13.4 Honors Advanced Algebra DATE: ________________

Compound Events

Compound event: an event that is made up of two or more events.

Independent events: events that have no effect on the outcome of each other. If two events, A and B, are said to be independent, they follow this formula: $P(A \text{ and } B) = P(A \cap B) = P(A) \bullet P(B)$

Dependent events: events that affect the outcome of each other.

Mutually exclusive events: events that cannot happen at the same time. If events A and B are mutually exclusive, they must follow this formula: $P(A \cap B) = 0, \text{ and } P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

Overlapping events: events that have outcomes in common. If events A and B are overlapping, they follow this formula:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Identifying Independent and Dependent Events

Are the outcomes of each trial independent or dependent events?

A **Choose a number tile from 12 tiles. Then spin a spinner.**

The choice of number tile does not affect the spinner result. The events are independant.

B **Pick one card from a set of 15 sequentially numbered cards. Then, without replacing the card, pick another card.**

The first card chosen affects possible outcomes of the second pick, so the events are dependant.

Finding the Probability of Independent Events

A desk drawer contains 5 red pens, 6 blue pens, 3 black pens, 24 silver paper clips, and 16 white paper clips. If you select a pen and a paper clip from the drawer without looking, what is the probability that you select a blue pen and a white paper clip?

Step 1 Let A = selecting a blue pen. Find the probability of A.

$P(A) = \frac{6}{14} = \frac{3}{7}$ $5+6+3=14=$ Total

Step 2 Let B = selecting a white paper clip. Find the probability of B.

$P(B) = \frac{16}{40} = \frac{2}{5}$ $24+16=40=$ Total

Step 3 Find $P(A \text{ and } B)$.

$P(A \text{ and } B) = P(A \cap B) = P(A) \cdot P(B) = \frac{3}{7} \cdot \frac{2}{5} = \frac{6}{35} \approx 0.171$ or 17.1%

The probability that you select a blue pen and a white paper clip is about 17.1%.

Using Probabilities to Test for Independence

Manufacturing **A factory foreman determines that on any given day there is a 15% chance that Machine A will malfunction, a 45% chance that Machine B will malfunction, and a 6.75% chance that both machines will malfunction. Are the events "Machine A malfunctions" and "Machine B malfunctions" independent events? Explain.**

A **What are the zeros of the function?**

Let A = Machine A malfunctions, and
B = Machine B malfunctions.

Step 1 Write each probability as a decimal.

$P(A) = 15\% = 0.15$

$P(B) = 45\% = 0.45$

$P(A \cap B) = 6.75\% = 0.675$

Step 2 Check whether the relationship $P(A \cap B) = P(A) \cdot P(B)$ is true.

$P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$

$0.0675 = 0.15 \cdot 0.45$

$0.0675 = 0.0675$ ✓ True

Because $P(A \cap B) = P(A) \cdot P(B)$, the events "Machine A malfunctions" and "Machine B malfunctions" are independent.

Finding the Probability of Mutually Exclusive Events

Athletics **Student athletes at a local high school may participate in only one sport each season. During the fall season, 28% of student athletes play basketball and 24% are on the swim team. What is the probability that a randomly selected student athlete plays basketball or is on the swim team?**

Because athletes participate in only one sport each season, the events are mutually exclusive. Use the formula $P(A \cup B) = P(A) + P(B)$.

P(basketball or swim team) $= P(\text{basketball}) + P(\text{swim team})$

$\cup$

$= 28\% + 24\% = 52\%$

The probability of an athlete either playing basketball or being on the swim team is 52%.

Finding Probabilities of Overlapping Events

What is the probability of rolling either an even number or a multiple of 3 when rolling a standard number cube?

Know	Need	Plan
You are rolling a standard number cube. The events are overlapping events because 6 is both even and a multiple of 3.	You need the probability of rolling an even number and the probability of rolling a multiple of 3.	Find the probabilities and use the formula for probabilities of overlapping events.

$S = \{1, 2, 3, 4, 5, 6\}$

$P(\text{even} \underset{\cup}{\text{or}} \text{ multiple of } 3) = P(\text{even}) + P(\text{multiple of } 3) - P(\text{even} \underset{\cap}{\text{and}} \text{ multiple of } 3)$

$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$

$= \frac{4}{6}$

$= 2/3$

The probability of rolling an even or a multiple of 3 is

Practice

1) Determine whether the outcomes of each trial are independent or dependent events?

a. A card is randomly chosen from a standard deck of cards, thenreplaced; another card is chosen at random.

b. Asking a student's age, and asking what year he or she expects to graduate.

a) Independent b) Dependent

2) You spin a spinner (circle divided into 4 equal sections labeled 1, 2, 3, and 4), without looking. You also choose a tile from a set of tiles numbered from 1 to 10. Find each probability. These are independent events

a. P(spinner lands on an odd number, and you choose an even number)

b. P(spinner lands on a number less than 4, and you choose a 9 or 10)

$A = \{1, 3\}$ $B = \{2, 4, 6, 8, 10\}$ $C = \{1, 2, 3\}$ $D = \{9, 10\}$

a) $P(A \cap B) = P(A) \cdot P(B) = \frac{2}{4} \cdot \frac{5}{10} = \frac{10}{40} = \frac{1}{4}$

b) $P(C \cap B) = P(C) \cdot P(B) = \frac{3}{4} \cdot \frac{2}{10} = \frac{6}{40} = \frac{3}{20}$

3) Use the given probabilities below to determine whether events A and B are independent.

a. $P(A) = 0.3,\ \ P(B) = 0.4,\ \ P(A \text{ and } B) = 0.7$ ∴ Not independent

$P(A \cap B) \overset{?}{=} P(A) \cdot P(B) \iff 0.7 \neq 0.3 \cdot 0.4 = 0.12$

b. $P(A) = \frac{2}{3},\ \ P(B) = \frac{4}{5},\ \ P(A \text{ and } B) = \frac{8}{15}$

$P(A \cap B) = P(A) \cdot P(B)$

$\frac{8}{15} = \frac{2}{3} \cdot \frac{4}{5}$

$\frac{8}{15} = \frac{8}{15}$ ✓ ∴ Yes, independent.

4) A bag contains 3 blue chips, 6 black chips, 2 green chips, and 4 red chips. Find the probability of each.

$\frac{3 + 6 + 2 + 4}{15}$

a. P(green chip or red chip) A = green chip, B = red chip

$P(A \cup B) = P(A) + P(B) = \frac{2}{15} + \frac{4}{15} = \frac{6}{15} = \frac{2}{5}$

b. P(blue, black, or red chip)

$P(C \cup D \cup E) = P(C) + P(D) + P(E) = \frac{3}{15} + \frac{6}{15} + \frac{4}{15} = 13/15$

5) A set of cards contains four suits (red, blue, green, and yellow). In each suit there are cards, numbered from 1 to 10. Find each probability. So 10 red, 10 blue, 10 green, and 10 yellow cards

a. P(green or yellow card, or card numbered 1) $= \frac{11}{20}$

b. P(red or blue card, or card less than 6) $= 3/4$

overlap

$P(A \cup B) = P(A) + P(B) - P(A \cup B) = 20/40 + \frac{4}{40} - \frac{2}{40} = \frac{22}{40} = \frac{11}{20}$

$P(C \cup D) = P(C) + P(D) - P(C \cap D) = \frac{20}{40} + \frac{20}{40} - \frac{10}{40} = \frac{30}{40} = \frac{3}{4}$

6) Suppose A and B are independent events. What is P(A∩B) if P(A) = 50% and P(B) = 25%?

Independent follows $P(A \cap B) = P(A) \cdot P(B) = 0.50 \cdot 0.25 = 0.125$ or $12.5\% = P(A \cap B)$

7) Suppose A and B are mutually exclusive events. What is P(A∪B) if P(A) = 0.6 and P(B) = 0.25?

8) Suppose A and B are overlapping events. What is P(A∪B) if P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{2}$, and P(A∩B) = $\frac{1}{5}$?

mutually exclusive: $P(A \cup B) = P(A) + P(B)$

$= 0.6 + 0.25$

$= 0.85$ or 85%

8) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{1}{3} + \frac{1}{2} - \frac{1}{5} = \frac{1}{3} \cdot \frac{10}{10} + \frac{1}{2} \cdot \frac{15}{15} - \frac{1}{5} \cdot \frac{6}{6}$

$= \frac{10}{30} + \frac{15}{30} - \frac{6}{30}$

$= \frac{25}{30} - \frac{6}{30} = \frac{19}{30}$

Overlapping events