

Comparing Conditional Probabilities

Note: Conditional probabilities are usually not reversible. $P(A|B) \neq P(B|A)$.

3. In a survey of pet owners, 45% own a dog, 27% own a cat, and 12% own both a dog and a cat. What is the conditional probability that a dog owner also owns a cat? What is the conditional probability that a cat owner also owns a dog?

4. In the same survey as the one in Problem 3 showed that 5% of the pet owners own a dog, a cat, and at least one other type of pet.
 - a. What is the conditional probability that a pet owner owns a cat and some other type of pet, given that he or she owns a dog?

 - b. What is the conditional probability that a pet owner owns a dog and some other type of pet, given that he or she owns a cat?

5. You roll two standard number cubes.
 - a. What is the probability that the sum is even, given that at least one number cube shows a 2?

 - b. What is the probability that at least one number shows a 2, given that the sum is even?

Conditional Probability

Because $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$, then $P(A \text{ and } B) = P(A) \cdot P(B|A)$. You can use this form of the conditional rule when you know the conditional. You can also combine conditional probabilities to find the probability of an event that can happen in more than one way.

Using a Tree Diagram

6. A college reported the following based on their graduation data.
- 70% of freshman had attended public schools
 - 60% of freshman who had attended public schools graduated within 5 years
 - 80% of other freshman graduated within 5 years

What percentage of freshman graduated within 5 years?

7. A soccer team wins 65% of its games on muddy fields and 30% of its games on dry fields. The probability of the field's being muddy for their next game is 70%. What is the probability the team will win their next game?

8. Suppose that your softball team has a 75% chance of making the playoffs. Your cross-town rivals have an 80% chance of making the playoffs. Teams that make the playoffs have a 25% chance of making the finals. Use this information to find the following probabilities.

- $P(\text{your team makes the playoffs and the finals})$
- $P(\text{cross – town rivals make the playoffs and the finals})$

Conditional Probability & Independence

If $P(B|A) = P(B)$, then the statement $P(A \text{ and } B) = P(A) \cdot P(B|A)$ simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is true if and only if A and B are independent events. This leads to another way to test events for independence:
Two events A and B are independent if and only if $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

Using Conditional Probability to Test for Independence

9. The two-way frequency table shows the number of juniors and seniors in a school who hold a part-time job. A teacher selects one at random. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

	Part-Time Job	No Part-Time Job	Totals
Juniors	71	104	175
Seniors	95	86	181
Totals	166	190	356

- a. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

Event A: The selected student is a senior.

Event B: The selected student has a part-time job.

- b. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

Event A: The selected student is a junior.

Event B: The selected student does not have a part-time job.