UNIT 1

Notes

Introduction

In this unit, students begin by applying the properties of real numbers to expressions, equalities, and inequalities, including absolute value inequalities and compound inequalities. Throughout the unit, students explore the relationship between linear equations and their graphs.

These explorations include modeling data with scatter plots and lines of regression, as well as linear programming and solving systems of equations. The unit concludes with instruction about operations on matrices and using matrices to solve systems of equations.

Assessment Options

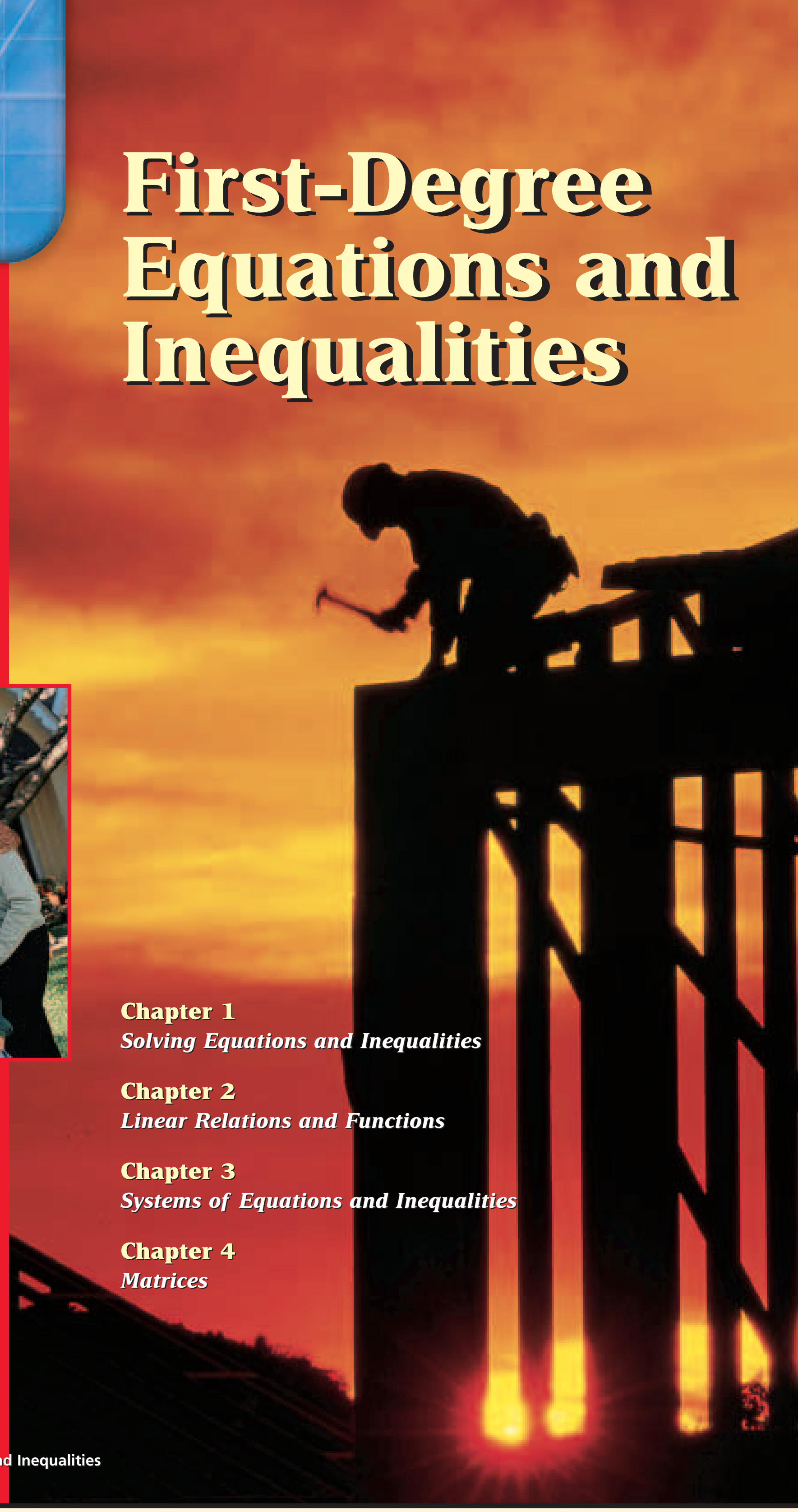
Unit 1 Test Pages 237–238 of the *Chapter 4 Resource Masters* may be used as a test or review for Unit 1. This assessment contains both multiple-choice and short answer items.

TestCheck and Worksheet Builder

This CD-ROM can be used to create additional unit tests and review worksheets.

UNIT 1

First-Degree Equations and Inequalities



You can model and analyze real-world situations by using algebra. In this unit, you will solve and graph linear equations and inequalities and use matrices.

Chapter 1
Solving Equations and Inequalities

Chapter 2
Linear Relations and Functions

Chapter 3
Systems of Equations and Inequalities

Chapter 4
Matrices

WebQuest Internet Project

Lessons in Home Buying, Selling

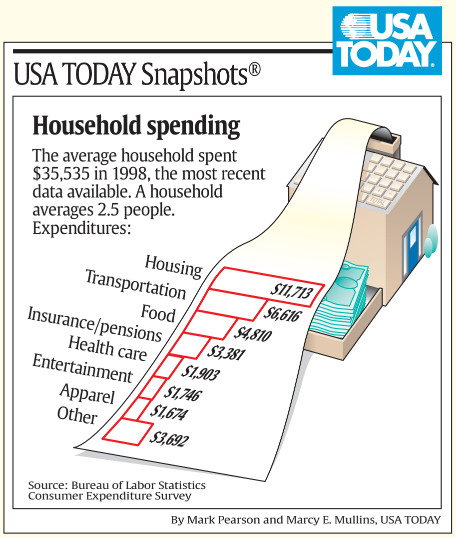
Source: *USA TODAY,* November 18, 1999

"'Buying a home,' says Housing and Urban Development Secretary Andrew Cuomo, 'is the most expensive, most complicated and most intimidating financial transaction most Americans ever make.'" In this project, you will be exploring how functions and equations relate to buying a home and your income.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.

Lesson	1-3	2-5	3-2	4-6
Page	27	84	120	192



Teaching Suggestions

Have students study the USA TODAY Snapshot®.

- Ask students to write an inequality using the data for two of the expenditure categories shown. **See students' work.**
- According to the data, what was the average cost per person for apparel in 1998? **$669.60**
- Point out to students that how they budget their money can affect their ability to buy a home. Their spending habits also affect what type of home they could afford.

Additional USA TODAY Snapshots® appearing in Unit 1:

Chapter 1	School shopping (p. 17)
	Just looking, thank you (p. 39)
Chapter 2	Cruises grow in popularity (p. 69)
	Cost of seeing the doctor (p. 84)
Chapter 3	Per-pupil spending is climbing (p. 135)
Chapter 4	Student-to-teacher ratios dropping (p. 206)

WebQuest Internet Project

A WebQuest is an online project in which students do research on the Internet, gather data, and make presentations using word processing, graphing, page-making, or presentation software. In each chapter, students advance to the next step in their WebQuest. At the end of Chapter 4, the project culminates with a presentation of their findings.

Teaching suggestions and sample answers are available in the *WebQuest and Project Resources.*

Chapter 1

Solving Equations and Inequalities

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days) Regular: Basic/Average	Regular: Advanced	Block: Basic/Average	Block: Advanced
1-1 Expressions and Formulas *(pp. 6–10)* • Use the order of operations to evaluate expressions. • Use formulas.	1	optional	0.5	optional
1-2 Properties of Real Numbers *(pp. 11–19)* • Classify real numbers. • Use the properties of real numbers to evaluate expressions. ***Follow-Up:*** Investigating Polygons and Patterns	2 (with 1-2 Follow-Up)	optional	0.5	optional
1-3 Solving Equations *(pp. 20–27)* • Translate verbal expressions into algebraic expressions and equations, and vice versa. • Solve equations using the properties of equality.	1	optional	1 (with 1-2 Follow-Up)	optional
1-4 Solving Absolute Value Equations *(pp. 28–32)* • Evaluate expressions involving absolute values. • Solve absolute value equations.	1	optional	0.5	optional
1-5 Solving Inequalities *(pp. 33–39)* • Solve inequalities. • Solve real-world problems involving inequalities.	1	optional	0.5	optional
1-6 Solving Compound and Absolute Value Inequalities *(pp. 40–46)* • Solve compound inequalities. • Solve absolute value inequalities.	1	optional	0.5	optional
Study Guide and **Practice Test** *(pp. 47–51)* **Standardized Test Practice** *(pp. 52–53)*	1	2	0.5	1
Chapter Assessment	1	1	0.5	0.5
TOTAL	**9**	**3**	**4.5**	**1.5**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter Resource Manager

Chapter 1 Resource Masters									
Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
1–2	3–4	5	6		SC 1, SM 91–96	1-1	1-1		graphing calculator, colored pencils
7–8	9–10	11	12	51		1-2	1-2		algebra tiles, index cards (*Follow:Up:* ruler or geometry software)
13–14	15–16	17	18	51, 53	GCS 27, SC 2	1-3	1-3		
19–20	21–22	23	24		GCS 28	1-4	1-4	1	
25–26	27–28	29	30	52		1-5	1-5	2	graphing calculator
31–32	33–34	35	36	52		1-6	1-6		masking tape
				38–50, 54–56					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 1

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students have worked with linear equations in previous classes and they should be familiar, to some extent, with some of the properties of equality and inequality. Also, in earlier grades students have used number lines and have related inequalities to intervals on number lines.

This Chapter

Students review the real number system and the order of operations. They begin to study formulas, evaluating expressions, and additive and multiplicative inverses. They see how properties of equality and properties of the real number system can be used to solve equations, and they study other topics related to linear equations, linear inequalities, and absolute value.

Future Connections

Equations, inequalities, and absolute value expressions appear throughout all levels of mathematics. Solving equations and inequalities and justifying mathematical steps on the basis of properties is at the center of all mathematical analysis and presentation.

1-1 Expressions and Formulas

An algebraic expression usually contains at least one variable and may also contain numbers and operations. The order of operations is a mathematical convention for deciding which operations are performed before others in an algebraic expression. That order is: evaluate powers; multiply and divide from left to right; and add and subtract from left to right. There is one more part to the convention: any grouping symbol (parentheses, brackets, braces, fraction bar) takes first priority. To evaluate an expression means to replace each variable with its given value and then follow the order of operations to simplify. A formula is an equation in which one variable is set equal to an algebraic expression.

1-2 Properties of Real Numbers

The set N of natural numbers is $\{1, 2, 3, \ldots\}$; add zero and the result is the set W of whole numbers. The set Z of integers is $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ and the numbers in the set Q of rational numbers have the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. The rationals, along with the set I of irrational numbers, make up the set R of real numbers. There is a one-to-one correspondence between the real numbers and the points on a line in that each real number corresponds to exactly one point on a line and each point on a line corresponds to exactly one real number.

Properties of real numbers are used to justify the steps of solving equations and describing mathematical relationships. These include the commutative and associative properties of addition and the commutative and associative properties of multiplication. Another property, the distributive property, relates addition and multiplication. The real numbers include an identity element for the operation of addition, an identity element for the operation of multiplication, an additive inverse for every real number, and a multiplicative inverse for every real number except 0.

1-3 Solving Equations

A mathematic sentence with an equal sign between two algebraic or arithmetic expressions is called an *equation*. To solve an equation requires a series of equations, equivalent to the given equation, that result in a final equation that isolates the variable on one side. That final equation presents the solution to the original equation. However, solutions should always be substituted into the original equation to check for correctness.

The rules for writing equivalent equations are called Properties of Equality. We can write the equation

$a = a$; given $a = b$ then we can write $b = a$; given $a = b$ and $b = c$ then we can write $a = c$. A fourth rule is Substitution: if $a = b$, then we can write an equation replacing a with b or b with a. Also, if $a = b$ we can write $a + c = b + c$, we can write $a - c = b - c$, we can write $a \cdot c = b \cdot c$, and, if $c \neq 0$, we can write $\frac{a}{c} = \frac{b}{c}$.

1-4 Solving Absolute Value Equations

The absolute value of a number is its distance from zero. Described algebraically, the definition of absolute value is $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$. The absolute value symbols are a grouping symbol like parentheses or a fraction bar. For example, to evaluate $2 \cdot |15 - 31|$, first calculate inside the symbols. So, $2 \cdot |15 - 31| = 2 \cdot |-16| = 2 \cdot (16)$ or 32.

The equation $|a - 6| = 4$ can be interpreted as *the distance between a and 6 is 4 units*. The value $a - 6$ can be 4 or -4, so if $a - 6 = 4$, then $a = 10$. If $a - 6 = -4$, then $a = 2$. The solution is {2, 10}. "No solution" can be written as { } or $\varnothing$, the symbols for the empty set.

1-5 Solving Inequalities

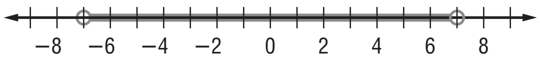
An inequality is a mathematical sentence with one of the symbols $<$, $\leq$, $>$, or $\geq$ between two expressions. Solving an inequality means writing a series of equivalent inequalities, ending with one that isolates the variable. The rules for writing equivalent inequalities are called properties of inequality. (The properties hold for all inequalities, but are usually expressed initially in terms of $>$.) If $a > b$, then we can write $a + c > b + c$ and $a - c > b - c$. Also, if $a > b$ and $c > 0$, then we can write $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$ or, if $c < 0$, we can write $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. In general, multiplying or dividing an inequality by a negative number *reverses* the order of the inequality. The Trichotomy Property states that for any two real numbers, either the values are equal or one value is greater than the other. In symbols, exactly one of these statements is true: $a < b$, $a = b$, or $a > b$.

When the solution to an inequality is graphed, an open circle indicates a value that is not included and a closed circle indicates a value that is included. Open circles are used with $<$ and $>$, and closed circles are used with $\leq$ and $\geq$. Solutions to inequalities are often written using set-builder notation, so a solution such as $x \geq 4$ would be written $\{x | x \geq 4\}$, read *the set of values x such that x is greater than or equal to 4.*

1-6 Solving Compound and Absolute Value Inequalities

There are important connections between compound inequalities and absolute value inequalities. An absolute value inequality using $<$ or $\leq$ is related to a compound inequality using the word *and*. For example, thinking of $|a| < 7$ as $|a - 0| < 7$, then the value of a is any number whose distance from 0 is less than 7 units.

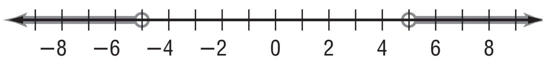
Possible values for a



$|a| < 7$ means $-7 < a$ *and* $a < 7$ or $-7 < a < 7$

An absolute value inequality using $>$ or $\geq$ is related to a compound inequality using the word *or*. For example, thinking of $|b| > 5$ as $|b - 0| > 5$, then the value of b is any number whose distance from 0 is more than 5.

Possible values for b



$|b| > 5$ means $b < -5$ *or* $b > 5$

To solve absolute value inequalities, use two patterns. One pattern is to rewrite $|A| < B$ as $-B < A$ *and* $A < B$ (or $-B < A < B$), so rewrite $|2x - 5| < 18$ as $-18 < 2x - 5$ *and* $2x - 5 < 18$. The solution is $-\frac{13}{2} < x < \frac{23}{2}$. The other pattern is to rewrite $|A| > B$ as $A < -B$ or $A > B$, so rewrite the inequality $|3x + 1| > 15$ as $3x + 1 < -15$ *or* $3x + 1 > 15$. The solution is $x < -\frac{16}{3}$ *or* $x > \frac{14}{3}$.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Solving Multi-Step Inequalities (Lesson 15)
- Solving Compound Inequalities (Lesson 16)

Chapter 1

DAILY INTERVENTION and Assessment

INTERVENTION

ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 5, 10, 18, 27, 32, 39 Practice Quiz 1, p. 18 Practice Quiz 2, p. 39	5-Minute Check Transparencies Quizzes, *CRM* pp. 51–52 Mid-Chapter Test, *CRM* p. 53 Study Guide and Intervention, *CRM* pp. 1–2, 7–8, 13–14, 19–20, 25–26, 31–32	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
Mixed Review	pp. 18, 27, 32, 39, 46	Cumulative Review, *CRM* p. 54	
Error Analysis	Find the Error, pp. 24, 43 Common Misconceptions, p. 12	Find the Error, *TWE* pp. 24, 44 Unlocking Misconceptions, *TWE* pp. 15, 18, 22 Tips for New Teachers, *TWE* pp. 10, 27	
Standardized Test Practice	pp. 10, 17, 23, 24, 27, 31, 32, 39, 46, 51, 52–53	*TWE* p. 23 Standardized Test Practice, *CRM* pp. 55–56	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 10, 17, 27, 31, 38, 45 Open Ended, pp. 8, 14, 24, 30, 37, 43	Modeling: *TWE* pp. 18, 32 Speaking: *TWE* pp. 10, 27 Writing: *TWE* pp. 39, 46 Open-Ended Assessment, *CRM* p. 49	
Chapter Assessment	Study Guide, pp. 47–50 Practice Test, p. 51	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 37–42 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 43–48 Vocabulary Test/Review, *CRM* p. 50	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*

The Princeton Review's *Cracking the ACT*

ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson	
1-4	**1**	*Solving Multi-Operational Equations IV*
1-5	**2**	*Solving Inequalities*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 5
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 8, 14, 24, 30, 37, 43)
- Writing in Math questions in every lesson, pp. 10, 17, 27, 31, 38, 45
- Reading Study Tip, pp. 11, 12, 34, 35
- WebQuest, p. 27

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 5, 47
- Study Notebook suggestions, pp. 8, 15, 19, 24, 30, 37, 43
- Modeling activities, pp. 18, 32
- Speaking activities, pp. 10, 27
- Writing activities, pp. 39, 46
- Differentiated Instruction, (Verbal/Linguistic), p. 29
- ELL Resources, pp. 4, 9, 17, 26, 29, 31, 38, 45, 47

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 1 Resource Masters,* pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 1 Resource Masters*, pp. 5, 11, 17, 23, 29, 35)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 1 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

The chart below correlates the objectives for each lesson to the NCTM Standards 2000. There is also space for you to reference your state and/or local objectives.

Lesson	NCTM Standards	Local Objectives
1-1	1, 2, 4, 8, 9	
1-2	1, 8, 9	
1-2 Follow-Up	1, 3, 9, 10	
1-3	1, 2, 4, 6, 8, 9	
1-4	1, 2, 8, 9, 10	
1-5	1, 2, 6, 8, 9	
1-6	1, 2, 6, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 1 Solving Equations and Inequalities

What You'll Learn

- **Lesson 1-1** Simplify and evaluate algebraic expressions.
- **Lesson 1-2** Classify and use the properties of real numbers.
- **Lesson 1-3** Solve equations.
- **Lesson 1-4** Solve absolute value equations.
- **Lessons 1-5 and 1-6** Solve and graph inequalities.

Key Vocabulary

- **order of operations** (p. 6)
- **algebraic expression** (p. 7)
- **Distributive Property** (p. 12)
- **equation** (p. 20)
- **absolute value** (p. 28)

Why It's Important

Algebra allows you to write expressions, equations, and inequalities that hold true for most or all values of variables. Because of this, algebra is an important tool for describing relationships among quantities in the real world. For example, the angle at which you view fireworks and the time it takes you to hear the sound are related to the width of the fireworks burst. A change in one of the quantities will cause one or both of the other quantities to change.

In Lesson 1-1, you will use the formula that relates these quantities.

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 1 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 1 test.

4 Chapter 1 Solving Equations and Inequalities

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lessons 1-1 through 1-3 — Operations with Rational Numbers

Simplify.

1. $20 - 0.16$ **19.84**
2. $12.2 + (-8.45)$ **3.75**
3. $-3.01 - 14.5$ **−17.51**
4. $-1.8 + 17$ **15.2**
5. $\frac{1}{4} - \frac{2}{3}$ $-\frac{5}{12}$
6. $\frac{3}{5} + (-6)$ $-5\frac{2}{5}$
7. $-7\frac{1}{2} + 5\frac{1}{3}$ $-2\frac{1}{6}$
8. $-11\frac{5}{8} - \left(-4\frac{3}{7}\right)$ $-7\frac{11}{56}$
9. $(0.15)(3.2)$ **0.48**
10. $2 \div (-0.4)$ **−5**
11. $(-1.21) \div (-1.1)$ **1.1**
12. $(-9)(0.036)$ **−0.324**
13. $-4 \div \frac{3}{2}$ $-2\frac{2}{3}$
14. $\left(\frac{5}{4}\right)\left(-\frac{3}{10}\right)$ $-\frac{3}{8}$
15. $\left(-2\frac{3}{4}\right)\left(-3\frac{1}{5}\right)$ $8\frac{4}{5}$
16. $7\frac{1}{8} \div (-2)$ $-3\frac{9}{16}$

For Lesson 1-1 — Powers

Evaluate each power.

17. 2^3 **8**
18. 5^3 **125**
19. $(-7)^2$ **49**
20. $(-1)^3$ **−1**
21. $(-0.8)^2$ **0.64**
22. $-(1.2)^2$ **−1.44**
23. $\left(\frac{2}{3}\right)^2$ $\frac{4}{9}$
24. $\left(-\frac{4}{11}\right)^2$ $\frac{16}{121}$

For Lesson 1-5 — Compare Real Numbers

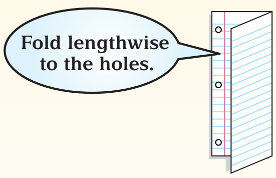
Identify each statement as *true* or *false*.

25. $-5 < -7$ **false**
26. $6 > -8$ **true**
27. $-2 \geq -2$ **true**
28. $-3 \geq -3.01$ **true**
29. $-9.02 < -9.2$ **false**
30. $\frac{1}{5} < \frac{1}{8}$ **false**
31. $\frac{2}{5} \geq \frac{16}{40}$ **true**
32. $\frac{3}{4} > 0.8$ **false**

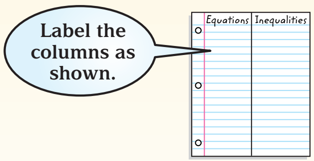
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about relations and functions. Begin with one sheet of notebook paper.

Step 1 Fold



Step 2 Cut and Label



Reading and Writing As you read and study the chapter, write notes, examples, and graphs in each column.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 1. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
1-2	Evaluating Square Roots (p. 10)
1-3	Evaluating Expressions (p. 18)
1-4	Additive Inverses (p. 27)
1-5	Solving Equations (p. 32)
1-6	Solving Absolute Value Equations (p. 39)

Each chapter opens with Prerequisite Skills practice for lessons in the chapter. More Prerequisite Skill practice can be found at the end of each lesson.

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Note-Taking and Charting Main Ideas Use this Foldable study guide for student notes about equations and inequalities. Note-taking is a skill that is based upon listening or reading for main ideas and then recording those ideas for future reference. In the columns of their Foldable, have students take notes about the processes and procedures that they learn. Encourage students to apply what they know and what they learn as they analyze and solve equations and inequalities.

Foldables™ are a unique way to enhance students' study skills. Encourage students to add to their Foldable as they work through the chapter, and use it to review for their chapter test.

1-1 Lesson Notes

1 Focus

5-Minute Check Transparency 1-1 Use as a quiz or review of prerequisite skills.

Mathematical Background notes are available for this lesson on p. 4C.

Building on Prior Knowledge

In previous courses, students have performed operations on integers and used the order of operations. In this lesson, they should realize that using formulas requires these skills.

How are formulas used by nurses?

Ask students:

- What are the units for the flow rate F? **drops per minute**
- Why is 12 hours multiplied by 60? **to convert the time from hours to minutes**
- **Medicine** What might happen if the flow rate is too fast or slow? **Too fast: the fluid might not be absorbed by the patient's body as expected; too slow: the medication might not be effective.**

Lessons open with a question that is designed to engage students in the mathematics of the lesson. These opening problems should also help to answer the question "When am I ever going to use this?"

1-1 Expressions and Formulas

What You'll Learn

- Use the order of operations to evaluate expressions.
- Use formulas.

Vocabulary

- order of operations
- variable
- algebraic expression
- formula

How are formulas used by nurses?

Intravenous or IV fluid must be given at a specific rate, neither too fast nor too slow. A nurse setting up an IV must control the flow rate F, in drops per minute. They use the formula $F = \frac{V \times d}{t}$, where V is the volume of the solution in milliliters, d is the drop factor in drops per milliliter, and t is the time in minutes. Suppose a doctor orders 1500 milliliters of IV saline to be given over 12 hours, or 12×60 minutes. Using a drop factor of 15 drops per milliliter, the expression $\frac{1500 \times 15}{12 \times 60}$ gives the correct flow rate for this patient's IV.

ORDER OF OPERATIONS A numerical expression such as $\frac{1500 \times 15}{12 \times 60}$ must have exactly one value. In order to find that value, you must follow the **order of operations**.

Key Concept — *Order of Operations*

Step 1 Evaluate expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars, as in $\frac{5+7}{2}$.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Grouping symbols can be used to change or clarify the order of operations. When calculating the value of an expression, begin with the innermost set of grouping symbols.

Example 1 *Simplify an Expression*

Find the value of $[2(10 - 4)^2 + 3] \div 5$.

$[2(10-4)^2 + 3] \div 5 = [2(6)^2 + 3] \div 5$	First subtract 4 from 10.
$= [2(36) + 3] \div 5$	Then square 6.
$= (72 + 3) \div 5$	Multiply 36 by 2.
$= 75 \div 5$	Add 72 and 3.
$= 15$	Finally, divide 75 by 5.

The value is 15.

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters
- Study Guide and Intervention, pp. 1–2
- Skills Practice, p. 3
- Practice, p. 4
- Reading to Learn Mathematics, p. 5
- Enrichment, p. 6

School-to-Career Masters, p. 1
Science and Mathematics Lab Manual, pp. 91–96

Transparencies

5-Minute Check Transparency 1-1
Real-World Transparency 1
Answer Key Transparencies

Technology

Interactive Chalkboard

Scientific calculators follow the order of operations.

Graphing Calculator Investigation

Order of Operations

Think and Discuss 2, 4, 5. See margin.

1. Simplify $8 - 2 \times 4 + 5$ using a graphing calculator. 5
2. Describe the procedure the calculator used to get the answer.
3. Where should parentheses be inserted in $8 - 2 \times 4 + 5$ so that the expression has each of the following values?
   a. -10 around $4 + 5$ **b.** 29 around $8 - 2$ **c.** -5 around $2 \times 4 + 5$
4. Evaluate $18^2 \div (2 \times 3)$ using your calculator. Explain how the answer was calculated.
5. If you remove the parentheses in Exercise 4, would the solution remain the same? Explain.

Variables are symbols, usually letters, used to represent unknown quantities. Expressions that contain at least one variable are called **algebraic expressions**. You can evaluate an algebraic expression by replacing each variable with a number and then applying the order of operations.

Example 2 Evaluate an Expression

Evaluate $x^2 - y(x + y)$ if $x = 8$ and $y = 1.5$.

$x^2 - y(x + y) = 8^2 - 1.5(8 + 1.5)$ Replace x with 8 and y with 1.5.

$= 64 - 1.5(8 + 1.5)$ Find 8^2.

$= 64 - 1.5(9.5)$ Add 8 and 1.5.

$= 64 - 14.25$ Multiply 1.5 and 9.5.

$= 49.75$ Subtract 14.25 from 64.

The value is 49.75.

Study Tip

Common Misconception

A common error in this type of problem is to subtract before multiplying.

$64 - 1.5(9.5) \neq 62.5(9.5)$

Remember to follow the order of operations.

Example 3 Expression Containing a Fraction Bar

Evaluate $\frac{a^3 + 2bc}{c^2 - 5}$ if $a = 2$, $b = -4$, and $c = -3$.

The fraction bar acts as both an operation symbol, indicating division, and as a grouping symbol. Evaluate the expressions in the numerator and denominator separately before dividing.

$\frac{a^3 + 2bc}{c^2 - 5} = \frac{2^3 + 2(-4)(-3)}{(-3)^2 - 5}$ $a = 2$, $b = -4$, and $c = -3$

$= \frac{8 + (-8)(-3)}{9 - 5}$ Evaluate the numerator and the denominator separately.

$= \frac{8 + 24}{9 - 5}$ Multiply -8 by -3.

$= \frac{32}{4}$ or 8 Simplify the numerator and the denominator. Then divide.

The value is 8.

www.algebra2.com/extra_examples

Graphing Calculator Investigation

Order of Operations To help find entry errors, have students work in pairs so one of them can watch as their partner performs the keystrokes to enter the expression. Sometimes it is necessary to use parentheses to obtain the correct answer with fractional expressions. For example, to evaluate $\frac{4(12)}{5(4)}$, you must enter 4 * 12/(5 * 4). Ask students why this is so.

2 Teach

ORDER OF OPERATIONS

In-Class Examples

Power Point®

1 Find the value of $[384 - 3(7 - 2)^3] \div 3$. 3

2 Evaluate $s - t(s^2 - t)$ if $s = 2$ and $t = 3.4$. -0.04

3 Evaluate $\frac{8xy + z^3}{y^2 + 5}$ if $x = 5$, $y = -2$, and $z = -1$. -9

Teaching Tip Ask students what sign the cube of a negative number has. negative sign

Answers

Graphing Calculator Investigation

2. The calculator multiplies 2 by 4, subtracts the result from 8, and then adds 5.
4. 54; The calculator found the square of 18 and divided it by the product of 2 and 3.
5. No; you would square 18 and then divide it by 2. The result would then be multiplied by 3.

Interactive Chalkboard

PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

FORMULAS

In-Class Example Power Point®

4 **GEOMETRY** Find the area of a trapezoid with base lengths of 13 meters and 25 meters and a height of 8 meters. **152 m²**

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
- copy several of the formulas (for example, the area of a trapezoid), and include notes about when the formula is used.
- make a sketch of a trapezoid and label the variables used in the formula for its area.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Examples illustrate all of the concepts taught in the lesson and closely mirror the exercises in the Guided Practice and Practice and Apply sections.

About the Exercises...

Organization by Objective
- **Order of Operations:** 16–37
- **Formulas:** 38–54

Odd/Even Assignments
Exercises 16–49 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 53 involves research on the Internet or other reference materials.

Assignment Guide
Basic: 17–33 odd, 37–47 odd, 53, 55–66

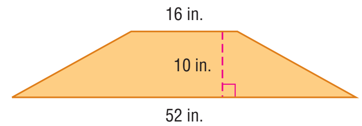
Average: 17–53 odd, 55–66

Advanced: 16–54 even, 55–58, (optional: 59–66)

FORMULAS A **formula** is a mathematical sentence that expresses the relationship between certain quantities. If you know the value of every variable in the formula except one, you can find the value of the remaining variable.

Example 4 Use a Formula

GEOMETRY The formula for the area A of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where h represents the height, and b_1 and b_2 represent the measures of the bases. Find the area of the trapezoid shown below.



Substitute each value given into the formula. Then evaluate the expression using the order of operations.

$A = \frac{1}{2}h(b_1 + b_2)$	Area of a trapezoid
$= \frac{1}{2}(10)(16 + 52)$	Replace h with 10, b_1 with 16, and b_2 with 52.
$= \frac{1}{2}(10)(68)$	Add 16 and 52.
$= 5(68)$	Divide 10 by 2.
$= 340$	Multiply 5 by 68.

The area of the trapezoid is 340 square inches.

Check for Understanding

Concept Check

1. **Describe** how you would evaluate the expression $a + b[(c + d) \div e]$ given values for a, b, c, d, and e.
   1. First, find the sum of *c* and *d*. Divide this sum by *e*. Multiply the quotient by *b*. Finally, add *a*.
2. **OPEN ENDED** Give an example of an expression where subtraction is performed before division and the symbols ( ), [ ], or { } are not used.
   2. Sample answer: $\frac{14 - 4}{5}$
3. **Determine** which expression below represents the amount of change someone would receive from a $50 bill if they purchased 2 children's tickets at $4.25 each and 3 adult tickets at $7 each at a movie theater. Explain.
   a. $50 - 2 \times 4.25 + 3 \times 7$
   b. $50 - (2 \times 4.25 + 3 \times 7)$
   c. $(50 - 2 \times 4.25) + 3 \times 7$
   d. $50 - (2 \times 4.25) + (3 \times 7)$
   b; See margin for explanation.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1, 3
10–12	2
13–15	4

Find the value of each expression.

4. $8(3 + 6)$ **72**
5. $10 - 8 \div 2$ **6**
6. $14 \cdot 2 - 5$ **23**
7. $[9 + 3(5 - 7)] \div 3$ **1**
8. $[6 - (12 - 8)^2] \div 5$ **−2**
9. $\frac{17(2 + 26)}{4}$ **119**

Evaluate each expression if $x = 4$, $y = -2$, and $z = 6$.

10. $z - x + y$ **0**
11. $x + (y - 1)^3$ **−23**
12. $x + [3(y + z) - y]$ **18**

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Suggest that students first rewrite an expression they are to evaluate and then write the value for each variable on top of that variable before they start to evaluate the expression. Students may find it helpful to use colored pencils to color code the values for the different variables in an expression.

Application

BANKING **For Exercises 13–15, use the following information.**
Simple interest is calculated using the formula $I = prt$, where p represents the principal in dollars, r represents the annual interest rate, and t represents the time in years. Find the simple interest I given each of the following values.

13. $p = \$1800$, $r = 6\%$, $t = 4$ years **\$432**
14. $p = \$5000$, $r = 3.75\%$, $t = 10$ years **\$1875**
15. $p = \$31{,}000$, $r = 2\frac{1}{2}\%$, $t = 18$ months **\$1162.50**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
16–37	1, 3
38–50	2, 3
51–54	4

Extra Practice

See page 828.

Find the value of each expression.

16. $18 + 6 \div 3$ **20**
17. $7 - 20 \div 5$ **3**
18. $3(8 + 3) - 4$ **29**
19. $(6 + 7)2 - 1$ **25**
20. $2(6^2 - 9)$ **54**
21. $-2(3^2 + 8)$ **−34**
22. $2 + 8(5) \div 2 - 3$ **19**
23. $4 + 64 \div (8 \times 4) \div 2$ **5**
24. $[38 - (8 - 3)] \div 3$ **11**
25. $10 - [5 + 9(4)]$ **−31**
26. $1 - \{30 \div [7 + 3(-4)]\}$ **7**
27. $12 + \{10 \div [11 - 3(2)]\}$ **14**
28. $\frac{1}{3}(4 - 7^2)$ **−15**
29. $\frac{1}{2}[9 + 5(-3)]$ **−3**
30. $\frac{16(9 - 22)}{4}$ **−52**
31. $\frac{45(4 + 32)}{10}$ **162**
32. $0.3(1.5 + 24) \div 0.5$ **15.3**
33. $1.6(0.7 + 3.3) \div 2.5$ **2.56**
★ 34. $\frac{1}{5} - \frac{20(81 \div 9)}{25}$ **−7**
★ 35. $\frac{12(52 \div 2^2)}{6} - \frac{2}{3}$ **$25\frac{1}{3}$**

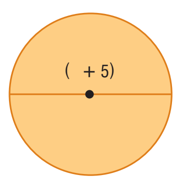
36. **BICYCLING** The amount of pollutants saved by riding a bicycle rather than driving a car is calculated by adding the organic gases, carbon monoxide, and nitrous oxides emitted. To find the pounds of pollutants created by starting a typical car 10 times and driving it for 50 miles, find the value of the expression $\frac{(52.84 \times 10) + (5.955 \times 50)}{454}$. **about 1.8 lb**

37. **NURSING** Determine the IV flow rate for the patient described at the beginning of the lesson by finding the value of $\frac{1500 \times 15}{12 \times 60}$. **31.25 drops per min**

Evaluate each expression if $w = 6$, $x = 0.4$, $y = \frac{1}{2}$, and $z = -3$.

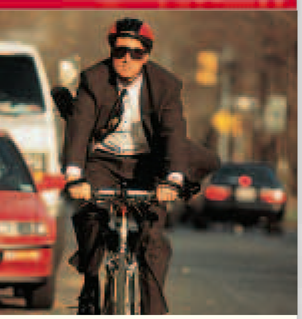
38. $w + x + z$ **3.4**
39. $w + 12 \div z$ **2**
40. $w(8 - y)$ **45**
41. $z(x + 1)$ **−4.2**
42. $w - 3x + y$ **5.3**
43. $5x + 2z$ **−4**
44. $z^4 - w$ **75**
45. $(5 - w)^2 + x$ **1.4**
46. $\frac{5wx}{z}$ **−4**
47. $\frac{2z - 15x}{3y}$ **−8**
★ 48. $(x - y)^2 - 2wz$ **36.01**
★ 49. $\frac{1}{y} + \frac{1}{w}$ **$2\frac{1}{6}$**

50. **GEOMETRY** The formula for the area A of a circle with diameter d is $A = \pi\left(\frac{d}{2}\right)^2$. Write an expression to represent the area of the circle. **$\pi\left(\frac{y+5}{2}\right)^2$**



★ 51. Find the value of ab^n if $n = 3$, $a = 2000$, and $b = -\frac{1}{5}$. **−16**

More About...



Bicycling

In order to increase awareness and acceptance of bicycling throughout the country, communities, corporations, clubs, and individuals are invited to join in sponsoring bicycling activities during the month of May, National Bike Month.

Source: League of American Bicyclists

www.algebra2.com/self_check_quiz

3. The sum of the cost of adult and children tickets should be subtracted from 50. Therefore parentheses need to be inserted around this sum to insure that this addition is done before subtraction.

Study Guide and Intervention, p. 1 (shown) and p. 2

Order of Operations

Order of Operations	1. 2. 3. 4.

Example 1 **Evaluate $[18 - (6 + 4)] \div 2$.**

$[18 - (6 + 4)] \div 2 = [18 - 10] \div 2$
$= 8 \div 2$
$= 4$

Example 2 **Evaluate $3x^2 + x(y - 5)$ if $x = 3$ and $y = 0.5$.**

Replace each variable with the given value.
$3x^2 + x(y - 5) = 3 \cdot (3)^2 + 3(0.5 - 5)$
$= 3 \cdot (9) + 3(-4.5)$
$= 27 - 13.5$
$= 13.5$

Exercises

Find the value of each expression.

1. $14 + (6 \div 2)$ **17**
2. $11 - (3 + 2)^2$ **−14**
3. $2 + (4 - 2)^3 - 6$ **4**
4. $9(3^2 + 6)$ **135**
5. $(5 + 2^3)^2 - 5^2$ **144**
6. $5^2 + \frac{1}{4} + 18 \div 2$ **34.25**
7. $\frac{16 + 2^3 \div 4}{1 - 2^2}$ **−6**
8. $(7 - 3^2)^2 + 6^2$ **40**
9. $20 \div 2^2 + 6$ **11**
10. $12 + 6 \div 3 - 2(4)$ **6**
11. $14 \div (8 - 20 \div 2)$ **−7**
12. $6(7) + 4 \div 4 - 5$ **38**
13. $8(4^2 \div 8 - 32)$ **−240**
14. $\frac{6 + 4 \div 2}{4 \div 6 - 1}$ **−24**
15. $\frac{6 + 9 \div 3 + 15}{8 - 2}$ **4**

Evaluate each expression if $a = 8.2$, $b = -3$, $c = 4$, and $d = -\frac{1}{2}$.

16. $\frac{ab}{d}$ **49.2**
17. $5(6c - 8b + 10d)$ **215**
18. $\frac{c^2 - 1}{b - d}$ **−6**
19. $ac - bd$ **31.3**
20. $(b - c)^2 + 4a$ **81.8**
21. $\frac{a}{d} + 6b - 5c$ **−54.4**
22. $3\left(\frac{c}{d}\right) - b$ **−21**
23. $cd + \frac{b}{d}$ **4**
24. $d(a + c)$ **−6.1**
25. $a + b \div c$ **7.45**
26. $b - c + 4 \div d$ **−15**
27. $\frac{a}{b + c} - d$ **8.7**

Skills Practice, p. 3 and Practice, p. 4 (shown)

Find the value of each expression.

1. $3(4 - 7) - 11$ **−20**
2. $4(12 - 4^2)$ **−16**
3. $1 + 2 - 3(4) \div 2$ **−3**
4. $12 - [20 - 2(6^2 \div 3 \times 2^2)]$ **88**
5. $20 \div (5 - 3) + 5^2(3)$ **85**
6. $(-2)^3 - (3)(8) + (5)(10)$ **18**
7. $18 - \{5 - [34 - (17 - 11)]\}$ **41**
8. $[4(5 - 3) - 2(4 - 8)] \div 16$ **1**
9. $\frac{1}{2}[6 - 4^2]$ **−5**
10. $\frac{1}{4}[-5 + 5(-3)]$ **−5**
11. $\frac{-8(13 - 37)}{6}$ **32**
12. $\frac{(-8)^2}{5 - 9} - (-1)^2 + 4(-9)$ **−53**

Evaluate each expression if $a = \frac{3}{4}$, $b = -8$, $c = -2$, $d = 3$, and $e = \frac{1}{3}$.

13. $ab^2 - d$ **45**
14. $(c + d)b$ **−8**
15. $\frac{ab}{c} + d^2$ **12**
16. $\frac{d(b - c)}{ac}$ **12**
17. $(b - de)e^2$ **−1**
18. $ac^3 - b^2de$ **−70**
19. $-b[a + (c - d)^2]$ **206**
20. $\frac{ac^4}{d} - \frac{c}{e^2}$ **22**
21. $9bc - \frac{1}{e}$ **141**
22. $2ab^2 - (d^3 - c)$ **67**

23. **TEMPERATURE** The formula $F = \frac{9}{5}C + 32$ gives the temperature in degrees Fahrenheit for a given temperature in degrees Celsius. What is the temperature in degrees Fahrenheit when the temperature is −40 degrees Celsius? **−40°F**

24. **PHYSICS** The formula $h = 120t - 16t^2$ gives the height h in feet of an object t seconds after it is shot upward from Earth's surface with an initial velocity of 120 feet per second. What will the height of the object be after 6 seconds? **144 ft**

25. **AGRICULTURE** Faith owns an organic apple orchard. From her experience the last few seasons, she has developed the formula $P = 20x - 0.01x^2 - 240$ to predict her profit P in dollars this season if her trees produce x bushels of apples. What is Faith's predicted profit this season if her orchard produces 300 bushels of apples? **\$4860**

Reading to Learn Mathematics, p. 5 — ELL

Pre-Activity **How are formulas used by nurses?**

Read the introduction to Lesson 1-1 at the top of page 6 in your textbook.

- Nurses use the formula $F = \frac{V \times d}{t}$ to control the flow rate for IVs. Name the quantity that each of the variables in this formula represents and the units in which each is measured.

  F represents the **flow rate** and is measured in **drops** per minute.

  V represents the **volume** of solution and is measured in **milliliters**.

  d represents the **drop factor** and is measured in **drops** per milliliter.

  t represents **time** and is measured in **minutes**.

- Write the expression that a nurse would use to calculate the flow rate of an IV if a doctor orders 1350 milliliters of IV saline to be given over 8 hours, with a drop factor of 20 drops per milliliter. Do not find the value of this expression. **$\frac{1350 \times 20}{8 \times 60}$**

Reading the Lesson

1. There is a customary order for grouping symbols. Brackets are used outside of parentheses. Braces are used outside of brackets. Identify the innermost expression(s) in each of the following expressions.
   a. $[(3 - 2^2) + 8] \div 4$ **$(3 - 2^2)$**
   b. $9 - [5(8 - 6) + 2(10 + 7)]$ **(8 − 6) and (10 + 7)**
   c. $\{14 - [8 + (3 - 12)^2]\} \div (6^3 - 100)$ **(3 − 12)**
2. Read the following instructions. Then use grouping symbols to show how the instructions can be put in the form of a mathematical expression.
   Multiply the difference of 13 and 5 by the sum of 9 and 21. Add the result to 10. Then divide what you get by 2. **[(13 − 5)(9 + 21) + 10] ÷ 2**
3. Why is it important for everyone to use the same order of operations for evaluating expressions? **Sample answer: If everyone did not use the same order of operations, different people might get different answers.**

Helping You Remember

4. Think of a phrase or sentence to help you remember the order of operations. **Sample answer: Please excuse my dear Aunt Sally. (parentheses; exponents; multiplication and division; addition and subtraction)**

Enrichment, p. 6

Significant Digits

All measurements are approximations. The **significant digits** of an approximate number are those which indicate the results of a measurement. For example, the mass of an object, measured to the nearest gram, is 210 grams. The measurement 210 g has 3 significant digits. The mass of the same object, measured to the nearest 100 g, is 200 g. The measurement 200 g has one significant digit.

1. Nonzero digits and zeros between significant digits are significant. For example, the measurement 9.071 m has 4 significant digits, 9, 0, 7, and 1.
2. Zeros at the end of a decimal fraction are significant. The measurement 0.050 mm has 2 significant digits, 5 and 0.
3. Underlined zeros in whole numbers are significant. The measurement 104,000 km has 5 significant digits, 1, 0, 4, 0, and 0.

In general, a computation involving multiplication or division of measurements *cannot* be more accurate than the least accurate measurement in the computation. Thus, the result of computation involving multiplication or division of measurements should be rounded to the number of significant digits in the least ...ent.

4 Assess

Open-Ended Assessment

Speaking Ask students to state various formulas they remember using in previous courses, and to explain what each variable represents (for example, $P = 2(\ell + w)$ to find the perimeter of a rectangle, where ℓ is the length and w is the width). Then have a volunteer suggest appropriate values for the variables in the formula. Ask the class as a whole to evaluate the given formula using the suggested values.

Tips for New Teachers

Intervention Students may be reluctant to take time to show all the steps they use when evaluating an expression, such as showing the substituted values before doing the computations. Help them see that these steps enable them to self-diagnose errors and to prevent calculation errors that might keep them from getting correct values.

New teachers, or teachers new to teaching mathematics, may especially appreciate the Tips for New Teachers.

Getting Ready for Lesson 1-2

PREREQUISITE SKILL Lesson 1-2 presents the properties of real numbers and the subsets of the real numbers, including irrationals. Remind students that the square root of a number is irrational if that number is not a perfect square. Exercises 59–66 should be used to determine your students' familiarity with evaluating square roots.

More About...

Fireworks

To estimate the width w in feet of a firework burst, use the formula $w = 20At$. In this formula, A is the estimated viewing angle of the firework display and t is the time in seconds from the instant you see the light until you hear the sound.

Source: www.efg2.com

52. **MEDICINE** Suppose a patient must take a blood pressure medication that is dispensed in 125-milligram tablets. The dosage is 15 milligrams per kilogram of body weight and is given every 8 hours. If the patient weighs 25 kilograms, how many tablets would be needed for a 30-day supply? Use the formula $n = 24d \div [8(b \times 15 \div 125)]$, where n is the number of tablets, d is the number of days the supply should last, and b is the body weight of the patient in kilograms. **30**

53. **MONEY** In 1950, the average price of a car was about \$2000. This may sound inexpensive, but the average income in 1950 was much less than it is now. To compare dollar amounts over time, use the formula $V = \frac{A}{S}C$, where A is the old dollar amount, S is the starting year's Consumer Price Index (CPI), C is the converting year's CPI, and V is the current value of the old dollar amount. Buying a car for \$2000 in 1950 was like buying a car for how much money in 2000? **\$8266.03**

Year	Average CPI
1950	42 1
1960	29 6
1970	38 8
1980	82 4
1990	130 7
2000	174 0

Source: U.S. Department of Labor

Online Research **Data Update** What is the current Consumer Price Index? Visit www.algebra2.com/data_update to learn more.

54. **FIREWORKS** Suppose you are about a mile from a fireworks display. You count 5 seconds between seeing the light and hearing the sound of the firework display. You estimate the viewing angle is about 4°. Using the information at the left, estimate the width of the firework display. **400 ft**

55. **CRITICAL THINKING** Write expressions having values from one to ten using exactly four 4s. You may use any combination of the operation symbols $+, -, \times, \div$, and/or grouping symbols, but no other numbers are allowed. An example of such an expression with a value of zero is $(4 + 4) - (4 + 4)$. **See margin.**

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

    How are formulas used by nurses?

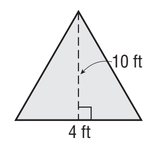
    Include the following in your answer:
    - an explanation of why a formula for the flow rate of an IV is more useful than a table of specific IV flow rates, and
    - a description of the impact of using a formula, such as the one for IV flow rate, incorrectly.

Standardized Test Practice

57. Find the value of $1 + 3(5 - 17) \div 2 \times 6$. **C**

    (A) -4 (B) 109
    (C) -107 (D) -144

58. The following are the dimensions of four rectangles. Which rectangle has the same area as the triangle at the right? **D**



    (A) 1.6 ft by 25 ft (B) 5 ft by 16 ft
    (C) 3.5 ft by 4 ft (D) 0.4 ft by 50 ft

Maintain Your Skills

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Evaluate each expression.**

59. $\sqrt{9}$ **3**
60. $\sqrt{16}$ **4**
61. $\sqrt{100}$ **10**
62. $\sqrt{169}$ **13**
63. $-\sqrt{4}$ **−2**
64. $-\sqrt{25}$ **−5**
65. $\sqrt{\frac{4}{9}}$ $\mathbf{\frac{2}{3}}$
66. $\sqrt{\frac{36}{49}}$ $\mathbf{\frac{6}{7}}$

Answers

55. Sample answer:
$4 - 4 + 4 \div 4 = 1$
$4 \div 4 + 4 \div 4 = 2$
$(4 + 4 + 4) \div 4 = 3$
$4 \times (4 - 4) + 4 = 4$
$(4 \times 4 + 4) \div 4 = 5$
$(4 + 4) \div 4 + 4 = 6$
$44 \div 4 - 4 = 7$
$(4 + 4) \times (4 \div 4) = 8$
$4 + 4 + 4 \div 4 = 9$
$(44 - 4) \div 4 = 10$

56. Nurses use formulas to calculate a drug dosage given a supply dosage and a doctor's drug order. They also use formulas to calculate IV flow rates. Answers should include the following.
- A table of IV flow rates is limited to those situations listed, while a formula can be used to find any IV flow rate.
- If a formula used in a nursing setting is applied incorrectly, a patient could die.

1-2 Properties of Real Numbers

What You'll Learn

- Classify real numbers.
- Use the properties of real numbers to evaluate expressions.

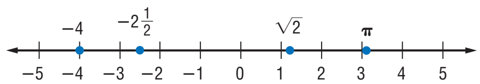
Vocabulary

- real numbers
- rational numbers
- irrational numbers

How is the Distributive Property useful in calculating store savings?

Manufacturers often offer coupons to get consumers to try their products. Some grocery stores try to attract customers by doubling the value of manufacturers' coupons. You can use the Distributive Property to calculate these savings.

REAL NUMBERS All of the numbers that you use in everyday life are **real numbers**. Each real number corresponds to exactly one point on the number line, and every point on the number line represents exactly one real number.



Real numbers can be classified as either **rational** or **irrational**.

Study Tip

Reading Math

A *ratio* is the comparison of two numbers by division.

Key Concept — *Real Numbers*

Rational Numbers

- **Words** A rational number can be expressed as a ratio $\frac{m}{n}$, where m and n are integers and n is not zero. The decimal form of a rational number is either a terminating or repeating decimal.
- **Examples** $\frac{1}{6}$, 1.9, 2.575757..., −3, $\sqrt{4}$, 0

Irrational Numbers

- **Words** A real number that is not rational is irrational. The decimal form of an irrational number neither terminates nor repeats.
- **Examples** $\sqrt{5}$, π, 0.010010001...

The sets of natural numbers, {1, 2, 3, 4, 5, ...}, whole numbers, {0, 1, 2, 3, 4, ...}, and integers, {..., −3, − 2, − 1, 0, 1, 2, ...} are all subsets of the rational numbers. The whole numbers are a subset of the rational numbers because every whole number n is equal to $\frac{n}{1}$.

1-2 Lesson Notes

1 Focus

5-Minute Check Transparency 1-2 Use as a quiz or review of Lesson 1-1.

Mathematical Background notes are available for this lesson on p. 4C.

Building on Prior Knowledge

In Lesson 1-1, students simplified and evaluated expressions. In this lesson, they broaden those skills to include using the real numbers and applying the commutative, associative, identity, inverse, and distributive properties of real numbers.

How is the Distributive Property useful in calculating store savings?

Ask students:

- In the list of Scanned Coupons and Bonus Coupons shown, what does 0.30 mean? **30¢**
- Why is there a negative sign after the decimal numbers? **The negative sign indicates that the amount is taken off or subtracted from the price.**

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters

- Study Guide and Intervention, pp. 7–8
- Skills Practice, p. 9
- Practice, p. 10
- Reading to Learn Mathematics, p. 11
- Enrichment, p. 12
- Assessment, p. 51

Teaching Algebra With Manipulatives Masters, p. 212

Transparencies

5-Minute Check Transparency 1-2
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

REAL NUMBERS

Teaching Tip Point out that a nonterminating decimal whose digits show a pattern but which has no repeating group of digits, such as the number 0.010010001… given in the Key Concepts examples on p. 11, is irrational. Another example is the number 1.232233222333….

In-Class Example

Power Point®

1 Name the sets of numbers to which each number belongs.

a. $-\frac{2}{3}$ **Q, R**

b. 9.999… **Q, R**

c. $\sqrt{6}$ **I, R**

d. $\sqrt{100}$ **N, W, Z, Q, R**

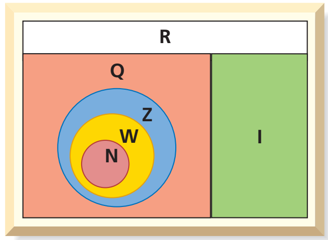
e. −23.3 **Q, R**

Reading Tip Ask students whether *fraction* and *rational number* mean the same thing. (No; 4 is not a fraction but it is a rational number. *Fraction* refers to the form of a number: $\frac{8}{4}$ is in the form of a fraction but it is a whole number in value.)

PROPERTIES OF REAL NUMBERS

Reading Tip Help students remember the names of properties by connecting the term *commutative* with "commuting, or moving from one position to another," and by connecting the term *associative* with "the people you associate with, or your group."

Study Tips offer students helpful information about the topics they are studying.



The Venn diagram shows the relationships among these sets of numbers.

R = reals	Q = rationals
I = irrationals	Z = integers
W = wholes	N = naturals

The square root of any whole number is either a whole number or it is irrational. For example, $\sqrt{36}$ is a whole number, but $\sqrt{35}$, since it lies between 5 and 6, must be irrational.

Study Tip

Common Misconception
Do not assume that a number is irrational because it is expressed using the square root symbol. Find its value first.

Example 1 Classify Numbers

Name the sets of numbers to which each number belongs.

a. $\sqrt{16}$

$\sqrt{16} = 4$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

b. −185 integers (Z), rationals (Q), and reals (R)

c. $\sqrt{20}$ irrationals (I) and reals (R)

$\sqrt{20}$ lies between 4 and 5 so it is not a whole number.

d. $-\frac{7}{8}$ rationals (Q) and reals (R)

e. $0.\overline{45}$ rationals (Q) and reals (R)

The bar over the 45 indicates that those digits repeat forever.

PROPERTIES OF REAL NUMBERS The real number system is an example of a mathematical structure called a *field*. Some of the properties of a field are summarized in the table below.

Study Tip

Reading Math
−a is read the opposite of a.

Key Concepts *Real Number Properties*

For any real numbers *a*, *b*, and *c*:

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	If $a \neq 0$, then $a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$.
Distributive	$a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	

Example 2 Identify Properties of Real Numbers

Name the property illustrated by each equation.

a. $(5 + 7) + 8 = 8 + (5 + 7)$

Commutative Property of Addition

The Commutative Property says that the order in which you add does not change the sum.

b. $3(4x) = (3 \cdot 4)x$

Associative Property of Multiplication

The Associative Property says that the way you group three numbers when multiplying does not change the product.

Example 3 Additive and Multiplicative Inverses

Identify the additive inverse and multiplicative inverse for each number.

a. $-1\frac{3}{4}$

Since $-1\frac{3}{4} + \left(1\frac{3}{4}\right) = 0$, the additive inverse of $-1\frac{3}{4}$ is $1\frac{3}{4}$.

Since $-1\frac{3}{4} = -\frac{7}{4}$ and $\left(-\frac{7}{4}\right)\left(-\frac{4}{7}\right) = 1$, the multiplicative inverse of $-1\frac{3}{4}$ is $-\frac{4}{7}$.

b. 1.25

Since $1.25 + (-1.25) = 0$, the additive inverse of 1.25 is -1.25.

The multiplicative inverse of 1.25 is $\frac{1}{1.25}$ or 0.8.

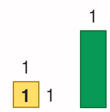
CHECK Notice that $1.25 \times 0.8 = 1$. ✓

You can model the Distributive Property using algebra tiles.

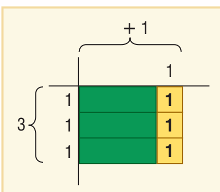
Algebra Activity

Distributive Property

- A 1 tile is a square that is 1 unit wide and 1 unit long. Its area is 1 square unit. An x tile is a rectangle that is 1 unit wide and x units long. Its area is x square units.



- To find the product $3(x + 1)$, model a rectangle with a width of 3 and a length of $x + 1$. Use your algebra tiles to mark off the dimensions on a product mat. Then make the rectangle with algebra tiles.
- The rectangle has 3 x tiles and 3 1 tiles. The area of the rectangle is $x + x + x + 1 + 1 + 1$ or $3x + 3$. Thus, $3(x + 1) = 3x + 3$.



Model and Analyze

Tell whether each statement is *true* or *false*. Justify your answer with algebra tiles and a drawing. **1–4. See pp. 53A–53B for drawings.**

1. $4(x + 2) = 4x + 2$ **false** **2.** $3(2x + 4) = 6x + 7$ **false**

3. $2(3x + 5) = 6x + 10$ **true** **4.** $(4x + 1)5 = 4x + 5$ **false**

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

2 Name the property illustrated by each equation.

a. $(-8 + 8) + 15 = 0 + 15$ **Additive Inverse Property**

b. $5(8 - 6) = 5(8) - 5(6)$ **Distributive Property**

3 Identify the additive inverse and multiplicative inverse for each number.

a. -7 **additive: 7; multiplicative: $-\frac{1}{7}$**

b. $\sqrt{\frac{1}{9}}$ **additive: $-\sqrt{\frac{1}{9}}$ or $-\frac{1}{3}$; multiplicative: 3**

Teaching Tip Make sure students understand that additive inverses must have a sum of 0 and that multiplicative inverses must have a product of 1.

In-Class Examples, which are included for every example in the Student Edition, exactly parallel the examples in the text. Teaching Tips about the Student Edition examples are included where appropriate.

Concept Check

Real Number Properties Ask students to name some mathematical operations that are *not* commutative and to give examples supporting their choices. **subtraction and division; $7 - 3 \neq 3 - 7$; $8 \div 2 \neq 2 \div 8$**

Algebra Activity

Materials: algebra tiles, product mat

- Have students verify with their tiles that the length of an x tile is not a multiple of the side length of a 1 tile.
- Suggest that students can verify they have modeled an expression like $2(3x + 5)$ correctly if they read the expression as "2 rows of 3 x tiles and 5 1 tiles." If they arrange their models like the one shown in the book, the rows of tiles can be "read" from left to right just as when reading the text.

In-Class Examples

Power Point®

4 **POSTAGE** Audrey went to a post office and bought eight 34¢ first-class stamps and eight 21¢ postcard stamps. How much did Audrey spend altogether on stamps? **8(0.34) + 8(0.21) or 8(0.34 + 0.21)**

Simplify $4(3a - b) + 2(b + 3a)$. **$18a - 2b$**

5 *Reading Tip* Help students recall the Distributive Property by connecting the name to "distributing or handing out papers, one to each person." Point out that the factor outside of the parentheses acts as a multiplier for each term within the parentheses.

Answer

2. A rational number is the ratio of two integers. Since $\sqrt{3}$ is not an integer, $\frac{\sqrt{3}}{2}$ is not a rational number.

The Distributive Property is often used in real-world applications.

Example 4 Use the Distributive Property to Solve a Problem

FOOD SERVICE A restaurant adds a 20% tip to the bills of parties of 6 or more people. Suppose a server waits on five such tables. The bill without the tip for each party is listed in the table. How much did the server make in tips during this shift?

Party 1	Party 2	Party 3	Party 4	Party 5
185 45	205 20	195 05	245 80	262 00

There are two ways to find the total amount of tips received.

Method 1

Multiply each dollar amount by 20% or 0.2 and then add.

$T = 0.2(185.45) + 0.2(205.20) + 0.2(195.05) + 0.2(245.80) + 0.2(262)$
$= 37.09 + 41.04 + 39.01 + 49.16 + 52.40$
$= 218.70$

Method 2

Add the bills of all the parties and then multiply the total by 0.2.

$T = 0.2(185.45 + 205.20 + 195.05 + 245.80 + 262)$
$= 0.2(1093.50)$
$= 218.70$

The server made \$218.70 during this shift.

Notice that both methods result in the same answer.

More About...

Food Service

Leaving a "tip" began in 18th century English coffee houses and is believed to have originally stood for "To Insure Promptness." Today, the American Automobile Association suggests leaving a 15% tip.

Source: Market Facts, Inc.

The properties of real numbers can be used to simplify algebraic expressions.

Example 5 Simplify an Expression

Simplify $2(5m + n) + 3(2m - 4n)$.

$2(5m + n) + 3(2m - 4n)$

$= 2(5m) + 2(n) + 3(2m) - 3(4n)$	Distributive Property
$= 10m + 2n + 6m - 12n$	Multiply.
$= 10m + 6m + 2n - 12n$	Commutative Property (+)
$= (10 + 6)m + (2 - 12)n$	Distributive Property
$= 16m - 10n$	Simplify.

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of each type of number. **Sample answers given.**
   a. natural **2** b. whole **5** c. integer **−11**
   d. rational **1.3** e. irrational **$\sqrt{2}$** f. real **−1.3**
2. **Explain** why $\frac{\sqrt{3}}{2}$ is *not* a rational number. **See margin.**
3. **Disprove** the following statement by giving a counterexample. A **counterexample** is a specific case that shows that a statement is false. Explain.
   Every real number has a multiplicative inverse.

3. 0; zero does not have a multiplicative inverse since $\frac{1}{0}$ is undefined.

Daily Intervention notes help you help students when they need it most. Differentiated Instruction suggestions are keyed to eight commonly-accepted learning styles.

DAILY INTERVENTION — Differentiated Instruction

Kinesthetic To model the Distributive Property, write 7(8 + 6) on the board. Then have a student distribute an index card with 7 on it to a student holding an index card with 8 written on it and also distribute an index card with 7 on it to a student holding an index card with 6 written on it. Ask each student holding 2 cards to name their product. Have the student who distributed the 7s find the sum of the products. Complete the equation on the board: 7(8 + 6) = 7(8) + 7(6).

Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
4–6	1
7–9	2
10–12	3
13–16	5
17, 18	4

Name the sets of numbers to which each number belongs.

4. -4 **Z, Q, R**
5. 45 **N, W, Z, Q, R**
6. $6.\overline{23}$ **Q, R**

Name the property illustrated by each equation.

7. $\frac{2}{3} \cdot \frac{3}{2} = 1$ **Mult. Iden.**
8. $(a + 4) + 2 = a + (4 + 2)$ **Assoc. (+)**
9. $4x + 0 = 4x$ **Add. Iden.**

Identify the additive inverse and multiplicative inverse for each number.

10. -8 **$8, -\frac{1}{8}$**
11. $\frac{1}{3}$ **$-\frac{1}{3}, 3$**
12. 1.5 **$-1.5, \frac{2}{3}$**

Simplify each expression.

13. $3x + 4y - 5x$ **$-2x + 4y$**
14. $9p - 2n + 4p + 2n$ **$13p$**
15. $3(5c + 4d) + 6(d - 2c)$ **$3c + 18d$**
16. $\frac{1}{2}(16 - 4a) - \frac{3}{4}(12 + 20a)$ **$-17a - 1$**

Application

BAND BOOSTERS **For Exercises 17 and 18, use the information below and in the table.**
Ashley is selling chocolate bars for $1.50 each to raise money for the band.

Ashley's Sales for One Week	
Day	**Bars Sold**
Monday	10
Tuesday	15
Wednesday	12
Thursday	8
Friday	19
Saturday	22
Sunday	31

17. Write an expression to represent the total amount of money Ashley raised during this week.
18. Evaluate the expression from Exercise 17 by using the Distributive Property. **$175.50**

17. 1.5(10 + 15 + 12 + 8 + 19 + 22 + 31) or 1.5(10) + 1.5(15) + 1.5(12) + 1.5(8) + 1.5(19) + 1.5(22) + 1.5(31)

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
19–27, 40–42, 59–62	1
28–39	2
43–48	3
63–65	4
49–58, 66–69	5

Extra Practice

See page 828.

Homework Help charts show students which examples to which they need to refer if they need additional practice. Extra Practice for every lesson is provided on pages 828–861.

Name the sets of numbers to which each number belongs. **19–26. See margin.**

19. 0
20. $-\frac{2}{9}$
21. $\sqrt{121}$
22. -4.55
23. $\sqrt{10}$
24. -31
25. $\frac{12}{2}$
★26. $\frac{3\pi}{2}$

★27. Name the sets of numbers to which all of the following numbers belong. Then arrange the numbers in order from least to greatest.
$2.\overline{49}$, $2.4\overline{9}$, 2.4, 2.49, $2.\overline{9}$ **Q, R; 2.4, 2.49, $2.\overline{49}$, $2.4\overline{9}$, $2.\overline{9}$**

Name the property illustrated by each equation. **31. Assoc. (+)**

28. $5a + (-5a) = 0$ **Add. Inv.**
29. $(3 \cdot 4) \cdot 25 = 3 \cdot (4 \cdot 25)$ **Assoc. (×)**
30. $-6xy + 0 = -6xy$ **Add. Iden.**
31. $[5 + (-2)] + (-4) = 5 + [-2 + (-4)]$
32. $(2 + 14) + 3 = 3 + (2 + 14)$ **Comm. (+)**
33. $\left(1\frac{2}{7}\right)\left(\frac{7}{9}\right) = 1$ **Mult. Inv.**
34. $2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$ **Dist.**
35. $ab = 1ab$ **Multi. Iden.**

NUMBER THEORY **For Exercises 36–39, use the properties of real numbers to answer each question.** **37. $-m$; Add. Inv.** **38. $\frac{1}{m}$; Multi. Inv.**

36. If $m + n = m$, what is the value of n? **0**
37. If $m + n = 0$, what is the value of n? What is n called with respect to m?
38. If $mn = 1$, what is the value of n? What is n called with respect to m?
39. If $mn = m$, what is the value of n? **1**

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
- copy the Venn diagram on p. 12, and add at least three examples for each set.
- copy the table of Real Number Properties and add examples that use whole numbers.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Real Numbers:** 19–27, 59–62
- **Properties of Real Numbers:** 28–58, 63–69

Exercises 19–26, 28–39, and 43–62 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 19–25 odd, 29–39 odd, 40–42, 43–57 odd, 59, 61, 63–64, 65, 67, 69, 70–73, 78–86

Average: 19–39 odd, 40–42, 43–61 odd, 63–65, 67–73, 78–86 (optional: 74–77)

Advanced: 20–38 even, 40–42, 44–62 even, 66–82 (optional: 83–86)

All: Practice Quiz 1 (1–10)

DAILY INTERVENTION

Unlocking Misconceptions

Positive Root Remind students that $\sqrt{9}$ means only the positive root, if one exists, so $\sqrt{9} = 3$. To indicate both roots of the equation $x^2 = 9$, the mathematical notation is $x = \pm\sqrt{9}$ or $x = \pm 3$.

Answers

19. W, Z, Q, R
20. Q, R
21. N, W, Z, Q, R
22. Q, R
23. I, R
24. Z, Q, R
25. N, W, Z, Q, R
26. I, R

Answers

65. $3\left(2\frac{1}{4}\right) + 2\left(1\frac{1}{8}\right)$

$= 3\left(2 + \frac{1}{4}\right) + 2\left(1 + \frac{1}{8}\right)$ Definition of a mixed number

$= 3(2) + 3\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{1}{8}\right)$ Distributive Property

$= 6 + \frac{3}{4} + 2 + \frac{1}{4}$ Multiply.

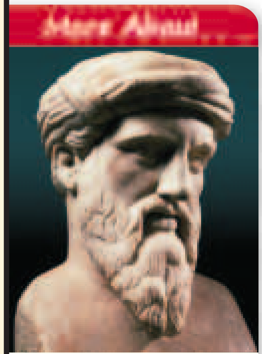
$= 6 + 2 + \frac{3}{4} + \frac{1}{4}$ Commutative Property of Addition

$= 8 + \frac{3}{4} + \frac{1}{4}$ Add.

$= 8 + \left(\frac{3}{4} + \frac{1}{4}\right)$ Associative Property of Addition

$= 8 + 1$ or 9 Add.

71. Answers should include the following.
- Instead of doubling each coupon value and then adding these values together, the Distributive Property could be applied allowing you to add the coupon values first and then double the sum.
- If a store had a 25% off sale on all merchandise, the Distributive Property could be used to calculate these savings. For example, the savings on a $15 shirt, $40 pair of jeans, and $25 pair of slacks could be calculated as $0.25(15) + 0.25(40) + 0.25(25)$ or as $0.25(15 + 40 + 25)$ using the Distributive Property.



Math History

Pythagoras (572–497 B.C.), was a Greek philosopher whose followers came to be known as the Pythagoreans. It was their knowledge of what is called the Pythagorean Theorem that led to the first discovery of irrational numbers.

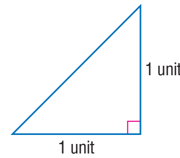
Source: *A History of Mathematics*

MATH HISTORY **For Exercises 40–42, use the following information.**
The Greek mathematician Pythagoras believed that all things could be described by numbers. By "number" he meant positive integers.

40. To what set of numbers was Pythagoras referring when he spoke of "numbers?" **natural numbers**

41. Use the formula $c = \sqrt{2s^2}$ to calculate the length of the hypotenuse c, or longest side, of this right triangle using s, the length of one leg. **$\sqrt{2}$ units**

42. Explain why Pythagoras could not find a "number" to describe the value of c. **The square root of 2 is irrational and therefore cannot be described by a natural number.**



Name the additive inverse and multiplicative inverse for each number.

43. -10 **$10; -\frac{1}{10}$** **44.** 2.5 **$-2.5; 0.4$** **45.** -0.125 **$0.125; -8$**

46. $-\frac{5}{8}$ **$\frac{5}{8}, -\frac{8}{5}$** **47.** $\frac{4}{3}$ **$-\frac{4}{3}, \frac{3}{4}$** **48.** $-4\frac{3}{5}$ **$4\frac{3}{5}; -\frac{5}{23}$**

Simplify each expression. **55. $-3.4m + 1.8n$** **56. $4.4p - 2.9q$**

49. $7a + 3b - 4a - 5b$ **$3a - 2b$** **50.** $3x + 5y + 7x - 3y$ **$10x + 2y$**

51. $3(15x - 9y) + 5(4y - x)$ **$40x - 7y$** **52.** $2(10m - 7a) + 3(8a - 3m)$ **$11m + 10a$**

53. $8(r + 7t) - 4(13t + 5r)$ **$-12r + 4t$** **54.** $4(14c - 10d) - 6(d + 4c)$ **$32c - 46d$**

55. $4(0.2m - 0.3n) - 6(0.7m - 0.5n)$ **56.** $7(0.2p + 0.3q) + 5(0.6p - q)$

57. $\frac{1}{4}(6 + 20y) - \frac{1}{2}(19 - 8y)$ **$-8 + 9y$** ★**58.** $\frac{1}{6}(3x + 5y) + \frac{2}{3}\left(\frac{3}{5}x - 6y\right)$ **$\frac{9}{10}x - \frac{19}{6}y$**

Determine whether each statement is *true* or *false*. If false, give a counterexample.

59. true
60. false; −3

59. Every whole number is an integer. **60.** Every integer is a whole number.

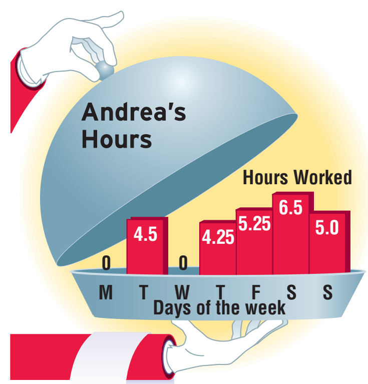
61. Every real number is irrational. **false; 6** **62.** Every integer is a rational number. **true**

WORK **For Exercises 63 and 64, use the information below and in the table.**
Andrea works as a hostess in a restaurant and is paid every two weeks.

63. $6.5(4.5 + 4.25 + 5.25 + 6.5 + 5)$ or $6.5(4.5) + 6.5(4.25) + (6.5)5.25 + 6.5(6.5) + 6.5(5)$

63. If Andrea earns $6.50 an hour, illustrate the Distributive Property by writing two expressions representing Andrea's pay last week.

64. Find the mean or average number of hours Andrea worked each day, to the nearest tenth of an hour. Then use this average to predict her pay for a two-week pay period. **3.6; $327.60**

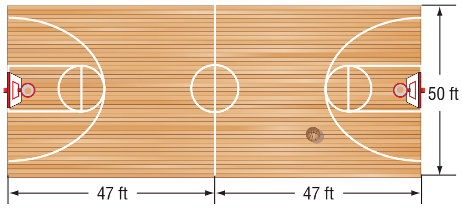


65. BAKING Mitena is making two types of cookies. The first recipe calls for $2\frac{1}{4}$ cups of flour, and the second calls for $1\frac{1}{8}$ cups of flour. If Mitena wants to make 3 batches of the first recipe and 2 batches of the second recipe, how many cups of flour will she need? Use the properties of real numbers to show how Mitena could compute this amount mentally. Justify each step. **See margin.**

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

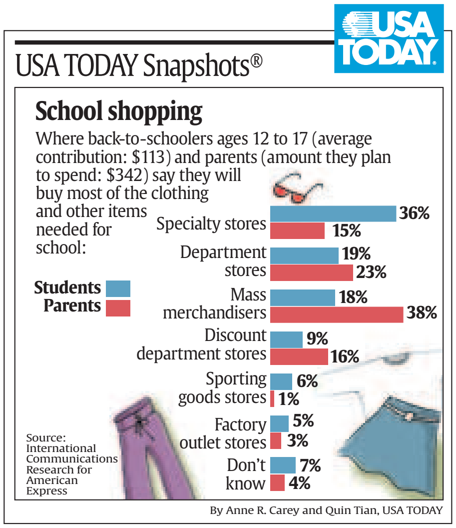
BASKETBALL For Exercises 66 and 67, use the diagram of an NCAA basketball court below.



66. Illustrate the Distributive Property by writing two expressions for the area of the basketball court. **50(47 + 47); 50(47) + 50(47)**

67. Evaluate the expression from Exercise 66 using the Distributive Property. What is the area of an NCAA basketball court? **4700 ft^2**

SCHOOL SHOPPING For Exercises 68 and 69, use the graph at the right.



68. Illustrate the Distributive Property by writing two expressions to represent the amount that the average student spends shopping for school at specialty stores and department stores.

68. $113(0.36 + 0.19); $113(0.36) + $113(0.19)

69. Evaluate the expression from Exercise 68 using the Distributive Property. **$62.15**

70. **CRITICAL THINKING** Is the Distributive Property also true for division? In other words, does $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$, $a \neq 0$? If so, give an example and explain why it is true. If not true, give a counterexample.

70. Yes; $\frac{6+8}{2} = \frac{6}{2} + \frac{8}{2} = 7$; dividing by a number is the same as multiplying by its reciprocal.

Standardized Test Practice exercises were created to closely parallel those on actual state proficiency tests and college entrance exams.

71. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How is the Distributive Property useful in calculating store savings?

Include the following in your answer:

- an explanation of how the Distributive Property could be used to calculate the coupon savings listed on a grocery receipt, and
- an example of how the Distributive Property could be used to calculate the savings from a clothing store sale where all items were discounted by the same percent.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

72. If a and b are natural numbers, then which of the following must also be a natural number? **B**

I. $a - b$ **II.** ab **III.** $\frac{a}{b}$

Ⓐ I only Ⓑ II only Ⓒ III only

Ⓓ I and II only Ⓔ II and III only

73. If $x = 1.4$, find the value of $27(x + 1.2) - 26(x + 1.2)$. **C**

Ⓐ 1 Ⓑ −0.4 Ⓒ 2.6 Ⓓ 65

Study Guide and Intervention, p. 7 (shown) and p. 8

Real Numbers All real numbers can be classified as either rational or irrational. The set of rational numbers includes several subsets: natural numbers, whole numbers, and integers.

R		
Q		n 0 $\frac{m}{n}$ m n
I		
N		1 2 3 4 5 6 7 8 9
W		0 1 2 3 4 5 6 7 8
Z		−3 −2 −1 0 1 2 3

Example **Name the sets of numbers to which each number belongs.**

a. $-\frac{11}{3}$ rationals (Q), reals (R)

b. $\sqrt{25}$
$\sqrt{25} = 5$ naturals (N), wholes (W), integers (Z), rationals (Q), reals (R)

Exercises

Name the sets of numbers to which each number belongs.

1. $\frac{6}{7}$ **Q, R** 2. $-\sqrt{81}$ **Z, Q, R** 3. 0 **W, Z, Q, R** 4. 192.0005 **Q, R**

5. 73 **N, W, Z, Q, R** 6. $34\frac{1}{2}$ **Q, R** 7. $\frac{\sqrt{36}}{9}$ **Q, R** 8. 26.1 **Q, R**

9. π **I, R** 10. $\frac{15}{3}$ **N, W, Z, Q, R** 11. $-4.\overline{17}$ **Q, R**

12. $\frac{\sqrt{25}}{5}$ **N, W, Z, Q, R** 13. −1 **Z, Q, R** 14. $\sqrt{42}$ **I, R**

15. −11.2 **Q, R** 16. $-\frac{8}{13}$ **Q, R** 17. $\frac{\sqrt{5}}{2}$ **I, R**

18. $33.\overline{3}$ **Q, R** 19. 894,000 **N, W, Z, Q, R** 20. −0.02 **Q, R**

Skills Practice, p. 9 and Practice, p. 10 (shown)

Name the sets of numbers to which each number belongs.

1. 6425 **N, W, Z, Q, R** 2. $\sqrt{7}$ **I, R** 3. 2π **I, R** 4. 0 **W, Z, Q, R**

5. $\sqrt{\frac{25}{36}}$ **Q, R** 6. $-\sqrt{16}$ **Z, Q, R** 7. −35 **Z, Q, R** 8. −31.8 **Q, R**

Name the property illustrated by each equation.

9. $5x \cdot (4y + 3x) = 5x \cdot (3x + 4y)$ **Comm. (+)**

10. $7x + (9x + 8) = (7x + 9x) + 8$ **Assoc. (+)**

11. $5(3x + y) = 5(3x + 1y)$ **Mult. Iden.**

12. $7n + 2n = (7 + 2)n$ **Distributive**

13. $3(2x)y = (3 \cdot 2)(xy)$ **Assoc. (×)**

14. $3x \cdot 2y = 3 \cdot 2 \cdot x \cdot y$ **Comm. (×)**

15. $(6 + -6)y = 0y$ **Add. Inv.**

16. $\frac{1}{4} \cdot 4y = 1y$ **Mult. Inv.**

17. $5(x + y) = 5x + 5y$ **Distributive**

18. $4n + 0 = 4n$ **Add. Iden.**

Name the additive inverse and multiplicative inverse for each number.

19. 0.4 **−0.4, 2.5** 20. −1.6 **1.6, −0.625**

21. $-\frac{11}{16}$ **$\frac{11}{16}, -\frac{16}{11}$** 22. $5\frac{5}{6}$ **$-5\frac{5}{6}, \frac{6}{35}$**

Simplify each expression.

23. $5x - 3y - 2x + 3y$ **$3x$**

24. $-11a - 13b + 7a - 3b$ **$-4a - 16b$**

25. $8x - 7y - (3 - 6y)$ **$8x - y - 3$**

26. $4c - 2c - (4c + 2c)$ **$-4c$**

27. $3(r - 10s) - 4(7s + 2r)$ **$-5r - 58s$**

28. $\frac{1}{5}(10a - 15) + \frac{1}{2}(8 + 4a)$ **$4a + 1$**

29. $2(4 - 2x + y) - 4(5 + x - y)$ **$-12 - 8x + 6y$**

30. $\frac{5}{6}\left(\frac{3}{5}x + 12y\right) - \frac{1}{4}(2x - 12y)$ **$13y$**

31. **TRAVEL** Olivia drives her car at 60 miles per hour for t hours. Ian drives his car at 50 miles per hour for $(t + 2)$ hours. Write a simplified expression for the sum of the distances traveled by the two cars. **$(110t + 100)$ mi**

32. **NUMBER THEORY** Use the properties of real numbers to tell whether the following statement is true or false: If $a > b$, it follows that $a\left(\frac{1}{a}\right) > b\left(\frac{1}{b}\right)$. Explain your reasoning. **false; counterexample: $5\left(\frac{1}{5}\right) \not> 4\left(\frac{1}{4}\right)$**

Reading to Learn Mathematics, p. 11 ELL

Pre-Activity **How is the Distributive Property useful in calculating store savings?**

Read the introduction to Lesson 1-2 at the top of page 11 in your textbook.

- Why are all of the amounts listed on the register slip at the top of page 11 followed by negative signs? **Sample answer: The amount of each coupon is subtracted from the total amount of purchases so that you save money by using coupons.**
- Describe two ways of calculating the amount of money you saved by using coupons if your register slip is the one shown on page 11. **Sample answer: Add all the individual coupon amounts or add the amounts for the scanned coupons and multiply the sum by 2.**

Reading the Lesson

1. Refer to the Key Concepts box on page 11. The numbers $2.\overline{57}$ and 0.010010001… both involve decimals that "go on forever." Explain why one of these numbers is rational and the other is irrational. **Sample answer: $2.\overline{57}$ = 2.5757… is a repeating decimal because there is a block of digits, 57, that repeats forever, so this number is rational. The number 0.010010001… is a non-repeating decimal because, although the digits follow a pattern, there is no block of digits that repeats. So this number is an irrational number.**

2. Write the Associative Property of Addition in symbols. Then illustrate this property by finding the sum 12 + 18 + 45 in two different ways. **$(a + b) + c = a + (b + c)$; Sample answer: (12 + 18) + 45 = 30 + 45 = 75; 12 + (18 + 45) = 12 + 63 = 75**

3. Consider the equations $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ and $(a \cdot b) \cdot c = c \cdot (a \cdot b)$. One of the equations uses the Associative Property of Multiplication and one uses the Commutative Property of Multiplication. How can you tell which property is being used in each equation? **The first equation uses the Associative Property of Multiplication. The quantities *a*, *b*, and *c* are used in the same order, but they are grouped differently on the two sides of the equation. The second equation uses the quantities in different orders on the two sides of the equation. So the second equation uses the Commutative Property of Multiplication.**

Helping You Remember

4. How can the meanings of the words *commuter* and *association* help you to remember the difference between the commutative and associative properties? **Sample answer: A commuter is someone who travels back and forth to work or another place, and the commutative property says you can switch the *order* when two numbers that are being added or multiplied. An association is a group of people who are connected or united, and the associative property says that you can switch the *grouping* when three numbers are added or multiplied.**

Enrichment, p. 12

Properties of a Group

A set of numbers forms a group with respect to an operation if for that operation the set has (1) the Closure Property, (2) the Associative Property, (3) a member which is an identity, and (4) an inverse for each member of the set.

Example 1 **Does the set {0, 1, 2, 3, …} form a group with respect to addition?**

Closure Property: For all numbers in the set, is $a + b$ in the set? 0 + 1 = 1, and 1 is in the set; 0 + 2 = 2, and 2 is in the set; and so on. The set has closure for addition.

Associative Property: For all numbers in the set, does $a + (b + c) = (a + b) + c$? $0 + (1 + 2) = (0 + 1) + 2$; $1 + (2 + 3) = (1 + 2) + 3$; and so on. The set is associative for addition.

Identity: Is there some number, i, in the set such that $i + a = a = a + i$ for all a? 0 + 1 = 1 = 1 + 0; 0 + 2 = 2 = 2 + 0; and so on. The identity for addition is 0.

Inverse: Does each number, a, have an inverse, a', such that $a' + a = a + a' = i$? The integer inverse of 3 is −3 since −3 …

4 Assess

Open-Ended Assessment

Modeling Ask students to give examples of each of the properties (identity, inverse, commutative, associative, and distributive) and examples for each set of numbers (reals, rationals, irrationals, integers, wholes, and naturals).

Getting Ready for Lesson 1-3

PREREQUISITE SKILL Lesson 1-3 presents translating verbal expressions into algebraic expressions and using the properties of equality to solve equations. After solving an equation, the solution is checked in the original equation by evaluating the expression on each side after replacing the variable with its numerical value. Use Exercises 83–86 to determine your students' familiarity with evaluating expressions.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 1-1 and 1-2. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 1-1 and 1-2) is available on p. 51 of the *Chapter 1 Resource Masters*.

Daily Intervention notes help you help students when they need it most. Unlocking Misconceptions suggestions help you analyze where students make common errors so you can point these trouble spots out to them.

Extending the Lesson

For Exercises 74–77, use the following information.
The product of any two whole numbers is always a whole number. So, the set of whole numbers is said to be *closed* under multiplication. This is an example of the **Closure Property**. State whether each statement is *true* or *false*. If false, give a counterexample. **75. False; $0 - 1 = -1$, which is not a whole number.**

74. The set of integers is closed under multiplication. **true**

75. The set of whole numbers is closed under subtraction.

76. The set of rational numbers is closed under addition. **true**

77. The set of whole numbers is closed under division.
False, $2 \div 3 = \frac{2}{3}$, which is not a whole number.

Maintain Your Skills

Mixed Review

Find the value of each expression. *(Lesson 1-1)*

78. $9(4 - 3)^5$ **9**

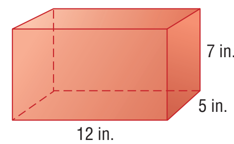
79. $5 + 9 \div 3(3) - 8$ **6**

Evaluate each expression if $a = -5$, $b = 0.25$, $c = \frac{1}{2}$, and $d = 4$. *(Lesson 1-1)*

80. $a + 2b - c$ **−5**

81. $b + 3(a + d)^3$ **−2.75**

82. **GEOMETRY** The formula for the surface area SA of a rectangular prism is $SA = 2\ell w + 2\ell h + 2wh$, where ℓ represents the length, w represents the width, and h represents the height. Find the surface area of the rectangular prism. *(Lesson 1-1)* **358 in^2**



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $a = 2$, $b = -\frac{3}{4}$, and $c = 1.8$.**
*(To review **evaluating expressions**, see Lesson 1-1.)*

83. $8b - 5$ **−11**

84. $\frac{2}{5}b + 1$ **$\frac{7}{10}$**

85. $1.5c - 7$ **−4.3**

86. $-9(a - 6)$ **36**

Two Quizzes in each chapter review skills and concepts presented in previous lessons.

Practice Quiz 1 — *Lessons 1-1 and 1-2*

Find the value of each expression. *(Lesson 1-1)*

1. $18 - 12 \div 3$ **14**
2. $-4 + 5(7 - 2^3)$ **−9**
3. $\frac{18 + 3 \times 4}{13 - 8}$ **6**
4. Evaluate $a^3 + b(9 - c)$ if $a = -2$, $b = \frac{1}{3}$, and $c = -12$. *(Lesson 1-1)* **−1**
5. **ELECTRICITY** Find the amount of current I (in amperes) produced if the electromotive force E is 2.5 volts, the circuit resistance R is 1.05 ohms, and the resistance r within a battery is 0.2 ohm. Use the formula $I = \frac{E}{R + r}$. *(Lesson 1-1)* **2 amperes**

Name the sets of numbers to which each number belongs. *(Lesson 1-2)*

6. 3.5 **Q, R**
7. $\sqrt{100}$ **N, W, Z, Q, R**
8. Name the property illustrated by $bc + (-bc) = 0$. *(Lesson 1-2)* **Add. Inv.**
9. Name the additive inverse and multiplicative inverse of $\frac{6}{7}$. *(Lesson 1-2)* **$-\frac{6}{7}, \frac{7}{6}$**
10. Simplify $4(14x - 10y) - 6(x + 4y)$. *(Lesson 1-2)* **$50x - 64y$**

DAILY INTERVENTION

Unlocking Misconceptions

Associative or Commutative Students sometimes use inappropriate visual cues to name properties. For example, they may think that an expression can only have two terms to be an example of commutativity. Suggest that students look first at the change from one expression to the other and ask themselves if it is a change in grouping (associativity) or in position (commutativity).

Algebra Activity

A Follow-Up of Lesson 1-2

Investigating Polygons and Patterns

Collect the Data

Use a ruler or geometry drawing software to draw six large polygons with 3, 4, 5, 6, 7, and 8 sides. The polygons do not need to be regular. Convex polygons, ones whose diagonals lie in the interior, will be best for this activity.

1. Copy the table below and complete the column labeled *Diagonals* by drawing the diagonals for all six polygons and record your results.

Figure Name	Sides (n)	Diagonals	Diagonals From One Vertex
	3	0	0
	4	2	1
	5	**5**	**2**
	6	**9**	**3**
	7	**14**	**4**
	8	**20**	**5**

Analyze the Data

2. Describe the pattern shown by the number of diagonals in the table above. **See pp. 53A–53B.**

3. Complete the last column in the table above by recording the number of diagonals that can be drawn from one vertex of each polygon.

4. Write an expression in terms of n that relates the number of diagonals from one vertex to the number of sides for each polygon. **$n - 3$**

5. If a polygon has n sides, how many vertices does it have? **n**

6. How many vertices does one diagonal connect? **2**

Algebra Activities use manipulatives and models to help students learn key concepts. There are teacher notes for every Algebra Activity in the Student Edition.

Make a Conjecture

7. Write a formula in terms of n for the number of diagonals of a polygon of n sides. (*Hint:* Consider your answers to Exercises 2, 3, and 4.) **$[n(n - 3)] \div 2$**

8. Draw a polygon with 10 sides. Test your formula for the decagon. **See pp. 53A–53B.**

9. Explain how your formula relates to the number of vertices of the polygon and the number of diagonals that can be drawn from each vertex. **See pp. 53A–53B.**

Extend the Activity

10. Draw 3 noncollinear dots on your paper. Determine the number of lines that are needed to connect each dot to every other dot. Continue by drawing 4 dots, 5 dots, and so on and finding the number of lines to connect them. **See pp. 53A–53B.**

11. Copy and complete the table at the right. **See table.**

12. Use any method to find a formula that relates the number of dots, x, to the number of lines, y. **$y = [x(x - 1)] \div 2$ or $y = 0.5x^2 - 0.5x$**

13. Explain why the formula works. **See pp. 53A–53B.**

Dots (x)	Connection Lines (y)
3	3
4	**6**
5	**10**
6	**15**
7	**21**
8	**28**

Resource Manager

Teaching Algebra with Manipulatives
- p. 213 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit
- ruler

Algebra Activity

A Follow-Up of Lesson 1-2

Getting Started

Objective Discover the relationship between the number of sides of a convex polygon and the total number of diagonals that can be drawn in the polygon.

Materials
ruler or geometry drawing software

Teach

- In Exercise 8, suggest to students that they draw a large decagon, draw all of its diagonals, and then carefully mark each diagonal as they count it.
- Guide students to recognize that each figure they create when connecting the dots in Exercises 10–13 is a polygon with all of its diagonals drawn. Relate this to the work in Exercises 1–9.

Assess

In Exercises 2–6, students should be able to see that there are consistent patterns in these relationships, and they should be able to make the generalizations that will form the parts of the formula. In Exercises 7–9, students should understand that the elements in the formula are not just arbitrary or mysterious, but are derived from the characteristics of the diagonals. They should also be able to apply the formula to a polygon with any number of sides.

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

1-3 Lesson Notes

1 Focus

5-Minute Check Transparency 1-3 Use as a quiz or review of Lesson 1-2.

Mathematical Background notes are available for this lesson on p. 4C.

Building on Prior Knowledge

In Lesson 1-2, students evaluated expressions with real numbers. In this lesson, they apply this skill to writing expressions and solving equations.

How can you find the most effective level of intensity for your workout?

Ask students:

- How can the expression $6 \times P \div (220 - A)$ be written as a ratio? $\frac{6P}{220 - A}$
- To achieve a 100% intensity level, the numerator and denominator of the ratio you just found must be equal. At what 10-second pulse count would you achieve a 100% intensity level? **Answers will vary.**
- **Fitness** Find your 10-second pulse count P after running in place for 30 seconds. What is your level of intensity for this value of P? **Answers will vary.**

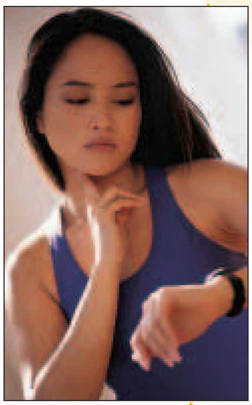
1-3 Solving Equations

Vocabulary
- open sentence
- equation
- solution

Vocabulary words are listed at the beginning of the lesson and are highlighted in yellow at point of use.

What You'll Learn
- Translate verbal expressions into algebraic expressions and equations, and vice versa.
- Solve equations using the properties of equality.

How can you find the most effective level of intensity for your workout?



When exercising, one goal is to find the best level of intensity as a percent of your maximum heart rate. To find the intensity level, multiply 6 and P, your 10-second pulse count. Then divide by the difference of 220 and your age A.

Multiply 6 and your pulse rate	and divide by	the difference of 220 and your age.
$6 \times P$	$\div$	$(220 - A)$

VERBAL EXPRESSIONS TO ALGEBRAIC EXPRESSIONS Verbal expressions can be translated into algebraic or mathematical expressions using the language of algebra. Any letter can be used as a variable to represent a number that is not known.

Example 1 Verbal to Algebraic Expression

Write an algebraic expression to represent each verbal expression.

a. **7 less than a number** $n - 7$

b. **three times the square of a number** $3x^2$

c. **the cube of a number increased by 4 times the same number** $p^3 + 4p$

d. **twice the sum of a number and 5** $2(y + 5)$

A mathematical sentence containing one or more variables is called an **open sentence**. A mathematical sentence stating that two mathematical expressions are equal is called an **equation**.

Example 2 Algebraic to Verbal Sentence

Write a verbal sentence to represent each equation.

a. $10 = 12 - 2$ Ten is equal to 12 minus 2.

b. $n + (-8) = -9$ The sum of a number and −8 is −9.

c. $\frac{n}{6} = n^2$ A number divided by 6 is equal to that number squared.

Open sentences are neither true nor false until the variables have been replaced by numbers. Each replacement that results in a true sentence is called a **solution** of the open sentence.

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters
- Study Guide and Intervention, pp. 13–14
- Skills Practice, p. 15
- Practice, p. 16
- Reading to Learn Mathematics, p. 17
- Enrichment, p. 18
- Assessment, pp. 51, 53

Graphing Calculator and Spreadsheet Masters, p. 27
School-to-Career Masters, p. 2
Teaching Algebra With Manipulatives Masters, pp. 214–215

Transparencies
5-Minute Check Transparency 1-3
Answer Key Transparencies

Technology
Interactive Chalkboard

PROPERTIES OF EQUALITY To solve equations, we can use properties of equality. Some of these *equivalence relations* are listed in the table below.

Key Concept — *Properties of Equality*

Property	Symbols	Examples
Reflexive	For any real number a, $a = a$.	$-7 + n = -7 + n$
Symmetric	For all real numbers a and b, if $a = b$, then $b = a$.	If $3 = 5x - 6$, then $5x - 6 = 3$.
Transitive	For all real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.	If $2x + 1 = 7$ and $7 = 5x - 8$, then $2x + 1 = 5x - 8$.
Substitution	If $a = b$, then a may be replaced by b and b may be replaced by a.	If $(4 + 5)m = 18$, then $9m = 18$.

Example 3 *Identify Properties of Equality*

Name the property illustrated by each statement.

a. **If $3m = 5n$ and $5n = 10p$, then $3m = 10p$.**

Transitive Property of Equality

b. **If $-11a + 2 = -3a$, then $-3a = -11a + 2$.**

Symmetric Property of Equality

Sometimes an equation can be solved by adding the same number to each side or by subtracting the same number from each side or by multiplying or dividing each side by the same number.

Key Concept — *Properties of Equality*

Addition and Subtraction Properties of Equality

- **Symbols** For any real numbers a, b, and c, if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- **Examples** If $x - 4 = 5$, then $x - 4 + 4 = 5 + 4$. If $n + 3 = -11$, then $n + 3 - 3 = -11 - 3$.

Multiplication and Division Properties of Equality

- **Symbols** For any real numbers a, b, and c, if $a = b$, then $a \cdot c = b \cdot c$ and, if $c \neq 0$, $\frac{a}{c} = \frac{b}{c}$.
- **Examples** If $\frac{m}{4} = 6$, then $4 \cdot \frac{m}{4} = 4 \cdot 6$. If $-3y = 6$, then $\frac{-3y}{-3} = \frac{6}{-3}$.

Example 4 *Solve One-Step Equations*

Solve each equation. Check your solution.

a. **$a + 4.39 = 76$**

$a + 4.39 = 76$	Original equation
$a + 4.39 - 4.39 = 76 - 4.39$	Subtract 4.39 from each side.
$a = 71.61$	Simplify.

The solution is 71.61.

(continued on the next page)

www.algebra2.com/extra_examples

Study Tip

Properties of Equality
These properties are also known as *axioms of equality*.

The Resource Manager lists all of the resources available for the lesson, including workbooks, blackline masters, transparencies, and technology.

Teaching Tip Suggest that students ask themselves these questions: "What is being shown on the left side of the equation in In-Class Example 4a at the right?" **5.48 is subtracted from s.** "What is the opposite or inverse of subtracting 5.48?" **Adding 5.48.** "What must be done to both sides of the equation $s - 5.48 = 0.02$ to get the variable s alone on one side of the equation?" **Add 5.48 to both sides and simplify the resulting equation.**

2 Teach

VERBAL EXPRESSIONS TO ALGEBRAIC EXPRESSIONS

In-Class Examples — Power Point®

1 Write an algebraic expression to represent each verbal expression.

a. 3 more than a number **$x + 3$**

b. six times the cube of a number **$6x^3$**

c. the square of a number decreased by the product of 5 and the number **$x^2 - 5x$**

d. twice the difference of a number and 6 **$2(x - 6)$**

2 Write a verbal sentence to represent each equation.

a. $14 + 9 = 23$ **The sum of 14 and 9 is 23.**

b. $6 = -5 + x$ **Six is equal to −5 plus a number.**

c. $7y - 2 = 19$ **Seven times a number minus 2 is 19.**

PROPERTIES OF EQUALITY

In-Class Examples — Power Point®

3 Name the property illustrated by each statement.

a. If $xy = 28$ and $x = 7$, then $7y = 28$. **Substitution Property of Equality**

b. $a - 2.03 = a - 2.03$ **Reflexive Property of Equality**

Reading Tip Help students remember the name of the Reflexive Property by relating $a = a$ to seeing your reflection in a mirror.

4 Solve each equation. Check your solution.

a. $s - 5.48 = 0.02$ **5.5**

b. $18 = \frac{1}{2}t$ **36**

In-Class Examples

Power Point®

5 Solve
$53 = 3(y - 2) - 2(3y - 1)$.
−19

6 **GEOMETRY** The area of a trapezoid is $A = \frac{1}{2}(b_1 + b_2)h$, where A is the area, b_1 is the length of one base, b_2 is the length of the other base, and h is the height of the trapezoid. Solve the formula for h.
$h = \dfrac{2A}{b_1 + b_2}$

Study Tip

Multiplication and Division Properties of Equality

Example 4b could also have been solved using the Division Property of Equality. Note that dividing each side of the equation by $-\frac{3}{5}$ is the same as multiplying each side by $-\frac{5}{3}$.

CHECK

$a + 4.39 = 76$	Original equation
$71.61 + 4.39 \stackrel{?}{=} 76$	Substitute 71.61 for *a*.
$76 = 76 \checkmark$	Simplify.

b. $-\frac{3}{5}d = 18$

$-\frac{3}{5}d = 18$	Original equation
$-\frac{5}{3}\left(-\frac{3}{5}\right)d = -\frac{5}{3}(18)$	Multiply each side by $-\frac{5}{3}$, the multiplicative inverse of $-\frac{3}{5}$.
$d = -30$	Simplify.

The solution is −30.

CHECK

$-\frac{3}{5}d = 18$	Original equation
$-\frac{3}{5}(-30) \stackrel{?}{=} 18$	Substitute −30 for *d*.
$18 = 18 \checkmark$	Simplify.

Sometimes you must apply more than one property to solve an equation.

Example 5 Solve a Multi-Step Equation

Solve $2(2x + 3) - 3(4x - 5) = 22$.

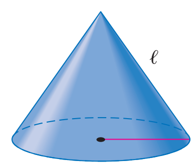
$2(2x + 3) - 3(4x - 5) = 22$	Original equation
$4x + 6 - 12x + 15 = 22$	Distributive and Substitution Properties
$-8x + 21 = 22$	Commutative, Distributive, and Substitution Properties
$-8x = 1$	Subtraction and Substitution Properties
$x = -\frac{1}{8}$	Division and Substitution Properties

The solution is $-\frac{1}{8}$.

You can use properties of equality to solve an equation or formula for a specified variable.

Example 6 Solve for a Variable

GEOMETRY **The surface area of a cone is $S = \pi r\ell + \pi r^2$, where S is the surface area, ℓ is the slant height of the cone, and r is the radius of the base. Solve the formula for ℓ.**



$S = \pi r\ell + \pi r^2$	Surface area formula
$S - \pi r^2 = \pi r\ell + \pi r^2 - \pi r^2$	Subtract πr^2 from each side.
$S - \pi r^2 = \pi r\ell$	Simplify.
$\frac{S - \pi r^2}{\pi r} = \frac{\pi r\ell}{\pi r}$	Divide each side by πr.
$\frac{S - \pi r^2}{\pi r} = \ell$	Simplify.

DAILY INTERVENTION

Unlocking Misconceptions

- **Solving Equations** Students may want to simplify, collect terms, and use the properties of equality to perform an operation on each side of an equation all in one or two steps. Help them see that it is more efficient to write down each step in the solution process than to have to solve the equation again because of a computational error.
- **Checking Solutions** Explain that checking solutions in order to discover possible errors is a vital procedure when you use math on the job.

Many standardized test questions can be solved by using properties of equality.

Standardized Test Practice

Example 7 Apply Properties of Equality

Multiple-Choice Test Item

If $3n - 8 = \frac{9}{5}$, what is the value of $3n - 3$?

(A) $\frac{34}{5}$ (B) $\frac{49}{15}$ (C) $-\frac{16}{5}$ (D) $-\frac{27}{5}$

Read the Test Item

You are asked to find the value of the expression $3n - 3$. Your first thought might be to find the value of n and then evaluate the expression using this value. Notice, however, that you are *not* required to find the value of n. Instead, you can use the Addition Property of Equality on the given equation to find the value of $3n - 3$.

Solve the Test Item

$3n - 8 = \frac{9}{5}$ Original equation

$3n - 8 + 5 = \frac{9}{5} + 5$ Add 5 to each side.

$3n - 3 = \frac{34}{5}$ $\frac{9}{5} + 5 = \frac{9}{5} + \frac{25}{5}$ or $\frac{34}{5}$

The answer is A.

The Princeton Review

Test-Taking Tip

If a problem seems to require lengthy calculations, look for a shortcut. There is probably a quicker way to solve it. Try using properties of equality.

To solve a word problem, it is often necessary to define a variable and write an equation. Then solve by applying the properties of equality.

Example 8 Write an Equation

HOME IMPROVEMENT Josh and Pam have bought an older home that needs some repair. After budgeting a total of $1685 for home improvements, they started by spending $425 on small improvements. They would like to replace six interior doors next. What is the maximum amount they can afford to spend on each door?

Explore Let c represent the cost to replace each door.

Plan Write and solve an equation to find the value of c.

The number of doors	times	the cost to replace each door	plus	previous expenses	equals	the total cost.
6	$\cdot$	c	$+$	425	$=$	1685

Solve

$6c + 425 = 1685$ Original equation

$6c + 425 - 425 = 1685 - 425$ Subtract 425 from each side.

$6c = 1260$ Simplify.

$\frac{6c}{6} = \frac{1260}{6}$ Divide each side by 6.

$c = 210$ Simplify.

They can afford to spend $210 on each door.

Examine The total cost to replace six doors at $210 each is 6(210) or $1260. Add the other expenses of $425 to that, and the total home improvement bill is 1260 + 425 or $1685. Thus, the answer is correct.

More About...

Home Improvement

Previously occupied homes account for approximately 85% of all U.S. home sales. Most homeowners remodel within 18 months of purchase. The top two remodeling projects are kitchens and baths.

Source: National Association of Remodeling Industry

In-Class Examples

Power Point®

7 If $4g + 5 = \frac{4}{9}$, what is the value of $4g - 2$? **B**

A $-\frac{41}{36}$ **B** $-\frac{59}{9}$

C $-\frac{41}{9}$ **D** $-\frac{67}{7}$

8 **HOME IMPROVEMENT** Carl wants to replace the five windows in the 2nd-story bedrooms of his house. His neighbor Will is a carpenter and he has agreed to help install them for $250. If Carl has budgeted $1000 for the total cost, what is the maximum amount he can spend on each window? **$150**

Teaching Tip Students, especially those with math anxiety, tend to omit the planning step. Encourage students to see that this step helps them find a way to write an equation, even if they only do the planning mentally.

Standardized Test Practice

Example 7 Point out to students that there are several ways to find the specified value. One alternate way would be to first solve the given equation for $3n$ and then subtract 3 from each side of that equation.

$3n - 8 = \frac{9}{5} \Rightarrow 3n = \frac{49}{5} \Rightarrow 3n - 3 = \frac{34}{5}$

Each chapter contains an example that gives students practice in solving problems on standardized tests. Standardized Test Practice suggestions give students additional methods for achieving success on standardized tests.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
- add the properties of equality given in this lesson to their list of real number properties from Lesson 1-2.
- include the formula in Example 6 in the list of formulas they began in Lesson 1-2.
- use the content of Example 7 to start a list of test-taking tips that they can review as they prepare for standardized tests.
- include any other item(s) that they find helpful in mastering the skills in this lesson

DAILY INTERVENTION

FIND THE ERROR Encourage students to use correct mathematical language to state the error. For example, *Crystal needed to use the Distributive Property on the right side of the equation before subtracting.*

Answer

3. His method can be confirmed by solving the equation using an alternative method.

$$C = \frac{5}{9}(F - 32)$$

$$C = \frac{5}{9}F - \frac{5}{9}(32)$$

$$C + \frac{5}{9}(32) = \frac{5}{9}F$$

$$\frac{9}{5}\left[C + \frac{5}{9}(32)\right] = F$$

$$\frac{9}{5}C + 32 = F$$

Check for Understanding

Concept Check

1. Sample answer: $2x = -14$

2. Sometimes true; only when the expression you are dividing by does not equal zero.

Find the Error exercises help students identify and address common errors before they occur.

1. **OPEN ENDED** Write an equation whose solution is -7.
2. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain.
   Dividing each side of an equation by the same expression produces an equivalent equation.
3. **FIND THE ERROR** Crystal and Jamal are solving $C = \frac{5}{9}(F - 32)$ for F.

Crystal	Jamal
$C = \frac{5}{9}(F - 32)$	$C = \frac{5}{9}(F - 32)$
$C + 32 = \frac{5}{9}F$	$\frac{9}{5}C = F - 32$
$\frac{9}{5}(C + 32) = F$	$\frac{9}{5}C + 32 = F$

Who is correct? Explain your reasoning. **Jamal; see margin for explanation.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
4, 5	1
6, 7	2
8, 9	3
10–15	4, 5
16, 17	6
18	7

Write an algebraic expression to represent each verbal expression.

4. five increased by four times a number **$5 + 4n$**
5. twice a number decreased by the cube of the same number **$2n - n^3$**

Write a verbal expression to represent each equation. **6–7. Sample answers given.**

6. $9n - 3 = 6$ **9 times a number decreased by 3 is 6.**
7. $5 + 3x^2 = 2x$ **5 plus 3 times the square of a number is twice that number.**

Name the property illustrated by each statement.

8. $(3x + 2) - 5 = (3x + 2) - 5$ **Reflexive (=)**
9. If $4c = 15$, then $4c + 2 = 15 + 2$. **Addition (=)**

Solve each equation. Check your solution.

10. $y + 14 = -7$ **-21**
11. $7 + 3x = 49$ **14**
12. $-4(b + 7) = -12$ **-4**
13. $7q + q - 3q = -24$ **-4.8**
14. $1.8a - 5 = -2.3$ **1.5**
15. $-\frac{3}{4}n + 1 = -11$ **16**

Solve each equation or formula for the specified variable.

16. $4y - 2n = 9$, for y **$y = \frac{9 + 2n}{4}$**
17. $I = prt$, for p **$p = \frac{I}{rt}$**

Standardized Test Practice

18. If $4x + 7 = 18$, what is the value of $12x + 21$? **D**
    (A) 2.75 (B) 32 (C) 33 (D) 54

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
19–28	1
29–34	2
35–40	3
41–56	4, 5
57–62	6
63–74	7

Extra Practice

See page 828.

Write an algebraic expression to represent each verbal expression.

19. the sum of 5 and three times a number **$5 + 3n$**
20. seven more than the product of a number and 10 **$10n + 7$**
21. four less than the square of a number **$n^2 - 4$**
22. the product of the cube of a number and -6 **$-6n^3$**
23. five times the sum of 9 and a number **$5(9 + n)$**
24. twice the sum of a number and 8 **$2(n + 8)$**
25. ★ the square of the quotient of a number and 4 **$\left(\frac{n}{4}\right)^2$**
26. ★ the cube of the difference of a number and 7 **$(n - 7)^3$**

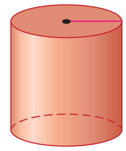
DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students work in pairs to read, discuss, and plan a solution strategy for real-world problems such as the one given in Example 8. This interaction can help students identify individual difficulties with word problems and also to discover new strategies used by other students.

GEOMETRY **For Exercises 27 and 28, use the following information.**

The formula for the surface area of a cylinder with radius r and height h is π times twice the product of the radius and height plus twice the product of π and the square of the radius.



27. Translate this verbal expression of the formula into an algebraic expression. **$2\pi rh + 2\pi r^2$**

28. Write an equivalent expression using the Distributive Property. **$2\pi r(h + r)$**

Write a verbal expression to represent each equation.

29. $x - 5 = 12$ **30.** $2n + 3 = -1$

31. $y^2 = 4y$ **32.** $3a^3 = a + 4$

33. $\frac{b}{4} = 2(b + 1)$ ★ **34.** $7 - \frac{1}{2}x = \frac{3}{x^2}$

Name the property illustrated by each statement.

35. If $[3(-2)]z = 24$, then $-6z = 24$. **Substitution (=)**

36. If $5 + b = 13$, then $b = 8$. **Subtraction (=)**

37. If $2x = 3d$ and $3d = -4$, then $2x = -4$. **Transitive (=)**

38. If $g - t = n$, then $g = n + t$. **Addition (=)**

39. If $14 = \frac{x}{2} + 11$, then $\frac{x}{2} + 11 = 14$. **Symmetric (=)**

★ **40.** If $y - 2 = -8$, then $3(y - 2) = 3(-8)$. **Multiplication (=)**

Solve each equation. Check your solution.

41. $2p + 15 = 29$ **7** **42.** $14 - 3n = -10$ **8**

43. $7a - 3a + 2a - a = 16$ **3.2** **44.** $x + 9x - 6x + 4x = 20$ **2.5**

45. $\frac{1}{9} - \frac{2}{3}b = \frac{1}{18}$ **$\frac{1}{12}$** **46.** $\frac{5}{8} + \frac{3}{4}x = \frac{1}{16}$ **$-\frac{3}{4}$**

47. $27 = -9(y + 5)$ **−8** **48.** $-7(p + 8) = 21$ **−11**

49. $3f - 2 = 4f + 5$ **−7** **50.** $3d + 7 = 6d + 5$ **$\frac{2}{3}$**

51. $4.3n + 1 = 7 - 1.7n$ **1** **52.** $1.7x - 8 = 2.7x + 4$ **−12**

★ **53.** $3(2z + 25) - 2(z - 1) = 78$ **$\frac{1}{4}$** ★ **54.** $4(k + 3) + 2 = 4.5(k + 1)$ **19**

★ **55.** $\frac{3}{11}a - 1 = \frac{7}{11}a + 9$ **$-\frac{55}{2}$** ★ **56.** $\frac{2}{5}x + \frac{3}{7} = 1 - \frac{4}{7}x$ **$\frac{10}{17}$**

Solve each equation or formula for the specified variable.

57. $d = rt$, for r **$\frac{d}{t} = r$** **58.** $x = \frac{-b}{2a}$, for a **$a = \frac{-b}{2x}$**

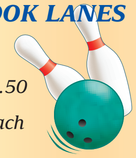
59. $V = \frac{1}{3}\pi r^2 h$, for h **$\frac{3V}{\pi r^2} = h$** **60.** $A = \frac{1}{2}h(a + b)$, for b **$\frac{2A}{h} - a = b$**

★ **61.** $\frac{a(b - 2)}{c - 3} = x$, for b **$b = \frac{x(c - 3)}{a} + 2$** ★ **62.** $x = \frac{y}{y + 4}$, for y **$\frac{4x}{1 - x} = y$**

Define a variable, write an equation, and solve the problem.

63. BOWLING Jon and Morgan arrive at Sunnybrook Lanes with $16.75. Find the maximum number of games they can bowl if they each rent shoes. **n = number of games; $2(1.50) + n(2.50) = 16.75$; 5**

SUNNYBROOK LANES



Shoe Rental: *$1.50*

Games: *$2.50 each*

www.algebra2.com/self_check_quiz

29–34. Sample answers are given.

29. 5 less than a number is 12.

30. Twice a number plus 3 is −1.

31. A number squared is equal to 4 times the number.

32. Three times the cube of a number is equal to the number plus 4.

33. A number divided by 4 is equal to twice the sum of that number and 1.

34. 7 minus half a number is equal to 3 divided by the square of *x*.

About the Exercises…

Organization by Objective

- **Verbal Expressions to Algebraic Expressions:** 19–34
- **Properties of Equality:** 35–74

Exercises 19–26 and 29–70 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 19–23 odd, 27–28, 29–39 odd, 41–51 odd, 57, 59, 63–69 odd, 75–89

Average: 19–25 odd, 27–28, 29–69 odd, 75–89

Advanced: 20–26 even, 30–70 even, 71–83 (optional: 84–89)

The Assignment Guides provide suggestions for exercises that are appropriate for basic, average, or advanced students. Many of the homework exercises are paired, so that students can do the odds one day and the evens the next day.

Study Guide and Intervention, p. 13 (shown) and p. 14

Verbal Expressions to Algebraic Expressions The chart suggests some ways to help you translate word expressions into algebraic expressions. Any letter can be used to represent a number that is not known.

Word Expression	Operation
$\frac{1}{2}$	

Example 1 **Write an algebraic expression to represent 18 less than the quotient of a number and 3.**

$\frac{n}{3} - 18$

Example 2 **Write a verbal sentence to represent $6(n - 2) = 14$.**

Six times the difference of a number and two is equal to 14.

Exercises

Write an algebraic expression to represent each verbal expression.

1. the sum of six times a number and 25 $6n + 25$
2. four times the sum of a number and 3 $4(n + 3)$
3. 7 less than fifteen times a number $15n - 7$
4. the difference of nine times a number and the quotient of 6 and the same number $9n - \frac{6}{n}$
5. the sum of 100 and four times a number $100 + 4n$
6. the product of 3 and the sum of 11 and a number $3(11 + n)$
7. four times the square of a number increased by five times the same number $4n^2 + 5n$
8. 23 more than the product of 7 and a number $7n + 23$

Write a verbal sentence to represent each equation. Sample answers are given.

9. $3n - 35 = 79$ The difference of three times a number and 35 is equal to 79.
10. $2(n^3 + 3n^2) = 4n$ Twice the sum of the cube of a number and three times the square of the number is equal to four times the number.
11. $\frac{5n}{n + 3} = n - 8$ The quotient of five times a number and the sum of the number and 3 is equal to the difference of the number and 8.

Skills Practice, p. 15 and Practice, p. 16 (shown)

Write an algebraic expression to represent each verbal expression.

1. 2 more than the quotient of a number and 5 $\frac{y}{5} + 2$
2. the sum of two consecutive integers $n + (n + 1)$
3. 5 times the sum of a number and 1 $5(m + 1)$
4. 1 less than twice the square of a number $2y^2 - 1$

Write a verbal expression to represent each equation. 5–8. Sample answers are given.

5. $5 - 2x = 4$ The difference of 5 and twice a number is 4.
6. $3y = 4y^3$ Three times a number is 4 times the cube of the number.
7. $3c = 2(c - 1)$ Three times a number is twice the difference of the number and 1.
8. $\frac{m}{5} = 3(2m + 1)$ The quotient of a number and 5 is 3 times the sum of twice the number and 1.

Name the property illustrated by each statement.

9. If $t - 13 = 52$, then $52 = t - 13$. Symmetric (=)
10. If $8(2q + 1) = 4$, then $2(2q + 1) = 1$. Division (=)
11. If $h + 12 = 22$, then $h = 10$. Subtraction (=)
12. If $4m = -15$, then $-12m = 45$. Multiplication (=)

Solve each equation. Check your solution.

13. $14 = 8 - 6r$ -1
14. $9 + 4n = -59$ -17
15. $\frac{3}{4} - \frac{1}{2}n = \frac{5}{8}$ $\frac{1}{4}$
16. $\frac{5}{6}s + \frac{3}{4} = \frac{11}{12}$ $\frac{1}{5}$
17. $-1.6r + 5 = -7.8$ 8
18. $6x - 5 = 7 - 9x$ $\frac{4}{5}$
19. $5(6 - 4v) = v + 21$ $\frac{3}{7}$
20. $6y - 5 = -3(2y + 1)$ $\frac{1}{6}$

Solve each equation or formula for the specified variable.

21. $E = mc^2$, for m $m = \frac{E}{c^2}$
22. $c = \frac{2d + 1}{3}$, for d $d = \frac{3c - 1}{2}$
23. $h = vt - gt^2$, for v $v = \frac{h + gt^2}{t}$
24. $E = \frac{1}{2}Iw^2 + U$, for I $I = \frac{2(E - U)}{w^2}$

Define a variable, write an equation, and solve the problem.

25. **GEOMETRY** The length of a rectangle is twice the width. Find the width if the perimeter is 60 centimeters. w = width; $2(2w) + 2w = 60$; 10 cm
26. **GOLF** Luis and three friends went golfing. Two of the friends rented clubs for $6 each. The total cost of the rented clubs and the green fees for each person was $76. What was the cost of the green fees for each person? g = green fees per person; $6(2) + 4g = 76$; $16

Reading to Learn Mathematics, p. 17 ELL

Pre-Activity **How can you find the most effective level of intensity for your workout?**

Read the introduction to Lesson 1-3 at the top of page 20 in your textbook.

- To find your target heart rate, what two pieces of information must you supply? age (A) and desired intensity level (I)
- Write an equation that shows how to calculate your target heart rate. $P = \frac{(220 - A) \cdot I}{6}$ or $P = (220 - A) \cdot I \div 6$

Reading the Lesson

1. a. How are algebraic expressions and equations alike? Sample answer: Both contain variables, constants, and operation signs.
   b. How are algebraic expressions and equations different? Sample answer: Equations contain equal signs; expressions do not.
   c. How are algebraic expressions and equations related? Sample answer: An equation is a statement that says that two algebraic expressions are equal.

Read the following problem and then write an equation that you could use to solve it. Do not actually solve the equation. In your equation, let *m* be the number of miles driven.

2. When Louisa rented a moving truck, she agreed to pay $28 per day plus $0.42 per mile. If she kept the truck for 3 days and the rental charges (without tax) were $153.72, how many miles did Louisa drive the truck? $3(28) + 0.42m = 153.72$

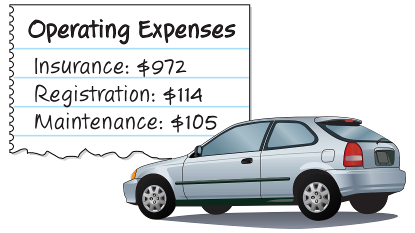
Helping You Remember

3. How can the words *reflection* and *symmetry* help you remember and distinguish between the reflexive and symmetric properties of equality? Think about how these words are used in everyday life or in geometry. Sample answer: When you look at your reflection, you are looking at yourself. The reflexive property says that every number is equal to itself. In geometry, symmetry with respect to a line means that the parts of a figure on the two sides of a line are identical. The symmetric property of equality allows you to interchange the two sides of an equation. The equal sign is like the line of symmetry.

For Exercises 64–70, define a variable, write an equation, and solve the problem.

64. **GEOMETRY** The perimeter of a regular octagon is 124 inches. Find the length of each side. s = length of a side; $8s = 124$; 15.5 in.

65. **CAR EXPENSES** Benito spent $1837 to operate his car last year. Some of these expenses are listed below. Benito's only other expense was for gasoline. If he drove 7600 miles, what was the average cost of the gasoline per mile? x = cost of gasoline per mile; $972 + 114 + 105 + 7600x = 1837$; 8.5¢



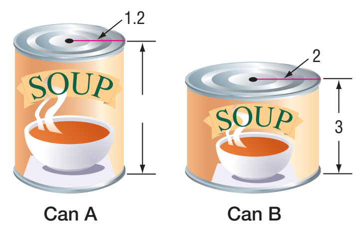
66. **SCHOOL** A school conference room can seat a maximum of 83 people. The principal and two counselors need to meet with the school's student athletes to discuss eligibility requirements. If each student must bring a parent with them, what is the maximum number of students that can attend each meeting? n = number of students that can attend each meeting; $2n + 3 = 83$; 40 students

67. **FAMILY** Chun-Wei's mother is 8 more than twice his age. His father is three years older than his mother is. If all three family members have lived 94 years, how old is each family member? a = Chun-Wei's age; $a + (2a + 8) + (2a + 8 + 3) = 94$; Chun-Wei: 15 yrs old, mother: 38 yrs old, father: 41 yrs old

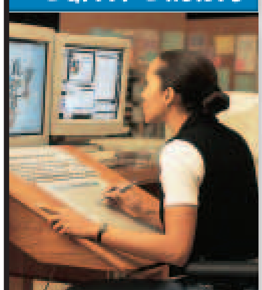
68. **SCHOOL TRIP** The Parent Teacher Organization has raised $1800 to help pay for a trip to an amusement park. They ask that there be one adult for every five students attending. Adult tickets cost $45 and student tickets cost $30. If the group wants to take 50 students, how much will each student need to pay so that adults agreeing to chaperone pay nothing? c = cost per student; $50(30 - c) + \frac{50}{5}(45) = 1800$; $3

69. **BUSINESS** A trucking company is hired to deliver 125 lamps for $12 each. The company agrees to pay $45 for each lamp that is broken during transport. If the trucking company needs to receive a minimum payment of $1364 for the shipment to cover their expenses, find the maximum number of lamps they can afford to break during the trip. n = number of lamps broken; $12(125) - 45n = 1365$; 3 lamps

70. **PACKAGING** Two designs for a soup can are shown at the right. If each can holds the same amount of soup, what is the height of can A? h = height of can A; $\pi(1.2^2)h = \pi(2^2)3$; $8\frac{1}{3}$ units



Career Choices



Industrial Design

Industrial designers use research on product use, marketing, materials, and production methods to create functional and appealing packaging designs.

Online Research

For information about a career as an industrial designer, visit: www.algebra2.com/careers

71. 15.1 mi/mo

RAILROADS **For Exercises 71–73, use the following information.**

The First Transcontinental Railroad was built by two companies. The Central Pacific began building eastward from Sacramento, California, while the Union Pacific built westward from Omaha, Nebraska. The two lines met at Promontory, Utah, in 1869, about 6 years after construction began.

71. The Central Pacific Company laid an average of 9.6 miles of track per month. Together the two companies laid a total of 1775 miles of track. Determine the average number of miles of track laid per month by the Union Pacific Company.

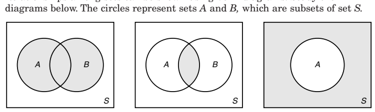
72. About how many miles of track did each company lay? See margin.

73. Why do you think the Union Pacific was able to lay track so much more quickly than the Central Pacific? See margin.

Enrichment, p. 18

Venn Diagrams

Relationships among sets can be shown using Venn diagrams. Study the diagrams below. The circles represent sets A and B, which are subsets of set S.



The union of A and B consists of all elements in *either* A or B.
The intersection of A and B consists of all elements in *both* A and B.
The complement of A consists of all elements *not* in A.

You can combine the operations of union, intersection, and finding the complement.

Example **Shade the region $(A \cap B)'$.**

Answers

72. Central: 690 mi.; Union: 1085 mi

73. The Central Pacific had to lay their track through the Rocky Mountains, while the Union Pacific mainly built track over flat prairie.

★ **74. MONEY** Allison is saving money to buy a video game system. In the first week, her savings were \$8 less than $\frac{2}{5}$ the price of the system. In the second week, she saved 50 cents more than $\frac{1}{2}$ the price of the system. She was still \$37 short. Find the price of the system.
\$295

75. CRITICAL THINKING Write a verbal expression to represent the algebraic expression $3(x - 5) + 4x(x + 1)$. **See margin.**

WebQuest

You can write and solve equations to determine the monthly payment for a home. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

76. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 53A–53B.**

How can you find the most effective level of intensity for your workout?

Include the following in your answer:
- an explanation of how to find the age of a person who is exercising at an 80% level of intensity I with a pulse count of 27, and
- a description of when it would be desirable to solve a formula like the one given for a specified variable.

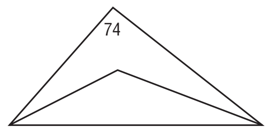
Standardized Test Practice

77. If $-6x + 10 = 17$, then $3x - 5 =$ **B**

Ⓐ $-\frac{7}{6}$. Ⓑ $-\frac{17}{2}$. Ⓒ 2. Ⓓ $\frac{19}{3}$. Ⓔ $\frac{5}{3}$.

78. In triangle PQR, $\overline{QS}$ and $\overline{SR}$ are angle bisectors and angle $P = 74°$. How many degrees are there in angle QSR? **D**

Ⓐ 106 Ⓑ 121 Ⓒ 125
Ⓓ 127 Ⓔ 143



Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 1-2)*

79. $2x + 9y + 4z - y - 8x$ **$-6x + 8y + 4z$**

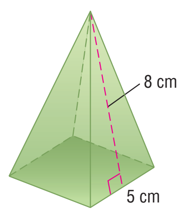
80. $4(2a + 5b) - 3(4b - a)$ **$11a + 8b$**

Evaluate each expression if $a = 3$, $b = -2$, and $c = 1.2$. *(Lesson 1-1)*

81. $a - [b(a - c)]$ **6.6**

82. $c^2 - ab$ **7.44**

83. GEOMETRY The formula for the surface area S of a regular pyramid is $S = \frac{1}{2}P\ell + B$, where P is the perimeter of the base, ℓ is the slant height, and B is the area of the base. Find the surface area of the square-based pyramid shown at the right. *(Lesson 1-1)* **105 cm²**



By having your students complete the Getting Ready exercises, you can target specific skills they will need for the next lesson.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Identify the additive inverse for each number or expression.** *(To review **additive inverses**, see Lesson 1-2.)*

84. 5 **−5**
85. −3 **3**
86. 2.5 **−2.5**
87. $\frac{1}{4}$ **$-\frac{1}{4}$**
88. $-3x$ **$3x$**
89. $5 - 6y$ **$-5 + 6y$**

4 Assess

Open-Ended Assessment

Speaking Have students discuss what difficulties they have with translating verbal problems into algebraic equations, including any anxieties that word problems may create. Ask students to share their strategies for overcoming these difficulties, using specific examples to illustrate their strategies.

Tips for New Teachers

Intervention Explain to students that they can solve verbal problems when they (1) face their anxiety about the words instead of avoiding the task, (2) ask questions about words they do not understand, and (3) take time to read, understand, and plan, using a sketch to help.

Getting Ready for Lesson 1-4

PREREQUISITE SKILL Lesson 1-4 presents solving equations that involve absolute value expressions. Solving equations often involves using additive inverses to isolate the variable on one side of an equation. Exercises 84–89 should be used to determine your students' familiarity with finding additive inverses.

Assessment Options

Quiz (Lesson 1–3) is available on p. 51 of the *Chapter 1 Resource Masters*.

Mid-Chapter Test (Lessons 1-1 through 1-3) is available on p. 53 of the *Chapter 1 Resource Masters*.

Answer

75. the product of 3 and the difference of a number and 5 added to the product of four times the number and the sum of the number and 1

Assessment Options lists the quizzes and tests that are available in the Chapter Resource Masters.

1-4 Lesson Notes

1 Focus

5-Minute Check Transparency 1-4 Use as a quiz or review of Lesson 1-3.

Mathematical Background notes are available for this lesson on p. 4D.

Building on Prior Knowledge

In Lesson 1-3, students wrote expressions and solved equations. In this lesson, they apply those skills to equations involving absolute values.

How can an absolute value equation describe the magnitude of an earthquake?

Ask students:

- In the absolute value equation $|E - 6.1| = 0.3$, what does the variable E represent? **the actual magnitude of the earthquake**
- What is the meaning of the number 0.3 in the equation? **the uncertainty of the estimated magnitude**
- What would the equation be for the magnitude of an earthquake estimated at 5.8 on the Richter scale? $|E - 5.8| = 0.3$

Key Concept boxes highlight definitions, formulas, and other important ideas. Multiple representations—words, symbols, examples, models—reach students of all learning styles

1-4 Solving Absolute Value Equations

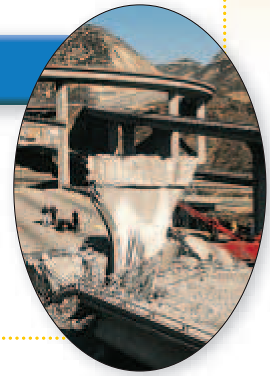
What You'll Learn

- Evaluate expressions involving absolute values.
- Solve absolute value equations.

Vocabulary

- absolute value
- empty set

How can an absolute value equation describe the magnitude of an earthquake?

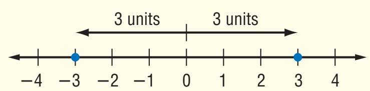


Seismologists use the Richter scale to express the magnitudes of earthquakes. This scale ranges from 1 to 10, 10 being the highest. The uncertainty in the estimate of a magnitude E is about plus or minus 0.3 unit. This means that an earthquake with a magnitude estimated at 6.1 on the Richter scale might actually have a magnitude as low as 5.8 or as high as 6.4. These extremes can be described by the absolute value equation $|E - 6.1| = 0.3$.

ABSOLUTE VALUE EXPRESSIONS The **absolute value** of a number is its distance from 0 on the number line. Since distance is nonnegative, the absolute value of a number is always nonnegative. The symbol $|x|$ is used to represent the absolute value of a number x.

Key Concept — *Absolute Value*

- **Words** For any real number a, if a is positive or zero, the absolute value of a is a. If a is negative, the absolute value of a is the opposite of a.
- **Symbols** For any real number a, $|a| = a$ if $a \geq 0$, and $|a| = -a$ if $a < 0$.
- **Model** $|-3| = 3$ and $|3| = 3$



When evaluating expressions that contain absolute values, the absolute value bars act as a grouping symbol. Perform any operations inside the absolute value bars first.

Example 1 *Evaluate an Expression with Absolute Value*

Evaluate $1.4 + |5y - 7|$ if $y = -3$.

$1.4 + |5y - 7| = 1.4 + |5(-3) - 7|$	Replace y with -3.
$= 1.4 + |-15 - 7|$	Simplify $5(-3)$ first.
$= 1.4 + |-22|$	Subtract 7 from -15.
$= 1.4 + 22$	$|-22| = 22$
$= 23.4$	Add.

The value is 23.4.

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters
- Study Guide and Intervention, pp. 19–20
- Skills Practice, p. 21
- Practice, p. 22
- Reading to Learn Mathematics, p. 23
- Enrichment, p. 24

Graphing Calculator and Spreadsheet Masters, p. 28

Transparencies

5-Minute Check Transparency 1-4
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 1
Interactive Chalkboard

ABSOLUTE VALUE EQUATIONS Some equations contain absolute value expressions. The definition of absolute value is used in solving these equations. For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $-a = b$. This second case is often written as $a = -b$.

Example 2 Solve an Absolute Value Equation

Solve $|x - 18| = 5$. Check your solutions.

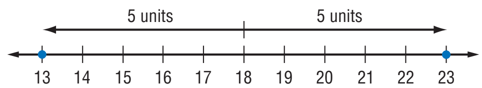
Case 1	$a = b$	or	**Case 2** $a = -b$
	$x - 18 = 5$		$x - 18 = -5$
	$x - 18 + 18 = 5 + 18$		$x - 18 + 18 = -5 + 18$
	$x = 23$		$x = 13$

CHECK

$\lvert x - 18\rvert = 5$	$\lvert x - 18\rvert = 5$
$\lvert 23 - 18\rvert \stackrel{?}{=} 5$	$\lvert 13 - 18\rvert \stackrel{?}{=} 5$
$\lvert 5\rvert \stackrel{?}{=} 5$	$\lvert -5\rvert \stackrel{?}{=} 5$
$5 = 5$ ✓	$5 = 5$ ✓

The solutions are 23 or 13. Thus, the solution set is {13, 23}.

On the number line, we can see that each answer is 5 units away from 18.



Because the absolute value of a number is always positive or zero, an equation like $|x| = -5$ is never true. Thus, it has no solution. The solution set for this type of equation is the **empty set**, symbolized by { } or ∅.

Study Tip

Common Misconception

For an equation like the one in Example 3, there is no need to consider the two cases. Remember to check your solutions in the original equation to prevent this error.

Example 3 No Solution

Solve $|5x - 6| + 9 = 0$.

$|5x - 6| + 9 = 0$ Original equation

$|5x - 6| = -9$ Subtract 9 from each side.

This sentence is *never* true. So the solution set is ∅.

It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions of the original equation.

Example 4 One Solution

Solve $|x + 6| = 3x - 2$. Check your solutions.

Case 1	$a = b$	or	**Case 2** $a = -b$
	$x + 6 = 3x - 2$		$x + 6 = -(3x - 2)$
	$6 = 2x - 2$		$x + 6 = -3x + 2$
	$8 = 2x$		$4x + 6 = 2$
	$4 = x$		$4x = -4$
			$x = -1$

There appear to be two solutions, 4 or −1.

(continued on the next page)

www.algebra2.com/extra_examples

2 Teach

ABSOLUTE VALUE EXPRESSIONS

In-Class Example Power Point®

1 Evaluate $2.7 + |6 - 2x|$ if $x = 4$. **4.7**

Teaching Tip Students may find it helpful to read the first absolute value bar as "the distance of" and the last absolute value bar as "from zero, without regard to direction." So, the expression $|6 - 2x|$ would be read as "the distance of the value of $6 - 2x$ from zero, without regard to direction."

ABSOLUTE VALUE EQUATIONS

In-Class Examples Power Point®

2 Solve $|y + 3| = 8$. Check your solutions. **{−11, 5}**

3 Solve $|6 - 4t| + 5 = 0$. **∅**

Teaching Tip Remind students to think about the meaning of the mathematical sentence before they begin their calculations and again when they evaluate the reasonableness of their solution.

4 Solve $|8 + y| = 2y - 3$. Check your solutions. **{11}**

Concept Check

Ask students if $-h$ must represent a negative number. **No, if h is negative then $-h$ is positive.** Have them find a value for h that makes this statement true: $|h| = -h$. **Zero and all negative numbers can be values for h.**

DAILY INTERVENTION

Differentiated Instruction ELL

Verbal/Linguistic Some students may think that the absolute value of x is always x. Suggest that they say in words the meaning of $|x|$ as "the distance of x from zero without regard to direction" to see that, for example, the distance of −3 from zero without regard to direction, cannot be −3. Suggest that they test some positive and negative values for the variable to show that the statement "the absolute value of x is always x" is not true.

ELL notations throughout the chapter indicate items that can assist English-Language Learners.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Absolute Values Expressions:** 17–28
- **Absolute Value Equations:** 29–49

Odd/Even Assignments

Exercises 17–48 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–25 odd, 29–43 odd, 47, 49, 50–54, 59–79

Average: 17–49 odd, 50–54, 59–79 (optional: 55–58)

Advanced: 18–48 even, 50–73 (optional: 74–79)

Answers

3. Always; since the opposite of 0 is still 0, this equation has only one case, $ax + b = 0$. The solution is $-\frac{b}{a}$.

CHECK

$|x + 6| = 3x - 2$ or $|x + 6| = 3x - 2$

$|4 + 6| \stackrel{?}{=} 3(4) - 2$ $\qquad$ $|-1 + 6| \stackrel{?}{=} 3(-1) - 2$

$|10| \stackrel{?}{=} 12 - 2$ $\qquad$ $|5| \stackrel{?}{=} -3 - 2$

$10 = 10$ ✓ $\qquad$ ~~$5 = -5$~~

Since $5 \neq -5$, the only solution is 4. Thus, the solution set is $\{4\}$.

Check for Understanding

Concept Check

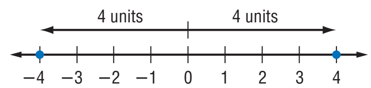
1. $|a| = -a$ when a is a negative number and the opposite of a negative number is positive.

2a. $|x| = 4$

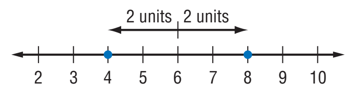
2b. $|x - 6| = 2$

1. **Explain** why if the absolute value of a number is always nonnegative, $|a|$ can equal $-a$.
2. **Write** an absolute value equation for each solution set graphed below.

a.



b.



3. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain. **See margin.**

For all real numbers a and b, $a \neq 0$, the equation $|ax + b| = 0$ will have one solution.

4. **OPEN ENDED** Write and evaluate an expression with absolute value. **Sample answer: $|4 - 6|$; 2**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
5–7	1
8–13	2–4
14–16	2

Evaluate each expression if $a = -4$ and $b = 1.5$.

5. $|a + 12|$ **8**
6. $|-6b|$ **9**
7. $-|a + 21|$ **−17**

Solve each equation. Check your solutions.

8. $|x + 4| = 17$ **{−21, 13}**
9. $|b + 15| = 3$ **{−18, −12}**
10. $|a - 9| = 20$ **{−11, 29}**
11. $|y - 2| = 34$ **{−32, 36}**
12. $|2w + 3| + 6 = 2$ **∅**
13. $|c - 2| = 2c - 10$ **{8}**

Application

FOOD **For Exercises 14–16, use the following information.**

A meat thermometer is used to assure that a safe temperature has been reached to destroy bacteria. Most meat thermometers are accurate to within plus or minus 2°F. **Source:** U.S. Department of Agriculture

Check for Understanding exercises are intended to be completed in class. Concept Check exercises ensure that students understand the concepts in the lesson. The other exercises are representative of the

14. The ham you are baking needs to reach an internal temperature of 160°F. If the thermometer reads 160°F, write an equation to determine the least and greatest temperatures of the meat. **$|x - 160| = 2$**
15. Solve the equation you wrote in Exercise 14. **least: 158°F; greatest: 162°F**
16. To what temperature reading should you bake a ham to ensure that the minimum internal temperature is reached? Explain. **162°F; This would ensure a minimum internal temperature of 160°F.**

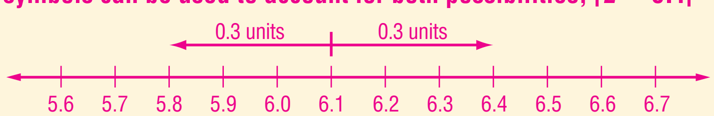
★ indicates increased difficulty

Practice and Apply

Evaluate each expression if $a = -5$, $b = 6$, and $c = 2.8$.

17. $|-3a|$ **15**
18. $|-4b|$ **24**
19. $|a + 5|$ **0**
20. $|2 - b|$ **4**
21. $|2b - 15|$ **3**
22. $|4a + 7|$ **13**
23. $-|18 - 5c|$ **−4**
24. $-|c - a|$ **−7.8**
25. $6 - |3c + 7|$ **−9.4**
26. $9 - |-2b + 8|$ **5**
★ 27. $3|a - 10| + |2a|$ **55**
★ 28. $|a - b| - |10c - a|$ **−22**

52. Answers should include the following.
- This equation needs to show that the difference of the estimate E from the originally stated magnitude of 6.1 could be plus 0.3 or minus 0.3, as shown in the graph below. Instead of writing two equations, $E - 6.1 = 0.3$ and $E - 6.1 = -0.3$, absolute value symbols can be used to account for both possibilities, $|E - 6.1| = 0.3$.



- Using an original magnitude of 5.9, the equation to represent the estimated extremes would be $|E - 5.9| = 0.3$.

Homework Help

For Exercises	See Examples
17–28	1
29–49	2–4

Extra Practice

See page 829.

Solve each equation. Check your solutions.

29. $|x - 25| = 17$ **{8, 42}**

30. $|y + 9| = 21$ **{12, −30}**

31. $|a + 12| = 33$ **{−45, 21}**

32. $2|b + 4| = 48$ **{−28, 20}**

33. $8|w - 7| = 72$ **{−2, 16}**

34. $|3x + 5| = 11$ **$\left\{2, -\frac{16}{3}\right\}$**

35. $|2z - 3| = 0$ **$\left\{\frac{3}{2}\right\}$**

36. $|6c - 1| = -2$ **∅**

37. $7|4x - 13| = 35$ **$\left\{2, \frac{9}{2}\right\}$**

38. $-3|2n + 5| = -9$ **{−4, −1}**

39. $-12|9x + 1| = 144$ **∅**

40. $|5x + 9| + 6 = 1$ **∅**

41. $|a - 3| - 14 = -6$ **{−5, 11}**

42. $3|p - 5| = 2p$ **{3, 15}**

43. $3|2a + 7| = 3a + 12$ **$\left\{-\frac{11}{3}, -3\right\}$**

44. $|3x - 7| - 5 = -3$ **$\left\{3, \frac{5}{3}\right\}$**

★ 45. $4|3t + 8| = 16t$ **{8}**

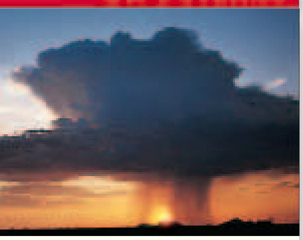
★ 46. $|15 + m| = -2m + 3$ **{−4}**

47. **COFFEE** Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee. **$|x - 200| = 5$; maximum: 205°F; minimum: 195°F**

48. **MANUFACTURING** A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bag the machine will approve. **$|x - 16| = 0.3$; heaviest: 16.3 oz, lightest: 15.7 oz**

49. **METEOROLOGY** The atmosphere of Earth is divided into four layers based on temperature variations. The troposphere is the layer closest to the planet. The average upper boundary of the layer is about 13 kilometers above Earth's surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere. **$|x - 13| = 5$; maximum: 18 km, minimum: 8 km**

More About



Meteorology

The troposphere is characterized by the density of its air and an average vertical temperature change of 6°C per kilometer. All weather phenomena occur within the troposphere.

Source: NASA

CRITICAL THINKING **For Exercises 50 and 51, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.**

50. If a and b are real numbers, then $|a + b| = |a| + |b|$.

51. If a, b, and c are real numbers, then $c|a + b| = |ca + cb|$. **sometimes; true only if $c \geq 0$**

50. sometimes; true only if $a \geq 0$ and $b \geq 0$ or if $a \leq 0$ and $b \leq 0$

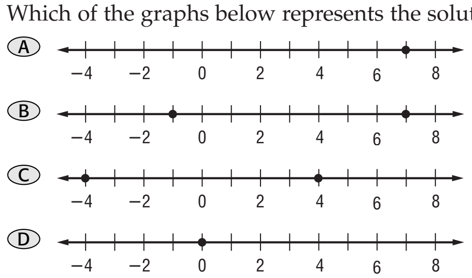
52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can an absolute value equation describe the magnitude of an earthquake?

Include the following in your answer:
- a verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible extremes in the variation of the earthquake's magnitude, and
- an equation to describe the extremes for a different magnitude.

Standardized Test Practice

53. Which of the graphs below represents the solution set for $|x - 3| - 4 = 0$? **B**



www.algebra2.com/self_check_quiz

There is a Study Guide and Intervention, Skills Practice, Practice, Reading to Learn Mathematics, and Enrichment Master for every lesson in the Student Edition. These masters can be found in the Chapter Resource Masters.

Study Guide and Intervention, p. 19 (shown) and p. 20

Absolute Value Expressions The **absolute value** of a number is the number of units it is from 0 on a number line. The symbol $|x|$ is used to represent the absolute value of a number x.

Absolute Value	• Words	a a a a a
	• Symbols	a $|a| = a$ $a \geq 0$ $|a| = -a$ $a < 0$

Example 1 **Evaluate $|-4| - |-2x|$ if $x = 6$.**

$|-4| - |-2x| = |-4| - |-2 \cdot 6|$
$= |-4| - |-12|$
$= 4 - 12$
$= -8$

Example 2 **Evaluate $|2x - 3y|$ if $x = -4$ and $y = 3$.**

$|2x - 3y| = |2(-4) - 3(3)|$
$= |-8 - 9|$
$= |-17|$
$= 17$

Exercises

Evaluate each expression if $w = -4$, $x = 2$, $y = \frac{1}{2}$, and $z = -6$.

1. $|2x - 8|$ **4**
2. $|6 + z| - |-7|$ **−7**
3. $5 + |w + z|$ **15**
4. $|x + 5| - |2w|$ **−1**
5. $|x| - |y| - |z|$ **$-4\frac{1}{2}$**
6. $|7 - x| + |3x|$ **11**
7. $|w - 4x|$ **12**
8. $|wz| - |xy|$ **23**
9. $|z| - 3|5yz|$ **−39**
10. $5|w| + 2|z - 2y|$ **34**
11. $|z| - 4|2z + y|$ **−40**
12. $10 - |xw|$ **2**
13. $|6y + z| + |yz|$ **6**
14. $3|wx| + \frac{1}{4}|4x + 8y|$ **27**
15. $7|yz| - 30$ **−9**
16. $14 - 2|w - xy|$ **4**
17. $|2x - y| + 5y$ **6**
18. $|xyz| + |wxz|$ **54**
19. $z|z| + x|x|$ **−32**
20. $12 - |10x - 10y|$ **−3**
21. $\frac{1}{2}|5z + 8w|$ **31**
22. $|yz - 4w| - w$ **17**
23. $\frac{3}{4}|wz| + \frac{1}{2}|8y|$ **20**
24. $xz - |xz|$ **−24**

Skills Practice, p. 21 and Practice, p. 22 (shown)

Evaluate each expression if $a = -1$, $b = -8$, $c = 5$, and $d = -1.4$.

1. $|6a|$ **6**
2. $|2b + 4|$ **12**
3. $-|10d + a|$ **−15**
4. $|17c| + |3b - 5|$ **114**
5. $-6|10a - 12|$ **−132**
6. $|2b - 1| - |-8b + 5|$ **−52**
7. $|5a - 7| + |3c - 4|$ **23**
8. $|1 - 7c| - |a|$ **33**
9. $-3|0.5c + 2| - |-0.5b|$ **−17.5**
10. $|4d| + |5 - 2a|$ **12.6**
11. $|a - b| + |b - a|$ **14**
12. $|2 - 2d| - 3|b|$ **−19.2**

Solve each equation. Check your solutions.

13. $|n - 4| = 13$ **{−9, 17}**
14. $|x - 13| = 2$ **{11, 15}**
15. $|2y - 3| = 29$ **{−13, 16}**
16. $7|x + 3| = 42$ **{−9, 3}**
17. $|3u - 6| = 42$ **{−12, 16}**
18. $|5x - 4| = -6$ **∅**
19. $-3|4x - 9| = 24$ **∅**
20. $-6|5 - 2y| = -9$ **$\left\{\frac{7}{4}, \frac{13}{4}\right\}$**
21. $|8 + p| = 2p - 3$ **{11}**
22. $|4w - 1| = 5w + 37$ **{−4}**
23. $4|2y - 7| + 5 = 9$ **{3, 4}**
24. $-2|7 - 3y| - 6 = -14$ **$\left\{1, \frac{11}{3}\right\}$**
25. $2|4 - s| = -3s$ **{−8}**
26. $5 - 3|2 + 2w| = -7$ **{−3, 1}**
27. $5|2r + 3| - 5 = 0$ **{−2, −1}**
28. $3 - 5|2d - 3| = 4$ **∅**

29. **WEATHER** A thermometer comes with a guarantee that the stated temperature differs from the actual temperature by no more than 1.5 degrees Fahrenheit. Write and solve an equation to find the minimum and maximum actual temperatures when the thermometer states that the temperature is 87.4 degrees Fahrenheit. **$|t - 87.4| = 1.5$; minimum: 85.9°F, maximum: 88.9°F**

30. **OPINION POLLS** Public opinion polls reported in newspapers are usually given with a margin of error. For example, a poll with a margin of error of ±5% is considered accurate to within plus or minus 5% of the actual value. A poll with a stated margin of error of ±3% predicts that candidate Tonwe will receive 51% of an upcoming vote. Write and solve an equation describing the minimum and maximum percent of the vote that candidate Tonwe is expected to receive. **$|x - 51| = 3$; minimum: 48%, maximum: 54%**

Reading to Learn Mathematics, p. 23 ELL

Pre-Activity **How can an absolute value equation describe the magnitude of an earthquake?**

Read the introduction to Lesson 1-4 at the top of page 28 in your textbook.

- What is a seismologist and what does magnitude of an earthquake mean? **a scientist who studies earthquakes; a number from 1 to 10 that tells how strong an earthquake is**
- Why is an absolute value equation rather than an equation without absolute value used to find the extremes in the actual magnitude of an earthquake in relation to its measured value on the Richter scale? **Sample answer: The actual magnitude can vary from the measured magnitude by up to 0.3 unit in either direction, so an absolute value equation is needed.**
- If the magnitude of an earthquake is estimated to be 6.9 on the Richter scale, it might actually have a magnitude as low as **6.6** or as high as **7.2**.

Reading the Lesson

1. Explain how $-a$ could represent a positive number. Give an example. **Sample answer: If a is negative, then $-a$ is positive. Example: If $a = -25$, then $-a = -(-25) = 25$.**
2. Explain why the absolute value of a number can never be negative. **Sample answer: The absolute value is the number of units it is from 0 on the number line. The number of units is never negative.**
3. What does the sentence $b \geq 0$ mean? **Sample answer: The number b is 0 or greater than 0.**
4. What does the symbol ∅ mean as a solution set? **Sample answer: If a solution set is ∅, then there are no solutions.**

Helping You Remember

5. How can the number line model for absolute value that is shown on page 28 of your textbook help you remember that many absolute value equations have two solutions? **Sample answer: The number line shows that for every positive number, there are two numbers that have that number as their absolute value.**

Enrichment, p. 24

Considering All Cases in Absolute Value Equations

You have learned that absolute value equations with one set of absolute value symbols have two cases that must be considered. For example, $|x + 3| = 5$ must be broken into $x + 3 = 5$ or $-(x + 3) = 5$. For an equation with two sets of absolute value symbols, four cases must be considered.

Consider the problem $|x + 2| + 3 = |x + 6|$. First we must write the equations for the case where $x + 6 \geq 0$ and where $x + 6 < 0$. Here are the equations for these two cases:

$|x + 2| + 3 = x + 6$

$|x + 2| + 3 = -(x + 6)$

Each of these equations also has two cases. By writing the equations for both cases of each equation above, you end up with the following four equations:

$x + 2 + 3 = x + 6$ $\quad$ $x + 2 + 3 = -(x + 6)$

$-(x + 2) + 3 = x + 6$ $\quad$ $-x - 2 + 3 = -(x + 6)$

... each ... equations and ... solutions in the original equation,

4 Assess

Open-Ended Assessment

Modeling Have students draw a number-line diagram like the one shown in Example 2 to model the equation $|x - 3| = 7$ and another number line to model the equation $|y| = 7$. You might suggest that students think of the equation $|y| = 7$ as $|y - 0| = 7$.

Each lesson ends with Open-Ended Assessment strategies for closing the lesson. These include writing, modeling, and speaking.

Getting Ready for Lesson 1-5

PREREQUISITE SKILL Lesson 1-5 presents solving inequalities using steps similar to those for solving equations. Exercises 74–79 should be used to determine your students' familiarity with solving equations.

54. Find the value of $-|-9| - |4| - 3|5 - 7|$. **A**

Ⓐ -19 Ⓑ -11 Ⓒ -7 Ⓓ 11

Extending the Lesson

For Exercises 55–58, consider the equation $|x + 1| + 2 = |x + 4|$.

55. To solve this equation, we must consider the case where $x + 4 \geq 0$ and the case where $x + 4 < 0$. Write the equations for each of these cases. **55.** $|x + 1| + 2 = x + 4$; $|x + 1| + 2 = -(x + 4)$

56. Notice that each equation you wrote in Exercise 55 has two cases. For each equation, write two other equations taking into consideration the case where $x + 1 \geq 0$ and the case where $x + 1 < 0$. **56.** $x + 1 + 2 = x + 4$; $-x - 1 + 2 = x + 4$; $x + 1 + 2 = -x - 4$; $-x - 1 + 2 = -x - 4$

57. Solve each equation you wrote in Exercise 56. Then, check each solution in the original equation, $|x + 1| + 2 = |x + 4|$. What are the solution(s) to this absolute value equation? **{−1.5}**

58. **MAKE A CONJECTURE** For equations with one set of absolute value symbols, two cases must be considered. For an equation with two sets of absolute value symbols, four cases must be considered. How many cases must be considered for an equation containing three sets of absolute value symbols? **8**

Maintain Your Skills

Mixed Review

Write an algebraic expression to represent each verbal expression. *(Lesson 1-3)*

59. twice the difference of a number and 11 **$2(n - 11)$**

60. the product of the square of a number and 5 **$5n^2$**

Solve each equation. Check your solution. *(Lesson 1-3)*

61. $3x + 6 = 22$ **$\frac{16}{3}$** **62.** $7p - 4 = 3(4 + 5p)$ **-2** **63.** $\frac{5}{7}y - 3 = \frac{3}{7}y + 1$ **14**

Name the property illustrated by each equation. *(Lesson 1-2)*

64. $(5 + 9) + 13 = 13 + (5 + 9)$ **Comm. (+)** **65.** $m(4 - 3) = m \cdot 4 - m \cdot 3$ **Dist.**

66. $\left(\frac{1}{4}\right)4 = 1$ **Mult. Inv.** **67.** $5x + 0 = 5x$ **Add. Iden.**

Determine whether each statement is *true* or *false*. If false, give a counterexample. *(Lesson 1-2)*

68. Every real number is a rational number. **false; $\sqrt{3}$**

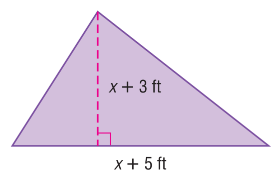
69. Every natural number is an integer. **true**

70. Every irrational number is a real number. **true**

71. Every rational number is an integer. **false; 1.2**

GEOMETRY **For Exercises 72 and 73, use the following information.**

The formula for the area A of a triangle is $A = \frac{1}{2}bh$, where b is the measure of the base and h is the measure of the height. *(Lesson 1-1)*



72. Write an expression to represent the area of the triangle above. **72.** $\frac{1}{2}(x + 3)(x + 5)$

73. Evaluate the expression you wrote in Exercise 72 for $x = 23$. **364 ft²**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.** *(To review **solving equations**, see page 20.)*

74. $14y - 3 = 25$ **2** **75.** $4.2x + 6.4 = 40$ **8** **76.** $7w + 2 = 3w - 6$ **-2**

77. $2(a - 1) = 8a - 6$ **$\frac{2}{3}$** **78.** $48 + 5y = 96 - 3y$ **6** **79.** $\frac{2x + 3}{5} = \frac{3}{10}$ **$-\frac{3}{4}$**

1-5 Solving Inequalities

What You'll Learn

- Solve inequalities.
- Solve real-world problems involving inequalities.

Vocabulary

- set-builder notation
- interval notation

How can inequalities be used to compare phone plans?

Kuni is trying to decide between two rate plans offered by a wireless phone company.

	Plan 1	Plan 2
Monthly Access Fee	\$35.00	\$55.00
Minutes Included	150	400
Additional Minutes	40¢	35¢

To compare these two rate plans, we can use inequalities. The monthly access fee for Plan 1 is less than the fee for Plan 2, \$35 < \$55. However, the additional minutes fee for Plan 1 is greater than that of Plan 2, \$0.40 > \$0.35.

SOLVE INEQUALITIES For any two real numbers, a and b, exactly one of the following statements is true.

$$a < b \qquad a = b \qquad a > b$$

This is known as the **Trichotomy Property** or the *property of order*.

Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.

Study Tip

Properties of Inequality
The properties of inequality are also known as *axioms* of inequality.

Key Concept — *Properties of Inequality*

Addition Property of Inequality

- **Words** For any real numbers, a, b, and c:
  If $a > b$, then $a + c > b + c$.
  If $a < b$, then $a + c < b + c$.
- **Example**
  $3 < 5$
  $3 + (-4) < 5 + (-4)$
  $-1 < 1$

Subtraction Property of Inequality

- **Words** For any real numbers, a, b, and c:
  If $a > b$, then $a - c > b - c$.
  If $a < b$, then $a - c < b - c$.
- **Example**
  $2 > -7$
  $2 - 8 > -7 - 8$
  $-6 > -15$

These properties are also true for ≤ and ≥ .

These properties can be used to solve inequalities. The solution sets of inequalities in one variable can then be graphed on number lines. Use a circle with an arrow to the left for < and an arrow to the right for > . Use and a dot with an arrow to the left for ≤ and an arrow to the right for ≥ .

1-5 Lesson Notes

1 Focus

5-Minute Check Transparency 1-5 Use as a quiz or review of Lesson 1-4.

Mathematical Background notes are available for this lesson on p. 4D.

Building on Prior Knowledge

In Lessons 1-3 and 1-4, students solved equations. In this lesson, students use similar steps to solve inequalities.

How can inequalities be used to compare phone plans?

Ask students:

- If Kuni knows that she will use no more than 150 minutes per month, which plan is best for her? **Plan 1**
- How much would she pay if she used 350 minutes under Plan 1? under Plan 2? **\$115; \$55**

Questions are provided at the beginning of each lesson to help you use the problem provided there to engage and inform students.

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters

- Study Guide and Intervention, pp. 25–26
- Skills Practice, p. 27
- Practice, p. 28
- Reading to Learn Mathematics, p. 29
- Enrichment, p. 30
- Assessment, p. 52

Transparencies

5-Minute Check Transparency 1-5
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 2
Interactive Chalkboard

A Four-step Teaching Plan shows you how to Focus, Teach, Practice/Apply, and Assess each lesson.

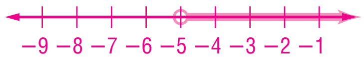
2 Teach

SOLVE INEQUALITIES

In-Class Example

Power Point®

1 Solve $4y - 3 < 5y + 2$. Graph the solution set on a number line. $y > -5$



Teaching Tip Ask students what difference it makes when you use the Addition and Subtraction Properties of Inequality whether the inequality sign is $<$, $>$, $\leq$, or $\geq$. **There is no difference in the calculations but there is a difference in the direction and beginning of the graph of the solution set.**

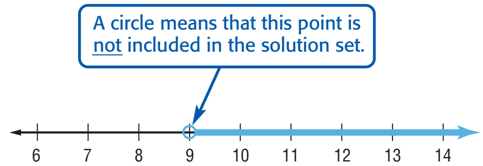
Example 1 Solve an Inequality Using Addition or Subtraction

Solve $7x - 5 > 6x + 4$. Graph the solution set on a number line.

$7x - 5 > 6x + 4$	Original inequality
$7x - 5 + (-6x) > 6x + 4 + (-6x)$	Add $-6x$ to each side.
$x - 5 > 4$	Simplify.
$x - 5 + 5 > 4 + 5$	Add 5 to each side.
$x > 9$	Simplify.

Any real number greater than 9 is a solution of this inequality.

The graph of the solution set is shown at the right.



CHECK Substitute 9 for x in $7x - 5 > 6x + 4$. The two sides should be equal. Then substitute a number greater than 9. The inequality should be true.

Multiplying or dividing each side of an inequality by a positive number does not change the truth of the inequality. However, multiplying or dividing each side of an inequality by a *negative* number requires that the order of the inequality be *reversed*. For example, to reverse $\leq$, replace it with $\geq$.

Key Concept — *Properties of Inequality*

Multiplication Property of Inequality

- **Words** For any real numbers, a, b, and c, where

	Words	Examples
c is positive:	if $a > b$, then $ac > bc$. if $a < b$, then $ac < bc$.	$-2 < 3$ $4(-2) < 4(3)$ $-8 < 12$
c is negative:	if $a > b$, then $ac < bc$. if $a < b$, then $ac > bc$.	$5 > -1$ $(-3)(5) < (-3)(1)$ $-15 < 3$

Division Property of Inequality

- **Words** For any real numbers, a, b, and c, where

	Words	Examples
c is positive:	if $a > b$, then $\frac{a}{c} > \frac{b}{c}$. if $a < b$, then $\frac{a}{c} < \frac{b}{c}$.	$-18 < -9$ $\frac{-18}{3} < \frac{-9}{3}$ $-6 < -3$
c is negative:	if $a > b$, then $\frac{a}{c} < \frac{b}{c}$. if $a < b$, then $\frac{a}{c} > \frac{b}{c}$.	$12 > 8$ $\frac{12}{-2} < \frac{8}{-2}$ $-6 < -4$

These properties are also true for $\leq$ and $\geq$.

Study Tip

Reading Math
$\{x \mid x > 9\}$ is read the set of all numbers x such that x is greater than 9.

The solution set of an inequality can be expressed by using **set-builder notation**. For example, the solution set in Example 1 can be expressed as $\{x \mid x > 9\}$.

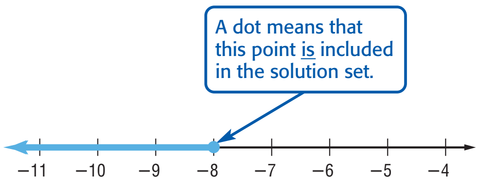
Example 2 Solve an Inequality Using Multiplication or Division

Solve $-0.25y \geq 2$. Graph the solution set on a number line.

$-0.25y \geq 2$	Original inequality
$\dfrac{-0.25y}{-0.25} \leq \dfrac{2}{-0.25}$	Divide each side by -0.25, reversing the inequality symbol.
$y \leq -8$	Simplify.

The solution set is $\{y \mid y \leq -8\}$.

The graph of the solution set is shown below.



Study Tip

Reading Math

The symbol $+\infty$ is read *positive infinity*, and the symbol $-\infty$ is read *negative infinity*.

The solution set of an inequality can also be described by using **interval notation**. The infinity symbols $+\infty$ and $-\infty$ are used to indicate that a set is unbounded in the positive or negative direction, respectively. To indicate that an endpoint *is not* included in the set, a parenthesis, (or), is used.

$x < 2$ — interval notation $(-\infty, 2)$

A bracket is used to indicate that the endpoint, -2, *is* included in the solution set below. Parentheses are always used with the symbols $+\infty$ and $-\infty$, because they do not include endpoints.

$x \geq -2$ — interval notation $[-2, +\infty)$

Study Tip

Solutions to Inequalities

When solving an inequality,

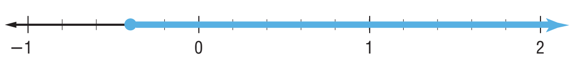
- if you arrive at a false statement, such as $3 > 5$, then the solution set for that inequality is the empty set, $\varnothing$.
- if you arrive at a true statement such as $3 > -1$, then the solution set for that inequality is the set of all real numbers.

Example 3 Solve a Multi-Step Inequality

Solve $-m \leq \frac{m+4}{9}$. Graph the solution set on a number line.

$-m \leq \dfrac{m+4}{9}$	Original inequality
$-9m \leq m + 4$	Multiply each side by 9.
$-10m \leq 4$	Add $-m$ to each side.
$m \geq -\dfrac{4}{10}$	Divide each side by -10, reversing the inequality symbol.
$m \geq -\dfrac{2}{5}$	Simplify.

The solution set is $\left[-\frac{2}{5}, +\infty\right)$ and is graphed below.

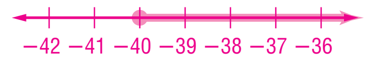


www.algebra2.com/extra_examples

In-Class Examples

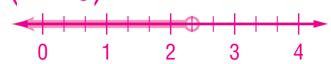
Power Point®

2 Solve $12 \geq -0.3p$. Graph the solution set on a number line. $\{p \mid p \geq -40\}$



Teaching Tip Remind students that when solving an inequality, in order to keep each intermediate inequality equivalent to the original, they must show both the division by a negative number and the reversal of the inequality sign in the same step.

3 Solve $-x > \frac{x-7}{2}$. Graph the solution set on a number line. $\left(-\infty, \frac{7}{3}\right)$



Concept Check

Ask students to name three different ways to show the solution of an inequality. **four possible responses: as a graph on a number line, as an inequality, using set-builder notation, using interval notation**

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students discuss the differences between solving an equation and solving an inequality and then how the solution processes are the same.

REAL-WORLD PROBLEMS WITH INEQUALITIES

In-Class Example Power Point®

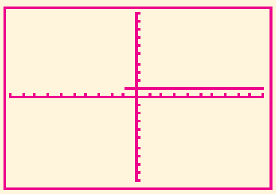
Teaching Tip To understand the situation given in Example 4, some students may find it helpful to make a sketch representing the elevator, the boxes, and the person.

4 **CONSUMER COSTS** Alida has at most \$10.50 to spend at a convenience store. She buys a bag of potato chips and a can of soda for \$1.55. If gasoline at this store costs \$1.35 per gallon, how many gallons of gasoline can Alida buy for her car, to the nearest tenth of a gallon? **no more than 6.6 gal**

Answer

Graphing Calculator Investigation

1. The graph is of the line $y = 1$, for $x \geq -1$.



Answers (p. 37)

4. $(-\infty, 1.5)$
5. $\left(-\infty, \frac{5}{3}\right]$
6. $[3, +\infty)$
7. $(6, +\infty)$
8. $(-\infty, -7)$
9. $(15, +\infty)$
10. $(-\infty, -24]$
11. $(-\infty, +\infty)$

Every effort is made to show the Answers to exercises (1) on the reduced Student Edition page, or (2) in the Teacher's margin of the Teacher's Wraparound Edition. However, answers that do not fit in either of these places can be found in pages at the end of each chapter.

REAL-WORLD PROBLEMS WITH INEQUALITIES Inequalities can be used to solve many verbal and real-world problems.

Study Tip

Inequality Phrases

- $<$ is less than; is fewer than
- $>$ is greater than; is more than
- $\leq$ is at most; is no more than; is less than or equal to
- $\geq$ is at least; is no less than; is greater than or equal to

Example 4 Write an Inequality

DELIVERIES Craig is delivering boxes of paper to each floor of an office building. Each box weighs 64 pounds, and Craig weighs 160 pounds. If the maximum capacity of the elevator is 2000 pounds, how many boxes can Craig safely take on each elevator trip?

Explore Let b = the number of boxes Craig can safely take on each trip. A maximum capacity of 2000 pounds means that this weight must be less than or equal to 2000.

Plan The total weight of the boxes is $64b$. Craig's weight plus the total weight of the boxes must be less than or equal to 2000. Write an inequality.

Craig's weight	plus	the weight of the boxes	is less than or equal to	2000.
160	$+$	$64b$	$\leq$	2000

Solve

$160 + 64b \leq 2000$	Original inequality
$160 - 160 + 64b \leq 2000 - 160$	Subtract 160 from each side.
$64b \leq 1840$	Simplify.
$\frac{64b}{64} \leq \frac{1840}{64}$	Divide each side by 64.
$b \leq 28.75$	Simplify.

Examine Since he cannot take a fraction of a box, Craig can take no more than 28 boxes per trip and still meet the safety requirements of the elevator.

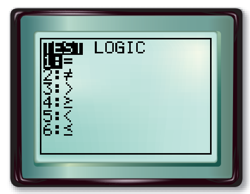
You can use a graphing calculator to find the solution set for an inequality.

Graphing Calculator Investigation

Solving Inequalities

The inequality symbols in the TEST menu on the TI-83 Plus are called *relational operators*. They compare values and return 1 if the test is true or 0 if the test is false.

You can use these relational operators to find the solution set of an inequality in one variable.



Think and Discuss **1. See margin.**

1. Clear the Y= list. Enter $11x + 3 \geq 2x - 6$ as Y1. Put your calculator in DOT mode. Then, graph in the standard viewing window. Describe the graph.
2. Using the TRACE function, investigate the graph. What values of x are on the graph? What values of y are on the graph? **all real numbers; 0 and 1**
3. Based on your investigation, what inequality is graphed? $x \geq -1$
4. Solve $11x + 3 \geq 2x - 6$ algebraically. How does your solution compare to the inequality you wrote in Exercise 3? **The solutions are the same.**

Graphing Calculator Investigation

Solving Inequalities After students enter $11x + 3$, have them press [2nd] [MATH] 4 to insert the $\geq$ symbol before entering $2x - 6$. The values of x for which 0 is returned (where the inequality is false) are not visible on the screen because they overlay part of the x-axis. To help students realize this fact, have them use the Trace feature to travel from positive values of x to increasingly negative values of x along the graph shown in the window.

Check for Understanding

Concept Check

1. Dividing by a number is the same as multiplying by its inverse.

1. **Explain** why it is not necessary to state a division property for inequalities.
2. **Write** an inequality using the $>$ symbol whose solution set is graphed below. Sample answer: $-2n > -6$

3. **OPEN ENDED** Write an inequality for which the solution set is the empty set. Sample answer: $x + 2 < x + 1$

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–11	1–3
12–14	4

4–11. See margin for interval notation. See pp. 53A–53B for graphs.

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

4. $a + 2 < 3.5$ $\{a \mid a < 1.5\}$
5. $5 \geq 3x$ $\left\{x \mid x \leq \frac{5}{3}\right\}$
6. $11 - c \leq 8$ $\{c \mid c \geq 3\}$
7. $4y + 7 > 31$ $\{y \mid y > 6\}$
8. $2w + 19 < 5$ $\{w \mid w < -7\}$
9. $-0.6p < -9$ $\{p \mid p > 15\}$
10. $\frac{n}{12} + 15 \leq 13$ $\{n \mid n \leq -24\}$
11. $\frac{5z + 2}{4} < \frac{5z}{4} + 2$ all real numbers

Define a variable and write an inequality for each problem. Then solve.

12. The product of 12 and a number is greater than 36. $12n > 36;\ n > 3$
13. Three less than twice a number is at most 5. $2n - 3 \leq 5;\ n \leq 4$

Application

14. **SCHOOL** The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need to make on the final exam to have a final grade of at least 80? at least 92

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–40	1–3
41–51	4

Extra Practice

See page 829.

15–38. See margin for interval notation. See pp. 53A–53B for graphs.

21. $\{k \mid k \geq -3.5\}$
23. $\{m \mid m > -4\}$
27. $\{n \mid n \geq 1.75\}$
28. $\left\{w \mid w > -\frac{1}{20}\right\}$
29. $\{x \mid x < -279\}$
30. $\{c \mid c > -18\}$
31. $\{d \mid d \geq -5\}$
32. $\{z \mid z > 2.6\}$
34. $\left\{a \mid a \geq \frac{5}{7}\right\}$

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

15. $n + 4 \geq -7$ $\{n \mid n \geq -11\}$
16. $b - 3 \leq 15$ $\{b \mid b \leq 18\}$
17. $5x < 35$ $\{x \mid x < 7\}$
18. $\frac{d}{2} > -4$ $\{d \mid d > -8\}$
19. $\frac{g}{-3} \geq -9$ $\{g \mid g \leq 27\}$
20. $-8p \geq 24$ $\{p \mid p \leq -3\}$
21. $13 - 4k \leq 27$
22. ★ $14 > 7y - 21$ $\{y \mid y < 5\}$
23. $-27 < 8m + 5$
24. $6b + 11 \geq 15$ $\left\{b \mid b \geq \frac{2}{3}\right\}$
25. $2(4t + 9) \leq 18$ $\{t \mid t \leq 0\}$
26. $90 \geq 5(2r + 6)$ $\{r \mid r \leq 6\}$
27. $14 - 8n \leq 0$
28. $-4(5w - 8) < 33$
29. $0.02x + 5.58 < 0$
30. $1.5 - 0.25c < 6$
31. $6d + 3 \geq 5d - 2$
32. $9z + 2 > 4z + 15$
33. $2(g + 4) < 3g - 2(g - 5)$ $\{g \mid g < 2\}$
34. $3(a + 4) - 2(3a + 4) \leq 4a - 1$
35. $y < \frac{-y + 2}{9}$ $\left\{y \mid y < \frac{1}{5}\right\}$
36. $\frac{1 - 4p}{5} < 0.2$ $\{p \mid p > 0\}$
37. ★ $\frac{4x + 2}{6} < \frac{2x + 1}{3}$ $\varnothing$
38. $12\left(\frac{1}{4} - \frac{n}{3}\right) \leq -6n$ $\left\{n \mid n \leq -\frac{3}{2}\right\}$

39. **PART-TIME JOB** David earns \$5.60 an hour working at Box Office Videos. Each week, 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least \$105 a week, solve the inequality $5.6x - 0.25(5.6x) \geq 105$ to determine how many hours he must work. at least 25 h

40. **STATE FAIR** Juan's parents gave him \$35 to spend at the State Fair. He spends \$13.25 for food. If rides at the fair cost \$1.50 each, solve the inequality $1.5n + 13.25 \leq 35$ to determine how many rides he can afford. no more than 14 rides

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 1.
- add the properties of inequality given in this lesson to their list of real number properties.
- write several examples of both set-builder notation and interval notation.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Solve Inequalities:** 15–40
- **Real-World Problems with Inequalities:** 41–51

Exercises 15–46 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 56–58 require a graphing calculator.

Assignment Guide

Basic: 15–35 odd, 39–43 odd, 47–49, 52–55, 59–72

Average: 15–47 odd, 48–49, 52–55, 59–72 (optional: 56–58)

Advanced: 16–46 even, 48–66 (optional: 67–72)

All: Practice Quiz 2 (1–5)

Answers

15. $[-11, +\infty)$
16. $(-\infty, 18]$
17. $(-\infty, 7)$
18. $(-8, +\infty)$
19. $(-\infty, 27]$
20. $(-\infty, -3]$
21. $[-3.5, +\infty)$
22. $(-\infty, 5)$
23. $(-4, +\infty)$
24. $\left[\frac{2}{3}, +\infty\right)$
25. $(-\infty, 0]$
26. $(-\infty, 6]$
27. $[1.75, +\infty)$
28. $\left(-\frac{1}{20}, +\infty\right)$
29. $(-\infty, -279)$
30. $(-18, +\infty)$
31. $[-5, +\infty)$
32. $(2.6, +\infty)$
33. $(-\infty, 2)$
34. $\left[\frac{5}{7}, +\infty\right)$
35. $\left(-\infty, \frac{1}{5}\right)$
36. $(0, +\infty)$
37. $\varnothing$
38. $\left(-\infty, -\frac{3}{2}\right]$

Lesson 1-5 Solving Inequalities 37

Study Guide and Intervention, p. 25 (shown) and p. 26

Solve Inequalities The following properties can be used to solve inequalities.

Addition and Subtraction Properties for Inequalities				Multiplication and Division Properties for Inequalities				
		a b c				a b c	c ≠ 0	
1.	$a < b$	$a + c < b + c$	$a - c < b - c$	1. c	$a < b$	$ac < bc$	$\frac{a}{c} < \frac{b}{c}$	
2.	$a > b$	$a + c > b + c$	$a - c > b - c$	2. c	$a > b$	$ac > bc$	$\frac{a}{c} > \frac{b}{c}$	
				3. c	$a < b$	$ac > bc$	$\frac{a}{c} > \frac{b}{c}$	
				4. c	$a > b$	$ac < bc$	$\frac{a}{c} < \frac{b}{c}$	

These properties are also true for ≤ and ≥.

Example 1 **Solve $2x + 4 > 36$. Then graph the solution set on a number line.**

$2x + 4 - 4 > 36 - 4$
$2x > 32$
$x > 16$

The solution set is $\{x \mid x > 16\}$.

13 14 15 16 17 18 19 20 21

Example 2 **Solve $17 - 3w \ge 35$. Then graph the solution set on a number line.**

$17 - 3w \ge 35$
$17 - 3w - 17 \ge 35 - 17$
$-3w \ge 18$
$w \le -6$

The solution set is $(-\infty, -6]$.

−9 −8 −7 −6 −5 −4 −3 −2 −1

Exercises

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

1. $7(7a - 9) \le 84$ $\{a \mid a \le 3\}$ or $(-\infty, 3]$
2. $3(9z + 4) > 35z - 4$ $\{z \mid z < 2\}$ or $(-\infty, 2)$
3. $5(12 - 3n) < 165$ $\{n \mid n > -7\}$ or $(-7, +\infty)$
4. $18 - 4k < 2(k + 21)$ $\{k \mid k > -4\}$ or $(-4, +\infty)$
5. $4(b - 7) + 6 < 22$ $\{b \mid b < 11\}$ or $(-\infty, 11)$
6. $2 + 3(m + 5) \ge 4(m + 3)$ $\{m \mid m \le 5\}$ or $(-\infty, 5]$
7. $4x - 2 > -7(4x - 2)$ $\left\{x \mid x > \frac{1}{2}\right\}$ or $\left(\frac{1}{2}, +\infty\right)$
8. $\frac{1}{3}(2y - 3) > y + 2$ $\{y \mid y < -9\}$ or $(-\infty, -9)$
9. $2.5d + 15 \le 75$ $\{d \mid d \le 24\}$ or $(-\infty, 24]$

Skills Practice, p. 27 and Practice, p. 28 (shown)

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

1. $8x - 6 \ge 10$ $\{x \mid x \ge 2\}$ or $[2, \infty)$
2. $23 - 4u < 11$ $\{u \mid u > 3\}$ or $(3, \infty)$
3. $-16 - 8r \ge 0$ $\{r \mid r \le -2\}$ or $(-\infty, -2]$
4. $14s < 9s + 5$ $\{s \mid s < 1\}$ or $(-\infty, 1)$
5. $9x - 11 > 6x - 9$ $\left\{x \mid x > \frac{2}{3}\right\}$ or $\left(\frac{2}{3}, \infty\right)$
6. $-3(4w - 1) > 18$ $\left\{w \mid w < -\frac{5}{4}\right\}$ or $\left(-\infty, -\frac{5}{4}\right)$
7. $1 - 8u \le 3u - 10$ $\{u \mid u \ge 1\}$ or $[1, \infty)$
8. $17.5 < 19 - 2.5x$ $\{x \mid x < 0.6\}$ or $(-\infty, 0.6)$
9. $9(2r - 5) - 3 < 7r - 4$ $\{r \mid r < 4\}$ or $(-\infty, 4)$
10. $1 + 5(x - 8) \le 2 - (x + 5)$ $\{x \mid x \le 6\}$ or $(-\infty, 6]$
11. $\frac{4x - 3}{2} \ge -3.5$ $\{x \mid x \ge -1\}$ or $[-1, \infty)$
12. $q - 2(2 - q) \le 0$ $\left\{q \mid q \le \frac{4}{3}\right\}$ or $\left(-\infty, \frac{4}{3}\right]$
13. $-36 - 2(w + 77) > -4(2w + 52)$ $\{w \mid w > -3\}$ or $(-3, \infty)$
14. $4n - 5(n - 3) > 3(n + 1) - 4$ $\{n \mid n < 4\}$ or $(-\infty, 4)$

Define a variable and write an inequality for each problem. Then solve.

15. Twenty less than a number is more than twice the same number. $n - 20 > 2n$; $n < -20$
16. Four times the sum of twice a number and −3 is less than 5.5 times that same number. $4[2n + (-3)] < 5.5n$; $n < 4.8$
17. **HOTELS** The Lincoln's hotel room costs $90 a night. An additional 10% tax is added. Hotel parking is $12 per day. The Lincoln's expect to spend $30 in tips during their stay. Solve the inequality $90x + 90(0.1)x + 12x + 30 \le 600$ to find how many nights the Lincoln's can stay at the hotel without exceeding total hotel costs of $600. **5 nights**
18. **BANKING** Jan's account balance is $3800. Of this, $750 is for rent. Jan wants to keep a balance of at least $500. Write and solve an inequality describing how much she can withdraw and still meet these conditions. $3800 - 750 - w \ge 500$; $w \le \$2550$

Reading to Learn Mathematics, p. 29 ELL

Pre-Activity **How can inequalities be used to compare phone plans?**

Read the introduction to Lesson 1-5 at the top of page 33 in your textbook.

- Write an inequality comparing the number of minutes per month included in the two phone plans. $150 < 400$ or $400 > 150$
- Suppose that in one month you use 230 minutes of airtime on your wireless phone. Find your monthly cost with each plan.

  Plan 1: $67 Plan 2: $55

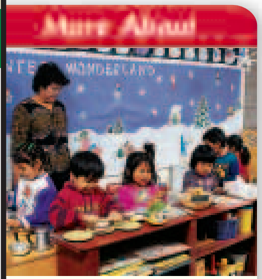
  Which plan should you choose? Plan 2

Reading the Lesson

1. There are several different ways to write or show inequalities. Write each of the following in interval notation.
   a. $\{x \mid x < -3\}$ $(-\infty, -3)$
   b. $\{x \mid x \ge 5\}$ $[5, +\infty)$
   c. −5 −4 −3 −2 −1 0 1 2 3 4 5 $(-\infty, 2]$
   d. −5 −4 −3 −2 −1 0 1 2 3 4 5 $(-1, +\infty)$
2. Show how you can write an inequality symbol followed by a number to describe each of the following situations.
   a. There are fewer than 600 students in the senior class. < 600
   b. A student may enroll in no more than six courses each semester. ≤ 6
   c. To participate in a concert, you must be willing to attend at least ten rehearsals. ≥ 10
   d. There is space for at most 165 students in the high school band. ≤ 165

Helping You Remember

3. One way to remember something is to explain it to another person. A common student error in solving inequalities is forgetting to reverse the inequality symbol when multiplying or dividing both sides of an inequality by a negative number. Suppose that your classmate is having trouble remembering this rule. How could you explain this rule to your classmate? **Sample answer: Draw a number line. Plot two positive numbers, for example, 3 and 8. Then plot their additive inverses, −3 and −8. Write an inequality that compares the positive numbers and one that compares the negative numbers. Notice that 8 > 3, but −8 < −3. The order changes when you multiply by −1.**



Child Care

In 1995, 55% of children ages three to five were enrolled in center-based child care programs. Parents cared for 26% of children, relatives cared for 19% of children, and non-relatives cared for 17% of children.

Source: National Center for Education Statistics

Define a variable and write an inequality for each problem. Then solve.

41. The sum of a number and 8 is more than 2. $n + 8 > 2$; $n > -6$
42. The product of −4 and a number is at least 35. $-4n \ge 35$; $n \le 8.75$
43. The difference of one half of a number and 7 is greater than or equal to 5.
44. One more than the product of −3 and a number is less than 16.
45. ★ Twice the sum of a number and 5 is no more than 3 times that same number increased by 11. $2(n + 5) \le 3n + 11$; $n \ge -1$
46. ★ 9 less than a number is at most that same number divided by 2. $n - 9 \le \frac{n}{2}$; $n \le 18$
47. **CHILD CARE** By Ohio law, when children are napping, the number of children per child care staff member may be as many as twice the maximum listed at the right. Write and solve an inequality to determine how many staff members are required to be present in a room where 17 children are napping and the youngest child is 18 months old. $2(7m) \ge 17$; $m \ge \frac{17}{14}$; at least 2 child care staff members

43. $\frac{1}{2}n - 7 \ge 5$; $n \ge 24$

44. $-3n + 1 < 16$; $n > -5$

Maximum Number of Children Per Child Care Staff Member	
5	12
2	12
6	12
	18
7	18
	30
8	30
	3

Source: Ohio Department of Job and Family Services

CAR SALES For Exercises 48 and 49, use the following information.
Mrs. Lucas earns a salary of $24,000 per year plus 1.5% commission on her sales. If the average price of a car she sells is $30,500, about how many cars must she sell to make an annual income of at least $40,000?

48. Write an inequality to describe this situation. $\$24{,}000 + 0.015(30{,}500n) \ge 40{,}000$
49. Solve the inequality and interpret the solution. $n \ge 34.97$; She must sell at least 35 cars.

TEST GRADES For Exercises 50 and 51, use the following information.
Ahmik's scores on the first four of five 100-point history tests were 85, 91, 89, and 94.

50. If a grade of at least 90 is an A, write an inequality to find the score Ahmik must receive on the fifth test to have an A test average. See margin.
51. Solve the inequality and interpret the solution. $s \ge 91$; Ahmik must score at least 91 on her next test to have an A test average.

52c. For all real numbers a, b, and c, if $a < b$ and $b < c$ then $a < c$.

52. **CRITICAL THINKING** Which of the following properties hold for inequalities? Explain your reasoning or give a counterexample.
    a. Reflexive b. Symmetric c. Transitive
    52a. It holds only for ≤ or ≥; $2 \not< 2$. 52b. $1 < 2$ but $2 \not< 1$

53. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. See pp. 53A–53B.

    How can inequalities be used to compare phone plans?

    Include the following in your answer:
    - an inequality comparing the number of minutes offered by each plan, and
    - an explanation of how Kuni might determine when Plan 1 might be cheaper than Plan 2 if she typically uses more than 150 but less than 400 minutes.

Enrichment, p. 30

Equivalence Relations

A relation R on a set A is an *equivalence relation* if it has the following properties.

Reflexive Property For any element a of set A, a R a.
Symmetric Property For all elements a and b of set A, if a R b, then b R a.
Transitive Property For all elements a, b, and c of set A, if a R b and b R c, then a R c.

Equality on the set of all real numbers is reflexive, symmetric, and transitive. Therefore, it is an equivalence relation.

In each of the following, a relation and a set are given. Write *yes* if the relation is an equivalence relation on the given set. If it is not, tell which of the properties it fails to exhibit.

1. <, {all numbers} no; reflexive, symmetric

Answer

50. $\frac{85 + 91 + 89 + 94 + s}{5} \ge 90$

Standardized Test Practice

54. If $4 - 5n \geq -1$, then n could equal all of the following EXCEPT **D**

Ⓐ $-\frac{1}{5}$. Ⓑ $\frac{1}{5}$. Ⓒ 1. Ⓓ 2.

55. If $a < b$ and $c < 0$, which of the following are true? **D**

I. $ac > bc$ **II.** $a + c < b + c$ **III.** $a - c > b - c$

Ⓐ I only Ⓑ II only Ⓒ III only

Ⓓ I and II only Ⓔ I, II, and III

Graphing Calculator

Use a graphing calculator to solve each inequality.

56. $-5x - 8 < 7$ **$x > -3$** 57. $-4(6x - 3) \leq 60$ **$x \geq -2$** 58. $3(x + 3) \geq 2(x + 4)$ **$x \geq -1$**

Maintain Your Skills

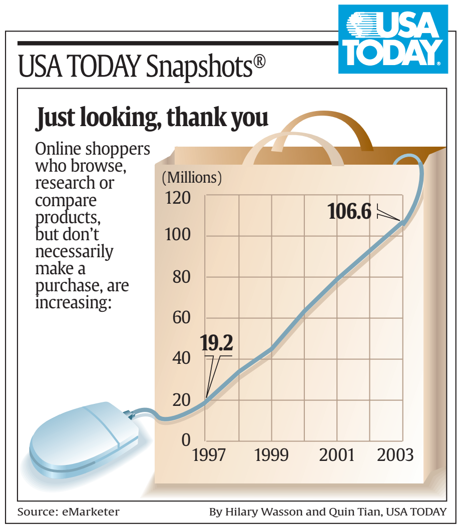
Mixed Review **Solve each equation. Check your solutions.** *(Lesson 1-4)*

59. $|x - 3| = 17$ **$\{-14, 20\}$** 60. $8|4x - 3| = 64$ 61. $|x + 1| = x$ **∅**

60. $\left\{-\frac{5}{4}, \frac{11}{4}\right\}$

62. **SHOPPING** On average, by how much did the number of people who just browse, but not necessarily buy, online increase each year from 1997 to 2003? Define a variable, write an equation, and solve the problem. *(Lesson 1-3)*

62. b = online browsers each year; $6b + 19.2 = 106.6$; about 14.6 million browsers each year



Name the sets of numbers to which each number belongs. *(Lesson 1-2)*

63. 31 **Q, R** 64. $-4.\overline{2}$ 65. $\sqrt{7}$ **I, R**

64. N, W, Z, Q, R

66. **BABY-SITTING** Jenny baby-sat for $5\frac{1}{2}$ hours on Friday night and 8 hours on Saturday. She charges \$4.25 per hour. Use the Distributive Property to write two equivalent expressions that represent how much money Jenny earned. *(Lesson 1-2)* **4.25(5.5 + 8); 4.25(5.5) + 4.25(8)**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solutions.**

*(To review **solving absolute value equations**, see Lesson 1-4.)*

67. $|x| = 7$ **$\{-7, 7\}$** 68. $|x + 5| = 18$ **$\{13, -23\}$** 69. $|5y - 8| = 12$ **$\left\{4, -\frac{4}{5}\right\}$**

70. $|2x - 36| = 14$ **$\{11, 25\}$** 71. $2|w + 6| = 10$ **$\{-11, -1\}$** 72. $|x + 4| + 3 = 17$ **$\{-18, 10\}$**

Practice Quiz 2 — Lessons 1-3 through 1-5

1. Solve $2d + 5 = 8d + 2$. Check your solution. *(Lesson 1-3)* **0.5**
2. Solve $s = \frac{1}{2}gt^2$ for g. *(Lesson 1-3)* **$\frac{2s}{t^2} = g$**
3. Evaluate $|x - 3y|$ if $x = -8$ and $y = 2$. *(Lesson 1-4)* **14**
4. Solve $3|3x + 2| = 51$. Check your solutions. *(Lesson 1-4)* **$\left\{-\frac{19}{3}, 5\right\}$**
5. Solve $2(m - 5) - 3(2m - 5) < 5m + 1$. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line. *(Lesson 1-5)* **See margin.**

4 Assess

Open-Ended Assessment

Writing Have students write their own list of tips for solving inequalities, including when to reverse the inequality sign and how to tell when the graph begins with a circle or with a dot.

Getting Ready for Lesson 1-6

PREREQUISITE SKILL Lesson 1-6 presents solving compound inequalities and absolute value inequalities. The procedure for solving absolute value inequalities are similar to those discussed for solving absolute value equations. Exercises 67–72 should be used to determine your students' familiarity with solving absolute value equations.

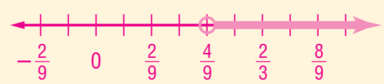
Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 1-3 through 1-5. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 1-4 and 1-5) is available on p. 52 of the *Chapter 1 Resource Masters*.

Answer (Practice Quiz 2)

5. $\left\{m \mid m > \frac{4}{9}\right\}$ or $\left(\frac{4}{9}, +\infty\right)$



Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

Glencoe's exclusive partnership with USA TODAY provides actual USA TODAY Snapshots® that illustrate mathematical concepts.

1-6 Lesson Notes

1 Focus

5-Minute Check Transparency 1-6 Use as a quiz or review of Lesson 1-5.

Mathematical Background notes are available for this lesson on p. 4D.

Building on Prior Knowledge

In Lesson 1-5 students solved inequalities, and in Lesson 1-4 they solved absolute value equations. In this lesson, they expand these skills to solving compound inequalities and absolute value inequalities.

How are compound inequalities used in medicine?

Ask students:

- If you are scheduled to have a glucose tolerance test at 10 A.M., at what hour should you begin fasting? **sometime between 6 P.M. and midnight**
- **Medicine** What does a glucose tolerance test measure? **how well the body processes sugar (glucose)**

1-6 Solving Compound and Absolute Value Inequalities

What You'll Learn

- Solve compound inequalities.
- Solve absolute value inequalities.

Vocabulary

- compound inequality
- intersection
- union

How are compound inequalities used in medicine?

One test used to determine whether a patient is diabetic and requires insulin is a glucose tolerance test. Patients start the test in a *fasting state*, meaning they have had no food or drink except water for at least 10 but no more than 16 hours. The acceptable number of hours h for fasting can be described by the following compound inequality.

$$h \geq 10 \text{ and } h \leq 16$$

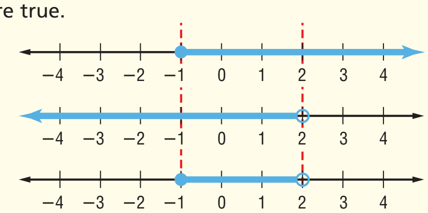
COMPOUND INEQUALITIES A **compound inequality** consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part of the inequality. The graph of a compound inequality containing *and* is the **intersection** of the solutions sets of the two inequalities.

Key Concept — "And" Compound Inequalities

- **Words** A compound inequality containing the word *and* is true if and only if *both* inequalities are true.
- **Example** $x \geq -1$

  $x < 2$

  $x \geq -1$ and $x < 2$



Study Tip

Interval Notation

The compound inequality $-1 \leq x < 2$ can be written as $[-1, 2)$, indicating that the solution set is the set of all numbers between -1 and 2, including -1, but not including 2.

Another way of writing $x \geq -1$ and $x < 2$ is $-1 \leq x < 2$. Both forms are read *x is greater than or equal to −1 and less than 2.*

Example 1 Solve an "and" Compound Inequality

Solve $13 < 2x + 7 \leq 17$. Graph the solution set on a number line.

Method 1

Write the compound inequality using the word *and*. Then solve each inequality.

$13 < 2x + 7$ and $2x + 7 \leq 17$

$6 < 2x$ $\qquad 2x \leq 10$

$3 < x$ $\qquad x \leq 5$

$$3 < x \leq 5$$

Method 2

Solve both parts at the same time by subtracting 7 from each part. Then divide each part by 2.

$13 < 2x + 7 \leq 17$

$6 < 2x \leq 10$

$3 < x \leq 5$

Resource Manager

Workbook and Reproducible Masters

Chapter 1 Resource Masters
- Study Guide and Intervention, pp. 31–32
- Skills Practice, p. 33
- Practice, p. 34
- Reading to Learn Mathematics, p. 35
- Enrichment, p. 36
- Assessment, p. 52

Teaching Algebra With Manipulatives Masters, p. 216

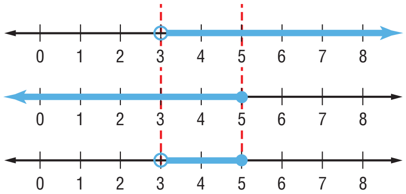
Transparencies

5-Minute Check Transparency 1-6
Answer Key Transparencies

Technology

Interactive Chalkboard

Graph the solution set for each inequality and find their intersection.



$x > 3$

$x \leq 5$

$3 < x \leq 5$

The solution set is $\{x \mid 3 < x \leq 5\}$.

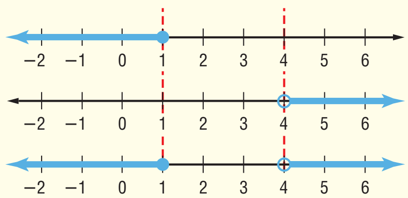
The graph of a compound inequality containing *or* is the **union** of the solution sets of the two inequalities.

Key Concept *"Or" Compound Inequalities*

- **Words** A compound inequality containing the word *or* is true if one or more of the inequalities is true.
- **Example** $x \leq 1$

$x > 4$

$x \leq 1$ or $x > 4$



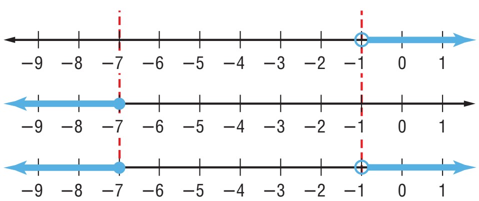
Example 2 Solve an "or" Compound Inequality

Solve $y - 2 > -3$ or $y + 4 \leq -3$. Graph the solution set on a number line.

Solve each inequality separately.

$y - 2 > -3$ or $y + 4 \leq -3$

$y > -1$ $\quad$ $y \leq -7$



$y > -1$

$y \leq -7$

$y > -1$ or $y \leq -7$

The solution set is $\{y \mid y > -1 \text{ or } y \leq -7\}$.

Study Tip

Interval Notation

In interval notation, the symbol for the union of the two sets is $\cup$. The compound inequality $y > -1$ or $y \leq -7$ is written as $(-\infty, -7] \cup (-1, +\infty)$, indicating that all values less than and including -7 are part of the solution set. In addition, all values greater than -1, not including -1, are part of the solution set.

ABSOLUTE VALUE INEQUALITIES In Lesson 1-4, you learned that the absolute value of a number is its distance from 0 on the number line. You can use this definition to solve inequalities involving absolute value.

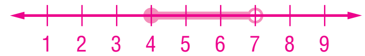
www.algebra2.com/extra_examples

2 Teach

COMPOUND INEQUALITIES

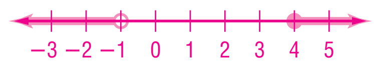
In-Class Examples Power Point®

1 Solve $10 \leq 3y - 2 < 19$. Graph the solution set on a number line. $\{y \mid 4 \leq y < 7\}$



Teaching Tip Remind students that the word *and* used in Method 1 means the values for $2x + 7$ must meet *both* conditions. That is, a value must be both greater than 13 and less than or equal to 17.

2 Solve $x + 3 < 2$ or $-x \leq -4$. Graph the solution set on a number line.
$\{x \mid x < -1 \text{ or } x \geq 4\}$



Reading Tip Students may make the mistake of wanting to associate *union* with the word *and* because union often indicates the joining of two or more things. As a memory device, point out that the word *or* begins with the letter *o* which is found in the word *union*, while *and* begins with the letter *a* which is not found in *union*.

Teacher to Teacher

Ron Millard — Shawnee Mission South H.S., Overland Park, KS

"To help make further work with absolute value more understandable, I teach my students to solve absolute value inequalities by using the definition of absolute value. Using this method, the statement $|3x - 12| \geq 6$ is rewritten as $3x - 12 \geq 6$ or $-(3x - 12) \geq 6$."

Teacher to Teacher features contain teaching suggestions from teachers who are creatively teaching Algebra in their classrooms.

ABSOLUTE VALUE INEQUALITIES

In-Class Examples Power Point®

3 Solve $3 > |d|$. Graph the solution set on a number line.
$\{d \mid -3 < d < 3\}$

4 Solve $3 < |d|$. Graph the solution set on a number line.
$\{d \mid d < -3 \text{ or } d > 3\}$

Reading Tip Make sure students understand the meaning of Examples 3 and 4 before they go on. Have them say the problem in words (for Example 3: "The distance of *a* from zero without regard to direction is less than 4.") and demonstrate where *a* can be located on a number line.

5 Solve $|2x - 2| \geq 4$. Graph the solution set on a number line.
$\{x \mid x \leq -1 \text{ or } x \geq 3\}$

Example 3 Solve an Absolute Value Inequality (<)

Solve $|a| < 4$. Graph the solution set on a number line.

You can interpret $|a| < 4$ to mean that the distance between a and 0 on a number line is less than 4 units. To make $|a| < 4$ true, you must substitute numbers for a that are fewer than 4 units from 0.

Notice that the graph of $|a| < 4$ is the same as the graph of $a > -4$ and $a < 4$.

All of the numbers between −4 and 4 are less than 4 units from 0. The solution set is $\{a \mid -4 < a < 4\}$.

Study Tip

Absolute Value Inequalities

Because the absolute value of a number is never negative,

- the solution of an inequality like $|a| < -4$ is the empty set.
- the solution of an inequality like $|a| > -4$ is the set of all real numbers.

Example 4 Solve an Absolute Value Inequality (>)

Solve $|a| > 4$. Graph the solution set on a number line.

You can interpret $|a| > 4$ to mean that the distance between a and 0 is greater than 4 units. To make $|a| > 4$ true, you must substitute values for a that are greater than 4 units from 0.

Notice that the graph of $|a| > 4$ is the same as the graph of $a > 4$ or $a < -4$.

All of the numbers *not* between −4 and 4 are greater than 4 units from 0. The solution set is $\{a \mid a > 4 \text{ or } a < -4\}$.

An absolute value inequality can be solved by rewriting it as a compound inequality.

Key Concept — *Absolute Value Inequalities*

- **Symbols** For all real numbers a and b, $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.
- **Examples** If $|2x + 1| < 5$, then $-5 < 2x + 1 < 5$.
  If $|2x + 1| > 5$, then $2x + 1 > 5$ or $2x + 1 < -5$.

These statements are also true for ≤ and ≥, respectively.

Example 5 Solve a Multi-Step Absolute Value Inequality

Solve $|3x - 12| \geq 6$. Graph the solution set on a number line.

$|3x - 12| \geq 6$ is equivalent to $3x - 12 \geq 6$ or $3x - 12 \leq -6$. Solve each inequality.

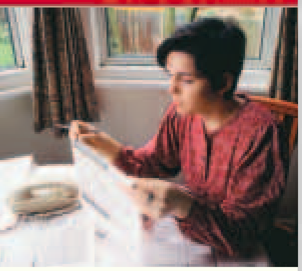
$3x - 12 \geq 6$	or	$3x - 12 \leq -6$
$3x \geq 18$		$3x \leq 6$
$x \geq 6$		$x \leq 2$

The solution set is $\{x \mid x \geq 6 \text{ or } x \leq 2\}$.

DAILY INTERVENTION — **Differentiated Instruction**

Kinesthetic Have students work in pairs to create a number line on the floor, perhaps using floor tiles and masking tape. Ask one partner to write or say an inequality such as $|x| < 5$ and then have the other partner walk from −5 to 5 on the number line to demonstrate the possible values for x.

More About



Job Hunting

When executives in a recent survey were asked to name one quality that impressed them the most about a candidate during a job interview, 32 percent said honesty and integrity.

Source: careerexplorer.net

Example 6 Write an Absolute Value Inequality

JOB HUNTING **To prepare for a job interview, Megan researches the position's requirements and pay. She discovers that the average starting salary for the position is \$38,500, but her actual starting salary could differ from the average by as much as \$2450.**

a. Write an absolute value inequality to describe this situation.

Let x = Megan's starting salary.

Her starting salary could differ from the average	by as much as	\$2450.
$\lvert 38{,}500 - x \rvert$	$\leq$	2450

b. Solve the inequality to find the range of Megan's starting salary.

Rewrite the absolute value inequality as a compound inequality. Then solve for x.

$$-2450 \leq 38{,}500 - x \leq 2450$$

$$-2450 - 38{,}500 \leq 38{,}500 - x - 38{,}500 \leq 2450 - 38{,}500$$

$$-40{,}950 \leq -x \leq -36{,}050$$

$$40{,}950 \geq x \geq 36{,}050$$

The solution set is $\{x \mid 36{,}050 \leq x \leq 40{,}950\}$. Thus, Megan's starting salary will fall between \$36,050 and \$40,950, inclusive.

In-Class Example

Power Point®

Teaching Tip Show students that $\lvert x - 38{,}500 \rvert \leq 2450$ will also work as the inequality for Example 6.

6 **HOUSING** According to a recent survey, the average monthly rent for a one-bedroom apartment in one city neighborhood is \$750. However, the actual rent for any given one-bedroom apartment might vary as much as \$250 from that average.

a. Write an absolute value inequality to describe this situation. $\lvert 750 - r \rvert \leq 250$

b. Solve the inequality to find the range of monthly rent. $\{r \mid 500 \leq r \leq 1000\}$; The actual rent falls between \$500 and \$1000.

Teaching Tip Suggest that students write some sample situations to help them understand problems that involve absolute value inequalities. In Example 6 for instance, students might ask themselves, "What are some possible salaries that fit this situation?"

Check for Understanding

Concept Check

1. **Write** a compound inequality to describe the following situation.
Buy a present that costs at least \$5 and at most \$15. $5 \leq c \leq 15$

2. **OPEN ENDED** Write a compound inequality whose graph is the empty set. Sample answer: $x < -3$ and $x > 2$

3. **FIND THE ERROR** Sabrina and Isaac are solving $\lvert 3x + 7 \rvert > 2$.

Sabrina	Isaac
$\lvert 3x + 7 \rvert > 2$	$\lvert 3x + 7 \rvert > 2$
$3x + 7 > 2$ or $3x + 7 < -2$	$-2 < 3x + 7 < 2$
$3x > 25$ $3x < -9$	$-9 < 3x < -5$
$x > -\frac{5}{3}$ $x < -3$	$-3 < x < -\frac{5}{3}$

Who is correct? Explain your reasoning.

3. Sabrina; an absolute value inequality of the form $\lvert a \rvert > b$ should be rewritten as an *or* compound inequality, $a > b$ or $a < b$.

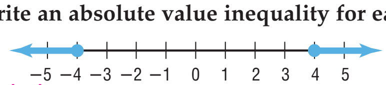
Guided Practice

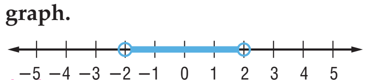
Write an absolute value inequality for each of the following. Then graph the solution set on a number line. 4–5. See margin for graphs.

4. all numbers between −8 and 8 $\lvert n \rvert < 8$

5. all numbers greater than 3 and less than −3 $\lvert n \rvert > 3$

Write an absolute value inequality for each graph.

6. 
$\lvert n \rvert \geq 4$

7. 
$\lvert n \rvert < 2$

GUIDED PRACTICE KEY	
Exercises	**Examples**
4, 5, 6, 7	3–5
8–13	1–5
14	6

3 Practice/Apply

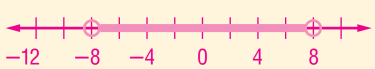
Study Notebook

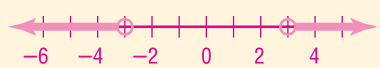
Have students—

- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 1.
- write a comparison between compound inequalities whose solutions involve the word "and," and compound inequalities whose solutions involve the word "or," including examples of both types.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Study Notebook tips offer suggestions for helping your students keep notes they can use to study this chapter.

Answers

4. 

5. 

About the Exercises...

Organization by Objective

- **Compound Inequalities:** 27–32, 45–47, 49–52
- **Absolute Value Inequalities:** 15–26, 33–44, 48

Odd/Even Assignments

Exercises 15–44 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 57–60 require a graphing calculator.

Assignment Guide

Basic: 15–23 odd, 27–39 odd, 45–47, 53–56, 61–75

Average: 15–45 odd, 46–47, 49–50, 53–56, 61–75 (optional: 57–60)

Advanced: 16–44 even, 48–75

DAILY INTERVENTION **FIND THE ERROR**

Have students use a finger to cover up "−2 <" in the second line of Isaac's solution. Ask them to compare the remaining inequality to the original, emphasizing the direction of the inequality symbols. Stress that Isaac's symbol should point in the same direction as the original symbol.

8–13. See margin for graphs.

8. $\{y \mid y > 4 \text{ or } y < -1\}$

Solve each inequality. Graph the solution set on a number line.

8. $y - 3 > 1$ or $y + 2 < 1$
9. $3 < d + 5 < 8$ $\{d \mid -2 < d < 3\}$
10. $|a| \geq 5$ $\{a \mid a \geq 5 \text{ or } a \leq -5\}$
11. $|g + 4| \leq 9$ $\{g \mid -13 \leq g \leq 5\}$
12. $|4k - 8| < 20$ $\{k \mid -3 < k < 7\}$
13. $|w| \geq -2$ all real numbers

Application

14. **FLOORING** Deion estimates that he will need between 55 and 60 ceramic tiles to retile his kitchen floor. If each tile costs \$6.25, write and solve a compound inequality to determine what the cost c of the tile could be.
$55 \leq \frac{c}{6.25} \leq 60$; $343.75 \leq c \leq 375$; between \$343.75 and \$375

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–26, 33–44	3–5
27–32, 51, 52	1, 2
45–50	6

Extra Practice

See page 829.

Write an absolute value inequality for each of the following. Then graph the solution set on a number line. 15–20. See margin for graphs.

15. all numbers greater than or equal to 5 or less than or equal to −5 $|n| \geq 5$
16. all numbers less than 7 and greater than −7 $|n| < 7$
17. all numbers between −4 and 4 $|n| < 4$
18. all numbers less than or equal to −6 or greater than or equal to 6 $|n| \geq 6$
19. all numbers greater than 8 or less than −8 $|n| > 8$
20. all number less than or equal to 1.2 and greater than or equal to −1.2 $|n| \leq 1.2$

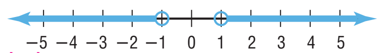
Write an absolute value inequality for each graph.

27–44. See pp. 53A–53B for graphs.

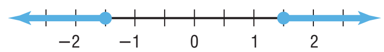
27. $\{p \mid p \leq 2 \text{ or } p \geq 8\}$

30. $\{c \mid c < -2 \text{ or } c \geq 1\}$

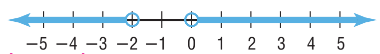
32. all real numbers

21. 
$|n| > 1$

22. 
$|n| \leq 5$

23. 
$|n| \geq 1.5$

24. 
$|n| < 6$

★ 25. 
$|n + 1| > 1$

★ 26. 
$|n - 1| \leq 3$

More About

Betta Fish

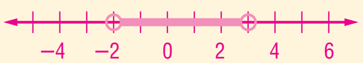
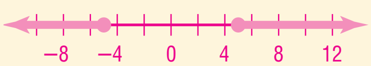
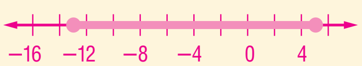
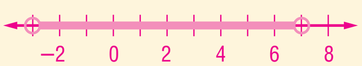
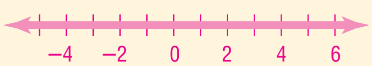
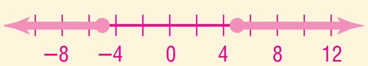
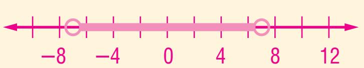
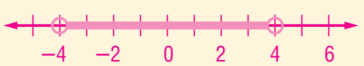
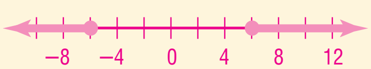
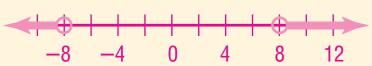
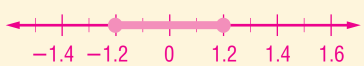
Adult Male Size: 3 inches
Water pH: 6.8–7.4
Temperature: 75–86°F
Diet: omnivore, prefers live foods
Tank Level: top dweller
Difficulty of Care: easy to intermediate
Life Span: 2–3 years
Source: www.about.com

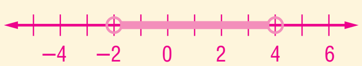
Solve each inequality. Graph the solution set on a number line.

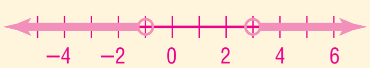
27. $3p + 1 \leq 7$ or $2p - 9 \geq 7$
28. $9 < 3t + 6 < 15$ $\{t \mid 1 < t < 3\}$
29. $-11 < -4x + 5 < 13$ $\{x \mid -2 < x < 4\}$
30. $2c - 1 < -5$ or $3c + 2 \geq 5$
31. $-4 < 4f + 24 < 4$ $\{f \mid -7 < f < -5\}$
32. $a + 2 > -2$ or $a - 8 < 1$
33. $|g| \leq 9$ $\{g \mid -9 \leq g \leq 9\}$
34. $|2m| \geq 8$ $\{m \mid m \geq 4 \text{ or } m \leq -4\}$
35. $|3k| < 0$ $\varnothing$
36. $|-5y| < 35$ $\{y \mid -7 < y < 7\}$
37. $|b - 4| > 6$ $\{b \mid b > 10 \text{ or } b > -2\}$
38. $|6r - 3| < 21$ $\{r \mid -3 < r < 4\}$
39. $|3w + 2| \leq 5$ $\left\{w \mid -\frac{7}{3} \leq w \leq 1\right\}$
40. $|7x| + 4 < 0$ $\varnothing$
41. ★ $|n| \geq n$ all real numbers
42. ★ $|n| \leq n$ $\{n \mid n \geq 0\}$
43. ★ $|2n - 7| \leq 0$ $\left\{n \mid n = \frac{7}{2}\right\}$
44. ★ $|n - 3| < n$ $\{n \mid n > 1.5\}$

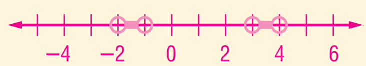
45. **BETTA FISH** A Siamese Fighting Fish, also known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta. $6.8 < x < 7.4$

Answers

8. 
9. 
10. 
11. 
12. 
13. 
15. 
16. 
17. 
18. 
19. 
20. 

53a. 

53b. 

53c. 

53d. $3 < |x + 2| \leq 8$ can be rewritten as $|x + 2| > 3$ and $|x + 2| \leq 8$. The solution of $|x + 2| > 3$ is $x > 1$ or $x < -5$. The solution of $|x + 2| \leq 8$ is $-10 \leq x \leq 6$. Therefore, the union of these two sets is ($x > 1$ or $x < -5$) and ($-10 \leq x \leq 6$). *(continued on the next page)*

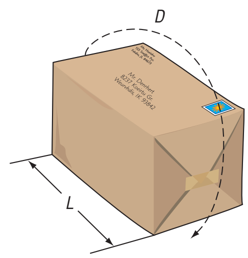
SPEED LIMITS **For Exercises 46 and 47, use the following information.**
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

46. Write an inequality to represent the allowable speed for a car on an interstate highway. **$45 \le s \le 65$**

47. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway. **$45 \le s \le 55$**

48. **HEALTH** *Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person's body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous. **$|t - 98.6| \ge 8$; $\{b \mid b > 106.6$ or $b < 90.6\}$**

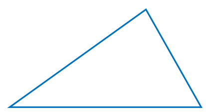
MAIL **For Exercises 49 and 50, use the following information.**
The U.S. Postal Service defines an oversized package as one for which the length L of its longest side plus the distance D around its thickest part is more than 108 inches and less than or equal to 130 inches.



49. Write a compound inequality to describe this situation. **108 in. $< L + D \le$ 130 in.**

50. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized. **84 in. $< L \le$ 106 in.**

GEOMETRY **For Exercises 51 and 52, use the following information.**
The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.



51. $a + b > c$, $a + c > b$, $b + c > a$

★ 51. Write three inequalities to express the relationships among the sides of $\triangle ABC$.

★ 52. Write a compound inequality to describe the range of possible measures for side c in terms of a and b. Assume that $a > b > c$. (*Hint*: Solve each inequality you wrote in Exercise 51 for c.) **$a - b < c < a + b$**

53. **CRITICAL THINKING** **Graph each set on a number line.** **a–d. See margin.**

a. $-2 < x < 4$

b. $x < -1$ or $x > 3$

c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (*Hint*: This is the intersection of the graphs in part **a** and part **b**.)

d. Solve $3 < |x + 2| \le 8$. Explain your reasoning and graph the solution set.

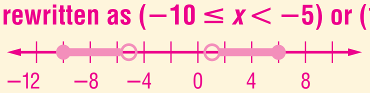
54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 53A–53B.**

How are compound inequalities used in medicine?

Include the following in your answer:
- an explanation as to when to use *and* and when to use *or* when writing a compound inequality,
- an alternative way to write $h \ge 10$ and $h \le 16$, and
- an example of an acceptable number of hours for this fasting state and a graph to support your answer.

www.algebra2.com/self_check_quiz

The union of the graph of $x > 1$ or $x < -5$ and the graph of $-10 \le x \le 6$ is shown below. From this we can see that the solution can be rewritten as $(-10 \le x < -5)$ or $(1 < x \le 6)$.



Study Guide and Intervention, p. 31 (shown) and p. 32

Compound Inequalities A compound inequality consists of two inequalities joined by the word *and* or the word *or*. To solve a compound inequality, you must solve each part separately.

And Compound Inequalities	$x > -4$ $x < 3$ (graph, –5 to 5)	
Or Compound Inequalities	$x \le -3$ $x > 1$ (graph, –5 to 5)	

Example 1 **Solve $-3 \le 2x + 5 \le 19$. Graph the solution set on a number line.**
$-3 \le 2x + 5$ and $2x + 5 \le 19$
$-8 \le 2x$ $\quad 2x \le 14$
$-4 \le x$ $\quad x \le 7$
$-4 \le x \le 7$

Example 2 **Solve $3y - 2 \ge 7$ or $2y - 1 \le -9$. Graph the solution set on a number line.**
$3y - 2 \ge 7$ or $2y - 1 \le -9$
$3y \ge 9$ or $2y \le -8$
$y \ge 3$ or $y \le -4$

Exercises

Solve each inequality. Graph the solution set on a number line.

1. $-10 < 3x + 2 \le 14$ **$\{x \mid -4 < x \le 4\}$**
2. $3a + 8 < 23$ or $\frac{1}{4}a - 6 > 7$ **$\{a \mid a < 5$ or $a > 52\}$**
3. $18 < 4x - 10 < 50$ **$\{x \mid 7 < x < 15\}$**
4. $5k + 2 < -13$ or $8k - 1 > 19$ **$\{k \mid k < -3$ or $k > 2.5\}$**
5. $100 \le 5y - 45 \le 225$ **$\{y \mid 29 \le y \le 54\}$**
6. $\frac{2}{3}b - 2 > 10$ or $\frac{3}{4}b + 5 < -4$ **$\{b \mid b < -12$ or $b > 18\}$**
7. $22 < 6w - 2 < 82$ **$\{w \mid 4 < w < 14\}$**
8. $4d - 1 > -9$ or $2d + 5 < 11$ **{all real numbers}**

Skills Practice, p. 33 and Practice, p. 34 (shown)

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

1. all numbers greater than 4 or less than –4 **$|n| > 4$**
2. all numbers between –1.5 and 1.5, including –1.5 and 1.5 **$|n| \le 1.5$**

Write an absolute value inequality for each graph.

3. **$|n| \ge 10$**
4. **$|n| < \frac{4}{3}$**

Solve each inequality. Graph the solution set on a number line.

5. $-8 \le 3y - 20 < 52$ **$\{y \mid 4 \le y < 24\}$**
6. $3(5x - 2) < 24$ or $6x - 4 > 4 + 5x$ **$\{x \mid x < 2$ or $x > 8\}$**
7. $2x - 3 > 15$ or $3 - 7x < 17$ **$\{x \mid x > -2\}$**
8. $15 - 5x \le 0$ and $5x + 6 \ge -14$ **$\{x \mid x \ge 3\}$**
9. $|2w| \ge 5$ **$\{w \mid w \le -\frac{5}{2}$ or $w \ge \frac{5}{2}\}$**
10. $|y + 5| < 2$ **$\{x \mid -7 < x < -3\}$**
11. $|x - 8| \ge 3$ **$\{x \mid x \le 5$ or $x \ge 11\}$**
12. $|2z - 2| \le 3$ **$\{z \mid -\frac{1}{2} \le z \le \frac{5}{2}\}$**
13. $|2x + 2| - 7 \le -5$ **$\{x \mid -2 \le x \le 0\}$**
14. $|x| > x - 1$ **all real numbers**
15. $|3b + 5| \le -2$ **$\varnothing$**
16. $|3n - 2| - 2 < 1$ **$\{n \mid -\frac{1}{3} < n < \frac{5}{3}\}$**

17. **RAINFALL** In 90% of the last 30 years, the rainfall at Shell Beach has varied no more than 6.5 inches from its mean value of 24 inches. Write and solve an absolute value inequality to describe the rainfall in the other 10% of the last 30 years. **$|r - 24| > 6.5$; $\{r \mid r < 17.5$ or $r > 30.5\}$**

18. **MANUFACTURING** A company's guidelines call for each can of soup produced not to vary from its stated volume of 14.5 fluid ounces by more than 0.08 ounces. Write and solve an absolute value inequality to describe acceptable can volumes. **$|v - 14.5| \le 0.08$; $\{v \mid 14.42 \le v \le 14.58\}$**

Reading to Learn Mathematics, p. 35 ELL

Pre-Activity **How are compound inequalities used in medicine?**

Read the introduction to Lesson 1-6 at the top of page 40 in your textbook.

- Five patients arrive at a medical laboratory at 11:30 A.M. for a glucose tolerance test. Each of them is asked when they last had something to eat or drink. Some of the patients are given the test and others are told that they must come back another day. Each of the patients is listed below with the times when they started to fast. (The P.M. times refer to the night before.) Which of the patients were accepted for the test?

Ora 5:00 A.M. Juanita 11:30 P.M. **Jason and Juanita**
Jason 1:30 A.M. Samir 5:00 P.M.

Reading the Lesson

1. a. Write a compound inequality that says, "x is greater than –3 *and* x is less than or equal to 4." **$-3 < x \le 4$**
   b. Graph the inequality that you wrote in part a on a number line.
2. Use a compound inequality and set-builder notation to describe the following graph. **$\{x \mid x \le -1$ or $x > 3\}$**
3. Write a statement equivalent to $|4x - 5| > 2$ that does not use the absolute value symbol. **$4x - 5 > 2$ or $4x - 5 < -2$**
4. Write a statement equivalent to $|3x + 7| < 8$ that does not use the absolute value symbol. **$-8 < 3x + 7 < 8$**

Helping You Remember

5. Many students have trouble knowing whether an absolute value inequality should be translated into an *and* or an *or* compound inequality. Describe a way to remember which of these applies to an absolute value inequality. Also describe how to recognize the difference from a number line graph. **Sample answer: If the absolute value quantity is followed by a < or ≤ symbol, the expression inside the absolute value bars must be *between* two numbers, so this becomes an *and* inequality. The number line graph will show a single interval *between* two numbers. If the absolute value quantity is followed by a > or ≥ symbol, it becomes an *or* inequality, and the graph will show two disconnected intervals with arrows going in opposite directions.**

Enrichment, p. 36

Conjunctions and Disjunctions

An absolute value inequality may be solved as a compound sentence.

Example 1 **Solve $|2x| < 10$.**
$|2x| < 10$ means $2x < 10$ and $2x > -10$.
Solve each inequality. $x < 5$ and $x > -5$.
Every solution for $|2x| < 10$ is a replacement for x that makes both $x < 5$ and $x > -5$ true.
A compound sentence that combines two statements by the word *and* is a *conjunction*.

Example 2 **Solve $|3x - 7| \ge 11$.**
$|3x - 7| \ge 11$ means $3x - 7 \ge 11$ or $3x - 7 \le -11$.
Solve each inequality. $3x \ge 18$ or $3x \le -4$

4 Assess

Open-Ended Assessment

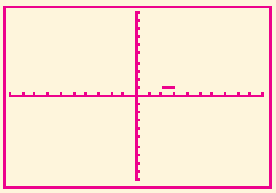
Writing Have students write a summary of the different kinds of inequalities they have seen in this chapter, with examples of each type and graphs of their solution sets.

Assessment Options

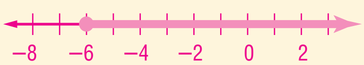
Quiz (Lesson 1-6) is available on p. 52 of the *Chapter 1 Resource Masters*.

Answers

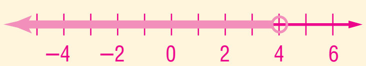
57.



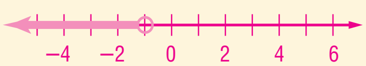
61.



62.



63.



Standardized Test Practice

55. **SHORT RESPONSE** Solve $|2x + 11| > 1$ for x. **$x > -5$ or $x < -6$**

56. If $5 < a < 7 < b < 14$, then which of the following best defines $\frac{a}{b}$? **D**

Ⓐ $\frac{5}{7} < \frac{a}{b} < \frac{1}{2}$ Ⓑ $\frac{5}{14} < \frac{a}{b} < \frac{1}{2}$

Ⓒ $\frac{5}{7} < \frac{a}{b} < 1$ Ⓓ $\frac{5}{14} < \frac{a}{b} < 1$

Graphing Calculator

LOGIC MENU **For Exercises 57–60, use the following information.**
You can use the operators in the LOGIC menu on the TI-83 Plus to graph compound and absolute value inequalities. To display the LOGIC menu, press [2] [ ] [▶].

57. Clear the Y= list. Enter $(5x + 2 > 12)$ and $(3x - 8 < 1)$ as Y1. With your calculator in DOT mode and using the standard viewing window, press [ ]. Make a sketch of the graph displayed. **See margin for sketch.**

58. Using the TRACE function, investigate the graph. Based on your investigation, what inequality is graphed? **$2 < x < 3$**

59. Write the expression you would enter for Y1 to find the solution set of the compound inequality $5x + 2 \geq 3$ or $5x + 2 \leq -3$. Then use the graphing calculator to find the solution set.

59. $(5x + 2 \geq 3)$ or $(5x + 2 \leq -3)$; $\{x \mid x \geq 0.2$ or $x \leq -1\}$

60. A graphing calculator can also be used to solve absolute value inequalities. Write the expression you would enter for Y1 to find the solution set of the inequality $|2x - 6| > 10$. Then use the graphing calculator to find the solution set. (*Hint*: The absolute value operator is item 1 on the MATH NUM menu.)
abs$(2x - 6) > 10$; $\{x \mid x < -2$ or $x > 8\}$

Maintain Your Skills

Mixed Review

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. *(Lesson 1-5)*

61–63. See margin for graphs.

61. $2d + 15 \geq 3$ **$d \geq -6$ or $[-6, +\infty)$**

62. $7x + 11 > 9x + 3$ **$x < 4$ or $(-\infty, 4)$**

63. $3n + 4(n + 3) < 5(n + 2)$ **$n < -1$ or $(-\infty, -1)$**

64. **CONTESTS** To get a chance to win a car, you must guess the number of keys in a jar to within 5 of the actual number. Those who are within this range are given a key to try in the ignition of the car. Suppose there are 587 keys in the jar. Write and solve an equation to determine the highest and lowest guesses that will give contestants a chance to win the car. *(Lesson 1-4)* **$|x - 587| = 5$; highest: 592 keys, lowest: 582 keys**

Solve each equation. Check your solutions.

65. $5|x - 3| = 65$ **{10, 16}**

66. $|2x + 7| = 15$ **{−11, 4}**

67. $|8c + 7| = -4$ **∅**

Name the property illustrated by each statement. *(Lesson 1-3)*

68. If $3x = 10$, then $3x + 7 = 10 + 7$. **Addition (=)**

69. If $-5 = 4y - 8$, then $4y - 8 = -5$. **Symmetric (=)**

70. If $-2x - 5 = 9$ and $9 = 6x + 1$, then $-2x - 5 = 6x + 1$. **Transitive (=)**

Simplify each expression. *(Lesson 1-2)*

71. $6a - 2b - 3a + 9b$ **$3a + 7b$**

72. $-2(m - 4n) - 3(5n + 6)$ **$-2m - 7n - 18$**

Find the value of each expression. *(Lesson 1-1)*

73. $6(5 - 8) \div 9 + 4$ **2**

74. $(3 + 7)^2 - 16 \div 2$ **92**

75. $\frac{7(1 - 4)}{8 - 5}$ **−7**

Key concepts from the lesson, one or two examples, and several practice problems are included in the Lesson-by-Lesson Review.

46 Chapter 1 Solving Equations and Inequalities

Chapter 1 Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 28)	equation (p. 20)	Reflexive Property (p. 21)
Addition Property	formula (p. 8)	set-builder notation (p. 34)
of Equality (p. 21)	Identity Property (p. 12)	solution (p. 20)
of Inequality (p. 33)	intersection (p. 40)	Substitution Property (p. 21)
algebraic expression (p. 7)	interval notation (p. 35)	Subtraction Property
Associative Property (p. 12)	Inverse Property (p. 12)	of Equality (p. 21)
Commutative Property (p. 12)	irrational numbers (p. 11)	of Inequality (p. 33)
compound inequality (p. 40)	Multiplication Property	Symmetric Property (p. 21)
counterexample (p. 14)	of Equality (p. 21)	Transitive Property (p. 21)
Distributive Property (p. 12)	of Inequality (p. 34)	Trichotomy Property (p. 33)
Division Property	open sentence (p. 20)	union (p. 41)
of Equality (p. 21)	order of operations (p. 6)	variable (p. 7)
of Inequality (p. 34)	rational numbers (p. 11)	
empty set (p. 29)	real numbers (p. 11)	

Choose the term from the list above that best matches each example.

1. $y > 3$ or $y < -2$ **compound inequality**
2. $0 + (-4b) = -4b$ **Iden. (+)**
3. $(m - 1)(-2) = -2(m - 1)$ **Comm. (×)**
4. $35x + 56 = 7(5x + 8)$ **Distributive**
5. $ab + 1 = ab + 1$ **Reflexive (=)**
6. If $2x = 3y - 4$, $3y - 4 = 7$, then $2x = 7$. **Trans. (=)**
7. $4(0.25) = 1$ **Multi. Inv.**
8. $2p + (4 + 9r) = (2p + 4) + 9r$ **Assoc. (+)**
9. $|5n|$ **absolute value**
10. $6y + 5z - 2(x + y)$ **algebraic expression**

Lesson-by-Lesson Review

1-1 Expressions and Formulas

See pages 6–10.

Concept Summary

- Order of Operations

Step 1 Simplify the expressions inside grouping symbols, such as parentheses, ( ), brackets, [ ], braces, { }, and fraction bars.

Step 2 Evaluate all powers.

Step 3 Do all multiplications and/or divisions from left to right.

Step 4 Do all additions and/or subtractions from left to right.

Example Evaluate $\frac{y^3}{3ab + 2}$ if $y = 4$, $a = -2$, and $b = -5$.

$\frac{y^3}{3ab + 2} = \frac{4^3}{3(-2)(-5) + 2}$ — $y = 4$, $a = -2$, and $b = -5$

$= \frac{64}{3(10) + 2}$ — Evaluate the numerator and denominator separately.

$= \frac{64}{32}$ or 2

www.algebra2.com/vocabulary_review

Chapter 1 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 1 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 1 is available on p. 50 of the *Chapter 1 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Since this is your students' first use of the Foldables, you may want to show some good examples, and ask volunteers to name the main ideas and procedures that they included. Then have everyone add any information they may have overlooked.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Exercises Find the value of each expression. *See Example 1 on page 6.*

11. $10 + 16 \div 4 + 8$ **22** **12.** $[21 - (9 - 2)] \div 2$ **7** **13.** $\frac{14(8 - 15)}{2}$ **−49**

Evaluate each expression if $a = 12$, $b = 0.5$, $c = -3$, and $d = \frac{1}{3}$.
See Examples 2 and 3 on page 7.

14. $6b - 5c$ **18** **15.** $c^3 + ad$ **−23** **16.** $\frac{9c + ab}{c}$ **7** **17.** $a[b^2(b + a)]$ **37.5**

1-2 Properties of Real Numbers

See pages 11–18.

Concept Summary

- Real numbers (R) can be classified as rational (Q) or irrational (I).
- Rational numbers can be classified as natural numbers (N), whole numbers (W), and/or integers (Z).
- Use the properties of real numbers to simplify algebraic expressions.

Example **Simplify $4(2b + 6c) + 3b - c$.**

$4(2b + 6c) + 3b - c$	$= 4(2b) + 4(6c) + 3b - c$	Distributive Property
	$= 8b + 24c + 3b - c$	Multiply.
	$= 8b + 3b + 24c - c$	Commutative Property (+)
	$= (8 + 3)b + (24 - 1)c$	Distributive Property
	$= 11b + 23c$	Add 3 to 8 and subtract 1 from 24.

Exercises Name the sets of numbers to which each value belongs.
See Example 1 on page 12.

18. $-\sqrt{9}$ **Z, Q, R** **19.** $1.\overline{6}$ **Q, R** **20.** $\frac{35}{7}$ **N, W, Z, Q, R** **21.** $\sqrt{18}$ **I, R**

Simplify each expression. *See Example 5 on page 14.*

22. $2m + 7n - 6m - 5n$ **$-4m + 2n$** **23.** $-5(a - 4b) + 4b$ **$-5a + 24b$** **24.** $2(5x + 4y) - 3(x + 8y)$ **$7x - 16y$**

1-3 Solving Equations

See pages 20–27.

Concept Summary

- Verbal expressions can be translated into algebraic expressions using the language of algebra, using variables to represent the unknown quantities.
- Use the properties of equality to solve equations.

Example **Solve $4(a + 5) - 2(a + 6) = 3$.**

$4(a + 5) - 2(a + 6) = 3$	Original equation
$4a + 20 - 2a - 12 = 3$	Distributive Property
$2a + 8 = 3$	Commutative, Distributive, and Substitution Properties
$2a = -5$	Subtraction Property (=)
$a = -2.5$	Division Property (=)

48 Chapter 1 Solving Equations and Inequalities

Exercises **Solve each equation. Check your solution.**
See Examples 3 and 4 on pages 21 and 22.

25. $x - 6 = -20$ **−14** **26.** $-\frac{2}{3}a = 14$ **−21** **27.** $7 + 5n = -58$ **−13**

28. $3w + 14 = 7w + 2$ **3** **29.** $5y + 4 = 2(y - 4)$ **−4** **30.** $\frac{n}{4} + \frac{n}{3} = \frac{1}{2}$ $\frac{6}{7}$

Solve each equation or formula for the specified variable. *See Example 5 on page 22.*

31. $Ax + By = C$ for x $x = \frac{C - By}{A}$ **32.** $\frac{a - 4b^2}{2c} = d$ for a $a = 2cd + 4b^2$ **33.** $A = p + prt$ for p $p = \frac{A}{1 + rt}$

1-4 Solving Absolute Value Equations

See pages 28–32.

Concept Summary

- For any real numbers a and b, where $b \geq 0$, if $|a| = b$, then $a = b$ or $a = -b$.

Example **Solve $|2x + 9| = 11$.**

Case 1 $a = b$ **or** **Case 2** $a = -b$

$2x + 9 = 11$ $2x + 9 = -11$

$2x = 2$ $2x = -20$

$x = 1$ $x = -10$

The solution set is $\{1, -10\}$. Check these solutions in the original equation.

Exercises **Solve each equation. Check your solutions.**
See Examples 1–4 on pages 28–30.

34. $|x + 11| = 42$ **{31, −53}** **35.** $3|x + 6| = 36$ **{6, −18}** **36.** $|4x - 5| = -25$ **∅**

37. $|x + 7| = 3x - 5$ **{6}** **38.** $|y - 5| - 2 = 10$ **{−7, 17}** **39.** $4|3x + 4| = 4x + 8$ $\left\{-\frac{3}{2}, -1\right\}$

1-5 Solving Inequalities

See pages 33–39.

Concept Summary

- Adding the same number to, or subtracting the same number from, each side of an inequality does not change the truth of the inequality.
- When you multiply or divide each side of an inequality by a negative number, the direction of the inequality symbol must be *reversed*.

Example **Solve $5 - 4a > 8$. Graph the solution set on a number line.**

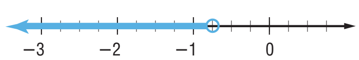
$5 - 4a > 8$ Original inequality

$-4a > 3$ Subtract 5 from each side.

$a < -\frac{3}{4}$ Divide each side by −4, reversing the inequality symbol.

The solution set is $\left\{a \middle| a < -\frac{3}{4}\right\}$.

The graph of the solution set is shown at the right.



Chapter 1 Study Guide and Review 49

Chapter 1 **For More ...**

- Extra Practice, see pages 828–829.
- Mixed Problem Solving, see page 862.

Exercises **Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line.**
See Examples 1–3 on pages 34–35. **40–45. See margin.**

40. $-7w > 28$ **41.** $3x + 4 \geq 19$ **42.** $\frac{n}{12} + 5 \leq 7$

43. $3(6 - 5a) < 12a - 36$ **44.** $2 - 3z \geq 7(8 - 2z) + 12$ **45.** $8(2x - 1) > 11x - 17$

1-6 Solving Compound and Absolute Value Inequalities

See pages 40–46.

Concept Summary

- The graph of an *and* compound inequality is the intersection of the solution sets of the two inequalities.
- The graph of an *or* compound inequality is the union of the solution sets of the two inequalities.
- For all real numbers a and b, $b > 0$, the following statements are true.
  1. If $|a| < b$ then $-b < a < b$.
  2. If $|a| > b$ then $a > b$ or $a < -b$.

Examples **Solve each inequality. Graph the solution set on a number line.**

1 $-19 < 4d - 7 \leq 13$

$-19 < 4d - 7 \leq 13$ Original inequality

$-12 < 4d \leq 20$ Add 7 to each part.

$-3 < d \leq 5$ Divide each part by 4.

The solution set is $\{x \mid -3 < d \leq 5\}$.

2 $|2x + 4| \geq 12$

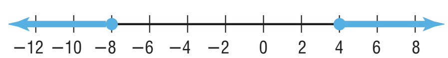
$|2x + 4| \geq 12$ is equivalent to $2x + 4 \geq 12$ or $2x + 4 \leq -12$.

$2x + 4 \geq 12$ or $2x + 4 \leq -12$ Original inequality

$2x \geq 8$ $2x \leq -16$ Subtract 4 from each side.

$x \geq 4$ $x \leq -8$ Divide each side by 2.

The solution set is $\{x \mid x \geq 4$ or $x \leq -8\}$.



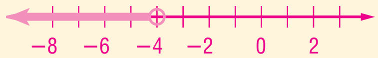
Exercises **Solve each inequality. Graph the solution set on a number line.**
See Examples 1–5 on pages 40–42. **46–51. See margin.**

46. $-1 < 3a + 2 < 14$ **47.** $-1 < 3(y - 2) \leq 9$ **48.** $|x| + 1 > 12$

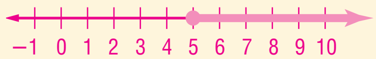
49. $|2y - 9| \leq 27$ **50.** $|5n - 8| > -4$ **51.** $|3b + 11| > 1$

Answers

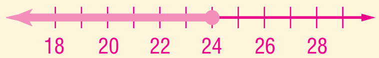
40. $\{w \mid w < -4\}$ or $(-\infty, -4)$



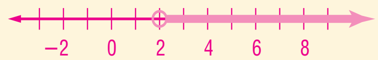
41. $\{x \mid x \geq 5\}$ or $[5, +\infty)$



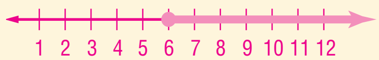
42. $\{n \mid n \leq 24\}$ or $(-\infty, 24]$



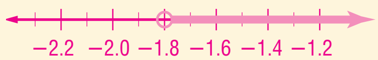
43. $\{a \mid a > 2\}$ or $(2, +\infty)$



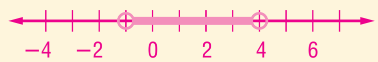
44. $\{z \mid z \geq 6\}$ or $[6, +\infty)$



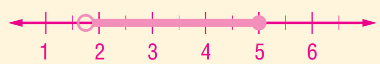
45. $\{x \mid x > -1.8\}$ or $(-1.8, +\infty)$



46. $\{a \mid -1 < a < 4\}$

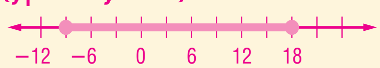


47. $\left\{y \middle| \frac{5}{3} < y \leq 5\right\}$



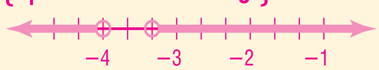
48. $\{x \mid x < -11$ or $x > 11\}$

49. $\{y \mid -9 \leq y \leq 18\}$



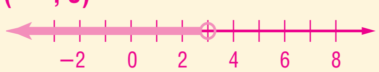
50. all real numbers

51. $\left\{b \middle| b < -4 \text{ or } b > -\frac{10}{3}\right\}$

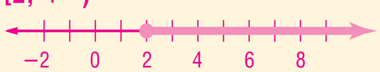


Answers (p. 51)

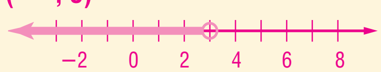
25. $(-\infty, 3)$



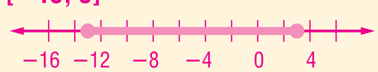
26. $[2, +\infty)$



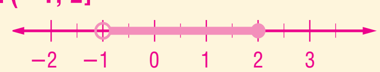
27. $(-\infty, 3)$



28. $[-13, 3]$



29. $(-1, 2]$



30. $\left\{y \middle| y < -\frac{4}{3} \text{ or } y > 2\right\}$

Chapter 1 Practice Test

Vocabulary and Concepts

Choose the term that best completes each sentence.

1. An algebraic (*equation*, *expression*) contains an equals sign.
2. (*Whole numbers*, *Rationals*) are a subset of the set of integers.
3. If $x + 3 = y$, then $y = x + 3$ is an example of the (*Transitive*, *Symmetric*) Property of Equality.

Skills and Applications

Find the value of each expression.

4. $[(3 + 6)^2 \div 3] \times 4$ **108**
5. $\frac{20 + 4 \times 3}{11 - 3}$ **4**
6. $0.5(2.3 + 25) \div 1.5$ **9.1**

Evaluate each expression if $a = -9$, $b = \frac{2}{3}$, $c = 8$, and $d = -6$.

7. $\frac{db + 4c}{a}$ $-\frac{28}{9}$
8. $\frac{a}{b^2} + c$ **−12.25**
9. $2b(4a + a^2)$ **60**

Name the sets of numbers to which each number belongs.

10. $\sqrt{17}$ **I, R**
11. 0.86 **Q, R**
12. $\sqrt{64}$ **N, W, Z, Q, R**

Name the property illustrated by each equation or statement. **14. Symm. (=)**

13. $(7 \cdot s) \cdot t = 7 \cdot (s \cdot t)$ **Assoc. (×)**
14. If $(r + s)t = rt + st$, then $rt + st = (r + s)t$.
15. $\left(3 \cdot \frac{1}{3}\right) \cdot 7 = \left(3 \cdot \frac{1}{3}\right) \cdot 7$ **Reflex. (=)**
16. $(6 - 2)a - 3b = 4a - 3b$ **Subst. (=)**
17. $(4 + x) + y = y + (4 + x)$ **Comm. (+)**
18. If $5(3) + 7 = 15 + 7$ and $15 + 7 = 22$, then $5(3) + 7 = 22$. **Trans. (=)**

Solve each equation. Check your solution(s). **21. all reals**

19. $5t - 3 = -2t + 10$ $\frac{13}{7}$
20. $2x - 7 - (x - 5) = 0$ **2**
21. $5m - (5 + 4m) = (3 + m) - 8$
22. $|8w + 2| + 2 = 0$ **∅**
23. $12\left|\frac{1}{2}y + 3\right| = 6$ **−7, −5**
24. $2|2y - 6| + 4 = 8$ **2, 4**

Solve each inequality. Describe the solution set using set builder or interval notation. Then graph the solution set on a number line. **27. $\{x \mid x < 3\}$** **25–30. See margin for interval notation and graphs.**

25. $4 > b + 1$ $\{b \mid b < 3\}$
26. $3q + 7 \geq 13$ $\{q \mid q \geq 2\}$
27. $5(3x - 5) + x < 2(4x - 1) + 1$
28. $|5 + k| \leq 8$ $\{k \mid -13 \leq k \leq 3\}$
29. $-12 < 7d - 5 \leq 9$ $\{d \mid -1 < d \leq 2\}$
30. $|3y - 1| > 5$ **See margin.**

For Exercises 31 and 32, define a variable, write an equation or inequality, and solve the problem. **31. m = miles traveled; $19.50 + 0.18m = 33$; 75 mi**

31. **CAR RENTAL** Mrs. Denney is renting a car that gets 35 miles per gallon. The rental charge is $19.50 a day plus 18¢ per mile. Her company will reimburse her for $33 of this portion of her travel expenses. If Mrs. Denney rents the car for 1 day, find the maximum number of miles that will be paid for by her company.
32. **SCHOOL** To receive a B in his English class, Nick must have an average score of at least 80 on five tests. He scored 87, 89, 76, and 77 on his first four tests. What must he score on the last test to receive a B in the class?

32. s = score on last test; $\frac{s + 87 + 89 + 76 + 77}{5} \geq 80$; at least 71

33. **STANDARDIZED TEST PRACTICE** If $\frac{a}{b} = 8$ and $ac - 5 = 11$, then $bc =$ **B**

(A) 93. (B) 2. (C) $\frac{5}{8}$. (D) cannot be determined

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Chapter 1 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 1 can be found on p. 50 of the *Chapter 1 Resource Masters*.

Chapter Tests There are six Chapter 1 Tests and an Open-Ended Assessment task available in the *Chapter 1 Resource Masters*.

Chapter 1 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	37–38
2A	MC	average	39–40
2B	MC	average	41–42
2C	FR	average	43–44
2D	FR	average	45–46
3	FR	advanced	47–48

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 1 can be found on p. 49 of the *Chapter 1 Resource Masters*. A sample scoring rubric for these tasks appears on p. A26.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction Translating words into algebraic expressions involves reading the words, deciding what they mean mathematically, and using the correct notation to write the translation. One way to build the skills involved is to go in the opposite direction, translating algebraic expressions into words.

Ask Students Write an expression or equation and create a word problem about it. Exchange your problem with a partner and translate what you receive into an expression or equation. Place your problem in your portfolio.

Chapter 1 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 1 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1

Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

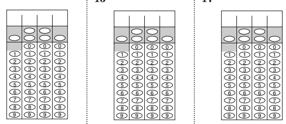
1 ○○○○ 4 ○○○○ 7 ○○○○ 9 ○○○○
2 ○○○○ 5 ○○○○ 8 ○○○○ 10 ○○○○
3 ○○○○ 6 ○○○○

Part 2 Short Response/Grid In

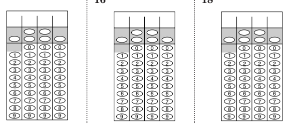
Solve the problem and write your answer in the blank.

For Questions 13–18, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11
12
13 15 17



14 16 18



Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

19 ○○○○ 21 ○○○○ 23 ○○○○
20 ○○○○ 22 ○○○○

Additional Practice

See pp. 55–56 in the *Chapter 1 Resource Masters* for additional standardized test practice.

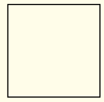
The items on the Standardized Test Practice pages were created to closely parallel those on actual state proficiency tests and college entrance exams, like PSAT, ACT and SAT.

Chapter 1 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In the square at the right, what is the value of x? **B**



Ⓐ 1 Ⓑ 2
Ⓒ 3 Ⓓ 4

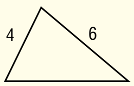
2. On a college math test, 18 students earned an A. This number is exactly 30% of the total number of students in the class. How many students are in the class? **D**

Ⓐ 5 Ⓑ 23
Ⓒ 48 Ⓓ 60

3. A student computed the average of her 7 test scores by adding the scores together and dividing this total by the number of tests. The average was 87. On her next test, she scored a 79. What is her new test average? **D**

Ⓐ 83 Ⓑ 84
Ⓒ 85 Ⓓ 86

4. If the perimeter of $\triangle PQR$ is 3 times the length of PQ, then PR = ____. **D**



Note: Figure not drawn to scale.

Ⓐ 4 Ⓑ 6
Ⓒ 7 Ⓓ 8

5. If a different number is selected from each of the three sets shown below, what is the greatest sum these 3 numbers could have? **C**

R = {3, 6, 7}; S = {2, 4, 7}; T = {1, 3, 7}

Ⓐ 13 Ⓑ 14
Ⓒ 17 Ⓓ 21

6. A pitcher contains a ounces of orange juice. If b ounces of juice are poured from the pitcher into each of c glasses, which expression represents the amount of juice remaining in the pitcher? **C**

Ⓐ $\frac{a}{b} + c$ Ⓑ $ab - c$
Ⓒ $a - bc$ Ⓓ $\frac{a}{bc}$

7. The sum of three consecutive integers is 135. What is the greatest of the three integers? **D**

Ⓐ 43 Ⓑ 44
Ⓒ 45 Ⓓ 46

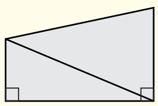
8. The ratio of girls to boys in a class is 5 to 4. If there are a total of 27 students in the class, how many are girls? **A**

Ⓐ 15 Ⓑ 12
Ⓒ 9 Ⓓ 5

9. For which of the following ordered pairs (x, y) is $x + y > 3$ and $x - y < -2$? **D**

Ⓐ (0, 3) Ⓑ (3, 4)
Ⓒ (5, 3) Ⓓ (2, 5)

10. If the area of $\triangle ABD$ is 280, what is the area of the polygon $ABCD$? **B**



Note: Figure not drawn to scale.

Ⓐ 560 Ⓑ 630
Ⓒ 700 Ⓓ 840

The Princeton Review Test-Taking Tip

Question 9 To solve equations or inequalities, you can replace the variables in the question with the values given in each answer choice. The answer chioce that results in true statements is the correct answer choice.

The Princeton Review *Log On* for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

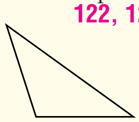
TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Part 2 Short Response/Grid In

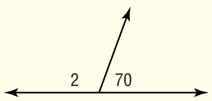
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. In the triangle below, x and y are integers. If $25 < y < 30$, what is one possible value of x? **122, 124, 126, or 128**



12. If n and p are each different positive integers and $n + p = 4$, what is one possible value of $3n + 4p$? **13 or 15**

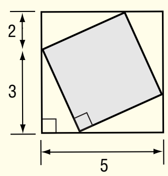
13. In the figure at the right, what is the value of x? **55**



14. One half quart of lemonade concentrate is mixed with $1\frac{1}{2}$ quarts of water to make lemonade for 6 people. If you use the same proportions of concentrate and water, how many quarts of lemonade concentrate are needed to make lemonade for 21 people? **1.75 or 7/4**

15. If 25 percent of 300 is equal to 500 percent of t, then t is equal to what number? **15**

16. In the figure below, what is the area of the shaded square in square units? **13**



17. There are 140 students in the school band. One of these students will be selected at random to be the student representative. If the probability that a brass player is selected is $\frac{2}{5}$, how many brass players are in the band? **56**

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18. A shelf holds fewer than 50 cans. If all of the cans on this shelf were put into stacks of five cans each, no cans would remain. If the same cans were put into stacks of three cans each, one can would remain. What is the greatest number of cans that could be on the shelf? **40**

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- (A) **the quantity in Column A is greater;**
- (B) **the quantity in Column B is greater;**
- (C) **the two quantities are equal;**
- (D) **the relationship cannot be determined from the information given.**

	Column A	Column B
19.	$\frac{\frac{3}{4}}{\left(\frac{3}{4}\right)^2}$	$\frac{4}{3}$

C

20.

+ 13	+ 14

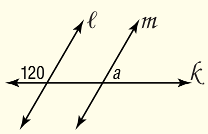
B

21. $0 < s < \frac{3}{4}$

1	3

D

22.



$\ell \parallel m$

120	2

C

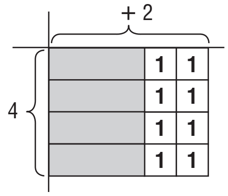
23. The average (arithmetic mean) of s and t is greater than the average of s and w.

B

Chapter 1 Standardized Test Practice 53

Page 13, Algebra Activity

1.



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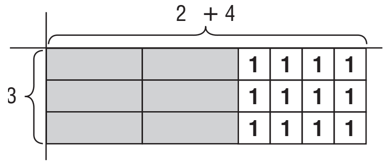

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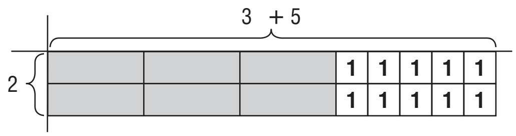

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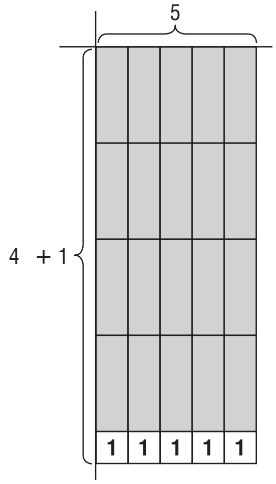


Page 19, Follow-Up of Lesson 1-2 Algebra Activity

2. 2

$0 + 2 = 2$ $2 + 3 = 5$ $5 + 4 = 9$

8. 35

10 $10 - 3$ $\div 2 = 35$

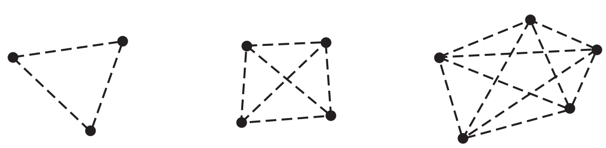
9. n n

$n - 3$ n $n - 3$

2

2 $y = n$ $n - 3$ $\div 2$

10.



13.

x $x - 1$

$y = x$ $x - 1$ $\div 2$

$y =$ x $x - 3$ $\div 2$ $+ x =$

0 5x^2 − 1 5x $+ x =$ 0 5x^2 − 0 5x $y =$ 0 5x^2 − 0 5x

Page 27, Lesson 1-3

76.

10-

A

0 80 I 27 P

$I = 6 \times P \div$ $220 - A$ A

6 28 0 8

220 −1 17 5

$17\frac{1}{2}$

A

P

A

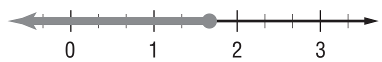
A

$220 - \frac{6P}{I}$

Pages 37–38, Lesson 1-5

4.

5.



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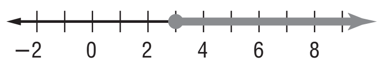

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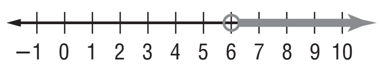

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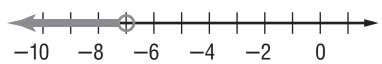

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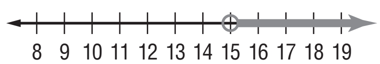

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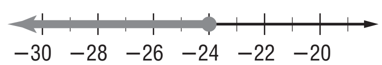

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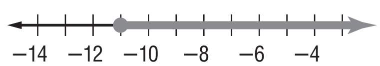

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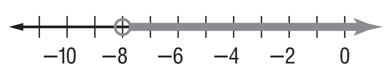

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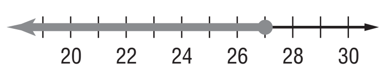
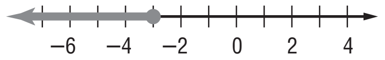
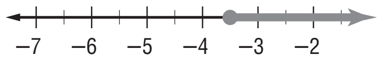


20. 

21. 

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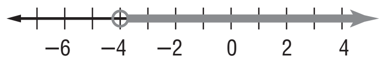
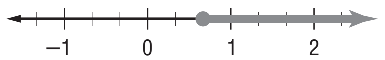
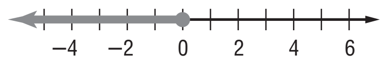
23. 

24. 

25. 

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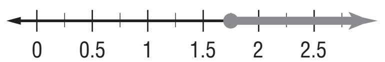
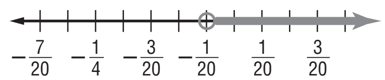
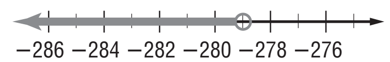
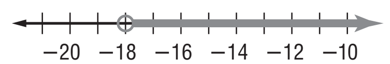
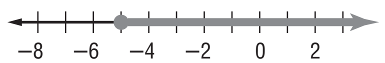
27. 

28. 

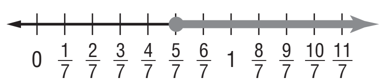
29. 

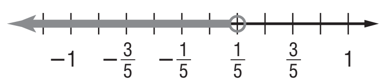
30. 

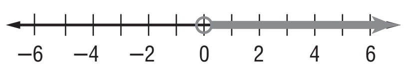
31. 

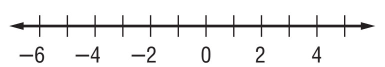
32. 

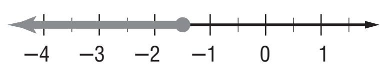
33. 

34. 

35. 

36. 

37. 

38. 

53.

150 < 400

n

1

2 *n* 1

35 40

150 35 + 0 4 *n* − 150

2 400

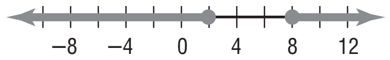
55 2

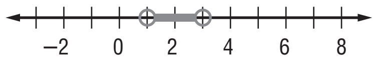
1 55 < 35 + 0 4 *n* − 150 *n*

n|*n* > 200

200 2

Pages 44–45, Lesson 1-6

27. 

28. 

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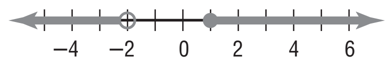
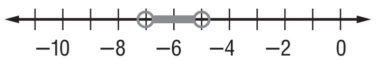
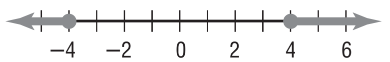
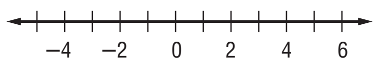
30. 

31. 

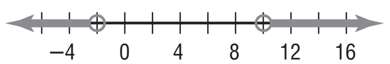
32. 

33. 

34. 

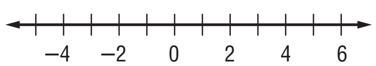
35. 

36. 

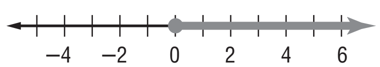
37. 

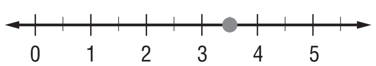
38. 

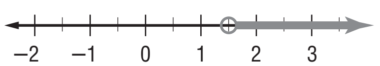
39. 

40. 

41. 

42. 

43. 

44. 

54.

and

or

10 ≤ *h* ≤ 16

12

10 ≤ *h* ≤ 16