Honors Advanced Algebra

Name: ______________________________

Period: ________ Date: ___________

13.6 Conditional Probability

Target 8.C. Explain conditional probability and independence using everyday examples of events based on the context of the problem.

Target 8.D. Compute probabilities of independent, dependent and compound events and use these to interpret data.

Conditional probabilities can be found using a formula.

Conditional Probability Formula
For any two events A and B, the probability of B occurring, given that event A has occurred, is $P(B\|A) = \frac{P(A \text{ and } B)}{P(A)}$, where $P(A) \neq 0$.

Using Conditional Probabilities

1. In a study designed to test the effectiveness of a new drug, half of the volunteers received the drug. The other half of the volunteers received a placebo, a tablet or pill containing no medication. The probability of a volunteer receiving the drug and getting well was 45%. What is the probability of someone getting well given that he receives the drug?

$\cap$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.45}{0.90} = 90\%$$

P(A) = half volunteers received drug = 50%

$P(A \cap B) = 45\%$ = prob volunteer receiving drug and getting well

2. Suppose that 62% of children are given a weekly allowance, and 38% of children do household chores to earn an allowance. What is the probability that a child does household chores, given that the child gets an allowance?

P(A) $\quad$ $P(A \cap B)$

do chores and earn allowance

P(child does chores | child gets allowance)

$= \dfrac{\text{P(child does chores and gets allowance)}}{\text{P(child gets allowance)}}$

$$= \frac{0.38}{0.62} = \frac{38}{62} = \boxed{\frac{19}{31}}$$

Comparing Conditional Probabilities

Note: Conditional probabilities are usually not reversible. $P(A|B) \neq P(B|A)$.

3. In a survey of pet owners, $\overset{P(A)}{\text{45\% own a dog}}$, $\overset{P(B)}{\text{27\% own a cat}}$, and $\overset{P(A \cap B)}{\text{12\% own both a dog and a cat}}$. What is the conditional probability that a dog owner also owns a cat? What is the conditional probability that a cat owner also owns a dog?

$$P(\overset{B}{\text{owns cat}} \mid \overset{A}{\text{already owns dog}}) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.45} = \boxed{0.267}$$

$$P(\overset{A}{\text{owns dog}} \mid \overset{B}{\text{already own cat}}) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.27} = \boxed{0.440}$$

Note $P(B|A) \neq P(A|B)$.

4. In the same survey as the one in Problem 3 showed that 5% of the pet owners own a dog a cat, and at least one other type of pet.
   a. What is the conditional probability that a pet owner owns a cat and some other type of pet, given that he or she owns a dog?

$$P(\text{owns cat} \overset{\text{and}}{/} \text{other pet} \mid \text{dog}) = \frac{0.05}{0.45} = \frac{5}{45} = \boxed{\frac{1}{9}}$$

   b. What is the conditional probability that a pet owner owns a dog and some other type of pet, given that he or she owns a cat?

$$P(\text{owns dog} \overset{\text{and}}{/} \text{other pet} \mid \text{cat}) = \frac{0.05}{0.27} = \boxed{\frac{5}{27}}$$

(11)

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

5. You roll two standard number cubes.
   a. What is the probability that the sum is even, given that at least one number cube shows a 2?

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{5/36}{11/36} = \frac{5}{36} \cdot \frac{36}{11} = \boxed{\frac{5}{11}}$$

   b. What is the probability that at least one number shows a 2, given that the sum is even?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{5/36}{18/36} = \boxed{\frac{5}{18}}$$

B

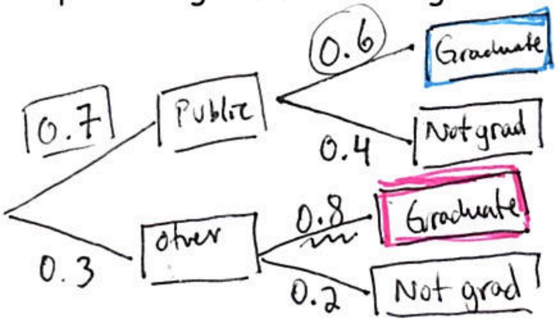
1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability
Because $P(B\|A) = \frac{P(A\ and\ B)}{P(A)}$, then $P(A\ and\ B) = P(A) \cdot P(B\|A)$. You can use this form of the conditional rule when you know the conditional. You can also combine conditional probabilities to find the probability of an event that can happen in more than one way.

Using a Tree Diagram

6. A college reported the following based on their graduation data.
   - 70% of freshman had attended public schools
   - 60% of freshman who had attended public schools graduated within 5 years
   - 80% of other freshman graduated within 5 years

   What percentage of freshman graduated within 5 years?



① $P(\text{Public} \cap \text{Graduate})$
$= P(\text{Grad} \mid \text{Public}) \cdot P(\text{Public})$
$= 0.6 \cdot 0.7$
$= 0.42$

② $P(\text{Graduate} \cap \text{Other})$
$= P(\text{Graduate} \mid \text{other}) \cdot P(\text{other})$
$= 0.8 \cdot 0.3 = 0.24$

$\therefore P(\text{Grad}) = P(\text{Public} \cap \text{Grad}) + P(\text{other} \cap \text{Grad}) = 0.42 + 0.24 = \boxed{66\%}$

7. A soccer team wins 65% of its games on muddy fields and 30% of its games on dry fields. The probability of the field's being muddy for their next game is 70%. What is the probability the team will win their next game? $P(\text{win}) = ?$ → So 30% being dry...

$P(\text{win}) = P(\text{Win} \mid \text{muddy}) \cdot P(\text{muddy}) + P(\text{win} \mid \text{dry}) \cdot P(\text{dry})$
$= 0.65 \cdot 0.70 + 0.30 \cdot 0.30$
$= 0.455 + 0.9 = \boxed{0.545}$

8. Suppose that your softball team has a 75% chance of making the playoffs. Your cross-town rivals have an 80% chance of making the playoffs. Teams that make the playoffs have a 25% chance of making the finals. Use this information to find the following probabilities.

   a. $P(\underbrace{your\ team\ makes\ the\ playoffs}_{A}\ \underbrace{and}_{\cap}\ \underbrace{the\ finals}_{B})$

   b. $P(\underbrace{cross-town\ rivals\ make\ the\ playoffs}_{A}\ \underbrace{and}_{\cap}\ \underbrace{the\ finals}_{B})$

a) $P(A \cap B) = P(A) \cdot P(B/A) = 0.75 \cdot 0.25 = 18.75\%$.

b) $P(A \cap B) = P(A) \cdot P(B/A) = 0.80 \cdot 0.25 = 20\%$.

Conditional Probability & Independence

If $P(B|A) = P(B)$, then the statement $P(A \; and \; B) = P(A) \cdot P(B|A)$ simplifies to $P(A \; and \; B) = P(A) \cdot P(B)$, which is true if and only if A and B are independent events. This leads to another way to test events for independence:

Two events A and B are independent if and only if $\boxed{P(A|B)} = \boxed{P(A)}$ and $\boxed{P(B|A)} = \boxed{P(B)}$.

Note! $P(B|A) = \frac{P(A \cap B)}{P(A)}$. If A, B indep.

$P(B|A) = \frac{P(A) \cdot P(B)}{P(A)} = P(B)$

Using Conditional Probability to Test for Independence

9. The two-way frequency table shows the number of juniors and seniors in a school who hold a part-time job. A teacher selects one at random. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

	Part-Time Job	No Part-Time Job	Totals
Juniors	71	104	175
Seniors	95	86	181
Totals	166	190	356

a. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

Event A: The selected student is a senior.

Event B: The selected student has a part-time job.

$\boxed{P(A|B)} = P(\text{senior} \mid \text{partime job}) = \boxed{95/166} \approx 0.57$

$\boxed{P(A)} = P(\text{senior}) = \frac{95+86}{356} = \boxed{\frac{181}{356}} \approx 0.51$

$\boxed{P(B|A)} = P(\text{partime job} \mid \text{senior}) = \boxed{\frac{95}{181}} \approx 0.52$

$\boxed{P(B)} = P(\text{part-time job}) = \frac{166}{356} = \boxed{\frac{83}{178}} \approx 0.47$

Since $P(A|B) \neq P(A)$ & $P(B|A) \neq P(B)$, A, B not independent!

b. Consider events A and B below. Are the events independent? Use probabilities to determine your answer.

Event A: The selected student is a junior.

Event B: The selected student does not have a part-time job.

You try it...

Ans: Events are not independent