Chapter

10

Exponential and Logarithmic Relations

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
10-1 **Exponential Functions** *(pp. 522–530)* ***Preview:*** Investigating Exponential Functions • Graph exponential functions. • Solve exponential equations and inequalities.	1	1	0.5	0.5
10-2 **Logarithms and Logarithmic Functions** *(pp. 531–540)* • Evaluate logarithmic expressions. • Solve logarithmic equations and inequalities. ***Follow-Up:*** Modeling Real-World Data: Curve Fitting	2	2 (with 10-2 Follow-Up)	1	1
10-3 **Properties of Logarithms** *(pp. 541–546)* • Simplify and evaluate expressions using the properties of logarithms. • Solve logarithmic equations using the properties of logarithms.	1	1	0.5	0.5
10-4 **Common Logarithms** *(pp. 547–553)* • Solve exponential equations and inequalities using common logarithms. • Evaluate logarithmic expressions using the Change of Base Formula. ***Follow-Up:*** Solving Exponential and Logarithmic Equations and Inequalities	1	1	0.5	0.5
10-5 **Base *e* and Natural Logarithms** *(pp. 554–559)* • Evaluate expressions involving the natural base and natural logarithms. • Solve exponential equations and inequalities using natural logarithms.	2 (with 10-4 Follow-Up)	2 (with 10-4 Follow-Up)	1 (with 10-4 Follow-Up)	1 (with 10-4 Follow-Up)
10-6 **Exponential Growth and Decay** *(pp. 560–565)* • Use logarithms to solve problems involving exponential decay. • Use logarithms to solve problems involving exponential growth.	1	1	0.5	0.5
Study Guide and **Practice Test** *(pp. 566–571)* **Standardized Test Practice** *(pp. 572–573)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**10**	**10**	**5**	**5**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter Resource Manager

CHAPTER 10 RESOURCE MASTERS									
Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
573–574	575–576	577	578		GCS 45	10-1	10-1		(*Preview:* paper, scissors, grid paper, calculator) graphing calculator, grid paper, string
579–580	581–582	583	584	623	SC 19	10-2	10-2		posterboard (*Follow-Up:* graphing calculator, grid paper)
585–586	587–588	589	590	623, 625		10-3	10-3		
591–592	593–594	595	596			10-4	10-4		(*Follow-Up:* graphing calculator)
597–598	599–600	601	602	624	SM 127–132	10-5	10-5	19	plastic coins, paper currency
603–604	605–606	607	608	624	GCS 46, SC 20	10-6	10-6		
				609–622, 626–628					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 10

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students have worked with exponents in many situations, including performing calculations, manipulating expressions, and applying properties. They have explored properties of inverses for operations and for functions, and they have solved many kinds of equations and inequalities.

This Chapter

Students are introduced to the term logarithm to solve for a variable that appears as an exponent. They explore the relationship between exponents and logarithms, and they use logarithms with two special bases, base 10 or common logarithms, and base *e* or natural logarithms. They apply the Change of Base Formula to rewrite a logarithm using a different base, and they apply appropriate formulas to solve problems involving exponential growth and exponential decay.

Future Connections

Students will continue to look at properties of and relationships between exponents and logarithms. They will apply formulas for exponential growth and exponential decay in science courses and in consumer situations. The natural-base exponential function will have an important role in precalculus and calculus topics.

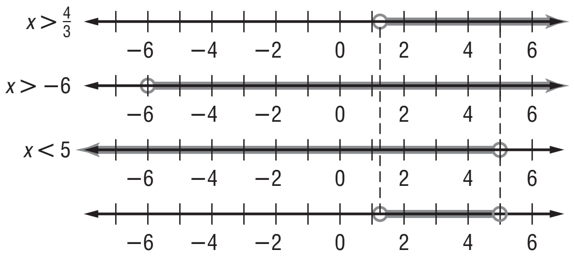
10-1 Exponential Functions

Examine the list of characteristics for an exponential function on page 524. The first characteristic states that an exponential function is continuous and one-to-one. The term *continuous* means that the function can be traced without lifting your pencil. The term *one-to-one* means that a horizontal line passing through the graph will intersect no more than one point on the graph. This characteristic is important for the development of the logarithmic function in Lesson 10-2, since only one-to-one functions can have inverses. The second characteristic listed is that the domain of the function is the set of all real numbers. This property is important because it means that $3^{\sqrt{5}}$ has meaning, since $\sqrt{5}$ is a real number and part of the domain of $y = 3^x$. The third and fourth characteristics of an exponential function are related. The x-axis is a *horizontal asymptote* of the graph of an exponential function. This means that the graph of this function approaches the horizontal line $x = 0$, getting closer and closer to this line but never crossing it. This restricts the graph of an exponential function to either Quadrants I and II, when a is positive, or to Quadrants III and IV, when a is negative. In terms of the range of the function, this means that when a is positive, all y values of the function will be positive, and when a is negative, all y values of the function will be negative. These two properties will also be important when considering the inverse of the exponential function. The last two properties are useful for graphing and writing exponential functions.

10-2 Logarithms and Logarithmic Functions

In the equation $y = \log_b x$, y is referred to as the *logarithm*, b is the *base*, and x is sometimes referred to as the *argument*. The definition of a logarithm given on page 532 indicates that a logarithm is an exponent.

When solving logarithmic equations and inequalities, it is important to remember that a defining characteristic of a logarithmic function is that its domain is the set of all *positive* numbers. This means that the logarithm of 0 or of a negative number for any base is undefined. It is very important to check possible solutions to logarithmic equations in the original equation, to be sure that they would not result in taking the logarithm of 0 or a negative number. For logarithmic inequalities, this fact will exclude not just one value from the solution set, but a range of values. In Example 8 on page 534, since the original inequality asks for the values $\log_{10}(3x - 4)$ and $\log_{10}(x + 6)$, we must solve two inequalities, $3x - 4 \le 0$ and $x + 6 \le 0$, to find what values must be excluded from the solution set we found using the Property of Inequality for Logarithmic Functions. Excluding the values such that $x \le \frac{4}{3}$ and $x \le -6$, the solution set is all x such that the following three inequalities are all satisfied: $x > \frac{4}{3}$, $x > -6$, and $x < 5$. To simplify this compound inequality, sketch all three inequalities, as shown below, and find where all three intersect.



The final number line shows that the solution set is the compound inequality $\frac{4}{3} < x < 5$.

10-3 Properties of Logarithms

The word logarithm is actually a contraction of "logical arithmetic." Logarithms were invented to make computation easier. Using logarithms, multiplication changes to addition, according to the Product Property of Logarithms, and division changes to subtraction, according to the Quotient Property of Logarithms. This is illustrated in Examples 1, 2, and 4 of Lesson 10-3. In these examples, students are given the approximate value of specific logarithms. Before the invention of the scientific calculator, these values took a good deal of time to compute. Rather than use the same arduous process to compute each and every logarithm one encountered, the properties of logarithms allowed the use of a relative few logarithmic values to compute others.

10-4 Common Logarithms

Before the invention of the scientific calculator, the appendices of algebra texts contained extensive tables of common logarithms of numbers. In order to read these tables, you had to understand the parts of a logarithm. Every logarithm has two parts, the *characteristic* and the *mantissa*. A mantissa is the logarithm of a number between 1 and 10. When the original number is expressed in scientific notation, the characteristic is the power of 10.

$$\begin{aligned}
645{,}000 &= 6.45 \cdot 10^5 && \text{Scientific notation}\\
\log 645{,}000 &= \log(6.45 \cdot 10^5) && \text{Property of Equality for Log Functions}\\
&= \log 6.45 + \log 10^5 && \text{Product Property}\\
&= \log 6.45 + 5 && \text{Inverse Property of Exponents and Logs}\\
&\approx 0.8096 + 5 && \log 6.45 \approx 0.8096\\
&\approx 5.8096 && \text{Simplify.}\\
\log 645{,}000 &\approx \underset{\substack{\uparrow\\ \text{characteristic}}}{5}.\underset{\substack{\uparrow\\ \text{mantissa}}}{8096}
\end{aligned}$$

10-5 Base *e* and Natural Logarithms

Exponentiation, which is the inverse operation of taking a logarithm, is sometimes referred to as finding the *antilogarithm*. That is, if $\log x = a$ then $x = \text{antilog } a$. Since antilogarithms mean the same operation as exponentiation, it follows that to find the antilogarithm of a common logarithm, you would use 2nd [10^x] on a graphing calculator. To find the antilogarithm of a natural logarithm, antiln a, you would use 2nd [e^x].

10-6 Exponential Growth and Decay

It is important to note that the variable r in the exponential decay formula $y = a(1 - r)^t$ and the variable k in the alternate exponential decay formula $y = ae^{-kt}$ are not equivalent. In a problem where a decay factor is given or asked for, the formula $y = a(1 - r)^t$ should be used and not the formula $y = ae^{-kt}$. The same is true of the exponential growth formulas $y = a(1 + r)^t$ and $y = ae^{kt}$.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Exponential Functions (Lesson 33)
- Growth and Decay (Lesson 34)

Chapter 10

DAILY INTERVENTION and Assessment

	Type	Student Edition	Teacher Resources	Technology/Internet
INTERVENTION	Ongoing	Prerequisite Skills, pp. 521, 530, 538, 546, 551, 559 Practice Quiz 1, p. 538 Practice Quiz 2, p. 559	5-Minute Check Transparencies Quizzes, *CRM* pp. 623–624 Mid-Chapter Test, *CRM* p. 625 Study Guide and Intervention, *CRM* pp. 573–574, 579–580, 585–586, 591–592, 597–598, 603–604	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
	Mixed Review	pp. 531, 538, 546, 551, 559, 565	Cumulative Review, *CRM* p. 626	
	Error Analysis	Find the Error, pp. 535, 544, 557 Common Misconceptions, p. 523	Find the Error, *TWE* pp. 535, 544, 557 Unlocking Misconceptions, *TWE* pp. 542, 548 Tips for New Teachers, *TWE* p. 534	
ASSESSMENT	Standardized Test Practice	pp. 530, 537, 538, 546, 551, 559, 562, 563, 564, 572–573	*TWE* p. 562 Standardized Test Practice, *CRM* pp. 627–628	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
	Open-Ended Assessment	Writing in Math, pp. 530, 537, 546, 551, 559, 564 Open Ended, pp. 527, 535, 544, 549, 557, 563	Modeling: *TWE* pp. 530, 565 Speaking: *TWE* pp. 546, 559 Writing: *TWE* pp. 538, 551 Open-Ended Assessment, *CRM* p. 621	
	Chapter Assessment	Study Guide, pp. 566–570 Practice Test, p. 571	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 609–614 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 615–620 Vocabulary Test/Review, *CRM* p. 622	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
10-5	**19** *Exponential and Logarithmic Functions*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 521
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 527, 535, 544, 549, 557, 563, 566)
- Writing in Math questions in every lesson, pp. 530, 537, 546, 551, 559, 564
- WebQuest, pp. 529, 565

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 521, 566
- Study Notebook suggestions, pp. 522, 527, 535, 544, 549, 557, 563
- Modeling activities, pp. 530, 565
- Speaking activities, pp. 546, 559
- Writing activities, pp. 538, 551
- ELL Resources, pp. 520, 529, 537, 545, 550, 558, 564, 566

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 10 Resource Masters,* pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 10 Resource Masters*, pp. 577, 583, 589, 595, 601, 607)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 10 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
10-1 Preview	1, 2, 3, 6, 7, 8, 10	
10-1	1, 2, 3, 4, 6, 8, 9, 10	
10-2	1, 2, 3, 4, 6, 7, 8, 9	
10-2 Follow-Up	1, 2, 3, 5, 6, 8, 10	
10-3	1, 2, 4, 6, 7, 8, 9	
10-4	1, 2, 4, 6, 8, 9	
10-4 Follow-Up	1, 2, 3	
10-5	1, 2, 3, 4, 6, 7, 8, 9	
10-6	1, 2, 4, 6, 8, 9	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 10 Exponential and Logarithmic Relations

What You'll Learn

- **Lessons 10-1 through 10-3** Simplify exponential and logarithmic expressions.
- **Lessons 10-1, 10-4, and 10-5** Solve exponential equations and inequalities.
- **Lessons 10-2 and 10-3** Solve logarithmic equations and inequalities.
- **Lesson 10-6** Solve problems involving exponential growth and decay.

Key Vocabulary

- exponential growth (p. 524)
- exponential decay (p. 524)
- logarithm (p. 531)
- common logarithm (p. 547)
- natural logarithm (p. 554)

Why It's Important

Exponential functions are often used to model problems involving growth and decay. Logarithms can also be used to solve such problems. *You will learn how a declining farm population can be modeled by an exponential function in Lesson 10-1.*

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 10 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 10 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 10.

Lessons 10-1 through 10-3 Multiply and Divide Monomials

Simplify. Assume that no variable equals 0. *(For review, see Lesson 5-1.)*

1. $x^5 \cdot x \cdot x^6$ **x^{12}**

2. $(3ab^4c^2)^3$ **$27a^3b^{12}c^6$**

3. $\frac{-36x^7y^4z^3}{21x^4y^9z^4}$ **$-\frac{12x^3}{7y^5z}$**

4. $\left(\frac{4ab^2}{64b^3c}\right)^2$ **$\frac{a^2}{256b^2c^2}$**

Lessons 10-2 and 10-3 Solve Inequalities

Solve each inequality. *(For review, see Lesson 1-5)*

5. $a + 4 < -10$ **$a < -14$**

6. $-5n \leq 15$ **$n \geq -3$**

7. $3y + 2 \geq -4$ **$y \geq -2$**

8. $15 - x > 9$ **$x < 6$**

Lessons 10-2 and 10-3 Inverse Functions

Find the inverse of each function. Then graph the function and its inverse. *(For review, see Lesson 7-8.)* **9. $f^{-1}(x) = -\frac{1}{2}x$** **9–12. See pp. 573A–573D for graphs.**

9. $f(x) = -2x$

10. $f(x) = 3x - 2$ **$f^{-1}(x) = \frac{x+2}{3}$**

11. $f(x) = -x + 1$ **$f^{-1}(x) = -x + 1$**

12. $f(x) = \frac{x-4}{3}$ **$f^{-1}(x) = 3x + 4$**

Lessons 10-2 and 10-3 Composition of Functions

Find $g[h(x)]$ and $h[g(x)]$. *(For review, see Lesson 7-7.)*

13. $h(x) = 3x + 4$ **$g[h(x)] = 3x + 2$**
$g(x) = x - 2$ **$h[g(x)] = 3x - 2$**

14. $h(x) = 2x - 7$ **$g[h(x)] = 10x - 35$**
$g(x) = 5x$ **$h[g(x)] = 10x - 7$**

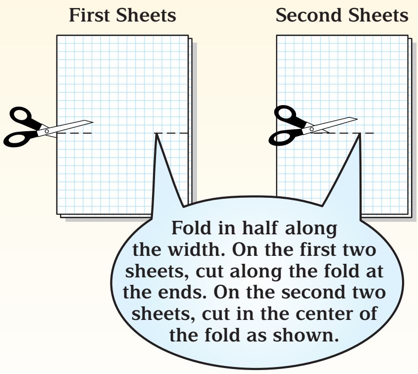
15. $h(x) = x - 4$ **$g[h(x)] = x^2 - 8x + 16$**
$g(x) = x^2$ **$h[g(x)] = x^2 - 4$**

16. $h(x) = 4x + 1$ **$g[h(x)] = -8x - 5$**
$g(x) = -2x - 3$ **$h[g(x)] = -8x - 11$**

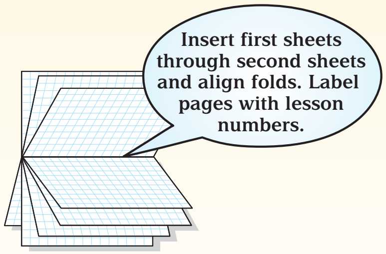
FOLDABLES™ Study Organizer

Make this Foldable to record information about exponential and logarithmic relations. Begin with four sheets of grid paper.

Step 1 Fold and Cut



Step 2 Fold and Label



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for each lesson.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 10. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
10-2	Composition of Functions (p. 530)
10-3	Multiplying and Dividing Monomials (p. 538)
10-4	Solving Logarithmic Equations and Inequalities (p. 546)
10-5	Logarithmic Equations (p. 551)
10-6	Exponential Equations and Inequalities (p. 559)

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Organization of Data and Journal Writing After students make their Foldable journals, have them label two pages for each lesson in Chapter 10. Writers' journals can be used by students to record the direction and progress of learning, to describe positive and negative experiences during learning, to write about personal associations and experiences called to mind during learning, and to list examples of ways in which new knowledge has or will be used in their daily life, as well as take notes, record key concepts, and write examples.

Algebra Activity

A Preview of Lesson 10-1

Getting Started

Objective Use paper stacking to explore an exponential function.

Materials
notebook paper
scissors
grid paper

Teach

- You may wish to do the example as a demonstration while students complete the table on the chalkboard.
- Students may recognize that the y value is doubled for each successive cut, but they may have to be led to realizing that this can be written in the form 2^x.
- Show students how to connect the points with a smooth curve, rather than connecting each pair of points with a straight line.

Assess

Have students work in small groups for **Exercises 1–9**. Observe students' work to determine if they are able to write the function in **Exercise 5**. Students should conclude after **Exercise 9** that exponential functions can increase faster than seems reasonable.

Algebra Activity

A Preview of Lesson 10-1

Investigating Exponential Functions

Collect the Data

Step 1 Cut a sheet of notebook paper in half.

Step 2 Stack the two halves, one on top of the other.

Step 3 Make a table like the one below and record the number of sheets of paper you have in the stack after one cut.

Number of Cuts	Number of Sheets
0	1
1	2
2	4

Step 4 Cut the two stacked sheets in half, placing the resulting pieces in a single stack. Record the number of sheets of paper in the new stack after 2 cuts.

Step 5 Continue cutting the stack in half, each time putting the resulting piles in a single stack and recording the number of sheets in the stack. Stop when the resulting stack is too thick to cut.

Analyze the Data

1. Write a list of ordered pairs (x, y), where x is the number of cuts and y is the number of sheets in the stack. Notice that the list starts with the ordered pair (0, 1), which represents the single sheet of paper before any cuts were made.

1. (0, 1), (1, 2), (2, 4), (3, 8), (4, 16), …

2. Continue the list, beyond the point where you stopped cutting, until you reach the ordered pair for 7 cuts. Explain how you calculated the last y values for your list, after you had stopped cutting.

2. (5, 32), (6, 64), (7, 128); The y value is found by raising 2 to the number of cuts.

3. Plot the ordered pairs in your list on a coordinate grid. Be sure to choose a scale for the y-axis so that you can plot all of the points. **See pp. 573A–573D.**
4. Describe the pattern of the points you have plotted. Do they lie on a straight line? **The points do not lie in a straight line. The slope increases as the x values increase.**

Make a Conjecture

5. Write a function that expresses y as a function of x. $y = 2^x$
6. Use a calculator to evaluate the function you wrote in Exercise 5 for $x = 8$ and $x = 9$. Does it give the correct number of sheets in the stack after 8 and 9 cuts? **256, 512; yes**
7. Notebook paper usually stacks about 500 sheets to the inch. How thick would your stack of paper be if you had been able to make 9 cuts? **about 1 in.**
8. Suppose each cut takes about 5 seconds. If you had been able to keep cutting, you would have made 36 cuts in three minutes. At 500 sheets to the inch, make a conjecture as to how thick you think the stack would be after 36 cuts. **Sample answer: 1 million ft**
9. Use your function from Exercise 5 to calculate the thickness of your stack after 36 cuts. Write your answer in miles. **2169 mi**

Resource Manager

Teaching Algebra with Manipulatives
- p. 1 (grid paper)
- p. 275 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit
- scissors
- coordinate grid stamp

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

10-1 Exponential Functions

What You'll Learn

- Graph exponential functions.
- Solve exponential equations and inequalities.

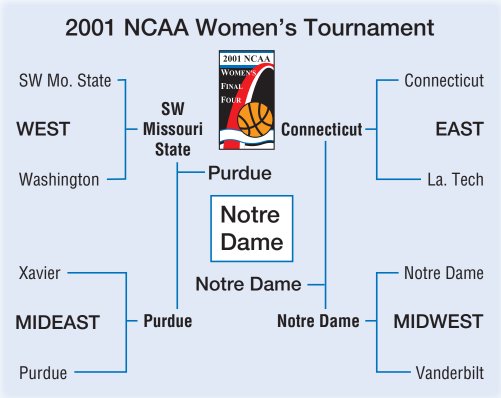
Vocabulary

- exponential function
- exponential growth
- exponential decay
- exponential equation
- exponential inequality

How does an exponential function describe tournament play?

The NCAA women's basketball tournament begins with 64 teams and consists of 6 rounds of play. The winners of the first round play against each other in the second round. The winners then move from the Sweet Sixteen to the Elite Eight to the Final Four and finally to the Championship Game.

The number of teams y that compete in a tournament of x rounds is $y = 2^x$.



Study Tip

Common Misconception

Be sure not to confuse polynomial functions and exponential functions. While $y = x^2$ and $y = 2^x$ each have an exponent, $y = x^2$ is a polynomial function and $y = 2^x$ is an exponential function.

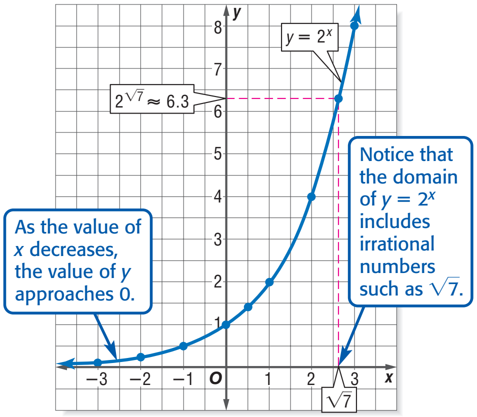
EXPONENTIAL FUNCTIONS In an exponential function like $y = 2^x$, the base is a constant, and the exponent is a variable. Let's examine the graph of $y = 2^x$.

Example 1 Graph an Exponential Function

Sketch the graph of $y = 2^x$. Then state the function's domain and range.

Make a table of values. Connect the points to sketch a smooth curve.

x	$y = 2^x$
-3	$2^{-3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
$\frac{1}{2}$	$2^{\frac{1}{2}} = \sqrt{2}$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$



As the value of x decreases, the value of y approaches 0.

Notice that the domain of $y = 2^x$ includes irrational numbers such as $\sqrt{7}$.

The domain is all real numbers, while the range is all positive numbers.

10-1 Lesson Notes

1 Focus

5-Minute Check Transparency 10-1 Use as a quiz or review of Chapter 9.

Mathematical Background notes are available for this lesson on p. 520C.

Building on Prior Knowledge

Ask students where they have heard the term *exponential* before and what they think it might mean. Students may have heard terms like *exponential growth* on a television news program and they might think that exponential means "enormous." Use students' answers to introduce the concept of exponential functions.

How does an exponential function describe tournament play?

Ask students:

- How many winners are there in the first round of the tournament? **32**
- After each round, how has the number of teams changed? **The number of teams remaining after each round is half the number of teams that played in that round.**
- If the tournament field was reduced to 32 teams, how many basketball games would have to be played by the tournament's winning team? **5 games**

Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters

- Study Guide and Intervention, pp. 573–574
- Skills Practice, p. 575
- Practice, p. 576
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Graphing Calculator and Spreadsheet Masters, p. 45

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Transparencies

5-Minute Check Transparency 10-1
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

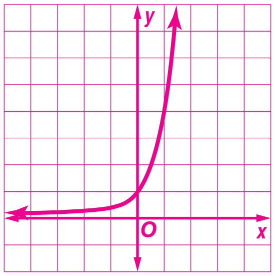
EXPONENTIAL FUNCTIONS

In-Class Example

Power Point®

Teaching Tip Watch for students who do not understand why the graph in Example 1 cannot simply be modeled by a quadratic, cubic, or quartic function. Point out that the graphs of $y = x^2$, $y = x^3$, and $y = x^4$ all pass through the point (0, 0) and not through the point (0, 1).

1 Sketch the graph of $y = 4^x$. Then state the function's domain and range.



The domain is all real numbers, while the range is all positive numbers.

Answers

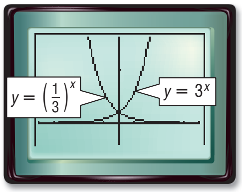
1. The shapes of the graphs are the same.
2. The asymptote for each graph is the x-axis and the y-intercept for each graph is 1.
3. The graphs are reflections of each other over the y-axis.
5. The graphs are reflections of each other over the x-axis.

You can use a TI-83 Plus graphing calculator to look at the graph of two other exponential functions, $y = 3x$ and $y = \left(\frac{1}{3}\right)^x$.

Graphing Calculator Investigation

Families of Exponential Functions

The calculator screen shows the graphs of $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$.



[−5, 5] scl: 1 by [−2, 8] scl: 1

Think and Discuss 1–3,5. See margin.

1. How do the shapes of the graphs compare?
2. How do the asymptotes and y-intercepts of the graphs compare?
3. Describe the relationship between the graphs.
4. Graph each group of functions on the same screen. Then compare the graphs, listing both similarities and differences in shape, asymptotes, domain, range, and y-intercepts.
   - **a.** $y = 2^x$, $y = 3^x$, and $y = 4^x$
   - **b.** $y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{3}\right)^x$, and $y = \left(\frac{1}{4}\right)^x$
   - **c.** $y = -3(2)^x$ and $y = 3(2)^x$; $y = -1(2)^x$ and $y = 2^x$.
5. Describe the relationship between the graphs of $y = -1(2)^x$ and $y = 2^x$.

4. See pp. 573A–573D.

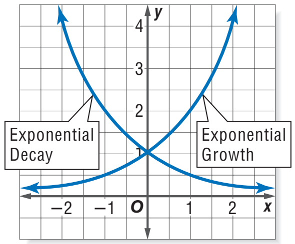
In general, an equation of the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$, is called an **exponential function** with base b. Exponential functions have the following characteristics.

1. The function is continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x-axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y-intercept is a.
6. The graphs of $y = ab^x$ and $y = a\left(\frac{1}{b}\right)^x$ are reflections across the y-axis.

Study Tip

Look Back
To review **continuous functions**, see page 63, Exercises 60 and 61. To review **one-to-one functions**, see Lesson 2-1.

There are two types of exponential functions: **exponential growth** and **exponential decay**. The base of an exponential growth function is a number greater than one. The base of an exponential decay function is a number between 0 and 1.



Study Tip

Exponential Growth and Decay
Notice that the graph of an exponential growth function *rises* from left to right. The graph of an exponential decay function *falls* from left to right.

Key Concept — Exponential Growth and Decay

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

Graphing Calculator Investigation

Families of Exponential Functions Have students begin by graphing the two functions separately, so they recognize that the two curves shown in the book are two distinct graphs. Students are used to seeing the U-shaped graphs of polynomial functions and might have difficulty separating the graphs visually. Also, a reminder about the meaning of the term *asymptotes* may be helpful for many students.

Example 2 Identify Exponential Growth and Decay

Determine whether each function represents exponential *growth* or *decay*.

Function	Exponential Growth or Decay?
a. $y = \left(\frac{1}{5}\right)^x$	The function represents exponential decay, since the base, $\frac{1}{5}$, is between 0 and 1.
b. $y = 3(4)^x$	The function represents exponential growth, since the base, 4, is greater than 1.
c. $y = 7(1.2)^x$	The function represents exponential growth, since the base, 1.2, is greater than 1.

Exponential functions are frequently used to model the growth or decay of a population. You can use the y-intercept and one other point on the graph to write the equation of an exponential function.

Example 3 Write an Exponential Function

FARMING **In 1983, there were 102,000 farms in Minnesota, but by 1998, this number had dropped to 80,000.**

a. Write an exponential function of the form $y = ab^x$ that could be used to model the farm population y of Minnesota. Write the function in terms of x, the number of years since 1983.

For 1983, the time x equals 0, and the initial population y is 102,000. Thus, the y-intercept, and value of a, is 102,000.

For 1998, the time x equals 1998 − 1983 or 15, and the population y is 80,000. Substitute these values and the value of a into an exponential function to approximate the value of b.

$y = ab^x$	Exponential function
$80{,}000 = 102{,}000b^{15}$	Replace x with 15, y with 80,000, and a with 102,000.
$0.78 \approx b^{15}$	Divide each side by 102,000.
$\sqrt[15]{0.78} \approx b$	Take the 15th root of each side.

To find the 15th root of 0.78, use selection 5: $\sqrt[x]{\ }$ under the MATH menu on the TI-83 Plus.

KEYSTROKES: 15 [MATH] 5 0.78 [ENTER] .9835723396

An equation that models the farm population of Minnesota from 1983 to 1998 is $y = 102{,}000(0.98)^x$.

b. Suppose the number of farms in Minnesota continues to decline at the same rate. Estimate the number of farms in 2010.

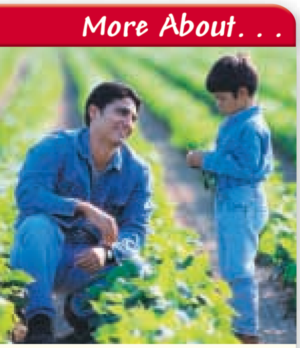
For 2010, the time x equals 2010 − 1983 or 27.

$y = 102{,}000(0.98)^x$	Modeling equation
$y = 102{,}000(0.98)^{27}$	Replace x with 27.
$y \approx 59{,}115$	Use a calculator.

The farm population in Minnesota will be about 59,115 in 2010.

www.algebra2.com/extra_examples

More About. . .



Farming

In 1999, 47% of the net farm income in the United States was from direct government payments. The USDA has set a goal of reducing this percent to 14% by 2005.

Source: USDA

TEACHING TIP

In Example 3, one of the given points is the y-intercept. You may wish to give your students a challenge problem in which any two points are given and students use a system of equations to find the equation of the exponential function.

In-Class Examples

Power Point®

2 Determine whether each function represents exponential *growth* or *decay*.

a. $y = (0.7)^x$ **The function represents exponential decay, since the base, 0.7, is between 0 and 1.**

b. $y = \frac{1}{2}(3)^x$ **The function represents exponential growth, since the base, 3, is greater than 1.**

c. $y = 10\left(\frac{4}{3}\right)^x$ **The function represents exponential growth, since the base, $\frac{4}{3}$, is greater than 1.**

3 **CELLULAR PHONES** In December of 1990, there were 5,283,000 cellular telephone subscribers in the United States. By December of 2000, this number had risen to 109,478, 000. **Source:** Cellular Telecommunications Industry Association

a. Write an exponential function of the form $y = ab^x$ that could be used to model the number of cellular telephone subscribers y in the U.S. Write the function in terms of x, the number of years since 1990. $\boldsymbol{y = 5{,}283{,}000(1.35)^x}$

b. Suppose the number of cellular telephone subscribers continues to increase at the same rate. Estimate the number of U.S. subscribers in 2010. **about 2,136,000,000 subscribers**

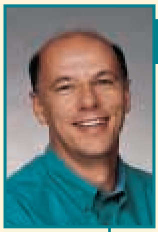
Interactive Chalkboard

PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

Teacher to Teacher



David S. Daniels Longmeadow H.S., Longmeadow, MA

"As a lead-in activity for exponential functions, have students flip 50 pennies and count the number of heads. Then have students remove those pennies that landed on heads and repeat the activity. Students should record their results and make a plot of the trial number versus the number of heads counted in that trial. The graph will model that of $y = \left(\frac{1}{2}\right)^x$."

EXPONENTIAL EQUATIONS AND INEQUALITIES

In-Class Examples

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4 Simplify each expression.

a. $5^{\sqrt{3}} \div 5^{\sqrt{2}}$ $5^{\sqrt{3}-\sqrt{2}}$

b. $\left(6^{\sqrt{5}}\right)^{\sqrt{6}}$ $6^{\sqrt{30}}$

5 Solve each equation.

a. $4^{9n-2} = 256$ $n = \frac{2}{3}$

b. $3^{5x} = 9^{2x-1}$ $x = -2$

Study Tip

Look Back

To review **Properties of Power**, see Lesson 5-1.

EXPONENTIAL EQUATIONS AND INEQUALITIES Since the domain of an exponential function includes irrational numbers such as $\sqrt{2}$, all the properties of rational exponents apply to irrational exponents.

Example 4 Simplify Expressions with Irrational Exponents

Simplify each expression.

a. $2^{\sqrt{5}} \cdot 2^{\sqrt{3}}$

$2^{\sqrt{5}} \cdot 2^{\sqrt{3}} = 2^{\sqrt{5}+\sqrt{3}}$ Product of Powers

b. $\left(7^{\sqrt{2}}\right)^{\sqrt{3}}$

$\left(7^{\sqrt{2}}\right)^{\sqrt{3}} = 7^{\sqrt{2}\cdot\sqrt{3}}$ Power of a Power

$= 7^{\sqrt{6}}$ Product of Radicals

The following property is useful for solving exponential equations. **Exponential equations** are equations in which variables occur as exponents.

Key Concept Property of Equality for Exponential Functions

- **Symbols** If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
- **Example** If $2^x = 2^8$, then $x = 8$.

Example 5 Solve Exponential Equations

Solve each equation.

a. $3^{2n+1} = 81$

$3^{2n+1} = 81$ Original equation

$3^{2n+1} = 3^4$ Rewrite 81 as 3^4 so each side has the same base.

$2n + 1 = 4$ Property of Equality for Exponential Functions

$2n = 3$ Subtract 1 from each side.

$n = \frac{3}{2}$ Divide each side by 2.

The solution is $\frac{3}{2}$.

CHECK $3^{2n+1} = 81$ Original equation

$3^{2\left(\frac{3}{2}\right)+1} \stackrel{?}{=} 81$ Substitute $\frac{3}{2}$ for n.

$3^4 \stackrel{?}{=} 81$ Simplify.

$81 = 81$ ✓ Simplify.

b. $4^{2x} = 8^{x-1}$

$4^{2x} = 8^{x-1}$ Original equation

$(2^2)^{2x} = (2^3)^{x-1}$ Rewrite each side with a base of 2.

$2^{4x} = 2^{3(x-1)}$ Power of a Power

$4x = 3(x-1)$ Property of Equality for Exponential Functions

$4x = 3x - 3$ Distributive Property

$x = -3$ Subtract $3x$ from each side.

The solution is -3.

The following property is useful for solving inequalities involving exponential functions or **exponential inequalities**.

Key Concept *Property of Inequality for Exponential Functions*

- **Symbols** If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.
- **Example** If $5^x < 5^4$, then $x < 4$.

This property also holds for ≤ and ≥.

Example 6 *Solve Exponential Inequalities*

Solve $4^{3p-1} > \frac{1}{256}$.

$4^{3p-1} > \frac{1}{256}$ Original inequality

$4^{3p-1} > 4^{-4}$ Rewrite $\frac{1}{256}$ as $\frac{1}{4^4}$ or 4^{-4} so each side has the same base.

$3p - 1 > -4$ Property of Inequality for Exponential Functions

$3p > -3$ Add 1 to each side.

$p > -1$ Divide each side by 3.

The solution set is $p > -1$.

CHECK Test a value of p greater than -1; for example, $p = 0$.

$4^{3p-1} > \frac{1}{256}$ Original inequality

$4^{3(0)-1} \overset{?}{>} \frac{1}{256}$ Replace p with 0.

$4^{-1} \overset{?}{>} \frac{1}{256}$ Simplify.

$\frac{1}{4} > \frac{1}{256}$ ✓ $a^{-1} = \frac{1}{a}$

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of a value of b for which $y = b^x$ represents exponential decay. **Sample answer: 0.8**

2. **Identify** each function as *linear*, *quadratic*, or *exponential*.

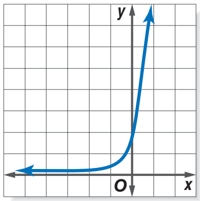
a. $y = 3x^2$ b. $y = 4(3)^x$ c. $y = 2x + 4$ d. $y = 4(0.2)^x + 1$

2a. quadratic
2b. exponential
2c. linear
2d. exponential

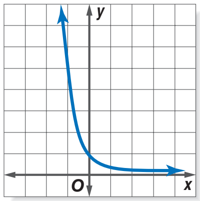
Match each function with its graph.

3. $y = 5^x$ **c** 4. $y = 2(5)^x$ **a** 5. $y = \left(\frac{1}{5}\right)^x$ **b**

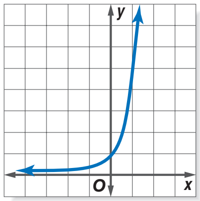
a.



b.



c.



Guided Practice

Sketch the graph of each function. Then state the function's domain and range.

6–7. See margin.

6. $y = 3(4)^x$ 7. $y = 2\left(\frac{1}{3}\right)^x$

In-Class Example

Power Point®

6 Solve $5^{3-2k} > \frac{1}{625}$. $k < \frac{7}{2}$

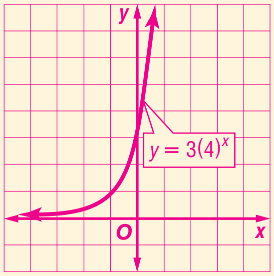
3 Practice/Apply

Study Notebook

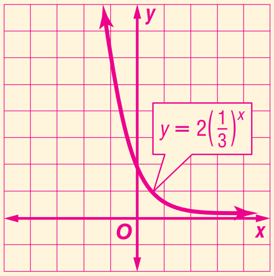
Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include examples of exponential growth and decay graphs and equations.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answers

6. $D = \{x | x \text{ is all real numbers.}\}$, $R = \{y | y > 0\}$



7. $D = \{x | x \text{ is all real numbers.}\}$, $R = \{y | y > 0\}$



DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Going around the room, have students count by ones beginning at 2, with each student calling out one number. Instruct them to record the number they called as n. Then have students find n^2 and 2^n. Now go around the room again and ask students to state their value of n^2 (for a class of 30 students, the recited numbers are all the squares from 4 to 961). Now have students state their values of 2^n (for a class of 30, the recited numbers are all the powers of 2 from 4 to 2^{31} or about 2×10^9).

About the Exercises…

Organization by Objective

- **Exponential Functions:** 21–38, 57–61
- **Exponential Equations and Inequalities:** 37–56, 62–66

Odd/Even Assignments

Exercises 21–56 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 61 involves research on the Internet or other reference materials. Exercises 71–75 require the use of graphing calculators.

Assignment Guide

Basic: 21, 23, 27–53 odd, 57–61, 68–70, 76–89

Average: 21–55 odd, 59–64, 67–70, 76–89 (optional: 71–75)

Advanced: 22–56 even, 61–86 (optional: 87–89)

GUIDED PRACTICE KEY

Exercises	Examples
6, 7	1
8–10	2
11, 12, 19, 20	3
13–15	4
16–18	5, 6

Determine whether each function represents exponential *growth* or *decay*.

8. $y = 2(7)^x$ **growth**
9. $y = (0.5)^x$ **decay**
10. $y = 0.3(5)^x$ **growth**

Write an exponential function whose graph passes through the given points.

11. (0, 3) and (−1, 6) $y = 3\left(\frac{1}{2}\right)^x$
12. (0, −18) and (−2, −2) $y = -18(3)^x$

Simplify each expression.

13. $2^{\sqrt{7}} \cdot 2^{\sqrt{7}}$ $2^{2\sqrt{7}}$ or $4^{\sqrt{7}}$
14. $(a^{\pi})^4$ $a^{4\pi}$
15. $81^{\sqrt{2}} \div 3^{\sqrt{2}}$ $3^{3\sqrt{2}}$ or $27^{\sqrt{2}}$

Solve each equation or inequality. Check your solution.

16. $2^{n+4} = \frac{1}{32}$ -9
17. $5^{2x+3} \le 125$ $x \le 0$
18. $9^{2y-1} = 27^y$ 2

Application

ANIMAL CONTROL **For Exercises 19 and 20, use the following information.** During the 19th century, rabbits were brought to Australia. Since the rabbits had no natural enemies on that continent, their population increased rapidly. Suppose there were 65,000 rabbits in Australia in 1865 and 2,500,000 in 1867.

19. Write an exponential function that could be used to model the rabbit population y in Australia. Write the function in terms of x, the number of years since 1865. $y = 65{,}000(6.20)^x$
20. Assume that the rabbit population continued to grow at that rate. Estimate the Australian rabbit population in 1872. **22,890,495,000**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
21–26	1
27–32	2
33–38, 57–66	3
39–44	4
45–56	5, 6

Extra Practice

See page 849.

Sketch the graph of each function. Then state the function's domain and range. **21–26. See pp. 573A–573D.**

21. $y = 2(3)^x$
22. $y = 5(2)^x$
23. $y = 0.5(4)^x$
24. $y = 4\left(\frac{1}{3}\right)^x$
★ 25. $y = -\left(\frac{1}{5}\right)^x$
★ 26. $y = -2.5(5)^x$

Determine whether each function represents exponential *growth* or *decay*.

27. $y = 10(3.5)^x$ **growth**
28. $y = 2(4)^x$ **growth**
29. $y = 0.4\left(\frac{1}{3}\right)^x$ **decay**
30. $y = 3\left(\frac{5}{2}\right)^x$ **growth**
31. $y = 30^{-x}$ **decay**
32. $y = 0.2(5)^{-x}$ **decay**

Write an exponential function whose graph passes through the given points.

33. (0, −2) and (−2, −32) $y = -2\left(\frac{1}{4}\right)^x$
34. (0, 3) and (1, 15) $y = 3(5)^x$
35. (0, 7) and (2, 63) $y = 7(3)^x$
36. (0, −5) and (−3, −135) $y = -5\left(\frac{1}{3}\right)^x$
37. (0, 0.2) and (4, 51.2) $y = 0.2(4)^x$
38. (0, −0.3) and (5, −9.6) $y = -0.3(2)^x$

Simplify each expression.

39. $\left(5^{\sqrt{2}}\right)^{\sqrt{8}}$ 5^4 or 625
40. $\left(x^{\sqrt{5}}\right)^{\sqrt{3}}$ $x^{\sqrt{15}}$
41. $7^{\sqrt{2}} \cdot 7^{3\sqrt{2}}$ $7^{4\sqrt{2}}$
42. $y^{3\sqrt{3}} \div y^{\sqrt{3}}$ $y^{2\sqrt{3}}$
43. $n^2 \cdot n^{\pi}$ $n^{2+\pi}$
44. $64^{\pi} \div 2^{\pi}$ $2^{5\pi}$

Solve each equation or inequality. Check your solution. **54.** $p \ge -2$

45. $3^{n-2} = 27$ 5
46. $2^{3x+5} = 128$ $\frac{2}{3}$
47. $5^{n-3} = \frac{1}{25}$ 1
48. $2^{2n} \le \frac{1}{16}$ $n \le -2$
49. $\left(\frac{1}{9}\right)^m = 81^{m+4}$ $-\frac{8}{3}$
50. $\left(\frac{1}{7}\right)^{y-3} = 343$ 0
51. $16^n < 8^{n+1}$ $n < 3$
52. $10^{x-1} = 100^{2x-3}$ $\frac{5}{3}$
53. $36^{2p} = 216^{p-1}$ -3
54. $32^{5p+2} \ge 16^{5p}$
★ 55. $3^{5x} \cdot 81^{1-x} = 9^{x-3}$ 10
56. $49^x = 7^{x^2-15}$ $-3, 5$

Answers (p. 529)

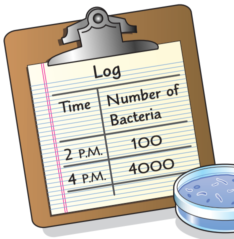
60. 9.67 million; 17.62 million; 32.12 million; These answers are in close agreement with the actual populations in those years.

61. 2144.97 million; 281.42 million; No, the growth rate has slowed considerably. The population in 2000 was much smaller than the equation predicts it would be.

WebQuest

The magnitude of an earthquake can be represented by an exponential equation. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

BIOLOGY **For Exercises 57 and 58, use the following information.**
The number of bacteria in a colony is growing exponentially.



57. Write an exponential function to model the population y of bacteria x hours after 2 P.M. $y = 100(6.32)^x$

58. How many bacteria were there at 7 P.M. that day? about 1,008,290

POPULATION **For Exercises 59–61, use the following information.**
Every ten years, the Bureau of the Census counts the number of people living in the United States. In 1790, the population of the U.S. was 3.93 million. By 1800, this number had grown to 5.31 million.

59. Write an exponential function that could be used to model the U.S. population y in millions for 1790 to 1800. Write the equation in terms of x, the number of decades x since 1790. $y = 3.93(1.35)^x$

60. Assume that the U.S. population continued to grow at that rate. Estimate the population for the years 1820, 1840, and 1860. Then compare your estimates with the actual population for those years, which were 9.64, 17.06, and 31.44 million, respectively. See margin.

61. **RESEARCH** Estimate the population of the U.S. in 2000. Then use the Internet or other reference to find the actual population of the U.S. in 2000. Has the population of the U.S. continued to grow at the same rate at which it was growing in the early 1800s? Explain. See margin.

62. Exponential; the base, $1 + \frac{r}{n}$, is fixed, but the exponent, nt, is variable since the time t can vary.

MONEY **For Exercises 62–64, use the following information.**
Suppose you deposit a principal amount of P dollars in a bank account that pays compound interest. If the annual interest rate is r (expressed as a decimal) and the bank makes interest payments n times every year, the amount of money A you would have after t years is given by $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$.

62. If the principal, interest rate, and number of interest payments are known, what type of function is $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$? Explain your reasoning.

63. Write an equation giving the amount of money you would have after t years if you deposit $1000 into an account paying 4% annual interest compounded quarterly (four times per year). $A(t) = 1000(1.01)^{4t}$

64. Find the account balance after 20 years. $2216.72

More About . . .

Computers

Since computers were invented, computational speed has multiplied by a factor of 4 about every three years.

Source: www.wired.com

COMPUTERS **For Exercises 65 and 66, use the information at the left.**

65. If a typical computer operates with a computational speed s today, write an expression for the speed at which you can expect an equivalent computer to operate after x three-year periods. $s \cdot 4^x$

★ 66. Suppose your computer operates with a processor speed of 600 megahertz and you want a computer that can operate at 4800 megahertz. If a computer with that speed is currently unavailable for home use, how long can you expect to wait until you can buy such a computer? 1.5 three-year periods or 4.5 yr

67. Sometimes; true when $b > 1$, but false when $b < 1$.

★ 67. **CRITICAL THINKING** Decide whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning.
For a positive base b other than 1, $b^x > b^y$ if and only if $x > y$.

www.algebra2.com/self_check_quiz

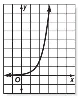
Study Guide and Intervention, p. 573 (shown) and p. 574

Exponential Functions An **exponential function** has the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

Properties of an Exponential Function	**1.** The function is continuous and one-to-one. **2.** The domain is the set of all real numbers. **3.** The x-axis is the asymptote of the graph. **4.** The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$. **5.** The graph contains the point $(0, a)$.
Exponential Growth and Decay	If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth. If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

Example 1 **Sketch the graph of $y = 0.1(4)^x$. Then state the function's domain and range.**
Make a table of values. Connect the points to form a smooth curve.

x	−1	0	1	2	3
y	0.025	0.1	0.4	1.6	6.4



The domain of the function is all real numbers, while the range is the set of all positive real numbers.

Example 2 **Determine whether each function represents exponential *growth* or *decay*.**

a. $y = 0.5(2)^x$
exponential growth, since the base, 2, is greater than 1

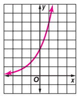
b. $y = -2.8(2)^x$
neither, since −2.8, the value of a is less than 0.

c. $y = 1.1(0.5)^x$
exponential decay, since the base, 0.5, is between 0 and 1

Exercises

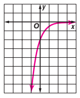
Sketch the graph of each function. Then state the function's domain and range.

1. $y = 3(2)^x$



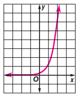
Domain: all real numbers; Range: all positive real numbers

2. $y = -2\left(\frac{1}{4}\right)^x$



Domain: all real numbers; Range: all negative real numbers

3. $y = 0.25(5)^x$



Domain: all real numbers; Range: all positive real numbers

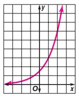
Determine whether each function represents exponential growth or decay.

4. $y = 0.3(1.2)^x$ growth
5. $y = -5\left(\frac{4}{5}\right)^x$ neither
6. $y = 3(10)^{-x}$ decay

Skills Practice, p. 575 and Practice, p. 576 (shown)

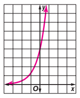
Sketch the graph of each function. Then state the function's domain and range.

1. $y = 1.5(2)^x$



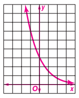
domain: all real numbers; range: all positive numbers

2. $y = 4(3)^x$



domain: all real numbers; range: all positive numbers

3. $y = 3(0.5)^x$



domain: all real numbers; range: all positive numbers

Determine whether each function represents exponential *growth* or *decay*.

4. $y = 5(0.6)^x$ decay
5. $y = 0.1(2)^x$ growth
6. $y = 5 \cdot 4^{-x}$ decay

Write an exponential function whose graph passes through the given points.

7. (0, 1) and (−1, 4) $y = \left(\frac{1}{4}\right)^x$
8. (0, 2) and (1, 10) $y = 2(5)^x$
9. (0, −3) and (1, −1.5) $y = -3(0.5)^x$
10. (0, 0.8) and (1, 1.6) $y = 0.8(2)^x$
11. (0, −0.4) and (2, −10) $y = -0.4(5)^x$
12. (0, π) and (3, 8π) $y = \pi(2)^x$

Simplify each expression.

13. $(2^{\sqrt{2}})^{\sqrt{8}}$ 16
14. $(n^{\sqrt{3}})^{\sqrt{75}}$ n^{15}
15. $y^{\sqrt{6}} \cdot y^{5\sqrt{6}}$ $y^{6\sqrt{6}}$
16. $13^{\sqrt{6}} \cdot 13^{\sqrt{24}}$ $13^{3\sqrt{6}}$
17. $n^3 \div n^{\pi}$ $n^{3-\pi}$
18. $125^{\sqrt{11}} \div 5^{\sqrt{11}}$ $5^{2\sqrt{11}}$

Solve each equation or inequality. Check your solution.

19. $3^{3x-5} > 81$ $x > 3$
20. $7^{6x} = 7^{2x-20}$ −5
21. $3^{6n-5} < 9^{4n-3}$ $n > \frac{1}{2}$
22. $9^{2x-1} = 27^{x+4}$ 14
23. $2^{3n-1} \geq \left(\frac{1}{8}\right)^n$ $n \geq \frac{1}{6}$
24. $16^{4n-1} = 128^{2n+1}$ $\frac{11}{2}$

BIOLOGY **For Exercises 25 and 26, use the following information.**
The initial number of bacteria in a culture is 12,000. The number after 3 days is 96,000.

25. Write an exponential function to model the population y of bacteria after x days.
$y = 12{,}000(2)^x$

26. How many bacteria are there after 6 days? 768,000

27. **EDUCATION** A college with a graduating class of 4000 students in the year 2002 predicts that it will have a graduating class of 4862 in 4 years. Write an exponential function to model the number of students y in the graduating class t years after 2002.
$y = 4000(1.05)^t$

Reading to Learn Mathematics, p. 577

ELL

Pre-Activity **How does an exponential function describe tournament play?**

Read the introduction to Lesson 10-1 at the top of page 523 in your textbook.

How many rounds of play would be needed for a tournament with 100 players? 7

Reading the Lesson

1. Indicate whether each of the following statements about the exponential function $y = 10^x$ is *true* or *false*.
   - **a.** The domain is the set of all positive real numbers. false
   - **b.** The y-intercept is 1. true
   - **c.** The function is one-to-one. true
   - **d.** The y-axis is an asymptote of the graph. false
   - **e.** The range is the set of all real numbers. false

2. Determine whether each function represents exponential *growth* or *decay*.
   - **a.** $y = 0.2(3)^x$. growth
   - **b.** $y = 3\left(\frac{2}{5}\right)^x$. decay
   - **c.** $y = 0.4(1.01)^x$. growth

3. Supply the reason for each step in the following solution of an exponential equation.

$9^{2x-1} = 27x$	Original equation
$(3^2)^{2x-1} = (3^3)^x$	Rewrite each side with a base of 3.
$3^{2(2x-1)} = 3^{3x}$	Power of a Power
$2(2x-1) = 3x$	Property of Equality for Exponential Functions
$4x - 2 = 3x$	Distributive Property
$x - 2 = 0$	Subtract $3x$ from each side.
$x = 2$	Add 2 to each side.

Helping You Remember

4. One way to remember that polynomial functions and exponential functions are different is to contrast the polynomial function $y = x^2$ and the exponential function $y = 2^x$. Tell at least three ways they are different.
Sample answer: In $y = x^2$, the variable x is a base, but in $y = 2^x$, the variable x is an exponent. The graph of $y = x^2$ is symmetric with respect to the y-axis, but the graph of $y = 2^x$ is not. The graph of $y = x^2$ touches the x-axis at (0, 0), but the graph of $y = 2^x$ has the x-axis as an asymptote. You can compute the value of $y = x^2$ mentally for $x = 100$, but you cannot compute the value of $y = 2^x$ mentally for $x = 100$.

Enrichment, p. 578

Finding Solutions of $x^y = y^x$

Perhaps you have noticed that if x and y are interchanged in equations such as $x = y$ and $xy = 1$, the resulting equation is equivalent to the original equation. The same is true of the equation $x^y = y^x$. However, finding solutions of $x^y = y^x$ and drawing its graph is not a simple process.

Solve each problem. Assume that x and y are positive real numbers.

1. If $a > 0$, will (a, a) be a solution of $x^y = y^x$? Justify your answer.
Yes, since $a^a = a^a$ must be true (Reflexive Prop. of Equality).

2. If $c > 0$, $d > 0$, and (c, d) is a solution of $x^y = y^x$, will (d, c) also be a solution? Justify your answer.
Yes; replacing x with d, y with c gives $d^c = c^d$; but if (c, d) is a solution, $c^d = d^c$. So, by the Symmetric Property of Equality, $d^c = c^d$ is true.

4 Assess

Open-Ended Assessment

Modeling Give students a sheet of grid paper and a length of string. Have students model the graph of the equation $y = \left(\frac{1}{2}\right)^x$. Have them check their model by graphing the equation on a graphing calculator.

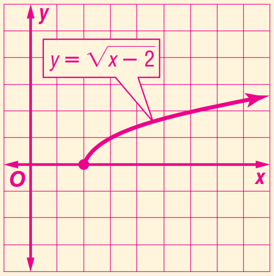
Getting Ready for Lesson 10-2

PREREQUISITE SKILL In Lesson 10-2, students will evaluate logarithmic expressions. Because logarithmic and exponential functions are inverses of each other, their composites are the identity function. Students must be familiar with compositions of functions in order to evaluate these inverse functions. Use Exercises 87–89 to determine your students' familiarity with composition of functions.

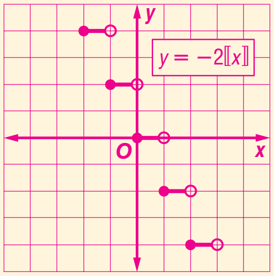
Answers

75. For $h > 0$, the graph of $y = 2^x$ is translated $|h|$ units to the right. For $h < 0$, the graph of $y = 2^x$ is translated $|h|$ units to the left. For $k > 0$, the graph of $y = 2^x$ is translated $|k|$ units up. For $k < 0$, the graph of $y = 2^x$ is translated $|k|$ units down.

80.



81.



68. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 573A–573D.**

How does an exponential function describe tournament play?

Include the following in your answer:

- an explanation of how you could use the equation $y = 2^x$ to determine the number of rounds of tournament play for 128 teams, and
- an example of an inappropriate number of teams for tournament play with an explanation as to why this number would be inappropriate.

Standardized Test Practice

69. If $4^{x+2} = 48$, then $4^x =$ **A**

Ⓐ 3.0. Ⓑ 6.4. Ⓒ 6.9. Ⓓ 12.0. Ⓔ 24.0.

70. **GRID IN** Suppose you deposit \$500 in an account paying 4.5% interest compounded semiannually. Find the dollar value of the account rounded to the nearest penny after 10 years. **780.25**

Graphing Calculator

FAMILIES OF GRAPHS **Graph each pair of functions on the same screen. Then compare the graphs, listing both similarities and differences in shape, asymptotes, domain, range, and y-intercepts.** **71–74. See pp. 573A–573D.**

71. $y = 2^x$ and $y = 2^x + 3$

72. $y = 3^x$ and $y = 3^{x+1}$

73. $y = \left(\frac{1}{5}\right)^x$ and $y = \left(\frac{1}{5}\right)^{x-2}$

74. $y = \left(\frac{1}{4}\right)^x$ and $y = \left(\frac{1}{4}\right)^x - 1$

75. Describe the effect of changing the values of h and k in the equation $y = 2^{x-h} + k$. **See margin.**

Maintain Your Skills

Mixed Review **Solve each equation or inequality. Check your solutions.** *(Lesson 9-6)*

76. $\frac{15}{p} + p = 16$ **1, 15**

77. $\frac{s-3}{s+4} = \frac{6}{s^2-16}$ **1, 6**

78. $\frac{2a-5}{a-9} + \frac{a}{a+9} = \frac{-6}{a^2-81}$ **$-\frac{13}{3}$, 3**

79. $\frac{x-2}{x} < \frac{x-4}{x-6}$ **$0 < x < 3$ or $x > 6$**

Identify each equation as a type of function. Then graph the equation. *(Lesson 9-5)*

80–82. See margin for graphs.

80. $y = \sqrt{x-2}$ **square root**

81. $y = -2[\![x]\!]$ **greatest integer**

82. $y = 8$ **constant**

Find the inverse of each matrix, if it exists. *(Lesson 4-7)*

83. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

84. $\begin{bmatrix} 2 & 4 \\ 5 & 10 \end{bmatrix}$ **does not exist**

85. $\begin{bmatrix} -5 & 6 \\ -11 & 3 \end{bmatrix}$ $\frac{1}{51}\begin{bmatrix} 3 & -6 \\ 11 & -5 \end{bmatrix}$

86. **ENERGY** A circular cell must deliver 18 watts of energy. If each square centimeter of the cell that is in sunlight produces 0.01 watt of energy, how long must the radius of the cell be? *(Lesson 5-8)* **about 23.94 cm**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find $g[h(x)]$ and $h[g(x)]$.**

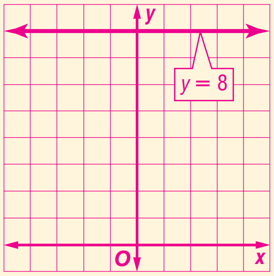
*(To review **composition of functions**, see Lesson 7-7.)* **87–89. See margin.**

87. $h(x) = 2x - 1$
$g(x) = x - 5$

88. $h(x) = x + 3$
$g(x) = x^2$

89. $h(x) = 2x + 5$
$g(x) = -x + 3$

82.



87. $g[h(x)] = 2x - 6$; $h[g(x)] = 2x - 11$

88. $g[h(x)] = x^2 + 6x + 9$; $h[g(x)] = x^2 + 3$

89. $g[h(x)] = -2x - 2$; $h[g(x)] = -2x + 11$

10-2 Logarithms and Logarithmic Functions

What You'll Learn

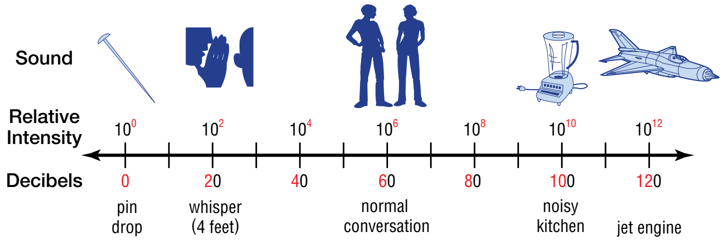
- Evaluate logarithmic expressions.
- Solve logarithmic equations and inequalities.

Vocabulary

- logarithm
- logarithmic function
- logarithmic equation
- logarithmic inequality

Why is a logarithmic scale used to measure sound?

Many scientific measurements have such an enormous range of possible values that it makes sense to write them as powers of 10 and simply keep track of their exponents. For example, the loudness of sound is measured in units called *decibels*. The graph shows the relative intensities and decibel measures of common sounds.



The decibel measure of the loudness of a sound is the exponent or logarithm of its relative intensity multiplied by 10.

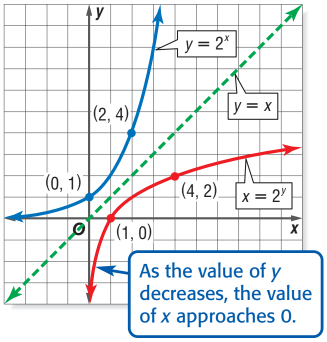
Study Tip

Look Back
To review **inverse functions**, see Lesson 7-8.

LOGARITHMIC FUNCTIONS AND EXPRESSIONS To better understand what is meant by a logarithm, let's look at the graph of $y = 2^x$ and its inverse. Since exponential functions are one-to-one, the inverse of $y = 2^x$ exists and is also a function. Recall that you can graph the inverse of a function by interchanging the x and y values in the ordered pairs of the function.

$y = 2^x$	
x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

$x = 2^y$	
x	y
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



The inverse of $y = 2^x$ can be defined as $x = 2^y$. Notice that the graphs of these two functions are reflections of each other over the line $y = x$.

In general, the inverse of $y = b^x$ is $x = b^y$. In $x = b^y$, y is called the **logarithm** of x. It is usually written as $y = \log_b x$ and is read *y equals log base b of x*.

10-2 Lesson Notes

1 Focus

5-Minute Check Transparency 10-2 Use as a quiz or review of Lesson 10-1.

Mathematical Background notes are available for this lesson on p. 520C.

Why is a logarithmic scale used to measure sound?

Ask students:

- On the number line shown, the scale along the bottom is 10 decibels per tick mark. What do you notice about the scale along the top for relative intensity? **The scale is not uniform; the relative intensity at the first tick mark is 10, at the second it is 100, at the third it is 1000, and so on.**
- If you draw a number line with a uniform scale whose tick marks are labeled from 0 to 10^{12}, what number is at the midpoint between 0 to 10^{12}? **5×10^{11}**
- Where does the point 1 million appear on your number line? **very close to the point for 0**
- Where does the point 100 appear on your number line? **very, very close to the point for 0**
- What problem arises with trying to represent the relative intensities on a standard number line? **Sample answer: The lesser intensities are so close together near 0 on the number line that they are difficult to represent accurately.**

Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters

- Study Guide and Intervention, pp. 579–580
- Skills Practice, p. 581
- Practice, p. 582
- Reading to Learn Mathematics, p. 583
- Enrichment, p. 584
- Assessment, p. 623

School-to-Career Masters, p. 19

Transparencies

5-Minute Check Transparency 10-2
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

LOGARITHMIC FUNCTIONS AND EXPRESSIONS

Teaching Tip After discussing the definition of logarithm at the bottom of p. 531, write the equation $y = 2x$ on the chalkboard and ask students to rewrite the equation with x in terms of y. $\left(x = \frac{1}{2}y\right)$ Repeat this for the equation $y = x^2$. $(x = \pm\sqrt{y})$ Now write the equation $y = 2^x$ on the chalkboard and ask students to rewrite this equation with x in terms of y. This will likely have students stymied. Explain that the rewritten equation is $x = \log_2 y$.

In-Class Examples

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1 Write each equation in exponential form.

a. $\log_3 9 = 2$ $\quad 3^2 = 9$

b. $\log_{10} \frac{1}{100} = -2$ $\quad 10^{-2} = \frac{1}{100}$

2 Write each equation in logarithmic form.

a. $5^3 = 125$ $\quad \log_5 125 = 3$

b. $27^{\frac{1}{3}} = 3$ $\quad \log_{27} 3 = \frac{1}{3}$

3 Evaluate $\log_3 243$. $\quad 5$

Key Concept — Logarithm with Base b

- **Words** Let b and x be positive numbers, $b \neq 1$. The *logarithm of x with base b* is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.
- **Symbols** Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.

Example 1 Logarithmic to Exponential Form

Write each equation in exponential form.

a. $\log_8 1 = 0$

$\log_8 1 = 0 \rightarrow 1 = 8^0$

b. $\log_2 \frac{1}{16} = -4$

$\log_2 \frac{1}{16} = -4 \rightarrow \frac{1}{16} = 2^{-4}$

Example 2 Exponential to Logarithmic Form

Write each equation in logarithmic form.

a. $10^3 = 1000$

$10^3 = 1000 \rightarrow \log_{10} 1000 = 3$

b. $9^{\frac{1}{2}} = 3$

$9^{\frac{1}{2}} = 3 \rightarrow \log_9 3 = \frac{1}{2}$

You can use the definition of logarithm to find the value of a logarithmic expression.

Example 3 Evaluate Logarithmic Expressions

Evaluate $\log_2 64$.

$\log_2 64 = y$ Let the logarithm equal y.

$64 = 2^y$ Definition of logarithm

$2^6 = 2^y$ $64 = 2^6$

$6 = y$ Property of Equality for Exponential Functions

So, $\log_2 64 = 6$.

The function $y = \log_b x$, where $b > 0$ and $b \neq 1$, is called a **logarithmic function**. As shown in the graph on the previous page, this function is the inverse of the exponential function $y = b^x$ and has the following characteristics.

1. The function is continuous and one-to-one.
2. The domain is the set of all positive real numbers.
3. The y-axis is an asymptote of the graph.
4. The range is the set of all real numbers.
5. The graph contains the point (1, 0). That is, the x-intercept is 1.

Since the exponential function $f(x) = b^x$ and the logarithmic function $g(x) = \log_b x$ are inverses of each other, their composites are the identity function. That is, $f[g(x)] = x$ and $g[f(x)] = x$.

$$f[g(x)] = x \qquad g[f(x)] = x$$
$$f(\log_b x) = x \qquad g(b^x) = x$$
$$b^{\log_b x} = x \qquad \log_b b^x = x$$

Study Tip

Look Back
To review **composition of functions**, see Lesson 7-7.

Thus, if their bases are the same, exponential and logarithmic functions "undo" each other. You can use this inverse property of exponents and logarithms to simplify expressions.

Example 4 Inverse Property of Exponents and Logarithms

Evaluate each expression.

a. $\log_6 6^8$

$\log_6 6^8 = 8$ $\quad \log_b b^x = x$

b. $3^{\log_3 (4x - 1)}$

$3^{\log_3 (4x - 1)} = 4x - 1$ $\quad b^{\log_b x} = x$

SOLVE LOGARITHMIC EQUATIONS AND INEQUALITIES

A **logarithmic equation** is an equation that contains one or more logarithms. You can use the definition of a logarithm to help you solve logarithmic equations.

Example 5 Solve a Logarithmic Equation

Solve $\log_4 n = \frac{5}{2}$.

$\log_4 n = \frac{5}{2}$ Original equation

$n = 4^{\frac{5}{2}}$ Definition of logarithm

$n = (2^2)^{\frac{5}{2}}$ $4 = 2^2$

$n = 2^5$ Power of a Power

$n = 32$ Simplify.

A **logarithmic inequality** is an inequality that involves logarithms. In the case of inequalities, the following property is helpful.

Key Concept Logarithmic to Exponential Inequality

- **Symbols** If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.
  If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.
- **Examples** $\log_2 x > 3$, $x > 2^3$ $\qquad$ $\log_3 x < 5$, $0 < x < 3^5$

Example 6 Solve a Logarithmic Inequality

Solve $\log_5 x < 2$. Check your solution.

$\log_5 x < 2$ Original inequality

$0 < x < 5^2$ Logarithmic to exponential inequality

$0 < x < 25$ Simplify.

The solution set is $\{x \mid 0 < x < 25\}$.

CHECK Try 5 to see if it satisfies the inequality.

$\log_5 x < 2$ Original inequality

$\log_5 5 \overset{?}{<} 2$ Substitute 5 for x.

$1 < 2$ ✓ $\log_5 5 = 1$ because $5^1 = 5$.

Study Tip

Special Values

If $b > 0$ and $b \neq 1$, then the following statements are true.

- $\log_b b = 1$ because $b^1 = b$.
- $\log_b 1 = 0$ because $b^0 = 1$.

www.algebra2.com/extra_examples

In-Class Example Power Point®

4 Evaluate each expression.

a. $\log_9 9^2$ **2**

b. $7^{\log_7 (x^2 - 1)}$ **$x^2 - 1$**

SOLVE LOGARITHMIC EQUATIONS AND INEQUALITIES

In-Class Examples Power Point®

5 Solve $\log_8 n = \frac{4}{3}$. **16**

6 Solve $\log_6 x > 3$. Check your solution. **$\{x \mid x > 216\}$**

In-Class Examples

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7 Solve $\log_4 x^2 = \log_4 (4x - 3)$. Check your solution. **1, 3**

8 Solve $\log_7 (2x + 8) > \log_7 (x + 5)$. Check your solution. $x > -3$

Tips for New Teachers

Intervention Students have not covered logarithmic functions before and are likely to find them confusing. Expect students to need extra time to absorb the material in this lesson before continuing with the rest of the chapter.

Use the following property to solve logarithmic equations that have logarithms with the same base on each side.

Key Concept — Property of Equality for Logarithmic Functions

- **Symbols** If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.
- **Example** If $\log_7 x = \log_7 3$, then $x = 3$.

Example 7 Solve Equations with Logarithms on Each Side

Solve $\log_5 (p^2 - 2) = \log_5 p$. Check your solution.

$\log_5 (p^2 - 2) = \log_5 p$	Original equation
$p^2 - 2 = p$	Property of Equality for Logarithmic Functions
$p^2 - p - 2 = 0$	Subtract p from each side.
$(p - 2)(p + 1) = 0$	Factor.
$p - 2 = 0$ or $p + 1 = 0$	Zero Product Property
$p = 2$ $\quad p = -1$	Solve each equation.

CHECK Substitute each value into the original equation.

$\log_5 (2^2 - 2) \stackrel{?}{=} \log_5 2$	Substitute 2 for p.
$\log_5 2 = \log_5 2$ ✓	Simplify.
$\log_5 [(-1)^2 - 2] \stackrel{?}{=} \log_5 (-1)$	Substitute -1 for p.

Since $\log_5 (-1)$ is undefined, -1 is an *extraneous* solution and must be eliminated. Thus, the solution is 2.

Study Tip

Extraneous Solutions

The domain of a logarithmic function does not include negative values. For this reason, be sure to check for extraneous solutions of logarithmic equations.

Use the following property to solve logarithmic inequalities that have the same base on each side. Exclude values from your solution set that would result in taking the logarithm of a number less than or equal to zero in the original inequality.

Key Concept — Property of Inequality for Logarithmic Functions

- **Symbols** If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.
- **Example** If $\log_2 x > \log_2 9$, then $x > 9$.

This property also holds for $\leq$ and $\geq$.

Example 8 Solve Inequalities with Logarithms on Each Side

Solve $\log_{10} (3x - 4) < \log_{10} (x + 6)$. Check your solution.

$\log_{10} (3x - 4) < \log_{10} (x + 6)$	Original inequality
$3x - 4 < x + 6$	Property of Inequality for Logarithmic Functions
$2x < 10$	Addition and Subtraction Properties of Inequalities
$x < 5$	Divide each side by 2.

We must exclude from this solution all values of x such that $3x - 4 \leq 0$ or $x + 6 \leq 0$. Thus, the solution set is $x > \frac{4}{3}$ and $x > -6$ and $x < 5$. This compound inequality simplifies to $\frac{4}{3} < x < 5$.

Study Tip

Look back

To review **compound inequalities**, see Lesson 1-6.

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Have students create colorful posters showing several equivalent exponential and logarithmic equations, such as $2^3 = 8$ and $3 = \log_2 8$. Suggest that students use a different color for each of the digits 2, 3, and 8 to help them visualize the relative locations of the digits in the pairs of equations.

Check for Understanding

Concept Check

1. **OPEN ENDED** Give an example of an exponential equation and its related logarithmic equation. **Sample answer: $x = 5^y$ and $y = \log_5 x$**

2. **Describe** the relationship between $y = 3^x$ and $y = \log_3 x$. **They are inverses.**

3. **FIND THE ERROR** Paul and Scott are solving $\log_3 x = 9$.

Paul

$\log_3 x = 9$

$3^x = 9$

$3^x = 3^2$

$x = 2$

Scott

$\log_3 x = 9$

$x = 3^9$

$x = 19{,}683$

Who is correct? Explain your reasoning. **Scott; see margin for explanation.**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4, 5	1
6, 7	2
8–11	3
12–17	4–7
18–20	4

Write each equation in logarithmic form.

4. $5^4 = 625$ **$\log_5 625 = 4$**

5. $7^{-2} = \frac{1}{49}$ **$\log_7 \frac{1}{49} = -2$**

Write each equation in exponential form.

6. $\log_3 81 = 4$ **$3^4 = 81$**

7. $\log_{36} 6 = \frac{1}{2}$ **$36^{\frac{1}{2}} = 6$**

Evaluate each expression.

8. $\log_4 256$ **4**

9. $\log_2 \frac{1}{8}$ **−3**

10. $3^{\log_3 21}$ **21**

11. $\log_5 5^{-1}$ **−1**

Solve each equation or inequality. Check your solutions.

12. $\log_9 x = \frac{3}{2}$ **27**

13. $\log_{\frac{1}{10}} x = -3$ **1000**

14. $\log_3 (2x - 1) \le 2$ **$\frac{1}{2} < x \le 5$**

15. $\log_5 (3x - 1) = \log_5 2x^2$ **$\frac{1}{2}$, 1**

16. $\log_2 (3x - 5) > \log_2 (x + 7)$ **$x > 6$**

17. $\log_b 9 = 2$ **3**

Application

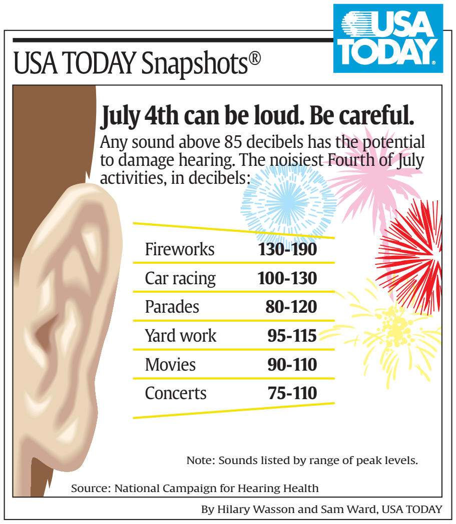
SOUND **For Exercises 18–20, use the following information.**
An equation for loudness L, in decibels, is $L = 10 \log_{10} R$, where R is the relative intensity of the sound.

18. Solve $130 = 10 \log_{10} R$ to find the relative intensity of a fireworks display with a loudness of 130 decibels. **10^{13}**

19. Solve $75 = 10 \log_{10} R$ to find the relative intensity of a concert with a loudness of 75 decibels. **$10^{7.5}$**

20. How many times more intense is the fireworks display than the concert? In other words, find the ratio of their intensities. **$10^{5.5}$ or about 316,228 times**

USA TODAY Snapshots®



July 4th can be loud. Be careful.

Any sound above 85 decibels has the potential to damage hearing. The noisiest Fourth of July activities, in decibels:

Fireworks	130-190
Car racing	100-130
Parades	80-120
Yard work	95-115
Movies	90-110
Concerts	75-110

Note: Sounds listed by range of peak levels.

Source: National Campaign for Hearing Health

By Hilary Wasson and Sam Ward, USA TODAY

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include examples of how to write logarithms in exponential form.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR

Review converting logarithms to exponential form. Also note that, according to Paul, $\log_3 x = 3^x$, which cannot be true.

Answer

3. The value of a logarithmic equation, 9, is the exponent of the equivalent exponential equation, and the base of the logarithmic expression, 3, is the base of the exponential equation. Thus $x = 3^9$ or 19,683.

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

About the Exercises...

Organization by Objective

- Logarithmic Functions and Expressions: 21–46, 66–71
- Solve Logarithmic Equations and Inequalities: 47–65

Odd/Even Assignments

Exercises 21–62 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 21–41 odd, 45–59 odd, 71–90

Average: 21–67 odd, 68, 69, 71–90

Advanced: 22–66 even, 68–84 (optional: 85–90)

All: Practice Quiz 1 (1–10)

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
21–26	1
27–32	2
33–46	3
47–62	4–7
63–65	4
68–70	5

Extra Practice

See page 849.

Write each equation in logarithmic form. 23. $\log_5 \frac{1}{125} = -3$

21. $8^3 = 512$ $\log_8 512 = 3$

22. $3^3 = 27$ $\log_3 27 = 3$

23. $5^{-3} = \frac{1}{125}$

24. $\left(\frac{1}{3}\right)^{-2} = 9$ $\log_{\frac{1}{3}} 9 = -2$

25. $100^{\frac{1}{2}} = 10$ $\log_{100} 10 = \frac{1}{2}$

26. $2401^{\frac{1}{4}} = 7$ $\log_{2401} 7 = \frac{1}{4}$

Write each equation in exponential form. 29. $4^{-1} = \frac{1}{4}$

27. $\log_5 125 = 3$ $5^3 = 125$

28. $\log_{13} 169 = 2$ $13^2 = 169$

29. $\log_4 \frac{1}{4} = -1$

30. $\log_{100} \frac{1}{10} = -\frac{1}{2}$ $100^{-\frac{1}{2}} = \frac{1}{10}$

31. $\log_8 4 = \frac{2}{3}$ $8^{\frac{2}{3}} = 4$

32. $\log_{\frac{1}{5}} 25 = -2$ $\left(\frac{1}{5}\right)^{-2} = 25$

Evaluate each expression.

33. $\log_2 16$ 4

34. $\log_{12} 144$ 2

35. $\log_{16} 4$ $\frac{1}{2}$

36. $\log_9 243$ $\frac{5}{2}$

37. $\log_2 \frac{1}{32}$ -5

38. $\log_3 \frac{1}{81}$ -4

39. $\log_5 5^7$ 7

40. $2^{\log_2 45}$ 45

41. $\log_{11} 11^{(n-5)}$ $n - 5$

42. $6^{\log_6 (3x+2)}$ $3x + 2$

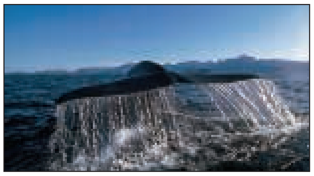
★ **43.** $\log_{10} 0.001$ -3

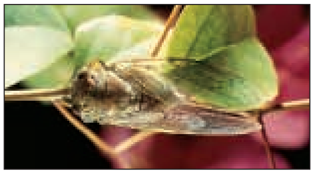
★ **44.** $\log_4 16^x$ $2x$

WORLD RECORDS **For Exercises 45 and 46, use the information given for Exercises 18–20 to find the relative intensity of each sound.** **Source:** *The Guinness Book of Records*

45. The loudest animal sounds are the low-frequency pulses made by blue whales when they communicate. These pulses have been measured up to 188 decibels. $10^{18.8}$

46. The loudest insect is the African cicada. It produces a calling song that measures 106.7 decibels at a distance of 50 centimeters. $10^{10.67}$





Solve each equation or inequality. Check your solutions.

47. $\log_9 x = 2$ 81

48. $\log_2 c > 8$ $c > 256$

49. $\log_{64} y \leq \frac{1}{2}$ $0 < y \leq 8$

50. $\log_{25} n = \frac{3}{2}$ 125

51. $\log_{\frac{1}{7}} x = -1$ 7

52. $\log_{\frac{1}{3}} p < 0$ $0 < p < 1$

53. $\log_2 (3x - 8) \geq 6$ $x \geq 24$

54. $\log_{10} (x^2 + 1) = 1$ ± 3

55. $\log_b 64 = 3$ 4

56. $\log_b 121 = 2$ 11

57. $\log_5 5^{6n+1} = 13$ 2

58. $\log_5 x = \frac{1}{2}$ $\sqrt{5}$

59. $\log_6 (2x - 3) = \log_6 (x + 2)$ 5

60. $\log_2 (4y - 10) \geq \log_2 (y - 1)$ $y \geq 3$

★ **61.** $\log_{10} (a^2 - 6) > \log_{10} a$ $a > 3$

★ **62.** $\log_7 (x^2 + 36) = \log_7 100$ ± 8

Show that each statement is true. 63–65. See margin.

★ **63.** $\log_5 25 = 2 \log_5 5$

★ **64.** $\log_{16} 2 \cdot \log_2 16 = 1$

★ **65.** $\log_7 [\log_3 (\log_2 8)] = 0$

Answers

63. $\log_5 25 \stackrel{?}{=} 2 \log_5 5$	Original equation
$\log_5 5^2 \stackrel{?}{=} 2 \log_5 5^1$	$25 = 5^2$ and $5 = 5^1$
$2 \stackrel{?}{=} 2(1)$	Inverse Property of Exponents and Logarithms
$2 = 2$ ✓	Simplify.
64. $\log_{16} 2 \cdot \log_2 16 \stackrel{?}{=} 1$	Original equation
$\log_{16} 16^{\frac{1}{4}} \cdot \log_2 2^4 \stackrel{?}{=} 1$	$2 = 16^{\frac{1}{4}}$ and $16 = 2^4$
$\frac{1}{4}(4) \stackrel{?}{=} 1$	Inverse Property of Exponents and Logarithms
$1 = 1$ ✓	

65. $\log_7 [\log_3 (\log_2 8)] \stackrel{?}{=} 0$	Original equation
$\log_7 [\log_3 (\log_2 2^3)] \stackrel{?}{=} 0$	$8 = 2^3$
$\log_7 (\log_3 3) \stackrel{?}{=} 0$	Inverse Property of Exponents and Logarithms
$\log_7 (\log_3 3^1) \stackrel{?}{=} 0$	$3 = 3^1$
$\log_7 1 \stackrel{?}{=} 0$	Inverse Property of Exponents and Logarithms
$\log_7 7^0 \stackrel{?}{=} 0$	$1 = 7^0$
$0 = 0$ ✓	Inverse Property of Exponents and Logarithms

66–67. See pp. 573A–573D.

66. a. Sketch the graphs of $y = \log_{\frac{1}{2}} x$ and $y = \left(\frac{1}{2}\right)^x$ on the same axes.

b. Describe the relationship between the graphs.

★ 67. a. Sketch the graphs of $y = \log_2 x + 3$, $y = \log_2 x - 4$, $y = \log_2 (x - 1)$, and $y = \log_2 (x + 2)$.

b. Describe this family of graphs in terms of its parent graph $y = \log_2 x$.

EARTHQUAKE **For Exercises 68 and 69, use the following information.** The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$, where x represents the amplitude of the seismic wave causing ground motion.

68. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 7 as an aftershock with a Richter scale rating of 4? 10^3 or 1000 as times great

69. How many times as great was the motion caused by the 1906 San Francisco earthquake that measured 8.3 on the Richter scale as that caused by the 2001 Bhuj, India, earthquake that measured 6.9? $10^{1.4}$ or about 25 times as great

More About. . .

Earthquake

The Loma Prieta earthquake measured 7.1 on the Richter scale and interrupted the 1989 World Series in San Francisco.

Source: U.S. Geological Survey

70. **NOISE ORDINANCE** A proposed city ordinance will make it illegal to create sound in a residential area that exceeds 72 decibels during the day and 55 decibels during the night. How many times more intense is the noise level allowed during the day than at night? $10^{1.7}$ or about 50 times

71. **CRITICAL THINKING** The value of $\log_2 5$ is between two consecutive integers. Name these integers and explain how you determined them. 2 and 3; Sample answer: 5 is between 2^2 and 2^3.

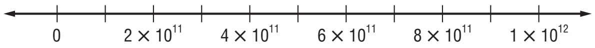
72. **CRITICAL THINKING** Using the definition of a logarithmic function where $y = \log_b x$, explain why the base b cannot equal 1. All powers of 1 are 1, so the inverse of $y = 1^x$ is not a function.

73. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 573A–573D.

Why is a logarithmic scale used to measure sound?

Include the following in your answer:

- the relative intensities of a pin drop, a whisper, normal conversation, kitchen noise, and a jet engine written in scientific notation,
- a plot of each of these relative intensities on the scale shown below, and



- an explanation as to why the logarithmic scale might be preferred over the scale shown above.

Standardized Test Practice

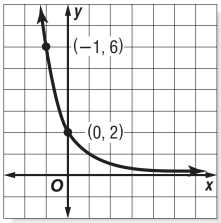
74. What is the equation of the function graphed at the right? B

Ⓐ $y = 2(3)^x$

Ⓑ $y = 2\left(\frac{1}{3}\right)^x$

Ⓒ $y = 3\left(\frac{1}{2}\right)^x$

Ⓓ $y = 3(2)^x$



www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 579 (shown) and p. 580

Logarithmic Functions and Expressions

Definition of Logarithm with Base b	Let b and x be positive numbers, $b \neq 1$. The logarithm of x with base b is denoted $\log_b x$ and is defined as the exponent y that makes the equation $b^y = x$ true.

The inverse of the exponential function $y = b^x$ is the **logarithmic function** $x = b^y$. This function is usually written as $y = \log_b x$.

Properties of Logarithmic Functions	1. The function is continuous and one-to-one. 2. The domain is the set of all positive real numbers. 3. The y-axis is an asymptote of the graph. 4. The range is the set of all real numbers. 5. The graph contains the point (0, 1).

Example 1 **Write an exponential equation equivalent to $\log_3 243 = 5$.**

$3^5 = 243$

Example 2 **Write a logarithmic equation equivalent to $6^{-3} = \frac{1}{216}$.**

$\log_6 \frac{1}{216} = -3$

Example 3 **Evaluate $\log_8 16$.**

$8^{\frac{4}{3}} = 16$, so $\log_8 16 = \frac{4}{3}$.

Exercises

Write each equation in logarithmic form.

1. $2^7 = 128$ $\log_2 128 = 7$
2. $3^{-4} = \frac{1}{81}$ $\log_3 \frac{1}{81} = -4$
3. $\left(\frac{1}{7}\right)^3 = \frac{1}{343}$ $\log_{\frac{1}{7}} \frac{1}{343} = 3$

Write each equation in exponential form.

4. $\log_{15} 225 = 2$ $15^2 = 225$
5. $\log_3 \frac{1}{27} = -3$ $3^{-3} = \frac{1}{27}$
6. $\log_4 32 = \frac{5}{2}$ $4^{\frac{5}{2}} = 32$

Evaluate each expression.

7. $\log_4 64$ 3
8. $\log_2 64$ 6
9. $\log_{100} 100{,}000$ 2.5
10. $\log_5 625$ 4
11. $\log_{27} 81$ $\frac{4}{3}$
12. $\log_{25} 5$ $\frac{1}{2}$
13. $\log_2 \frac{1}{128}$ −7
14. $\log_{10} 0.00001$ −5
15. $\log_4 \frac{1}{32}$ −2.5

Skills Practice, p. 581 and Practice, p. 582 (shown)

Write each equation in logarithmic form.

1. $5^3 = 125$ $\log_5 125 = 3$
2. $7^0 = 1$ $\log_7 1 = 0$
3. $3^4 = 81$ $\log_3 81 = 4$
4. $3^{-4} = \frac{1}{81}$ $\log_3 \frac{1}{81} = -4$
5. $\left(\frac{1}{4}\right)^3 = \frac{1}{64}$ $\log_{\frac{1}{4}} \frac{1}{64} = 3$
6. $7776^{\frac{1}{5}} = 6$ $\log_{7776} 6 = \frac{1}{5}$

Write each equation in exponential form.

7. $\log_6 216 = 3$ $6^3 = 216$
8. $\log_2 64 = 6$ $2^6 = 64$
9. $\log_3 \frac{1}{81} = -4$ $3^{-4} = \frac{1}{81}$
10. $\log_{10} 0.00001 = -5$ $10^{-5} = 0.00001$
11. $\log_{25} 5 = \frac{1}{2}$ $25^{\frac{1}{2}} = 5$
12. $\log_{32} 8 = \frac{3}{5}$ $32^{\frac{3}{5}} = 8$

Evaluate each expression.

13. $\log_3 81$ 4
14. $\log_{10} 0.0001$ −4
15. $\log_2 \frac{1}{16}$ −4
16. $\log_{\frac{1}{3}} 27$ −3
17. $\log_9 1$ 0
18. $\log_8 4$ $\frac{2}{3}$
19. $\log_7 \frac{1}{49}$ −2
20. $\log_6 6^4$ 4
21. $\log_3 \frac{1}{3}$ −1
22. $\log_4 \frac{1}{256}$ −4
23. $\log_9 9^{(n + 1)}$ $n + 1$
24. $2^{\log_2 32}$ 32

Solve each equation or inequality. Check your solutions.

25. $\log_{10} n = -3$ $\frac{1}{1000}$
26. $\log_4 x > 3$ $x > 64$
27. $\log_4 x = \frac{3}{2}$ 8
28. $\log_{\frac{1}{5}} x = -3$ 125
29. $\log_7 q < 0$ $0 < q < 1$
30. $\log_6 (2y + 8) \geq 2$ $y \geq 14$
31. $\log_y 16 = -4$ $\frac{1}{2}$
32. $\log_n \frac{1}{8} = -3$ 2
33. $\log_b 1024 = 5$ 4
34. $\log_8 (3x + 7) < \log_8 (7x + 4)$ $x > \frac{3}{4}$
35. $\log_7 (8x + 20) = \log_7 (x + 6)$ −2
36. $\log_3 (x^2 - 2) = \log_3 x$ 2

37. **SOUND** Sounds that reach levels of 130 decibels or more are painful to humans. What is the relative intensity of 130 decibels? 10^{13}

38. **INVESTING** Maria invests $1000 in a savings account that pays 8% interest compounded annually. The value of the account A at the end of five years can be determined from the equation $\log A = \log[1000(1 + 0.08)^5]$. Find the value of A to the nearest dollar. $1469

Reading to Learn Mathematics, p. 583

ELL

Pre-Activity **Why is a logarithmic scale used to measure sound?**

Read the introduction to Lesson 10-2 at the top of page 531 in your textbook.

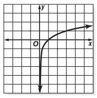
How many times louder than a whisper is normal conversation? 10^4 or 10,000 times

Reading the Lesson

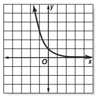
1. a. Write an exponential equation that is equivalent to $\log_3 81 = 4$. $3^4 = 81$
   b. Write a logarithmic equation that is equivalent to $25^{-\frac{1}{2}} = \frac{1}{5}$. $\log_{25} \frac{1}{5} = -\frac{1}{2}$
   c. Write an exponential equation that is equivalent to $\log_4 1 = 0$. $4^0 = 1$
   d. Write a logarithmic equation that is equivalent to $10^{-3} = 0.001$. $\log_{10} 0.001 = -3$
   e. What is the inverse of the function $y = 5^x$? $y = \log_5 x$
   f. What is the inverse of the function $y = \log_{10} x$? $y = 10^x$

2. Match each function with its graph.
   a. $y = 3^x$ IV
   b. $y = \log_3 x$ I
   c. $y = \left(\frac{1}{3}\right)^x$ II

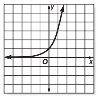
I.



II.



III.



3. Indicate whether each of the following statements about the exponential function $y = \log_5 x$ is *true* or *false*.
   a. The y-axis is an asymptote of the graph. true
   b. The domain is the set of all real numbers. false
   c. The graph contains the point (5, 0). false
   d. The range is the set of all real numbers. true
   e. The y-intercept is 1. false

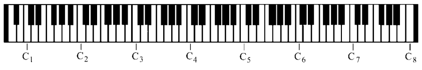
Helping You Remember

4. An important skill needed for working with logarithms is changing an equation between logarithmic and exponential forms. Using the words *base*, *exponent*, and *logarithm*, describe an easy way to remember and apply the part of the definition of logarithm that says, "$\log_b x = y$ if and only if $b^y = x$." Sample answer: In these equations, *b* stands for *base*. In log form, *b* is the subscript, and in exponential form, *b* is the number that is raised to a power. A *logarithm* is an *exponent*, so *y*, which is the log in the first equation, becomes the exponent in the second equation.

Enrichment, p. 584

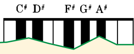
Musical Relationships

The frequencies of notes in a musical scale that are one octave apart are related by an exponential equation. For the eight C notes on a piano, the equation is $C_n = C_1 2^{n-1}$, where C_n represents the frequency of note C_n.



1. Find the relationship between C_1 and C_2. $C_2 = 2C_1$
2. Find the relationship between C_1 and C_4. $C_4 = 8C_1$

The frequencies of consecutive notes are related by a common ratio r. The general equation is $f_n = f_1 r^{n-1}$.



3. If the fre

4 Assess

Open-Ended Assessment

Writing Have students write a step-by-step explanation of the procedure for solving a logarithmic equation such as $\log_8 n = \frac{7}{3}$.

Getting Ready for Lesson 10-3

PREREQUISITE SKILL In Lesson 10-3, students will evaluate expressions using the properties of logarithms. Because these properties are related to exponential properties, students must be familiar with exponential properties when multiplying or dividing terms with like bases. Use Exercises 85–90 to determine your students' familiarity with multiplying and dividing monomials.

Assessment Options

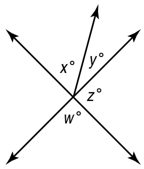
Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 10-1 and 10-2. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 10-1 and 10-2) is available on p. 623 of the *Chapter 10 Resource Masters*.

75. In the figure at the right, if $y = \frac{2}{7}x$ and $z = 3w$, then $x =$ D

Ⓐ 14. Ⓑ 20.

Ⓒ 28. Ⓓ 35.



Maintain Your Skills

Mixed Review

Simplify each expression. *(Lesson 10-1)*

76. $x^{\sqrt{6}} \cdot x^{\sqrt{6}}$ $x^{2\sqrt{6}}$

77. $\left(b^{\sqrt{6}}\right)^{\sqrt{24}}$ b^{12}

Solve each equation. Check your solutions. *(Lesson 9-6)* **79.** $-3, \frac{14}{5}$

78. $\frac{2x+1}{x} - \frac{x+1}{x-4} = \frac{-20}{x^2-4x}$ $\varnothing$

79. $\frac{2a-5}{a-9} - \frac{a-3}{3a+2} = \frac{5}{3a^2-25a-18}$

Solve each equation by using the method of your choice. Find exact solutions. *(Lesson 6-5)*

80. $9y^2 = 49$ $\pm\frac{7}{3}$

81. $2p^2 = 5p + 6$ $\frac{5 \pm \sqrt{73}}{4}$

Simplify each expression. *(Lesson 9-2)* **83.** $\frac{6x-58}{(x-3)(x+3)(x+7)}$

82. $\frac{3}{2y} + \frac{4}{3y} - \frac{7}{5y}$ $\frac{43}{30y}$

83. $\frac{x-7}{x^2-9} - \frac{x-3}{x^2+10x+21}$

84. BANKING Donna Bowers has $4000 she wants to save in the bank. A certificate of deposit (CD) earns 8% annual interest, while a regular savings account earns 3% annual interest. Ms. Bowers doesn't want to tie up all her money in a CD, but she has decided she wants to earn $240 in interest for the year. How much money should she put in to each type of account? (*Hint:* Use Cramer's Rule.) *(Lesson 4-4)* **$2400, CD; $1600, savings**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify. Assume that no variable equals zero.**
*(To review **multiplying and dividing monomials**, see Lesson 5-1.)*

85. $x^4 \cdot x^6$ x^{10}

86. $(y^3)^8$ y^{24}

87. $(2a^2b)^3$ $8a^6b^3$

88. $\frac{a^4n^7}{a^3n}$ an^6

89. $\frac{x^5yz^2}{x^2y^3z^5}$ $\frac{x^3}{y^2z^3}$

90. $\left(\frac{b^7}{a^4}\right)^0$ 1

Practice Quiz 1

Lessons 10-1 and 10-2

1. Determine whether $5(1.2)^x$ represents exponential *growth* or *decay*. *(Lesson 10-1)* **growth**

2. Write an exponential function whose graph passes through (0, 2) and (2, 32). $y = 2(4)^x$

3. Write an equivalent logarithmic equation for $4^6 = 4096$. *(Lesson 10-2)* $\log_4 4096 = 6$

4. Write an equivalent exponential equation for $\log_9 27 = \frac{3}{2}$. *(Lesson 10-2)* $9^{\frac{3}{2}} = 27$

Evaluate each expression. *(Lesson 10-2)*

5. $\log_8 16$ $\frac{4}{3}$

6. $\log_4 4^{15}$ 15

Solve each equation or inequality. Check your solutions. *(Lessons 10-1 and 10-2)*

7. $3^{4x} = 3^{3-x}$ $\frac{3}{5}$

8. $3^{2n} \le \frac{1}{9}$ $n \le -1$

9. $\log_2 (x + 6) > 5$ $x > 26$

10. $\log_5 (4x - 1) = \log_5 (3x + 2)$ 3

Graphing Calculator Investigation

A Follow-Up of Lesson 10-2

Modeling Real-World Data: Curve Fitting

We are often confronted with data for which we need to find an equation that best fits the information. We can find exponential and logarithmic functions of best fit using a TI-83 Plus graphing calculator.

Example

The population per square mile in the United States has changed dramatically over a period of years. The table shows the number of people per square mile for several years.

U.S. Population Density

Year	People per square mile	Year	People per square mile
1790	4.5	1900	21.5
1800	6.1	1910	26.0
1810	4.3	1920	29.9
1820	5.5	1930	34.7
1830	7.4	1940	37.2
1840	9.8	1950	42.6
1850	7.9	1960	50.6
1860	10.6	1970	57.5
1870	10.9	1980	64.0
1880	14.2	1990	70.3
1890	17.8	2000	80.0

Source: Northeast-Midwest Institute

a. Use a graphing calculator to enter the data and draw a scatter plot that shows how the number of people per square mile is related to the year.

Step 1 Enter the year into L1 and the people per square mile into L2.

KEYSTROKES: *See pages 87 and 88 to review how to enter lists.*

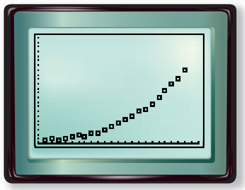
Be sure to clear the Y= list. Use the ▶ key to move the cursor from L1 to L2.

Step 2 Draw the scatter plot.

KEYSTROKES: *See pages 87 and 88 to review how to graph a scatter plot.*

Make sure that Plot 1 is on, the scatter plot is chosen, Xlist is L1, and Ylist is L2. Use the viewing window [1780, 2020] with a scale factor of 10 by [0, 115] with a scale factor of 5.

We see from the graph that the equation that best fits the data is a curve. Based on the shape of the curve, try an exponential model.



[1780, 2020] scl: 10 by [0, 115] scl: 5

Step 3 To determine the exponential equation that best fits the data, use the exponential regression feature of the calculator.

KEYSTROKES: STAT ▶ 0 2nd [L1] , 2nd [L2] ENTER

The equation is $y = 1.835122 \times 10^{-11}(1.014700091)^x$.

(continued on the next page)

www.algebra2.com/other_calculator_keystrokes

Graphing Calculator Investigation

A Follow-Up of Lesson 10-2

Getting Started

Turning Off Stat Plots Before Step 1, students should use the keystrokes 2nd [STAT PLOT] and check that both plot 2 and plot 3 are turned off.

Diagnostics Display Students should have the calculator set to DiagnosticOn. To set the calculator for diagnostics, use 2nd [CATALOG], move the cursor down to DiagnosticOn, and press ENTER twice.

Teach

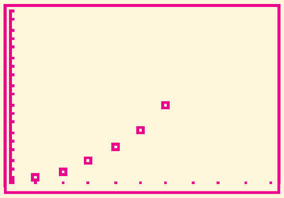
- When students begin the exercises, they should clear lists L1 and L2. They should also enter appropriate settings for the graphing window.
- Point out that the table of data is arranged in two "double" columns.
- Suggest that students compare their graphs to the one shown.
- Have students estimate the population density in 2010 and 2050. How soon will the population density be twice what it was in 2000? **about 2040**
- If you have time, consider extending this activity into a discussion of how life in the future will be different as the result of the increasing population density. Ask students to think about the effect on transportation, housing, crime rates, and so on. You may wish to team-teach with a social studies teacher.

Assess

In **Exercise 2**, make sure students can explain why their equation of best fit is a good choice. In **Exercise 3**, students' answers may vary slightly. When you discuss **Exercise 6**, you may want to ask for any ideas students have about how to use the calculator to judge the relative merits of various models (quadratic, cubic, quartic, and exponential).

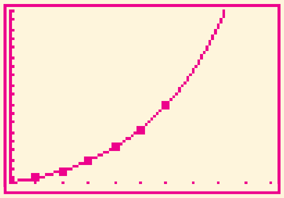
Answers

1.



[0, 50] scl: 5 by [30, 400] scl: 20

2.



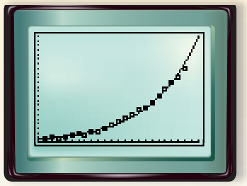
[0, 50] scl: 5 by [30, 400] scl: 20

Graphing Calculator Investigation

The calculator also reports an r value of 0.991887235. Recall that this number is a correlation coefficient that indicates how well the equation fits the data. A perfect fit would be $r = 1$. Therefore, we can conclude that this equation is a pretty good fit for the data.

To check this equation visually, overlap the graph of the equation with the scatter plot.

KEYSTROKES: Y= VARS 5 ▶ ▶ 1 GRAPH

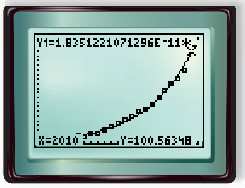


[1780, 2020] scl: 10 by [0, 115] scl: 5

b. If this trend continues, what will be the population per square mile in 2010?

To determine the population per square mile in 2010, from the graphics screen, find the value of y when $x = 2010$.

KEYSTROKES: 2nd [CALC] 1 2010 ENTER



```
Y1=1.8351221071296E-11*
X=2010          Y=100.56348
```

[1780, 2020] scl: 10 by [0, 115] scl: 5

The calculator returns a value of approximately 100.6. If this trend continues, in 2010, there will be approximately 100.6 people per square mile.

Exercises

In 1985, Erika received $30 from her aunt and uncle for her seventh birthday. Her father deposited it into a bank account for her. Both Erika and her father forgot about the money and made no further deposits or withdrawals. The table shows the account balance for several years.

Elapsed Time (years)	Balance
0	$30.00
5	$41.10
10	$56.31
15	$77.16
20	$105.71
25	$144.83
30	$198.43

1. Use a graphing calculator to draw a scatter plot for the data. **See margin.**
2. Calculate and graph the curve of best fit that shows how the elapsed time is related to the balance. Use ExpReg for this exercise. **See margin.**
3. Write the equation of best fit. $y = 29.99908551(1.06500135)^x$
4. Write a sentence that describes the fit of the graph to the data. **This equation is a good fit because $r \approx 1$.**
5. Based on the graph, estimate the balance in 41 years. Check this using the CALC value. **After 41 years she will have approximately $397.**
6. Do you think there are any other types of equations that would be good models for these data? Why or why not? **A quadratic equation might be a good model for this example because the shape is close to a portion of a parabola.**

10-3 Properties of Logarithms

What You'll Learn

- Simplify and evaluate expressions using the properties of logarithms.
- Solve logarithmic equations using the properties of logarithms.

How are the properties of exponents and logarithms related?

In Lesson 5-1, you learned that the product of powers is the sum of their exponents.

$$9 \cdot 81 = 3^2 \cdot 3^4 \text{ or } 3^{2+4}$$

In Lesson 10-2, you learned that logarithms *are* exponents, so you might expect that a similar property applies to logarithms. Let's consider a specific case. Does $\log_3 (9 \cdot 81) = \log_3 9 + \log_3 81$?

$$\begin{aligned} \log_3 (9 \cdot 81) &= \log_3 (3^2 \cdot 3^4) && \text{Replace 9 with } 3^2 \text{ and 81 with } 3^4. \\ &= \log_3 3^{(2+4)} && \text{Product of Powers} \\ &= 2 + 4 \text{ or } 6 && \text{Inverse property of exponents and logarithms} \end{aligned}$$

$$\begin{aligned} \log_3 9 + \log_3 81 &= \log_3 3^2 + \log_3 3^4 && \text{Replace 9 with } 3^2 \text{ and 81 with } 3^4. \\ &= 2 + 4 \text{ or } 6 && \text{Inverse property of exponents and logarithms} \end{aligned}$$

So, $\log_3 (9 \cdot 81) = \log_3 9 + \log_3 81$.

PROPERTIES OF LOGARITHMS Since logarithms are exponents, the properties of logarithms can be derived from the properties of exponents. The example above and other similar examples suggest the following property of logarithms.

Key Concept *Product Property of Logarithms*

- **Words** The logarithm of a product is the sum of the logarithms of its factors.
- **Symbols** For all positive numbers m, n, and b, where $b \neq 1$, $\log_b mn = \log_b m + \log_b n$.
- **Example** $\log_3 (4)(7) = \log_3 4 + \log_3 7$

To show that this property is true, let $b^x = m$ and $b^y = n$. Then, using the definition of logarithm, $x = \log_b m$ and $y = \log_b n$.

$$\begin{aligned} b^x b^y &= mn && \\ b^{x+y} &= mn && \text{Product of Powers} \\ \log_b b^{x+y} &= \log_b mn && \text{Property of Equality for Logarithmic Functions} \\ x + y &= \log_b mn && \text{Inverse Property of Exponents and Logarithms} \\ \log_b m + \log_b n &= \log_b mn && \text{Replace } x \text{ with } \log_b m \text{ and } y \text{ with } \log_b n. \end{aligned}$$

You can use the Product Property of Logarithms to approximate logarithmic expressions.

10-3 Lesson Notes

1 Focus

5-Minute Check Transparency 10-3 Use as a quiz or review of Lesson 10-2.

Mathematical Background notes are available for this lesson on p. 520D.

How are the properties of exponents and logarithms related?

Ask students:

- How do you know that logarithms are exponents? **Sample answer: The logarithm of a number is equal to the power (or exponent) when the number is rewritten in exponential form.**
- Since $\log_3 (9 \cdot 81) = \log_3 729$, how could $\log_3 729$ have been used in the justification that $\log_3 (9 \cdot 81) = \log_3 9 + \log_3 81$? **After stating that $\log_3 (9 \cdot 81) = \log_3 729$, then the statements $\log_3 729 = \log_3 3^6$ and $\log_3 3^6 = 6$ could be used to justify that $\log_3 (9 \cdot 81) = 6$.**

Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters

- Study Guide and Intervention, pp. 585–586
- Skills Practice, p. 587
- Practice, p. 588
- Reading to Learn Mathematics, p. 589
- Enrichment, p. 590
- Assessment, pp. 623, 625

Transparencies

5-Minute Check Transparency 10-3
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

PROPERTIES OF LOGARITHMS

In-Class Examples

Power Point®

Teaching Tip When discussing the Product Property of Logarithms, point out that the logarithms used in the example ($\log_3 (4)(7) = \log_3 4 + \log_3 7$) show the property applies to all logarithms and not just those that can be simplified. Be sure students did not get this impression from the earlier example where it was shown that $\log_3 (9 \cdot 81) = \log_3 9 + \log_3 81$.

1 Use $\log_5 2 \approx 0.4307$ to approximate the value of $\log_5 250$. **3.4307**

Teaching Tip Some students may wonder how the approximation for $\log_2 3$ was determined since on most calculators the log button calculates only logarithms of base 10. State that $\log_2 3 = \frac{\log_{10} 3}{\log_{10} 2}$, which can be evaluated using a calculator. Stress that this procedure will be formally discussed in Lesson 10-4.

2 Use $\log_6 8 \approx 1.1606$ and $\log_6 32 \approx 1.9343$ to approximate the value of $\log_6 4$. **0.7737**

3 **SOUND** The sound made by a lawnmower has a relative intensity of 10^9 or 90 decibels. Would the sound of ten lawnmowers running at that same intensity be ten times as loud or 900 decibels? Explain your reasoning. **No; the sound of ten lawnmowers is perceived to be only 10 decibels louder than the sound of one lawnmower, or 100 decibels.**

TEACHING TIP

The value of R in Example 3 is determined by finding the ratio of the intensity I of the sound in watts per square meter to the intensity I_0 of 10^{-12} watts per square meter. The intensity I_0 corresponds to the threshold of hearing. Thus a formula that relates the intensity of a sound in watts per square meter to its loudness in decibels is $L = 10 \log_{10} \frac{I}{I_0}$.

Career Choices

Sound Technician

Sound technicians produce movie sound tracks in motion picture production studios, control the sound of live events such as concerts, or record music in a recording studio.

Online Research For information about a career as a sound technician, visit: www.algebra2.com/careers

Example 1 Use the Product Property

Use $\log_2 3 \approx 1.5850$ to approximate the value of $\log_2 48$.

$\log_2 48 = \log_2 (2^4 \cdot 3)$ Replace 48 with $16 \cdot 3$ or $2^4 \cdot 3$.

$= \log_2 2^4 + \log_2 3$ Product Property

$= 4 + \log_2 3$ Inverse Property of Exponents and Logarithms

$\approx 4 + 1.5850$ or 5.5850 Replace $\log_2 3$ with 1.5850.

Thus, $\log_2 48$ is approximately 5.5850.

Recall that the quotient of powers is found by subtracting exponents. The property for the logarithm of a quotient is similar.

Key Concept — Quotient Property of Logarithms

- **Words** The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
- **Symbols** For all positive numbers m, n, and b, where $b \neq 1$, $\log_b \frac{m}{n} = \log_b m - \log_b n$.

You will show that this property is true in Exercise 47.

Example 2 Use the Quotient Property

Use $\log_3 5 \approx 1.4650$ and $\log_3 20 \approx 2.7268$ to approximate $\log_3 4$.

$\log_3 4 = \log_3 \frac{20}{5}$ Replace 4 with the quotient $\frac{20}{5}$.

$= \log_3 20 - \log_3 5$ Quotient Property

$\approx 2.7268 - 1.4650$ or 1.2618 $\log_3 20 = 2.7268$ and $\log_3 5 = 1.4650$

Thus, $\log_3 4$ is approximately 1.2618.

CHECK Using the definition of logarithm and a calculator, $3^{1.2618} \approx 4$. ✓

Example 3 Use Properties of Logarithms

SOUND **The loudness L of a sound in decibels is given by $L = 10 \log_{10} R$, where R is the sound's relative intensity. Suppose one person talks with a relative intensity of 10^6 or 60 decibels. Would the sound of ten people each talking at that same intensity be ten times as loud or 600 decibels? Explain your reasoning.**

Let L_1 be the loudness of one person talking. → $L_1 = 10 \log_{10} 10^6$

Let L_2 be the loudness of ten people talking. → $L_2 = 10 \log_{10} (10 \cdot 10^6)$

Then the increase in loudness is $L_2 - L_1$.

$L_2 - L_1 = 10 \log_{10} (10 \cdot 10^6) - 10 \log_{10} 10^6$ Substitute for L_1 and L_2.

$= 10(\log_{10} 10 + \log_{10} 10^6) - 10 \log_{10} 10^6$ Product Property

$= 10 \log_{10} 10 + 10 \log_{10} 10^6 - 10 \log_{10} 10^6$ Distributive Property

$= 10 \log_{10} 10$ Subtract.

$= 10(1)$ or 10 Inverse Property of Exponents and Logarithms

The sound of two people talking is perceived by the human ear to be only about 10 decibels louder than the sound of one person talking, or 70 decibels.

DAILY INTERVENTION — Unlocking Misconceptions

Power Property After you have discussed the Power Property of Logarithms on p. 543, clarify that the property works for logarithms because they are equivalent to exponents. Stress that students should not read a statement such as $\log_2 5^3 = 3 \log_2 5$ and conclude that $5^3 = 3 \times 5$.

Recall that the power of a power is found by multiplying exponents. The property for the logarithm of a power is similar.

Key Concept — *Power Property of Logarithms*

- **Words** The logarithm of a power is the product of the logarithm and the exponent.
- **Symbols** For any real number p and positive numbers m and b, where $b \neq 1$, $\log_b m^p = p \log_b m$.

You will show that this property is true in Exercise 50.

Example 4 *Power Property of Logarithms*

Given $\log_4 6 \approx 1.2925$, approximate the value of $\log_4 36$.

$\log_4 36 = \log_4 6^2$	Replace 36 with 6^2.
$= 2 \log_4 6$	Power Property
$\approx 2(1.2925)$ or 2.585	Replace $\log_4 6$ with 1.2925.

SOLVE LOGARITHMIC EQUATIONS You can use the properties of logarithms to solve equations involving logarithms.

Example 5 *Solve Equations Using Properties of Logarithms*

Solve each equation.

a. $3 \log_5 x - \log_5 4 = \log_5 16$

$3 \log_5 x - \log_5 4 = \log_5 16$	Original equation
$\log_5 x^3 - \log_5 4 = \log_5 16$	Power Property
$\log_5 \frac{x^3}{4} = \log_5 16$	Quotient Property
$\frac{x^3}{4} = 16$	Property of Equality for Logarithmic Functions
$x^3 = 64$	Multiply each side by 4.
$x = 4$	Take the cube root of each side.

The solution is 4.

b. $\log_4 x + \log_4 (x - 6) = 2$

$\log_4 x + \log_4 (x - 6) = 2$	Original equation
$\log_4 x(x - 6) = 2$	Product Property
$x(x - 6) = 4^2$	Definition of logarithm
$x^2 - 6x - 16 = 0$	Subtract 16 from each side.
$(x - 8)(x + 2) = 0$	Factor.
$x - 8 = 0$ or $x + 2 = 0$	Zero Product Property
$x = 8$ $\quad x = -2$	Solve each equation.

CHECK Substitute each value into the original equation.

$\log_4 8 + \log_4 (8 - 6) \stackrel{?}{=} 2$

$\log_4 8 + \log_4 2 \stackrel{?}{=} 2$

$\log_4 (8 \cdot 2) \stackrel{?}{=} 2$

$\log_4 16 \stackrel{?}{=} 2$

$2 = 2 \checkmark$

$\log_4 (-2) + \log_4 (-2 - 6) \stackrel{?}{=} 2$

$\log_4 (-2) + \log_4 (-8) \stackrel{?}{=} 2$

Since $\log_4 (-2)$ and $\log_4 (-8)$ are undefined, -2 is an extraneous solution and must be eliminated.

The only solution is 8.

www.algebra2.com/extra_examples

In-Class Example

Power Point®

4 Given $\log_5 6 \approx 1.1133$, approximate the value of $\log_5 216$. **3.3399**

SOLVE LOGARITHMIC EQUATIONS

In-Class Example

Power Point®

5 Solve each equation.

a. $4 \log_2 x - \log_2 5 = \log_2 125$ **5**

b. $\log_8 x + \log_8 (x - 12) = 2$ **16**

Study Tip

Checking Solutions

It is wise to check all solutions to see if they are valid since the domain of a logarithmic function is not the complete set of real numbers.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Right after discussing Example 5, have pairs of students rework both parts of the example together without looking at the solution in the text. Have the partners take turns explaining the solution steps to each other.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- summarize the properties of logarithms they learned in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR When discussing the error made by Clemente, remind students that logarithms are exponents. Adding $\log_7 6 + \log_7 3$ as $\log_7 (6 + 3)$ is similar to saying that $x^2 + x^3 = x^{2+3}$ or x^5, which students should recognize as being untrue because x^2 and x^3 are unlike terms.

About the Exercises...

Organization by Objective

- **Properties of Logarithms:** 13–20, 37–46
- **Solve Logarithmic Equations:** 21–34

Odd/Even Assignments

Exercises 13–34 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 46 involves research on the Internet or other reference materials.

Assignment Guide

Basic: 13–17 odd, 21–31 odd, 35–40, 47–66

Average: 13–33 odd, 35–43, 47–66

Advanced: 14–34 even, 35, 36, 41–62 (optional: 63–66)

Check for Understanding

Concept Check

1. properties of exponents

2. Sample answer: $2 \log_3 x + \log_3 5$; $\log_3 5x^2$

1. **Name** the properties that are used to derive the properties of logarithms.
2. **OPEN ENDED** Write an expression that can be simplified by using two or more properties of logarithms. Then simplify it.
3. **FIND THE ERROR** Umeko and Clemente are simplifying $\log_7 6 + \log_7 3 - \log_7 2$.

Umeko

$\log_7 6 + \log_7 3 - \log_7 2$

$= \log_7 18 - \log_7 2$

$= \log_7 9$

Clemente

$\log_7 6 + \log_7 3 - \log_7 2$

$= \log_7 9 - \log_7 2$

$= \log_7 7$ or 1

Who is correct? Explain your reasoning. **Umeko; see margin for explanation.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	1, 2, 4
7–10	5
11, 12	3

Use $\log_3 2 \approx 0.6310$ and $\log_3 7 \approx 1.7712$ to approximate the value of each expression.

4. $\log_3 \frac{7}{2}$ **1.1402**
5. $\log_3 18$ **2.6310**
6. $\log_3 \frac{2}{3}$ **−0.3690**

Solve each equation. Check your solutions.

7. $\log_3 42 - \log_3 n = \log_3 7$ **6**
8. $\log_2 3x + \log_2 5 = \log_2 30$ **2**
9. $2 \log_5 x = \log_5 9$ **3**
10. $\log_{10} a + \log_{10} (a + 21) = 2$ **4**

Application

MEDICINE **For Exercises 11 and 12, use the following information.**
The pH of a person's blood is given by $\text{pH} = 6.1 + \log_{10} B - \log_{10} C$, where B is the concentration of bicarbonate, which is a base, in the blood and C is the concentration of carbonic acid in the blood. **11. $\text{pH} = 6.1 + \log_{10} \frac{B}{C}$**

11. Use the Quotient Property of Logarithms to simplify the formula for blood pH.
12. Most people have a blood pH of 7.4. What is the approximate ratio of bicarbonate to carbonic acid for blood with this pH? **20:1**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
13–20	1, 2, 4
21–34	5
37–45	3

Extra Practice

See page 850.

Use $\log_5 2 \approx 0.4307$ and $\log_5 3 \approx 0.6826$ to approximate the value of each expression.

13. $\log_5 9$ **1.3652**
14. $\log_5 8$ **1.2921**
15. $\log_5 \frac{2}{3}$ **−0.2519**
16. $\log_5 \frac{3}{2}$ **0.2519**
17. $\log_5 50$ **2.4307**
18. $\log_5 30$ **2.1133**
★19. $\log_5 0.5$ **−0.4307**
★20. $\log_5 \frac{10}{9}$ **0.0655**

Solve each equation. Check your solutions.

21. $\log_3 5 + \log_3 x = \log_3 10$ **2**
22. $\log_4 a + \log_4 9 = \log_4 27$ **3**
23. $\log_{10} 16 - \log_{10} 2t = \log_{10} 2$ **4**
24. $\log_7 24 - \log_7 (y + 5) = \log_7 8$ **−2**
25. $\log_2 n = \frac{1}{4} \log_2 16 + \frac{1}{2} \log_2 49$ **14**
26. $2 \log_{10} 6 - \frac{1}{3} \log_{10} 27 = \log_{10} x$ **12**
27. $\log_{10} z + \log_{10} (z + 3) = 1$ **2**
28. $\log_6 (a^2 + 2) + \log_6 2 = 2$ **±4**
29. $\log_2 (12b - 21) - \log_2 (b^2 - 3) = 2$ **∅**
30. $\log_2 (y + 2) - \log_2 (y - 2) = 1$ **6**
31. $\log_3 0.1 + 2 \log_3 x = \log_3 2 + \log_3 5$ **10**
32. $\log_5 64 - \log_5 \frac{8}{3} + \log_5 2 = \log_5 4p$ **12**

Answer

3. Clemente incorrectly applied the product and quotient properties of logarithms.

$\log_7 6 + \log_7 3 = \log_7 (6 \cdot 3)$ or $\log_7 18$ Product Property of Logarithms

$\log_7 18 - \log_7 2 = \log_7 (18 \div 2)$ or $\log_7 9$ Quotient Property of Logarithms

35. False; $\log_2(2^2+2^3) = \log_2 12$, $\log_2 2^2 + \log_2 2^3 = 2 + 3$ or 5, and $\log_2 12 \neq 5$ since $2^5 \neq 12$.

39. about 0.4214 kilocalorie per gram

40. about 0.8429 kilocalories per gram

More About. . .

Star Light

The Greek astronomer Hipparchus made the first known catalog of stars. He listed the brightness of each star on a scale of 1 to 6, the brightest being 1. With no telescope, he could only see stars as dim as the 6th magnitude.

Source: NASA

Solve for *n*. 34. $\frac{1}{2}(x-1)$

★ 33. $\log_a 4n - 2\log_a x = \log_a x$ $\frac{x^3}{4}$

★ 34. $\log_b 8 + 3\log_b n = 3\log_b(x-1)$

CRITICAL THINKING **Tell whether each statement is *true* or *false*. If true, show that it is true. If false, give a counterexample.**

35. For all positive numbers *m*, *n*, and *b*, where $b \neq 1$, $\log_b(m+n) = \log_b m + \log_b n$.

36. For all positive numbers *m*, *n*, *x*, and *b*, where $b \neq 1$, $n\log_b x + m\log_b x = (n+m)\log_b x$. See pp. 573A–573D.

37. **EARTHQUAKES** The great Alaskan earthquake in 1964 was about 100 times more intense than the Loma Prieta earthquake in San Francisco in 1989. Find the difference in the Richter scale magnitudes of the earthquakes. 2

BIOLOGY **For Exercises 38–40, use the following information.**

The energy *E* (in kilocalories per gram molecule) needed to transport a substance from the outside to the inside of a living cell is given by $E = 1.4(\log_{10} C_2 - \log_{10} C_1)$, where C_1 is the concentration of the substance outside the cell and C_2 is the concentration inside the cell.

38. Express the value of *E* as one logarithm. $E = 1.4\log\frac{C_2}{C_1}$

39. Suppose the concentration of a substance inside the cell is twice the concentration outside the cell. How much energy is needed to transport the substance on the outside of the cell to the inside? (Use $\log_{10} 2 \approx 0.3010$.)

40. Suppose the concentration of a substance inside the cell is four times the concentration outside the cell. How much energy is needed to transport the substance from the outside of the cell to the inside?

SOUND **For Exercises 41–43, use the formula for the loudness of sound in Example 3 on page 542. Use $\log_{10} 2 \approx 0.3010$ and $\log_{10} 3 \approx 0.47712$.**

41. A certain sound has a relative intensity of *R*. By how many decibels does the sound increase when the intensity is doubled? 3

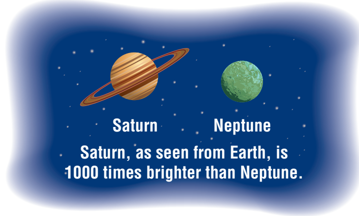
42. A certain sound has a relative intensity of *R*. By how many decibels does the sound decrease when the intensity is halved? 3

★ 43. A stadium containing 10,000 cheering people can produce a crowd noise of about 90 decibels. If every one cheers with the same relative intensity, how much noise, in decibels, is a crowd of 30,000 people capable of producing? Explain your reasoning. About 95 decibels; see margin for explanation.

STAR LIGHT **For Exercises 44–46, use the following information.**

The brightness, or apparent magnitude, *m* of a star or planet is given by the formula $m = 6 - 2.5\log_{10}\frac{L}{L_0}$, where *L* is the amount of light coming to Earth from the star or planet and L_0 is the amount of light from a sixth magnitude star.

The crescent moon is about 100 times brighter than the brightest star, Sirius.



Saturn, as seen from Earth, is 1000 times brighter than Neptune.

★ 44. Find the difference in the magnitudes of Sirius and the crescent moon. 5

★ 45. Find the difference in the magnitudes of Saturn and Neptune. 7.5

46. **RESEARCH** Use the Internet or other reference to find the magnitude of the dimmest stars that we can now see with ground-based telescopes. about 22

www.algebra2.com/self_check_quiz

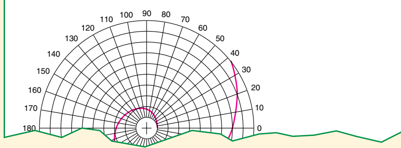
Answer

43. $L = 10\log_{10} R$, where *L* is the loudness of the sound in decibels and *R* is the relative intensity of the sound. Since the crowd increased by a factor of 3, we assume that the intensity also increases by a factor of 3. Thus, we need to find the loudness of $3R$. $L = 10\log_{10} 3R$; $L = 10(\log_{10} 3 + \log_{10} R)$
$L = 10\log_{10} 3 + 10\log_{10} R$;
$L \approx 10(0.4771) + 90$; $L \approx 4.771 + 90$ or about 95

Enrichment, p. 590

Spirals

Consider an angle in standard position with its vertex at a point *O* called the pole. Its initial side is on a coordinatized axis called the *polar axis.* A point *P* on the terminal side of the angle is named by the *polar coordinates* (r, θ), where *r* is the directed distance of the point from *O* and θ is the measure of the angle. Graphs in this system may be drawn on polar coordinate paper such as the kind shown below.



Study Guide and Intervention, p. 585 (shown) and p. 586

Properties of Logarithms Properties of exponents can be used to develop the following properties of logarithms.

Product Property of Logarithms	For all positive numbers *m*, *n*, and *b*, where $b \neq 1$, $\log_b mn = \log_b m + \log_b n$.
Quotient Property of Logarithms	For all positive numbers *m*, *n*, and *b*, where $b \neq 1$, $\log_b \frac{m}{n} = \log_b m - \log_b n$.
Power Property of Logarithms	For any real number *p* and positive numbers *m* and *b*, where $b \neq 1$, $\log_b m^p = p\log_b m$.

Example **Use $\log_3 28 \approx 3.0331$ and $\log_3 4 \approx 1.2619$ to approximate the value of each expression.**

a. $\log_3 36$

$$\begin{aligned}\log_3 36 &= \log_3(3^2 \cdot 4)\\ &= \log_3 3^2 + \log_3 4\\ &= 2 + \log_3 4\\ &\approx 2 + 1.2619\\ &\approx 3.2619\end{aligned}$$

b. $\log_3 7$

$$\begin{aligned}\log_3 7 &= \log_3\left(\frac{28}{4}\right)\\ &= \log_3 28 - \log_3 4\\ &\approx 3.0331 - 1.2619\\ &\approx 1.7712\end{aligned}$$

c. $\log_3 256$

$$\begin{aligned}\log_3 256 &= \log_3(4^4)\\ &= 4 \cdot \log_3 4\\ &\approx 4(1.2619)\\ &\approx 5.0476\end{aligned}$$

Exercises

Use $\log_{12} 3 \approx 0.4421$ and $\log_{12} 7 \approx 0.7831$ to evaluate each expression.

1. $\log_{12} 21$ 1.2252
2. $\log_{12}\frac{7}{3}$ 0.3410
3. $\log_{12} 49$ 1.5662
4. $\log_{12} 36$ 1.4421
5. $\log_{12} 63$ 1.6673
6. $\log_{12}\frac{27}{49}$ −0.2399
7. $\log_{12}\frac{81}{49}$ 0.2022
8. $\log_{12} 16{,}807$ 3.9155
9. $\log_{12} 441$ 2.4504

Use $\log_5 3 \approx 0.6826$ and $\log_5 4 \approx 0.8614$ to evaluate each expression.

10. $\log_5 12$ 1.5440
11. $\log_5 100$ 2.8614
12. $\log_5 0.75$ −0.1788
13. $\log_5 144$ 3.0880
14. $\log_5\frac{27}{16}$ 0.3250
15. $\log_5 375$ 3.6826
16. $\log_5 1.\overline{3}$ 0.1788
17. $\log_5\frac{9}{16}$ −0.3576
18. $\log_5\frac{81}{5}$ 1.7304

Skills Practice, p. 587 and Practice, p. 588 (shown)

Use $\log_{10} 5 \approx 0.6990$ and $\log_{10} 7 \approx 0.8451$ to approximate the value of each expression.

1. $\log_{10} 35$ 1.5441
2. $\log_{10} 25$ 1.3980
3. $\log_{10}\frac{7}{5}$ 0.1461
4. $\log_{10}\frac{5}{7}$ −0.1461
5. $\log_{10} 245$ 2.3892
6. $\log_{10} 175$ 2.2431
7. $\log_{10} 0.2$ −0.6990
8. $\log_{10}\frac{25}{7}$ 0.5529

Solve each equation. Check your solutions.

9. $\log_7 n = \frac{2}{3}\log_7 8$ 4
10. $\log_{10} u = \frac{3}{2}\log_{10} 4$ 8
11. $\log_6 x + \log_6 9 = \log_6 54$ 6
12. $\log_8 48 - \log_8 w = \log_8 4$ 12
13. $\log_9(3u + 14) - \log_9 5 = \log_9 2u$ 2
14. $4\log_2 x + \log_2 5 = \log_2 405$ 3
15. $\log_3 y = -\log_3 16 + \frac{1}{3}\log_3 64$ $\frac{1}{4}$
16. $\log_2 d = 5\log_2 2 - \log_2 8$ 4
17. $\log_{10}(3m - 5) + \log_{10} m = \log_{10} 2$ 2
18. $\log_{10}(b + 3) + \log_{10} b = \log_{10} 4$ 1
19. $\log_8(t + 10) - \log_8(t - 1) = \log_8 12$ 2
20. $\log_3(a + 3) + \log_3(a + 2) = \log_3 6$ 0
21. $\log_{10}(r + 4) - \log_{10} r = \log_{10}(r + 1)$ 2
22. $\log_4(x^2 - 4) - \log_4(x + 2) = \log_4 1$ 3
23. $\log_{10} 4 + \log_{10} w = 2$ 25
24. $\log_8(n - 3) + \log_8(n + 4) = 1$ 4
25. $3\log_5(x^2 + 9) - 6 = 0$ ±4
26. $\log_{16}(9x + 5) - \log_{16}(x^2 - 1) = \frac{1}{2}$ 3
27. $\log_6(2x - 5) + 1 = \log_6(7x + 10)$ 8
28. $\log_2(5y + 2) - 1 = \log_2(1 - 2y)$ 0
29. $\log_{10}(c^2 - 1) - 2 = \log_{10}(c + 1)$ 101
30. $\log_7 x + 2\log_7 x - \log_7 3 = \log_7 72$ 6

31. SOUND The loudness *L* of a sound in decibels is given by $L = 10\log_{10} R$, where *R* is the sound's relative intensity. If the intensity of a certain sound is tripled, by how many decibels does the sound increase? about 4.8 db

32. EARTHQUAKES An earthquake rated at 3.5 on the Richter scale is felt by many people, and an earthquake rated at 4.5 may cause local damage. The Richter scale magnitude reading *m* is given by $m = \log_{10} x$, where *x* represents the amplitude of the seismic wave causing ground motion. How many times greater is the amplitude of an earthquake that measures 4.5 on the Richter scale than one that measures 3.5? 10 times

Reading to Learn Mathematics, p. 589

ELL

Pre-Activity **How are the properties of exponents and logarithms related?**

Read the introduction to Lesson 10-3 at the top of page 541 in your textbook.

Find the value of $\log_5 125$. 3 Find the value of $\log_5 5$. 1

Find the value of $\log_5(125 \div 5)$. 2

Which of the following statements is true? B

A. $\log_5(125 \div 5) = (\log_5 125) \div (\log_5 5)$

B. $\log_5(125 \div 5) = \log_5 125 - \log_5 5$

Reading the Lesson

1. Each of the properties of logarithms can be stated in words or in symbols. Complete the statements of these properties in words.

a. The logarithm of a quotient is the __difference__ of the logarithms of the __numerator__ and the __denominator__.

b. The logarithm of a power is the __product__ of the logarithm of the base and the __exponent__.

c. The logarithm of a product is the __sum__ of the logarithms of its __factors__.

2. State whether each of the following equations is *true* or *false*. If the statement is true, name the property of logarithms that is illustrated.

a. $\log_3 10 = \log_3 30 - \log_3 3$ true; Quotient Property
b. $\log_4 12 = \log_4 4 + \log_4 8$ false
c. $\log_2 81 = 2\log_2 9$ true; Power Property
d. $\log_8 30 = \log_8 5 \cdot \log_8 6$ false

3. The algebraic process of solving the equation $\log_2 x + \log_2(x + 2) = 3$ leads to "$x = -4$ or $x = 2$." Does this mean that both −4 and 2 are solutions of the logarithmic equation? Explain your reasoning. Sample answer: No; 2 is a solution because it checks: $\log_2 2 + \log_2(2 + 2) = \log_2 2 + \log_2 4 = 1 + 2 = 3$. However, because $\log_2(-4)$ and $\log_2(-2)$ are undefined, −4 is an extraneous solution and must be eliminated. The only solution is 2.

Helping You Remember

4. A good way to remember something is to relate it something you already know. Use words to explain how the Product Property for exponents can help you remember the product property for logarithms. Sample answer: When you *multiply* two numbers or expressions with the same base, you *add* the exponents and keep the *same base*. Logarithms are exponents, so to find the logarithm of a *product*, you *add* the logarithms of the factors, keeping the *same base*.

4 Assess

Open-Ended Assessment

Speaking Ask students to explain the Product Property, Quotient Property, and Power Property of Logarithms in their own words. Encourage them to use specific examples for clarification.

Getting Ready for Lesson 10-4

PREREQUISITE SKILL Students will use common logarithms to solve exponential equations and inequalities in Lesson 10-4. The solution techniques involve using the skills they learned when solving logarithmic equations and inequalities. Use Exercises 63–66 to determine your students' familiarity with solving logarithmic equations and inequalities.

Assessment Options

Quiz (Lesson 10-3) is available on p. 623 of the *Chapter 10 Resource Masters*.

Mid-Chapter Test (Lessons 10-1 through 10-3) is available on p. 625 of the *Chapter 10 Resource Masters*.

47. **CRITICAL THINKING** Use the properties of exponents to prove the Quotient Property of Logarithms. **See margin.**

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 573A–573D.**

How are the properties of exponents and logarithms related?

Include the following in your answer:

- examples like the one shown at the beginning of the lesson illustrating the Quotient Property and Power Property of Logarithms, and
- an explanation of the similarity between one property of exponents and its related property of logarithms.

Standardized Test Practice

49. Simplify $2 \log_5 12 - \log_5 8 - 2 \log_5 3$. **A**

Ⓐ $\log_5 2$ Ⓑ $\log_5 3$ Ⓒ $\log_5 0.5$ Ⓓ 1

50. **SHORT RESPONSE** Show that $\log_b m^p = p \log_b m$ for any real number p and positive number m and b, where $b \neq 1$. **See margin.**

Maintain Your Skills

Mixed Review

Evaluate each expression. *(Lesson 10-2)*

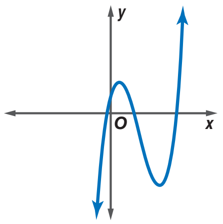
51. $\log_3 81$ **4**   52. $\log_9 \frac{1}{729}$ **−3**   53. $\log_7 7^{2x}$ **2x**

Solve each equation or inequality. Check your solutions. *(Lesson 10-1)*

54. $3^{5n+3} = 3^{33}$ **6**   55. $7^a = 49^{-4}$ **−8**   56. $3^{d+4} > 9^d$ **$d < 4$**

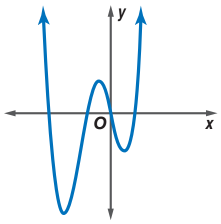
Determine whether each graph represents an odd-degree polynomial function or an even-degree polynomial function. Then state how many real zeros each function has. *(Lesson 7-1)*

57.



odd; 3

58.



even; 4

Simplify each expression. *(Lesson 9-1)*

59. $\frac{39a^3b^4}{13a^4b^3}$ **$\frac{3b}{a}$**   60. $\frac{k+3}{5kl} \cdot \frac{10kl}{k+3}$ **2**   61. $\frac{5y-15z}{42x^2} \div \frac{y-3z}{14x}$ **$\frac{5}{3x}$**

62. **PHYSICS** If a stone is dropped from a cliff, the equation $t = \frac{1}{4}\sqrt{d}$ represents the time t in seconds that it takes for the stone to reach the ground. If d represents the distance in feet that the stone falls, find how long it would take for a stone to fall from a 150-foot cliff. *(Lesson 5-6)* **3.06 s**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation or inequality. Check your solutions.**

*(To review **solving logarithmic equations and inequalities**, see Lesson 10-2.)*

63. $\log_3 x = \log_3 (2x - 1)$ **1**   64. $\log_{10} 2^x = \log_{10} 32$ **5**

65. $\log_2 3x > \log_2 5$ **$x > \frac{5}{3}$**   66. $\log_5 (4x + 3) < \log_5 11$ **$-\frac{3}{4} < x < 2$**

Answers

47. Let $b^x = m$ and $b^y = n$. Then $\log_b m = x$ and $\log_b n = y$.

$\frac{b^x}{b^y} = \frac{m}{n}$

$b^{x-y} = \frac{m}{n}$ Quotient Property

$\log_b b^{x-y} = \log_b \frac{m}{n}$ Property of Equality for Logarithmic Equations

$x - y = \log_b \frac{m}{n}$ Inverse Property of Exponents and Logarithms

$\log_b m - \log_b n = \log_b \frac{m}{n}$ Replace x with $\log_b m$ and y with $\log_b n$.

50. Let $b^x = m$, then $\log_b m = x$.

$(b^x)^p = m^p$

$b^{xp} = m^p$ Product of Powers

$\log_b b^{xp} = \log_b m^p$ Property of Equality for Logarithmic Equations

$xp = \log_b m^p$ Inverse Property of Exponents and Logarithms

$p \log_b m = \log_b m^p$ Replace x with $\log_b m$.

10-4 Common Logarithms

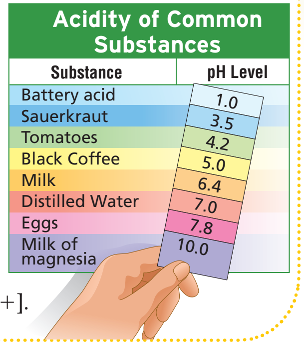
What You'll Learn

- Solve exponential equations and inequalities using common logarithms.
- Evaluate logarithmic expressions using the Change of Base Formula.

Why is a logarithmic scale used to measure acidity?

The pH level of a substance measures its acidity. A low pH indicates an acid solution while a high pH indicates a basic solution. The pH levels of some common substances are shown.

The pH level of a substance is given by pH $= -\log_{10}[H+]$, where $H+$ is the substance's hydrogen ion concentration in moles per liter. Another way of writing this formula is pH $= -\log[H+]$.



Acidity of Common Substances

Substance	pH Level
Battery acid	1.0
Sauerkraut	3.5
Tomatoes	4.2
Black Coffee	5.0
Milk	6.4
Distilled Water	7.0
Eggs	7.8
Milk of magnesia	10.0

COMMON LOGARITHMS You have seen that the base 10 logarithm function, $y = \log_{10} x$, is used in many applications. Base 10 logarithms are called **common logarithms**. Common logarithms are usually written without the subscript 10.

$$\log_{10} x = \log x, x > 0$$

Most calculators have a LOG key for evaluating common logarithms.

Example 1 Find Common Logarithms

Use a calculator to evaluate each expression to four decimal places.

a. log 3 **KEYSTROKES:** LOG 3 ENTER .4771212547 about 0.4771

b. log 0.2 **KEYSTROKES:** LOG 0.2 ENTER −.6989700043 about −0.6990

Sometimes an application of logarithms requires that you use the inverse of logarithms, or exponentiation.

$$10^{\log x} = x$$

Example 2 Solve Logarithmic Equations Using Exponentiation

EARTHQUAKES **The amount of energy E, in ergs, that an earthquake releases is related to its Richter scale magnitude M by the equation $\log E = 11.8 + 1.5M$. The Chilean earthquake of 1960 measured 8.5 on the Richter scale. How much energy was released?**

$\log E = 11.8 + 1.5M$	Write the formula.
$\log E = 11.8 + 1.5(8.5)$	Replace M with 8.5.
$\log E = 24.55$	Simplify.
$10^{\log E} = 10^{24.55}$	Write each side using exponents and base 10.
$E = 10^{24.55}$	Inverse Property of Exponents and Logarithms
$E \approx 3.55 \times 10^{24}$	Use a calculator.

The amount of energy released by this earthquake was about 3.55×10^{24} ergs.

Vocabulary

- common logarithm
- Change of Base Formula

Study Tip

Technology
Nongraphing scientific calculators often require entering the number followed by the function, for example, 3 LOG.

10-4 Lesson Notes

1 Focus

5-Minute Check Transparency 10-4 Use as a quiz or review of Lesson 10-3.

Mathematical Background notes are available for this lesson on p. 520D.

Why is a logarithmic scale used to measure acidity?

Ask students:

- What does pH level measure? **the acidity of a substance**
- Where have you heard of pH levels? **Sample answer: in soap and shampoo commercials**
- Distilled water has a neutral pH. That is, it is neither acidic nor basic. Use the chart to determine the pH level for a neutral substance. **7.0**

2 Teach

COMMON LOGARITHMS

Teaching Tip Stress that when the base of a logarithm is not shown, the base is assumed to be 10.

Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters

- Study Guide and Intervention, pp. 591–592
- Skills Practice, p. 593
- Practice, p. 594
- Reading to Learn Mathematics, p. 595
- Enrichment, p. 596

Transparencies

5-Minute Check Transparency 10-4
Real-World Transparency 10
Answer Key Transparencies

Technology

Interactive Chalkboard

In-Class Examples

Power Point®

1 Use a calculator to evaluate each expression to four decimal places.

a. log 6 **about 0.7782**

b. log 0.35 **about −0.4559**

2 **EARTHQUAKE** Refer to Example 2. The San Fernando Valley earthquake of 1994 measured 6.6 on the Richter scale. How much energy did this earthquake release? **about 5.01×10^{21} ergs**

Teaching Tip After discussing In-Class Example 2, have students compare the Richter scale magnitudes of the Chilean and San Fernando Valley earthquakes. **$8.5 \div 6.6 \approx 1.29$; the Chilean magnitude was about 29% greater.** Then have them compare the energy released by the Chilean earthquake to the energy released by the San Fernando Valley earthquake. **$3.55 \times 10^{24} \div 5.01 \times 10^{21} \approx 708.58$; the Chilean earthquake released more than 6 times as much energy.** Point out that these results demonstrate the nonlinear nature of the equation that models the amount of energy released.

3 Solve $5^x = 62$. **about 2.5643**

4 Solve $2^{7x} > 3^{5x-3}$. **$\{x \mid x < 5.1415\}$**

Study Tip

Using Logarithms
When you use the Property for Logarithmic Functions as in the second step of Example 3, this is sometimes referred to as *taking the logarithm of each side.*

Example 3 Solve Exponential Equations Using Logarithms

Solve $3^x = 11$.

$3^x = 11$	Original equation
$\log 3^x = \log 11$	Property of Equality for Logarithmic Functions
$x \log 3 = \log 11$	Power Property of Logarithms
$x = \dfrac{\log 11}{\log 3}$	Divide each side by log 3.
$x \approx \dfrac{1.0414}{0.4771}$	Use a calculator.
$x \approx 2.1828$	The solution is approximately 2.1828.

CHECK You can check this answer using a calculator or by using estimation. Since $3^2 = 9$ and $3^3 = 27$, the value of x is between 2 and 3. In addition, the value of x should be closer to 2 than 3, since 11 is closer to 9 than 27. Thus, 2.1828 is a reasonable solution. ✓

Example 4 Solve Exponential Inequalities Using Logarithms

Solve $5^{3y} < 8^{y-1}$.

$5^{3y} < 8^{y-1}$	Original inequality
$\log 5^{3y} < \log 8^{y-1}$	Property of Inequality for Logarithmic Functions
$3y \log 5 < (y - 1) \log 8$	Power Property of Logarithms
$3y \log 5 < y \log 8 - \log 8$	Distributive Property
$3y \log 5 - y \log 8 < -\log 8$	Subtract y log 8 from each side.
$y(3 \log 5 - \log 8) < -\log 8$	Distributive Property
$y < \dfrac{-\log 8}{3 \log 5 - \log 8}$	Divide each side by 3 log 5 − log 8.
$y < \dfrac{-(0.9031)}{3(0.6990) - 0.9031}$	Use a calculator.
$y < -0.7564$	The solution set is $\{y \mid y < -0.7564\}$.

CHECK Test $y = -1$.

$5^{3y} < 8^{y-1}$	Original inequality
$5^{3(-1)} < 8^{(-1)-1}$	Replace y with 1.
$5^{-3} < 8^{-2}$	Simplify.
$\dfrac{1}{125} < \dfrac{1}{64}$ ✓	Negative Exponent Property

CHANGE OF BASE FORMULA The **Change of Base Formula** allows you to write equivalent logarithmic expressions that have different bases.

Key Concept — *Change of Base Formula*

- **Symbols** For all positive numbers, a, b and n, where $a \neq 1$ and $b \neq 1$,

$$\log_a n = \frac{\log_b n}{\log_b a} \begin{array}{l} \leftarrow \text{log base } b \text{ of original number} \\ \leftarrow \text{log base } b \text{ of old base} \end{array}$$

- **Example** $\log_5 12 = \dfrac{\log_{10} 12}{\log_{10} 5}$

DAILY INTERVENTION — Unlocking Misconceptions

Change of Base As you discuss the Change of Base Formula, point out that the base b that students are changing to does not have to be 10. Any base could be used; however, b is most commonly 10 because this allows for the logarithms to be evaluated with a calculator.

To prove this formula, let $\log_a n = x$.

$a^x = n$ Definition of logarithm

$\log_b a^x = \log_b n$ Property of Equality for Logarithms

$x \log_b a = \log_b n$ Power Property of Logarithms

$x = \frac{\log_b n}{\log_b a}$ Divide each side by $\log_b a$.

$\log_a n = \frac{\log_b n}{\log_b a}$ Replace x with $\log_a n$.

This formula makes it possible to evaluate a logarithmic expression of any base by translating the expression into one that involves common logarithms.

Example 5 Change of Base Formula

Express $\log_4 25$ in terms of common logarithms. Then approximate its value to four decimal places.

$\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4}$ Change of Base Formula

≈ 2.3219 Use a calculator.

The value of $\log_4 25$ is approximately 2.3219.

Check for Understanding

Concept Check

1. **Name** the base used by the calculator LOG key. What are these logarithms called? **10; common logarithms**
2. **OPEN ENDED** Give an example of an exponential equation requiring the use of logarithms to solve. Then solve your equation.

2. Sample answer: $5^x = 2$; $x \approx 0.4307$

3. **Explain** why you must use the Change of Base Formula to find the value of $\log_2 7$ on a calculator. **A calculator is not programmed to find base 2 logarithms.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	1
7–12	3, 4
13–15	5
16	2

Use a calculator to evaluate each expression to four decimal places.

4. log 4 **0.6021**
5. log 23 **1.3617**
6. log 0.5 **−0.3010**

Solve each equation or inequality. Round to four decimal places. **12. $\{p \mid p \leq 4.8188\}$**

7. $9^x = 45$ **1.7325**
8. $4^{5n} > 30$ **$\{n \mid n > 0.4907\}$**
9. $3.1^{a-3} = 9.42$ **4.9824**
10. $11^{x^2} = 25.4$ **±1.1615**
11. $7^{t-2} = 5^t$ **11.5665**
12. $4^{p-1} \leq 3^p$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

13. $\log_7 5$ **$\frac{\log 5}{\log 7}$; 0.8271**
14. $\log_3 42$ **$\frac{\log 42}{\log 3}$; 3.4022**
15. $\log_2 9$ **$\frac{\log 9}{\log 2}$; 3.1699**

Application

16. **DIET** Sandra's doctor has told her to avoid foods with a pH that is less than 4.5. What is the hydrogen ion concentration of foods Sandra is allowed to eat? Use the information at the beginning of the lesson. **at least 0.00003 mole per liter**

★ **indicates increased difficulty**

Practice and Apply

Use a calculator to evaluate each expression to four decimal places.

17. log 5 **0.6990**
18. log 12 **1.0792**
19. log 7.2 **0.8573**
20. log 2.3 **0.3617**
21. log 0.8 **−0.0969**
22. log 0.03 **−1.5229**

www.algebra2.com/extra_examples

CHANGE OF BASE FORMULA

In-Class Example

Power Point®

5 Express $\log_3 18$ in terms of common logarithms. Then approximate its value to four decimal places. **$\log_3 18 = \frac{\log_{10} 18}{\log_{10} 3}$; $\log_3 18 \approx 2.6309$**

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include an example of an exponential inequality that they solved, and an example showing how to use the Change of Base Formula.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective

- **Common Logarithms:** 17–44, 51–55
- **Change of Base Formula:** 45–50

Odd/Even Assignments

Exercises 17–52 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–41 odd, 45, 47, 51, 56, 59–77

Average: 17–51 odd, 56–77

Advanced: 18–52 even, 53–74 (optional: 72–77)

DAILY INTERVENTION

Differentiated Instruction

Naturalist Have interested students research earthquakes and their Richter scale measurements. Have them calculate the amount of energy released by three earthquakes they found to be of interest. Have students compare the energy released to the amount of destruction caused and share their findings with the class.

Study Guide and Intervention, p. 591 (shown) and p. 592

Common Logarithms Base 10 logarithms are called **common logarithms**. The expression $\log_{10} x$ is usually written without the subscript as $\log x$. Use the LOG key on your calculator to evaluate common logarithms.

The relation between exponents and logarithms gives the following identity.

Inverse Property of Logarithms and Exponents	$10^{\log x} = x$

Example 1 **Evaluate log 50 to four decimal places.**

Use the LOG key on your calculator. To four decimal places, log 50 = 1.6990.

Example 2 **Solve $3^{2x + 1} = 12$.**

$$\begin{aligned} 3^{2x+1} &= 12 && \text{Original equation}\\ \log 3^{2x+1} &= \log 12 && \text{Property of Equality for Logarithms}\\ (2x+1)\log 3 &= \log 12 && \text{Power Property of Logarithms}\\ 2x+1 &= \frac{\log 12}{\log 3} && \text{Divide each side by log 3.}\\ 2x &= \frac{\log 12}{\log 3} - 1 && \text{Subtract 1 from each side.}\\ x &= \frac{1}{2}\left(\frac{\log 12}{\log 3} - 1\right) && \text{Multiply each side by } \tfrac{1}{2}.\\ x &\approx 0.6309 \end{aligned}$$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. log 18 **1.2553**
2. log 39 **1.5911**
3. log 120 **2.0792**
4. log 5.8 **0.7634**
5. log 42.3 **1.6263**
6. log 0.003 **−2.5229**

Solve each equation or inequality. Round to four decimal places.

7. $4^{3x} = 12$ **0.5975**
8. $6^{x+2} = 18$ **−0.3869**
9. $5^{4x-2} = 120$ **1.2437**
10. $7^{3x-1} \geq 21$ **$\{x \mid x \geq 0.8549\}$**
11. $2.4^{x+4} = 30$ **−0.1150**
12. $6.5^{2x} \geq 200$ **$\{x \mid x \geq 1.4153\}$**
13. $3.6^{4x-1} = 85.4$ **1.1180**
14. $2^{x+5} = 3^{x-2}$ **13.9666**
15. $9^{3x} = 4^{5x+2}$ **−8.1595**
16. $6^{x-5} = 2^{7x+3}$ **−3.6069**

Skills Practice, p. 593 and Practice, p. 594 (shown)

Use a calculator to evaluate each expression to four decimal places.

1. log 101 **2.0043**
2. log 2.2 **0.3424**
3. log 0.05 **−1.3010**

Use the formula pH = −log[H+] to find the pH of each substance given its concentration of hydrogen ions.

4. milk: $[H+] = 2.51 \times 10^{-7}$ mole per liter **6.6**
5. acid rain: $[H+] = 2.51 \times 10^{-6}$ mole per liter **5.6**
6. black coffee: $[H+] = 1.0 \times 10^{-5}$ mole per liter **5.0**
7. milk of magnesia: $[H+] = 3.16 \times 10^{-11}$ mole per liter **10.5**

Solve each equation or inequality. Round to four decimal places.

8. $2^x < 25$ **$\{x \mid x < 4.6439\}$**
9. $5^a = 120$ **2.9746**
10. $6^z = 45.6$ **2.1319**
11. $9^m \geq 100$ **$\{m \mid m \geq 2.0959\}$**
12. $3.5^x = 47.9$ **3.0885**
13. $8.2^y = 64.5$ **1.9802**
14. $2^{b+1} \leq 7.31$ **$\{b \mid b \leq 1.8699\}$**
15. $4^{2x} = 27$ **1.1887**
16. $2^{a-4} = 82.1$ **10.3593**
17. $9^{z-2} > 38$ **$\{z \mid z > 3.6555\}$**
18. $5^{w+3} = 17$ **−1.2396**
19. $30^{x^2} = 50$ **±1.0725**
20. $5^{x^2-3} = 72$ **±2.3785**
21. $4^{2x} = 9^{x+1}$ **3.8188**
22. $2^{n+1} = 5^{2n-1}$ **0.9117**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23. $\log_5 12$ **$\frac{\log_{10} 12}{\log_{10} 5}$; 1.5440**
24. $\log_8 32$ **$\frac{\log_{10} 32}{\log_{10} 8}$; 1.6667**
25. $\log_{11} 9$ **$\frac{\log_{10} 9}{\log_{10} 11}$; 0.9163**
26. $\log_2 18$ **$\frac{\log_{10} 18}{\log_{10} 2}$; 4.1699**
27. $\log_9 6$ **$\frac{\log_{10} 6}{\log_{10} 9}$; 0.8155**
28. $\log_7 \sqrt{8}$ **$\frac{\log_{10} 8}{2 \log_{10} 7}$; 0.5343**

29. **HORTICULTURE** Siberian irises flourish when the concentration of hydrogen ions [H+] in the soil is not less than 1.58×10^{-8} mole per liter. What is the pH of the soil in which these irises will flourish? **7.8 or less**

30. **ACIDITY** The pH of vinegar is 2.9 and the pH of milk is 6.6. How many times greater is the hydrogen ion concentration of vinegar than of milk? **about 5000**

31. **BIOLOGY** There are initially 1000 bacteria in a culture. The number of bacteria doubles each hour. The number of bacteria N present after t hours is $N = 1000(2)^t$. How long will it take the culture to increase to 50,000 bacteria? **about 5.6 h**

32. **SOUND** An equation for loudness L in decibels is given by $L = 10 \log R$, where R is the sound's relative intensity. An air-raid siren can reach 150 decibels and jet engine noise can reach 120 decibels. How many times greater is the relative intensity of the air-raid siren than that of the jet engine noise? **1000**

Reading to Learn Mathematics, p. 595

ELL

Pre-Activity **Why is a logarithmic scale used to measure acidity?**

Read the introduction to Lesson 10-4 at the top of page 547 in your textbook.

Which substance is more acidic, milk or tomatoes? **tomatoes**

Reading the Lesson

1. Rhonda used the following keystrokes to enter an expression on her graphing calculator:

LOG 17) ENTER

The calculator returned the result 1.230448921.

Which of the following conclusions are correct? **a, c, and d**

a. The base 10 logarithm of 17 is about 1.2304.
b. The base 17 logarithm of 10 is about 1.2304.
c. The common logarithm of 17 is about 1.230449.
d. $10^{1.230448921}$ is very close to 17.
e. The common logarithm of 17 is exactly 1.230448921.

2. Match each expression from the first column with an expression from the second column that has the same value.

a. $\log_2 2$ **iv**
b. log 12 **iii**
c. $\log_3 1$ **i**
d. $\log_5 \frac{1}{5}$ **v**
e. log 1000 **ii**

i. $\log_4 1$
ii. $\log_2 8$
iii. $\log_{10} 12$
iv. $\log_5 5$
v. log 0.1

3. Calculators do not have keys for finding base 8 logarithms directly. However, you can use a calculator to find $\log_8 20$ if you apply the **change of base** formula.

Which of the following expressions are equal to $\log_8 20$? **B and C**

A. $\log_{20} 8$
B. $\frac{\log_{10} 20}{\log_{10} 8}$
C. $\frac{\log 20}{\log 8}$
D. $\frac{\log 8}{\log 20}$

Helping You Remember

4. Sometimes it is easier to remember a formula if you can state it in words. State the change of base formula in words. **Sample answer: To change the logarithm of a number from one base to another, divide the log of the original number in the old base by the log of the new base in the old base.**

Homework Help

For Exercises	See Examples
17–22	1
23–44,	3, 4
53–57	
45–50	5
51–55	2

Extra Practice

See page 850.

ACIDITY **For Exercises 23–26, use the information at the beginning of the lesson to find the pH of each substance given its concentration of hydrogen ions.**

23. ammonia: $[H+] = 1 \times 10^{-11}$ mole per liter **11**
24. vinegar: $[H+] = 6.3 \times 10^{-3}$ mole per liter **2.2**
25. lemon juice: $[H+] = 7.9 \times 10^{-3}$ mole per liter **2.1**
26. orange juice: $[H+] = 3.16 \times 10^{-4}$ mole per liter **3.5**

Solve each equation or inequality. Round to four decimal places.

27. $6^x \geq 42$ **$\{x \mid x \geq 2.0860\}$**
28. $5^x = 52$ **2.4550**
29. $8^{2a} < 124$ **$\{a \mid a < 1.1590\}$**
30. $4^{3p} = 10$ **0.5537**
31. $3^{n+2} = 14.5$ **0.4341**
32. $9^{z-4} = 6.28$ **4.8362**
33. $8.2^{n-3} = 42.5$ **4.7820**
34. $2.1^{t-5} = 9.32$ **8.0086**
35. $20^{x^2} = 70$ **±1.1909**
36. $2^{x^2-3} = 15$ **±2.6281**
37. $8^{2n} > 52^{4n+3}$ **$\{n \mid n > -1.0178\}$**
38. $2^{2x+3} = 3^{3x}$ **1.0890**
39. $16^{d-4} = 3^{3-d}$ **3.7162**
40. $7^{p+2} \leq 13^{5-p}$ **$\{p \mid p \leq 1.9803\}$**
41. $5^{5y-2} = 2^{2y+1}$ **0.5873**
42. $8^{2x-5} = 5^{x+1}$ **4.7095**

★ 43. $2^n = \sqrt{3^{n-2}}$ **−7.6377**

★ 44. $4^x = \sqrt{5^{x+2}}$ **2.7674**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

45. $\log_2 13$
46. $\log_5 20$
47. $\log_7 3$
48. $\log_3 8$

★ 49. $\log_4 (1.6)^2$

★ 50. $\log_6 \sqrt{5}$

45. $\frac{\log 13}{\log 2} \approx 3.7004$
46. $\frac{\log 20}{\log 5} \approx 1.8614$
47. $\frac{\log 3}{\log 7} \approx 0.5646$
48. $\frac{\log 8}{\log 3} \approx 1.8928$
49. $\frac{2 \log 1.6}{\log 4} \approx 0.6781$
50. $\frac{0.5 \log 5}{\log 6} \approx 0.4491$

For Exercises 51 and 52, use the information presented at the beginning of the lesson.

51. **POLLUTION** The acidity of water determines the toxic effects of runoff into streams from industrial or agricultural areas. A pH range of 6.0 to 9.0 appears to provide protection for freshwater fish. What is this range in terms of the water's hydrogen ion concentration? **between 0.000000001 and 0.000001 mole per liter**

52. **BUILDING DESIGN** The 1971 Sylmar earthquake in Los Angeles had a Richter scale magnitude of 6.3. Suppose an architect has designed a building strong enough to withstand an earthquake 50 times as intense as the Sylmar quake. Find the magnitude of the strongest quake this building is designed to withstand. **8**

More About. . .

Pollution

As little as 0.9 milligram per liter of iron at a pH of 5.5 can cause fish to die.

Source: Kentucky Water Watch

ASTRONOMY **For Exercises 53–55, use the following information.**

Some stars appear bright only because they are very close to us. Absolute magnitude M is a measure of how bright a star would appear if it were 10 parsecs, about 32 light years, away from Earth. A lower magnitude indicates a brighter star. Absolute magnitude is given by $M = m + 5 - 5 \log d$, where d is the star's distance from Earth measured in parsecs and m is its apparent magnitude.

53. Sirius and Vega are two of the brightest stars in Earth's sky. The apparent magnitude of Sirius is −1.44 and of Vega is 0.03. Which star appears brighter?

53. Sirius

54. Sirius is 2.64 parsecs from Earth while Vega is 7.76 parsecs from Earth. Find the absolute magnitude of each star. **Sirius: 1.45, Vega: 0.58**

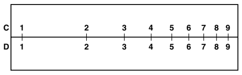
55. Which star is actually brighter? That is, which has a lower absolute magnitude? **Vega**

56. **CRITICAL THINKING**
    a. Without using a calculator, find the value of $\log_2 8$ and $\log_8 2$. **$3; \frac{1}{3}$**
    b. Without using a calculator, find the value of $\log_9 27$ and $\log_{27} 9$. **$\frac{3}{2}; \frac{2}{3}$**
    c. Make and prove a conjecture as to the relationship between $\log_a b$ and $\log_b a$. **See margin.**

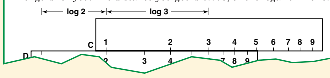
Enrichment, p. 596

The Slide Rule

Before the invention of electronic calculators, computations were often performed on a slide rule. A slide rule is based on the idea of logarithms. It has two movable rods labeled with C and D scales. Each of the scales is logarithmic.



To multiply 2 × 3 on a slide rule, move the C rod to the right as shown below. You can find 2 × 3 by adding log 2 to log 3, and the slide rule adds the lengths for you. The distance you get is 0.778, or the logarithm of 6.



MONEY **For Exercises 57 and 58, use the following information.**
If you deposit P dollars into a bank account paying an annual interest rate r (expressed as a decimal), with n interest payments each year, the amount A you would have after t years is $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Marta places \$100 in a savings account earning 6% annual interest, compounded quarterly.

57. If Marta adds no more money to the account, how long will it take the money in the account to reach \$125? **about 3.75 yr or 3 yr 9 mo**

58. How long will it take for Marta's money to double? **about 11.64 yr or 11 yr 8 mo**

59. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

Why is a logarithmic scale used to measure acidity?

Include the following in your answer:
- the hydrogen ion concentration of three substances listed in the table, and
- an explanation as to why it is important to be able to distinguish between a hydrogen ion concentration of 0.00001 mole per liter and 0.0001 mole per liter.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

60. **QUANTITATIVE COMPARISION** **Compare the quantity in Column A and the quantity in Column B. Then determine whether:** **A**

Ⓐ the quantity in Column A is greater,
Ⓑ the quantity in Column B is greater,
Ⓒ the two quantities are equal, or
Ⓓ the relationship cannot be determined from the information given.

Column A	Column B
$\log 10^3$	$\log 10^2$

61. If $2^4 = 3^x$, then what is the value of x? **C**

Ⓐ 0.63 Ⓑ 2.34 Ⓒ 2.52 Ⓓ 4

Maintain Your Skills

Mixed Review

Use $\log_7 2 \approx 0.3562$ and $\log_7 3 \approx 0.5646$ to approximate the value of each expression. *(Lesson 10-3)*

62. $\log_7 16$ **1.4248** **63.** $\log_7 27$ **1.6938** **64.** $\log_7 36$ **1.8416**

Solve each equation or inequality. Check your solutions. *(Lesson 10-2)*

65. $\log_4 r = 3$ **64** **66.** $\log_8 z \leq -2$ **67.** $\log_3 (4x - 5) = 5$ **62**

66. $\left\{z \mid 0 < z \leq \frac{1}{64}\right\}$

68. Use synthetic substitution to find $f(-2)$ for $f(x) = x^3 + 6x - 2$. *(Lesson 7-4)* **−22**

Factor completely. If the polynomial is not factorable, write prime. *(Lesson 5-4)*

69. $3d^2 + 2d - 8$ **$(d + 2)(3d - 4)$**
70. $42pq - 35p + 18q - 15$ **$(7p + 3)(6q - 5)$**
71. $13xyz + 3x^2z + 4k$ **prime**

Getting Ready for the Next Lesson

PREREQUISITE SKILLS **Write an equivalent exponential equation.**
*(For review of **logarithmic equations**, see Lesson 10-2.)*

72. $\log_2 3 = x$ **$2^x = 3$** **73.** $\log_3 x = 2$ **$3^2 = x$** **74.** $\log_5 125 = 3$ **$5^3 = 125$**

Write an equivalent logarithmic equation.
*(For review of **logarithmic equations**, see Lesson 10-2.)*

75. $5^x = 45$ **$\log_5 45 = x$** **76.** $7^3 = x$ **$\log_7 x = 3$** **77.** $b^y = x$ **$\log_b x = y$**

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Ask students to explain in writing what it means to use the Change of Base Formula. They should include comments about why this formula is useful.

Getting Ready for Lesson 10-5

PREREQUISITE SKILL In Lesson 10-5, students will solve exponential equations and inequalities using natural logarithms and the skills they learned solving common logarithmic equations and inequalities. Students should be confident when converting between exponential and logarithmic equations before proceeding. Use Exercises 72–77 to determine your students' familiarity with converting between exponential and logarithmic equations.

Answers

56c. conjecture: $\log_a b = \frac{1}{\log_b a}$;

proof:

$\log_a b \stackrel{?}{=} \frac{1}{\log_b a}$	Original statement
$\frac{\log_b b}{\log_b a} \stackrel{?}{=} \frac{1}{\log_b a}$	Change of Base Formula
$\frac{1}{\log_b a} = \frac{1}{\log_b a}$ ✓	Inverse Property of Exponents and Logarithms

59. Comparisons between substances of different acidities are more easily distinguished on a logarithmic scale. Answers should include the following.

- Sample answer:
  Tomatoes: 6.3×10^{-5} mole per liter
  Milk: 3.98×10^{-7} mole per liter
  Eggs: 1.58×10^{-8} mole per liter
- Those measurements correspond to pH measurements of 5 and 4, indicating a weak acid and a stronger acid. On the logarithmic scale we can see the difference in these acids, whereas on a normal scale, these hydrogen ion concentrations would appear nearly the same. For someone who has to watch the acidity of the foods they eat, this could be the difference between an enjoyable meal and heartburn.

Graphing Calculator Investigation

A Follow-Up of Lesson 10-4

Getting Started

Using Parentheses In Step 1 of Example 1, remind students that they must also use parentheses around the fraction $\frac{1}{2}$.

Teach

- Before discussing Example 1, use a simple equation such as $2x = 6$ to show students how the equation can be solved by graphing. Graph the equations $y = 2x$ and $y = 6$ and then identify the point of intersection of the graphs.
- Ask students why it is necessary in Step 1 to enter the equations using parentheses around the exponents.
- Have students substitute the solution to Example 1 into the original equation to verify that it is correct.
- In Example 2, make sure students understand why the equations must be rewritten using the Change of Base Formula.
- Students can find the solution set for Example 2 without using the shading options. Simply have them use the intersect feature, noting that the graph of Y1 intersects or is above the graph of Y2 at and to the right of $x = 0.5$.

Graphing Calculator

A Follow-Up of Lesson 10-4

Solving Exponential and Logarithmic Equations and Inequalities

You can use a TI-83 Plus graphing calculator to solve exponential and logarithmic equations and inequalities. This can be done by graphing each side of the equation separately and using the intersect feature on the calculator.

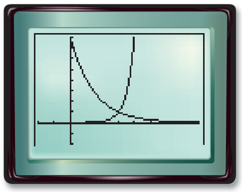
Example 1

Solve $2^{3x-9} = \left(\frac{1}{2}\right)^{x-3}$ by graphing.

Step 1 *Graph each side of the equation.*

- Graph each side of the equation as a separate function. Enter 2^{3x-9} as Y1. Enter $\left(\frac{1}{2}\right)^{x-3}$ as Y2. Be sure to include the added parentheses around each exponent. Then graph the two equations.

KEYSTROKES: *See pages 87 and 88 to review graphing equations.*

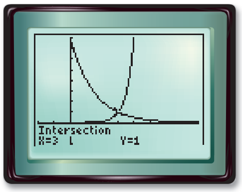


[−2, 8] scl: 1 by [−2, 8] scl: 1

Step 2 *Use the intersect feature.*

- You can use the intersect feature on the CALC menu to approximate the ordered pair of the point at which the curves cross.

KEYSTROKES: *See page 115 to review how to use the intersect feature.*



Intersection
X=3 Y=1

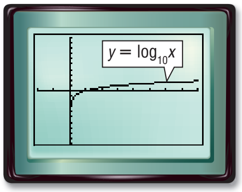
[−2, 8] scl: 1 by [−2, 8] scl: 1

The calculator screen shows that the x-coordinate of the point at which the curves cross is 3. Therefore, the solution of the equation is 3.

The TI-83 Plus has $y = \log_{10} x$ as a built-in function. Enter $\boxed{\text{Y=}}$ $\boxed{\text{LOG}}$ $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{\text{GRAPH}}$ to view this graph. To graph logarithmic functions with bases other than 10, you must use the Change of Base Formula,

$$\log_a n = \frac{\log_b n}{\log_b a}.$$

For example, $\log_3 x = \frac{\log_{10} x}{\log_{10} 3}$, so to graph $y = \log_3 x$ you must enter $\boxed{\text{LOG}}$ $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{\text{LOG}}$ 3 $\boxed{)}$ as Y1.



[−2, 8] scl: 1 by [−5, 5] scl: 1

www.algebra2.com/other_calculator_keystrokes

Investigation

Example 2

Solve $\log_2 2x \geq \log_{\frac{1}{2}} 2x$ by graphing.

Step 1 Rewrite the problem as a system of common logarithmic inequalities.

- The first inequality is $\log_2 2x \geq y$ or $y \leq \log_2 2x$. The second inequality is $y \geq \log_{\frac{1}{2}} 2x$.
- Use the Change of Base Formula to create equations that can be entered into the calculator.

$$\log_2 2x = \frac{\log 2x}{\log 2} \qquad \log_{\frac{1}{2}} 2x = \frac{\log 2x}{\log \frac{1}{2}}$$

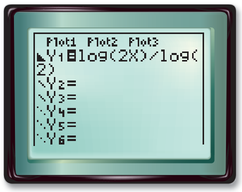
Thus, the two inequalities are $y \leq \frac{\log 2x}{\log 2}$ and $y \geq \frac{\log 2x}{\log \frac{1}{2}}$.

Step 2 Enter the first inequality.

- Enter $y \leq \frac{\log 2x}{\log 2}$ as Y1. Since the inequality includes *less than*, shade below the curve.

KEYSTROKES: Y= LOG 2 X,T,θ,n) ÷ LOG 2)

Use the arrow and ENTER keys to choose the shade below icon, ▙.



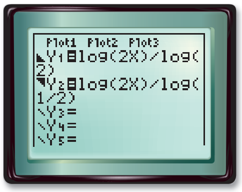
```
 Plot1 Plot2 Plot3
▙Y1■log(2X)/log(
2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
```

Step 3 Enter the second inequality.

- Enter $y \geq \frac{\log 2x}{\log \frac{1}{2}}$ as Y2. Since the inequality includes *greater than*, shade above the curve.

KEYSTROKES: LOG 2 X,T,θ,n) ÷ LOG 1 ÷ 2) GRAPH

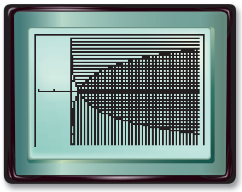
Use the arrow and ENTER keys to choose the shade above icon, ◥.



```
 Plot1 Plot2 Plot3
▙Y1■log(2X)/log(
2)
◥Y2■log(2X)/log(
1/2)
\Y3=
\Y4=
\Y5=
```

Step 4 Graph the inequalities.

KEYSTROKES: GRAPH



[−2, 8] scl: 1 by [−5, 5] scl: 1

The x values of the points in the region where the shadings overlap is the solution set of the original inequality. Using the calculator's **intersect** feature, you can conclude that the solution set is $\{x \mid x \geq 0.5\}$.

Exercises

Solve each equation or inequality by graphing. **7. $x \geq 6$**

1. $3.5^{x+2} = 1.75^{x+3}$ **−1.2**

2. $-3^{x+4} = -0.5^{2x+3}$ **−2.6**

3. $6^{2-x} - 4 = -0.25^{x-2.5}$ **1.8**

4. $3^x - 4 = 5^{\frac{x}{2}}$ **2**

5. $\log_2 3x = \log_3 (2x + 2)$ **0.7**

6. $2^{x-2} \geq 0.5^{x-3}$ **$x \geq 2.5$**

7. $\log_3 (3x - 5) \geq \log_3 (x + 7)$

8. $5^{x+3} \leq 2^{x+4}$ **$x \leq -2.24$**

9. $\log_2 2x \leq \log_4 (x + 3)$ **$0 < x \leq 1$**

Assess

In **Exercise 9**, check that students record the inequalities in the solution set correctly. In particular, students must include the fact that x must be greater than 0.

10-5 Lesson Notes

1 Focus

5-Minute Check Transparency 10-5 Use as a quiz or review of Lesson 10-4.

Mathematical Background notes are available for this lesson on p. 520D.

How is the natural base *e* used in banking?

Ask students:

- What is the formula calculating? **the amount of money in the account**
- Why does the interest increase as the time between compounding periods decreases? **Sample answer: Interest is earned not just on the initial $1 but also on the total interest that has accrued. As the compounding occurs more often, the amount of money earning interest grows faster.**

10-5 Base *e* and Natural Logarithms

What You'll Learn

- Evaluate expressions involving the natural base and natural logarithms.
- Solve exponential equations and inequalities using natural logarithms.

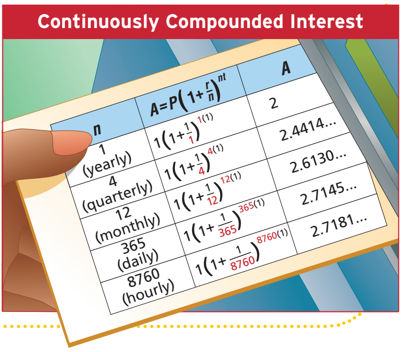
How is the natural base *e* used in banking?

Vocabulary

- natural base, *e*
- natural base exponential function
- natural logarithm
- natural logarithmic function

Suppose a bank compounds interest on accounts *continuously*, that is, with no waiting time between interest payments. In order to develop an equation to determine continuously compounded interest, examine what happens to the value A of an account for increasingly larger numbers of compounding periods n. Use a principal P of $1, an interest rate r of 100% or 1, and time t of 1 year.

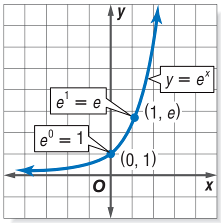
Continuously Compounded Interest



n	$A=P\left(1+\frac{r}{n}\right)^{nt}$	A
1 (yearly)	$1\left(1+\frac{1}{1}\right)^{1(1)}$	2
4 (quarterly)	$1\left(1+\frac{1}{4}\right)^{4(1)}$	2.4414...
12 (monthly)	$1\left(1+\frac{1}{12}\right)^{12(1)}$	2.6130...
365 (daily)	$1\left(1+\frac{1}{365}\right)^{365(1)}$	2.7145...
8760 (hourly)	$1\left(1+\frac{1}{8760}\right)^{8760(1)}$	2.7181...

BASE *e* AND NATURAL LOGARITHMS In the table above, as n increases, the expression $1\left(1+\frac{1}{n}\right)^{n(1)}$ or $\left(1+\frac{1}{n}\right)^{n}$ approaches the irrational number 2.71828... . This number is referred to as the **natural base, *e***.

An exponential function with base e is called a **natural base exponential function**. The graph of $y = e^x$ is shown at the right. Natural base exponential functions are used extensively in science to model quantities that grow and decay continuously.



Most calculators have an e^x function for evaluating natural base expressions.

Study Tip

Simplifying Expressions with e

You can simplify expressions involving e in the same manner in which you simplify expressions involving π.

Examples:

- $\pi^2 \cdot \pi^3 = \pi^5$
- $e^2 \cdot e^3 = e^5$

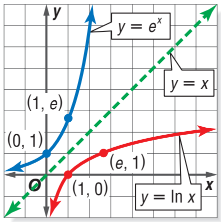
Example 1 Evaluate Natural Base Expressions

Use a calculator to evaluate each expression to four decimal places.

a. e^2 KEYSTROKES: 2nd [e^x] 2 ENTER 7.389056099 about 7.3891

b. $e^{-1.3}$ KEYSTROKES: 2nd [e^x] -1.3 ENTER .272531793 about 0.2725

The logarithm with base e is called the **natural logarithm**, sometimes denoted by $\log_e x$, but more often abbreviated $\ln x$. The **natural logarithmic function**, $y = \ln x$, is the inverse of the natural base exponential function, $y = e^x$. The graph of these two functions shows that $\ln 1 = 0$ and $\ln e = 1$.



Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters

- Study Guide and Intervention, pp. 597–598
- Skills Practice, p. 599
- Practice, p. 600
- Reading to Learn Mathematics, p. 601
- Enrichment, p. 602
- Assessment, p. 624

Science and Mathematics Lab Manual, pp. 127–132

Transparencies

5-Minute Check Transparency 10-5
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 19
Interactive Chalkboard

Most calculators have an LN key for evaluating natural logarithms.

Example 2 Evaluate Natural Logarithmic Expressions

Use a calculator to evaluate each expression to four decimal places.

a. **ln 4** KEYSTROKES: LN 4 ENTER 1.386294361 about 1.3863

b. **ln 0.05** KEYSTROKES: LN 0.05 ENTER −2.995732274 about −2.9957

You can write an equivalent base *e* exponential equation for a natural logarithmic equation and vice versa by using the fact that $\ln x = \log_e x$.

Example 3 Write Equivalent Expressions

Write an equivalent exponential or logarithmic equation.

a. $e^x = 5$

$e^x = 5 \rightarrow \log_e 5 = x$

$\ln 5 = x$

b. $\ln x \approx 0.6931$

$\ln x \approx 0.6931 \rightarrow \log_e x \approx 0.6931$

$x \approx e^{0.6931}$

Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to "undo" each other.

$$e^{\ln x} = x \qquad \ln e^x = x$$

Example 4 Inverse Property of Base e and Natural Logarithms

Evaluate each expression.

a. $e^{\ln 7}$

$e^{\ln 7} = 7$

b. $\ln e^{4x+3}$

$\ln e^{4x+3} = 4x + 3$

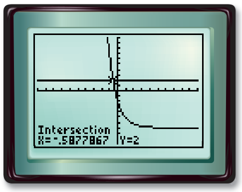
EQUATIONS AND INEQUALITIES WITH *e* AND ln Equations and inequalities involving base *e* are easier to solve using natural logarithms than using common logarithms. All of the properties of logarithms that you have learned apply to natural logarithms as well.

Example 5 Solve Base e Equations

Solve $5e^{-x} - 7 = 2$.

$5e^{-x} - 7 = 2$	Original equation
$5e^{-x} = 9$	Add 7 to each side.
$e^{-x} = \frac{9}{5}$	Divide each side by 5.
$\ln e^{-x} = \ln \frac{9}{5}$	Property of Equality for Logarithms
$-x = \ln \frac{9}{5}$	Inverse Property of Exponents and Logarithms
$x = -\ln \frac{9}{5}$	Divide each side by −1.
$x \approx -0.5878$	Use a calculator.

The solution is about −0.5878.



```
Intersection
X=-.5877867   Y=2
```

CHECK You can check this value by substituting −0.5878 into the original equation or by finding the intersection of the graphs of $y = 5e^{-x} - 7$ and $y = 2$.

www.algebra2.com/extra_examples

2 Teach

BASE e AND NATURAL LOGARITHMS

Teaching Tip Stress that *e* is a constant like π, and not a variable like *x* or *y*.

In-Class Examples

Power Point®

1 Use a calculator to evaluate each expression to four decimal places.

a. $e^{0.5}$ **about 1.6487**

b. e^{-8} **about 0.0003**

2 Use a calculator to evaluate each expression to four decimal places.

a. ln 3 **about 1.0986**

b. $\ln \frac{1}{4}$ **about −1.3863**

3 Write an equivalent exponential or logarithmic equation.

a. $e^x = 23$ $\ln 23 = x$

b. $\ln x \approx 1.2528$ $x \approx e^{1.2528}$

4 Evaluate each expression.

a. $e^{\ln 21}$ **21**

b. $\ln e^{x^2 - 1}$ $x^2 - 1$

EQUATIONS AND INEQUALITIES WITH e AND ln

In-Class Example

Power Point®

5 Solve $3e^{-2x} + 4 = 10$. $x \approx -0.3466$

In-Class Examples

Power Point®

6 **SAVINGS** Suppose you deposit \$700 into an account paying 6% annual interest, compounded continuously.

a. What is the balance after 8 years? **\$1131.25**

b. How long will it take for the balance in your account to reach at least \$2000? **at least 17.5 years**

7 Solve each equation or inequality.

a. $\ln 3x = 0.5$ **about 0.5496**

b. $\ln(2x - 3) < 2.5$ **$1.5 < x < 7.5912$**

Study Tip

Continuously Compounded Interest

Although no banks actually pay interest compounded continuously, the equation $A = Pe^{rt}$ is so accurate in computing the amount of money for quarterly compounding, or daily compounding, that it is often used for this purpose.

When interest is compounded continuously, the amount A in an account after t years is found using the formula $A = Pe^{rt}$, where P is the amount of principal and r is the annual interest rate.

Example 6 Solve Base e Inequalities

SAVINGS **Suppose you deposit \$1000 in an account paying 5% annual interest, compounded continuously.**

a. What is the balance after 10 years?

$A = Pe^{rt}$	Continuous compounding formula
$= 1000e^{(0.05)(10)}$	Replace P with 1000, r with 0.05, and t with 10.
$= 1000e^{0.5}$	Simplify.
≈ 1648.72	Use a calculator.

The balance after 10 years would be \$1648.72.

b. How long will it take for the balance in your account to reach at least \$1500?

$$\underbrace{\text{The balance}}_{A}\ \underbrace{\text{is at least}}_{\geq}\ \underbrace{\$1500.}_{1500}$$

$A \geq 1500$	Write an inequality.
$1000e^{(0.05)t} \geq 1500$	Replace A with $1000e^{(0.05)t}$.
$e^{(0.05)t} \geq 1.5$	Divide each side by 1000.
$\ln e^{(0.05)t} \geq \ln 1.5$	Property of Equality for Logarithms
$0.05t \geq \ln 1.5$	Inverse Property of Exponents and Logarithms
$t \geq \frac{\ln 1.5}{0.05}$	Divide each side by 0.05.
$t \geq 8.11$	Use a calculator.

It will take at least 8.11 years for the balance to reach \$1500.

Study Tip

Equations with ln

As with other logarithmic equations, remember to check for extraneous solutions.

Example 7 Solve Natural Log Equations and Inequalities

Solve each equation or inequality.

a. $\ln 5x = 4$

$\ln 5x = 4$	Original equation
$e^{\ln 5x} = e^4$	Write each side using exponents and base e.
$5x = e^4$	Inverse Property of Exponents and Logarithms
$x = \frac{e^4}{5}$	Divide each side by 5.
$x \approx 10.9196$	Use a calculator.

The solution is 10.9196. Check this solution using substitution or graphing.

b. $\ln(x - 1) > -2$

$\ln(x - 1) > -2$	Original inequality
$e^{\ln(x - 1)} > e^{-2}$	Write each side using exponents and base e.
$x - 1 > e^{-2}$	Inverse Property of Exponents and Logarithms
$x > e^{-2} + 1$	Add 1 to each side.
$x > 1.1353$	Use a calculator.

The solution is all numbers greater than about 1.1353. Check this solution using substitution.

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Using plastic coins and paper currency, have pairs of students begin with \$10, choose an interest rate, and calculate how much they will have after 5, 10, 15, and 20 years. After each calculation, have students model the amount with their money to help them visualize the growth over time.

Check for Understanding

Concept Check

1. **Name** the base of natural logarithms. **the number e**

2. **OPEN ENDED** Give an example of an exponential equation that requires using natural logarithms instead of common logarithms to solve. **Sample answer: $e^x = 8$**

3. **FIND THE ERROR** Colby and Elsu are solving $\ln 4x = 5$.

3. Elsu; Colby tried to write each side as a power of 10. Since the base of the natural logarithmic function is e, he should have written each side as a power of e; $10^{\ln 4x} \neq 4x$.

Colby

$\ln 4x = 5$

$10^{\ln 4x} = 10^5$

$4x = 100{,}000$

$x = 25{,}000$

Elsu

$\ln 4x = 5$

$e^{\ln 4x} = e^5$

$4x = e^5$

$x = \frac{e^5}{4}$

$x \approx 37.1033$

Who is correct? Explain your reasoning.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1, 2
8, 9	3
10, 11	4
12–17	5–7
18, 19	5

Use a calculator to evaluate each expression to four decimal places.

4. e^6 **403.4288**
5. $e^{-3.4}$ **0.0334**
6. $\ln 1.2$ **0.1823**
7. $\ln 0.1$ **−2.3026**

Write an equivalent exponential or logarithmic equation.

8. $e^x = 4$ **$x = \ln 4$**
9. $\ln 1 = 0$ **$e^0 = 1$**

Evaluate each expression.

10. $e^{\ln 3}$ **3**
11. $\ln e^{5x}$ **$5x$**

Solve each equation or inequality. **15. $0 < x < 403.4288$**

12. $e^x > 30$ **$x > 3.4012$**
13. $2e^x - 5 = 1$ **1.0986**
14. $3 + e^{-2x} = 8$ **−0.8047**
15. $\ln x < 6$
16. $2 \ln 3x + 1 = 5$ **2.4630**
17. $\ln x^2 = 9$ **±90.0171**

Application

ALTITUDE **For Exercises 18 and 19, use the following information.**
The altimeter in an airplane gives the altitude or height h (in feet) of a plane above sea level by measuring the outside air pressure P (in kilopascals). The height and air pressure are related by the model $P = 101.3\, e^{-\frac{h}{26{,}200}}$

18. $h = -26{,}200 \ln \frac{P}{101.3}$

18. Find a formula for the height in terms of the outside air pressure.

19. Use the formula you found in Exercise 18 to approximate the height of a plane above sea level when the outside air pressure is 57 kilopascals. **about 15,066 ft**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
20–29	1, 2
30–33	3
34–37	4
38–53	5–7
54–57	6
58–61	3, 5

Extra Practice

See page 850.

Use a calculator to evaluate each expression to four decimal places.

20. e^4 **54.5982**
21. e^5 **148.4132**
22. $e^{-1.2}$ **0.3012**
23. $e^{0.5}$ **1.6487**
24. $\ln 3$ **1.0986**
25. $\ln 10$ **2.3026**
26. $\ln 5.42$ **1.6901**
27. $\ln 0.03$ **−3.5066**

28. **SAVINGS** If you deposit \$150 in a savings account paying 4% interest compounded continuously, how much money will you have after 5 years? Use the formula presented in Example 6. **\$183.21**

29. **PHYSICS** The equation $\ln \frac{I_0}{I} = 0.014d$ relates the intensity of light at a depth of d centimeters of water I with the intensity in the atmosphere I_0. Find the depth of the water where the intensity of light is half the intensity of the light in the atmosphere. **about 49.5 cm**

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 10.
- include examples of how to evaluate expressions containing natural logarithms.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Make sure students can identify what Colby did incorrectly. Point out that raising both terms to base 10 is not incorrect but the step that follows incorrectly states that $10^{\ln 4x} = 4x$.

About the Exercises…

Organization by Objective
- **Base e and Natural Logarithms:** 20–37
- **Equations and Inequalities with e and ln:** 38–61

Odd/Even Assignments
Exercises 20–53 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 21–51 odd, 54–59, 62–80

Average: 21–53 odd, 54–59, 62–80

Advanced: 20–52 even, 54–74 (optional: 75–80)

All: Practice Quiz 2 (1–5)

Study Guide and Intervention, p. 597 (shown) and p. 598

Base e and Natural Logarithms The irrational number $e \approx 2.71828\ldots$ often occurs as the base for exponential and logarithmic functions that describe real-world phenomena.

Natural Base e	As n increases, $\left(1 + \frac{1}{n}\right)^n$ approaches $e \approx 2.71828\ldots$. $\ln x = \log_e x$

The functions $y = e^x$ and $y = \ln x$ are inverse functions.

Inverse Property of Base e and Natural Logarithms	$e^{\ln x} = x \qquad \ln e^x = x$

Natural base expressions can be evaluated using the e^x and ln keys on your calculator.

Example 1 **Evaluate ln 1685.**

Use a calculator.
$\ln 1685 \approx 7.4295$

Example 2 **Write a logarithmic equation equivalent to $e^{2x} = 7$.**

$e^{2x} = 7 \rightarrow \log_e 7 = 2x$ or $2x = \ln 7$

Example 3 **Evaluate $\ln e^{18}$.**

Use the Inverse Property of Base e and Natural Logarithms.
$\ln e^{18} = 18$

Exercises

Use a calculator to evaluate each expression to four decimal places.

1. $\ln 732$ 6.5958
2. $\ln 84{,}350$ 11.3427
3. $\ln 0.735$ -0.3079
4. $\ln 100$ 4.6052
5. $\ln 0.0824$ -2.4962
6. $\ln 2.388$ 0.8705
7. $\ln 128{,}245$ 11.7617
8. $\ln 0.00614$ -5.0929

Write an equivalent exponential or logarithmic equation.

9. $e^{15} = x$ $\ln x = 15$
10. $e^{3x} = 45$ $3x = \ln 45$
11. $\ln 20 = x$ $e^x = 20$
12. $\ln x = 8$ $x = e^8$
13. $e^{-5x} = 0.2$ $-5x = \ln 0.2$
14. $\ln (4x) = 9.6$ $4x = e^{9.6}$
15. $e^{8.2} = 10x$ $\ln 10x = 8.2$
16. $\ln 0.0002 = x$ $e^x = 0.0002$

Evaluate each expression.

17. $\ln e^3$ 3
18. $e^{\ln 42}$ 42
19. $e^{\ln 0.5}$ 0.5
20. $\ln e^{16.2}$ 16.2

Skills Practice, p. 599 and Practice, p. 600 (shown)

Use a calculator to evaluate each expression to four decimal places.

1. $e^{1.5}$ 4.4817
2. $\ln 8$ 2.0794
3. $\ln 3.2$ 1.1632
4. $e^{-0.6}$ 0.5488
5. $e^{4.2}$ 66.6863
6. $\ln 1$ 0
7. $e^{-2.5}$ 0.0821
8. $\ln 0.037$ -3.2968

Write an equivalent exponential or logarithmic equation.

9. $\ln 50 = x$ $e^x = 50$
10. $\ln 36 = 2x$ $e^{2x} = 36$
11. $\ln 6 \approx 1.7918$ $e^{1.7918} \approx 6$
12. $\ln 9.3 \approx 2.2300$ $e^{2.2300} \approx 9.3$
13. $e^x = 8$ $x = \ln 8$
14. $e^5 = 10x$ $5 = \ln 10x$
15. $e^{-x} = 4$ $x = -\ln 4$
16. $e^2 = x + 1$ $2 = \ln (x + 1)$

Evaluate each expression.

17. $e^{\ln 12}$ 12
18. $e^{\ln 3x}$ $3x$
19. $\ln e^{-1}$ -1
20. $\ln e^{-2y}$ $-2y$

Solve each equation or inequality.

21. $e^x < 9$ $\{x|x < 2.1972\}$
22. $e^{-x} = 31$ -3.4340
23. $e^x = 1.1$ 0.0953
24. $e^x = 5.8$ 1.7579
25. $2e^x - 3 = 1$ 0.6931
26. $5e^x + 1 \geq 7$ $\{x|x \geq 0.1823\}$
27. $4 + e^x = 19$ 2.7081
28. $-3e^x + 10 < 8$ $\{x|x > -0.4055\}$
29. $e^{3x} = 8$ 0.6931
30. $e^{-4x} = 5$ -0.4024
31. $e^{0.5x} = 6$ 3.5835
32. $2e^{5x} = 24$ 0.4970
33. $e^{2x} + 1 = 55$ 1.9945
34. $e^{3x} - 5 = 32$ 1.2036
35. $9 + e^{2x} = 10$ 0
36. $e^{-3x} + 7 \geq 15$ $\{x|x \leq -0.6931\}$
37. $\ln 4x = 3$ 5.0214
38. $\ln (-2x) = 7$ -548.3166
39. $\ln 2.5x = 10$ 8810.5863
40. $\ln (x - 6) = 1$ 8.7183
41. $\ln (x + 2) = 3$ 18.0855
42. $\ln (x + 3) = 5$ 145.4132
43. $\ln 3x + \ln 2x = 9$ 36.7493
44. $\ln 5x + \ln x = 7$ 14.8097

INVESTING For Exercises 45 and 46, use the formula for continuously compounded interest, $A = Pe^{rt}$, where P is the principal, r is the annual interest rate, and t is the time in years.

45. If Sarita deposits $1000 in an account paying 3.4% annual interest compounded continuously, what is the balance in the account after 5 years? $1185.30

46. How long will it take the balance in Sarita's account to reach $2000? about 20.4 yr

47. **RADIOACTIVE DECAY** The amount of a radioactive substance y that remains after t years is given by the equation $y = ae^{kt}$, where a is the initial amount present and k is the decay constant for the radioactive substance. If $a = 100$, $y = 50$, and $k = -0.035$, find t. about 19.8 yr

Reading to Learn Mathematics, p. 601

ELL

Pre-Activity **How is the natural base e used in banking?**

Read the introduction to Lesson 10-5 at the top of page 554 in your textbook.

Suppose that you deposit $675 in a savings account that pays an annual interest rate of 5%. In each case listed below, indicate which method of compounding would result in more money in your account at the end of one year.

a. annual compounding or monthly compounding monthly
b. quarterly compounding or daily compounding daily
c. daily compounding or continuous compounding continuous

Reading the Lesson

1. Jagdish entered the following keystrokes in his calculator:

LN 5) ENTER

The calculator returned the result 1.609437912. Which of the following conclusions are correct? d and f

a. The common logarithm of 5 is about 1.6094.
b. The natural logarithm of 5 is exactly 1.609437912.
c. The base 5 logarithm of e is about 1.6094.
d. The natural logarithm of 5 is about 1.609438.
e. $10^{1.609437912}$ is very close to 5.
f. $e^{1.609437912}$ is very close to 5.

2. Match each expression from the first column with its value in the second column. Some choices may be used more than once or not at all.

a. $e^{\ln 5}$ IV	I. 1
b. $\ln 1$ V	II. 10
c. $e^{\ln e}$ VI	III. -1
d. $\ln e^5$ IV	IV. 5
e. $\ln e$ I	V. 0
f. $\ln \left(\frac{1}{e}\right)$ III	VI. e

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose that you are studying with a classmate who is puzzled when asked to evaluate $\ln e^3$. How would you explain to him an easy way to figure this out? **Sample answer: *ln* means *natural log*. The natural log of e^3 is the power to which you raise e to get e^3. This is obviously 3.**

More About. . .

Money

To determine the doubling time on an account paying an interest rate r that is compounded *annually*, investors use the "Rule of 72." Thus, the amount of time needed for the money in an account paying 6% interest compounded annually to double is $\frac{72}{6}$ or 12 years.

Source: www.datachimp.com

Write an equivalent exponential or logarithmic equation.

30. $e^{-x} = 5$ $-x = \ln 5$
31. $e^2 = 6x$ $2 = \ln 6x$
32. $\ln e = 1$ $e^1 = e$
33. $\ln 5.2 = x$ $e^x = 5.2$

Evaluate each expression.

34. $e^{\ln 0.2}$ 0.2
35. $e^{\ln y}$ y
36. $\ln e^{-4x}$ $-4x$
37. $\ln e^{45}$ 45

Solve each equation or inequality.

38. $3e^x + 1 = 5$ 0.2877
39. $2e^x - 1 = 0$ -0.6931
40. $e^x < 4.5$ $x < 1.5041$
41. $e^x > 1.6$ $x > 0.4700$
42. $-3e^{4x} + 11 = 2$ 0.2747
43. $8 + 3e^{3x} = 26$ 0.5973
44. $e^{5x} \geq 25$ $x \geq 0.6438$
45. $e^{-2x} \leq 7$ $x \geq -0.9730$
46. $\ln 2x = 4$ 27.2991
47. $\ln 3x = 5$ 49.4711
48. $\ln (x + 1) = 1$ 1.7183
49. $\ln (x - 7) = 2$ 14.3891
50. $\ln x + \ln 3x = 12$ 232.9197
51. $\ln 4x + \ln x = 9$ 45.0086

★ 52. $\ln (x^2 + 12) = \ln x + \ln 8$ 2, 6

★ 53. $\ln x + \ln (x + 4) = \ln 5$ 1

MONEY **For Exercises 54–57, use the formula for continuously compounded interest found in Example 6.** 55. $t = \frac{100 \ln 2}{r}$

54. If you deposit $100 in an account paying 3.5% interest compounded continuously, how long will it take for your money to double? about 19.8 yr

55. Suppose you deposit A dollars in an account paying an interest rate r as a percent, compounded continuously. Write an equation giving the time t needed for your money to double, or the *doubling time*.

56. Explain why the equation you found in Exercise 55 might be referred to as the "Rule of 70." $100 \ln 2 \approx 70$

57. **MAKE A CONJECTURE** State a rule that could be used to approximate the amount of time t needed to triple the amount of money in a savings account paying r percent interest compounded continuously. $t = \frac{110}{r}$

POPULATION **For Exercises 58 and 59, use the following information.**

In 2000, the world's population was about 6 billion. If the world's population continues to grow at a constant rate, the future population P, in billions, can be predicted by $P = 6e^{0.02t}$, where t is the time in years since 2000. 58. about 7.33 billion

58. According to this model, what will the world's population be in 2010?

59. Some experts have estimated that the world's food supply can support a population of, at most, 18 billion. According to this model, for how many more years will the world's population remain at 18 billion or less? about 55 yr

Online Research **Data Update** What is the current world population? Visit www.algebra2.com/data_update to learn more.

RUMORS **For Exercises 60 and 61, use the following information.**

The number of people H who have heard a rumor can be approximated by $H = \frac{P}{1 + (P - S)e^{-0.35t}}$, where P is the total population, S is the number of people who start the rumor, and t is the time in minutes. Suppose two students start a rumor that the principal will let everyone out of school one hour early that day.

60. If there are 1600 students in the school, how many students will have heard the rumor after 10 minutes? about 32 students

61. How much time will pass before half of the students have heard the rumor? about 21 min

62. **CRITICAL THINKING** Determine whether the following statement is *sometimes*, *always*, or *never* true. Explain your reasoning. Always; see pp. 573A-573D.

For all positive numbers x and y, $\frac{\log x}{\log y} = \frac{\ln x}{\ln y}$.

Enrichment, p. 602

Approximations for π and e

The following expression can be used to approximate e. If greater and greater values of n are used, the value of the expression approximates e more and more closely.

$$\left(1 + \frac{1}{n}\right)^n$$

Another way to approximate e is to use this infinite sum. The greater the value of n, the closer the approximation.

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \ldots + \frac{1}{2 \cdot 3 \cdot 4 \cdot \ldots \cdot n} + \ldots$$

In a similar manner, π can be approximated using an infinite product discovered by the English mathematician John Wallis (1616–1703).

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \ldots \cdot \frac{2n}{2n - 1} \cdot \frac{2n}{2n + 1} \ldots$$

Solve each problem.

1. Use a calculator with an e^x key to find e to 7 decimal places. 2.7182818

63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How is the natural base e used in banking?

Include the following in your answer:

- an explanation of how to calculate the value of an account whose interest is compounded continuously, and
- an explanation of how to use natural logarithms to find when the account will have a specified value.

Standardized Test Practice

64. If $e^x \neq 1$ and $e^{x^2} = \frac{1}{(\sqrt{2})^x}$, what is the value of x? **B**

Ⓐ −1.41 Ⓑ −0.35 Ⓒ 1.00 Ⓓ 1.10

65. **SHORT RESPONSE** The population of a certain country can be modeled by the equation $P(t) = 40\ e^{0.02t}$, where P is the population in millions and t is the number of years since 1900. When will the population be 100 million, 200 million, and 400 million? What do you notice about these time periods? **1946, 1981, 2015; It takes between 34 and 35 years for the population to double.**

Maintain Your Skills

Mixed Review

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places. *(Lesson 10-4)*

66. $\log_4 68$ **67.** $\log_6 0.047$ **68.** $\log_{50} 23$

66. $\frac{\log 68}{\log 4} = 3.0437$

67. $\frac{\log 0.047}{\log 6} = -1.7065$

68. $\frac{\log 23}{\log 50} = 0.8015$

Solve each equation. Check your solutions. *(Lesson 10-3)*

69. $\log_3 (a + 3) + \log_3 (a - 3) = \log_3 16$ **5** **70.** $\log_{11} 2 + 2 \log_{11} x = \log_{11} 32$ **4**

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation. *(Lesson 9-4)*

71. $mn = 4$ **inverse, 4** **72.** $\frac{a}{b} = c$ **joint, 1** **73.** $y = -7x$ **direct, −7**

74. **COMMUNICATION** A microphone is placed at the focus of a parabolic reflector to collect sounds for the television broadcast of a football game. The focus of the parabola that is the cross section of the reflector is 5 inches from the vertex. The latus rectum is 20 inches long. Assuming that the focus is at the origin and the parabola opens to the right, write the equation of the cross section. *(Lesson 8-2)* $x = \frac{1}{20}y^2 - 5$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation or inequality.**
*(To review **exponential equations and inequalities**, see Lesson 10-1.)*

75. $2^x = 10$ **3.32** **76.** $5^x = 12$ **1.54** **77.** $6^x = 13$ **1.43**

78. $2(1 + 0.1)^x = 50$ **323.49** **79.** $10(1 + 0.25)^x = 200$ **13.43** **80.** $400(1 - 0.2)^x = 50$ **9.32**

Practice Quiz 2

Lessons 10-3 through 10-5

1. Express $\log_4 5$ in terms of common logarithms. Then approximate its value to four decimal places. *(Lesson 10-4)* $\frac{\log 5}{\log 4}$; **1.1610**

2. Write an equivalent exponential equation for $\ln 3x = 2$. *(Lesson 10-5)* $e^2 = 3x$

Solve each equation or inequality. *(Lesson 10-3 through 10-5)*

3. $\log_2 (9x + 5) = 2 + \log_2 (x^2 - 1)$ **3** **4.** $2^{x-3} > 5$ $x > 5.3219$ **5.** $2e^x - 1 = 7$ **1.3863**

4 Assess

Open-Ended Assessment

Speaking Ask students to explain how evaluating expressions involving base e and natural logarithms is similar to evaluating expressions involving common logarithms and base 10, and also how they differ.

Getting Ready for Lesson 10-6

PREREQUISITE SKILL Students will encounter exponential growth and decay problems in Lesson 10-6. They will be required to solve exponential equations and inequalities. Use Exercises 75–80 to determine your students' familiarity with solving exponential equations and inequalities.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 10-3 through 10-5. Lesson numbers are given to the right of the exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 10-4 and 10-5) is available on p. 624 of the *Chapter 10 Resource Masters*.

Answer

63. The number e is used in the formula for continuously compounded interest, $A = Pe^{rt}$. Although no banks actually pay interest compounded continually, the equation is so accurate in computing the amount of money for quarterly compounding, or daily compounding, that it is often used for this purpose. Answers should include the following.

- If you know the annual interest rate r and the principal P, the value of the account after t years is calculated by multiplying P times e raised to the r times t power. Use a calculator to find the value of e^{rt}.
- If you know the value A you wish the account to achieve, the principal P, and the annual interest rate r, the time t needed to achieve this value is found by first taking the natural logarithm of A minus the natural logarithm of P. Then, divide this quantity by r.

10-6 Lesson Notes

1 Focus

5-Minute Check Transparency 10-6 Use as a quiz or review of Lesson 10-5.

Mathematical Background notes are available for this lesson on p. 520D.

How can you determine the current value of your car?

Ask students:

- What kinds of items increase in value? **Sample answer: artwork, some trading cards, some collectibles**
- During which year does the car depreciate the most? **first year**
- How can the amount of depreciation be different each year when the percent of decrease is always the same? **In the first year, the car depreciates 16% of its value at the beginning of that year (when it was new). This is when its value is greatest, so the amount of depreciation is also the greatest during this year. The amount of depreciation decreases each year because the value of the car at the beginning of each year is less than it was at the beginning of the previous year.**

10-6 Exponential Growth and Decay

What You'll Learn

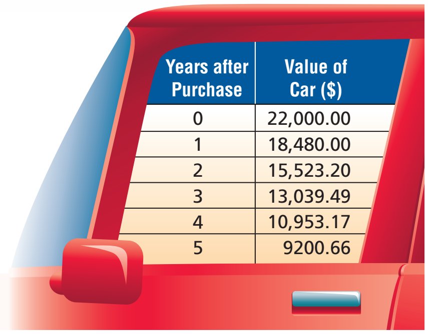
- Use logarithms to solve problems involving exponential decay.
- Use logarithms to solve problems involving exponential growth.

Vocabulary

- rate of decay
- rate of growth

How can you determine the current value of your car?

Certain assets, like homes, can *appreciate* or increase in value over time. Others, like cars, *depreciate* or decrease in value with time. Suppose you buy a car for $22,000 and the value of the car decreases by 16% each year. The table shows the value of the car each year for up to 5 years after it was purchased.



Years after Purchase	Value of Car ($)
0	22,000.00
1	18,480.00
2	15,523.20
3	13,039.49
4	10,953.17
5	9200.66

EXPONENTIAL DECAY The depreciation of the value of a car is an example of exponential decay. When a quantity *decreases* by a fixed percent each year, or other period of time, the amount y of that quantity after t years is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent of decrease expressed as a decimal. The percent of decrease r is also referred to as the **rate of decay**.

Example 1 Exponential Decay of the Form $y = a(1 - r)^t$

CAFFEINE **A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?**

Study Tip

Rate of Change
Remember to rewrite the rate of change as a decimal before using it in the formula.

Explore The problem gives the amount of caffeine consumed and the rate at which the caffeine is eliminated. It asks you to find the time it will take for half of the caffeine to be eliminated from a person's body.

Plan Use the formula $y = a(1 - r)^t$. Let t be the number of hours since drinking the coffee. The amount remaining y is half of 130 or 65.

Solve

$$
\begin{aligned}
y &= a(1 - r)^t && \text{Exponential decay formula}\\
65 &= 130(1 - 0.11)^t && \text{Replace } y \text{ with 65, } a \text{ with 130, and } r \text{ with 11\% or 0.11.}\\
0.5 &= (0.89)^t && \text{Divide each side by 130.}\\
\log 0.5 &= \log (0.89)^t && \text{Property of Equality for Logarithms}\\
\log 0.5 &= t \log (0.89) && \text{Product Property for Logarithms}\\
\frac{\log 0.5}{\log 0.89} &= t && \text{Divide each side by log 0.89.}\\
5.9480 &\approx t && \text{Use a calculator.}
\end{aligned}
$$

Resource Manager

Workbook and Reproducible Masters

Chapter 10 Resource Masters
- Study Guide and Intervention, pp. 603–604
- Skills Practice, p. 605
- Practice, p. 606
- Reading to Learn Mathematics, p. 607
- Enrichment, p. 608
- Assessment, p. 624

Graphing Calculator and Spreadsheet Masters, p. 46
School-to-Career Masters, p. 20
Teaching Algebra With Manipulatives Masters, p. 278

Transparencies

5-Minute Check Transparency 10-6
Answer Key Transparencies

Technology

Interactive Chalkboard

It will take approximately 6 hours for half of the caffeine to be eliminated from a person's body.

Examine Use the formula to find how much of the original 130 milligrams of caffeine would remain after 6 hours.

$y = a(1 - r)^t$ Exponential decay formula

$y = 130(1 - 0.11)^6$ Replace a with 130, r with 0.11, and t with 6.

$y \approx 64.6$ Use a calculator.

Half of 130 is 65, so the answer seems reasonable.

Another model for exponential decay is given by $y = ae^{-kt}$, where k is a constant. This is the model preferred by scientists. Use this model to solve problems involving radioactive decay.

Example 2 Exponential Decay of the Form $y = ae^{-kt}$

PALEONTOLOGY The *half-life* of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years. That is, every 5760 years half of a mass of Carbon-14 decays away.

a. What is the value of k for Carbon-14?

To determine the constant k for Carbon-14, let a be the initial amount of the substance. The amount y that remains after 5760 years is then represented by $\frac{1}{2}a$ or $0.5a$.

$y = ae^{-kt}$ Exponential decay formula

$0.5a = ae^{-k(5760)}$ Replace y with $0.5a$ and t with 5760.

$0.5 = e^{-5760k}$ Divide each side by a.

$\ln 0.5 = \ln e^{-5760k}$ Property of Equality for Logarithmic Functions

$\ln 0.5 = -5760k$ Inverse Property of Exponents and Logarithms

$\frac{\ln 0.5}{-5760} = k$ Divide each side by -5760.

$0.00012 \approx k$ Use a calculator.

The constant for Carbon-14 is 0.00012. Thus, the equation for the decay of Carbon-14 is $y = ae^{-0.00012t}$, where t is given in years.

b. A paleontologist examining the bones of a woolly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

Let a be the initial amount of Carbon-14 in the animal's body. Then the amount y that remains after t years is 3% of a or $0.03a$.

$y = ae^{-0.00012t}$ Formula for the decay of Carbon-14

$0.03a = ae^{-0.00012t}$ Replace y with $0.03a$.

$0.03 = e^{-0.00012t}$ Divide each side by a.

$\ln 0.03 = \ln e^{-0.00012t}$ Property of Equality for Logarithms

$\ln 0.03 = -0.00012t$ Inverse Property of Exponents and Logarithms

$\frac{\ln 0.03}{-0.00012} = t$ Divide each side by -0.00012.

$29{,}221 \approx t$ Use a calculator.

The mammoth lived about 29,000 years ago.

Career Choices

Paleontologist

Paleontologists study fossils found in geological formations. They use these fossils to trace the evolution of plant and animal life and the geologic history of Earth.

Online Research For information about a career as a paleontologist, visit: www.algebra2.com/careers

Source: U.S. Department of Labor

2 Teach

EXPONENTIAL DECAY

Teaching Tip In Example 1, point out that you are calculating how long until half the caffeine has been eliminated, which also means half the caffeine remains. If the value to be found is something other than half, students must be careful that they use the formula correctly.

In-Class Examples

Power Point®

1 **CAFFEINE** Refer to Example 1. How long will it take for 90% of this caffeine to be eliminated from a person's body? **about 20 h**

2 **GEOLOGY** The half-life of Sodium-22 is 2.6 years.

a. What is the value of k for Sodium-22? **about 0.2666**

b. A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of Earth? **about 9 years ago**

DAILY INTERVENTION

Differentiated Instruction

Logical Have students work in pairs or small groups. Ask them to examine the growth and decay formulas used in Examples 1–4 and to discuss how the equations are related. In particular, ask them to discuss how they can identify which equations are used for exponential decay situations (minus/negative sign) and which are used for exponential growth.

EXPONENTIAL GROWTH

In-Class Examples Power Point®

3 The population of a city of one million is increasing at a rate of 3% per year. If the population continues to grow at this rate, in how many years will the population have doubled? **D**

A 4 years **B** 5 years

C 20 years **D** 23 years

4 **POPULATION** As of 2000, Nigeria had an estimated population of 127 million people and the United States had an estimated population of 278 million people. The populations of Nigeria and the United States can be modeled by $N(t) = 127e^{0.026t}$ and $U(t) = 278e^{0.009t}$, respectively. According to these models, when will Nigeria's population be more than the population of the United States? **after 46 years or in 2046**

EXPONENTIAL GROWTH When a quantity *increases* by a fixed percent each time period, the amount y of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent of increase expressed as a decimal. The percent of increase r is also referred to as the **rate of growth**.

Standardized Test Practice

Example 3 Exponential Growth of the Form $y = a(1 + r)^t$

Multiple-Choice Test Item

In 1910, the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in 2010?

Ⓐ 138,000 Ⓑ 531,845

Ⓒ 1,063,690 Ⓓ 1.4×10^{11}

Read the Test Item

You need to find the population of the city 2010 − 1910 or 100 years later. Since the population is growing at a fixed percent each year, use the formula $y = a(1 + r)^t$.

Solve the Test Item

$y = a(1 + r)^t$ Exponential growth formula

$y = 120{,}000(1 + 0.015)^{100}$ Replace a = 120,000, r with 0.015, and t with 2010 − 1910 or 100.

$y = 120{,}000(1.015)^{100}$ Simplify.

$y \approx 531{,}845.48$ Use a calculator.

The answer is B.

The Princeton Review

Test-Taking Tip

To change a percent to a decimal, drop the percent symbol and move the decimal point two places to the left.

1.5% = 0.015

Another model for exponential growth, preferred by scientists, is $y = ae^{kt}$, where k is a constant. Use this model to find the constant k.

Example 4 Exponential Growth of the Form $y = ae^{kt}$

POPULATION As of 2000, China was the world's most populous country, with an estimated population of 1.26 billion people. The second most populous country was India, with 1.01 billion. The populations of India and China can be modeled by $I(t) = 1.01e^{0.015t}$ and $C(t) = 1.26e^{0.009t}$, respectively. According to these models, when will India's population be more than China's?

You want to find t such that $I(t) > C(t)$.

$I(t) > C(t)$

$1.01e^{0.015t} > 1.26e^{0.009t}$ Replace $I(t)$ with $1.01e^{0.015t}$ and $C(t)$ with $1.26e^{0.009t}$

$\ln 1.01e^{0.015t} > \ln 1.26e^{0.009t}$ Property of Inequality for Logarithms

$\ln 1.01 + \ln e^{0.015t} > \ln 1.26 + \ln e^{0.009t}$ Product Property of Logarithms

$\ln 1.01 + 0.015t > \ln 1.26 + 0.009t$ Inverse Property of Exponents and Logarithms

$0.006t > \ln 1.26 - \ln 1.01$ Subtract 0.009 from each side.

$t > \dfrac{\ln 1.26 - \ln 1.01}{0.006}$ Divide each side by 0.006.

$t > 36.86$ Use a calculator.

After 37 years or in 2037, India will be the most populous country in the world.

Standardized Test Practice

Example 3 On all standardized tests, students should look to identify any answer choices that can be logically eliminated. In Example 3, students can quickly determine that since 1% of 120,000 is 1200 and therefore 1.5% is 1800, over the 100 years from 1920 to 2010 the city's population will have increased by more than 1800(100) or 180,000 people. So Choice A is much too low. Choice D can also be eliminated, because 1.4×10^{11} written in standard notation is 140,000,000,000 (which is 140 billion). That's more than the population of the entire planet! So, the answer must be either Choice B or Choice C.

Check for Understanding

Concept Check

1. **Write** a general formula for exponential growth and decay where r is the percent of change. **$y = a(1 + r)^t$, where $r > 0$ represents exponential growth and $r < 0$ represents exponential decay**
2. **Explain** how to solve $y = (1 + r)^t$ for t. **See margin.**
3. **OPEN ENDED** Give an example of a quantity that grows or decays at a fixed rate. **Sample answer: money in a bank**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	2
7, 8	4
9	1, 3

SPACE **For Exercises 4–6, use the following information.**
A radioisotope is used as a power source for a satellite. The power output P (in watts) is given by $P = 50e^{-\frac{t}{250}}$, where t is the time in days.

4. Is the formula for power output an example of exponential growth or decay? Explain your reasoning. **Decay; the exponent is negative.**
5. Find the power available after 100 days. **about 33.5 watts**
6. Ten watts of power are required to operate the equipment in the satellite. How long can the satellite continue to operate? **about 402 days**

POPULATION GROWTH **For Exercises 7 and 8, use the following information.**
The city of Raleigh, North Carolina, grew from a population of 212,000 in 1990 to a population of 259,000 in 1998.

7. Write an exponential growth equation of the form $y = ae^{kt}$ for Raleigh, where t is the number of years after 1990. **$y = 212{,}000e^{0.025t}$**
8. Use your equation to predict the population of Raleigh in 2010. **about 349,529 people**

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

9. Suppose the weight of a bar of soap decreases by 2.5% each time it is used. If the bar weighs 95 grams when it is new, what is its weight to the nearest gram after 15 uses? **C**

Ⓐ 57.5 g Ⓑ 59.4 g Ⓒ 65 g Ⓓ 93 g

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
10	1
12–14,	2
11, 17–20	3
15, 16	4

Extra Practice

See page 851.

10. **COMPUTERS** Zeus Industries bought a computer for \$2500. It is expected to depreciate at a rate of 20% per year. What will the value of the computer be in 2 years? **\$1600**
11. **REAL ESTATE** The Martins bought a condominium for \$85,000. Assuming that the value of the condo will appreciate at most 5% a year, how much will the condo be worth in 5 years? **at most \$108,484.93**
12. **MEDICINE** Radioactive iodine is used to determine the health of the thyroid gland. It decays according to the equation $y = ae^{-0.0856t}$, where t is in days. Find the half-life of this substance. **about 8.1 days**
13. **PALEONTOLOGY** A paleontologist finds a bone that might be a dinosaur bone. In the laboratory, she finds that the Carbon-14 found in the bone is $\frac{1}{12}$ of that found in living bone tissue. Could this bone have belonged to a dinosaur? Explain your reasoning. (*Hint:* The dinosaurs lived from 220 million years ago to 63 million years ago.) **No; the bone is only about 21,000 years old, and dinosaurs died out 63,000,000 years ago.**
14. **ANTHROPOLOGY** An anthropologist finds there is so little remaining Carbon-14 in a prehistoric bone that instruments cannot measure it. This means that there is less than 0.5% of the amount of Carbon-14 the bones would have contained when the person was alive. How long ago did the person die?

14. more than 44,000 years ago

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 10.
- record the formulas for exponential growth and decay.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Exponential Decay:** 10–20
- **Exponential Growth:** 10–20

Assignment Guide

Basic: 11, 13, 17, 18, 21–40

Average: 11, 13, 15–18, 21–40

Advanced: 10–14 even, 15, 16, 19–40

Answer

2. Take the common logarithm of each side, use the Power Property to write $\log (1 + r)^t$ as $t \log (1 + r)$, and then divide each side by the quantity $\log (1 + r)$.

Study Guide and Intervention, p. 603 (shown) and p. 604

Exponential Decay Depreciation of value and radioactive decay are examples of **exponential decay**. When a quantity decreases by a fixed percent each time period, the amount of the quantity after t time periods is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent decrease expressed as a decimal.

Another exponential decay model often used by scientists is $y = ae^{-kt}$, where k is a constant.

Example **CONSUMER PRICES As technology advances, the price of many technological devices such as scientific calculators and camcorders goes down. One brand of hand-held organizer sells for $89.**

a. If its price decreases by 6% per year, how much will it cost after 5 years?

Use the exponential decay model with initial amount $89, percent decrease 0.06, and time 5 years.

$y = a(1 - r)^t$ Exponential decay formula

$y = 89(1 - 0.06)^5$ $a = 89, r = 0.06, t = 5$

$y = \$65.32$

After 5 years the price will be $65.32.

b. After how many years will its price be $50?

To find when the price will be $50, again use the exponential decay formula and solve for t.

$y = a(1 - r)^t$ Exponential decay formula

$50 = 89(1 - 0.06)^t$ $y = 50, a = 89, r = 0.06$

$\frac{50}{89} = (0.94)^t$ Divide each side by 89.

$\log\left(\frac{50}{89}\right) = \log (0.94)^t$ Property of Equality for Logarithms

$\log\left(\frac{50}{89}\right) = t \log 0.94$ Power Property

$t = \frac{\log\left(\frac{50}{89}\right)}{\log 0.94}$ Divide each side by log 0.94.

$t \approx 9.3$

The price will be $50 after about 9.3 years.

Exercises

1. **BUSINESS** A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a $500 item drop below $100? 6 weeks

CARBON DATING Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of Carbon-14, t is the number of years ago the animal lived, and y is the remaining amount after t years.

2. How old is a fossil remain that has lost 95% of its Carbon-14? about 25,000 years old
3. How old is a skeleton that has 95% of its Carbon-14 remaining? about 427.5 years old

Skills Practice, p. 605 and Practice, p. 606 (shown)

Solve each problem.

1. **INVESTING** The formula $A = P\left(1 + \frac{r}{2}\right)^{2t}$ gives the value of an investment after t years in an account that earns an annual interest rate r compounded twice a year. Suppose $500 is invested at 6% annual interest compounded twice a year. In how many years will the investment be worth $1000? about 11.7 yr
2. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000 if the growth rate per hour is 85%? about 7.5 h
3. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant k in the decay formula for the substance. about 0.02166
4. **DEPRECIATION** A piece of machinery valued at $250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to $100,000? about 7.2 yr
5. **INFLATION** For Dave to buy a new car comparably equipped to the one he bought 8 years ago would cost $12,500. Since Dave bought the car, the inflation rate for cars like his has been at an average annual rate of 5.1%. If Dave originally paid $8400 for the car, how long ago did he buy it? about 8 yr
6. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt-60, is radioactive and has a half-life of 5.7 years. Cobalt-60 is used to trace the path of nonradioactive substances in a system. What is the value of k for Cobalt-60? about 0.1216
7. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use $k = 0.00012$. about 50,000 yr
8. **POPULATION** The population of rabbits in an area is modeled by the growth equation $P(t) = 8e^{0.26t}$, where P is in thousands and t is in years. How long will it take for the population to reach 25,000? about 4.4 yr
9. **DEPRECIATION** A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally $12,000, in how many months is it worth $7350? about 12 mo
10. **BIOLOGY** In a laboratory, a culture increases from 30 to 195 organisms in 5 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$? about 45.4%

Reading to Learn Mathematics, p. 607

ELL

Pre-Activity **How can you determine the current value of your car?**

Read the introduction to Lesson 10-6 at the top of page 560 in your textbook.

- Between which two years shown in the table did the car depreciate by the greatest amount?
  between years 0 and 1
- Describe two ways to calculate the value of the car 6 years after it was purchased. (Do not actually calculate the value.)
  Sample answer: 1. Multiply $9200.66 by 0.16 and subtract the result from $9200.66. 2. Multiply $9200.66 by 0.84.

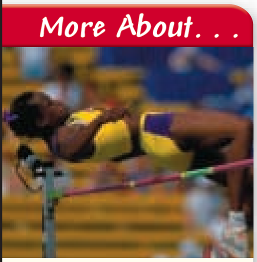
Reading the Lesson

1. State whether each situation is an example of exponential *growth* or *decay*.
   a. A city had 42,000 residents in 1980 and 128,000 residents in 2000. growth
   b. Raul compared the value of his car when he bought it new to the value when he traded ';lpit in six years later. decay
   c. A paleontologist compared the amount of carbon-14 in the skeleton of an animal when it died to the amount 300 years later. decay
   d. Maria deposited $750 in a savings account paying 4.5% annual interest compounded quarterly. She did not make any withdrawals or further deposits. She compared the balance in her passbook immediately after she opened the account to the balance 3 years later. growth
2. State whether each equation represents exponential growth or decay.
   a. $y = 5e^{0.15t}$ growth
   b. $y = 1000(1 - 0.05)^t$ decay
   c. $y = 0.3e^{-1200t}$ decay
   d. $y = 2(1 + 0.0001)^t$ growth

Helping You Remember

3. Visualizing their graphs is often a good way to remember the difference between mathematical equations. How can your knowledge of the graphs of exponential equations from Lesson 10-1 help you to remember that equations of the form $y = a(1 + r)^t$ represent exponential growth, while equations of the form $y = a(1 - r)^t$ represent exponential decay?
   Sample answer: If $a > 0$, the graph of $y = ab^x$ is always increasing if $b > 1$ and is always decreasing if $0 < b < 1$. Since r is always a positive number, if $b = 1 + r$, the base will be greater than 1 and the function will be increasing (growth), while if $b = 1 - r$, the base will be less than 1 and the function will be decreasing (decay).

More About. . .



Olympics

The women's high jump competition first took place in the USA in 1895, but it did not become an Olympic event until 1928.

Source: www.princeton.edu

BIOLOGY **For Exercises 15 and 16, use the following information.**
Bacteria usually reproduce by a process known as *binary fission*. In this type of reproduction, one bacterium divides, forming two bacteria. Under ideal conditions, some bacteria reproduce every 20 minutes. 15. about 0.0347

15. Find the constant k for this type of bacteria under ideal conditions.
16. Write the equation for modeling the exponential growth of this bacterium.
    $y = ae^{0.0347t}$

ECONOMICS **For Exercises 17 and 18, use the following information.**
The annual Gross Domestic Product (GDP) of a country is the value of all of the goods and services produced in the country during a year. During the period 1985–1999, the Gross Domestic Product of the United States grew about 3.2% per year, measured in 1996 dollars. In 1985, the GDP was $5717 billion.

17. Assuming this rate of growth continues, what will the GDP of the United States be in the year 2010? $12,565 billion
18. In what year will the GDP reach $20 trillion? about 2025
19. ★ **OLYMPICS** In 1928, when the high jump was first introduced as a women's sport at the Olympic Games, the winning women's jump was 62.5 inches, while the winning men's jump was 76.5 inches. Since then, the winning jump for women has increased by about 0.38% per year, while the winning jump for men has increased at a slower rate, 0.3%. If these rates continue, when will the women's winning high jump be higher than the men's? after the year 2182
20. ★ **HOME OWNERSHIP** The Mendes family bought a new house 10 years ago for $120,000. The house is now worth $191,000. Assuming a steady rate of growth, what was the yearly rate of appreciation? 4.7%
21. **CRITICAL THINKING** The half-life of Radium is 1620 years. When will a 20-gram sample of Radium be completely gone? Explain your reasoning.
    Never; theoretically, the amount left will always be half of the previous amount.
22. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson. See margin.

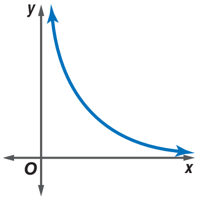
    How can you determine the current value of your car?

    Include the following in your answer:
    - a description of how to find the percent decrease in the value of the car each year, and
    - a description of how to find the value of a car for any given year when the rate of depreciation is known.

Standardized Test Practice

23. **SHORT RESPONSE** An artist creates a sculpture out of salt that weighs 2000 pounds. If the sculpture loses 3.5% of its mass each year to erosion, after how many years will the statue weigh less than 1000 pounds? about 19.5 yr
24. The curve shown at the right represents a portion of the graph of which function? D

    Ⓐ $y = 50 - x$ Ⓑ $y = \log x$
    Ⓒ $y = e^{-x}$ Ⓓ $xy = 5$



Enrichment, p. 608

Effective Annual Yield

When interest is compounded more than once per year, the effective annual yield is higher than the annual interest rate. The effective annual yield, E, is the interest rate that would give the same amount of interest if the interest were compounded once per year. If P dollars are invested for one year, the value of the investment at the end of the year is $A = P(1 + E)$. If P dollars are invested for one year at a nominal rate r compounded n times per year, the value of the investment at the end of the year is $A = P\left(1 + \frac{r}{n}\right)^n$. Setting the amounts equal and solving for E will produce a formula for the effective annual yield.

$$P(1 + E) = P\left(1 + \frac{r}{n}\right)^n$$
$$1 + E = \left(1 + \frac{r}{n}\right)^n$$
$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

Maintain Your Skills

Mixed Review **Write an equivalent exponential or logarithmic equation.** *(Lesson 10-5)*

25. $e^3 = y$ $\ln y = 3$

26. $e^{4n-2} = 29$ $\ln 29 = 4n - 2$

27. $\ln 4 + 2 \ln x = 8$ $4x^2 = e^8$

Solve each equation or inequality. Round to four decimal places. *(Lesson 10-4)*

28. $16^x = 70$ 1.5323

29. $2^{3p} > 1000$ $p > 3.3219$

30. $\log_b 81 = 2$ 9

BUSINESS **For Exercises 31–33, use the following information.**
The board of a small corporation decided that 8% of the annual profits would be divided among the six managers of the corporation. There are two sales managers and four nonsales managers. Fifty percent of the amount would be split equally among all six managers. The other 50% would be split among the four nonsales managers. Let p represent the annual profits of the corporation. *(Lesson 9-2)*

31. Write an expression to represent the share of the profits each nonsales manager will receive. $\frac{0.5(0.08p)}{6} + \frac{0.5(0.08p)}{4}$

32. Simplify this expression. $\frac{p}{60}$

33. Write an expression in simplest form to represent the share of the profits each sales manager will receive. $\frac{p}{150}$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. *(Lesson 8-6)*

34. $4y^2 - 3x^2 + 8y - 24x = 50$ hyperbola

35. $7x^2 - 42x + 6y^2 - 24y = -45$ ellipse

36. $y^2 + 3x - 8y = 4$ parabola

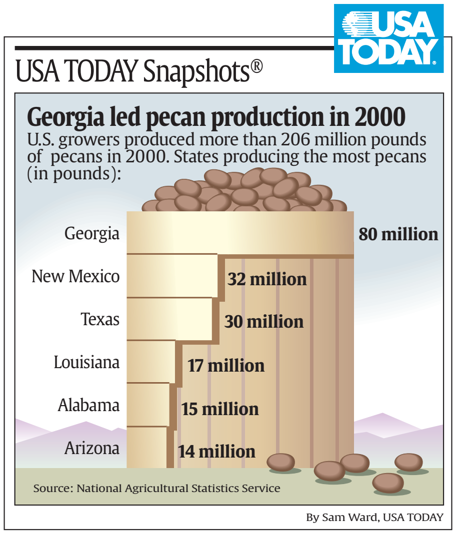
37. $x^2 + y^2 - 6x + 2y + 5 = 0$ circle

AGRICULTURE **For Exercises 38–40, use the graph at the right.** *(Lesson 5-1)*

38. Write the number of pounds of pecans produced by U.S. growers in 2000 in scientific notation. 2.06×10^8

39. Write the number of pounds of pecans produced by the state of Georgia in 2000 in scientific notation. 8×10^7

40. What percent of the overall pecan production for 2000 can be attributed to Georgia? about 38.8%



Source: National Agricultural Statistics Service

By Sam Ward, USA TODAY

WebQuest Internet Project

On Quake Anniversary, Japan Still Worries

It is time to complete your project. Use the information and data you have gathered about earthquakes to prepare a research report or Web page. Be sure to include graphs, tables, diagrams, and any calculations you need for the earthquake you chose.

www.algebra2.com/webquest

4 Assess

Open-Ended Assessment

Modeling Using manipulatives, ask students to demonstrate why the amount of compound interest earned annually increases each year. Have students relate this to their understanding of exponential growth.

Assessment Options

Quiz (Lesson 10-6) is available on p. 624 of the *Chapter 10 Resource Masters*.

Answer

22. Answers should include the following.
    - Find the absolute value of the difference between the price of the car for two consecutive years. Then divide this difference by the price of the car for the earlier year.
    - Find 1 minus the rate of decrease in the value of the car as a decimal. Raise this value to the number of years it has been since the car was purchased, and then multiply by the original value of the car.

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

Chapter 10 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 10 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 10 is available on p. 622 of the *Chapter 10 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 10 Study Guide and Review

Vocabulary and Concept Check

Change of Base Formula (p. 548)
common logarithm (p. 547)
exponential decay (p. 524)
exponential equation (p. 526)
exponential function (p. 524)
exponential growth (p. 524)
exponential inequality (p. 527)
logarithm (p. 531)
logarithmic equation (p. 533)
logarithmic function (p. 532)
logarithmic inequality (p. 533)
natural base, *e* (p. 554)
natural base exponential function (p. 554)
natural logarithm (p. 554)
natural logarithmic function (p. 554)
Power Property of Logarithms (p. 543)
Product Property of Logarithms (p. 541)
Property of Equality for Exponential Functions (p. 526)
Property of Equality for Logarithmic Functions (p. 534)
Property of Inequality for Exponential Functions (p. 527)
Property of Inequality for Logarithmic Functions (p. 534)
Quotient Property of Logarithms (p. 542)
rate of decay (p. 560)
rate of growth (p. 562)

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

1. If $24^{2y+3} = 24^{y-4}$, then $2y + 3 = y - 4$ by the *Property of Equality for Exponential Functions* . **true**
2. The number of bacteria in a petri dish over time is an example of *exponential decay*. **false; exponential growth**
3. The *natural logarithm* is the inverse of the exponential function with base 10.
4. The *Power Property of Logarithms* shows that ln 9 < ln 81.
5. If a savings account yields 2% interest per year, then 2% is the *rate of growth*.
6. Radioactive half-life is an example of *exponential decay*. **true**
7. The inverse of an exponential function is a *composite function*.
8. The *Quotient Property of Logarithms* is shown by $\log_4 2x = \log_4 2 + \log_4 x$.
9. The function $f(x) = 2(5)^x$ is an example of a *quadratic function*. **false; exponential function**

3. false; common logarithm
4. false; Property of Inequality for Logarithms
5. true
7. false; logarithmic function
8. false; Product Property of Logarithms

Lesson-by-Lesson Review

10-1 Exponential Functions

See pages 523–530.

Concept Summary

- An exponential function is in the form $y = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.
- The function $y = ab^x$ represents exponential growth for $a > 0$ and $b > 1$, and exponential decay for $a > 0$ and $0 < b < 1$.
- Property of Equality for Exponential Functions: If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.
- Property of Inequality for Exponential Functions: If $b > 1$, then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.

www.algebra2.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Have students look through the chapter to make sure they have included notes and examples for each lesson in this chapter in their Foldable.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Example **Solve $64 = 2^{3n+1}$ for n.**

$64 = 2^{3n+1}$ Original equation

$2^6 = 2^{3n+1}$ Rewrite 64 as 2^6 so each side has the same base.

$6 = 3n + 1$ Property of Equality for Exponential Functions

$\frac{5}{3} = n$ The solution is $\frac{5}{3}$.

Exercises **Determine whether each function represents exponential *growth* or *decay*.** *See Example 2 on page 525.*

10. $y = 5(0.7)^x$ **decay**

11. $y = \frac{1}{3}(4)^x$ **growth**

Write an exponential function whose graph passes through the given points. *See Example 3 on page 525.*

12. (0, −2) and (3, −54) $y = -2(3)^x$

13. (0, 7) and (1, 1.4) $y = 7\left(\frac{1}{5}\right)^x$

Solve each equation or inequality. *See Examples 5 and 6 on pages 526 and 527.*

14. $9^x = \frac{1}{81}$ -2

15. $2^{6x} = 4^{5x+2}$ -1

16. $49^{3p+1} = 7^{2p-5}$ $-\frac{7}{4}$

17. $9^{x^2} \le 27^{x^2-2}$ $x \le -\sqrt{6}$ or $x \ge \sqrt{6}$

10-2 Logarithms and Logarithmic Functions

See pages 531–538.

Concept Summary	Examples
• Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that $\log_b x = y$ if and only if $b^y = x$.	$\log_7 x = 2 \rightarrow 7^2 = x$
• Logarithmic to exponential inequality: If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$.	$\log_2 x > 5 \rightarrow x > 2^5$ $\log_3 x < 4 \rightarrow 0 < x < 3^4$
If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$.	
• Property of Equality for Logarithmic Functions: If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.	If $\log_5 x = \log_5 6$, then $x = 6$.
• Property of Inequality for Logarithmic Functions: If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.	If $\log_4 x > \log_4 10$, then $x > 10$.

Examples **1** **Solve $\log_9 n > \frac{3}{2}$.**

$\log_9 n > \frac{3}{2}$ Original inequality

$n > 9^{\frac{3}{2}}$ Logarithmic to exponential inequality

$n > (3^2)^{\frac{3}{2}}$ $9 = 3^2$

$n > 3^3$ Power of a Power

$n > 27$ Simplify.

2 **Solve $\log_3 12 = \log_3 2x$.**

$\log_3 12 = \log_3 2x$ Original equation

$12 = 2x$ Property of Equality for Logarithmic Functions

$6 = x$ Divide each side by 2.

Exercises **Write each equation in logarithmic form.** *See Example 1 on page 532.*

18. $7^3 = 343$ **$\log_7 343 = 3$** 19. $5^{-2} = \frac{1}{25}$ **$\log_5 \frac{1}{25} = -2$** 20. $4^{\frac{3}{2}} = 8$ **$\log_4 8 = \frac{3}{2}$**

Write each equation in exponential form. *See Example 2 on page 532.* **23. $6^{-2} = \frac{1}{36}$**

21. $\log_4 64 = 3$ **$4^3 = 64$** 22. $\log_8 2 = \frac{1}{3}$ **$8^{\frac{1}{3}} = 2$** 23. $\log_6 \frac{1}{36} = -2$

Evaluate each expression. *See Examples 3 and 4 on pages 532 and 533.*

24. $4^{\log_4 9}$ **9** 25. $\log_7 7^{-5}$ **-5** 26. $\log_{81} 3$ **$\frac{1}{4}$** 27. $\log_{13} 169$ **2**

Solve each equation or inequality. *See Examples 5–8 on pages 533 and 534.*

28. $\log_4 x = \frac{1}{2}$ **2**

29. $\log_{81} 729 = x$ **$\frac{3}{2}$**

30. $\log_b 9 = 2$ **3**

31. $\log_8 (3y - 1) < \log_8 (y + 5)$ **$\frac{1}{3} < y < 3$**

32. $\log_5 12 < \log_5 (5x - 3)$ **$x > 3$**

33. $\log_8 (x^2 + x) = \log_8 12$ **$-4, 3$**

10-3 Properties of Logarithms

See pages 541–546.

Concept Summary

- The logarithm of a product is the sum of the logarithms of its factors.
- The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
- The logarithm of a power is the product of the logarithm and the exponent.

Example **Use $\log_{12} 9 \approx 0.884$ and $\log_{12} 18 \approx 1.163$ to approximate the value of $\log_{12} 2$.**

$\log_{12} 2 = \log_{12} \frac{18}{9}$ Replace 2 with $\frac{18}{9}$.

$= \log_{12} 18 - \log_{12} 9$ Quotient Property

$\approx 1.163 - 0.884$ or 0.279 Replace $\log_{12} 9$ with 0.884 and $\log_{12} 18$ with 1.163.

Exercises **Use $\log_9 7 \approx 0.8856$ and $\log_9 4 \approx 0.6309$ to approximate the value of each expression.** *See Examples 1 and 2 on page 542.*

34. $\log_9 28$ **1.5165** 35. $\log_9 49$ **1.7712** 36. $\log_9 144$ **2.2618**

Solve each equation. *See Example 5 on page 543.*

37. $\log_2 y = \frac{1}{3} \log_2 27$ **3**

38. $\log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x$ **14**

39. $2 \log_2 x - \log_2 (x + 3) = 2$ **6**

40. $\log_3 x - \log_3 4 = \log_3 12$ **48**

41. $\log_6 48 - \log_6 \frac{16}{5} + \log_6 5 = \log_6 5x$ **15**

42. $\log_7 m = \frac{1}{3} \log_7 64 + \frac{1}{2} \log_7 121$ **44**

10-4 Common Logarithms

See pages 547–551.

Concept Summary

- Base 10 logarithms are called common logarithms and are usually written without the subscript 10: $\log_{10} x = \log x$.
- You use the inverse of logarithms, or exponentiation, to solve equations or inequalities involving common logarithms: $10^{\log x} = x$.
- The Change of Base Formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$ ← log base *b* original number ← log base *b* old base

Example **Solve $5^x = 7$.**

$5^x = 7$	Original equation
$\log 5^x = \log 7$	Property of Equality for Logarithmic Functions
$x \log 5 = \log 7$	Power Property of Logarithms
$x = \dfrac{\log 7}{\log 5}$	Divide each side by log 5.
$x \approx \dfrac{0.8451}{0.6990}$ or 1.2090	Use a calculator.

Exercises **Solve each equation or inequality. Round to four decimal places.** *See Examples 3 and 4 on page 548.*

43. $2^x = 53$ **5.7279**

44. $2.3^{x^2} = 66.6$ **±2.2452**

45. $3^{4x - 7} < 4^{2x + 3}$ $\boldsymbol{x < 7.3059}$

46. $6^{3y} = 8^{y - 1}$ **−0.6309**

47. $12^{x - 5} \geq 9.32$ $\boldsymbol{x \geq 5.8983}$

48. $2.1^{x - 5} = 9.32$ **8.0086**

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places. *See Example 5 on page 549.*

49. $\log_4 11$ $\dfrac{\log 11}{\log 4}$; **1.7297**

50. $\log_2 15$ $\dfrac{\log 15}{\log 2}$; **3.9069**

51. $\log_{20} 1000$ $\dfrac{\log 1000}{\log 20}$; **2.3059**

10-5 Base e and Natural Logarithms

See pages 554–559.

Concept Summary

- You can write an equivalent base *e* exponential equation for a natural logarithmic equation and vice versa by using the fact that $\ln x = \log_e x$.
- Since the natural base function and the natural logarithmic function are inverses, these two functions can be used to “undo” each other. $e^{\ln x} = x$ and $\ln e^x = x$

Example **Solve $\ln(x + 4) > 5$.**

$\ln(x + 4) > 5$	Original inequality
$e^{\ln(x + 4)} > e^5$	Write each side using exponents and base e.
$x + 4 > e^5$	Inverse Property of Exponents and Logarithms
$x > e^5 - 4$	Subtract 4 from each side.
$x > 144.4132$	Use a calculator.

Chapter 10 **For More ...**

- Extra Practice, see pages 849–851.
- Mixed Problem Solving, see page 871.

Exercises **Write an equivalent exponential or logarithmic equation.**
See Example 3 on page 555.

52. $e^x = 6$ **$\ln 6 = x$**

53. $\ln 7.4 = x$ **$e^x = 7.4$**

Evaluate each expression. *See Example 4 on page 555.*

54. $e^{\ln 12}$ **12**

55. $\ln e^{7x}$ **$7x$**

Solve each equation or inequality.
See Examples 5 and 7 on pages 555 and 556.

56. $2e^x - 4 = 1$ **0.9163**

57. $e^x > 3.2$ **$x > 1.1632$**

58. $-4e^{2x} + 15 = 7$ **0.3466**

59. $\ln 3x \leq 5$ **$0 < x \leq 49.4711$**

60. $\ln (x - 10) = 0.5$ **11.6487**

61. $\ln x + \ln 4x = 10$ **74.2066**

10-6 Exponential Growth and Decay

See pages 560–565.

Concept Summary

- Exponential decay: $y = a(1 - r)^t$ or $y = ae^{-kt}$
- Exponential growth: $y = a(1 + r)^t$ or $y = ae^{kt}$

Example **BIOLOGY** **A certain culture of bacteria will grow from 500 to 4000 bacteria in 1.5 hours. Find the constant k for the growth formula. Use $y = ne^{kt}$.**

$y = ae^{kt}$ Exponential growth formula

$4000 = 500e^{k(1.5)}$ Replace y with 4000, a with 500, and t with 1.5.

$8 = e^{1.5k}$ Divide each side by 500.

$\ln 8 = \ln e^{1.5k}$ Property of Equality for Logarithmic Functions

$\ln 8 = 1.5k$ Inverse Property of Exponents and Logarithms

$\frac{\ln 8}{1.5} = k$ Divide each side by 1.5.

$1.3863 \approx k$ Use a calculator.

Exercises *See Examples 1–4 on pages 560–562.*

62. **BUSINESS** Able Industries bought a fax machine for $250. It is expected to depreciate at a rate of 25% per year. What will be the value of the fax machine in 3 years? **$105.47**

63. **BIOLOGY** For a certain strain of bacteria, k is 0.872 when t is measured in days. How long will it take 9 bacteria to increase to 738 bacteria? **5.05 days**

64. **CHEMISTRY** Radium-226 decomposes radioactively. Its half-life, the time it takes for half of the sample to decompose, is 1800 years. Find the constant k in the decay formula for this compound. **about −0.000385**

65. **POPULATION** The population of a city 10 years ago was 45,600. Since then, the population has increased at a steady rate each year. If the population is currently 64,800, find the annual rate of growth for this city. **about 3.6%**

Chapter 10 Practice Test

Vocabulary and Concepts

Choose the term that best completes each sentence.

1. The equation $y = 0.3(4)^x$ is an exponential (*growth*, *decay*) function.
2. The logarithm of a quotient is the (*sum*, *difference*) of the logarithms of the numerator and the denominator.
3. The base of a natural logarithm is (*10*, *e*) .

Skills and Applications

4. Write $3^7 = 2187$ in logarithmic form. $\log_3 2187 = 7$
5. Write $\log_8 16 = \frac{4}{3}$ in exponential form. $8^{\frac{4}{3}} = 16$
6. Write an exponential function whose graph passes through (0, 0.4) and (2, 6.4). $y = 0.4(4)^x$
7. Express $\log_3 5$ in terms of common logarithms. $\frac{\log 5}{\log 3}$
8. Evaluate $\log_2 \frac{1}{32}$. -5

Use $\log_4 7 \approx 1.4037$ and $\log_4 3 \approx 0.7925$ to approximate the value of each expression.

9. $\log_4 21$ 2.1962
10. $\log_4 \frac{7}{12}$ -0.3888

Simplify each expression.

11. $\left(3^{\sqrt{8}}\right)^{\sqrt{2}}$ 81
12. $81^{\sqrt{5}} \div 3^{\sqrt{5}}$ $3^{3\sqrt{5}}$

Solve each equation or inequality. Round to four decimal places if necessary. 17. 108 19. 2, 6 22. 15

13. $2^{x-3} = \frac{1}{16}$ -1
14. $27^{2p+1} = 3^{4p-1}$ -2
15. $\log_2 x < 7$ $0 < x < 128$
16. $\log_m 144 = -2$ $\frac{1}{12}$
17. $\log_3 x - 2\log_3 2 = 3\log_3 3$
18. $\log_9 (x + 4) + \log_9 (x - 4) = 1$ 5
19. $\log_5 (8y - 7) = \log_5 (y^2 + 5)$
20. $\log_3 3^{(4x-1)} = 15$ 4
21. $7.6^{x-1} = 431$ 3.9910
22. $\log_2 5 + \frac{1}{3}\log_2 27 = \log_2 x$
23. $3^x = 5^{x-1}$ 3.1507
24. $4^{2x-3} = 9^{x+3}$ 18.6848
25. $e^{3y} > 6$ $y > 0.5973$
26. $2e^{3x} + 5 = 11$ 0.3662
27. $\ln 3x - \ln 15 = 2$ 36.9453

COINS **For Exercises 28 and 29, use the following information.**
You buy a commemorative coin for \$25. The value of the coin increases 3.25% per year.

28. How much will the coin be worth in 15 years? \$40.39
29. After how many years will the coin have doubled in value? 22

30. **QUANTITATIVE COMPARISION** **Compare the quantity in Column A and the quantity in Column B. Then determine whether:** B

Ⓐ **the quantity in Column A is greater,**

Ⓑ **the quantity in Column B is greater,**

Ⓒ **the two quantities are equal, or**

Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B
\$100 was deposited in an account 5 years ago.	
the current value of the account if the annual interest rate is 3% compounded quarterly	the current value of the account if the annual interest rate is 3% compounded continuously

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Chapter 10 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 10 can be found on p. 622 of the *Chapter 10 Resource Masters*.

Chapter Tests There are six Chapter 10 Tests and an Open-Ended Assessment task available in the *Chapter 10 Resource Masters*.

Chapter 10 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	609–610
2A	MC	average	611–612
2B	MC	average	613–614
2C	FR	average	615–616
2D	FR	average	617–618
3	FR	advanced	619–620

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 10 can be found on p. 621 of the *Chapter 10 Resource Masters*. A sample scoring rubric for these tasks appears on p. A25.

Unit 3 Test A unit test/review can be found on pp. 629–630 of the *Chapter 10 Resource Masters*.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction In mathematics, exponential functions can be used to model real-world problems. The solution to the exponential function provides a solution to the real-world problem.

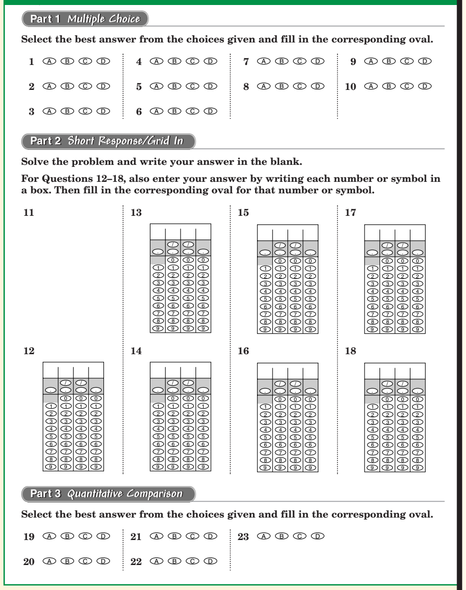
Ask Students Find a real-world problem modeled by an exponential function from your work in this chapter and show how you solved it. Explain how the function models the real-world situation and what could be gained by understanding the real-world problem better. Place your work in your portfolio.

Chapter 10 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 10 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1



Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ
4 Ⓐ Ⓑ Ⓒ Ⓓ
5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ
7 Ⓐ Ⓑ Ⓒ Ⓓ
8 Ⓐ Ⓑ Ⓒ Ⓓ
9 Ⓐ Ⓑ Ⓒ Ⓓ
10 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 12–18, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11
13
15
17
12
14
16
18

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

19 Ⓐ Ⓑ Ⓒ Ⓓ
20 Ⓐ Ⓑ Ⓒ Ⓓ
21 Ⓐ Ⓑ Ⓒ Ⓓ
22 Ⓐ Ⓑ Ⓒ Ⓓ
23 Ⓐ Ⓑ Ⓒ Ⓓ

Additional Practice

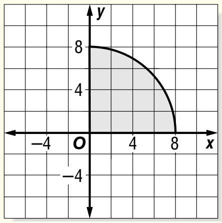
See pp. 627–628 in the *Chapter 10 Resource Masters* for additional standardized test practice.

Chapter 10 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The arc shown is part of a circle. Find the area of the shaded region. **B**



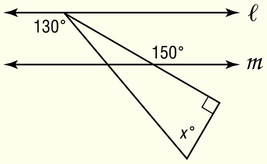
Ⓐ 8π units^2

Ⓑ 16π units^2

Ⓒ 32π units^2

Ⓓ 64π units^2

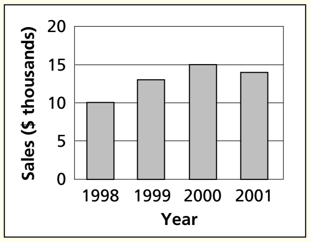
2. If line ℓ is parallel to line m in the figure below, what is the value of x? **D**



Ⓐ 40 Ⓑ 50

Ⓒ 60 Ⓓ 70

3. According to the graph, what was the percent of increase in sales from 1998 to 2000? **D**



Ⓐ 5% Ⓑ 15% Ⓒ 25% Ⓓ 50%

4. What is the x-intercept of the line described by the equation $y = 2x + 5$? **B**

Ⓐ -5 Ⓑ $-\frac{5}{2}$

Ⓒ 0 Ⓓ $\frac{5}{2}$

5. $\frac{(xy)^2z^0}{y^2x^3} =$ **D**

Ⓐ $\frac{1}{x^2y}$ Ⓑ $\frac{z}{x^2}$

Ⓒ $\frac{z}{x}$ Ⓓ $\frac{1}{x}$

6. If $\frac{v^2 - 36}{6 - v} = 10$, then $v =$ **A**

Ⓐ $-16.$ Ⓑ $-4.$

Ⓒ 4. Ⓓ 8.

7. The expression $\frac{1}{3}\sqrt{45}$ is equivalent to **A**

Ⓐ $\sqrt{5}.$ Ⓑ $3\sqrt{5}.$

Ⓒ 5. Ⓓ 15.

8. What are all the values for x such that $x^2 < 3x + 18$? **B**

Ⓐ $x < -3$ Ⓑ $-3 < x < 6$

Ⓒ $x > -3$ Ⓓ $x < 6$

9. If $f(x) = 2x^3 - 18x$, what are all the values of x at which $f(x) = 0$? **B**

Ⓐ 0, 3 Ⓑ −3, 0, 3

Ⓒ −6, 0, 6 Ⓓ −3, 2, 3

10. Which of the following is equal to $\frac{17.5(10^{-2})}{500(10^{-4})}$? **D**

Ⓐ $0.035(10^{-2})$ Ⓑ $0.35(10^{-2})$

Ⓒ $0.0035(10^2)$ Ⓓ $0.035(10^2)$

The Princeton Review Test-Taking Tip

Question 7 You can use estimates to help you eliminate answer choices. For example, in Question 7, you can estimate that $\frac{1}{3}\sqrt{45}$ is less than $\frac{1}{3}\sqrt{49}$, which is $\frac{7}{3}$ or $2\frac{1}{3}$. Eliminate choices C and D.

Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

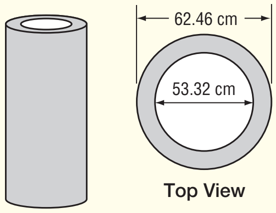
Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

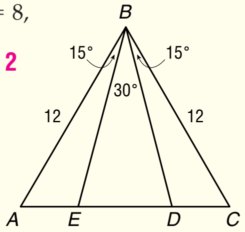
11. If the outer diameter of a cylindrical tank is 62.46 centimeters and the inner diameter is 53.32 centimeters, what is the thickness of the tank? **4.57 cm**



12. What number added to 80% of itself is equal to 45? **25**

13. Of 200 families surveyed, 95% have at least one TV and 60% of those with TVs have more than 2 TVs. If 50 families have exactly 2 TVs, how many families have exactly 1 TV? **26**

14. In the figure, if $ED = 8$, what is the measure of line segment AE? **2**



15. If $a \leftrightarrow b$ is defined as $a - b + ab$, find the value of $4 \leftrightarrow 2$. **10**

16. If $6(m + k) = 26 + 4(m + k)$, what is the value of $m + k$? **13**

17. If $y = 1 - x^2$ and $-3 \leq x \leq 1$, what number is found by subtracting the *least* possible value of y from the *greatest* possible value of y? **9**

18. If $f(x) = (x - \pi)(x - 3)(x - e)$, what is the difference between the greatest and least roots of $f(x)$? Round to the nearest hundredth. **.42**

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Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

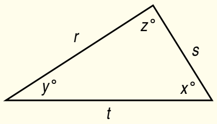
19. $-1 < xy < 0$

D

Column A	Column B
$x + y$	xy

20.

C

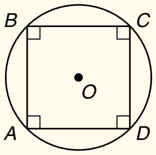


$z = x + y$

t	$\sqrt{r^2 + s^2}$

21.

A



circumference of circle $O = 8\pi$

perimeter of square $ABCD$	16

22.

$$x - y + z = 5$$
$$x + y + z = 9$$

A

$x + z$	6

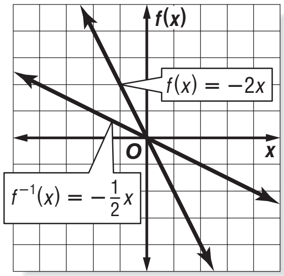
23. $nx \neq 0$

B

$-2nx$	$(x - n)^2$

Page 521, Chapter 10 Getting Started

9.



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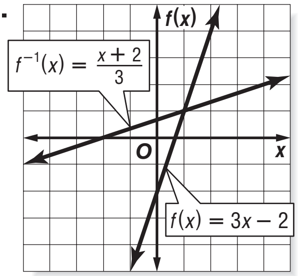

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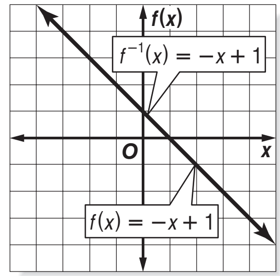

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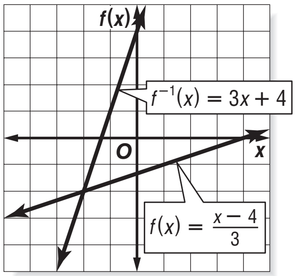
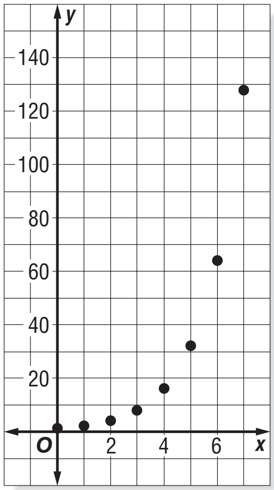


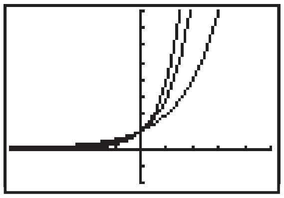
Page 522, Preview of Lesson 10-1 Algebra Activity

3. Sample graph:



Page 524, Lesson 10-1 Graphing Calculator Investigation

4a. As the value of x increases, the value of y for the graph of $y = 4^x$ increases faster than for the graph of $y = 3^x$, and the value of y for the graph of $y = 3^x$ increases faster than for the graph of $y = 2^x$. The graphs have the same domain, all real numbers, and range, $y > 0$. They have the same asymptote, the x-axis, and the same y-intercept, 1.



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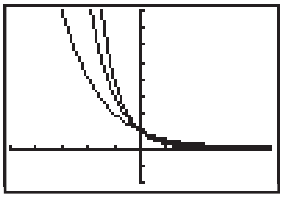
4b. As the value of x increases, the value of y for the graph of $y = \left(\frac{1}{3}\right)^x$ decreases faster than for the graph of $y = \left(\frac{1}{2}\right)^x$, and the value of y for the graph of $y = \left(\frac{1}{4}\right)^x$ decreases faster than for the graph of $y = \left(\frac{1}{3}\right)^x$. The graphs have the same domain, all real numbers, and range, $y > 0$. They have the same asymptote, the x-axis, and the same y-intercept, 1.



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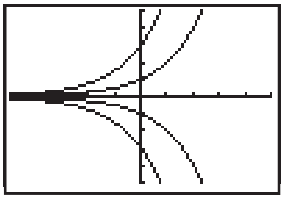
4c. The graph of $y = 3(2)^x$ moves down and to the right more quickly than the graph of $y = -1(2)^x$. The graph of $y = 3(2)^x$ moves up and to the right more quickly than the graph of $y = 2^x$. All of the graphs have the same domain, all real numbers, and asymptote, the x-axis, but the range of $y = -3(2)^x$ and $y = -1(2)^x$ is $y < 0$, while the range of $y = 2^x$ and $y = 3(2)^x$ is $y > 0$. The y-intercept of $y = -3(2)^x$ is -3, of $y = -1(2)^x$ is -1, of $y = 2^x$ is 1, and of $y = 3(2)^x$ is 3.



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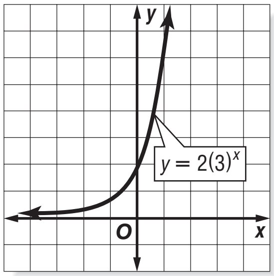
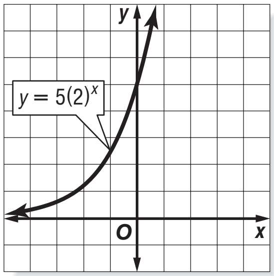
Pages 528–530, Lesson 10-1

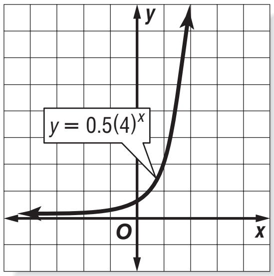
21. D = $\{x | x$ is all real numbers.$\}$,
R = $\{y | y > 0\}$



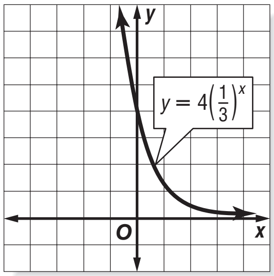
22. D = $\{x | x$ is all real numbers.$\}$,
R = $\{y | y > 0\}$



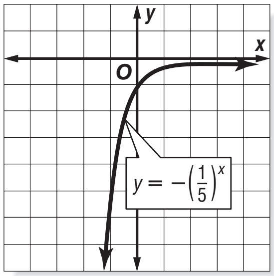
23. D = $\{x | x$ is all real numbers.$\}$,
R = $\{y | y > 0\}$



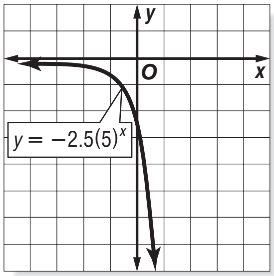
24. D = $\{x | x$ is all real numbers.$\}$,
R = $\{y | y > 0\}$



25. D = $\{x | x$ is all real numbers.$\}$, R = $\{y | y < 0\}$



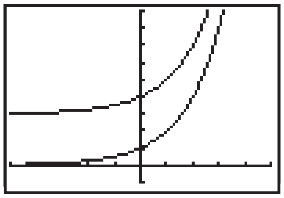
26. D = $\{x | x$ is all real numbers.$\}$, R = $\{y | y < 0\}$



68. The number of teams y that could compete in a tournament in which x rounds are played can be expressed as $y = 2^x$. The 2 teams that make it to the final round got there as a result of winning games played with 2 other teams, for a total of $2 \cdot 2 = 2^2$ or 4 games played in the previous round or semifinal round. Answers should include the following.

- Rewrite 128 as a power of 2, 2^7. Substitute 2^7 for y in the equation $y = 2^x$. Then, using the Property of Equality for Exponents, x must be 7. Therefore, 128 teams would need to play 7 rounds of tournament play.
- Sample answer: 52 would be an inappropriate number of teams to play in this type of tournament because 52 is not a power of 2.

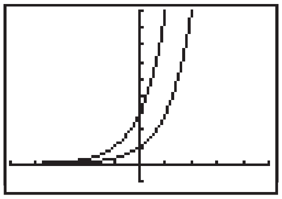
71.



[−5, 5] scl: 1 by [−1, 9] scl: 1

The graphs have the same shape. The graph of $y = 2^x + 3$ is the graph of $y = 2^x$ translated three units up. The asymptote for the graph of $y = 2^x$ is the line $y = 0$ and for $y = 2^x + 3$ is the line $y = 3$. The graphs have the same domain, all real numbers, but the range of $y = 2^x$ is $y > 0$ and the range of $y = 2^x + 3$ is $y > 3$. The y-intercept of the graph of $y = 2^x$ is 1 and for the graph of $y = 2^x + 3$ is 4.

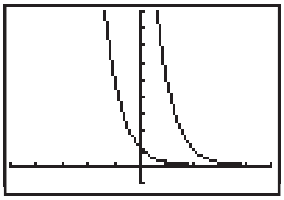
72.



[−5, 5] scl: 1 by [−1, 9] scl: 1

The graphs have the same shape. The graph of $y = 3^{x+1}$ is the graph of $y = 3^x$ translated one unit to the left. The asymptote for the graph of $y = 3^x$ and for $y = 3^{x+1}$ is the line $y = 0$. The graphs have the same domain, all real numbers, and range, $y > 0$. The y-intercept of the graph of $y = 3^x$ is 1 and for the graph of $y = 3^{x+1}$ is 3.

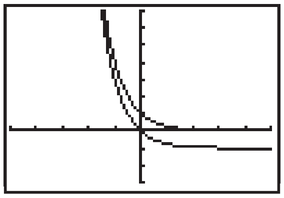
73.



[−5, 5] scl: 1 by [−1, 9] scl: 1

The graphs have the same shape. The graph of $y = \left(\frac{1}{5}\right)^{x-2}$ is the graph of $y = \left(\frac{1}{5}\right)^x$ translated two units to the right. The asymptote for the graph of $y = \left(\frac{1}{5}\right)^x$ and for $y = \left(\frac{1}{5}\right)^{x-2}$ is the line $y = 0$. The graphs have the same domain, all real numbers, and range, $y > 0$. The y-intercept of the graph of $y = \left(\frac{1}{5}\right)^x$ is 1 and for the graph of $y = \left(\frac{1}{5}\right)^{x-2}$ is 25.

74.



[−5, 5] scl: 1 by [−3, 7] scl: 1

The graphs have the same shape. The graph of $y = \left(\frac{1}{4}\right)^x - 1$ is the graph of $y = \left(\frac{1}{4}\right)^x$ translated one unit down. The asymptote for the graph of $y = \left(\frac{1}{4}\right)^x$ is the line $y = 0$ and for the graph of $y = \left(\frac{1}{4}\right)^x - 1$ is the line $y = -1$. The graphs have the same domain, all real numbers, but the range of $y = \left(\frac{1}{4}\right)^x$ is $y > 0$ and of $y = \left(\frac{1}{4}\right)^x - 1$ is $y > -1$. The y-intercept of the graph of $y = \left(\frac{1}{4}\right)^x$ is 1 and for the graph of $y = \left(\frac{1}{4}\right)^x - 1$ is 0.

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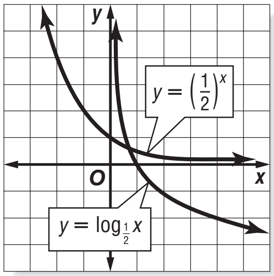
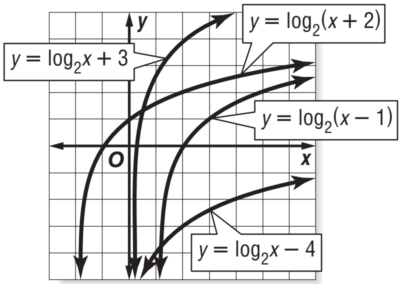
66a.



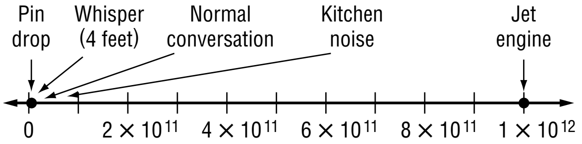
66b. The graphs are reflections of each other over the line $y = x$.

67a.



67b. The graph of $y = \log_2 x + 3$ is the graph of $y = \log_2 x$ translated 3 units up. The graph of $y = \log_2 x - 4$ is the graph of $y = \log_2 x$ translated 4 units down. The graph of $\log_2 (x - 1)$ is the graph of $y = \log_2 x$ translated 1 unit to the right. The graph of $\log_2 (x + 2)$ is the graph of $y = \log_2 x$ translated 2 units to the left.

73. A logarithmic scale illustrates that values next to each other vary by a factor of 10. Answers should include the following.

- Pin drop: 1×10^0; Whisper: 1×10^2; Normal conversation: 1×10^6; Kitchen noise: 1×10^{10}; Jet engine: 1×10^{12}
- 

- On the scale shown above, the sound of a pin drop and the sound of normal conversation appear not to differ by much at all, when in fact they do differ in terms of the loudness we perceive. The first scale shows this difference more clearly.

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36. $n \log_b x + m \log_b x \overset{?}{=} (n + m) \log_b x$

$\log_b x^n + \log_b x^m \overset{?}{=} (n + m) \log_b x$	Power Property of Logarithms
$\log_b (x^n \cdot x^m) \overset{?}{=} (n + m) \log_b x$	Product Property of Logarithms
$\log_b (x^{n+m}) \overset{?}{=} (n + m) \log_b x$	Product of Powers Property
$(n + m) \log_b x = (n + m) \log_b x$ ✓	Power Property of Logarithms

48. Since logarithms are exponents, the properties of logarithms are similar to the properties of exponents. The Product Property states that to multiply two powers that have the same base, add the exponents. Similarly, the logarithm of a product is the sum of the logarithms of its factors. The Quotient Property states that to divide two powers that have the same base, subtract their exponents. Similarly, the logarithm of a quotient is the difference of the logarithms of the numerator and the denominator. The Power Property states that to find the power of a power, multiply the exponents. Similarly, the logarithm of a power is the product of the logarithm and the exponent. Answers should include the following.

- Quotient Property:

$\log_2 \left(\frac{32}{8}\right) = \log_2 \left(\frac{2^5}{2^3}\right)$	Replace 32 with 2^5 and 8 with 2^3.
$= \log_2 2^{(5-3)}$	Quotient of Powers
$= 5 - 3$ or 2	Inverse Property of Exponents and Logarithms
$\log_2 32 - \log_2 8 = \log_2 2^5 - \log_2 2^3$	Replace 32 with 2^5 and 8 with 2^3.
$= 5 - 3$ or 2	Inverse Property of Exponents and Logarithms

So, $\log_2 \left(\frac{32}{8}\right) = \log_2 32 - \log_2 8$.

Power Property:

$\log_3 9^4 = \log_3 (3^2)^4$	Replace 9 with 3^2.
$= \log_3 3^{(2 \cdot 4)}$	Power of a Power
$= 2 \cdot 4$ or 8	Inverse Property of Exponents and Logarithms
$4 \log_3 9 = (\log_3 9) \cdot 4$	Commutative Property (×)
$= (\log_3 3^2) \cdot 4$	Replace 9 with 3^2.
$= 2 \cdot 4$ or 8	Inverse Property of Exponents and Logarithms

So, $\log_3 9^4 = 4 \log_3 9$.

- The Product of Powers Property and Product Property of Logarithms both involve the addition of exponents, since logarithms are exponents.

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62.

$\frac{\log x}{\log y} \overset{?}{=} \frac{\ln x}{\ln y}$	Original statement
$\frac{\log x}{\log y} \overset{?}{=} \frac{\frac{\log x}{\log e}}{\frac{\log y}{\log e}}$	Change of Base Formula
$\frac{\log x}{\log y} \overset{?}{=} \frac{\log x}{\log e} \cdot \frac{\log e}{\log y}$	Multiply $\frac{\log x}{\log e}$ by the reciprocal of $\frac{\log y}{\log e}$.
$\frac{\log x}{\log y} = \frac{\log x}{\log y}$	Simplify.

Notes