9.4. Advanced Algebra

DATE: 4/14

Direct, Joint, & Inverse Variation

Target 8A. Classify an equation as direct, inverse, or joint variation
Target 8B. Create equations to solve direct, inverse, or joint variation problems.

Direct Variation: y varies directly as x if there is some nonzero constant k such that $y = kx$, where k is called the ***constant of variation***.

$\frac{y}{x} = \frac{kx}{x} \rightarrow \frac{y}{x} = k$

1. If y varies directly as x and y = 12 when x = -3, find y when x = 16.

$\frac{y_1}{x_1} = \frac{y_2}{x_2} \Rightarrow \frac{12}{-3} = \frac{y}{16} \Rightarrow 12 \cdot 16 = -3 \cdot y \Rightarrow \frac{192}{-3} = \frac{-3y}{-3}$ $\quad -64 = y$

2. If y varies directly as x and y = -15 when x = 5, find y when x = 3.

$\frac{y_1}{x_1} = \frac{y_2}{x_2} \Rightarrow \frac{-15}{5} = \frac{y}{3} \Rightarrow -15 \cdot 3 = 5 \cdot y \Rightarrow \frac{-45}{5} = \frac{5y}{5} \Rightarrow -9 = y$

3. If y varies directly as x and y = 15 when x = 3, find y when x = 12.

$\frac{y_1}{x_2} = \frac{y_2}{x_2} \Rightarrow \frac{15}{3} = \frac{y}{12} \Rightarrow 15 \cdot 12 = 3 \cdot y \Rightarrow \frac{180}{3} = \frac{3y}{3} \Rightarrow 60 = y$

Joint Variation: y varies jointly as x and z if there is some number k such that $y = kxz$ where $k \neq 0$.

$\frac{y}{xz} = \frac{kxz}{xz} \rightarrow \frac{y}{xz} = k$

4. Suppose y varies jointly as x and z. Find y when x = 8 and z = 3, if y = 16 when z = 2 and x = 5.

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2} \Rightarrow \frac{16}{5 \cdot 2} = \frac{y}{8 \cdot 3} \Rightarrow \frac{16}{10} = \frac{y}{24} \Rightarrow 16 \cdot 24 = 10 \cdot y$

$\Rightarrow \frac{384}{10} = \frac{10y}{10}$

$38.4 = y$

5. Suppose y varies jointly as x and z. Find y when x = 10 and z = 5, if y = 2 when z = 8 and x = 3.

$\frac{y_1}{x_1 z_1} = \frac{y_2}{x_2 z_2} \Rightarrow \frac{2}{3 \cdot 8} = \frac{y}{10 \cdot 5} \Rightarrow \frac{2}{24} = \frac{y}{50} \Rightarrow 2 \cdot 50 = 24y$

$\Rightarrow \frac{100}{24} = \frac{24y}{24}$

$4.1\overline{6} = y$

Inverse Variation: y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$ where $x \neq 0$ and $y \neq 0$.

6. If r varies inversely as t and r = 18 when t = -3, find r when t = -11.

$r_1 t_1 = r_2 t_2 \Rightarrow 18(-3) = r\cdot(-11)$

$\Rightarrow \frac{-54}{-11} = \frac{-11r}{-11}$

$\frac{54}{11} = r$

7. If y varies inversely as x and y = -14 when x = 12, find x when y = 21.

$x_1 y_1 = x_2 y_2 \Rightarrow (12)(-14) = x(21)$

$\Rightarrow \frac{-168}{21} = \frac{21x}{21}$

$-8 = x$

State whether each equation represents direct, joint, or inverse variation. Then name the constant of variation (k).

8. $ab = 20$

9. $\frac{y}{x} = -0.5$

10. $A = \frac{1}{2}bh$

7. If y varies inversely as x and y = -14 when x = 12, find x when y = 21.

State whether each equation represents direct, joint, or inverse variation. Then name the constant of variation (k).

8. $ab = 20$

Inverse

$k = 20$

9. $\frac{y}{x} = -0.5$

Direct

$k = -0.5$

10. $A = \frac{1}{2}bh$

Joint

$k = \frac{1}{2}$

11. $V = \pi r^2 h$

Joint

$k = \pi$

12. $c = 12m$

Direct

$k = 12$

13. $p = \frac{4}{s}$

Inverse

$k = 4$

14. $y = 2rt$

Joint

$k = 2$

15. $R = \frac{8}{w}$

Inverse

$k = 8$

16. $\frac{a}{b} = \frac{1}{3}$

Direct, $k = \frac{1}{3}$