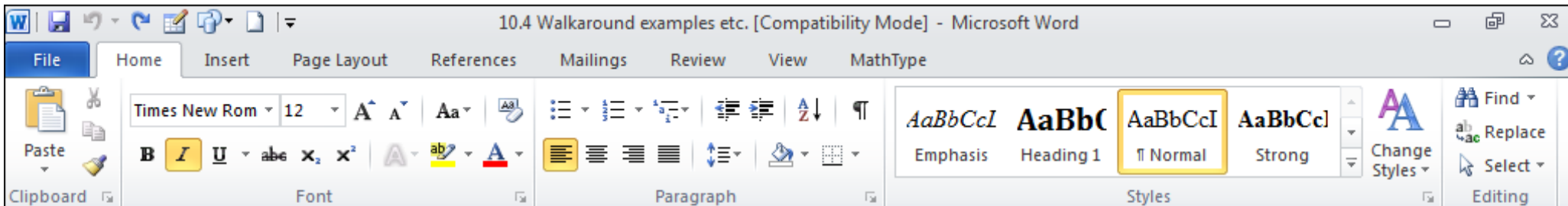


Postulate: If a line is perpendicular to a radius, then it is tangent to the circle.

Example

The radii of two concentric circles are 16 and 20. Find CD.

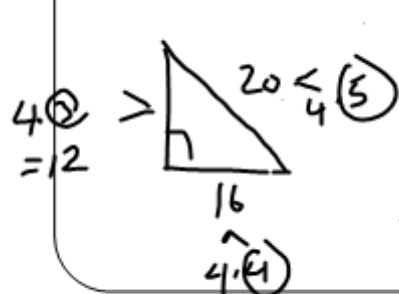




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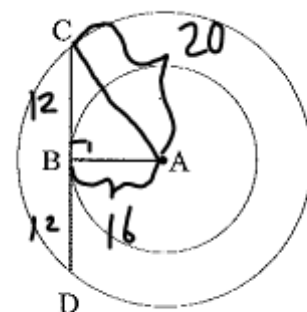
Example

The radii of two concentric circles are 16 and 20. Find CD.



(3, 4, 5)

$$\therefore CD = 2(12) = 24$$

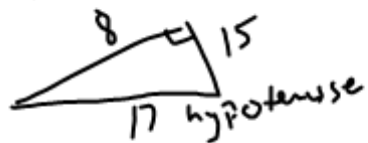


Tangent-Tangent Theorem: If two tangents are drawn to a circle from an external point, then those segments are congruent.

Example

Given: $AX = x^2 - 10x + 24$
 $BX = x^2 + 2x$
 $AY = 7.5x$

Find XY.



$$\overline{AX} \cong \overline{BX}$$

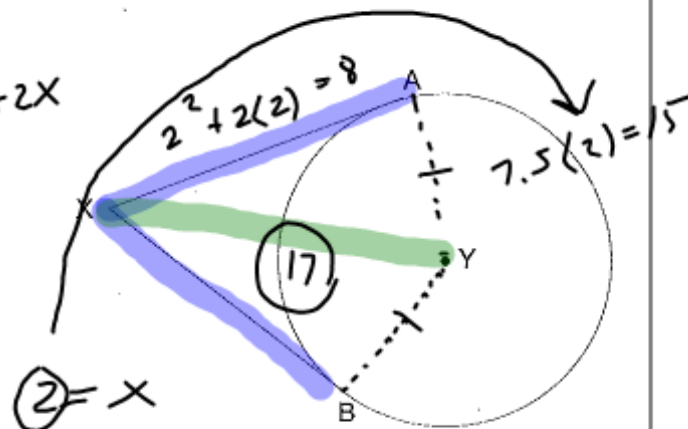
$$x^2 - 10x + 24 = x^2 + 2x$$

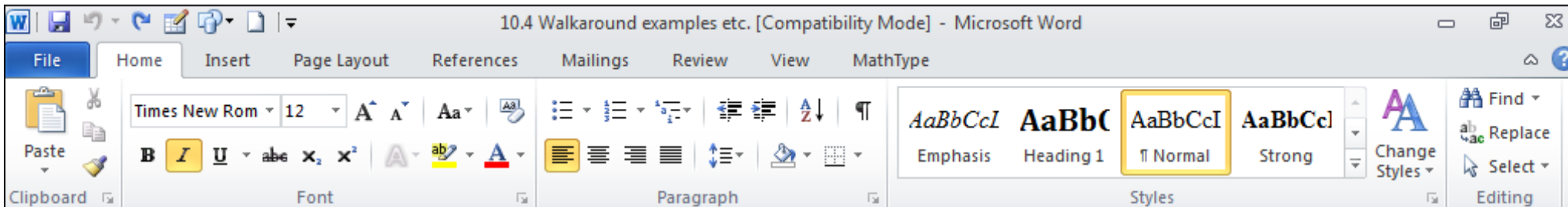
$$-10x + 24 = 2x$$

$$+10x \quad +10x$$

$$\frac{24}{12} = \frac{12x}{12}$$

$$2 = x$$





Common Tangent Procedure

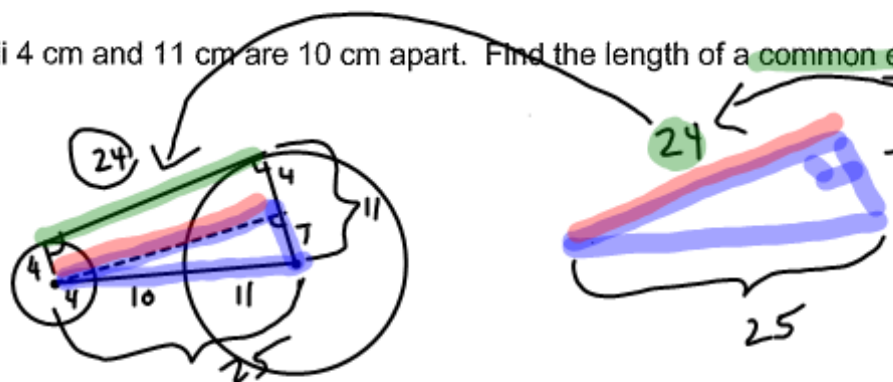
- 1) Draw the segment of centers.
- 2) Draw the radii to the points of contact.
- 3) Through the center of the smaller circle draw a line parallel to the common tangent.
- 4) Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.
- 5) Use the Pythagorean Theorem and properties of a rectangle.

↳ Follow this procedure when practicing

Common Tangent Problems

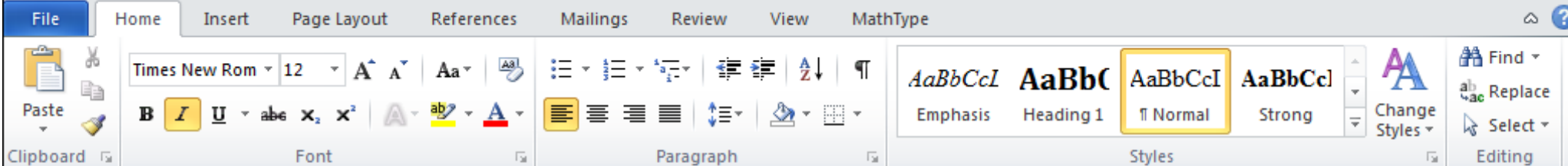
Example

1. Two circles with radii 4 cm and 11 cm are 10 cm apart. Find the length of a common external tangent.

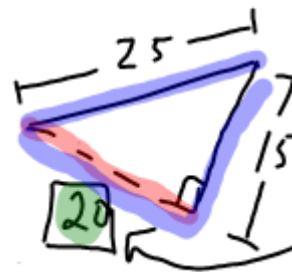
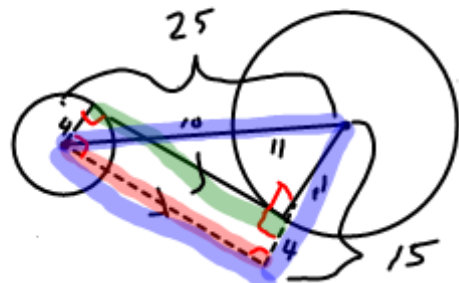


Since red seg is || to green

2. Two circles with radii 4 cm and 11 cm are 10 cm apart. Find the length of a common internal tangent.



2. Two circles with radii 4 cm and 11 cm are 10 cm apart. Find the length of a **common internal tangent**.

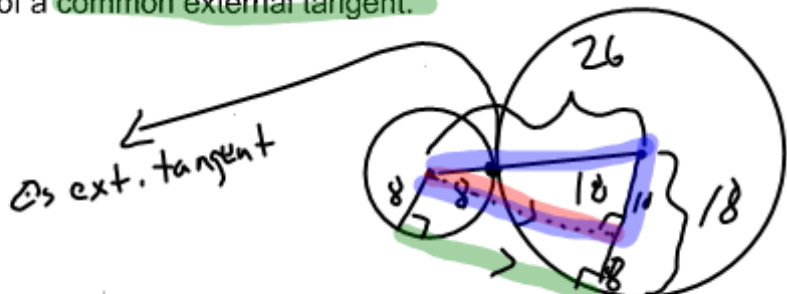


$$\begin{matrix} 25 & 15 \\ \hat{5} & \hat{5} \end{matrix}$$

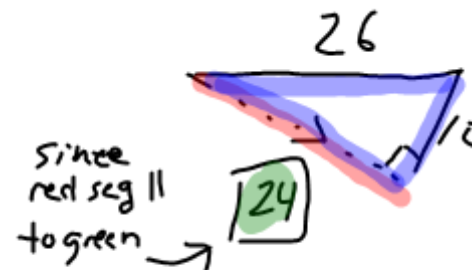
$$\begin{matrix} 20 \\ \wedge \\ 5 \cdot 4 \end{matrix}$$

since red seg. \parallel to green

3. A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a **common external tangent**.



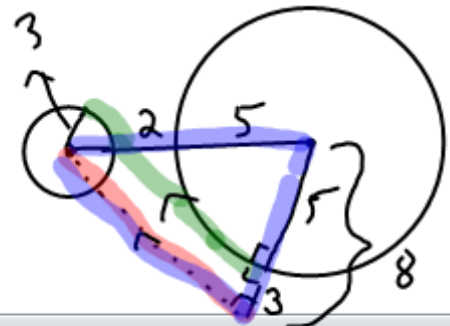
As ext. tangent



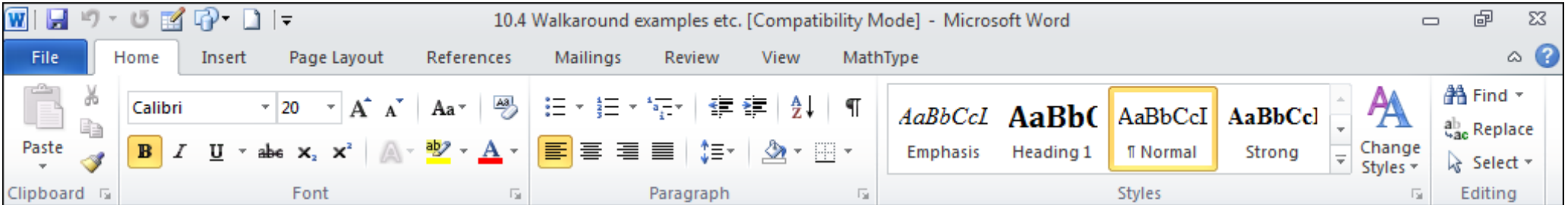
since red seg \parallel to green

$$\begin{matrix} 26^2 \\ 10^2 + b^2 = 26^2 \\ 100 + b^2 = 676 \\ -100 & -100 \\ \hline b^2 = 576 \\ b = 24 \end{matrix}$$

4. The centers of two circles with radii of 3 in and 5 in are 10 in apart. Find the length of a **common internal tangent**.



since red seg. \parallel to green,



Utilize tangent-tangent Theorem

Walk-Around Problems

Example

1. Given: $AB = 16$
 $BC = 12$
 $CD = 20$

Find AD.

$AD = 24 - x + x$

$\therefore AD = 24 \checkmark$

Easy Method

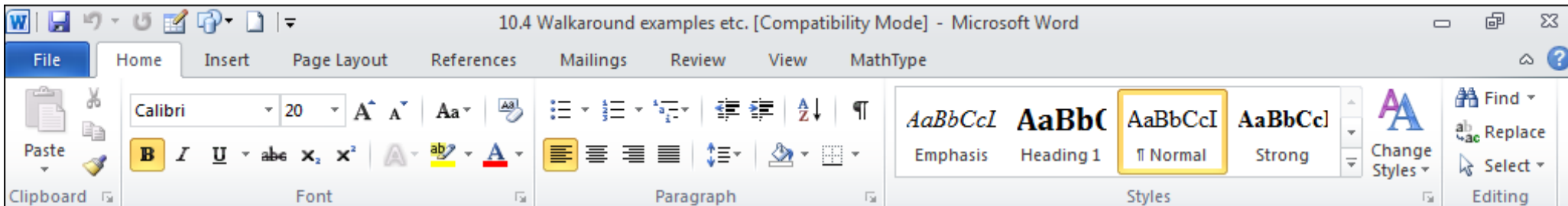
$$AB + DC = BC + AD$$

$$16 + 20 = 12 + x$$

$$\begin{array}{r} 36 \\ -12 \\ \hline 24 = x \end{array} \therefore AD = 24$$

$$20 - (-4 + x) = 20 + 4 - x = 24 - x$$

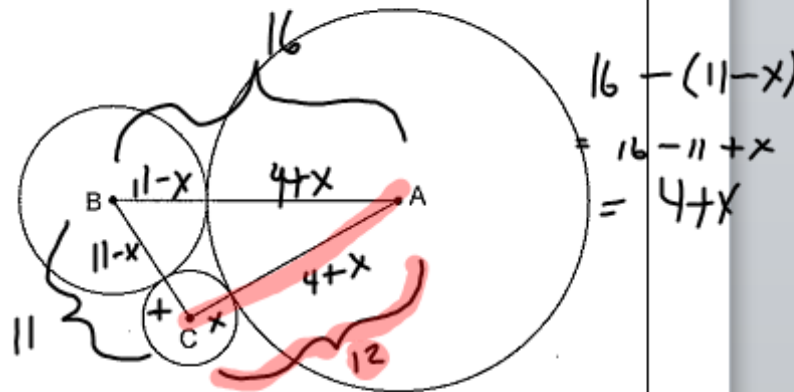
$$12 - (16 - x) = 12 - 16 + x = -4 + x$$



2. Given: $AB = 16$
 $BC = 11$
 $AC = 12$

Find the radius of circle A, B, and C.

Choose a \odot and label the radius of that \odot with x (or any other variable!)



$$\therefore 12 = x + 4 + x$$

$$12 = 2x + 4$$

$$8 = 2x$$

$$\boxed{4 = x}$$

Radius of $\odot C$ is $\boxed{4}$

— 11 — $\odot B$ is $11 - x = 11 - 4 = \boxed{7}$

— 11 — $\odot A$ is $4 + x = 4 + 4 = \boxed{8}$