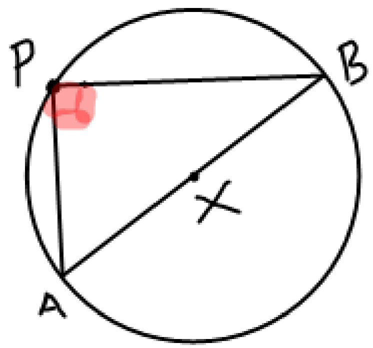
Useful Angle-Arc Theorems

5/10/13

① An $\angle$ inscribed in a semicircle is a right $\angle$.

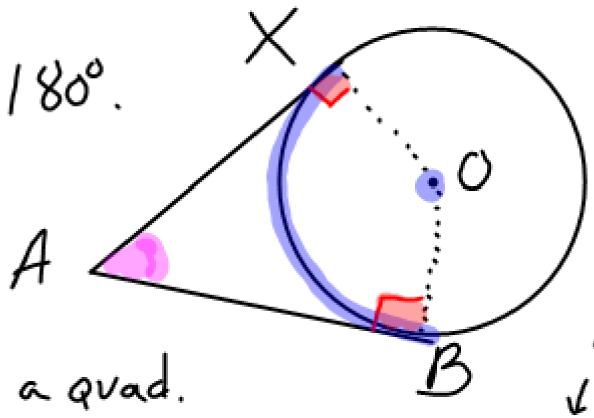


Pf: Since $\angle APB$ is an inscribed $\angle$, we must show $\angle APB$ is 90°.

$\angle APB$ is on ⊙ $\Big\}$ $\angle APB = \frac{1}{2} m\widehat{AB} = \frac{1}{2}(180^\circ) = 90^\circ$ ✓

② The sum of measures of tangent–tangent $\angle$ and its minor arc is 180°.

Must show $\angle A + \underbrace{m\widehat{XB}}_{\text{(minor)}} = 180^\circ$.



Pf: We know $\overline{AX}$ and $\overline{AB}$ are tangent to ⊙O. Then $\angle AXO \cong \angle ABO = 90^\circ$. AXOB is a quad.

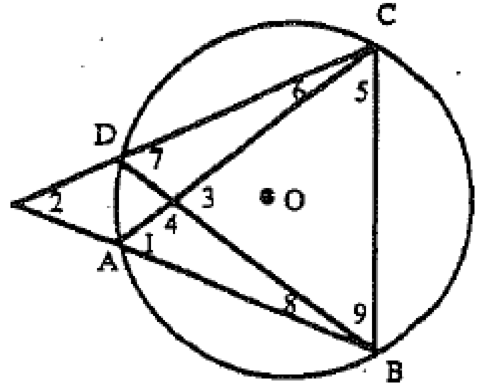
So $\angle A + \angle AXO + \angle ABO + \angle XOB = 360$ $\longrightarrow$ Recall $S_i = 180(\overset{\text{4 sides}}{4}-2) = 360$

$\angle A + 90 + 90 + \angle XOB = 360$

$\angle A + 180 + \angle XOB = 360 \Rightarrow \angle A + \underbrace{\angle XOB}_{\text{AT center}} = 180 \Rightarrow \angle A + m\widehat{XB} = 180^\circ$ ✓

Name Key

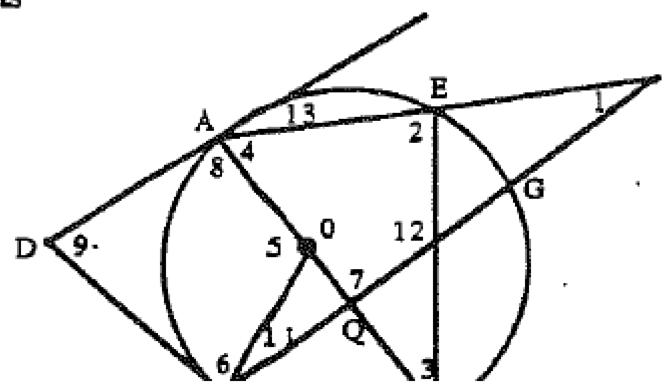
Given: m∠BDC = 50
m $\overset{\frown}{DA}$=30
m $\overset{\frown}{CD}$=90
⊙O



m∠1 = 50
m∠2 = 35
m∠3 = 65
m∠4 = 115
m∠5 = 70
m∠6 = 15
m∠7 = 50
m∠8 = 15
m∠9 = 45

Given: $\overrightarrow{DA}$ and $\overrightarrow{DC}$ are tangents
m $\overset{\frown}{EA}$ = 86
m $\overset{\frown}{GE}$ = 56
m $\overset{\frown}{AC}$ = 110

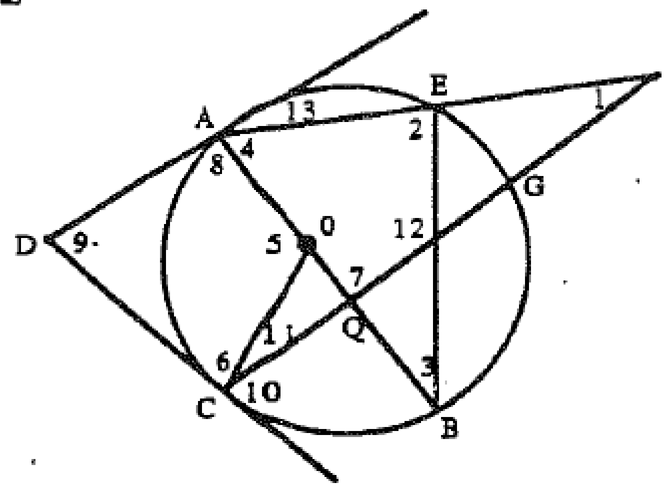
⊙O



m∠1 = ________
m∠2 = ________
m∠3 = ________
m∠4 = ________
m∠5 = ________
m∠6 = ________
m∠7 = ________
m∠8 = ________
m∠9 = ________
m∠10

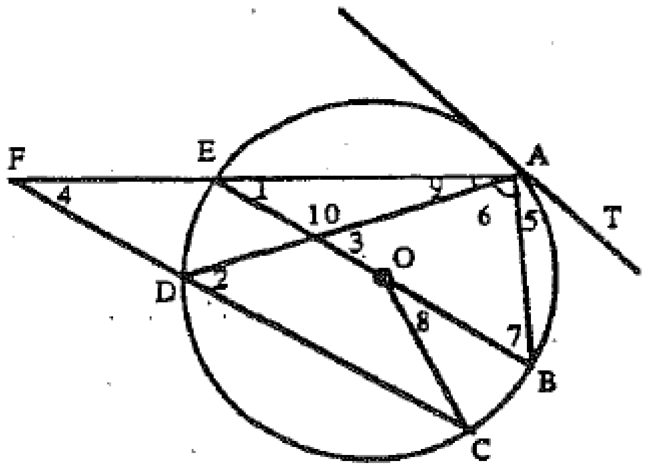
Given: $\overrightarrow{DA}$ and $\overrightarrow{DC}$ are tangents
m$\widehat{EA}$ = 86
m$\widehat{GE}$ = 56
m$\widehat{AC}$ = 110

⊙O



m∠1 = 27
m∠2 = 90
m∠3 = 43
m∠4 = 47
m∠5 = 110
m∠6 = 90
m∠7 = 106
m∠8 = 90
m∠9 = 70
m∠10 = 54
m∠11 = 36
m∠12 = 117
m∠13 = 43

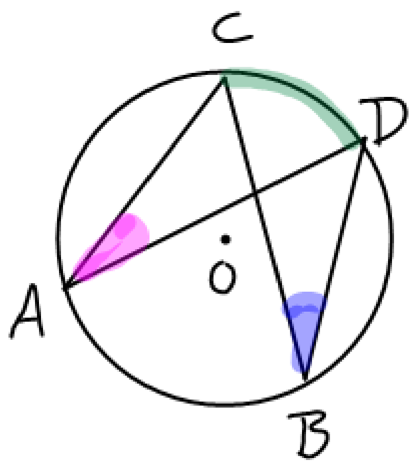
Given: ⊙O, $\overleftrightarrow{AT}$ is a tangent
m$\widehat{AB}$ = 80
m$\widehat{BC}$ = 20
m$\widehat{DE}$ = 50



m∠1 = 40
m∠2 = 50
m∠3 = 65
m∠4 = 25
m∠5 = 40
m∠6 = 65
m∠7 = 50
m∠8 = 20
m∠9 = 25
m∠10 = 115

(3) Given: $\angle A$, $\angle B$ are inscribed $\angle$s intercepting $\overset{\frown}{CD}$. (same arc)

Conclusion: $\underline{\angle A \cong \angle B}$.

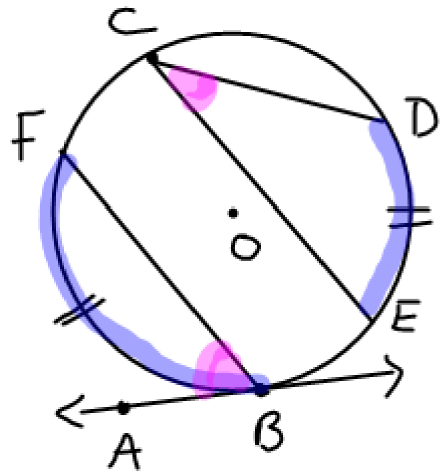


Pf: Easy! Think vertex location. Then transitive property.

(4) Given: $\overleftrightarrow{AB}$ tangent at pt. B

$\overset{\frown}{AB} \cong \overset{\frown}{DE}$ (congruent arcs)

Conclusion: $\underline{\angle ABF \cong \angle C}$



Pf: Can you prove it?