Chapter 12

Probability and Statistics

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days) Regular Basic/Average	Regular Advanced	Block Basic/Average	Block Advanced
12-1 The Counting Principle *(pp. 632–637)* • Solve problems involving independent events. • Solve problems involving dependent events.	1	1	0.5	0.5
12-2 Permutations and Combinations *(pp. 638–643)* • Solve problems involving linear permutations. • Solve problems involving combinations.	1	1	0.5	0.5
12-3 Probability *(pp. 644–650)* • Find the probability and odds of events. • Create and use graphs of probability distributions.	1	1	0.5	0.5
12-4 Multiplying Probabilities *(pp. 651–657)* • Find the probability of two independent events. • Find the probability of two dependent events.	1	1	0.5	0.5
12-5 Adding Probabilities *(pp. 658–663)* • Find the probability of mutually exclusive events. • Find the probability of inclusive events.	1	1	0.5	0.5
12-6 Statistical Measures *(pp. 664–670)* • Use measures of central tendency to represent a set of data. • Find measures of variation for a set of data.	1	1	0.5	0.5
12-7 The Normal Distribution *(pp. 671–675)* • Determine whether a set of data appears to be normally distributed or skewed. • Solve problems involving normally distributed data.	2	1	1.5	0.5
12-8 Binomial Experiments *(pp. 676–681)* • Use binomial expansions to find probabilities. • Find probabilities for binomial experiments. ***Follow-Up:*** Simulations	2	2 (with 12-8 (Follow-Up)	1	1.5 (with 12-8 (Follow-Up)
12-9 Sampling and Error *(pp. 682–686)* • Determine whether a sample is unbiased. • Find margins of sampling error. ***Follow-Up:*** Testing Hypotheses	1	2 (with 12-9 (Follow-Up)	0.5	1
Study Guide and **Practice Test** *(pp. 687–693)* **Standardized Test Practice** *(pp. 694–695)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	13	13	7	7

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter Resource Manager

Chapter 12 Resource Masters

Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
699–700	701–702	703	704			12-1	12-1		restaurant menu
705–706	707–708	709	710		GCS 50	12-2	12-2	22	index cards
711–712	713–714	715	716	767		12-3	12-3		
717–718	719–720	721	722		SC 23	12-4	12-4	23	compass, protractor
723–724	725–726	727	728	767, 769	GCS 49	12-5	12-5		colored chips, index cards
729–730	731–732	733	734		SM 115–118	12-6	12-6		graphing calculator
735–736	737–738	739	740	768		12-7	12-7		measuring tape
741–742	743–744	745	746		SC 24	12-8	12-8		ball (*Follow-Up:* die, tally sheet, grid paper)
747–748	749–750	751	752	768	SM 57–62	12-9	12-9		(*Follow-Up:* ruler)
				753–766, 770–772					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 12

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Some of the notation used in this chapter will be familiar, including factorials and binomial expansion. Also, some of the content will be familiar, including simple probability, relative frequency, and finding means, medians, and modes.

This Chapter

Students learn to represent counting situations using permutations and combinations. They describe the likelihood of single events using odds and probability, and they calculate probabilities for pairs of dependent or independent events, mutually exclusive or inclusive events, and binomial experiments. They calculate the central tendency and variation of data sets by calculating means, medians, variance, and standard deviations, and they explore normal distributions, skewed distributions, and sampling error.

Future Connections

Students will continue to use permutations, combinations, and probabilities in their math classes. They will study the mathematical underpinnings of statistical ideas in later math courses, and they will apply those statistical ideas in courses on behavioral science, psychology, economics, and many other fields.

12-1 The Counting Principle

In this lesson students investigate the Fundamental Counting Principle. The Fundamental Counting Principle states that the total number of options for a succession of choices is the product of the number of options for the individual choices. Students use exponents and factorials to express answers to counting problems.

12-2 Permutations and Combinations

The real-world situations in this lesson involve selecting some number of objects from a larger group of objects. If the order of selection is one of the attributes that differentiates among the selected objects, then the selection is called a permutation. If the order does not differentiate among the selected objects, then the selection is called a combination. As students analyze and apply the formulas for permutations, they consider situations in which some of the items in the large group are duplicates. Students also explore the relationship between permutations and combinations, which can be represented by the formula $C(n, r) = \frac{P(n, r)}{r!}$.

12-3 Probability

In this lesson, students analyze the likelihood that a particular event will happen. The likelihood of an event can be described in terms of odds and probability. Some of the mathematical properties of these expressions are that the odds of success and the odds of failure for any given event are reciprocals, that each probability is a number between 0 and 1, inclusive, and that if you add the probability of success and the probability of failure for any given event, the sum is 1. As students explore these descriptions of likelihood, they compare the probabilities for all the events in a sample space. They investigate the probabilities by looking at tables of probability distributions and by graphing those distributions as relative-frequency histograms.

12-4 Multiplying Probabilities

In this lesson, students consider the likelihood that two events will both happen and determine how that likelihood is related to the probabilities of the separate events. If two events A and B are independent, then the probability that both A and B occur is the product of the individual probabilities. If the two events are dependent, then the probability of both occurring is the product of the probability of A occurring times the probability of B occurring given that A occurred. Students explore problems in which they calculate values of $P(A)$, $P(B)$, and $P(B \text{ following } A)$, and use those values to calculate the value $P(A \text{ and } B)$.

12-5 Adding Probabilities

This lesson considers the likelihood that at least one of two events will happen, and relates that likelihood to the probabilities of the separate events. If it is not possible that two events A and B both occur, then A and B are mutually exclusive and $P(A \text{ or } B)$ is $P(A) + P(B)$. If two events are not mutually exclusive, then $P(A \text{ or } B)$ is the probability that A will happen, plus the probability that B will happen, minus the probability that both will happen. Formulas can clarify the relationship between mutually exclusive and inclusive events. Starting with $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, if $P(A \text{ and } B) = 0$ then the events cannot both happen, so they are mutually exclusive. In that case, $P(A \text{ or } B) = P(A) + P(B)$.

12-6 Statistical Measures

Students investigate how the values of a data set are distributed. They will choose the most appropriate measure of central tendency for a given set of data. For the dispersion of the data, they find the *variance* by using a formula whose key step is to look at how the individual data values differ from the mean of the set. They also calculate the *standard deviation*, which is the square root of the variance.

12-7 The Normal Distribution

For a large data set, the heights of the bars of a relative-frequency histogram can be replaced with a curve. A curve is a normal distribution curve if the probability distribution curve is symmetric and the mean, median, and mode are indicated by the peak of the curve. Another condition for a distribution to be normal involves the percent of data values that are within one, two, or three standard deviations of the mean. A data set with a long tail above the mean is positively skewed, while a data set with a long tail below the mean is negatively skewed.

12-8 Binomial Experiments

One or more terms of the binomial expansion $(p + q)^n$ can be used to calculate the probability for a binomial experiment. In a binomial experiment there are exactly two outcomes for each trial, there is a fixed number of trials, each trial is independent, and the probability of success or failure is the same for each trial. Tossing a coin five times is an example of a binomial experiment because each of these conditions is met.

12-9 Sampling and Error

In this lesson, students investigate sampling. They discuss how the response from a sample reflects what the responses might be from the entire population. If everyone in the population has an equal chance to be in the sample, then the sample is called an unbiased or random sample. For unbiased samples, students will describe the difference between sample and population responses by calculating the margin of sampling error (ME). If some percent p of people in a sample answer a question in a particular way, then for that question the percent of the population expected to answer the same way will be in the interval $p \pm ME$. A formula lets students calculate the ME based on the sample size and the value of p.

Chapter 12

DAILY INTERVENTION and Assessment

INTERVENTION / ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 631, 637, 643, 650, 657, 663, 670, 675, 680 Practice Quiz 1, p. 650 Practice Quiz 2, p. 670	5-Minute Check Transparencies Quizzes, *CRM* pp. 767–768 Mid-Chapter Test, *CRM* p. 769 Study Guide and Intervention, *CRM* pp. 699–700, 705–706, 711–712, 717–718, 723–724, 729–730, 735–736, 741–742, 747–748	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
Mixed Review	pp. 637, 643, 650, 657, 663, 670, 675, 681, 685	Cumulative Review, *CRM* p. 770	
Error Analysis	Find the Error, pp. 654, 660 Common Misconceptions, p. 659	Find the Error, *TWE* pp. 654, 660 Unlocking Misconceptions, *TWE* p. 639 Tips for New Teachers, *TWE* pp. 648, 668	
Standardized Test Practice	pp. 633, 634, 636, 642, 649, 657, 662, 669, 675, 680, 685, 693, 694–695	*TWE* p. 633 Standardized Test Practice, *CRM* pp. 771–772	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 636, 642, 649, 657, 662, 669, 675, 679, 685 Open Ended, pp. 634, 641, 647, 654, 660, 666, 673, 678, 683	Modeling: *TWE* pp. 650, 663 Speaking: *TWE* pp. 643, 657, 680, 684 Writing: *TWE* pp. 637, 670, 675 Open-Ended Assessment, *CRM* p. 765	
Chapter Assessment	Study Guide, pp. 687–692 Practice Test, p. 693	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 753–758 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 759–764 Vocabulary Test/Review, *CRM* p. 766	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
12-2	**22** *Combinations and Permutations*
12-4	**23** *Integration: Introduction to Probability*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 631
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 634, 641, 647, 654, 660, 666, 673, 678, 683, 687)
- Writing in Math questions in every lesson, pp. 636, 642, 649, 657, 662, 669, 675, 679, 685
- Reading Study Tip, pp. 633, 638, 644, 646, 665, 669
- WebQuest, pp. 635, 685

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 631, 687
- Study Notebook suggestions, pp. 635, 641, 647, 654, 660, 667, 673, 678, 681, 684, 686
- Modeling activities, pp. 650, 663
- Speaking activities, pp. 643, 657, 680, 684
- Writing activities, pp. 637, 670, 675
- Differentiated Instruction, (Verbal/Linguistic), p. 683
- ELL Resources, pp. 630, 636, 642, 649, 656, 662, 669, 674, 679, 683, 685, 687

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 12 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 12 Resource Masters*, pp. 703, 709, 715, 721, 727, 733, 739, 745, 751)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 12 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
12-1	1, 5, 6, 8, 9, 10	
12-2	1, 5, 6, 8, 9, 10	
12-3	1, 5, 6, 8, 9, 10	
12-4	1, 5, 6, 8, 9, 10	
12-5	1, 5, 6, 8, 9, 10	
12-6	1, 5, 6, 8, 9, 10	
12-7	1, 5, 6, 8, 9, 10	
12-8	1, 5, 6, 8, 9, 10	
12-8 Follow-Up	1, 5, 6, 9, 10	
12-9	1, 5, 6, 8, 9, 10	
12-9 Follow-Up	5, 7, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 12 Probability and Statistics

What You'll Learn

- **Lessons 12-1 and 12-2** Solve problems involving independent events, dependent events, permutations, and combinations.
- **Lessons 12-3, 12-4, 12-5, and 12-8** Find probability and odds.
- **Lesson 12-6** Find statistical measures.
- **Lesson 12-7** Use the normal distribution.
- **Lesson 12-9** Determine whether a sample is unbiased.

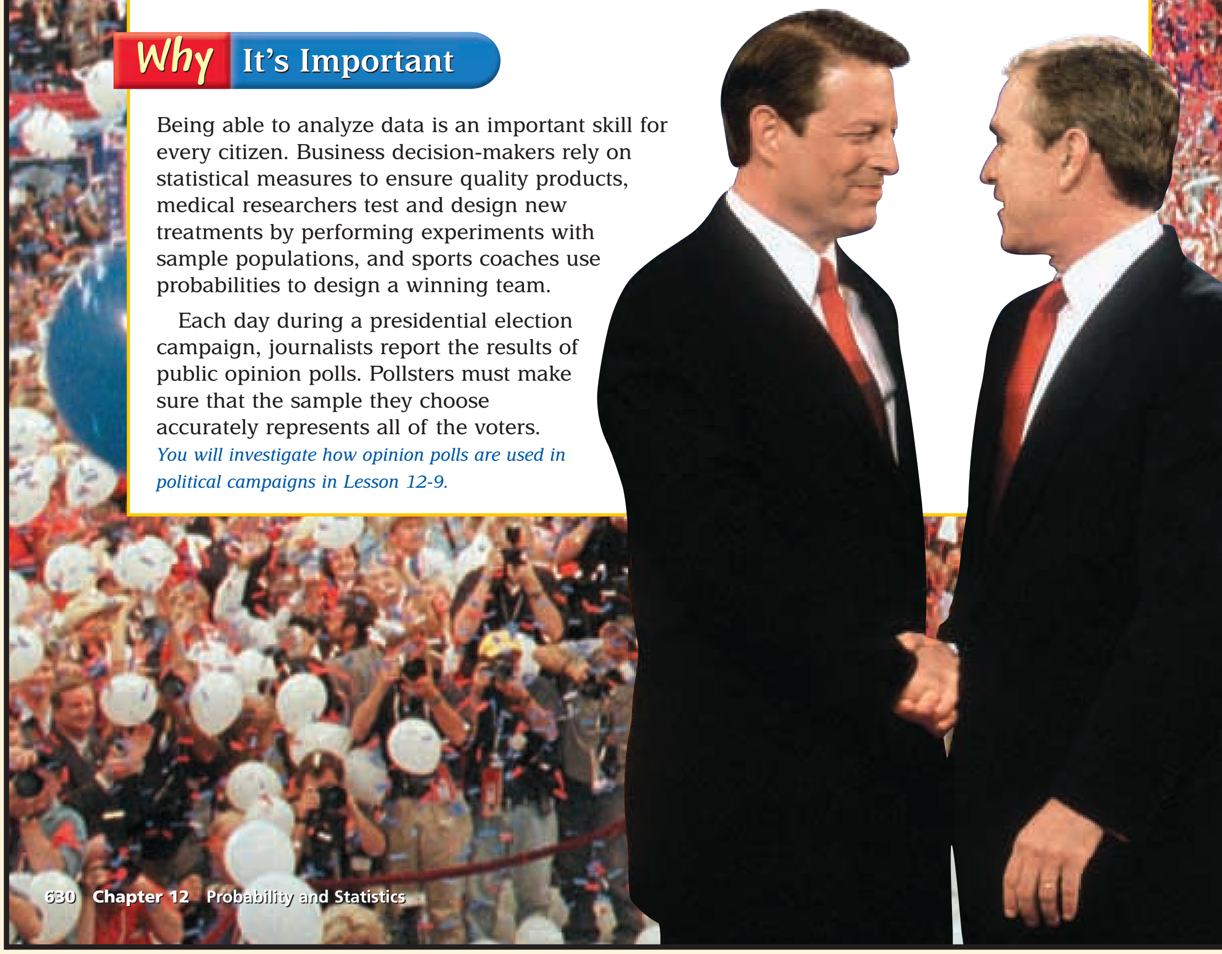
Key Vocabulary

- **permutation** (p. 638)
- **combination** (p. 640)
- **probability** (p. 644)
- **measures of central tendency** (p. 664)
- **measures of variation** (p. 665)

Why It's Important

Being able to analyze data is an important skill for every citizen. Business decision-makers rely on statistical measures to ensure quality products, medical researchers test and design new treatments by performing experiments with sample populations, and sports coaches use probabilities to design a winning team.

Each day during a presidential election campaign, journalists report the results of public opinion polls. Pollsters must make sure that the sample they choose accurately represents all of the voters. *You will investigate how opinion polls are used in political campaigns in Lesson 12-9.*



Vocabulary Builder ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 12 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 12 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 12.

For Lesson 12-3 — **Find Simple Probability**

Find each probability if a die is rolled once. **6.** $\frac{5}{6}$

1. $P(2)$ $\frac{1}{6}$ **2.** $P(5)$ $\frac{1}{6}$ **3.** $P(\text{even number})$ $\frac{1}{2}$

4. $P(\text{odd number})$ $\frac{1}{2}$ **5.** $P(\text{numbers less than 5})$ $\frac{2}{3}$ **6.** $P(\text{numbers greater than 1})$

For Lesson 12-6 — **Box-and-Whisker Plots**

Make a box-and-whisker plot for each set of data. *(For review, see pages 826 and 827.)*

7. {24, 32, 38, 38, 26, 33, 37, 39, 23, 31, 40, 21} **7–10. See margin.**

8. {25, 46, 31, 53, 39, 59, 48, 43, 68, 64, 29}

9. {51, 69, 46, 27, 60, 53, 55, 39, 81, 54, 46, 23}

10. {13.6, 15.1, 14.9, 15.7, 16.0, 14.1, 16.3, 14.3, 13.8}

For Lesson 12-6 — **Evaluate Expressions**

Evaluate $\sqrt{\frac{(a-b)^2+(c-b)^2}{d}}$ for each set of values. *(For review, see Lesson 5-6.)*

11. $a = 4, b = 7, c = 1, d = 5$ **3** **12.** $a = 2, b = 6, c = 9, d = 5$ $\sqrt{5}$

13. $a = 5, b = 1, c = 7, d = 4$ $\sqrt{13}$ **14.** $a = 3, b = 4, c = 11, d = 10$ $\sqrt{5}$

For Lesson 12-8 **15.** $a^3 + 3a^2b + 3ab^2 + b^3$ — **Expand Binomials**

Expand each binomial. *(For review, see Lesson 5-2.)* **16.** $c^4 + 4c^3d + 6c^2d^2 + 4cd^3 + d^4$

15. $(a + b)^3$ **16.** $(c + d)^4$ **17.** $(m - n)^5$ **18.** $(x + y)^6$

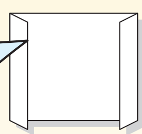
17. $m^5 - 5m^4n + 10m^3n^2 - 10m^2n^3 + 5mn^4 - n^5$

18. $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

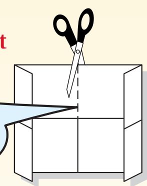
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about probability and statistics. Begin with one sheet of 11" by 17" paper.

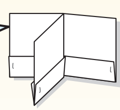
Step 1 Fold



Step 2 Fold and Cut



Step 3 Staple and Label



Reading and Writing As you read and study the chapter, you can write notes and examples on index cards and store the cards in the Foldable pockets.

Getting Started

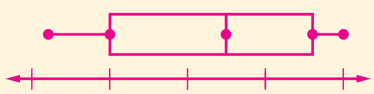
This section provides a review of the basic concepts needed before beginning Chapter 12. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
12-2	Factorials (p. 637)
12-3	Evaluating Expressions (p. 643)
12-6	Mean, Median, Mode, and Range (p. 663)
12-8	Binomial Expansions (p. 675)
12-9	Radical Expressions (p. 680)

Answers

7.



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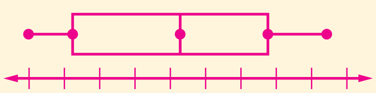

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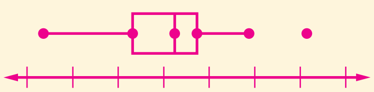

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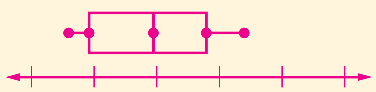


FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Organization of Data and Statistics in Writing After students make their Foldable, have them label the four pockets with the key topics of this chapter—The Counting Principle, Permutations and Combinations, Probability, and Statistics. Throughout the chapter, students might record examples of probability and statistics they see in everyday print (newspapers, magazines, and advertisements). They should note how writers use statistics to prove or disprove points of view and discuss the ethical responsibilities writers have when using statistics.

12-1 Lesson Notes

1 Focus

5-Minute Check Transparency 12-1 Use as a quiz or review of Chapter 11.

Mathematical Background notes are available for this lesson on p. 630C.

How can you count the maximum number of license plates a state can issue?

Ask students:

- How many letters are there on the license plate? how many digits? **3; 3**
- How many possibilities are there to fill the first place on this plate? **26 (assuming all letters are possibilities)**
- How many possibilities are there to fill the fourth place on this plate? **10 (assuming all digits are possibilities)**

12-1 The Counting Principle

What You'll Learn

- Solve problems involving independent events.
- Solve problems involving dependent events.

Vocabulary

- outcomes
- sample space
- event
- independent events
- Fundamental Counting Principle
- dependent events

How can you count the maximum number of license plates a state can issue?

Most states have letters and digits on their license plates. The number of possible plates is too great to count by listing all of the possibilities. It is much more efficient to count the number of possibilities by using the Fundamental Counting Principle.

INDEPENDENT EVENTS An **outcome** is the result of a single trial. For example, the trial of flipping a coin once has two outcomes: head or tail. The set of all possible outcomes is called the **sample space**. An **event** consists of one or more outcomes of a trial. The choices of letters and digits to be put on a license plate are called **independent events** because each letter or digit chosen does *not* affect the choices for the others.

For situations in which the number of choices leads to a small number of total possibilities, you can use a tree diagram or a table to count them.

Example 1 Independent Events

FOOD A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or a sesame seed bun. How many different combinations of meat and a bun are possible?

First, note that the choice of the type of meat does not affect the choice of the type of bun, so these events are independent.

Method 1 Tree Diagram

Let H represent hamburger, C, chicken, F, fish, P, plain, and S, sesame seed. Make a tree diagram in which the first row shows the choice of meat and the second row shows the choice of bun.

Meat	H		C		F	
Bun	P	S	P	S	P	S
Possible Combinations	HP	HS	CP	CS	FP	FS

There are six possible outcomes.

Method 2 Make a Table

Make a table in which each row represents a type of meat and each column represents a type of bun.

This method also shows that there are six outcomes.

		Bun	
		Plain	Sesame
Meat	Hamburger	HP	HS
	Chicken	CP	CS
	Fish	FP	FS

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 699–700
- Skills Practice, p. 701
- Practice, p. 702
- Reading to Learn Mathematics, p. 703
- Enrichment, p. 704

Transparencies

5-Minute Check Transparency 12-1
Answer Key Transparencies

Technology

Interactive Chalkboard

Notice that in Example 1, there are 3 ways to choose the type of meat, 2 ways to choose the type of bun, and $3 \cdot 2$ or 6 total ways to choose a combination of the two. This illustrates the **Fundamental Counting Principle**.

Key Concept — Fundamental Counting Principle

- **Words** If event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in $m \cdot n$ ways.
- **Example** If event M can occur in 2 ways and event N can occur in 3 ways, then M followed by N can occur in $2 \cdot 3$ or 6 ways.

This rule can be extended to any number of events.

Standardized Test Practice

Example 2 Fundamental Counting Principle

Multiple-Choice Test Item

Kim won a contest on a radio station. The prize was a restaurant gift certificate and tickets to a sporting event. She can select one of three different restaurants and tickets to a football, baseball, basketball, or hockey game. How many different ways can she select a restaurant followed by a sporting event?

(A) 7 (B) 12 (C) 15 (D) 16

Read the Test Item

Her choice of a restaurant does not affect her choice of a sporting event, so these events are independent.

Solve the Test Item

There are 3 ways she can choose a restaurant and there are 4 ways she can choose the sporting event. By the Fundamental Counting Principle, there are $3 \cdot 4$ or 12 total ways she can choose her two prizes. The answer is B.

The Princeton Review

Test-Taking Tip

Remember that you can check your answer by making a tree diagram or a table showing the outcomes.

The Fundamental Counting Principle can be used to count the number of outcomes possible for any number of successive events.

Example 3 More than Two Independent Events

COMMUNICATION **Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?**

The choice of any digit does not affect the other two digits, so the choices of the digits are independent events.

There are 10 possible first digits in the code, 10 possible second digits, and 10 possible third digits. So, there are $10 \cdot 10 \cdot 10$ or 1000 possible different code numbers.

Study Tip

Reading Math

Independent and *dependent* have the same meaning in mathematics as they do in ordinary language.

DEPENDENT EVENTS Some situations involve dependent events. With **dependent events**, the outcome of one event *does* affect the outcome of another event. The Fundamental Counting Principle applies to dependent events as well as independent events.

www.algebra2.com/extra_examples

2 Teach

INDEPENDENT EVENTS

In-Class Examples Power Point®

1 A sandwich menu offers customers a choice of white, wheat, or rye bread with one spread chosen from butter, mustard, or mayonnaise. How many different combinations of bread and spread are possible? **9**

Teaching Tip Make sure students know how to read a tree diagram so that they can identify the possibilities.

2 For their vacation, the Murray family is choosing a trip to the beach or to the mountains. They can select their transportation from a car, plane, or train. How many different ways can they select a destination followed by a means of transportation? **C**

A 2 **B** 5

C 6 **D** 9

3 How many codes are possible if the code is just two digits? **100**

Standardized Test Practice

Example 2 Have students draw tree diagrams to show the possible prize outcomes. Make sure students recognize that which restaurant is chosen has no affect on the choice of sporting event Kim attends.

Interactive Chalkboard

PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

DEPENDENT EVENTS

In-Class Example Power Point®

4 Refer to the table in Example 4 in the Student Edition. How many different schedules could a student have who is planning to take only 4 different classes? **24**

Example 4 Dependent Events

SCHOOL **Charlita wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?**

When Charlita schedules a given class for a given period, she cannot schedule that class for any other period. Therefore, the choices of which class to schedule each period are dependent events.

There are 6 classes Charlita can take during first period. That leaves 5 classes she can take second period. After she chooses which classes to take the first two periods, there are 4 remaining choices for third period, and so on.

Period	1st	2nd	3rd	4th	5th	6th
Number of Choices	6	5	4	3	2	1

There are $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ or 720 schedules that Charlita could have.
Note that $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$.

Study Tip

Look Back
To review **factorials**, see Lesson 11-7.

Concept Summary — Independent and Dependent Events

- **Words** If the outcome of an event does *not* affect the outcome of another event, the two events are *independent*.
- **Example** Tossing a coin and rolling a die are independent events.

- **Words** If the outcome of an event *does* affect the outcome of another event, the two events are *dependent*.
- **Example** Taking a piece of candy from a jar and then taking a second piece without replacing the first are dependent events because taking the first piece affects what is available to be taken next.

Check for Understanding

Concept Check

1. **List** the possible outcomes when a coin is tossed three times. Use H for heads and T for tails. **HHH, HHT, HTH, HTT, THH, THT, TTH, TTT**
2. **OPEN ENDED** Describe a situation in which you can use the Fundamental Counting Principle to show that there are 18 total possibilities.
3. **Explain** how choosing to buy a car or a pickup truck and then selecting the color of the vehicle could be dependent events. **The available colors for the car could be different from those for the truck.**

2. Sample answer: buying a shirt that comes in 3 sizes and 6 colors

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1–4

State whether the events are *independent* or *dependent*.

4. choosing the color and size of a pair of shoes **independent**
5. choosing the winner and runner-up at a dog show **dependent**

Solve each problem.

6. An ice cream shop offers a choice of two types of cones and 15 flavors of ice cream. How many different 1-scoop ice cream cones can a customer order? **30**
7. Lance's math quiz has eight true-false questions. How many different choices for giving answers to the eight questions are possible? **256**
8. For a college application, Macawi must select one of five topics on which to write a short essay. She must also select a different topic from the list for a longer essay. How many ways can she choose the topics for the two essays? **20**

Standardized Test Practice

9. A bookshelf holds 4 different biographies and 5 different mystery novels. How many ways can one book of each type be selected? **D**

   (A) 1 (B) 9 (C) 10 (D) 20

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students work in pairs or small groups. Give each group a menu from a neighborhood restaurant, or have them design a brief menu. Then ask each group to use their menu to write, and answer, four problems similar to Examples 1 through 4. Have groups exchange problems and solve.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
10–23, 25–27	1–4

Extra Practice

See page 854.

12. independent

State whether the events are *independent* or *dependent*.

10. choosing a president, vice president, secretary, and treasurer for Student Council, assuming that a person can hold only one office **dependent**

11. selecting a fiction book and a nonfiction book at the library **independent**

12. Each of six people guess the total number of points scored in a basketball game. Each person writes down his or her guess without telling what it is.

13. The letters A through Z are written on pieces of paper and placed in a jar. Four of them are selected one after the other without replacing any of them. **dependent**

Solve each problem.

14. Tim wants to buy one of three different albums he sees in a music store. Each is available on tape and on CD. From how many combinations of album and format does he have to choose? **6**

15. A video store has 8 new releases this week. Each is available on videotape and on DVD. How many ways can a customer choose a new release and a format to rent? **16**

16. Carlos has homework to do in math, chemistry, and English. How many ways can he choose the order in which to do his homework? **6**

17. The menu for a banquet has a choice of 2 types of salad, 5 main courses, and 3 desserts. How many ways can a salad, main course, and dessert be selected to form a meal? **30**

18. A golf club manufacturer makes drivers with 4 different shaft lengths, 3 different lofts, 2 different grips, and 2 different club head materials. How many different combinations are possible? **48**

19. Each question on a five-question multiple-choice quiz has answer choices labeled A, B, C, and D. How many different ways can a student answer the five questions? **1024**

★**20.** How many ways can six different books be arranged on a shelf if one of the books is a dictionary and it must be on an end? **240**

★**21.** In how many orders can eight actors be listed in the opening credits of a movie if the leading actor must be listed first or last? **10,080**

22. **PASSWORDS** Abby is registering at a Web site. She must select a password containing 6 numerals to be able to use the site. How many passwords are allowed if no digit may be used more than once? **151,200**

23. **ENTERTAINMENT** Solve the problem in the comic strip below. Assume that the books are all different. **362,880**

Peanuts®

24. **CRITICAL THINKING** The members of the Math Club need to elect a president and a vice-president. They determine that there are a total of 272 ways that they can fill the positions with two different members. How many people are in the Math Club? **17**

www.algebra2.com/self_check_quiz

WebQuest

You can use the Fundamental Counting Principle to list possible outcomes in games. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include their own examples of both independent and dependent events.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Independent Events:** 11, 12
- **Dependent Events:** 10, 13

Odd/Even Assignments
Exercises 10–21 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 28 involves research on the Internet or other reference materials.

Assignment Guide

Basic: 11–19 odd, 23–25, 29–31, 34–63

Average: 11–25 odd, 29–31, 34–63 (optional: 32, 33)

Advanced: 10–24 even, 26–55 (optional: 56–63)

Lesson 12-1 The Counting Principle 635

Study Guide and Intervention, p. 699 (shown) and p. 700

Independent Events If the outcome of one event does not affect the outcome of another event and vice versa, the events are called **independent events**.

Fundamental Counting Principle	If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.

Example **FOOD For the Breakfast Special at the Country Pantry, customers can choose their eggs scrambled, fried, or poached, whole wheat or white toast, and either orange, apple, tomato, or grapefruit juice. How many different Breakfast Specials can a customer order?**

A customer's choice of eggs does not affect his or her choice of toast or juice, so the events are independent. There are 3 ways to choose eggs, 2 ways to choose toast, and 4 ways to choose juice. By the Fundamental Counting Principle, there are $3 \cdot 2 \cdot 4$ or 24 ways to choose the Breakfast Special.

Exercises

Solve each problem.

1. The Palace of Pizza offers small, medium, or large pizzas with 14 different toppings available. How many different one-topping pizzas do they serve? **42**
2. The letters A, B, C, and D are used to form four-letter passwords for entering a computer file. How many passwords are possible if letters can be repeated? **256**
3. A restaurant serves 5 main dishes, 3 salads, and 4 desserts. How many different meals could be ordered if each has a main dish, a salad, and a dessert? **60**
4. Marissa brought 8 T-shirts and 6 pairs of shorts to summer camp. How many different outfits consisting of a T-shirt and a pair of shorts does she have? **48**
5. There are 6 different packages available for school pictures. The studio offers 5 different backgrounds and 2 different finishes. How many different options are available? **60**
6. How many 5-digit even numbers can be formed using the digits 4, 6, 7, 2, 8 if digits can be repeated? **2500**
7. How many license plate numbers consisting of three letters followed by three numbers are possible when repetition is allowed? **17,576,000**
8. How many 4-digit positive even integers are there? **4500**

Skills Practice, p. 701 and Practice, p. 702 (shown)

State whether the events are *independent* or *dependent*.

1. choosing an ice cream flavor and choosing a topping for the ice cream **independent**
2. choosing an offensive player of the game and a defensive player of the game in a professional football game **independent**
3. From 15 entries in an art contest, a camp counselor chooses first, second, and third place winners. **dependent**
4. Jillian is selecting two more courses for her block schedule next semester. She must select one of three morning history classes and one of two afternoon math classes. **independent**

Solve each problem.

5. A briefcase lock has 3 rotating cylinders, each containing 10 digits. How many numerical codes are possible? **1000**
6. A golf club manufacturer makes irons with 7 different shaft lengths, 3 different grips, 5 different lies, and 2 different club head materials. How many different combinations are offered? **210**
7. There are five different routes that a commuter can take from her home to the office. In how many ways can she make a round trip if she uses a different route coming than going? **20**
8. In how many ways can the four call letters of a radio station be arranged if the first letter must be W or K and no letters repeat? **27,600**
9. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and any digit can be repeated? **8,000,000**
10. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and any digit can be repeated? **9,000,000**
11. How many 7-digit phone numbers can be formed if the first digit cannot be 0 or 1, and if no digit can be repeated? **483,840**
12. How many 7-digit phone numbers can be formed if the first digit cannot be 0, and if no digit can be repeated? **544,320**
13. How many 6-character passwords can be formed if the first character is a digit and the remaining 5 characters are letters that can be repeated? **118,813,760**
14. How many 6-character passwords can be formed if the first and last characters are digits and the remaining characters are letters? Assume that any character can be repeated. **45,697,600**

Reading to Learn Mathematics, p. 703 ELL

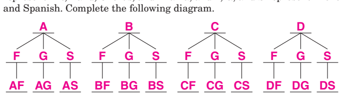
Pre-Activity **How can you count the maximum number of license plates a state can issue?**

Read the introduction to Lesson 12-1 at the top of page 632 in your textbook.

Assume that all Florida license plates have three letters followed by three digits, and that there are no rules against using the same letter or number more than once. How many choices are there for each letter? for each digit? **26; 10**

Reading the Lesson

1. Shamim is signing up for her classes. Most of her classes are required, but she has two electives. For her arts class, she can chose between Art, Band, Chorus, or Drama. For her language class, she can choose between French, German, and Spanish.
   a. To organize her choices, Shamim decides to make a tree diagram. Let A, B, C, and D represent Art, Band, Chorus, and Drama, and F, G, and S represent French, German, and Spanish. Complete the following diagram.



   b. How could Shamim have found the number of possible combinations without making a tree diagram? **Sample answer: Multiply the number of choices for her arts class by the number of choices for her language class: $3 \times 4 = 12$.**
2. A jar contains 6 red marbles, 4 blue marbles, and 3 yellow marbles. Indicate whether the events described are *dependent* or *independent*.
   a. A marble is drawn out of the jar and is not replaced. A second marble is drawn. **dependent**
   b. A marble is drawn out of the jar and is put back in. The jar is shaken. A second marble is drawn. **independent**

Helping You Remember

3. One definition of *independent* is "not determined or influenced by someone or something else." How can this definition help you remember the difference between *independent* and *dependent* events? **Sample answer: If the outcome of one event does *not* affect or influence the outcome of another, the events are independent. If the outcome of one event *does* affect or influence the outcome of another, the events are dependent.**

25. **HOME SECURITY** How many different 5-digit codes are possible using the keypad shown at the right if the first digit cannot be 0 and no digit may be used more than once? **27,216**

More About. . .

Area Codes

Before 1995, area codes had the following format.

(XYZ)

X = 2, 3, …, or 9

Y = 0 or 1

Z = 0, 1, 2, …, or 9

Source: www.nanpa.com

AREA CODES **For Exercises 26 and 27, refer to the information about telephone area codes at the left.**

26. How many area codes were possible before 1995? **160**
27. In 1995, the restriction on the middle digit was removed, allowing any digit in that position. How many total codes were possible after this change was made? **800**
28. **RESEARCH** Use the Internet or other resource to find the configuration of letters and numbers on license plates in your state. Then find the number of possible plates. **See students' work.**
29. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

   How can you count the maximum number of license plates a state can issue?

   Include the following in your answer:
   - an explanation of how to use the Fundamental Counting Principle to find the number of different license plates in a state such as Florida, which has 3 letters followed by 3 numbers, and
   - a way that a state can increase the number of possible plates without increasing the length of the plate number.

Standardized Test Practice

30. How many numbers between 100 and 999, inclusive, have 7 in the tens place? **A**

   (A) 90 (B) 100 (C) 110 (D) 120

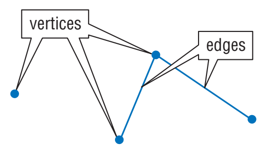
31. A coin is tossed four times. How many possible sequences of heads or tails are possible? **C**

   (A) 4 (B) 8 (C) 16 (D) 32

Extending the Lesson

For Exercises 32 and 33, use the following information.

A **finite graph** is a collection of points, called **vertices**, and segments, called **edges**, connecting the vertices. For example, the graph shown at the right has 4 vertices and 2 edges.



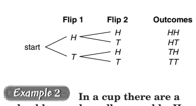
32. Suppose a graph has 10 vertices and each pair of vertices is connected by exactly one edge. Find the number of edges in the graph. (*Hint:* If you use the Fundamental Counting Principle, be sure to count each edge only once.) **45**
33. **TRANSPORTATION** The table shows the distances in miles of the roads between some towns. Draw a graph in which the vertices represent the towns and the edges are labeled with the lengths of the roads. Use your graph to find the length of the shortest route from Greenville to Red Rock. **20 mi**

Route	Miles
Greenville to Roseburg	14
Greenville to Bluemont	12
Greenville to Whiteston	9
Roseburg to Bluemont	8
Bluemont to Whiteston	5
Roseburg to Red Rock	7
Bluemont to Red Rock	9
Whiteston to Red Rock	11

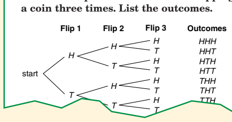
Enrichment, p. 704

Tree Diagrams and the Power Rule

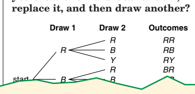
If you flip a coin once, there are two possible outcomes: heads showing (H) or tails showing (T). The tree diagram to the right shows the four (2^2) possible outcomes if you flip a coin twice.



Example 1 **Draw a tree diagram to show all the possible outcomes for flipping a coin three times. List the outcomes.**



Example 2 **In a cup there are a red, a blue, and a yellow marble. How many possible outcomes are there if you draw one marble at random, replace it, and then draw another?**



Maintain Your Skills

Mixed Review

34. Prove that $4 + 7 + 10 \cdots + (3n + 1) = \frac{n(3n+5)}{2}$ for all positive integers n. *(Lesson 11-8)* **See pp. 695A–695B.**

Find the indicated term of each expansion. *(Lesson 11-7)*

35. third term of $(x + y)^8$ **$28x^6y^2$**

36. fifth term of $(2a - b)^7$ **$280a^3b^4$**

Evaluate each expression. *(Lesson 10-2)*

37. $\log_2 128$ **7**

38. $\log_3 243$ **5**

39. $\log_9 3$ **$\frac{1}{2}$**

Simplify each expression. *(Lesson 9-1)*

40. $-\frac{x^2 - y^2}{x + y} \cdot \frac{1}{x - y}$ **-1**

41. $\dfrac{\frac{x^2}{x^2 - 25y^2}}{\frac{x}{5y - x}}$ **$-\frac{x}{x + 5y}$**

42. **CARTOGRAPHY** Edison is located at (9, 3) in the coordinate system on a road map. Kettering is located at (12, 5) on the same map. Each side of a square on the map represents 10 miles. To the nearest mile, what is the distance between Edison and Kettering? *(Lesson 8-1)* **36 mi**

Solve each equation. *(Lesson 7-3)*

43. $x^4 - 5x^2 + 4 = 0$ **$\pm 1, \pm 2$**

44. $y^4 + 4y^3 + 4y^2 = 0$ **$0, -2$**

Write an equation for the parabola with the given vertex that passes through the given point. *(Lesson 6-6)*

45. vertex (3, 2)
point (5, 6)
$y = (x - 3)^2 + 2$

46. vertex (−1, 4)
point (−2, 2)
$y = -2(x + 1)^2 + 4$

47. vertex (0, 8)
point (4, 0)
$y = -\frac{1}{2}x^2 + 8$

Solve each equation. *(Lesson 5-8)*

48. $\sqrt{2x + 1} = 3$ **4**

49. $3 + \sqrt{x + 1} = 5$ **3**

50. $\sqrt{x} + \sqrt{x + 5} = 5$ **4**

Find the inverse of each matrix, if it exists. *(Lesson 4-7)*

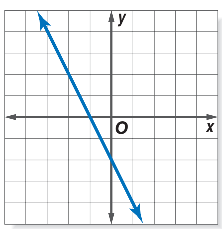
51. $\begin{bmatrix} 3 & 1 \\ -4 & 1 \end{bmatrix}$ **$\frac{1}{7}\begin{bmatrix} 1 & -1 \\ 4 & 3 \end{bmatrix}$**

52. $\begin{bmatrix} 4 & -5 \\ 2 & -1 \end{bmatrix}$ **$\frac{1}{6}\begin{bmatrix} -1 & 5 \\ -2 & 4 \end{bmatrix}$**

53. $\begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$ **no inverse exists**

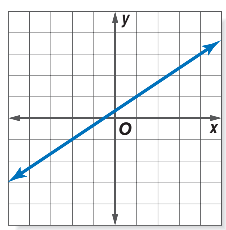
Write an equation in slope-intercept form for each graph. *(Lesson 2-4)*

54.



$y = -2x - 2$

55.



$y = \frac{2}{3}x + \frac{1}{3}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression.**
*(To review **factorials**, see Lesson 11-7.)*

56. $\frac{5!}{2!}$ **60**

57. $\frac{6!}{4!}$ **30**

58. $\frac{7!}{3!}$ **840**

59. $\frac{6!}{1!}$ **720**

60. $\frac{4!}{2!2!}$ **6**

61. $\frac{6!}{2!4!}$ **15**

62. $\frac{8!}{3!5!}$ **56**

63. $\frac{5!}{5!0!}$ **1**

4 Assess

Open-Ended Assessment

Writing Ask students to write a brief explanation of the difference between independent and dependent events, and to give several examples for each.

Getting Ready for Lesson 12-2

PREREQUISITE SKILL Lesson 12-2 presents solving problems involving permutations and combinations. Students will use their familiarity with evaluating expressions involving factorials as they apply formulas for permutations and combinations. Exercises 56–63 should be used to determine your students' familiarity with evaluating factorials.

Answer

29. The maximum number of license plates is a product with factors of 26s and 10s, depending on how many letters are used and how many digits are used. Answers should include the following.

- **There are 26 choices for the first letter, 26 for the second, and 26 for the third. There are 10 choices for the first number, 10 for the second, and 10 for the third. By the Fundamental Counting Principle, there are $26^3 \cdot 10^3$ or 17,576,000 possible license plates.**
- **Replace positions containing numbers with letters.**

Lesson 12-1 The Counting Principle 637

12-2 Lesson Notes

1 Focus

5-Minute Check Transparency 12-2 Use as a quiz or review of Lesson 12-1.

Mathematical Background notes are available for this lesson on p. 630C.

How do permutations and combinations apply to softball?

Ask students:

- Is a lineup or batting order for the first batters of A, B, C, and D different from a lineup of D, C, B, A? **Yes, the order matters.**
- Is the number of ways, 840, equal to either 7! or 4!? **no**
- How could you write $7 \cdot 6 \cdot 5 \cdot 4$ as an expression in terms of 7! and 4!? $\frac{7!}{(7-4)!}$

12-2 Permutations and Combinations

What You'll Learn

- Solve problems involving linear permutations.
- Solve problems involving combinations.

Vocabulary

- permutation
- linear permutation
- combination

How do permutations and combinations apply to softball?

When the manager of a softball team fills out her team's lineup card before the game, the order in which she fills in the names is important because it determines the order in which the players will bat.

Suppose she has 7 possible players in mind for the top 4 spots in the lineup. You know from the Fundamental Counting Principle that there are $7 \cdot 6 \cdot 5 \cdot 4$ or 840 ways that she could assign players to the top 4 spots.

PERMUTATIONS When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**. In a permutation, the *order* of the objects is very important. The arrangement of objects or people in a line is called a **linear permutation**.

Notice that $7 \cdot 6 \cdot 5 \cdot 4$ is the product of the first 4 factors of 7!. You can rewrite this product in terms of 7!.

$$7 \cdot 6 \cdot 5 \cdot 4 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \quad \text{Multiply by } \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or 1.}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} \text{ or } \frac{7!}{3!} \quad 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \text{ and } 3! = 3 \cdot 2 \cdot 1$$

Notice that 3! is the same as $(7 - 4)!$.

Study Tip

Reading Math
The expression $P(n, r)$ is read *the number of permutations of n objects taken r at a time*. It is sometimes written as ${}_nP_r$.

The number of ways to arrange 7 people or objects taken 4 at a time is written $P(7, 4)$. The expression for the softball lineup above is a case of the following formula.

Key Concept *Permutations*

The number of permutations of n distinct objects taken r at a time is given by

$$P(n, r) = \frac{n!}{(n - r)!}.$$

Example 1 *Permutation*

FIGURE SKATING **There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?**

Since each winner will receive a different medal, order is important. You must find the number of permutations of 10 things taken 3 at a time.

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 705–706
- Skills Practice, p. 707
- Practice, p. 708
- Reading to Learn Mathematics, p. 709
- Enrichment, p. 710

Graphing Calculator and Spreadsheet Masters, p. 50

Transparencies

5-Minute Check Transparency 12-2
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 22
Interactive Chalkboard

$$P(n, r) = \frac{n!}{(n-r)!}$$ Permutation formula

$$P(10, 3) = \frac{10!}{(10-3!)}$$ $n = 10, r = 3$

$$= \frac{10!}{7!}$$ Simplify.

$$= \frac{10 \cdot 9 \cdot 8 \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{6}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{4}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{1}}}{\underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{6}} \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{1}}} \text{ or } 720$$ Divide by common factors.

The gold, silver, and bronze medals can be awarded in 720 ways.

Notice that in Example 1, all of the factors of $(n - r)!$ are also factors of $n!$. Instead of writing all of the factors, you can also evaluate the expression in the following way.

$$\frac{10!}{(10-3)!} = \frac{10!}{7!}$$ Simplify.

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$$ $\frac{7!}{7!} = 1$

$$= 10 \cdot 9 \cdot 8 \text{ or } 720$$ Multiply.

Suppose you want to rearrange the letters of the word *geometry* to see if you can make a different word. If the two *e*'s were not identical, the eight letters in the word could be arranged in $P(8, 8)$ or 8! ways. To account for the identical *e*'s, divide $P(8, 8)$ or 40,320 by the number of arrangements of *e*. The two *e*'s can be arranged in $P(2, 2)$ or 2! ways.

$$\frac{P(8, 8)}{P(2, 2)} = \frac{8!}{2!}$$ Divide.

$$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} \text{ or } 20{,}160$$ Simplify.

Thus, there are 20,160 ways to arrange the letters in *geometry*.

When some letters or objects are alike, use the rule below to find the number of permutations.

Key Concept — **Permutations with Repetitions**

The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

This rule can be extended to any number of objects that are repeated.

Example 2 Permutation with Repetition

How many different ways can the letters of the word *MISSISSIPPI* be arranged?

The second, fifth, eighth, and eleventh letters are each I.

The third, fourth, sixth, and seventh letters are each S.

The ninth and tenth letters are each P.

You need to find the number of permutations of 11 letters of which 4 of one letter, 4 of another letter, and 2 of another letter are the same.

$$\frac{11!}{4!4!2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!4!2!} \text{ or } 34{,}650$$

There are 34,650 ways to arrange the letters.

www.algebra2.com/extra_examples

2 Teach

PERMUTATIONS

In-Class Examples Power Point®

1 Eight people enter the Best Pie contest. How many ways can blue, red, and green ribbons be awarded? **336**

2 How many different ways can the letters of the word *BANANA* be arranged? **60**

DAILY INTERVENTION

Unlocking Misconceptions

Discuss the notation for $P(n, r)$. Students might reasonably think that this expression could be an ordered pair, or function notation. However, when the P is used, the expression is probably for permutation notation, which can also be written as ${}_nP_r$.

COMBINATIONS

In-Class Examples

Power Point®

3 Five cousins at a family reunion decide that three of them will go to pick up a pizza. How many ways can they choose the three people to go? **10**

4 Six cards are drawn from a standard deck of cards. How many hands consist of two hearts and four spades? **55,770**

Study Tip

Permutations and Combinations

- If order in an arrangement *is* important, the arrangement is a *permutation*.
- If order is *not* important, the arrangement is a *combination*.

COMBINATIONS An arrangement or selection of objects in which order is *not* important is called a **combination**. The number of combinations of n objects taken r at a time is written $C(n, r)$. *It is sometimes written* $_nC_r$.

You know that there are $P(n, r)$ ways to select r objects from a group of n if the order is important. There are $r!$ ways to order the r objects that are selected, so there are $r!$ permutations that are all the same combination. Therefore,

$$C(n, r) = \frac{P(n, r)}{r!} \text{ or } \frac{n!}{(n-r)!r!}.$$

Key Concept — Combinations

The number of combinations of n distinct objects taken r at a time is given by

$$C(n, r) = \frac{n!}{(n-r)!r!}.$$

Example 3 Combination

A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they choose the two students?

Since the order they choose the students is not important, you must find the number of combinations of 7 students taken 2 at a time.

$C(n, r) = \frac{n!}{(n-r)!r!}$ Combination formula

$C(7, 2) = \frac{7!}{(7-2)!2!}$ $n = 7$ and $r = 2$

$= \frac{7!}{5!2!}$ or 21 Simplify.

There are 21 possible ways to choose the two students.

In more complicated situations, you may need to multiply combinations and/or permutations.

Study Tip

Deck of Cards

In this text, a *standard deck of cards* always means a deck of 52 playing cards. There are 4 suits—clubs (black), diamonds (red), hearts (red), and spades (black)—with 13 cards in each suit.

Example 4 Multiple Events

Five cards are drawn from a standard deck of cards. How many hands consist of three clubs and two diamonds?

By the Fundamental Counting Principle, you can multiply the number of ways to select three clubs and the number of ways to select two diamonds.

Only the cards in the hand matter, not the order in which they were drawn, so use combinations.

C(13, 3) Three of 13 clubs are to be drawn.

C(13, 2) Two of 13 diamonds are to be drawn.

$C(13, 3) \cdot C(13, 2) = \frac{13!}{(13-3)!3!} \cdot \frac{13!}{(13-2)!2!}$ Combination formula

$= \frac{13!}{10!3!} \cdot \frac{13!}{11!2!}$ Subtract.

$= 286 \cdot 78$ or 22,308 Simplify.

There are 22,308 hands consisting of 3 clubs and 2 diamonds.

DAILY INTERVENTION — Differentiated Instruction

Visual/Spatial Have students model the various problems by writing letters, names, or other labels on index cards. After students have tried to model and tally possible combinations, they will soon realize that the formulas save lots of time.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe a situation in which the number of outcomes is given by $P(6, 3)$. **See margin.**
2. **Show** that $C(n, n - r) = C(n, r)$. **See margin.**
3. **Determine** whether the statement $C(n, r) = P(n, r)$ is *sometimes*, *always*, or *never* true. Explain your reasoning. **Sometimes; the statement is true when $r = 1$.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6, 7, 11	3
8, 9	1, 3
10	2

Evaluate each expression.

4. $P(5, 3)$ **60**
5. $P(6, 3)$ **120**
6. $C(4, 2)$ **6**
7. $C(6, 1)$ **6**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

8. choosing 2 different pizza toppings from a list of 6 **combination; 15**
9. seven shoppers in line at a checkout counter **permutation; 5040**
10. an arrangement of the letters in the word *intercept* **permutation; 90,720**

Application

11. **SCHOOL** The principal at Cobb County High School wants to start a mentoring group. He needs to narrow his choice of students to be mentored to six from a group of nine. How many ways can a group of six be selected? **84**

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
12–15	1
16–19	3
20, 21, 32–35	4
22–31	1–3

Extra Practice
See page 854.

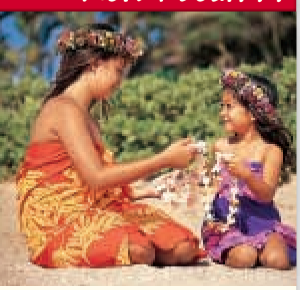
Evaluate each expression.

12. $P(8, 2)$ **56**
13. $P(9, 1)$ **9**
14. $P(7, 5)$ **2520**
15. $P(12, 6)$ **665,280**
16. $C(5, 2)$ **10**
17. $C(8, 4)$ **70**
18. $C(12, 7)$ **792**
19. $C(10, 4)$ **210**
★20. $C(12, 4) \cdot C(8, 3)$ **27,720**
★21. $C(9, 3) \cdot C(6, 2)$ **1260**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. **22. permutation; 5040** **26. combination; 220**

22. the winner and first, second, and third runners-up in a contest with 10 finalists
23. selecting two of eight employees to attend a business seminar **combination; 28**
24. an arrangement of the letters in the word *algebra* **permutation; 2520**
25. placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf **permutation; 120**
26. selecting nine books to check out of the library from a reading list of twelve
27. an arrangement of the letters in the word *parallel* **permutation; 3360**
28. choosing two CDs to buy from ten that are on sale **combination; 45**
29. selecting three of fifteen flavors of ice cream at the grocery store **combination; 455**
30. **MOVIES** The manager of a four-screen movie theater is deciding which of 12 available movies to show. The screens are in rooms with different seating capacities. How many ways can he show four different movies on the screens? **11,880**
31. **LANGUAGES** How many different arrangements of the letters of the Hawaiian word *aloha* are possible? **60**
32. **GOVERNMENT** How many ways can five members of the 100-member United States Senate be chosen to be put on a committee? **75,287,520**

More About. . .



Languages

The Hawaiian language consists of only twelve letters, the vowels a, e, i, o, and u and the consonants h, k, l, m, n, p, and w.

Source: www.andhawaii.com

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include their own examples for different kinds of permutations and combinations.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Permutations:** 12–15, 22, 24, 25, 27
- **Combinations:** 16–21, 23, 26, 28, 29

Odd/Even Assignments
Exercises 12–29 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–19 odd, 23–31 odd, 37–40, 44–72

Average: 13–35 odd, 37–40, 44–72 (optional: 41–43)

Advanced: 12–36 even, 37–68 (optional: 69–72)

Answers

1. Sample answer: There are six people in a contest. How many ways can the first, second, and third prizes be awarded?
2. $C(n, n - r)$

$$= \frac{n!}{[n - (n - r)]!(n - r)!}$$
$$= \frac{n!}{r!(n - r)!}$$
$$= \frac{n!}{(n - r)!r!}$$
$$= C(n, r)$$

Teacher to Teacher

Harry Rattien   Townsend Harris H.S. at Queens College, Flushing, NY

I use the following mnemonic device to help my students remember the difference between permutations and combinations.

Permutation → place   Combination → choose

Study Guide and Intervention, p. 705 (shown) and p. 706

Permutations When a group of objects or people are arranged in a certain order, the arrangement is called a **permutation**.

Permutations	The number of permutations of n distinct objects taken r at a time is given by $P(n, r) = \frac{n!}{(n-r)!}$.
Permutations with Repetitions	The number of permutations of n objects of which p are alike and q are alike is $\frac{n!}{p!q!}$.

The rule for permutations with repetitions can be extended to any number of objects that are repeated.

Example **From a list of 20 books, each student must choose 4 books for book reports. The first report is a traditional book report, the second a poster, the third a newspaper interview with one of the characters, and the fourth a timeline of the plot. How many different orderings of books can be chosen?**

Since each book report has a different format, order is important. You must find the number of permutations of 20 objects taken 4 at a time.

$P(n, r) = \frac{n!}{(n-r)!}$ Permutation formula

$P(20, 4) = \frac{20!}{(20-4)!}$ $n = 20$, $r = 4$

$= \frac{20!}{16!}$ Simplify.

$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot \cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}{\cancel{16} \cdot \cancel{15} \cdot \ldots \cdot \cancel{1}}$ Divide by common factors.

$= 116{,}280$

Books for the book reports can be chosen 116,280 ways.

Exercises

Evaluate each expression.

1. $P(6, 3)$ **120**
2. $P(8, 5)$ **6720**
3. $P(9, 4)$ **3024**
4. $P(11, 6)$ **332,640**

How many different ways can the letters of each word be arranged?

5. MOM **3**
6. MONDAY **720**
7. STEREO **360**

8. **SCHOOL** The high school chorus has been practicing 12 songs, but there is time for only 5 of them at the spring concert. How may different orderings of 5 songs are possible? **95,040**

Skills Practice, p. 707 and Practice, p. 708 (shown)

Evaluate each expression.

1. $P(8, 6)$ **20,160**
2. $P(9, 7)$ **181,440**
3. $P(3, 3)$ **6**
4. $P(4, 3)$ **24**
5. $P(4, 1)$ **4**
6. $P(7, 2)$ **42**
7. $C(8, 2)$ **28**
8. $C(11, 3)$ **165**
9. $C(20, 18)$ **190**
10. $C(9, 9)$ **1**
11. $C(3, 1)$ **3**
12. $C(9, 3) \cdot C(6, 2)$ **1260**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.

13. selecting a 4-person bobsled team from a group of 9 athletes **combination; 126**
14. an arrangement of the letters in the word *Canada* **permutation; 120**
15. arranging 4 charms on a bracelet that has a clasp, a front, and a back **permutation; 24**
16. selecting 3 desserts from 10 desserts that are displayed on a dessert cart in a restaurant **combination; 120**
17. an arrangement of the letters in the word *annually* **permutation; 5040**
18. forming a 2-person sales team from a group of 12 salespeople **combination; 66**
19. making 5-sided polygons by choosing any 5 of 11 points located on a circle to be the vertices **combination; 462**
20. seating 5 men and 5 women alternately in a row, beginning with a woman **permutation; 14,400**
21. **STUDENT GROUPS** Farmington High is planning its academic festival. All math classes will send 2 representatives to compete in the math bowl. How many different groups of students can be chosen from a class of 16 students? **120**
22. **PHOTOGRAPHY** A photographer is taking pictures of a bride and groom and their 6 attendants. If she takes photographs of 3 people in a group, how many different groups can she photograph? **56**
23. **AIRLINES** An airline is hiring 5 flight attendants. If 8 people apply for the job, how many different groups of 5 attendants can the airline hire? **56**
24. **SUBSCRIPTIONS** A school librarian would like to buy subscriptions to 7 new magazines. Her budget, however, will allow her to buy only 4 new subscriptions. How many different groups of 4 magazines can she choose from the 7 magazines? **35**

Reading to Learn Mathematics, p. 709 ELL

Pre-Activity **How do permutations and combinations apply to softball?**

Read the introduction to Lesson 12-2 at the top of page 638 in your textbook.

Suppose that 20 students enter a math contest. In how many ways can first, second, and third places be awarded? (Write your answer as a product. Do not calculate the product.) **20 · 19 · 18**

Reading the Lesson

1. Indicate whether each situation involves a *permutation* or a *combination*.
   a. choosing five students from a class to work on a special project **combination**
   b. arranging five pictures in a row on a wall **permutation**
   c. drawing a hand of 13 cards from a 52-card deck **combination**
   d. arranging the letters of the word *algebra* **permutation**
2. Write an expression that can be used to calculate each of the following.
   a. number of combinations of n distinct objects taken r at a time $\frac{n!}{(n-r)!r!}$
   b. number of permutations of n objects of which p are alike and q are alike $\frac{n!}{p!q!}$
   c. number of permutations of n distinct objects taken r at a time $\frac{n!}{(n-r)!}$
3. Five cards are drawn from a standard deck of cards. Suppose you are asked to determine how many possible hands consist of one heart, two diamonds, and two spades.
   a. Which of the following would you use to solve this problem: *Fundamental Counting Principle*, *permutations*, or *combinations*? (More than one of these may apply.)
   Fundamental Counting Principle, combinations
   b. Write an expression that involves the notation $P(n, r)$ and/or $C(n, r)$ that you would use to solve this problem. (Do not do any calculations.)
   $C(13, 1) \cdot C(13, 2) \cdot C(13, 2)$

Helping You Remember

4. Many students have trouble knowing when to use permutations and when to use combinations to solve counting problems. How can the idea of *order* help you to remember the difference between permutations and combinations?
   Sample answer: A permutation is an arrangement of objects in which order is important. A combination is a selection of objects in which order is *not* important.

More About. . .

Card Games

Hanafuda cards are often called "flower cards" because each suit is depicted by a different flower. Each flower is representative of the calendar month in which the flower blooms.

Source: www.gamesdomain.com

★ 33. How many ways can a hand of five cards consisting of four cards from one suit and one card from another suit be drawn from a standard deck of cards? **111,540**

★ 34. How many ways can a hand of five cards consisting of three cards from one suit and two cards from another suit be drawn from a standard deck of cards? **267,696**

35. **LOTTERIES** In a multi-state lottery, the player must guess which five of forty-nine white balls numbered from 1 to 49 will be drawn. The order in which the balls are drawn does not matter. The player must also guess which one of forty-two red balls numbered from 1 to 42 will be drawn. How many ways can the player fill out a lottery ticket? **80,089,128**

36. **CARD GAMES** *Hanafuda* is a Japanese game that uses a deck of cards made up of 12 suits, with each suit having four cards. How many 7-card hands can be formed so that 3 are from one suit and 4 are from another? **528**

37. **CRITICAL THINKING** Show that $C(n-1, r) + C(n-1, r-1) = C(n, r)$. **See pp. 695A–695B.**

38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 695A–695B.**

How do permutations and combinations apply to softball?

Include the following in your answer:
- an explanation of how to find the number of 9-person lineups that are possible, and
- an explanation of how many ways there are to choose 9 players if 16 players show up for a game.

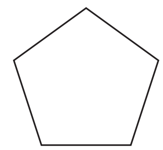
Standardized Test Practice

39. How many ways can eight runners in an Olympic race finish in first, second, and third places? **D**

Ⓐ 8 Ⓑ 24 Ⓒ 56 Ⓓ 336

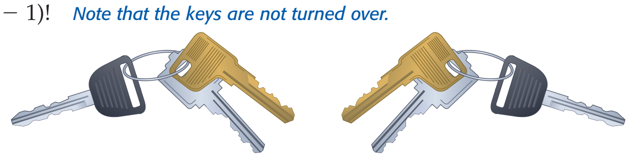
40. How many diagonals can be drawn in the pentagon? **A**

Ⓐ 5 Ⓑ 10
Ⓒ 15 Ⓓ 20



Extending the Lesson

When n distinct objects are arranged in a circle, there are n ways that the arrangement can be rotated to obtain an arrangement that is really the same as the original. For example, the two arrangements of three objects shown below are the same. Therefore, the number of **circular permutations** of n distinct objects is $\frac{n!}{n}$ or $(n-1)!$ *Note that the keys are not turned over.*



Find the number of possibilities for each situation.

41. a basketball huddle of 5 players **24**
42. four different dishes on a revolving tray in the middle of a table at a Chinese restaurant **6**
43. six quarters with designs from six different states arranged in a circle on top of your desk **120**

Enrichment, p. 710

Combinations and Pascal's Triangle

Pascal's triangle is a special array of numbers invented by Blaise Pascal (1623–1662). The values in Pascal's triangle can be found using the combinations shown below.

1. Evaluate the expression in each cell of the triangle.

C(1,0) **1** | C(1,1) **1**

C(2,0) **1** | C(2,1) **2** | C(2,2) **1**

C(3,0) **1** | C(3,1) **3** | C(3,2) **3** | C(3,3) **1**

C(4,1) | C(4,2) | C(4,3) | C(4,4)

Maintain Your Skills

Mixed Review

44. Darius can do his homework in pencil or pen, using lined or unlined paper, and on one or both sides of each page. How many ways can he prepare his homework? *(Lesson 12-1)* **8**

45. A customer in an ice cream shop can order a sundae with a choice of 10 flavors of ice cream, a choice of 4 flavors of sauce, and with or without a cherry on top. How many different sundaes are possible? *(Lesson 12-1)* **80**

Find a counterexample to each statement. *(Lesson 11-8)*

46. $1 + 2 + 3 + \ldots + n = 2n - 1$

47. $5^n + 1$ is divisible by 6.

46. Sample answer: $n = 3$
47. Sample answer: $n = 2$

Solve each equation or inequality. *(Lesson 10-5)*

48. $3e^x + 1 = 2$ **-1.0986**

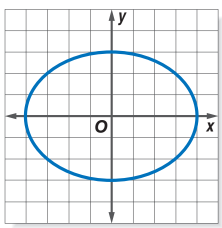
49. $e^{2x} > 5$ **$x > 0.8047$**

50. $\ln(x - 1) = 3$ **21.0855**

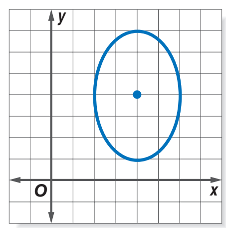
51. **CONSTRUCTION** A painter works on a job for 10 days and is then joined by an associate. Together they finish the job in 6 more days. The associate could have done the job in 30 days. How long would it have taken the painter to do the job alone? *(Lesson 9-6)* **20 days**

Write an equation for each ellipse. *(Lesson 8-4)*

52.



53.



52. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

53. $\frac{(y-4)^2}{9} + \frac{(x-4)^2}{4} = 1$

Find $p(-1)$ and $p(5)$ for each function. *(Lesson 7–1)*

54. $p(x) = \frac{1}{2}x^2 + 3x - 1$ **$-\frac{7}{2}; \frac{53}{2}$**

55. $p(x) = x^4 - 4x^3 + 2x - 7$ **$-4; 128$**

Solve each equation by factoring. *(Lesson 6-3)*

56. $x^2 - 16 = 0$ **$\{-4, 4\}$**

57. $x^2 - 3x - 10 = 0$ **$\{-2, 5\}$**

58. $3x^2 + 8x - 3 = 0$ **$\left\{-3, \frac{1}{3}\right\}$**

Simplify. *(Lesson 5-6)*

59. $\sqrt{128}$ **$8\sqrt{2}$**

60. $\sqrt{3x^6y^4}$ **$|x^3|y^2\sqrt{3}$**

61. $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$ **$4\sqrt{5}$**

Solve each system of equations by using inverse matrices. *(Lesson 4-8)*

62. $x + 2y = 5$
$3x - 3y = -12$ **$(-1, 3)$**

63. $5a + 2b = 4$
$-3a + b = 2$ **$(0, 2)$**

Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

64. $(2, 1), (5, -3)$ **$-\frac{4}{3}$**

65. $(0, 4), (7, -2)$ **$-\frac{6}{7}$**

66. $(5, 3), (2, 3)$ **0**

Solve each equation. Check your solutions. *(Lesson 1-4)*

67. $|x - 4| = 11$ **$\{-7, 15\}$**

68. $|2x + 2| = -3$ **$\varnothing$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL Evaluate the expression $\frac{x}{x + y}$ for the given values of x and y. *(To review **evaluating expressions**, see Lesson 1-1.)*

69. $x = 3, y = 2$ **$\frac{3}{5}$**

70. $x = 4, y = 4$ **$\frac{1}{2}$**

71. $x = 2, y = 8$ **$\frac{1}{5}$**

72. $x = 5, y = 10$ **$\frac{1}{3}$**

4 Assess

Open-Ended Assessment

Speaking Ask students to work with a partner. One writes an expression, such as $C(3, 2)$, and hands it to the other, who reads the notation aloud (for example, "the number of combinations of 3 things taken 2 at a time") and calculates the value. **3** The partners discuss and correct this value as necessary. Then they exchange roles.

Getting Ready for Lesson 12-3

PREREQUISITE SKILL Lesson 12-3 presents finding the probability and odds of events. Students will use their familiarity with evaluating rational expressions as they apply probability formulas. Exercises 69–72 should be used to determine your students' familiarity with evaluating rational expressions.

Lesson 12-2 Permutations and Combinations 643

12-3 Lesson Notes

1 Focus

5-Minute Check Transparency 12-3 Use as a quiz or review of Lesson 12-2.

Mathematical Background notes are available for this lesson on p. 630C.

What do probability and odds tell you about life's risks?

Ask students:

- On average, out of 750,000 people, how many will be struck by lightning each year? **1**
- If there are 260 million people in the United States, how many people will be struck by lightning each year? **about 347**
- Does probability say anything about where or why an event occurs? **no**

12-3 Probability

What You'll Learn

- Find the probability and odds of events.
- Create and use graphs of probability distributions.

Vocabulary

- probability
- success
- failure
- random
- odds
- random variable
- probability distribution
- relative-frequency histogram

What do probability and odds tell you about life's risks?

The risk of getting struck by lightning in any given year is 1 in 750,000. The chances of surviving a lightning strike are 3 in 4. These risks and chances are a way of describing the probability of an event. The **probability** of an event is a ratio that measures the chances of the event occurring.

PROBABILITY AND ODDS Mathematicians often use tossing of coins and rolling of dice to illustrate probability. When you toss a coin, there are only two possible outcomes—heads or tails. A desired outcome is called a **success**. Any other outcome is called a **failure**.

Key Concept *Probability of Success and Failure*

If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows.

$$P(S) = \frac{s}{s+f} \qquad P(F) = \frac{f}{s+f}$$

The probability of an event occurring is always between 0 and 1, inclusive. The closer the probability of an event is to 1, the more likely the event is to occur. The closer the probability of an event is to 0, the less likely the event is to occur.

Study Tip

Reading Math
When P is followed by an event in parentheses, P stands for *probability*. When there are two numbers in parentheses, P stands for *permutations*.

Example 1 *Probability*

When two coins are tossed, what is the probability that both are tails?

You can use a tree diagram to find the sample space.

First coin	H		T	
Second coin	H	T	H	T
Possible outcomes	HH	HT	TH	TT

There are 4 possible outcomes. You can confirm this using the Fundamental Counting Principle. There are 2 possible results for the first coin and 2 for the second coin, so there are $2 \cdot 2$ or 4 possible outcomes. Only one of these outcomes, TT, is a success, so $s = 1$. The other three outcomes are failures, so $f = 3$.

$P(\text{two tails}) = \frac{s}{s+f}$ Probability formula

$= \frac{1}{1+3}$ or $\frac{1}{4}$ $s = 1, f = 3$

The probability of tossing two heads is $\frac{1}{4}$. *This probability can also be written as a decimal, 0.25, or as a percent, 25%.*

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 711–712
- Skills Practice, p. 713
- Practice, p. 714
- Reading to Learn Mathematics, p. 715
- Enrichment, p. 716
- Assessment, p. 767

Transparencies

5-Minute Check Transparency 12-3
Answer Key Transparencies

Technology

Interactive Chalkboard

In more complicated situations, you may need to use permutations and/or combinations to count the outcomes. When all outcomes have an equally likely chance of occurring, we say that the outcomes occur at **random**.

Example 2 Probability with Combinations

Monifa has a collection of 32 CDs—18 R&B and 14 rap. As she is leaving for a trip, she randomly chooses 6 CDs to take with her. What is the probability that she selects 3 R&B and 3 rap?

Step 1 Determine how many 6-CD selections meet the conditions.

$C(18, 3)$ Select 3 R&B CDs. Their order does not matter.
$C(14, 3)$ Select 3 rap CDs.

Step 2 Use the Fundamental Counting Principle to find the number of successes.

$$C(18, 3) \cdot C(14, 3) = \frac{18!}{15!3!} \cdot \frac{14!}{11!3!} \text{ or } 297{,}024$$

Step 3 Find the total number, $s + f$, of possible 6-CD selections.

$$C(32, 6) = \frac{32!}{26!6!} \text{ or } 906{,}192 \qquad s + f = 906{,}192$$

Step 4 Determine the probability.

$$P(\text{3 R\&B CDs and 3 rap CDs}) = \frac{s}{s+f} \qquad \text{Probability formula}$$
$$= \frac{297{,}024}{906{,}192} \qquad \text{Substitute.}$$
$$\approx 0.32777 \qquad \text{Use a calculator.}$$

The probability of selecting 3 R&B CDs and 3 rap CDs is about 0.32777 or 33%.

Another way to measure the chance of an event occurring is with odds. The **odds** that an event will occur can be expressed as the ratio of the number of successes to the number of failures.

Key Concept — Odds

The odds that an event will occur can be expressed as the ratio of the number of ways it can succeed to the number of ways it can fail. If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows.

Odds of success $= s:f$ Odds of failure $= f:s$

Example 3 Odds

LIFE EXPECTANCY **According to the U.S. National Center for Health Statistics, the chances of a male born in 1990 living to be at least 65 years of age are about 3 in 4. For females, the chances are about 17 in 20.**

a. What are the odds of a male living to be at least 65?

Three out of four males will live to be at least 65, so the number of successes (living to 65) is 3. The number of failures is 4 − 3 or 1.

$$\text{odds of a male living to 65} = s:f \qquad \text{Odds formula}$$
$$= 3:1 \qquad s = 3, f = 1$$

The odds of a male living to at least 65 are 3:1.

www.algebra2.com/extra_examples

2 Teach

PROBABILITY AND ODDS

In-Class Examples Power Point®

1 When three coins are tossed, what is the probability that all three are heads? $\frac{1}{8}$ **or 12.5%**

2 Roman has a collection of 26 books—16 are fiction and 10 are nonfiction. He randomly chooses 8 books to take with him on vacation. What is the probability that he chooses 4 fiction and 4 nonfiction? **0.24464 or 24.5%**

3 Using the statistics in Example 3 in the Student Edition, what are the odds that a male born in 1990 will die before age 65? **1:3** a female born in 1990? **3:17**

PROBABILITY DISTRIBUTIONS

In-Class Example Power Point®

4 Use the table and graph in Example 4 in the Student Edition.

a. Use the graph to determine which outcomes are least likely. What is their probability? **The least likely outcomes are 2 and 12, with a probability of $\frac{1}{36}$ for each.**

b. Use the table to find $P(S = 11)$. What other sum has the same probability? **The probability of a sum of 11 is $\frac{1}{18}$, which is the same as that for a sum of 3.**

c. What are the odds of rolling a sum of 5? **1:8**

b. What are the odds of a female living to be at least 65?

Seventeen out of twenty females will live to be at least 65, so the number of successes in this case is 17. The number of failures is 20 − 17 or 3.

odds of a female living to be 65 $= s:f$ Odds formula

$= 17:3$ $s = 17, f = 3$

The odds of a female living to at least 65 are 17:3.

PROBABILITY DISTRIBUTIONS Many experiments, such as rolling a die, have numerical outcomes. A **random variable** is a variable whose value is the numerical outcome of a random event. For example, when rolling a die we can let the random variable D represent the number showing on the die. Then D can equal 1, 2, 3, 4, 5, or 6. A **probability distribution** for a particular random variable is a function that maps the sample space to the probabilities of the outcomes in the sample space. The table below illustrates the probability distribution for rolling a die.

D = Roll	1	2	3	4	5	6
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$P(D = 4) = \frac{1}{6}$

Study Tip

Reading Math

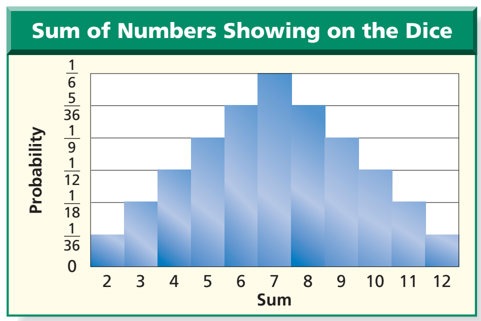
The notation $P(X = n)$ is used with random variables. $P(D = 4) = \frac{1}{6}$ is read *the probability that D equals 4 is one sixth.*

To help visualize a probability distribution, you can use a table of probabilities or a graph, called a **relative-frequency histogram**.

Example 4 Probability Distribution

Suppose two dice are rolled. The table and the relative-frequency histogram show the distribution of the sum of the numbers rolled. *You will be asked to verify some of these probabilities in Exercise 3.*

S = Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$



a. Use the graph to determine which outcome is most likely. What is its probability?

The greatest probability in the graph is $\frac{1}{6}$. The most likely outcome is a sum of 7 and its probability is $\frac{1}{6}$.

b. Use the table to find $P(S = 9)$. What other sum has the same probability?

According to the table, the probability of a sum of 9 is $\frac{1}{9}$. The other outcome with a probability of $\frac{1}{9}$ is 5.

DAILY INTERVENTION

Differentiated Instruction

Naturalist Ask students to find examples outside the classroom of odds and probabilities, perhaps from statistics on natural disasters or weather reports. Have them share these examples with the class.

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c. **What are the odds of rolling a sum of 7?**

Step 1 Identify *s* and *f*.

$P(\text{rolling a } 7) = \frac{1}{6}$

$= \frac{s}{s+f}$ $s = 1, f = 5$

Step 2 Find the odds.

Odds $= s:f$

$= 1:5$

So, the odds of rolling a sum of 7 are 1:5.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe an event that has a probability of 0 and an event that has a probability of 1. **See margin.**
2. **Write** the probability of an event whose odds are 3:2. $\frac{3}{5}$
3. **Verify** the probabilities given for sums of 2 and 3 in Example 4. **See margin.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	2
7–12	3
13, 14	4
15–18	1

Suppose you select 2 letters at random from the word *compute*. Find each probability.

4. $P(\text{2 vowels})$ $\frac{1}{7}$
5. $P(\text{2 consonants})$ $\frac{2}{7}$
6. $P(\text{1 vowel, 1 consonant})$ $\frac{4}{7}$

Find the odds of an event occurring, given the probability of the event.

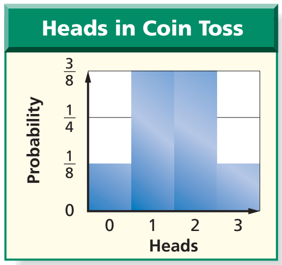
7. $\frac{8}{9}$ 8:1
8. $\frac{1}{6}$ 1:5
9. $\frac{2}{9}$ 2:7

Find the probability of an event occurring, given the odds of the event.

10. 6:5 $\frac{6}{11}$
11. 10:1 $\frac{10}{11}$
12. 2:5 $\frac{2}{7}$

The table and the relative-frequency histogram show the distribution of the number of heads when 3 coins are tossed. Find each probability.

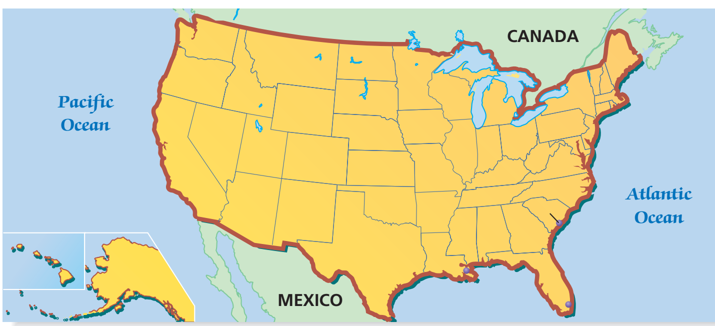
H = Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



13. $P(H = 0)$ $\frac{1}{8}$
14. $P(H = 2)$ $\frac{3}{8}$

Application

GEOGRAPHY **For Exercises 15–18, find each probability if a state is chosen at random from the 50 states.**



15. $P(\text{next to the Pacific Ocean})$ $\frac{1}{10}$
16. $P(\text{has at least five neighboring states})$ $\frac{21}{50}$
17. $P(\text{borders Mexico})$ $\frac{2}{25}$
18. $P(\text{is surrounded by water})$ $\frac{1}{50}$

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Probability and Odds:** 19–53
- **Probability Distributions:** 55–60

Odd/Even Assignments

Exercises 19–29, 34–49, and 55–60 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 19–59 odd, 62–65, 70–82

Average: 19–61 odd, 62–65, 70–82 (optional: 66–69)

Advanced: 20–60 even, 61–78 (optional: 79–83)

All: Practice Quiz 1 (1–10)

Answers

1. Sample answer: The event *July comes before June* has a probability of 0. The event *June comes before July* has a probability of 1.

3. There are 6 · 6 or 36 possible outcomes for the two dice. Only 1 outcome, 1 and 1, results in a sum of 2, so $P(2) = \frac{1}{36}$. There are 2 outcomes, 1 and 2 as well as 2 and 1, that result in a sum of 3, so $P(3) = \frac{2}{36}$ or $\frac{1}{18}$.

Tips for New Teachers

Intervention Students may be confused about the nature of odds and probability. Lead students in a discussion about the difference between theoretical and experimental probability. If you have a state lottery, this may be an opportunity to examine mistaken beliefs about chance. Be sensitive to the fact that some students may have cultural or familial prohibitions against cards, dice, or gambling of any kind. Explain that historically the laws of probability were actually developed in the context of gambling, but they are now used in many other ways, including medicine and meteorology.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
19–33, 54	1, 2
34–53	3
55–60	4

Extra Practice
See page 854.

Ebony has 4 male kittens and 7 female kittens. She picks up 2 kittens to give to a friend. Find the probability of each selection.

19. P(2 male) $\frac{6}{55}$
20. P(2 female) $\frac{21}{55}$
21. P(1 of each) $\frac{28}{55}$

Bob is moving and all of his CDs are mixed up in a box. Twelve CDs are rock, eight are jazz, and five are classical. If he reaches in the box and selects them at random, find each probability.

22. P(3 jazz) $\frac{14}{575}$
23. P(3 rock) $\frac{11}{115}$
24. P(1 classical, 2 jazz) $\frac{7}{115}$
25. P(2 classical, 1 rock) $\frac{6}{115}$
26. P(1 jazz, 2 rock) $\frac{132}{575}$
27. P(1 classical, 1 jazz, 1 rock) $\frac{24}{115}$
28. P(2 rock, 2 classical) $\frac{6}{115}$
29. P(2 jazz, 1 reggae) 0

30. **LOTTERIES** The state of Florida has a lottery in which 6 numbers out of 53 are drawn at random. What is the probability of a given ticket matching all 6 numbers in any order? $\frac{1}{22,957,480}$

More About. . .

Entrance Tests

In addition to the MCAT, most medical schools require applicants to have had one year each of biology, physics, and English, and two years of chemistry in college.

ENTRANCE TESTS **For Exercises 31–33, use the table that shows the college majors of the students who took the Medical College Admission Test (MCAT) in April 2000.**
If a student taking the test were randomly selected, find each probability. Express as decimals rounded to the nearest thousandth.

Major	Students
biological sciences	15,819
humanities	963
math or statistics	179
physical sciences	2770
social sciences	2482
specialized health sciences	1431
other	1761

31. P(math or statistics) 0.007
32. P(biological sciences) 0.623
33. P(physical sciences) 0.109

Find the odds of an event occurring, given the probability of the event.

34. $\frac{1}{2}$ 1:1
35. $\frac{3}{8}$ 3:5
36. $\frac{11}{12}$ 11:1
37. $\frac{5}{8}$ 5:3
38. $\frac{4}{7}$ 4:3
39. $\frac{1}{5}$ 1:4
40. $\frac{4}{11}$ 4:7
41. $\frac{3}{4}$ 3:1

Find the probability of an event occurring, given the odds of the event.

42. 6:1 $\frac{6}{7}$
43. 3:7 $\frac{3}{10}$
44. 5:6 $\frac{5}{11}$
45. 4:5 $\frac{4}{9}$
46. 9:8 $\frac{9}{17}$
47. 1:8 $\frac{1}{9}$
48. 7:9 $\frac{7}{16}$
49. 3:2 $\frac{3}{5}$

50. **GENEOLOGY** The odds that an American is of English ancestry are 1:9. What is the probability that an American is of English ancestry? $\frac{1}{10}$

GENETICS **For Exercises 51 and 52, use the following information.**
Eight out of 100 males and 1 out of 1000 females have some form of color blindness.

51. What are the odds of a male being color-blind? 2:23
52. What are the odds of a female being color-blind? 1:999

53. **EDUCATION** Josefina's guidance counselor estimates that the probability she will get a college scholarship is $\frac{4}{5}$. What are the odds that she will *not* earn a scholarship? 1:4

Answer

63. Probability and odds are good tools for assessing risk. Answers should include the following.

- P(struck by lightning) $= \frac{s}{s+f} = \frac{1}{750,000}$, so Odds = 1:(750,000 − 1) or 1:749,999.

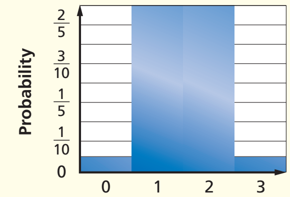
  P(surviving a lightning strike) $= \frac{s}{s+f} = \frac{3}{4}$, so Odds = 3:(4 − 3) or 3:1.
- In this case, success is being struck by lightning or surviving the lightning strike. Failure is not being struck by lightning or not surviving the lightning strike.

★54. **CARD GAMES** The game of euchre is played using only the 9s, 10s, jacks, queens, kings, and aces from a standard deck of cards. Find the probability of being dealt a 5-card euchre hand containing all four suits. $\frac{540}{1771}$

Three students are selected at random from a group of 3 sophomores and 3 juniors. The table and relative-frequency histogram show the distribution of the number of sophomores chosen. Find each probability.

Sophomores	0	1	2	3
Probability	$\frac{1}{20}$	$\frac{9}{20}$	$\frac{9}{20}$	$\frac{1}{20}$

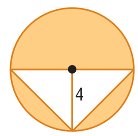
Number of Sophomores



55. P(0 sophomores) $\frac{1}{20}$
56. P(1 sophomore) $\frac{9}{20}$
57. P(2 sophomores) $\frac{9}{20}$
58. P(3 sophomores) $\frac{1}{20}$
59. P(2 juniors) $\frac{9}{20}$
60. P(1 junior) $\frac{9}{20}$

★61. **WRITING** Josh types the 5 entries in the bibliography of his term paper in random order, forgetting that they should be in alphabetical order by author. What is the probability that he actually typed them in alphabetical order? $\frac{1}{120}$

62. **CRITICAL THINKING** Find the probability that a point chosen at random in the figure is in the shaded region. Write your answer in terms of π. $\frac{\pi - 1}{\pi}$



63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

What do probability and odds tell you about life's risks?

Include the following in your answer:
- the odds of being struck by lightning and surviving the lightning strike, and
- a description of the meaning of *success* and *failure* in this case.

Standardized Test Practice

64. $\frac{6!}{2!} = ?$ **C**

(A) 3 (B) 60 (C) 360 (D) 720

65. A jar contains 4 red marbles, 3 green marbles, and 2 blue marbles. If a marble is drawn at random, what is the probability that it is not green? **D**

(A) $\frac{2}{9}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Extending the Lesson

Theoretical probability is determined using mathematical methods and assumptions about the fairness of coins, dice, and so on. **Experimental probability** is determined by performing experiments and observing the outcomes.

Determine whether each probability is *theoretical* or *experimental*. Then find the probability.

66. Two dice are rolled. What is the probability that the sum will be 12?

66. theoretical; $\frac{1}{36}$

67. A baseball player has 126 hits in 410 at-bats this season. What is the probability that he gets a hit in his next at-bat? **experimental; about 0.307**

68. A bird watcher observes that 5 out of 25 birds in a garden are red. What is the probability that the next bird to fly into the garden will be red? **experimental;** $\frac{1}{5}$

69. A hand of 2 cards is dealt from a standard deck of cards. What is the probability that both cards are clubs? **theoretical;** $\frac{1}{17}$

www.algebra2.com/self_check_quiz

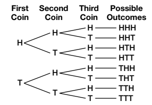
Study Guide and Intervention, p. 711 (shown) and p. 712

Probability and Odds In probability, a desired outcome is called a **success**; any other outcome is called a **failure**.

Probability of Success and Failure	If an event can succeed in s ways and fail in f ways, then the probabilities of success, $P(S)$, and of failure, $P(F)$, are as follows. $P(S) = \frac{s}{s+f}$ and $P(F) = \frac{f}{s+f}$.
Definition of Odds	If an event can succeed in s ways and fail in f ways, then the odds of success and of failure are as follows. Odds of success = $s:f$ Odds of failure = $f:s$

Example 1 **When 3 coins are tossed, what is the probability that at least 2 are heads?**

You can use a tree diagram to find the sample space.



Of the 8 possible outcomes, 4 have at least 2 heads. So the probability of tossing at least 2 heads is $\frac{4}{8}$ or $\frac{1}{2}$.

Example 2 **What is the probability of picking 4 fiction books and 2 biographies from a best-seller list that consists of 12 fiction books and 6 biographies?**

By the Fundamental Counting Principle, the number of successes is $C(12, 4) \cdot C(6, 2)$.
The total number of selections, $s + f$, of 6 books is $C(18, 6)$.

$P(\text{4 fiction, 2 biography}) = \frac{C(12, 4) \cdot C(6, 2)}{C(18, 6)}$ or about 0.40

The probability of selecting 4 fiction books and 2 biographies is about 40%.

Exercises

Find the odds of an event occurring, given the probability of the event.

1. $\frac{3}{7}$ 3:4
2. $\frac{4}{5}$ 4:1
3. $\frac{2}{13}$ 2:11
4. $\frac{1}{15}$ 1:14

Find the probability of an event occurring, given the odds of the event.

5. 10:1 $\frac{10}{11}$
6. 2:5 $\frac{2}{7}$
7. 4:9 $\frac{4}{13}$
8. 8:3 $\frac{8}{11}$

One bag of candy contains 15 red candies, 10 yellow candies, and 6 green candies. Find the probability of each selection.

9. picking a red candy $\frac{15}{31}$
10. not picking a yellow candy $\frac{21}{31}$
11. picking a green candy $\frac{6}{31}$
12. not picking a red candy $\frac{16}{31}$

Skills Practice, p. 713 and Practice, p. 714 (shown)

A bag contains 1 green, 4 red, and 5 yellow balls. Two balls are selected at random. Find the probability of each selection.

1. P(2 red) $\frac{2}{15}$
2. P(1 red and 1 yellow) $\frac{4}{9}$
3. P(1 green and 1 yellow) $\frac{1}{9}$
4. P(2 green) 0
5. P(2 red and 1 yellow) 0
6. P(1 red and 1 green) $\frac{4}{45}$

A bank contains 3 pennies, 8 nickels, 4 dimes, and 10 quarters. Two coins are selected at random. Find the probability of each selection.

7. P(2 pennies) $\frac{1}{100}$
8. P(2 dimes) $\frac{1}{50}$
9. P(1 nickel and 1 dime) $\frac{8}{75}$
10. P(1 quarter and 1 penny) $\frac{1}{10}$
11. P(1 quarter and 1 nickel) $\frac{4}{15}$
12. P(2 dimes and 1 quarter) 0

Henrico visits a home decorating store to choose wallpapers for his new house. The store has 28 books of wallpaper samples, including 10 books of WallPride samples and 18 books of Deluxe Wall Coverings samples. The store will allow Henrico to bring 4 books home for a few days so he can decide which wallpapers he wants to buy. If Henrico randomly chooses 4 books to bring home, find the probability of each selection.

13. P(4 WallPride) $\frac{2}{195}$
14. P(2 WallPride and 2 Deluxe) $\frac{153}{455}$
15. P(1 WallPride and 3 Deluxe) $\frac{544}{1365}$
16. P(3 WallPride and 1 Deluxe) $\frac{48}{455}$

For Exercises 17–20, use the table that shows the range of verbal SAT scores for freshmen at a small liberal arts college. If a freshman student is chosen at random, find each probability. Express as decimals rounded to the nearest thousandth.

Range	400–449	450–499	500–549	550–559	600–649	650+
Number of Students	129	275	438	602	620	412

17. P(400–449) 0.052
18. P(550–559) 0.243
19. P(at least 650) 0.166

Find the odds of an event occurring, given the probability of the event.

20. $\frac{4}{11}$ 4:7
21. $\frac{12}{13}$ 12:1
22. $\frac{5}{99}$ 5:94
23. $\frac{1}{1000}$ 1:999
24. $\frac{5}{16}$ 5:11
25. $\frac{3}{95}$ 3:92
26. $\frac{9}{70}$ 9:61
27. $\frac{8}{15}$ 8:7

Find the probability of an event occurring, given the odds of the event.

28. 2:23 $\frac{2}{25}$
29. 2:5 $\frac{2}{7}$
30. 15:1 $\frac{15}{16}$
31. 9:7 $\frac{9}{16}$
32. 11:14 $\frac{11}{25}$
33. 1000:1 $\frac{1000}{1001}$
34. 12:17 $\frac{12}{29}$
35. 8:13 $\frac{8}{21}$

Reading to Learn Mathematics, p. 715 ELL

Pre-Activity **What do probability and odds tell you about life's risks?**

Read the introduction to Lesson 12-3 at the top of page 644 in your textbook.

What is the probability that a person will *not* be struck by lightning in a given year? $\frac{749,999}{750,000}$

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.
   a. If an event can never occur, its probability is a negative number. false
   b. If an event is certain to happen, its probability is 1. true
   c. If an event can succeed in s ways and fail in f ways, then the probability of success is $\frac{s}{f}$. false
   d. If an event can succeed in s ways and fail in f ways, then the odds against the event are $s:f$. false
   e. A probability distribution is a function in which the domain is the sample space of an experiment. true
2. A weather forecast says that the chance of rain tomorrow is 40%.
   a. Write the probability that it will rain tomorrow as a fraction in lowest terms. $\frac{2}{5}$
   b. Write the probability that it will not rain tomorrow as a fraction in lowest terms. $\frac{3}{5}$
   c. What are the odds in favor of rain? 2:3
   d. What are the odds against rain? 3:2
3. Refer to the table in Example 4 on page 646 in your textbook.
   a. What other sum has the same probability as a sum of 11? 3
   b. What are the odds of rolling a sum of 8? 5:31
   c. What are the odds against rolling a sum of 9? 8:1

Helping You Remember

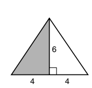
4. A good way to remember something is to explain it to someone else. Suppose that your friend Roberto is having trouble remembering the difference between probability and odds. What would you tell him to help him remember this easily?

   Sample answer: Probability gives the ratio of successes to the total number of outcomes, while odds gives the ratio of successes to failures.

Enrichment, p. 716

Geometric Probability

If a dart, thrown at random, hits the triangular board shown at the right, what is the chance that it will hit the shaded region? This chance, also called a probability, can be determined by comparing the area of the shaded region to the area of the board. This ratio indicates what fraction of the tosses should hit in the shaded region.



$$\frac{\text{area of shaded region}}{\text{area of triangular board}} = \frac{\frac{1}{2}(4)(6)}{\frac{1}{2}(8)(6)} = \frac{12}{24} \text{ or } \frac{1}{2}$$

In general, if S is a subregion of some region R, then the probability, $P(S)$, that a point, chosen at random, belongs to subregion S is given by the following.

$$P(S) = \frac{\text{area of subregion } S}{\text{are of region } R}$$

4 Assess

Open-Ended Assessment

Modeling Have students create a simple probability experiment using manipulatives and classroom objects. Have students first calculate the probability and then perform the experiment to verify their calculations.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 12-1 through 12-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 12-1 through 12-3) is available on p. 767 of the *Chapter 12 Resource Masters*.

Getting Ready for Lesson 12-4

BASIC SKILL Lesson 12-4 presents finding the probability of two events. Students will use their familiarity with multiplying fractions as they calculate probabilities. Exercises 79–83 should be used to determine your students' familiarity with multiplying rational expressions.

Maintain Your Skills

Mixed Review

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. *(Lesson 12-2)*

70. arranging 5 different books on a shelf **permutation; 120**

71. arranging the letters of the word *arrange* **permutation; 1260**

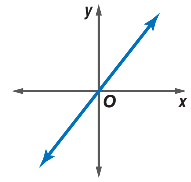
72. picking 3 apples from the last 7 remaining at the grocery store **combination; 35**

73. A mail-order computer company offers a choice of 4 amounts of memory, 2 sizes of hard drives, and 2 sizes of monitors. How many different systems are available to a customer? *(Lesson 12-1)* **16**

74. How many ways can 4 different gifts be placed into 4 different gift bags if each bag gets exactly 1 gift? *(Lesson 12-1)* **24**

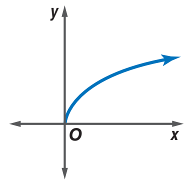
Identify the type of function represented by each graph. *(Lesson 9-5)*

75.



direct variation

76.



square root

Solve each matrix equation. *(Lesson 4-1)*

77. $[x \quad y] = [y \quad 4]$ **(4, 4)**

78. $\begin{bmatrix} 3y \\ 2x \end{bmatrix} = \begin{bmatrix} x + 8 \\ y - x \end{bmatrix}$ **(1, 3)**

Getting Ready for the Next Lesson

BASIC SKILL **Find each product if $a = \frac{3}{5}$, $b = \frac{2}{7}$, $c = \frac{3}{4}$, and $d = \frac{1}{3}$.**

79. ab $\frac{6}{35}$

80. bc $\frac{3}{14}$

81. cd $\frac{1}{4}$

82. bd $\frac{2}{21}$

83. ac $\frac{9}{20}$

Practice Quiz 1 — *Lessons 12-1 through 12-3*

1. At the Burger Bungalow, you can order your hamburger with or without cheese, with or without onions or pickles, and either rare, medium, or well-done. How many different ways can you order your hamburger? *(Lesson 12-1)* **24**

2. For a particular model of car, a dealer offers 3 sizes of engines, 2 types of stereos, 18 body colors, and 7 upholstery colors. How many different possibilities are available for that model? *(Lesson 12-1)* **756**

3. How many codes consisting of a letter followed by 3 digits can be made if no digit can be used more than once? *(Lesson 12-1)* **18,720**

Evaluate each expression. *(Lesson 12-2)*

4. $P(12, 3)$ **1320**

5. $C(8, 3)$ **56**

Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities. *(Lesson 12-2)*

6. 8 cars in a row parked next to a curb **permutation; 40,320**

7. a hand of 6 cards from a standard deck of cards **combination; 20,358,520**

Two cards are drawn from a standard deck of cards. Find each probability. *(Lesson 12-3)*

8. $P(\text{2 aces})$ $\frac{1}{221}$

9. $P(\text{1 heart, 1 club})$ $\frac{13}{102}$

10. $P(\text{1 queen, 1 king})$ $\frac{8}{663}$

650 Chapter 12 Probability and Statistics

12-4 Multiplying Probabilities

What You'll Learn

- Find the probability of two independent events.
- Find the probability of two dependent events.

Vocabulary

- area diagram

How does probability apply to basketball?

Reggie Miller of the Indiana Pacers is one of the best free-throw shooters in the National Basketball Association. The table shows the five highest season free-throw statistics of his career. For any year, you can determine the probability that Miller will make two free throws in a row based on the probability of his making one free throw.

Season	FT%
1990–91	91.8
1993–94	90.8
1998–99	91.5
1999–00	92.9
2000–01	92.8

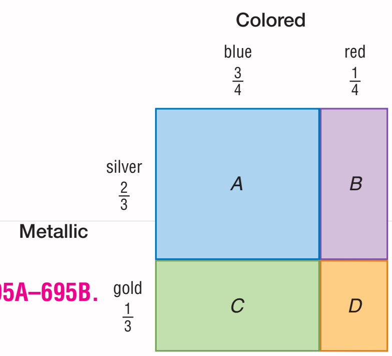
Source: *Sporting News*

PROBABILITY OF INDEPENDENT EVENTS In a situation with two events like shooting a free throw and then shooting another one, you can find the probability of *both* events occurring if you know the probability of each event occurring. You can use an **area diagram** to model the probability of the two events occurring at the same time.

Algebra Activity

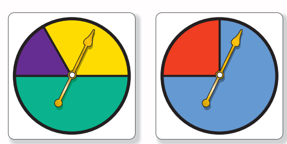
Area Diagrams

Suppose there are 1 red and 3 blue paper clips in one drawer and 1 gold and 2 silver paper clips in another drawer. The area diagram represents the probabilities of choosing one colored paper clip and one metallic paper clip if one of each is chosen at random. For example, rectangle A represents drawing 1 silver clip and 1 blue clip.



Model and Analyze **1, 4. See pp. 695A–695B.**

1. Find the areas of rectangles A, B, C, and D, and explain what each area represents. **2. $\frac{1}{6}$** **3. 1; 1; 1; The sum of the probabilities must be 1.**
2. What is the probability of choosing a red paper clip and a silver paper clip?
3. What are the length and width of the whole square? What is the area? Why does the area need to have this value?
4. Make an area diagram that represents the probability of each outcome if you spin each spinner once. Label the diagram and describe what the area of each rectangle represents.



12-4 Lesson Notes

1 Focus

5-Minute Check Transparency 12-4 Use as a quiz or review of Lesson 12-3.

Mathematical Background notes are available for this lesson on p. 630D.

How does probability apply to basketball?

Ask students:

- Ask a volunteer to explain what a free throw is, for the benefit of any students who might not be familiar with the game of basketball.
- Based on the information in this table, out of 10 free throws, how many would you expect Miller to make in the 2001-2002 season? **9**

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 717–718
- Skills Practice, p. 719
- Practice, p. 720
- Reading to Learn Mathematics, p. 721
- Enrichment, p. 722

School-to-Career Masters, p. 23

Teaching Algebra With Manipulatives Masters, pp. 291, 292–293

Transparencies

5-Minute Check Transparency 12-4

Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 23

Interactive Chalkboard

2 Teach

PROBABILITY OF INDEPENDENT EVENTS

In-Class Examples

Power Point®

1 Gerardo has 9 dimes and 7 pennies in his pocket. He randomly selects one coin, looks at it, and replaces it. He then randomly selects another coin. What is the probability that both of the coins he selects are dimes? $\frac{81}{256}$

2 When three dice are rolled, what is the probability that two dice show a 5 and the third die shows an even number? $\frac{1}{72}$

Teaching Tip To verify that students understand the notation in the Key Concept box, have them read aloud the expression $P(A \text{ and } B) = P(A) \cdot P(B)$ and ask them to explain it.

In Exercise 4 of the activity, spinning one spinner has no effect on the second spinner. These events are independent.

> **Key Concept** — *Probability of Two Independent Events*
>
> If two events, A and B, are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

This formula can be applied to any number of independent events.

Study Tip

Alternative Method
You could use the Fundamental Counting Principle to find the number of successes and the number of total outcomes.
both regular $= 8 \cdot 8$ or 64
total outcomes $= 13 \cdot 13$ or 169
So, $P(\text{both reg.}) = \frac{64}{169}$.

Example 1 Two Independent Events

At a picnic, Julio reaches into an ice-filled cooler containing 8 regular soft drinks and 5 diet soft drinks. He removes a can, then decides he is not really thirsty, and puts it back. What is the probability that Julio and the next person to reach into the cooler both randomly select a regular soft drink?

Explore These events are independent since Julio replaced the can that he removed. The outcome of the second person's selection is not affected by Julio's selection.

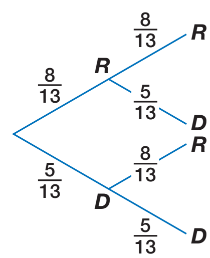
Plan Since there are 13 cans, the probability of each person's getting a regular soft drink is $\frac{8}{13}$.

Solve
$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular})$ — Probability of independent events
$= \frac{8}{13} \cdot \frac{8}{13}$ or $\frac{64}{169}$ — Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{64}{169}$ or about 0.38.

Examine You can verify this result by making a tree diagram that includes probabilities. Let R stand for regular and D stand for diet.

$P(R, R) = \frac{8}{13} \cdot \frac{8}{13}$



The formula for the probability of independent events can be extended to any number of independent events.

Example 2 Three Independent Events

In a board game, three dice are rolled to determine the number of moves for the players. What is the probability that the first die shows a 6, the second die shows a 6, and the third die does not?

Let A be the event that the first die shows a 6. → $P(A) = \frac{1}{6}$

Let B be the event that the second die shows a 6. → $P(B) = \frac{1}{6}$

Let C be the event that the third die does *not* show a 6. → $P(C) = \frac{5}{6}$

Algebra Activity

Materials (optional): paper clips in red, blue, gold, and silver; spinner with circle whose segments can be changed

Suggest that students use the least common denominator of the probabilities to choose the length of the side of the square for their area diagram. For example, when representing probabilities of $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{3}$, a square with sides of 6 centimeters works well.

652 Chapter 12 Probability and Statistics

$P(A, B, \text{and } C) = P(A) \cdot P(B) \cdot P(C)$ Probability of independent events

$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}$ or $\frac{5}{216}$ Substitute and multiply.

The probability that the first die shows a 6 and the second die does not is $\frac{5}{36}$.

PROBABILITY OF DEPENDENT EVENTS In Example 1, what is the probability that both people select a regular soft drink if Julio does not put his back in the cooler? In this case, the two events are dependent because the outcome of the first event affects the outcome of the second event.

Study Tip

Conditional Probability

The event of getting a regular soft drink the second time *given* that Julio got a regular soft drink the first time is called a *conditional probability*.

First selection	Second selection	
$P(\text{regular}) = \frac{8}{13}$	$P(\text{regular}) = \frac{7}{12}$	Notice that when Julio removes his can, there is not only one fewer regular soft drink but also one fewer drink in the cooler.

$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular following regular})$

$= \frac{8}{13} \cdot \frac{7}{12}$ or $\frac{14}{39}$ Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{14}{39}$ or about 0.36.

Key Concept *Probability of Two Dependent Events*

If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

This formula can be extended to any number of dependent events.

Example 3 Two Dependent Events

The host of a game show is drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, 6 show *television*, 3 show *vacation*, and 1 shows *car*. If the host draws the chips at random and does not replace them, find each probability.

Because the first chip is not replaced, the events are dependent. Let T represent a television, V a vacation, and C a car.

a. a vacation, then a car

$P(V, \text{then } C) = P(V) \cdot P(C \text{ following } V)$ Dependent events

$= \frac{3}{10} \cdot \frac{1}{9}$ or $\frac{1}{30}$ After the first chip is drawn, there are 9 left.

The probability of a vacation and then a car is $\frac{1}{30}$ or about 0.03.

b. two televisions

$P(T, \text{then } T) = P(T) \cdot P(T \text{ following } T)$ Dependent events

$= \frac{6}{10} \cdot \frac{5}{9}$ or $\frac{1}{3}$ If the first chip shows television, then 5 of the remaining 9 show television.

The probability of the host drawing two televisions is $\frac{1}{3}$.

www.algebra2.com/extra_examples

PROBABILITY OF DEPENDENT EVENTS

In-Class Example Power Point®

3 Refer to Example 3 in the Student Edition. The next week, the host of the game show draws from a bag of 20 chips, of which 11 say *computer*, 8 say *trip*, and 1 says *truck*. Drawing at random and without replacement, find each of the following probabilities.

a. a computer, then a truck **$\frac{11}{380}$ or about 0.03**

b. two trips **$\frac{14}{95}$ or about 0.15**

In-Class Example

Power Point®

4 Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart, another heart, and a spade in that order. $\frac{13}{850}$ **or about 0.015**

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Ask students to describe a situation in which Tabitha would be correct. **Sample answer: Once a number is rolled with the die, that number roll is considered invalid and the die must be rolled again until a valid number is rolled.**

Example 4 Three Dependent Events

Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a diamond, a club, and another diamond in that order.

Since the cards are not replaced, the events are dependent. Let D represent a diamond and C a club.

$$P(D, C, D) = P(D) \cdot P(C \text{ following } D) \cdot P(D \text{ following } D \text{ and } C)$$

$$= \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} \text{ or } \frac{13}{850}$$

If the first two cards are a diamond and a club, then 12 of the remaining cards are diamonds.

The probability is $\frac{13}{850}$ or about 0.015.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe two real-life events that are dependent.
2. **Write** a formula for $P(A, B, C, \text{and } D)$ if A, B, C, and D are independent.
3. **FIND THE ERROR** Mario and Tabitha are calculating the probability of getting a 4 and then a 2 if they roll a die twice.

Mario

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{6}$

$= \frac{1}{36}$

Tabitha

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{5}$

$= \frac{1}{30}$

Who is correct? Explain your reasoning. **Mario; the probabilities of rolling a 4 and rolling a 2 are both $\frac{1}{6}$.**

1. Sample answer: putting on your socks, and then your shoes

2. $P(A, B, C, \text{and } D) = P(A) \cdot P(B) \cdot P(C) \cdot P(D)$

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5, 9, 12	1, 2
6–8, 10, 13	3
11	4

A die is rolled twice. Find each probability.

4. $P(5, \text{then } 1)$ $\frac{1}{36}$
5. $P(\text{two even numbers})$ $\frac{1}{4}$

Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.

6. $P(\text{two hearts})$ $\frac{1}{17}$
7. $P(\text{ace, then king})$ $\frac{4}{663}$

There are 8 action, 3 romantic comedy, and 5 children's DVDs on a shelf. Suppose two DVDs are selected at random from the shelf. Find each probability.

8. $P(\text{2 action DVDs})$, if no replacement occurs $\frac{7}{30}$
9. $P(\text{2 action DVDs})$, if replacement occurs $\frac{1}{4}$
10. $P(\text{a romantic comedy DVD, then a children's DVD})$, if no replacement occurs $\frac{1}{16}$

Determine whether the events are *independent* or *dependent*. Then find the probability.

11. Yana has 7 blue pens, 3 black pens, and 2 red pens in his desk drawer. If he selects three pens at random with no replacement, what is the probability that he will first select a blue pen, then a black pen, and then another blue pen? **11. dependent; $\frac{21}{220}$**
12. A black die and a white die are rolled. What is the probability that a 3 shows on the black die and a 5 shows on the white die? **independent; $\frac{1}{36}$**

DAILY INTERVENTION

Differentiated Instruction

Naturalist Have students investigate how probability can be used to report the results of Mendel's famous experiments with seeds, and how it is used today by botanists who are developing desired characteristics in flowers and vegetables.

654 Chapter 12 Probability and Statistics

Application 13. **ELECTIONS** Tami, Sonia, Malik, and Roger are the four candidates for student council president. If their names are placed in random order on the ballot, what is the probability that Malik's name will be first on the ballot followed by Sonia's name second? $\frac{1}{12}$

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–19, 36–39, 44–46	1, 2
20–29	1, 3
30–35	1–4
40–43	3

Extra Practice

See page 855.

A die is rolled twice. Find each probability.

14. P(2, then 3) $\frac{1}{36}$
15. P(no 6s) $\frac{25}{36}$
16. P(two 4s) $\frac{1}{36}$
17. P(1, then any number) $\frac{1}{6}$
18. P(two of the same number) $\frac{1}{6}$
19. P(two different numbers) $\frac{5}{6}$

The tiles *A, B, G, I, M, R,* and *S* of a word game are placed face down in the lid of the game. If two tiles are chosen at random, find each probability.

20. P(R, then S), if no replacement occurs $\frac{1}{42}$
21. P(A, then M), if replacement occurs $\frac{1}{49}$
22. P(2 consonants), if replacement occurs $\frac{25}{49}$
23. P(2 consonants), if no replacement occurs $\frac{10}{21}$

★ 24. P(B, then D), if replacement occurs 0

★ 25. P(selecting the same letter twice), if no replacement occurs 0

Ashley takes her 3-year-old brother Alex into an antique shop. There are 4 statues, 3 picture frames, and 3 vases on a shelf. Alex accidentally knocks 2 items off the shelf and breaks them. Find each probability.

26. P(breaking 2 vases) $\frac{1}{15}$
27. P(breaking 2 statues) $\frac{2}{15}$
28. P(breaking a picture frame, then a vase) $\frac{1}{10}$
29. P(breaking a statue, then a picture frame) $\frac{2}{15}$

Determine whether the events are *independent* or *dependent*. Then find the probability.

30. There are 3 miniature chocolate bars and 5 peanut butter cups in a candy dish. Judie chooses 2 of them at random. What is the probability that she chooses 2 miniature chocolate bars? dependent; $\frac{3}{28}$
31. A bowl contains 4 peaches and 5 apricots. Maxine randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were apricots? independent; $\frac{25}{81}$
32. A bag contains 7 red, 4 blue, and 6 yellow marbles. If 3 marbles are selected in succession, what is the probability of selecting blue, then yellow, then red, if replacement occurs each time? independent; $\frac{168}{4913}$
33. Joe's wallet contains three \$1 bills, four \$5 bills, and two \$10 bills. If he selects three bills in succession, find the probability of selecting a \$10 bill, then a \$5 bill, and then a \$1 bill if the bills are not replaced. dependent; $\frac{1}{21}$
34. What is the probability of getting heads each time if a coin is tossed 5 times?

34. independent; $\frac{1}{32}$

★ 35. When Diego plays his favorite video game, the odds are 3 to 4 that he will reach the highest level of the game. What is the probability that he will reach the highest level each of the next four times he plays? dependent; $\frac{81}{2401}$

www.algebra2.com/self_check_quiz

About the Exercises...

Organization by Objective

- **Probability of Independent Events:** 14–19, 21, 22, 24, 28, 29, 31, 32, 34
- **Probability of Dependent Events:** 20, 23, 25–27, 30, 33, 35, 40–43

Odd/Even Assignments

Exercises 14–35 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

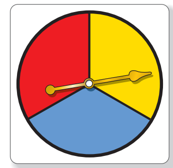
Basic: 15–23 odd, 27–33 odd, 45, 50–77

Average: 15–35 odd, 41–45 odd, 50–77

Advanced: 14–34 even, 36–39, 40–46 even, 47–71 (optional: 72–77)

Lesson 12-4 Multiplying Probabilities 655

For Exercises 36–39, suppose you spin the spinner twice.



36. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning red and then blue. $\frac{1}{9}$; See pp. 695A–695B for diagram.

37. Sketch an area diagram of the outcomes. Shade the region on your area diagram corresponding to getting the same color twice. See pp. 695A–695B.

38. What is the probability that you get the same color on both spins? $\frac{1}{3}$

39. If you spin the same color twice, what is the probability that the color is red? $\frac{1}{3}$

Find each probability if 13 cards are drawn from a standard deck of cards and no replacement occurs.

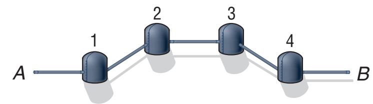
★ 40. P(all clubs) $\frac{1}{635{,}013{,}559{,}600}$

★ 41. P(all black cards) $\frac{19}{1{,}160{,}054}$

★ 42. P(all one suit) $\frac{1}{158{,}753{,}389{,}900}$

★ 43. P(no aces) $\frac{6327}{20{,}825}$

44. **UTILITIES** A city water system includes a sequence of 4 pumps as shown below. Water enters the system at point A, is pumped through the system by pumps at locations 1, 2, 3, and 4, and exits the system at point B.



If the probability of failure for any one pump is $\frac{1}{100}$, what is the probability that water will flow all the way through the system from A to B? $\left(\frac{99}{100}\right)^4$ or about 0.96

More About. . .

Spelling

The National Spelling Bee has been held every year since 1925, except for 1943-1945. Of the first 76 champions, 42 were girls and 34 were boys.

Source: www.spellingbee.com

45. **SPELLING** Suppose a contestant in a spelling bee has a 93% chance of spelling any given word correctly. What is the probability that he or she spells the first five words in a bee correctly and then misspells the sixth word? about 4.87%

★ 46. **LITERATURE** The following quote is from *The Mirror Crack'd*, which was written by Agatha Christie in 1962.

> "I think you're begging the question," said Haydock, "and I can see looming ahead one of those terrible exercises in probability where six men have white hats and six men have black hats and you have to work it out by mathematics how likely it is that the hats will get mixed up and in what proportion. If you start thinking about things like that, you would go round the bend. Let me assure you of that!"

If the twelve hats are all mixed up and each man randomly chooses a hat, what is the probability that the first three men get their own hats? Assume that no replacement occurs. $\frac{1}{1320}$

For Exercises 47–49, use the following information.

You have a bag containing 10 marbles. In this problem, a *cycle* means that you draw a marble, record its color, and put it back.

47. You go through the cycle 10 times. If you do not record any black marbles, can you conclude that there are no black marbles in the bag? no

48. Can you conclude that there are none if you repeat the cycle 50 times? no

49. How many times do you have to repeat the cycle to be certain that there are no black marbles in the bag? Explain your reasoning. See margin.

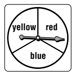
50. **CRITICAL THINKING** If one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance of the whole string lighting? 21

Study Guide and Intervention, p. 717 (shown) and p. 718

Probability of Independent Events

Probability of Two Independent Events	If two events, A and B, are independent, then the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

Example **In a board game each player has 3 different-colored markers. To move around the board the player first spins a spinner to determine which piece can be moved. He or she then rolls a die to determine how many spaces that colored piece should move. On a given turn what is the probability that a player will be able to move the yellow piece more than 2 spaces?**



Let A be the event that the spinner lands on yellow, and let B be the event that the die shows a number greater than 2. The probability of A is $\frac{1}{3}$, and the probability of B is $\frac{2}{3}$.

$P(A \text{ and } B) = P(A) \cdot P(B)$ — Probability of independent events

$= \frac{1}{3} \cdot \frac{2}{3}$ or $\frac{2}{9}$ — Substitute and multiply.

The probability that the player can move the yellow piece more than 2 spaces is $\frac{2}{9}$.

Exercises

A die is rolled 3 times. Find the probability of each event.

1. a 1 is rolled, then a 2, then a 3 $\frac{1}{216}$
2. a 1 or a 2 is rolled, then a 3, then a 5 or a 6 $\frac{1}{54}$
3. 2 odd numbers are rolled, then a 6 $\frac{1}{24}$
4. a number less than 3 is rolled, then a 3, then a number greater than 3 $\frac{1}{36}$
5. A box contains 5 triangles, 6 circles, and 4 squares. If a figure is removed, replaced, and a second figure is picked, what is the probability that a triangle and then a circle will be picked? $\frac{2}{15}$ or about 0.13
6. A bag contains 5 red marbles and 4 white marbles. A marble is selected from the bag, then replaced, and a second selection is made. What is the probability of selecting 2 red marbles? $\frac{25}{81}$ or about 0.31
7. A jar contains 7 lemon jawbreakers, 3 cherry jawbreakers, and 8 rainbow jawbreakers. What is the probability of selecting 2 lemon jawbreakers in succession providing the jawbreaker drawn first is then replaced before the second is drawn? $\frac{49}{324}$ or about 0.15

Skills Practice, p. 719 and Practice, p. 720 (shown)

A die is rolled three times. Find each probability.

1. P(three 4s) $\frac{1}{216}$
2. P(no 4s) $\frac{125}{216}$
3. P(2, then 3, then 1) $\frac{1}{216}$
4. P(three different even numbers) $\frac{1}{36}$
5. P(any number, then 5, then 5) $\frac{1}{36}$
6. P(even number, then odd number, then 1) $\frac{1}{24}$

There are 3 nickels, 2 dimes, and 5 quarters in a purse. Three coins are selected in succession at random. Find the probability.

7. P(nickel, then dime, then quarter), if no replacement occurs $\frac{1}{24}$
8. P(nickel, then dime, then quarter), if replacement occurs $\frac{3}{100}$
9. P(2 nickels, then 1 quarter), if no replacement occurs $\frac{1}{24}$
10. P(3 dimes), if replacement occurs $\frac{1}{125}$
11. P(3 dimes), if no replacement occurs 0

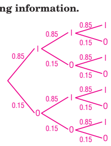
For Exercises 12 and 13, determine whether the events are *independent* or *dependent*. Then find each probability.

12. Serena is creating a painting. She wants to use 2 more colors. She chooses randomly from 6 shades of red, 10 shades of green, 4 shades of yellow, 4 shades of purple, and 6 shades of blue. What is the probability that she chooses 2 shades of green? dependent; $\frac{3}{29}$
13. Kershel's mother is shopping at a bakery. The owner offers Kershel a cookie from a jar containing 22 chocolate chip cookies, 18 sugar cookies, and 15 oatmeal cookies. Without looking, Kershel selects one, drops it back in, and then randomly selects another. What is the probability that neither selection was a chocolate chip cookie? independent; $\frac{9}{25}$
14. METEOROLOGY The Fadeeva's are planning a 3-day vacation to the mountains. A long-range forecast reports that the probability of rain each day is 10%. Assuming that the daily probabilities of rain are independent, what is the probability that there is no rain on the first two days, but that it rains on the third day? $\frac{81}{1000}$

RANDOM NUMBERS **For Exercises 15 and 16, use the following information.**

Anita has a list of 20 jobs around the house to do, and plans to do 3 of them today. She assigns each job a number from 1 to 20, and sets her calculator to generate random numbers from 1 to 20, which can reoccur. Of the jobs, 3 are outside, and the rest are inside.

15. Sketch a tree diagram showing all of the possibilities that the first three numbers generated correspond to inside jobs or outside jobs. Use it to find the probability that the first two numbers correspond to inside jobs, and the third to an outside job. 0.108375



16. What is the probability that the number generated corresponds to an outside job three times in a row? 0.003375

Reading to Learn Mathematics, p. 721 ELL

Pre-Activity **How does probability apply to basketball?**

Read the introduction to Lesson 12-4 at the top of page 651 in your textbook.

Write the probability that Reggie Miller made a free-throw shot during the 1998−99 season as a fraction in lowest terms. (Your answer should not include a decimal.) $\frac{183}{200}$

Reading the Lesson

1. A bag contains 4 yellow balls, 5 red balls, 1 white ball, and 2 black balls. A ball is drawn from the bag and is not replaced. A second ball is drawn.

   a. Let Y be the event "first ball is yellow" and B be the event "second ball is black." Are these events *independent* or *dependent*? dependent

   b. Tell which formula you would use to find the probability that the first ball is yellow and the second ball is black. C

      A. $P(Y \text{ and } B) = \frac{P(Y)}{P(Y) + P(B)}$

      B. $P(Y \text{ and } B) = P(Y) \cdot P(B)$

      C. $P(Y \text{ and } B) = P(Y) \cdot P(B \text{ following } Y)$

   c. Which equation shows the correct calculation of this probability? B

      A. $\frac{1}{3} + \frac{2}{11} = \frac{17}{33}$ B. $\frac{1}{3} \cdot \frac{2}{11} = \frac{2}{33}$

      C. $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ D. $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

   d. Which equation shows the correct calculation of the probability that if three balls are drawn in succession without replacement, all three will be red? B

      A. $\frac{5}{12} \cdot \frac{5}{12} \cdot \frac{5}{12} = \frac{125}{1728}$ B. $\frac{5}{12} \cdot \frac{4}{11} \cdot \frac{3}{10} = \frac{1}{22}$

      C. $\frac{5}{12} + \frac{4}{11} + \frac{3}{10} = \frac{713}{660}$

Helping You Remember

2. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both independent and dependent events? Explain your reasoning. Sample answer: Just remember the formula for dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$. When the events are independent, $P(B \text{ following } A) = P(B)$, so the formula for dependent events simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is the correct formula for independent events.

Enrichment, p. 722

Conditional Probability

Suppose a pair of dice is thrown. It is known that the sum is greater than seven. Find the probability that the dice match.

The probability of an event given the occurrence of another event is called *conditional probability*. The conditional probability of event A, the dice match, given event B, their sum is greater than seven, is denoted $P(A/B)$.

There are 15 sums greater than seven and there are 36 possible pairs altogether.

$P(B) = \frac{15}{36}$

There are three matching pairs greater than seven.

$P(A \text{ and } B) = \frac{3}{36}$

$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$

$P(A/B) = \frac{\frac{3}{36}}{\frac{15}{36}}$ or $\frac{1}{5}$

...bability is $\frac{1}{...}$

Answer

49. Sample answer: As the number of trials increases, the results become more reliable. However, you cannot be absolutely certain that there are no black marbles in the bag without looking at all of the marbles.

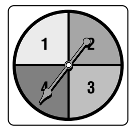
51. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. See pp. 695A–695B.

How does probability apply to basketball?

Include the following in your answer:
- an explanation of how a value such as one of those in the table at the beginning of the lesson could be used to find the chances of Reggie Miller making 0, 1, or 2 of 2 successive free throws, assuming the 2 free throws are independent, and
- a possible psychological reason why 2 free throws on the same trip to the foul line might not be independent.

Standardized Test Practice

52. The spinner is spun four times. What is the probability that the spinner lands on 2 each time? **D**



(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) $\frac{1}{8}$ (D) $\frac{1}{16}$

53. A coin is tossed and a die is rolled. What is the probability of a head and a 3? **C**

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) $\frac{1}{12}$ (D) $\frac{1}{24}$

Maintain Your Skills

Mixed Review

A gumball machine contains 7 red, 8 orange, 9 purple, 7 white, and 5 yellow gumballs. Tyson buys 3 gumballs. Find each probability, assuming that the machine dispenses the gumballs at random. *(Lesson 12-3)*

54. P(3 red) $\frac{1}{204}$ 55. P(2 white, 1 purple) $\frac{3}{340}$

56. P(1 purple, 1 orange, 1 yellow) $\frac{1}{119}$

57. **PHOTOGRAPHY** A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a row if the bride and groom stand in the middle? *(Lesson 12-2)* **1440 ways**

Solve each equation. Check your solutions. *(Lesson 10-3)*

58. $\log_5 5 + \log_5 x = \log_5 30$ **6** 59. $\log_{16} c - 2\log_{16} 3 = \log_{16} 4$ **36**

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials. *(Lesson 7-4)*

60. $x^3 - x^2 - 10x + 6; x + 3$ 61. $x^3 - 7x^2 + 12x; x - 3$

60. $x^2 - 4x + 2$
61. $x, x - 4$

Graph each inequality. *(Lesson 6-7)* **62–64. See margin.**

62. $y \leq x^2 + x - 2$ 63. $y < x^2 - 4$ 64. $y > x^2 - 3x$

Simplify. *(Lesson 5-5)*

65. $\sqrt{(153)^2}$ **153** 66. $\sqrt[3]{-729}$ **−9** 67. $\sqrt[16]{b^{16}}$ $|b|$ 68. $\sqrt{25a^8b^6}$ $5a^4|b^3|$

Solve each system of equations. *(Lesson 3-2)*

69. $z = 4y - 2$, $z = -y + 3$ **(1, 2)** 70. $j - k = 4$, $2j + k = 35$ **(13, 9)** 71. $3x + 1 = -y - 1$, $2y = -4x$ **(−2, 4)**

Getting Ready for the Next Lesson

BASIC SKILL **Find each sum if $a = \frac{1}{2}$, $b = \frac{1}{6}$, $c = \frac{2}{3}$, and $d = \frac{3}{4}$.**

72. $a + b$ $\frac{2}{3}$ 73. $b + c$ $\frac{5}{6}$ 74. $a + d$ $\frac{5}{4}$

75. $b + d$ $\frac{11}{12}$ 76. $c + a$ $1\frac{1}{6}$ 77. $c + d$ $1\frac{5}{12}$

4 Assess

Open-Ended Assessment

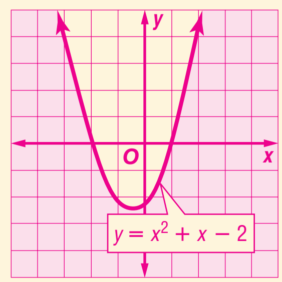
Speaking Ask students working in small groups to write an original problem (using objects or situations at school) that involves two selections. Have them create two versions, one with and one without replacement. Ask each group to present their problems to the class and to lead a discussion to compare the two solutions.

Getting Ready for Lesson 12-5

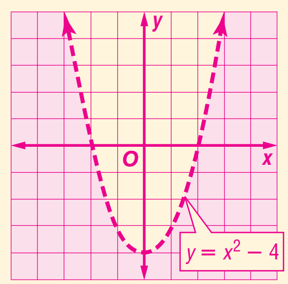
BASIC SKILL Lesson 12-5 presents finding the probability of mutually exclusive events. Students will use their familiarity with adding fractions as they calculate these probabilities. Exercises 72–77 should be used to determine your students' familiarity with adding fractions.

Answers

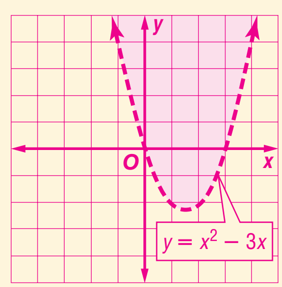
62.



63.



64.



12-5 Lesson Notes

1 Focus

5-Minute Check Transparency 12-5 Use as a quiz or review of Lesson 12-4.

Mathematical Background notes are available for this lesson on p. 630D.

How does probability apply to your personal habits?

Ask students:

- Which of these activities would have the greatest probability of being reported by a randomly selected person? **brushing teeth**
- Which of these activities would have the least probability of being reported by a randomly selected person? **Preparing clothes and taking medication have the same least probability.**

12-5 Adding Probabilities

What You'll Learn

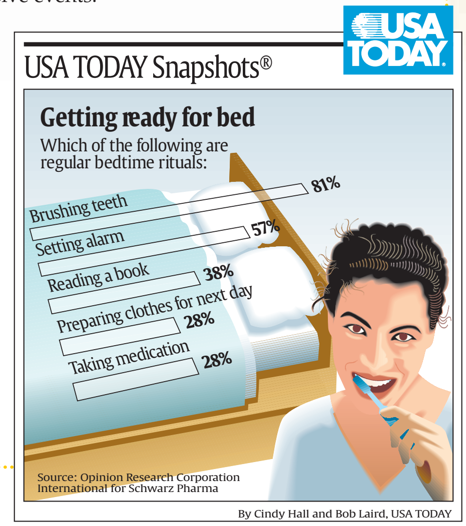
- Find the probability of mutually exclusive events.
- Find the probability of inclusive events.

Vocabulary

- simple event
- compound event
- mutually exclusive events
- inclusive events

How does probability apply to your personal habits?

The graph shows the results of a survey about bedtime rituals. Determining the probability that a randomly selected person reads a book or brushes his or her teeth before going to bed requires adding probabilities.



MUTUALLY EXCLUSIVE EVENTS When you roll a die, an event such as rolling a 1 is called a **simple event** because it consists of only one event. An event that consists of two or more simple events is called a **compound event**. For example, the event of rolling an odd number or a number greater than 5 is a compound event because it consists of the simple events rolling a 1, rolling a 3, rolling a 5, or rolling a 6.

When there are two events, it is important to understand how they are related before finding the probability of one or the other event occurring. Suppose you draw a card from a standard deck of cards. What is the probability of drawing a 2 or an ace? Since a card cannot be both a 2 *and* an ace, these are called **mutually exclusive events**. That is, the two events cannot occur at the same time. The probability of drawing a 2 or an ace is found by adding their individual probabilities.

$$P(2 \text{ or ace}) = P(2) + P(\text{ace}) \quad \text{Add probabilities.}$$
$$= \frac{4}{52} + \frac{4}{52} \quad \text{There are 4 twos and 4 aces in a deck.}$$
$$= \frac{8}{52} \text{ or } \frac{2}{13} \quad \text{Simplify.}$$

The probability of drawing a 2 or an ace is $\frac{2}{13}$.

Key Concept — Probability of Mutually Exclusive Events

- **Words** If two events, *A* and *B*, are mutually exclusive, then the probability that *A* or *B* occurs is the sum of their probabilities.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B)$

This formula can be extended to any number of mutually exclusive events.

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 723–724
- Skills Practice, p. 725
- Practice, p. 726
- Reading to Learn Mathematics, p. 727
- Enrichment, p. 728
- Assessment, pp. 767, 769

Graphing Calculator and Spreadsheet Masters, p. 49

Transparencies

5-Minute Check Transparency 12-5
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 1 *Two Mutually Exclusive Events*

Keisha has a stack of 8 baseball cards, 5 basketball cards, and 6 soccer cards. If she selects a card at random from the stack, what is the probability that it is a baseball or a soccer card?

These are mutually exclusive events, since the card cannot be both a baseball card *and* a soccer card. Note that there is a total of 19 cards.

$P(\text{baseball or soccer}) = P(\text{baseball}) + P(\text{soccer})$ Mutually exclusive events

$= \frac{8}{19} + \frac{6}{19} \text{ or } \frac{14}{19}$ Substitute and add.

The probability that Keisha selects a baseball or a soccer card is $\frac{14}{19}$.

Example 2 *Three Mutually Exclusive Events*

There are 7 girls and 6 boys on the junior class homecoming committee. A subcommittee of 4 people is being chosen at random to decide the theme for the class float. What is the probability that the subcommittee will have at least 2 girls?

At least 2 girls means that the subcommittee may have 2, 3, or 4 girls. It is not possible to select a group of 2 girls, a group of 3 girls, and a group of 4 girls all in the same 4-member subcommittee, so the events are mutually exclusive. Add the probabilities of each type of committee.

$P(\text{at least 2 girls}) = P(\text{2 girls}) + P(\text{3 girls}) + P(\text{4 girls})$

2 girls, 2 boys; 3 girls, 1 boy; 4 girls, 0 boys

$= \frac{C(7, 2) \cdot C(6, 2)}{C(13, 4)} + \frac{C(7, 3) \cdot C(6, 1)}{C(13, 4)} + \frac{C(7, 4) \cdot C(6, 0)}{C(13, 4)}$

$= \frac{315}{715} + \frac{210}{715} + \frac{35}{715} \text{ or } \frac{112}{143}$ Simplify.

The probability of at least 2 girls on the subcommittee is $\frac{112}{143}$ or about 0.78.

Study Tip

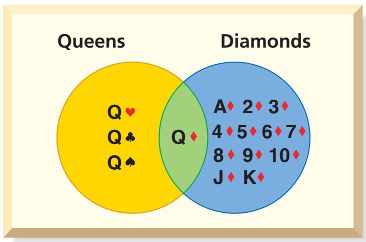
Choosing a Committee

$C(13, 4)$ refers to choosing 4 subcommittee members from 13 committee members. Since order does not matter, the number of combinations is found.

INCLUSIVE EVENTS What is the probability of drawing a queen or a diamond from a standard deck of cards? Since it is possible to draw a card that is both a queen and a diamond, these events are *not* mutually exclusive. These are called **inclusive events**.

P(queen)	**P(diamond)**	**P(diamond, queen)**
$\frac{4}{52}$	$\frac{13}{52}$	$\frac{1}{52}$
1 queen in each suit	diamonds	queen of diamonds

In the first two fractions above, the probability of drawing the queen of diamonds is counted twice, once for a queen and once for a diamond. To find the correct probability, you must subtract P(queen of diamonds) from the sum of the first two probabilities.



Study Tip

Common Misconception

In mathematics, unlike everyday language, the expression *A* or *B* allows the possibility of both *A* and *B* occurring.

www.algebra2.com/extra_examples

2 Teach

MUTUALLY EXCLUSIVE EVENTS

In-Class Examples Power Point®

1 Sylvia has a stack of playing cards consisting of 10 hearts, 8 spades, and 7 clubs. If she selects a card at random from this stack, what is the probability that it is a heart or a club? $\frac{17}{25}$

2 The Film Club makes a list of 9 comedies and 5 adventure movies they want to see. They plan to select 4 titles at random to show this semester. What is the probability that at least two of the films they select are comedies? $\frac{906}{1001}$ or about 0.91

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

INCLUSIVE EVENTS

In-Class Example

Power Point®

3 There are 2400 subscribers to an Internet service provider. Of these, 1200 own Brand A computers, 500 own Brand B, and 100 own both A and B. What is the probability that a subscriber selected at random owns either Brand A or Brand B? $\frac{2}{3}$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- add a representative problem for each of the probability situations in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Discuss whether the two events are inclusive or exclusive. Some students may feel that rain Saturday reduces the chance of rain on Sunday. Some students may think the two events are independent. Encourage interested students to research the science of weather forecasting.

$$P(\text{queen or diamond}) = P(\text{queen}) + P(\text{diamond}) - P(\text{queen of diamonds})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} \text{ or } \frac{4}{13}$$

The probability of drawing a queen or a diamond is $\frac{4}{13}$.

Key Concept — *Probability of Inclusive Events*

- **Words** If two events, A and B, are inclusive, then the probability that A or B occurs is the sum of their probabilities decreased by the probability of both occurring.
- **Symbols** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example 3 Inclusive Events

EDUCATION **The enrollment at Southburg High School is 1400. Suppose 550 students take French, 700 take algebra, and 400 take both French and algebra. What is the probability that a student selected at random takes French or algebra?**

Since some students take both French and algebra, the events are inclusive.

$P(\text{French}) = \frac{550}{1400}$ $\quad P(\text{algebra}) = \frac{700}{1400}$ $\quad P(\text{French and algebra}) = \frac{400}{1400}$

$$P(\text{French or algebra}) = P(\text{French}) + P(\text{algebra}) - P(\text{French and algebra})$$

$$= \frac{550}{1400} + \frac{700}{1400} - \frac{400}{1400} \text{ or } \frac{17}{28}$$ Substitute and simplify.

The probability that a student selected at random takes French or algebra is $\frac{17}{28}$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe two mutually exclusive events and two inclusive events.
2. **Draw** a Venn diagram to illustrate Example 3. **See margin.**
3. **FIND THE ERROR** Refer to the comic below.

1. Sample answer: mutually exclusive events: tossing a coin and rolling a die; inclusive events: drawing a 7 and a diamond from a standard deck of cards

The Born Loser®

Why is the weather forecaster's prediction incorrect? **The events are not mutually exclusive, so the chance of rain is less than 100%.**

Guided Practice **A die is rolled. Find each probability.**

4. $P(1 \text{ or } 6)$ $\frac{1}{3}$
5. $P(\text{at least } 5)$ $\frac{1}{3}$
6. $P(\text{less than } 3)$ $\frac{1}{3}$
7. $P(\text{prime})$ $\frac{1}{2}$
8. $P(\text{even or prime})$ $\frac{5}{6}$
9. $P(\text{multiple of 2 or 3})$ $\frac{2}{3}$

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students reflect on the definitions, skills, and formulas they have learned in these first five lessons on probability. Ask them to write an entry in their notes that describes their reaction to this topic in general, and to indicate which kinds of problems they find the most interesting, and which they find the most challenging.

GUIDED PRACTICE KEY	
Exercises	Examples
4–7, 12	1, 2
8–11	3

A card is drawn from a standard deck of cards. Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

10. *P*(6 or king) **mutually exclusive; $\frac{2}{13}$**

11. *P*(queen or spade) **inclusive; $\frac{4}{13}$**

Application

12. **SCHOOL** There are 8 girls and 8 boys on the student senate. Three of the students are seniors. What is the probability that a person selected from the student senate is not a senior? **$\frac{13}{16}$**

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
13–22, 33–42	1, 2
23–26	1–3
27–32, 43–46	3

Extra Practice

See page 855.

18. $\frac{105}{143}$ **21.** $\frac{38}{143}$

Lisa has 9 rings in her jewelry box. Five are gold and 4 are silver. If she randomly selects 3 rings to wear to a party, find each probability.

13. *P*(2 silver or 2 gold) **1**

14. *P*(all gold or all silver) **$\frac{1}{6}$**

15. *P*(at least 2 gold) **$\frac{25}{42}$**

16. *P*(at least 1 silver) **$\frac{37}{42}$**

Seven girls and six boys walk into a video store at the same time. There are five salespeople available to help them. Find the probability that the salespeople will first help the given numbers of girls and boys.

17. *P*(4 girls or 4 boys) **$\frac{35}{143}$**

18. *P*(3 girls or 3 boys)

★ **19.** *P*(all girls or all boys) **$\frac{3}{143}$**

20. *P*(at least 3 girls) **$\frac{84}{143}$**

21. *P*(at least 4 girls or at least 4 boys)

★ **22.** *P*(at least 2 boys) **$\frac{32}{39}$**

For Exercises 23–26, determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability. **24. inclusive; $\frac{1}{2}$**

23. There are 3 literature books, 4 algebra books, and 2 biology books on a shelf. If a book is randomly selected, what is the probability of selecting a literature book or an algebra book? **mutually exclusive; $\frac{7}{9}$**

24. A die is rolled. What is the probability of rolling a 5 or a number greater than 3?

25. In the Math Club, 7 of the 20 girls are seniors, and 4 of the 14 boys are seniors. What is the probability of randomly selecting a boy or a senior to represent the Math Club at a statewide math contest? **inclusive; $\frac{21}{34}$**

26. A card is drawn from a standard deck of cards. What is the probability of drawing an ace or a face card? (*Hint:* A face card is a jack, queen, or king.) **mutually exclusive; $\frac{4}{13}$**

27. One tile with each letter of the alphabet is placed in a bag, and one is drawn at random. What is the probability of selecting a vowel or a letter from the word *equation*? **$\frac{4}{13}$**

28. Each of the numbers from 1 to 30 is written on a card and placed in a bag. If one card is drawn at random, what is the probability that the number is a multiple of 2 or a multiple of 3? **$\frac{2}{3}$**

Two cards are drawn from a standard deck of cards. Find each probability.

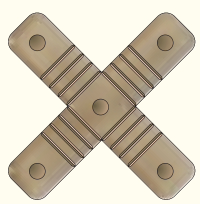
29. *P*(both kings or both black) **$\frac{55}{221}$**

30. *P*(both kings or both face cards) **$\frac{11}{221}$**

31. *P*(both face cards or both red) **$\frac{188}{663}$**

32. *P*(both either red or a king) **$\frac{63}{221}$**

More About. . .



World Cultures

Totolospi is a Hopi game of chance. The players use cane dice, which have both a flat side and a round side, and a counting board inscribed in stone.

WORLD CULTURES **For Exercises 33–36, refer to the information at the left.** When tossing 3 cane dice, if three round sides land up, the player advances 2 lines. If three flat sides land up, the player advances 1 line. If a combination is thrown, the player loses a turn. Find each probability.

33. *P*(advancing 2 lines) **$\frac{1}{8}$**

34. *P*(advancing 1 line) **$\frac{1}{8}$**

35. *P*(advancing at least 1 line) **$\frac{1}{4}$**

36. *P*(losing a turn) **$\frac{3}{4}$**

www.algebra2.com\self_check_quiz

About the Exercises...

Organization by Objective

- **Mutually Exclusive Events:** 13–23, 26, 33–42
- **Inclusive Events:** 24, 25, 27–32, 43–46

Odd/Even Assignments

Exercises 13–42 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

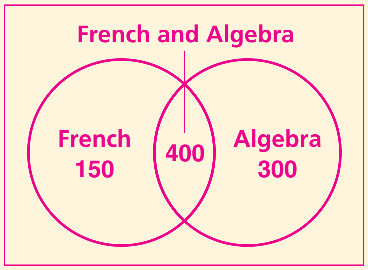
Basic: 13–17 odd, 21–39 odd, 47–75

Average: 13–43 odd, 47–75

Advanced: 14–42 even, 44–69 (optional: 70–75)

Answer

2.



Study Guide and Intervention, p. 723 (shown) and p. 724

Mutually Exclusive Events Events that cannot occur at the same time are called mutually exclusive events.

Probability of Mutually Exclusive Events	If two events, A and B, are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

This formula can be extended to any number of mutually exclusive events.

Example 1 **To choose an afternoon activity, summer campers pull slips of paper out of a hat. Today there are 25 slips for a nature walk, 35 slips for swimming, and 30 slips for arts and crafts. What is the probability that a camper will pull a slip for a nature walk or for swimming?**

These are mutually exclusive events. Note that there is a total of 90 slips.

$$P(\text{nature walk or swimming}) = P(\text{nature walk}) + P(\text{swimming}) = \frac{25}{90} + \frac{35}{90} \text{ or } \frac{2}{3}$$

The probability of a camper's pulling out a slip for a nature walk or for swimming is $\frac{2}{3}$.

Example 2 **By the time one tent of 6 campers gets to the front of the line, there are only 10 nature walk slips and 15 swimming slips left. What is the probability that more than 4 of the 6 campers will choose a swimming slip?**

$$P(\text{more than 4 swimmers}) = P(\text{5 swimmers}) + P(\text{6 swimmers}) = \frac{C(10, 1) \cdot C(15, 5)}{C(25, 6)} + \frac{C(10, 0) \cdot C(15, 6)}{C(25, 6)} \approx 0.2$$

The probability of more than 4 of the campers swimming is about 0.2.

Exercises

Find each probability.

1. A bag contains 45 dyed eggs: 15 yellow, 12 green, and 18 red. What is the probability of selecting a green or a red egg? $\frac{2}{3}$
2. The letters from the words LOVE and LIVE are placed on cards and put in a box. What is the probability of selecting an L or an O from the box? $\frac{3}{8}$
3. A pair of dice is rolled, and the two numbers are added. What is the probability that the sum is either a 5 or a 7? $\frac{5}{18}$ or about 0.28
4. A bowl has 10 whole wheat crackers, 16 sesame crackers, and 14 rye crisps. If a person picks a cracker at random, what is the probability of picking either a sesame cracker or a rye crisp? $\frac{3}{4}$
5. An art box contains 12 colored pencils and 20 pastels. If 5 drawing implements are chosen at random, what is the probability that at least 4 of them are pastels? about 0.37

Skills Practice, p. 725 and Practice, p. 726 (shown)

An urn contains 7 white marbles and 5 blue marbles. Four marbles are selected without replacement. Find each probability.

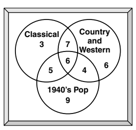
1. P(4 white or 4 blue) $\frac{8}{99}$
2. P(exactly 3 white) $\frac{35}{99}$
3. P(at least 3 white) $\frac{14}{33}$
4. P(fewer than 3 white) $\frac{19}{33}$
5. P(3 white or 3 blue) $\frac{49}{99}$
6. P(no white or no blue) $\frac{8}{99}$

Jason and Maria are playing a board game in which three dice are tossed to determine a player's move. Find each probability.

7. P(two 5s) $\frac{5}{72}$
8. P(three 5s) $\frac{1}{216}$
9. P(at least two 5s) $\frac{2}{27}$
10. P(no 5s) $\frac{125}{216}$
11. P(one 5) $\frac{25}{72}$
12. P(one 5 or two 5s) $\frac{5}{12}$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

13. A clerk chooses 4 CD players at random for floor displays from a shipment of 24 CD players. If 15 of the players have a blue case and the rest have a red case, what is the probability of choosing 4 players with a blue case or 4 players with a red case? mutual. exclus.; $\frac{71}{506}$
14. A department store employs 28 high school students, all juniors and seniors. Six of the 12 seniors are females and 12 of the juniors are males. One student employee is chosen at random. What is the probability of selecting a senior or a female? inclusive; $\frac{4}{7}$
15. A restaurant has 5 pieces of apple pie, 4 pieces of chocolate cream pie, and 3 pieces of blueberry pie. If Janine selects a piece of pie at random for dessert, what is the probability that she selects either apple or chocolate cream? mutually exclusive; $\frac{3}{4}$
16. At a statewide meeting, there are 20 school superintendents, 13 principals, and 6 assistant principals. If one of these people is chosen at random, what is the probability that he or she is either a principal or an assistant principal? mutually exclusive; $\frac{19}{39}$
17. An airline has one bank of 13 telephones at a reservations office. Of the 13 operators who work there, 8 take reservations for domestic flights and 5 take reservations for international flights. Seven of the operators taking domestic reservations and 3 of the operators taking international reservations are female. If an operator is chosen at random, what is the probability that the person chosen takes domestic reservations or is a male? inclusive; $\frac{10}{13}$
18. **MUSIC** Forty senior citizens were surveyed about their music preferences. The results are displayed in the Venn diagram. If a senior citizen from the survey group is selected at random, what is the probability that he or she likes only country and western music? What is the probability that he or she likes classical and/or country, but not 1940's pop? $\frac{3}{20}$; $\frac{2}{5}$



Reading to Learn Mathematics, p. 727 ELL

Pre-Activity **How does probability apply to your personal habits?**

Read the introduction to Lesson 12-5 at the top of page 658 in your textbook.

Why do the percentages shown on the bar graph add up to more than 100%? Sample answer: Many people do more than one of the listed bedtime rituals.

Reading the Lesson

1. Indicate whether the events in each pair are *inclusive* or *mutually exclusive*.
   a. Q: drawing a queen from a standard deck of cards
      D: drawing a diamond from a standard deck of cards inclusive
   b. J: drawing a jack from a standard deck of cards
      K: drawing a king from a standard deck of cards mutually exclusive
2. Marla took a quiz on this lesson that contained the following problem.

   Each of the integers from 1 through 25 is written on a slip of paper and placed in an envelope. If one slip is drawn at random, what is the probability that it is odd or a multiple of 5?

   Here is Marla's work.

   $$P(\text{odd}) = \frac{13}{25} \qquad P(\text{multiple of 5}) = \frac{5}{25} \text{ or } \frac{1}{5}$$
   $$P(\text{odd or multiple of 5}) = P(\text{odd}) + P(\text{multiple of 5}) = \frac{13}{25} + \frac{5}{25} = \frac{18}{25}$$

   a. Why is Marla's work incorrect? Sample answer: Marla used the formula for mutually exclusive events, but the events are inclusive. She should use the formula for inclusive events so that the odd multiples of 5 will not be counted twice.

   b. Show the corrected work.

   $$P(\text{odd or multiple of 5}) = P(\text{odd}) + P(\text{multiple of 5}) - P(\text{odd multiple of 5}) = \frac{13}{25} + \frac{5}{25} - \frac{3}{25} = \frac{15}{25} = \frac{3}{5}$$

Helping You Remember

3. Some students have trouble remembering a lot of formulas, so they try to keep the number of formulas they have to know to a minimum. Can you learn just one formula that will allow you to find probabilities for both mutually exclusive and inclusive events? Explain your reasoning. Sample answer: Just remember the formula for inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. When the events are mutually exclusive, $P(A \text{ and } B) = 0$, so the formula for inclusive events simplifies to $P(A \text{ and } B) = P(A) + P(B)$, which is the correct formula for mutually exclusive events.

More About. . .

Recycling

The United States recycles 28% of its waste.

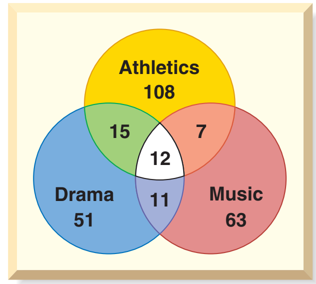
Source: The U.S. Environmental Protection Agency

For Exercises 37–42, use the following information.

Each of the numbers 1 through 30 is written on a table tennis ball and placed in a wire cage. Each of the numbers 20 through 45 is written on a table tennis ball and placed in a different wire cage. One ball is chosen at random from each spinning cage. Find each probability.

37. P(each is a 25) $\frac{1}{780}$
38. P(neither is a 20) $\frac{145}{156}$
39. P(exactly one is a 30) $\frac{9}{130}$
40. P(exactly one is a 40) $\frac{1}{26}$
41. ★ P(the numbers are equal) $\frac{11}{780}$
42. ★ P(the sum is 30) $\frac{1}{78}$
43. **RECYCLING** In one community, 300 people were surveyed to see if they would participate in a curbside recycling program. Of those surveyed, 134 said they would recycle aluminum cans, and 108 said they would recycle glass. Of those, 62 said they would recycle both. What is the probability that a randomly selected member of the community would recycle aluminum or glass? $\frac{121}{150}$

SCHOOL **For Exercises 44–46, use the Venn diagram that shows the number of participants in extracurricular activities for a junior class of 324 students.** Determine each probability if a student is selected at random from the class.



44. P(drama or music) $\frac{53}{108}$
45. P(drama or athletics) $\frac{17}{27}$
46. ★ P(athletics and drama, or music and athletics) $\frac{17}{162}$
47. **CRITICAL THINKING** Consider the following probability equation.

    $$P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B)$$

    A textbook gives this equation for events A and B that are mutually exclusive or inclusive. Is this correct? Explain. See margin.
48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 695A–695B.

    How does probability apply to your personal habits?

    Include the following in your answer:
    - an explanation of whether the events listed in the graphic are mutually exclusive or inclusive, and
    - an explanation of how to determine the probability that a randomly selected person reads a book or brushes his or her teeth before going to bed if in a survey of 2000 people, 600 said that they do both.

Standardized Test Practice

49. In a jar of red and white gumballs, the ratio of white gumballs to red gumballs is 5:4. If the jar contains a total of 180 gumballs, how many of them are red? C

    (A) 45 (B) 64 (C) 80 (D) 100

50. $\langle x \rangle = \frac{1}{2}x$ if x is composite. $\langle x \rangle = 2x$ if x is prime. What is the value of $\langle 7 \rangle + \langle 18 \rangle$? A

    (A) 23 (B) 46 (C) 50 (D) 64

Enrichment, p. 728

Probability and Tic-Tac-Toe

What would be the chances of winning at tic-tac-toe if it were turned into a game of pure chance? To find out, the nine cells of the tic-tac-toe board are numbered from 1 to 9 and nine chips (also numbered from 1 to 9) are put into a bag. Player A draws a chip at random and enters an X in the corresponding cell. Player B does the same and enters an O.

To solve the problem, assume that both players draw all their chips without looking and all X and O entries are made at the same time. There are four possible outcomes: a draw, A wins, B wins, and either A or B can win.

There are 16 arrangements that result in a draw. Reflections and rotations must be counted as shown below.

```
o x o        x o x        o o x
x o x  4     o o x  4     x x o  8
x o x        x x o        o x x
```

There are 36 arrangements in which either player may win because both players have winning triples.

Maintain Your Skills

Mixed Review A die is rolled three times. Find each probability. *(Lesson 12-4)*

51. P(1, then 2, then 3) $\frac{1}{216}$

52. P(no 4s) $\frac{125}{216}$

53. P(three 1s) $\frac{1}{216}$

54. P(three even numbers) $\frac{1}{8}$

Find the odds of an event occurring, given the probability of the event. *(Lesson 12-3)*

55. $\frac{4}{5}$ **4:1**

56. $\frac{1}{9}$ **1:8**

57. $\frac{2}{7}$ **2:5**

58. $\frac{5}{8}$ **5:3**

Find the sum of each series. *(Lessons 11-2 and 11-4)*

59. $2 + 4 + 8 + \cdots + 128$ **254**

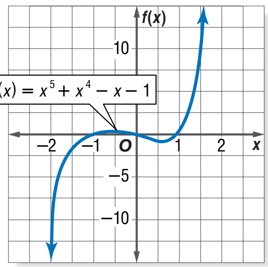
60. $\sum_{n=1}^{3}(5n - 2)$ **24**

Find the exact solution(s) of each system of equations. *(Lesson 8-7)*

61. $y = -10$
$y^2 = x^2 + 36$ **(±8, −10)**

62. $x^2 = 144$
$x^2 + y^2 = 169$ **(±12, ±5)**

63. Use the graph of the polynomial function at the right to determine at least one binomial factor of the polynomial. Then find all factors of the polynomial. *(Lesson 7-4)*
$(x + 1)^2(x - 1)(x^2 + 1)$



Find the maxima and minima of each function. Round to the nearest hundredth. *(Lesson 6-2)*

64. $f(x) = x^3 + 2x^2 - 5$

65. $f(x) = x^3 + 3x^2 + 2x + 1$

64. min: (0, −5); max: (−1.33, −3.81)
65. min: (−0.42, 0.62); max: (−1.58, 1.38)

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region. *(Lesson 3-4)* **66–67. See margin for graphs.**

66. $y \geq x - 2$
$x \geq 0$
$y \leq 2 - x$
$f(x, y) = 3x + y$
(0, 2), (2, 0), (0, −2); max: $f(2, 0) = 6$; min: $f(0, -2) = -2$

67. $y \geq 2x - 3$
$1 \leq x \leq 3$
$y \leq x + 2$
$f(x, y) = x + 4y$
(1, 3), (1, −1), (3, 3), (3, 5); max: $f(3, 5) = 23$; min: $f(1, -1) = -3$

SPEED SKATING **For Exercises 68 and 69, use the following information.**
In the 1988 Winter Olympics, Bonnie Blair set a world record for women's speed skating by skating approximately 12.79 meters per second in the 500-meter race. *(Lesson 2-6)*

68. Suppose she could maintain that speed. Write an equation that represents how far she could travel in t seconds. **$d = 12.79t$**

69. What type of equation is the one in Exercise 68? **direct variation**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the mean, median, mode, and range for each set of data. Round to the nearest hundredth, if necessary.** **70–75. See margin.**
*(To review **mean**, **median**, **mode**, and **range**, see pages 822 and 823.)*

70. 298, 256, 399, 388, 276

71. 3, 75, 58, 7, 34

72. 4.8, 5.7, 2.1, 2.1, 4.8, 2.1

73. 80, 50, 65, 55, 70, 65, 75, 50

74. 61, 89, 93, 102, 45, 89

75. 13.3, 15.4, 12.5, 10.7

4 Assess

Open-Ended Assessment

Modeling Ask students to use colored chips, index cards, and other objects to design and model two probability problems—one that involves mutually exclusive events and the other inclusive events. Have students explain their problems to a partner.

Assessment Options

Quiz (Lessons 12-4 and 12-5) is available on p. 767 of the *Chapter 12 Resource Masters*.

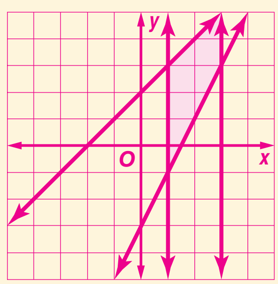
Mid-Chapter Test (Lessons 12-1 through 12-5) is available on p. 769 of the *Chapter 12 Resource Masters*.

Getting Ready for Lesson 12-6

PREREQUISITE SKILL Lesson 12-6 presents using measures of central tendency and variation for a set of data. Students will use their familiarity with mean, median, mode, and range as they calculate standard deviation. Exercises 70–75 should be used to determine your students' familiarity with finding mean, median, mode, and range for a set of values.

Answers

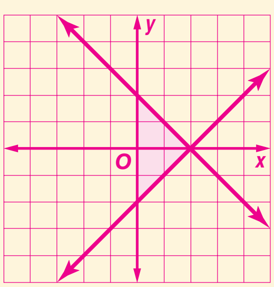
67.



70. 323.4, 298, no mode, 143
71. 35.4, 34, no mode, 72
72. 3.6, 3.45, 2.1, 3.6
73. 63.75, 65, 50 and 65, 30
74. 79.83, 89, 89, 57
75. 12.98, 12.9, no mode, 4.7

47. Subtracting $P(A \text{ and } B)$ from each side and adding $P(A \text{ or } B)$ to each side results in the equation $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. This is the equation for the probability of inclusive events. If A and B are mutually exclusive, then $P(A \text{ and } B) = 0$, so the equation simplifies to $P(A \text{ or } B) = P(A) + P(B)$, which is the equation for the probability of mutually exclusive events. Therefore, the equation is correct in either case.

66.



12-6 Lesson Notes

1 Focus

5-Minute Check Transparency 12-6 Use as a quiz or review of Lesson 12-5.

Mathematical Background notes are available for this lesson on p. 630D.

What statistics should a teacher tell the class after a test?

Ask students:

- Why is it helpful to put a list in order when studying data? **Sample answer: If the data are in order, it is much easier to find the lowest value, median, mode, and highest value.**
- What observations can you make about this data without doing any calculations, or using only mental math? **Sample answers may include: greatest and least values (94 to 19) and the range (75), as well as the fact that 19 and 34 seem to be outliers**

12-6 Statistical Measures

What You'll Learn

- Use measures of central tendency to represent a set of data.
- Find measures of variation for a set of data.

Vocabulary

- measure of central tendency
- measure of variation
- dispersion
- variance
- standard deviation

What statistics should a teacher tell the class after a test?

On Mr. Dent's most recent Algebra 2 test, his students earned the following scores.

72	70	77	76	90	68	81	86	34	94
71	84	89	67	19	85	75	66	80	94

When his students ask how they did on the test, which measure of central tendency should Mr. Dent use to describe the scores?

MEASURES OF CENTRAL TENDENCY Sometimes it is convenient to have one number that describes a set of data. This number is called a **measure of central tendency**, because it represents the center or middle of the data. The most commonly used measures of central tendency are the *mean*, *median*, and *mode*.

When deciding which measure of central tendency to use to represent a set of data, look closely at the data itself.

Study Tip

Look Back
To review **outliers**, see Lesson 2-5.

Concept Summary — Measures of Tendency

Use	When . . .
mean	the data are spread out, and you want an average of the values.
median	the data contain outliers.
mode	the data are tightly clustered around one or two values.

Example 1 Choose a Measure of Central Tendency

SWEEPSTAKES A sweepstakes offers a first prize of $10,000, two second prizes of $100, and one hundred third prizes of $10.

a. Which measure of central tendency best represents the available prizes?

Since 100 of the 103 prizes are $10, the mode ($10) best represents the available prizes. Notice that in this case the median is the same as the mode.

b. Which measure of central tendency would the organizers of the sweepstakes be most likely to use in their advertising?

The organizers would be most likely to use the mean (about $109) to make people think they had a better chance of winning more money.

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters
- Study Guide and Intervention, pp. 729–730
- Skills Practice, p. 731
- Practice, p. 732
- Reading to Learn Mathematics, p. 733
- Enrichment, p. 734

Science and Mathematics Lab Manual, pp. 115–118

Transparencies

5-Minute Check Transparency 12-6
Answer Key Transparencies

Technology

Interactive Chalkboard

MEASURES OF VARIATION

Measures of variation or **dispersion** measure how spread out or scattered a set of data is. The simplest measure of variation to calculate is the *range*, the difference between the greatest and the least values in a set of data. Variance and standard deviation are measures of variation that indicate how much the data values differ from the mean.

Study Tip

Reading Math

The symbol σ is the lower case Greek letter *sigma*. $\bar{x}$ is read *x bar*.

To find the **variance** σ^2 of a set of data, follow these steps.

1. Find the mean, $\bar{x}$.
2. Find the difference between each value in the set of data and the mean.
3. Square each difference.
4. Find the mean of the squares.

The **standard deviation** σ is the square root of the variance.

TEACHING TIP

In this text, assume that students are being asked to find the standard deviation of a population, for which the formula has been given.

Key Concept — *Standard Deviation*

If a set of data consists of the n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the standard deviation σ is given by the following formula.

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}}$$

Example 2 Standard Deviation

STATES **The table shows the populations in millions of 11 eastern states as of the 2000 Census. Find the variance and standard deviation of the data to the nearest tenth.**

State	Population	State	Population	State	Population
NY	19.0	MD	5.3	RI	1.0
PA	12.3	CT	3.4	DE	0.8
NJ	8.4	ME	1.3	VT	0.6
MA	6.3	NH	1.2	—	—

Source: U.S. Census Bureau

Step 1 Find the mean. Add the data and divide by the number of items.

$$\bar{x} = \frac{19.0 + 12.3 + 8.4 + 6.3 + 5.3 + 3.4 + 1.3 + 1.2 + 1.0 + 0.8 + 0.6}{11}$$

$\approx 5.4\overline{18}$ The mean is about 5.4 people.

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$ Variance formula

$$\approx \frac{(19.0 - 5.4)^2 + (12.3 - 5.4)^2 + \cdots + (0.8 - 5.4)^2 + (0.6 - 5.4)^2}{11}$$

$\approx \frac{344.4}{11}$ Simplify.

$\approx 31.3\overline{09}$ The variance is about 31.3 people.

Step 3 Find the standard deviation.

$\sigma^2 \approx 31.3$ Take the square root of each side.

$\sigma \approx 5.594640292$ The standard deviation is about 5.6 people.

www.algebra2.com/extra_examples

2 Teach

MEASURES OF CENTRAL TENDENCY

In-Class Example (PowerPoint®)

1 A new Internet company has 3 employees who are paid \$300,000, 10 who are paid \$100,000, and 60 who are paid \$50,000.

a. Which measure of central tendency best represents the pay at this company? **mode or median**

b. Which measure of central tendency would recruiters for this company be most likely to use to attract job applicants? **mean**

MEASURES OF VARIATION

In-Class Example (PowerPoint®)

2 **RIVERS** This table shows the length in thousands of miles of some of the longest rivers in the world. Find the standard deviation for these data. **1.05**

River	Length (thousands of miles)
Nile	4.16
Amazon	4.08
Missouri	2.35
Rio Grande	1.90
Danube	1.78

Teaching Tip Explain to students that the standard deviation is a number representing the typical or representative variation for the data items in that set. It tells how far a data value will typically be from the mean of the entire data set.

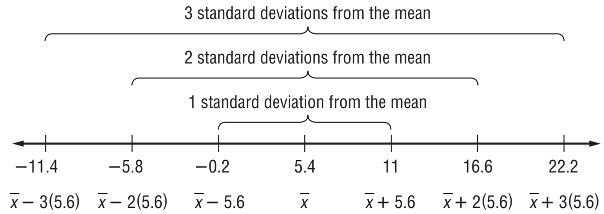
Teaching Tip Tell students that, for a normal distribution, 68.3% of the data is always within one standard deviation of the mean; 95.4% is always within two standard deviations, and 99.7% is always within three standard deviations, because of the way standard deviation is defined.

Answers

2. **Sample answer: The variance of the set {0, 1} is 0.25 and the standard deviation is 0.5.**

3. $\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Most of the members of a set of data are within 1 standard deviation of the mean. The populations of the states in Example 2 can be broken down as shown below.



Looking at the original data, you can see that most of the states' populations were between 2.4 million and 20.2 million. That is, the majority of members of the data set were within 1 standard deviation of the mean.

You can use a TI-83 Plus graphing calculator to find statistics for the data in Example 2.

Graphing Calculator Investigation

One-Variable Statistics

The TI-83 Plus can compute a set of one-variable statistics from a list of data. These statistics include the mean, variance, and standard deviation. Enter the data into L1.

KEYSTROKES: STAT ENTER 19.0 ENTER 12.3 ENTER ...

Then use STAT ▶ 1 ENTER to show the statistics. The mean $\bar{x}$ is about 5.4, the sum of the values $\sum x$ is 59.6, the standard deviation σx is about 5.6, and there are $n = 11$ data items. If you scroll down, you will see the least value (minX = .6), the three quartiles (1, 3.4, and 8.4), and the greatest value (maxX = 19).

Think and Discuss

1. Find the variance of the data set. **about 31.36**
2. Enter the data set in list L1 but without the outlier 19.0. What are the new mean, median, and standard deviation? **4.06, 2.35, about 3.8**
3. Did the mean or median change less when the outlier was deleted? **median**

Check for Understanding

Concept Check

1. **OPEN ENDED** Give a sample set of data with a variance and standard deviation of 0. **Sample answer: {10, 10, 10, 10, 10, 10}**
2. **Find a counterexample** for the following statement. **See margin.**
   The standard deviation of a set of data is always less than the variance.
3. **Write** the formula for standard deviation using sigma notation. (*Hint:* To review sigma notation, see Lesson 11-5.) **See margin.**

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	2
7, 8	1

Guided Practice

Find the variance and standard deviation of each set of data to the nearest tenth.

4. {48, 36, 40, 29, 45, 51, 38, 47, 39, 37} **40, 6.3**
5. {321, 322, 323, 324, 325, 326, 327, 328, 329, 330} **8.2, 2.9**
6. {43, 56, 78, 81, 47, 42, 34, 22, 78, 98, 38, 46, 54, 67, 58, 92, 55} **424.3, 20.6**

Graphing Calculator Investigation

Statistics On the TI-83 Plus calculator, press STAT and 1 (to select Edit) followed by ENTER to enter the values in L1. Then press STAT ▶ 1 and ENTER to display the statistics for L1. For Exercise 2, students can edit the list in L1 to delete 19.0. Then recalculate the statistics.

666 Chapter 12 Probability and Statistics

Application

EDUCATION **For Exercises 7 and 8, use the following information.**
The table below shows the amounts of money spent on education per student in 1998 in two regions of the United States.

Pacific States		Southwest Central States	
State	Expenditures per Student ($)	State	Expenditures per Student ($)
Alaska	10,650	Texas	6291
California	5345	Arkansas	5222
Washington	6488	Louisiana	5194
Oregon	6719	Oklahoma	4634

Source: National Education Association

7. Find the mean for each region. **$7300.50, $5335.25**

8. For which region is the mean more representative of the data? Explain.

8. The mean is more representative for the southwest central states because the data for the Pacific states contains the most extreme value, $10,650.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
17–26	1
9–16, 27–33	2

Extra Practice

See page 855.

Find the variance and standard deviation of each set of data to the nearest tenth.

9. {400, 300, 325, 275, 425, 375, 350} **2500, 50**

10. {5, 4, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 9} **1.6, 1.3**

11. {2.4, 5.6, 1.9, 7.1, 4.3, 2.7, 4.6, 1.8, 2.4} **3.1, 1.7**

12. {4.3, 6.4, 2.9, 3.1, 8.7, 2.8, 3.6, 1.9, 7.2} **4.8, 2.2**

13. {234, 345, 123, 368, 279, 876, 456, 235, 333, 444} **37,691.2, 194.1**

14. {13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 67, 56, 34, 99, 44, 55} **569.4, 23.9**

★ **15.**

Stem	Leaf
4	4 5 6 7 7
5	3 5 6 7 8 9
6	7 7 8 9 9 9 $4\|5 = 45$

82.9, 9.1

★ **16.**

Stem	Leaf
5	7 7 7 8 9
6	3 4 5 5 6 7
7	2 3 4 5 6 $6\|3 = 63$

43.6, 6.6

BASKETBALL **For Exercises 17 and 18, use the following information.**
The table below shows the rebounding totals for the 2000 Los Angeles Sparks.

306	179	205	194	105	55	122	32	23	16	23

Source: WNBA

17. Find the mean, median, and mode of the data to the nearest tenth. **114.5, 105, 23**

18. Which measure of central tendency best represents the data? Explain.

18. The mean and median both seem to represent the center of the data.

Online Research **Data Update** For the latest rebounding statistics for both women's and men's professional basketball, visit: www.algebra2.com/data_update

EDUCATION **For Exercises 19 and 20, use the following information. The Millersburg school board is negotiating a pay raise with the teacher's union. Three of the administrators have salaries of $80,000 each. However, a majority of the teachers have salaries of about $35,000 per year.**

19. You are a member of the school board and would like to show that the current salaries are reasonable. Would you quote the mean, median, or mode as the "average" salary to justify your claim? Explain. **Mean; it is highest.**

20. You are the head of the teacher's union and maintain that a pay raise is in order. Which of the mean, median, or mode would you quote to justify your claim? Explain your reasoning. **See margin.**

More About. . .

Basketball

Natalie Williams of the Utah Starzz led the Women's National Basketball Association in rebounding in 2000 with 336 rebounds in 29 games, an average of about 11.6 rebounds per game.
Source: WNBA

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Measures of Central Tendency:** 17–26
- **Measures of Variation:** 9–16, 27–33

Odd/Even Assignments
Exercises 9–16 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 9–13 odd, 17, 18, 21–23, 27–30, 34 –41, 45–64

Average: 9–15 odd, 19–23, 31–41, 45–64 (optional: 42–44)

Advanced: 10–16 even, 19, 20, 24–26, 31–58 (optional: 59–64)

All: Practice Quiz 2 (1–10)

Answer

20. Mode; it is lower and is what most employees make. It reflects the most representative worker.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Assign students to partners, one who is fairly new to the graphing calculator and the other who is confident in calculator skills. Have them work together to address difficulties using the calculator.

Tips for New Teachers

Intervention Scientific calculators, as well as graphing calculators, have special keys and functions that can be used to find mean, median, and standard deviation. Since the scientific calculator costs only a fraction of the graphing calculator, more students may have their own calculator of this type.

Answers

34. Different scales are used on the vertical axes.
35. Sample answer: The first graph might be used by a sales manager to show a salesperson that he or she does not deserve a big raise. It appears that sales are steady but not increasing fast enough to warrant a big raise.
36. Sample answer: The second graph might be shown by the company owner to a prospective buyer of the company. It looks like there is a dramatic rise in sales.
39. The statistic(s) that best represent a set of test scores depends on the distribution of the particular set of scores. Answers should include the following.
    - mean, 73.9; median, 76.5; mode, 94
    - The mode is not representative at all because it is the highest score. The median is more representative than the mean because it is influenced less than the mean by the two very low scores of 34 and 19.
44. The mean deviations would be greater for the greater standard deviation and lower for the groups of data that have the smaller standard deviation.

22. Mode; it is the least expensive price.
23. Mean or median; they are nearly equal and are more representative of the prices than the mode.

ADVERTISING **For Exercises 21–23, use the following information.**
A camera store placed an ad in the newspaper showing five digital cameras for sale. The ad says, "Our digital cameras average $695." The prices of the digital cameras are $1200, $999, $1499, $895, $695, $1100, $1300, and $695.

21. Find the mean, median, and mode of the prices. **$1047.88, $1049.50, $695**

22. Which measure is the store using in its ad? Why did they choose this measure?

23. As a consumer, which measure would you want to see advertised? Explain.

More About. . .

Shopping
While the Mall of America does not have the most gross leasable area, it is the largest fully enclosed retail and entertainment complex in the United States.
Source: Mall of America

SHOPPING MALLS **For Exercises 24–26, use the following information.**
The table lists the areas of some large shopping malls in the United States.

	Mall	Gross Leasable Area (ft^2)
1	Del Amo Fashion Center, Torrance, CA	3,000,000
2	South Coast Plaza/Crystal Court, Costa Mesa, CA	2,918,236
3	Mall of America, Bloomington, MN	2,472,500
4	Lakewood Center Mall, Lakewood, CA	2,390,000
5	Roosevelt Field Mall, Garden City, NY	2,300,000
6	Gurnee Mills, Gurnee, IL	2,200,000
7	The Galleria, Houston, TX	2,100,000
8	Randall Park Mall, North Randall, OH	2,097,416
9	Oakbrook Shopping Center, Oak Brook, IL	2,006,688
10	Sawgrass Mills, Sunrise, FL	2,000,000
10	The Woodlands Mall, The Woodlands, TX	2,000,000
10	Woodfield, Schaumburg, IL	2,000,000

Source: Blackburn Marketing Service

24. 2,290,403; 2,150,000; 2,000,000

24. Find the mean, median, and mode of the gross leasable areas.

25. You are a realtor who is trying to lease mall space in different areas of the country to a large retailer. Which measure would you talk about if the customer felt that the malls were too large for his store? Explain. **Mode; it is lowest.**

26. Which measure would you talk about if the customer had a large inventory? Explain. **Mean; it is highest.**

FOOTBALL **For Exercises 27–30, use the weights in pounds of the starting offensive linemen of the football teams from three high schools.**

Jackson	Washington	King
170, 165, 140, 188, 195	144, 177, 215, 225, 197	166, 175, 196, 206, 219

27. Find the standard deviation of the weights for Jackson High. **19.3**

28. Find the standard deviation of the weights for Washington High. **28.9**

29. Find the standard deviation of the weights for King High **19.5**

30. Which team had the most variation in weights? How do you think this variation will impact their play? **Washington; see students' work.**

SCHOOL **For Exercises 31–33, use the frequency table at the right that shows the scores on a multiple-choice test.**

Score	Frequency
90	3
85	2
80	3
75	7
70	6
65	4

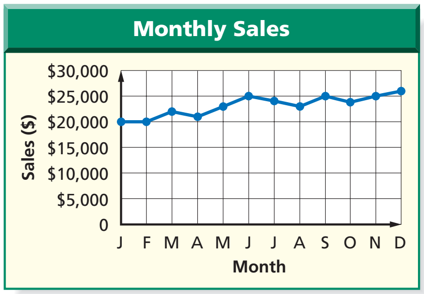
31. 59.8, 7.7

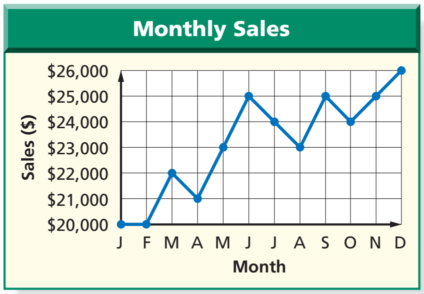
31. Find the variance and standard deviation of the scores.

★ **32.** What percent of the scores are within one standard deviation of the mean? **64%**

★ **33.** What percent of the scores are within two standard deviations of the mean? **100%**

For Exercises 34–36, consider the two graphs below. **34–36. See margin.**





34. Explain why the graphs made from the same data look different.

35. Describe a situation where the first graph might be used.

36. Describe a situation where the second graph might be used.

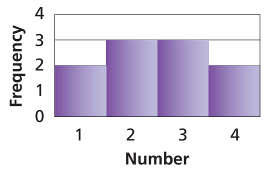
CRITICAL THINKING **For Exercises 37 and 38, consider the two sets of data.**

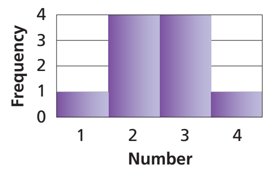
A = {1, 2, 2, 2, 2, 3, 3, 3, 3, 4}, B = {1, 1, 2, 2, 2, 3, 3, 3, 4, 4}

37. Find the mean, median, variance, and standard deviation of each set of data to the nearest tenth. **A: 2.5, 2.5, 0.7, 0.8; B: 2.5, 2.5, 1.1, 1.0**

38. The first histogram is lower in the middle and higher on the ends, so it represents data that are more spread out. Since set B has the greater standard deviation, set B corresponds to the first histogram and set A corresponds to the second.

38. Explain how you can tell which histogram below goes with each data set without counting the frequencies in the sets.





39. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. **See margin.**

What statistics should a teacher tell the class after a test?

Include the following in your answer:
- the mean, median, and mode of the given data set, and
- which measure of central tendency you think best represents the test scores and why.

Standardized Test Practice

40. What is the mean of the numbers represented by $x + 1$, $3x - 2$, and $2x - 5$? **A**

(A) $2x - 2$ (B) $\frac{6x - 7}{3}$ (C) $\frac{x + 1}{3}$ (D) $x + 4$

41. Manuel got scores of 92, 85, and 84 on three successive tests. What score must he get on a fourth test in order to have an average of 90? **D**

(A) 96 (B) 97 (C) 98 (D) 99

Extending the Lesson

Mean deviation is another method of dispersion. It is the mean of the deviations of the data from the mean of the data. If a set of data consists of n values $x_1, x_2, \ldots, x_n$ and has mean $\bar{x}$, then the mean deviation is given by the following formula.

$$MD = \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \cdots + |x_n - \bar{x}|}{n} \text{ or } \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

Study Tip

Reading Math
Mean deviation is also sometimes called *mean absolute deviation.*

Find the mean deviation of each set of data to the nearest tenth.

42. {95, 91, 88, 86} **3**

43. {10.4, 11.4, 16.2, 14.9, 13.5} **1.9**

44. Suppose two sets of data have the same mean and different standard deviations. Describe their mean deviations. **See margin.**

Study Guide and Intervention, p. 729 (shown) and p. 730

Measures of Central Tendency

	Use	When
Measures of Central Tendency	mean	the data are spread out and you want an average of values
	median	the data contain outliers
	mode	the data are tightly clustered around one or two values

Example **Find the mean, median, and mode of the following set of data: {42, 39, 35, 40, 38, 35, 45}.**

To find the mean, add the values and divide by the number of values.

$$\text{mean} = \frac{42 + 39 + 35 + 40 + 38 + 35 + 45}{7} \approx 39.14.$$

To find the median, arrange the values in ascending or descending order and choose the middle value. (If there is an even number of values, find the mean of the two middle values.) In this case, the median is 39.

To find the mode, take the most common value. In this case, the mode is 35.

Exercises

Find the mean, median, and mode of each set of data. Round to the nearest hundredth, if necessary.

1. {238, 261, 245, 249, 255, 262, 241, 245} **249.5; 247; 245**

2. {9, 13, 8, 10, 11, 9, 12, 16, 10, 9} **10.7; 10; 9**

3. {120, 108, 145, 129, 102, 132, 134, 118, 108, 142} **123.8; 124.5; 108**

4. {68, 54, 73, 58, 63, 72, 65, 70, 61} **64.89; 65; no mode**

5. {34, 49, 42, 38, 40, 45, 34, 28, 43, 30} **38.3; 39; 34**

6. The table at the right shows the populations of the six New England capitals. Which would be the most appropriate measure of central tendency to represent the data? Explain why and find that value.
Source: www.factfinder.census.gov **There is no mode. The population of Boston is an outlier and would raise the mean too high. The median, 79,500, would be the best choice.**

City	Population (rounded to the nearest 1000)
Augusta, ME	19,000
Boston, MA	589,000
Concord, NH	37,000
Hartford, CT	122,000
Montpelier, VT	8,000
Providence, RI	174,000

Skills Practice, p. 731 and Practice, p. 732 (shown)

Find the variance and standard deviation of each set of data to the nearest tenth.

1. {47, 61, 93, 22, 82, 22, 37} **673.1, 25.9**

2. {10, 10, 54, 39, 96, 91, 91, 18} **1228.6, 35.1**

3. {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5} **1.6, 1.2**

4. {1100, 725, 850, 335, 700, 800, 950} **49,150.0; 221.7**

5. {3.4, 7.1, 8.5, 5.1, 4.7, 6.3, 9.9, 8.4, 3.6} **4.7, 2.2**

6. {2.8, 0.5, 1.9, 0.8, 1.9, 1.5, 3.3, 2.6, 0.7, 2.5} **0.8, 0.9**

7. HEALTH CARE Eight physicians with 15 patients on a hospital floor see these patients an average of 18 minutes a day. The 22 nurses on the same floor see the patients an average of 3 hours a day. As a hospital administrator, would you quote the mean, median, or mode as an indicator of the amount of daily medical attention the patients on this floor receive? Explain. **Either the median or the mode; they are equal and higher than the mean, which is lowered by the smaller amount of time the physicians spend with the patients.**

For Exercises 8–10, use the frequency table that shows the percent of public school teachers in the U. S. in 1999 who used computers or the Internet at school for various administrative and teaching activities.

Activity	Percent Using Computer or Internet
Create instructional materials	39
Administrative record keeping	34
Communicate with colleagues	23
Gather information for planning lessons	16
Multimedia classroom presentations	8
Access research and best practices for teaching	8
Communicate with parents or students	8
Access model lesson plans	6

Source: National Assessment of Educational Progress

8. Find the mean, median, and mode of the data. **17.75%, 12%, 8%**

9. Suppose you believe teachers use computers or the Internet too infrequently. Which measure would you quote as the "average?" Explain. **Mode; it is lowest.**

10. Suppose you believe teachers use computers or the Internet too often. Which measure would you quote as the "average?" Explain. **Mean; it is highest.**

For Exercises 11 and 12, use the frequency table that shows the number of games played by 24 American League baseball players between opening day, 2001 and September 8, 2001.

No. of Games	Frequency
141	4
140	3
139	4
138	5
137	2
136	3
135	3

Source: Major League Baseball

11. Find the mean, median, mode, and standard deviation of the number of games played to the nearest tenth. **138.2, 138; 138, 2.0**

12. For how many players is the number of games within one standard deviation of the mean? **14**

Reading to Learn Mathematics, p. 733

ELL

Pre-Activity **What statistics should a teacher tell the class after a test?**

Read the introduction to Lesson 12-6 at the top of page 664 in your textbook.

There is more than one way to give an "average" score for this test. Three measures of central tendency for these scores are 94, 76.5 and 73.9. Can you tell which of these is the mean, the median, and the mode without doing any calculations? Explain your answer.

Sample answer: Yes. The mode must be one of the scores, so it must be an integer. The median must be either one of the scores or halfway between two of the scores, so it must be an integer or a decimal ending with .5. Therefore, 94 is the mode, 76.5 is the median, and 73.9 is the mean.

Reading the Lesson

1. Match each measure with one of the six descriptions of how to find measures of central tendency and variation.

a. median **vi** **b.** mode **i** **c.** range **iv**

d. variance **iii** **e.** mean **ii** **f.** standard deviation **v**

i. Find the most commonly occurring values or values in a set of data.

ii. Add the data and divide by the number of items.

iii. Find the mean of the squares of the differences between each value in the set of data and the mean.

iv. Find the difference between the largest and smallest values in the set of data.

v. Take the positive square root of the variance.

vi. If there is an odd number of items in a set of data, take the middle one. If there is an even number of items, add the two middle items and divide by 2.

Helping You Remember

2. It is usually easier to remember a complicated procedure if you break it down into steps. Write the procedure for finding the standard deviation for a set of data in a series of brief, numbered steps.

Sample answer:
1. Find the mean.
2. Find the difference between each value and the mean.
3. Square each difference.
4. Find the mean of the squares.
5. Take the positive square root.

Enrichment, p. 734

Probabilities in Genetics

Genes are the units which transmit hereditary traits. The possible forms which a gene may take, **dominant** and **recessive,** are called **alleles.** A particular trait is determined by two alleles, one from the female parent and one from the male parent. If an organism has the trait which is dominant, it may have either two dominant alleles or one dominant and one recessive allele. If the organism has the trait which is recessive, it must have two recessive alleles.

Example **Consider a plant in which tall stems, *T*, are dominant to short stems, *t*. What is the probability of obtaining a long-stemmed plant if two long-stemmed plants both with the genetic formula *Tt* are crossed?**

	T	*t*
T	*TT*	*Tt*

A *Punnett square* is a chart used to determine the possible combinations of characteristics among offspring.

4 Assess

Open-Ended Assessment

Writing Ask students to write a brief explanation of what standard deviation is and how it can be used. Have them include at least one example.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 12-4 through 12-6. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Getting Ready for Lesson 12-7

BASIC SKILL Lesson 12-7 presents data that are normally distributed. Students will use percents to calculate ranges of data within a normal distribution. Exercises 59–64 should be used to determine your students' familiarity with finding percents.

Maintain Your Skills

Mixed Review

Determine whether the events are mutually exclusive or inclusive. Then find the probability. *(Lesson 12-5)*

45. A card is drawn from a standard deck of cards. What is the probability that it is a 5 or a spade? **inclusive; $\frac{4}{13}$**

46. A jar of change contains 5 quarters, 8 dimes, 10 nickels, and 19 pennies. If a coin is pulled from the jar at random, what is the probability that it is a nickel or a dime? **mutually exclusive; $\frac{3}{7}$**

Two cards are drawn from a standard deck of cards. Find each probability. *(Lesson 12-4)*

47. P(ace, then king) if replacement occurs **$\frac{1}{169}$**

48. P(ace, then king) if no replacement occurs **$\frac{4}{663}$**

49. P(heart, then club) if no replacement occurs **$\frac{13}{204}$**

50. P(heart, then club) if replacement occurs **$\frac{1}{16}$**

51. Find the coordinates of the vertices and foci and the slopes of the asymptotes for the hyperbola given by $\frac{y^2}{81} - \frac{x^2}{25} = 1$. *(Lesson 8-5)* **$(0, \pm 9)$; $(0, \pm\sqrt{106})$; $\pm\frac{9}{5}$**

If $f(x) = x - 7$, $g(x) = 4x^2$, and $h(x) = 2x + 1$, find each value. *(Lesson 7-7)*

52. $f[g(-1)]$ **-3**
53. $h[f(15)]$ **17**
54. $f \circ h(2)$ **-2**

55. BUSINESS The Energy Booster Company keeps their stock of Health Aid liquid in a rectangular tank whose sides measure $x - 1$ centimeters, $x + 3$ centimeters, and $x - 2$ centimeters. Suppose they would like to bottle their Health Aid in $x - 3$ containers of the same size. How much liquid in cubic centimeters will remain unbottled? *(Lesson 7-2)* **12 cm^3**

Use Cramer's Rule to solve each system of equations. *(Lesson 4-6)*

56. $2x + 6y = 28$, $-x - 4y = -20$ **$(-4, 6)$**
57. $7c - 3d = -8$, $4c + d = 9$ **$(1, 5)$**
58. $m - 2n = -7$, $-3m + n = -4$ **$(3, 5)$**

Getting Ready for the Next Lesson

BASIC SKILL **Find each percent.**

59. 68% of 200 **136**
60. 68% of 500 **340**
61. 95% of 400 **380**
62. 95% of 500 **475**
63. 99% of 400 **396**
64. 99% of 500 **495**

Practice Quiz 2 — *Lessons 12-4 through 12-6*

A bag contains 5 red marbles, 3 green marbles, and 2 blue marbles. Two marbles are drawn at random from the bag. Find each probability. *(Lesson 12-4)*

1. P(red, then green) if replacement occurs **$\frac{3}{20}$**
2. P(red, then green) if no replacement occurs **$\frac{1}{6}$**
3. P(2 red) if no replacement occurs **$\frac{2}{9}$**
4. P(2 red) if replacement occurs **$\frac{1}{4}$**

A twelve-sided die has sides numbered 1 through 12. The die is rolled once. Find each probability. *(Lesson 12-5)*

5. P(4 or 5) **$\frac{1}{6}$**
6. P(even or a multiple of 3) **$\frac{2}{3}$**
7. P(odd or a multiple of 4) **$\frac{3}{4}$**

Find the variance and standard deviation of each set of data to the nearest tenth. *(Lesson 12-6)*

8. {5, 8, 2, 9, 4} **6.6, 2.6**
9. {16, 22, 18, 31, 25, 22} **23.6, 4.9**
10. {425, 400, 395, 415, 420} **134.0, 11.6**

670 Chapter 12 Probability and Statistics

12-7 The Normal Distribution

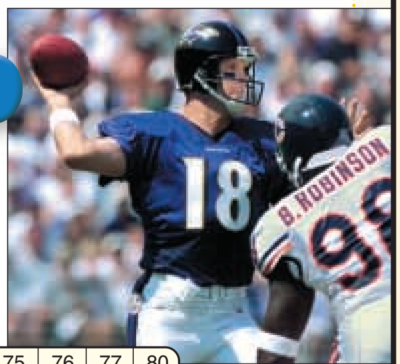
What You'll Learn

- Determine whether a set of data appears to be normally distributed or skewed.
- Solve problems involving normally distributed data.

Vocabulary

- discrete probability distribution
- continuous probability distribution
- normal distribution
- skewed distribution

How are the heights of professional athletes distributed?



The frequency table below lists the heights of the 2001 Baltimore Ravens. The table shows the heights of the players, but it does not show how these heights compare to the height of an average player. To make that comparison, you can determine how the heights are distributed.

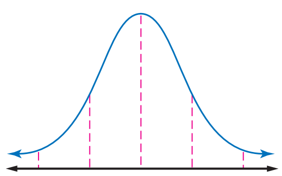
Height (in.)	67	69	70	71	72	73	74	75	76	77	80
Frequency	1	1	4	4	10	6	6	8	7	5	1

Source: www.ravenszone.net

NORMAL AND SKEWED DISTRIBUTIONS The probability distributions you have studied thus far are **discrete probability distributions** because they have only a finite number of possible values. A discrete probability distribution can be represented by a histogram. For a **continuous probability distribution**, the outcome can be any value in an interval of real numbers. Continuous probability distributions are represented by curves instead of histograms.

The curve at the right represents a continuous probability distribution. Notice that the curve is symmetric. Such a curve is often called a *bell curve*. Many distributions with symmetric curves or histograms are **normal distributions**.

Normal Distribution



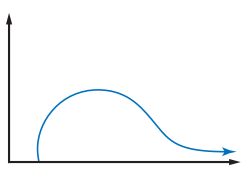
Study Tip

Skewed Distributions

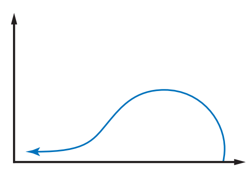
In a positively skewed distribution, the long tail is in the positive direction. These are sometimes said to be *skewed to the right*. In a negatively skewed distribution, the long tail is in the negative directon. These are sometimes said to be *skewed to the left*.

A curve or histogram that is not symmetric represents a **skewed distribution**. For example, the distribution for a curve that is high at the left and has a tail to the right is said to be *positively skewed*. Similarly, the distribution for a curve that is high at the right and has a tail to the left is said to be *negatively skewed*.

Positively Skewed



Negatively Skewed



12-7 Lesson Notes

1 Focus

5-Minute Check Transparency 12-7 Use as a quiz or review of Lesson 12-6.

Mathematical Background notes are available for this lesson on p. 630D.

How are the heights of professional athletes distributed?

Ask students:

- What is the greatest height listed in feet and inches? **6 ft 8 in.**
- How many players were exactly 6 feet tall? **10**

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 735–736
- Skills Practice, p. 737
- Practice, p. 738
- Reading to Learn Mathematics, p. 739
- Enrichment, p. 740
- Assessment, p. 768

Transparencies

5-Minute Check Transparency 12-7
Real-World Transparency 12
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

NORMAL AND SKEWED DISTRIBUTIONS

In-Class Example Power Point®

1 Determine whether the data {31, 37, 35, 36, 34, 36, 32, 36, 33, 32, 34, 34, 35, 34} appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. **normally distributed**

USE NORMAL DISTRIBUTIONS

In-Class Example Power Point®

2 Students counted the number of candies in 100 small packages. They found that the number of candies per package was normally distributed, with a mean of 23 candies per package and a standard deviation of 1 piece of candy.

a. About how many packages had between 24 and 22 candies? **about 68 packages**

b. What is the probability that a package selected at random had more than 25 candies? **about 2.5%**

Study Tip

Normal Distribution

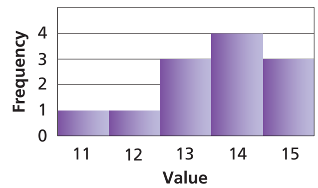
If you randomly select an item from data that are normally distributed, the probability that the one you pick will be within one standard deviation of the mean is 0.68. If you do this 1000 times, about 683 of those picked will be within one standard deviation of the mean.

Example 1 Classify a Data Distribution

Determine whether the data {14, 15, 12, 11, 13, 13, 14, 15, 14, 12, 13, 14, 15} appear to be *positively skewed, negatively skewed,* or *normally distributed.*

Make a frequency table for the data. Then use the table to make a histogram.

Value	11	12	13	14	15
Frequency	1	1	3	4	3

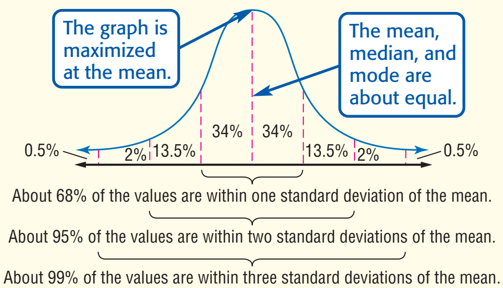


Since the histogram is high at the right and has a tail to the left, the data are negatively skewed.

USE NORMAL DISTRIBUTIONS Normal distributions occur quite frequently in real life. Standardized test scores, the lengths of newborn babies, the useful life and size of manufactured items, and production levels can all be represented by normal distributions. In all of these cases, the number of data values must be large for the distribution to be approximately normal.

Key Concept — Normal Distribution

Normal distributions have these properties.

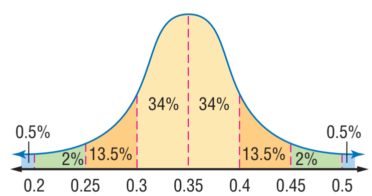


Example 2 Normal Distribution

PHYSIOLOGY The reaction times for a hand-eye coordination test administered to 1800 teenagers are normally distributed with a mean of 0.35 second and a standard deviation of 0.05 second.

a. About how many teens had reaction times between 0.25 and 0.45 second?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.



The values 0.25 and 0.45 are 2 standard deviations *below and above* the mean, respectively. Therefore, about 95% of the data are between 0.25 and 0.45.

$1800 \times 95\% = 1710$ Multiply 1800 by 0.95.

About 1710 of the teenagers had reaction times between 0.25 and 0.45 second.

DAILY INTERVENTION — Differentiated Instruction

Kinesthetic Ask students to measure carefully the distance around the wrists of 15 classmates to the nearest tenth of a centimeter and find the mean and standard deviation for their data. Then have them determine if this data is normally distributed, or positively or negatively skewed.

672 Chapter 12 Probability and Statistics

b. What is the probability that a teenager selected at random had a reaction time greater than 0.4 second?

The value 0.4 is one standard deviation above the mean. You know that about 100% − 68% or 32% of the data are more than one standard deviation away from the mean. By the symmetry of the normal curve, half of 32%, or 16%, of the data are more than one standard deviation above the mean.

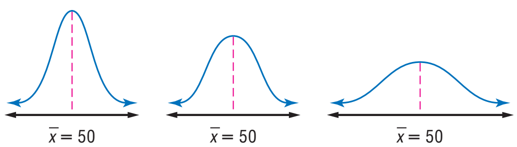
The probability that a teenager selected at random had a reaction time greater than 0.4 second is about 16% or 0.16.

Check for Understanding

Concept Check

1. **OPEN ENDED** Sketch a positively skewed graph. Describe a situation in which you would expect data to be distributed this way. **See margin.**
2. **Compare and contrast** the means and standard deviations of the graphs.

See margin.



3. **Explain** how to find what percent of a set of normally distributed data is more than 3 standard deviations above the mean. **See margin.**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4	1
5–11	2

4. The table at the right shows female mathematics SAT scores in 2000. Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.
normally distributed

Score	Percent of Females
200–299	3
300–399	14
400–499	33
500–599	31
600–699	15
700–800	4

Source: www.collegeboard.org

For Exercises 5–7, use the following information.
Mrs. Sung gave a test in her trigonometry class. The scores were normally distributed with a mean of 85 and a standard deviation of 3.

5. What percent would you expect to score between 82 and 88? **68%**
6. What percent would you expect to score between 88 and 91? **13.5%**
7. What is the probability that a student chosen at random scored between 79 and 91? **95%**

Application

QUALITY CONTROL **For Exercises 8–11, use the following information.**
The useful life of a radial tire is normally distributed with a mean of 30,000 miles and a standard deviation of 5000 miles. The company makes 10,000 tires a month.

8. About how many tires will last between 25,000 and 35,000 miles? **6800**
9. About how many tires will last more than 40,000 miles? **250**
10. About how many tires will last less than 25,000 miles? **1600**
11. What is the probability that if you buy a radial tire at random, it will last between 20,000 and 35,000 miles? **81.5%**

www.algebra2.com/extra_examples

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- draw graphs of normally distributed, positively skewed, and negatively skewed sets of data.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective
- **Normal and Skewed Distributions:** 12–14
- **Use Normal Distributions:** 15–26

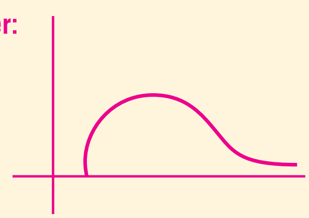
Odd/Even Assignments
Exercises 12, 13, and 15–26 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13, 15–21, 27–44
Average: 13, 22–44
Advanced: 12, 14, 22–41 (optional: 42–44)

Answers

1. Sample answer:



the use of cassettes since CDs were introduced

2. The mean of the three graphs is the same, but the standard deviations are different. The first graph has the least standard deviation, the standard deviation of the middle graph is slightly greater, and the standard deviation of the last graph is greatest.

3. Since 99% of the data is within 3 standard deviations of the mean, 1% of the data is more than 3 standard deviations from the mean. By symmetry, half of this, or 0.5%, is more than 3 standard deviations above the mean.

Practice and Apply

Homework Help

For Exercises	See Examples
12–14	1
15–26	2

Extra Practice

See page 856.

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

12.

U.S. Population	
Age	**Percent**
0–19	28.7
20–39	29.3
40–59	25.5
60–79	13.3
80–99	3.2
100+	0.0

Source: U.S. Census Bureau

positively skewed

13.

Record Low Temperatures in the 50 States	
Temperature (°F)	**Number of States**
−80 to −65	4
−64 to −49	12
−48 to −33	19
−32 to −17	12
−16 to −1	2
0 to 15	1

Source: *The World Almanac*

13. normally distributed

14. **SCHOOL** The frequency table at the right shows the grade-point averages (GPAs) of the juniors at Stanhope High School. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain. **Negatively skewed; the histogram is high at the right and has a tail to the left.**

GPA	Frequency
0.0–0.4	4
0.5–0.9	4
1.0–1.4	2
1.5–1.9	32
2.0–2.4	96
2.5–2.9	91
3.0–3.4	110
3.5–4.0	75

FOOD **For Exercises 15–17, use the following information.**

The shelf life of a particular dairy product is normally distributed with a mean of 12 days and a standard deviation of 3.0 days.

15. About what percent of the products last between 9 and 15 days? **68%**
16. About what percent of the products last between 12 and 15 days? **34%**
17. About what percent of the products last less than 3 days? **0.5%**
18. About what percent of the products last more than 15 days? **16%**

VENDING **For Exercises 19–21, use the following information.**

The vending machine in the school cafeteria usually dispenses about 6 ounces of soft drink. Lately, it is not working properly, and the variability of how much of the soft drink it dispenses has been getting greater. The amounts are normally distributed with a standard deviation of 0.2 ounce.

19. What percent of the time will you get more than 6 ounces of soft drink? **50%**
20. What percent of the time will you get less than 6 ounces of soft drink? **50%**
21. What percent of the time will you get between 5.6 and 6.4 ounces of soft drink? **95%**

MANUFACTURING **For Exercises 22–24, use the following information.**

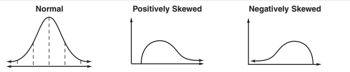
A company manufactures 1000 CDs per hour that are supposed to be 120 millimeters in diameter. These CDs are made for drives 122 millimeters wide. The sizes of CDs made by this company are normally distributed with a standard deviation of 1 millimeter. **22. 50%**

22. What percent of the CDs would you expect to be greater than 120 millimeters?
23. In one hour, how many CDs would you expect to be between 119 and 122 millimeters? **815**
24. About how many CDs per hour will be too large to fit in the drives? **25**

Study Guide and Intervention, p. 735 (shown) and p. 736

Normal and Skewed Distributions A **continuous probability** distribution is represented by a curve.

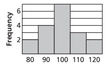
Types of Continuous Distributions



Example **Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.**
{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2

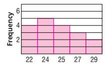


Then use the data to make a histogram.
Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

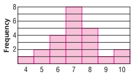
1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22} **positively skewed**



2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2

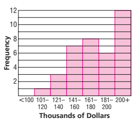
normally distributed



3.

Housing Price	No. of Houses Sold
less than $100,000	0
$100,00–$120,000	1
$121,00–$140,000	3
$141,00–$160,000	7
$161,00–$180,000	8
$181,00–$200,000	6
over $200,000	12

negatively skewed



Skills Practice, p. 737 and Practice, p. 738 (shown)

Determine whether the data in each table appear to be positively skewed, negatively skewed, or normally distributed.

1.

Time Spent at a Museum Exhibit	
Minutes	**Frequency**
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

normally distributed

2.

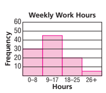
Average Age of High School Principals	
Age in Years	**Number**
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

negatively skewed

For Exercises 3 and 4, use the frequency table that shows the number of hours worked per week by 100 high school seniors.

Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

3. Make a histogram of the data.



4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain. **Positively skewed; the histogram is high at the left and has a tail to the right.**

TESTING **For Exercises 5–10, use the following information.**

The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

5. About what percent of the scores are between 70 and 130? **95%**
6. About what percent of the scores are between 85 and 130? **81.5%**
7. About what percent of the scores are over 115? **16%**
8. About what percent of the scores are lower than 85 or higher than 115? **32%**
9. If 80 people take the test, how many would you expect to score higher than 130? **2**
10. If 75 people take the test, how many would you expect to score lower than 85? **12**
11. **TEMPERATURE** The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2°. If you prefer to swim when the temperature is at least 77.8°, about what percent of the days does the temperature meet your preference? **84%**

Reading to Learn Mathematics, p. 739 ELL

Pre-Activity **How are the heights of professional athletes distributed?**

Read the introduction to Lesson 12-7 at the top of page 671 in your textbook.

There were 53 players on the team and the mean height was approximately 73.6. About what fraction of the players' heights are between 72 and 75, inclusive? **Sample answer: about $\frac{2}{3}$**

Reading the Lesson

1. Indicate whether each of the following statements is *true* or *false*.
   a. In a continuous probability distribution, there is a finite number of possible outcomes. **false**
   b. Every normal distribution can be represented by a bell curve. **true**
   c. A distribution that is represented by a curve that is high at the left and has a tail to the right is negatively skewed. **false**
   d. A normal distribution is an example of a skewed distribution. **false**
2. Ms. Rose gave the same quiz to her two geometry classes. She recorded the following scores.

First-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	0	1	0	3	4	5	7	4	3	2

Fifth-period class:

Score	0	1	2	3	4	5	6	7	8	9	10
Frequency	0	0	0	0	3	4	9	7	6	1	0

In each class, 30 students took the quiz. The mean score for each class was 6.4. Which set of scores has the greater standard deviation? (Answer this question without doing any calculations.) Explain your answer.

First period class; sample answer: The scores are more spread out from the mean than for the fifth period class.

Helping You Remember

3. Many students have trouble remembering how to determine if a curve represents a distribution that is *positively skewed* or *negatively skewed*. What is an easy way to remember this?

Sample answer: Follow the tail! If the tail is on the right (positive direction), the distribution is positively skewed. If the tail is on the left (negative direction), the distribution is negatively skewed.

Enrichment, p. 740

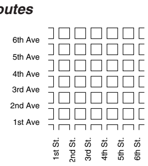
Street Networks: Finding All Possible Routes

A section of a city is laid out in square blocks. Going north from the intersection of First Avenue and First Street, the avenues are 1st, 2nd, 3rd, and so on. Going east, the streets are numbered in the same way.

Factorials can be used to find the number, $r(e, n)$, of different routes between two intersections. The formula is shown below.

$$r(e, n) = \frac{[(e - 1) + (n - 1)]!}{(e - 1)!(n - 1)!}$$

The number of streets going east is e; the number of avenues going north is n.



The following problems examine the possible routes from one location to another. Assume that you never use a route that is unnecessarily long. Assume that $e \geq 1$ and $n \geq 1$.

Answer

27. The mean would increase by 25; the standard deviation would not change; and the graph would be translated 25 units to the right.

More About. . .

Health

A systolic blood pressure below 130 is normal and between 130 and 139 is "high normal."

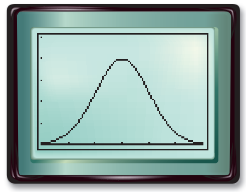
Source: National Institutes of Health

HEALTH **For Exercises 25 and 26, use the following information.**
A recent study showed that the systolic blood pressure of high school students ages 14–17 is normally distributed with a mean of 120 and a standard deviation of 12. Suppose a high school has 800 students.

25. About what percent of the students have blood pressures below 108? **16%**

26. About how many students have blood pressures between 108 and 144? **652**

27. CRITICAL THINKING The graphing calculator screen shows the graph of a normal distribution for a large set of test scores whose mean is 500 and whose standard deviation is 100. If every test score in the data set were increased by 25 points, describe how the mean, standard deviation, and graph of the data would change. **See margin.**



[200, 800] scl: 100 by [0, 0.005] scl: 0.001

28. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How are the heights of professional athletes distributed?

Include the following items in your answer:
- a histogram of the given data, and
- an explanation of whether you think the data are normally distributed.

Standardized Test Practice

29. If $x + y = 5$ and $xy = 6$, what is the value of $x^2 + y^2$? **A**

(A) 13 (B) 17 (C) 25 (D) 37

30. Which of the following is not the square of a rational number? **D**

(A) 0.04 (B) 0.16 (C) $\frac{4}{9}$ (D) $\frac{2}{3}$

Maintain Your Skills

Mixed Review

Find the variance and standard deviation of each set of data to the nearest tenth. *(Lesson 12-6)*

31. {7, 16, 9, 4, 12, 3, 9, 4} **17.5, 4.2**

32. {12, 14, 28, 19, 11, 7, 10} **42.5, 6.5**

A card is drawn from a standard deck of cards. Find each probability. *(Lesson 12-5)*

33. P(jack or queen) $\frac{2}{13}$

34. P(ace or heart) $\frac{4}{13}$

35. P(2 or face card) $\frac{4}{13}$

Find all of the rational zeros for each function. *(Lesson 7-6)* **39.** $\frac{1}{4}$, **1**

36. $f(x) = x^3 + 4x^2 - 5x$ **−5, 0, 1**

37. $p(x) = x^3 - 3x^2 - 10x + 24$ **−3, 2, 4**

38. $h(x) = x^4 - 2x^2 + 1$ **1, −1**

39. $f(x) = 4x^4 - 13x^3 - 13x^2 + 28x - 6$

METEOROLOGY **For excercises 40 and 41, use the following information.**
Weather forecasters can determine the approximate time that a thunderstorm will last if they know the diameter d of the storm in miles. The time t in hours can be found by using the formula $216t^2 = d^3$. *(Lesson 6-2)*

40. Graph $y = 216t^2 - 5^3$ and use it to estimate how long a thunderstorm will last if its diameter is 5 miles. **See margin for graph; about 45 min.**

41. Find how long a thunderstorm will last if its diameter is 5 miles and compare this time with your estimate in Exercise 40. **0.76 h**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the indicated term of each expression.** **42.** $21a^5b^2$
(For review of ***binomial expansions****, see Lesson 5-2.)* **43.** $56c^5d^3$ **44.** $126x^5y^4$

42. third term of $(a + b)^7$ **43.** fourth term of $(c + d)^8$ **44.** fifth term of $(x + y)^9$

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Have students in small groups discuss the meaning of standard deviation and normally distributed and skewed data. Ask them to write their own definitions for these terms in informal, but accurate, language.

Assessment Options

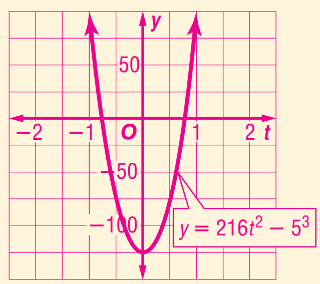
Quiz (Lessons 12-6 and 12-7) is available on p. 768 of the *Chapter 12 Resource Masters*.

Getting Ready for Lesson 12-8

PREREQUISITE SKILL Lesson 12-8 presents finding probabilities by using binomial expansions. Students will use their familiarity with finding terms of a binomial expansion as they determine probabilities. Exercises 42–44 should be used to determine your students' familiarity with finding a specified term of a binomial expansion.

Answer

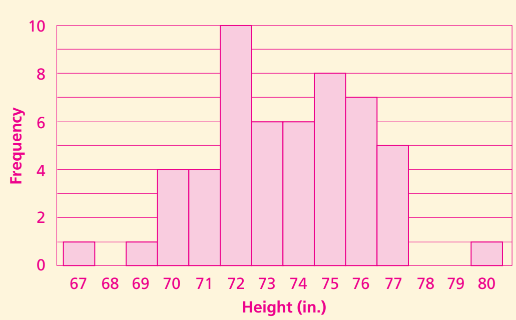
40.



Answer

28. If a large enough group of athletes is studied, some of the characteristics may be normally distributed; others may have skewed distributions. Answers should include the following.

- **See graph at the right.**
- **Since the histogram has two peaks, the data may not be normally distributed. This may be due to players who play certain positions tending to be of similar large sizes while players who play the other positions tend to be of similar smaller sizes.**



12-8 Binomial Experiments

What You'll Learn

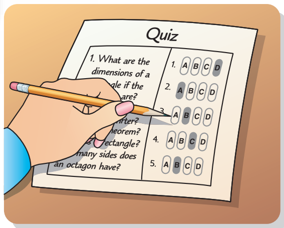
- Use binomial expansions to find probabilities.
- Find probabilities for binomial experiments.

Vocabulary

- binomial experiment

How can you determine whether guessing is worth it?

What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice quiz if you guess at every question?



Study Tip

Look Back
To review the **Binomial Theorem**, see Lesson 11-7.

BINOMIAL EXPANSIONS You can use the Binomial Theorem to find probabilities in certain situations where there are two possible outcomes. The 5 possible ways of getting 4 questions right r and 1 question wrong w are shown at the right. This chart shows the combination of 5 things (answer choices) taken 4 at a time (right answers) or $C(5, 4)$.

w	r	r	r	r
r	w	r	r	r
r	r	w	r	r
r	r	r	w	r
r	r	r	r	w

The terms of the binomial expansion of $(r + w)^5$ can be used to find the probabilities of each combination of right and wrong.

$$(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$$

Coefficient	Term	Meaning
$C(5, 5) = 1$	r^5	1 way to get all 5 questions right
$C(5, 4) = 5$	$5r^4w$	5 ways to get 4 questions right and 1 question wrong
$C(5, 3) = 10$	$10r^3w^2$	10 ways to get 3 questions right and 2 questions wrong
$C(5, 2) = 10$	$10r^2w^3$	10 ways to get 2 questions right and 3 questions wrong
$C(5, 1) = 5$	$5rw^4$	5 ways to get 1 question right and 4 questions wrong
$C(5, 0) = 1$	w^5	1 way to get all 5 questions wrong

The probability of getting a question right that you guessed on is $\frac{1}{4}$. So, the probability of getting the question wrong is $\frac{3}{4}$. To find the probability of getting 4 questions right and 1 question wrong, substitute $\frac{1}{4}$ for r and $\frac{3}{4}$ for w in the term $5r^4w$.

$$P(\text{4 right, 1 wrong}) = 5r^4w$$

$$= 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) \qquad r = \frac{1}{4},\ w = \frac{3}{4}$$

$$= \frac{15}{1024} \qquad \text{Multiply.}$$

The probability of getting exactly 4 questions correct is $\frac{15}{1024}$ or about 1.5%.

12-8 Lesson Notes

1 Focus

5-Minute Check Transparency 12-8 Use as a quiz or review of Lesson 12-7.

Mathematical Background notes are available for this lesson on p. 630D.

Building on Prior Knowledge

In Chapter 11, students learned to use the Binomial Theorem. In this lesson, students will use the Binomial Theorem to find probabilities.

How can you determine whether guessing is worth it?

Ask students:

- How many choices are there for each question? **4**
- If you guess at random, without being able to eliminate any of the choices, what is the probability of selecting the correct answer on one question? **1 out of 4 or 25%**

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 741–742
- Skills Practice, p. 743
- Practice, p. 744
- Reading to Learn Mathematics, p. 745
- Enrichment, p. 746

School-to-Career Masters, p. 24
Teaching Algebra With Manipulatives Masters, p. 294

Transparencies

5-Minute Check Transparency 12-8
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 1 Binomial Theorem

If a family has 4 children, what is the probability that they have 3 boys and 1 girl?

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is $\frac{1}{2}$, and the probability of a girl g is $\frac{1}{2}$.

$(b + g)^4 = b^4 + 4b^3g + 6b^2g^2 + 4bg^3 + g^4$

The term $4b^3g$ represents 3 boys and 1 girl.

$$P(3 \text{ boys}, 1 \text{ girl}) = 4b^3g$$
$$= 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) \quad b = \frac{1}{2}, g = \frac{1}{2}$$
$$= \frac{1}{4} \quad \text{Multiply.}$$

The probability of 3 boys and 1 girl is $\frac{1}{4}$ or 25%.

BINOMIAL EXPERIMENTS Problems like Example 1 that can be solved using binomial expansion are called **binomial experiments**.

Key Concept — Binomial Experiments

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

A binomial experiment is sometimes called a *Bernoulli experiment*.

Suppose that in the application at the beginning of the lesson, the first 3 questions are answered correctly. Then the last 2 are answered incorrectly. The probability of this occurring is $\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$ or $\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$. In general, there are $C(5, 3)$ ways to arrange 3 correct answers among the 5 questions, so the probability of exactly 3 correct answers is given by $C(5, 3)\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2$.

More About. . .

Sports

The National Hockey League record for most goals in a game by one player is seven. A player has scored five or more goals in a game 53 times in league history.

Source: NHL

Example 2 Binomial Experiment

SPORTS **Suppose that when hockey star Jaromir Jagr takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game one night.**

a. What is the probability that he will score exactly 2 goals?

The probability that he scores a goal on a given shot is $\frac{1}{7}$. The probability that he does not score on a given shot is $\frac{6}{7}$. There are $C(6, 2)$ ways to choose the 2 shots that score.

$$P(2 \text{ goals}) = C(6, 2)\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4 \quad \text{If he scores on 2 shots, he fails to score on 4 shots.}$$
$$= \frac{6 \cdot 5}{2}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4 \quad C(6, 2) = \frac{6!}{4!2!}$$
$$= \frac{19{,}440}{117{,}649} \quad \text{Simplify.}$$

The probability that Jagr will score exactly 2 goals is $\frac{19{,}440}{117{,}649}$ or about 0.17.

www.algebra2.com/extra_examples

2 Teach

BINOMIAL EXPANSIONS

In-Class Example Power Point®

Teaching Tip This example assumes that the chance for having a boy is 1 out of 2. Actually, from a biological standpoint, this is not quite accurate. In the U.S., about 1050 males are born for each 1000 females.

1 If a family has 4 children, what is the probability that they have 2 girls and 2 boys? **37.5%**

BINOMIAL EXPERIMENTS

In-Class Example Power Point®

2 A report said that approximately 1 out of 6 cars sold in a certain year was green. Suppose a salesperson sells 7 cars per week.

a. What is the probability that this salesperson will sell exactly 3 green cars in a week? **about 0.078**

b. What is the probability that this salesperson will sell at least 3 green cars in a week? **about 0.096**

DAILY INTERVENTION — Differentiated Instruction

Kinesthetic Have students in small groups do a binomial experiment by tossing a ball into the wastebasket about 20 times to establish the probability of scoring a goal. Then have them find the probability that they will score exactly 4 goals in 8 tries.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 12.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Binomial Expansions:** 12–37
- **Binomial Experiments:** 12–37

Odd/Even Assignments

Exercises 12–33 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 42–43 require a graphing calculator.

Assignment Guide

Basic: 13–31 odd, 35, 38–41, 44–56

Average: 13–35 odd, 36–41, 44–56 (optional: 42, 43)

Advanced: 12–34 even, 36–50 (optional: 51–56)

b. What is the probability that he will score at least 2 goals?

Instead of adding the probabilities of getting exactly 2, 3, 4, 5, and 6 goals, it is easier to subtract the probabilities of getting exactly 0 or 1 goal from 1.

$$P(\text{at least 2 goals}) = 1 - P(0\text{ goals}) - P(1\text{ goal})$$
$$= 1 - C(6, 0)\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^6 - C(6, 1)\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^5$$
$$= 1 - \frac{46{,}656}{117{,}649} - \frac{46{,}656}{117{,}649} \quad \text{Simplify.}$$
$$= \frac{24{,}337}{117{,}649} \quad \text{Subtract.}$$

The probability that Jagr will score at least 2 goals is $\frac{24{,}337}{117{,}649}$ or about 0.21.

Check for Understanding

Concept Check

1. **OPEN ENDED** Describe a situation for which the P(2 or more) can be found by using a binomial expansion. **1. Sample answer: In a 5-card hand, what is the probability that at least 2 cards are hearts?**
2. Refer to the application at the beginning of the lesson. List the possible sequences of 3 right answers and 2 wrong answers. **2. RRRWW, RRWRW, RRWWR, RWRRW, RWRWR, RWWRR, WRRRW, WRRWR, WRWRR, WWRRR**
3. **Explain** why each experiment is not binomial.
   a. rolling a die and recording whether a 1, 2, 3, 4, 5, or 6 comes up **3a. Each trial has more than two possible outcomes.**
   b. tossing a coin repeatedly until it comes up heads **The number of trials is not fixed.**
   c. removing marbles from a bag and recording whether each one is black or white, if no replacement occurs **The trials are not independent.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–11	1, 2

Find each probability if a coin is tossed 3 times.

4. $P(\text{exactly 2 heads})$ $\frac{3}{8}$
5. $P(0\text{ heads})$ $\frac{1}{8}$
6. $P(\text{at least 1 head})$ $\frac{7}{8}$

Four cards are drawn from a standard deck of cards. Each card is replaced before the next one is drawn. Find each probability.

7. $P(4\text{ jacks})$ $\frac{1}{28{,}561}$
8. $P(\text{exactly 3 jacks})$ $\frac{48}{28{,}561}$
9. $P(\text{at most 1 jack})$ $\frac{27{,}648}{28{,}561}$

Application

SPORTS **For Exercises 10 and 11, use the following information.**
Jessica Mendoza of Stanford University was the 2000 NCAA women's softball batting leader with an average of .475. This means that the probability of her getting a hit in a given at-bat was 0.475.

10. Find the probability of her getting 4 hits in 4 at-bats. **about 0.05**
11. Find the probability of her getting exactly 2 hits in 4 at-bats. **about 0.37**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
12–37	1, 2

Extra Practice

See page 856.

Find each probability if a coin is tossed 4 times.

12. $P(4\text{ tails})$ $\frac{1}{16}$
13. $P(0\text{ tails})$ $\frac{1}{16}$
14. $P(\text{exactly 2 tails})$ $\frac{3}{8}$
15. $P(\text{exactly 1 tail})$ $\frac{1}{4}$
16. $P(\text{at least 3 tails})$ $\frac{5}{16}$
17. $P(\text{at most 2 tails})$ $\frac{11}{16}$

Find each probability if a die is rolled 5 times.

18. $P(\text{exactly one 5})$ $\frac{3125}{7776}$
19. $P(\text{exactly three 5s})$ $\frac{125}{3888}$
20. $P(\text{at most two 5s})$ $\frac{625}{648}$
21. $P(\text{at least three 5s})$ $\frac{23}{648}$

As an apartment manager, Jackie Thomas is responsible for showing prospective renters different models of apartments. When showing a model, the probability that she selects the correct key from her set is $\frac{1}{4}$. If she shows 5 models in a day, find each probability.

22. P(never the correct key) $\frac{243}{1024}$
23. P(always the correct key) $\frac{1}{1024}$
24. P(correct exactly 4 times) $\frac{15}{1024}$
25. P(correct exactly 2 times) $\frac{135}{512}$
26. P(no more than 2 times correct) $\frac{459}{512}$
27. P(at least 3 times correct) $\frac{53}{512}$

Prisana guesses at all 10 true/false questions on her history test. Find each probability.

28. P(exactly 6 correct) $\frac{105}{512}$
29. P(exactly 4 correct) $\frac{105}{512}$
30. P(at most half correct) $\frac{319}{512}$
31. P(at least half correct) $\frac{319}{512}$

If a thumbtack is dropped, the probability of it landing point-up is 0.4. If 12 tacks are dropped, find each probability.

★ 32. P(at least 9 points up) **about 0.02**
★ 33. P(at most 4 points up) **about 0.44**

34. **CARS** According to a recent survey, about 1 in 3 new cars is leased rather than bought. What is the probability that 3 of 7 randomly-selected new cars are leased? $\frac{560}{2187}$

35. **INTERNET** In 2001, it was estimated that 32.5% of U.S. adults use the Internet. What is the probability that exactly 2 out of 5 randomly-selected U.S. adults use the Internet? **about 0.32**

WORLD CULTURES For Exercises 36 and 37, use the following information.
The Cayuga Indians played a game of chance called *Dish*, in which they used 6 flattened peach stones blackened on one side. They placed the peach stones in a wooden bowl and tossed them. The winner was the first person to get a prearranged number of points. The table below shows the points that were given for each toss. Assume that each face (black or neutral) of each stone has an equal chance of showing up.

36. Copy and complete the table by finding the probability of each outcome.

Outcome	Points	Probability
6 black	5	1/64
5 black, 1 neutral	1	3/32
4 black, 2 neutral	0	15/64
3 black, 3 neutral	0	5/16
2 black, 4 neutral	0	15/64
1 black, 5 neutral	1	3/32
6 neutral	5	1/64

37. Find the probability that a player gets at least 1 point for a toss. $\frac{1}{4}$

38. **CRITICAL THINKING** Write an expression for the probability of exactly m successes in n trials of a binomial experiment where the probability of success in a given trial is p. $C(n, m)p^m(1 - p)^{n-m}$

39. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 695A–695B.

How can you determine whether guessing is worth it?

Include the following in your answer:
- an explanation of how to find the probability of getting any number of questions right on a 5-question multiple-choice quiz, and
- the probability of each score.

www.algebra2.com/self_check_quiz

More About. . .

Internet

The word *Internet* was virtually unknown until the mid-1980s. By 1997, 19 million Americans were using the Internet. That number tripled in 1998 and passed 100 million in 1999.
Source: UCLA

Study Guide and Intervention, p. 741 (shown) and p. 742

Binomial Expansions For situations with only 2 possible outcomes, you can use the Binomial Theorem to find probabilities. The coefficients of terms in a binomial expansion can be found by using combinations.

Example **What is the probability that 3 coins show heads and 3 show tails when 6 coins are tossed?**
There are 2 possible outcomes that are equally likely: heads (H) and tails (T). The tosses of 6 coins are independent events. When $(H + T)^6$ is expanded, the term containing H^3T^3, which represents 3 heads and 3 tails, is used to get the desired probability. By the Binomial Theorem the coefficient of H^3T^3 is $C(6, 3)$.

$$P(\text{3 heads, 3 tails}) = \frac{6!}{3!3!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3 \quad P(H) = \frac{1}{2} \text{ and } P(T) = \frac{1}{2}$$
$$= \frac{20}{64}$$
$$= \frac{5}{16}$$

The probability of getting 3 heads and 3 tails is $\frac{5}{16}$ or 0.3125.

Exercises

Find each probability if a coin is tossed 8 times.

1. P(exactly 5 heads) about 22%
2. P(exactly 2 heads) about 11%
3. P(even number of heads) 50%
4. P(at least 6 heads) about 14%

Mike guesses on all 10 questions of a true-false test. If the answers true and false are evenly distributed, find each probability.

5. Mike gets exactly 8 correct answers. $\frac{45}{1024}$ or 0.044
6. Mike gets at most 3 correct answers. $\frac{11}{64}$ or 0.172
7. A die is tossed 4 times. What is the probability of tossing exactly two sixes? $\frac{25}{216}$ or 0.116

Skills Practice, p. 743 and Practice, p. 744 (shown)

Find each probability if a coin is tossed 6 times.

1. P(exactly 3 tails) $\frac{5}{16}$
2. P(exactly 5 tails) $\frac{3}{32}$
3. P(0 tails) $\frac{1}{64}$
4. P(at least 4 heads) $\frac{11}{32}$
5. P(at least 4 tails) $\frac{11}{32}$
6. P(at most 2 tails) $\frac{11}{32}$

The probability of Chris making a free throw is $\frac{2}{3}$. If she shoots 5 times, find each probability.

7. P(all missed) $\frac{1}{243}$
8. P(all made) $\frac{32}{243}$
9. P(exactly 2 made) $\frac{40}{243}$
10. P(exactly 1 missed) $\frac{80}{243}$
11. P(at least 3 made) $\frac{64}{81}$
12. P(at most 2 made) $\frac{17}{81}$

When Tarin and Sam play a certain board game, the probability that Tarin will win a game is $\frac{3}{4}$. If they play 5 games, find each probability.

13. P(Sam wins only once) $\frac{405}{1024}$
14. P(Tarin wins exactly twice) $\frac{45}{512}$
15. P(Sam wins exactly 3 games) $\frac{45}{512}$
16. P(Sam wins at least 1 game) $\frac{781}{1024}$
17. P(Tarin wins at least 3 games) $\frac{459}{512}$
18. P(Tarin wins at most 2 games) $\frac{53}{512}$

19. **SAFETY** In August 2001, the American Automobile Association reported that 73% of Americans use seat belts. In a random selection of 10 Americans in 2001, what is the probability that exactly half of them use seat belts? **Source:** AAA about 7.5%

HEALTH For Exercises 20 and 21, use the following information.
In 2001, the American Heart Association reported that 50 percent of the Americans who receive heart transplants are ages 50–64 and 20 percent are ages 35–49. **Source:** American Heart Association

20. In a randomly selected group of 10 heart transplant recipients, what is the probability that at least 8 of them are ages 50–64? $\frac{7}{128}$
21. In a randomly selected group of 5 heart transplant recipients, what is the probability that 2 of them are ages 35–49? $\frac{128}{625}$

Reading to Learn Mathematics, p. 745 ELL

Pre-Activity **How can you determine whether guessing is worth it?**

Read the introduction to Lesson 12-8 at the top of page 676 in your textbook.

Suppose you are taking a 50-question multiple-choice test in which there are 5 answer choices for each question. You are told that no points will be deducted for wrong answers. Should you guess the answers to the questions you do not know? Explain your reasoning. **Sample answer: Yes; the probability of guessing the right answer to a question is $\frac{1}{5}$, so you have a chance to get some points by guessing, and you have nothing to lose.**

Reading the Lesson

1. Indicate whether each of the following is a *binomial experiment* or *not a binomial experiment*. If the experiment is not a binomial experiment, explain why.
   a. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time. **binomial experiment**
   b. A pair of dice is thrown 5 times and the sum of the numbers that come up is recorded each time. **Not a binomial experiment; there are more than two possible outcomes for each trial.**
   c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded. **Not a binomial experiment; the trials are not independent (or, the probabilities for the two trials are not the same).**
   d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded. **binomial experiment**
2. Len randomly guesses the answers to all 6 multiple-choice questions on his chemistry test. Each question has 5 choices. Which of the following expressions gives the probability that he will get at least 4 of the answers correct? **B**

   A. $P(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + P(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + P(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

   B. $C(6, 4)\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^2 + C(6, 5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^1 + C(6, 6)\left(\frac{1}{5}\right)^6\left(\frac{4}{5}\right)^0$

   C. $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4 + C(6, 5)\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^5 + C(6, 6)\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^6$

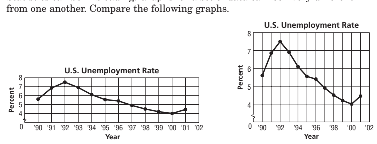
Helping You Remember

3. Some students have trouble remembering how to calculate binomial probabilities. What is an easy way to remember which numbers to put into an expression like $C(6, 4)\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^4$? **Sample answer: The binomial coefficient is $C(n, r)$, where n is the number of trials and r is the number of successes. The probability of success is raised to the rth power and the probability of failure is raised to the $(n - r)$th power.**

Enrichment, p. 746

Misuses of Statistics

Statistics can be misleading. Graphs for a set of data can look very different from one another. Compare the following graphs.



Notice that the two graphs show the same data, but the spacing in the vertical and horizontal scales differs. Scales can be cramped or spread out to make a graph that gives a certain impression. Which graph would you use to

4 Assess

Open-Ended Assessment

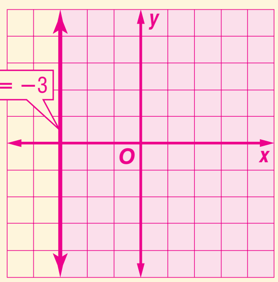
Speaking Ask students to use their own families, for example, 2 boys and a girl, and find the probabilities for that particular group of siblings. Then have students explain the steps they used.

Getting Ready for Lesson 12-9

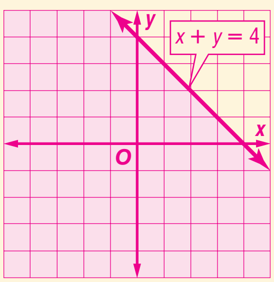
PREREQUISITE SKILL Lesson 12-9 presents finding sources of bias and sampling error. Students will use their familiarity with evaluating radical expressions as they find the margin of error. Exercises 51–54 should be used to determine your students' familiarity with finding the value of a radical expression.

Answers

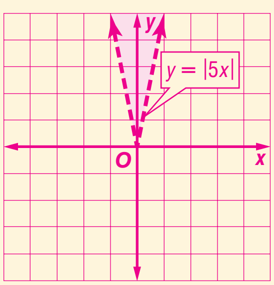
48.



49.

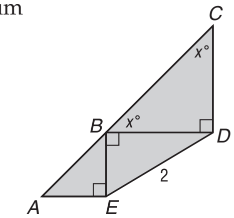


50.



Standardized Test Practice

40. **GRID IN** In the figure, if $DE = 2$, what is the sum of the area of $\triangle ABE$ and the area of $\triangle BCD$? **2**



41. What is the net result if a discount of 5% is applied to a bill of $340.60? **B**

Ⓐ $306.54 Ⓑ $323.57
Ⓒ $335.60 Ⓓ $357.63

Graphing Calculator

BINOMIAL DISTRIBUTION You can use a TI-83 Plus to investigate the graph of a binomial distribution.

Step 1 Enter the number of trials in L1. Start with 10 trials.

KEYSTROKES: STAT 1 ▲ 2nd [LIST] ▶ 5 X,T,θ,n , X,T,θ,n , 0 , 10) ENTER

Step 2 Calculate the probability of success for each trial in L2.

KEYSTROKES: ▶ ▲ 2nd [DISTR] 0 10 , .5 , 2nd [L1]) ENTER

Step 3 Graph the histogram.

KEYSTROKES: 2nd [STATPLOT]

Use the arrow and ENTER keys to choose ON, the histogram, L1 as the Xlist, and L2 as the frequency. Use the window [0, 10] scl:1 by [0, 0.5] scl: 0.1.

42. See students' work.

42. Replace the 10 in the keystrokes for steps 1 and 2 to graph the binomial distribution for several values of n less than or equal to 47. You may have to adjust your viewing window to see all of the histogram. Make sure Xscl is 1.

43. What type of distribution does the binomial distribution start to resemble as n increases? **normal distribution**

Maintain Your Skills

Mixed Review

For Exercises 44–46, use the following information.
A set of 400 test scores is normally distributed with a mean of 75 and a standard deviation of 8. *(Lesson 12-7)*

44. What percent of the test scores lie between 67 and 83? **68%**

45. How many of the test scores are greater than 91? **10**

46. What is the probability that a randomly-selected score is less than 67? **16%**

47. A salesperson had sales of $11,000, $15,000, $11,000, $16,000, $12,000, and $12,000 in the last six months. Which measure of central tendency would he be likely to use to represent these data when he talks with his supervisor? Explain. *(Lesson 12-6)* **Mean; it is highest.**

Graph each inequality. *(Lesson 2-7)* **48–50. See margin.**

48. $x \geq -3$ **49.** $x + y \leq 4$ **50.** $y > |5x|$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate** $2\sqrt{\frac{p(1-p)}{n}}$ **for the given values of p and n. Round to the nearest thousandth, if necessary.** *(For review of **radical expressions**, see Lesson 5-6.)*

51. $p = 0.5, n = 100$ **0.1** **52.** $p = 0.5, n = 400$ **0.05**

53. $p = 0.25, n = 500$ **0.039** **54.** $p = 0.75, n = 1000$ **0.027**

55. $p = 0.3, n = 500$ **0.041** **56.** $p = 0.6, n = 1000$ **0.031**

Answers (p. 681)

5. The class results should be better since it is a much larger set of data.

7. Sample answer: Put 20 marbles—5 red, 3, yellow, 3 blue, 3 green, 3 orange, and 3 black—into a bag. The red will represent Amazing Amy, and the other colors will represent each of the other prizes.

Algebra Activity

A Follow-Up of Lesson 12-8

Simulations

A **simulation** uses a probability experiment to mimic a real-life situation. You can use a simulation to solve the following problem.

A brand of cereal is offering one of six different prizes in every box. If the prizes are equally and randomly distributed within the cereal boxes, how many boxes, on average, would you have to buy in order to get a complete set of the six prizes?

Collect the Data

Work in pairs or small groups to complete steps 1 through 4.

Step 1 Use the six numbers on a die to represent the six different prizes.

Step 2 Roll the die and record which prize was in the first box of cereal. Use a tally sheet like the one shown.

Step 3 Continue to roll the die and record the prize number until you have a complete set of prizes. Stop as soon as you have a complete set. This is the end of one trial in your simulation. Record the number of boxes required for this trial.

Step 4 Repeat steps 1, 2, and 3 until your group has carried out 25 trials. Use a new tally sheet for each trial.

Simulation Tally Sheet	
Prize Number	Boxes Purchased
1	
2	
3	
4	
5	
6	
Total Needed	

Analyze the Data **1–2. See pp. 695A–695B.**

1. Create two different statistical graphs of the data collected for 25 trials.
2. Determine the mean, median, maximum, minimum, and standard deviation of the total number of boxes needed in the 25 trials.
3. Combine the small-group results and determine the mean, median, maximum, minimum, and standard deviation of the number of boxes required for all the trials conducted by the class. **See students' work.**

Make a Conjecture

4. If you carry out 25 additional trials, will your results be the same as in the first 25 trials? Explain. **Probably not; the outcomes of the trials are random since you are rolling a die.**
5. Should the small-group results or the class results give a better idea of the average number of boxes required to get a complete set of superheroes? Explain. **See margin.**
6. If there were 8 superheroes instead of 6, would you need to buy more boxes of cereal or fewer boxes of cereal on average? **more**
7. What if one of the 6 prizes was more common than the other 5? For instance, suppose that one prize, Amazing Amy, appears in 25% of all the boxes and the other 5 prizes are equally and randomly distributed among the remaining 75% of the boxes? Design and carry out a new simulation to predict the average number of boxes you would need to buy to get a complete set. Include some measures of central tendency and dispersion with your data. **See margin.**

Algebra Activity

A Follow-Up of Lesson 12-8

Getting Started

Objective Simulate a real-life situation, collect data, and do a statistical analysis.

Materials
one die for each group

Teach

- Ask students why rolling a die can simulate this problem. **because it has 6 random outcomes**
- Ask students before they collect their data if they would expect every group in the class to have the same results. **Probably not, since you are finding experimental and not theoretical probabilities.**
- Have students complete the simulation to collect data and then complete Exercises 1–7.

Assess

- In **Exercises 1–3**, students should be able to collect and organize data in a usable form and find various statistical measures.
- In **Exercises 4–7**, students should conclude that the greater the number of trials, the closer the experimental probabilities will be to the theoretical probabilities. They should also recognize that changes in the parameters of the experiment affect the outcomes.

Resource Manager

Teaching Algebra with Manipulatives
- p. 22 (master for die patterns)
- p. 295 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit
- dice

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

12-9 Lesson Notes

1 Focus

5-Minute Check Transparency 12-9 Use as a quiz or review of Lesson 12-8.

Mathematical Background notes are available for this lesson on p. 630D.

How are opinion polls used in political campaigns?

Ask students:

- Do the results of this poll indicate beyond doubt that Bush will be the victor? **No, the 7% undecided could be the deciding margin in the actual election.**
- What is the difference between the Other category and the Undecided category? **Those in the Other category are probably going to vote for a candidate other than Bush or Gore, while those in the Undecided category might be added to the total for one of those two.**

12-9 Sampling and Error

What You'll Learn

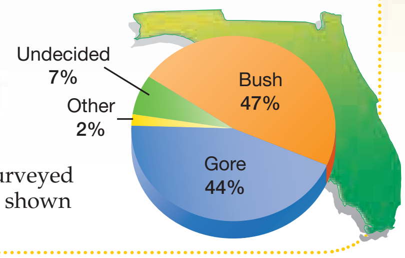
- Determine whether a sample is unbiased.
- Find margins of sampling error.

Vocabulary

- unbiased sample
- margin of sampling error

How are opinion polls used in political campaigns?

About a month before the 2000 presidential election, Mason-Dixon Polling & Research surveyed the preferences of Florida voters. The results shown were published in the *Orlando Sentinel*.



BIAS When polling organizations want to find how the public feels about an issue, they do not have the time or money to ask everyone. Instead, they obtain their results by polling a small portion of the population. To be sure that the results are representative of the population, they need to make sure that this portion is a random or **unbiased sample** of the population. A sample of size n is random when every possible sample of size n has an equal chance of being selected.

Example 1 Biased and Unbiased Samples

State whether each method would produce a random sample. Explain.

a. asking every tenth person coming out of a health club how many times a week they exercise to determine how often people in the city exercise

This would not result in a random sample because the people surveyed would probably exercise more often than the average person.

b. surveying people going into an Italian restaurant to find out people's favorite type of food

This would probably not result in a random sample because the people surveyed would probably be more likely than others to prefer Italian food.

MARGIN OF ERROR As the size of a sample increases, it more accurately reflects the population. If you sampled only three people and two prefer Brand A, you could say, "Two out of three people chose Brand A over any other brand," but you may not be giving a true picture of how the total population would respond. The **margin of sampling error (ME)** gives a limit on the difference between how a sample responds and how the total population would respond.

Key Concept *Margin of Sampling Error*

If the percent of people in a sample responding in a certain way is p and the size of the sample is n, then 95% of the time, the percent of the population responding in that same way will be between $p - ME$ and $p + ME$, where

$$ME = 2\sqrt{\frac{p(1-p)}{n}}.$$

That is, the probability is 0.95 that $p \pm ME$ will contain the true population results.

Resource Manager

Workbook and Reproducible Masters

Chapter 12 Resource Masters

- Study Guide and Intervention, pp. 747–748
- Skills Practice, p. 749
- Practice, p. 750
- Reading to Learn Mathematics, p. 751
- Enrichment, p. 752
- Assessment, p. 768

Science and Mathematics Lab Manual, pp. 57–62

Transparencies

5-Minute Check Transparency 12-9
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 2 Find a Margin of Error

In a survey of 1000 randomly selected adults, 37% answered "yes" to a particular question. What is the margin of error?

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$= 2\sqrt{\frac{0.37(1-0.37)}{1000}}$ $p = 37\%$ or 0.37, $n = 1000$

≈ 0.030535 Use a calculator.

The margin of error is about 3%. This means that there is a 95% chance that the percent of people in the whole population who would answer "yes" is between 37 − 3 or 34% and 37 + 3 or 40%.

Published survey results often include the margin of error for the data. You can use this information to determinine the sample size.

Example 3 Analyze a Margin of Error

HEALTH **In a recent Gallup Poll, 25% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 3%.**

a. What does the 3% indicate about the results?

The 3% means that the probability is 95% that the percent of people in the population who had smoked cigarettes in the past week was between 25 − 3 or 22% and 25 + 3 or 28%.

b. How many people were surveyed?

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ Formula for margin of sampling error

$0.03 = 2\sqrt{\frac{0.25(1-0.25)}{n}}$ $ME = 0.03$, $p = 0.25$

$0.015 = \sqrt{\frac{0.25(0.75)}{n}}$ Divide each side by 2.

$0.000225 = \frac{0.25(0.75)}{n}$ Square each side.

$n = \frac{0.25(0.75)}{0.000225}$ Multiply by n and divide by 0.000225.

$n \approx 833.33$ Use a calculator.

About 833 people were surveyed.

More About. . .

Health

The percent of smokers in the United States population declined from 38.7% in 1985 to 25.8% in 1999. New therapies, like the nicotine patch, are helping more people to quit.

Source: U.S. Department of Health and Human Services

Check for Understanding

Concept Check

1–3. See pp. 695A–695B.

1. **Describe** how sampling techniques can influence the results of a survey.
2. **OPEN ENDED** Give an example of a good sample and a bad sample. Explain your reasoning.
3. **Explain** what happens to the margin of sampling error when the size of the sample n increases. Why does this happen?

www.algebra2.com/extra_examples

2 Teach

BIAS

In-Class Example Power Point®

1 State whether each method would produce a random sample. Explain.

a. surveying people going into an action movie to find out the most popular kind of movie **No; they will most likely think action movies are the most popular kind of movie.**

b. calling every 10th person on the list of subscribers to a newspaper to ask about the quality of the delivery service **Yes; no obvious bias exists in calling every 10th subscriber.**

MARGIN OF ERROR

In-Class Examples Power Point®

2 In a survey of 100 randomly selected adults, 37% answered "yes" to a particular question. What is the margin of error? **0.09656 or about 10%**

3 HEALTH In an earlier survey, 30% of the people surveyed said they had smoked cigarettes in the past week. The margin of error was 2%.

a. What does the 2% indicate about the results? **There is a 95% chance that the percent of people in the population who had smoked cigarettes in the past week was between 28% and 32%.**

b. How many people were surveyed? **2100**

DAILY INTERVENTION

Differentiated Instruction ELL

Verbal/Linguistic Have students in small groups design a survey question and practice asking it in such a way that there is bias built into the tone of voice and facial expression of the questioner. Then have them try out the question on other groups to see if they get a high percentage of the answer that the bias is designed to elicit.

3 Practice/Apply

Study Notebook

Have students—
- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 12.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Bias:** 11–14
- **Margin of Error:** 15–28

Odd/Even Assignments
Exercises 11–24 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 11–25 odd, 28–38
Average: 11–27 odd, 28–38
Advanced: 12–26 even, 28–38

4 Assess

Open-Ended Assessment

Speaking Ask students to explain why a larger sample will result in a lower margin of error, if the percent stays the same.

Assessment Options

Quiz (Lessons 12-8 and 12-9) is available on p. 768 of the *Chapter 12 Resource Masters*.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6–8	2
9, 10	3

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

4. the government sending a tax survey to everyone whose social security number ends in a particular digit **Yes; last digits of social security numbers are random.**

5. surveying students in the honors chemistry classes to determine the average time students in your school study each week **No; these students probably study more than average.**

For Exercises 6–8, find the margin of sampling error to the nearest percent.

6. $p = 72\%$, $n = 100$ **about 9%**
7. $p = 31\%$, $n = 500$ **about 4%**

8. In a survey of 520 randomly-selected high school students, 68% of those surveyed stated that they were involved in extracurricular activities at their school. **about 4%**

Application

MEDIA **For Exercises 9 and 10, use the following information.**
According to a survey in *American Demographics*, 77% of Americans age 12 or older said they listen to the radio every day. Suppose the survey had a margin of error of 5%.

9. What does the 5% indicate about the results? **See margin.**
10. How many people were surveyed? **about 283**

Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1
15–26	2
27, 28	3

Extra Practice
See page 856.

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer. **11–14. See margin for explanations.**

11. pointing with your pencil at a class list with your eyes closed as a way to find a sample of students in your class **no**

12. putting the names of all seniors in a hat, then drawing names from the hat to select a sample of seniors **yes**

13. calling every twentieth person listed in the telephone book to determine which political candidate is favored **yes**

14. finding the heights of all the boys in a freshman physical education class to determine the average height of all the boys in your school **no**

For Exercises 15–24, find the margin of sampling error to the nearest percent.

15. $p = 81\%$, $n = 100$
16. $p = 16\%$, $n = 400$
17. $p = 54\%$, $n = 500$
18. $p = 48\%$, $n = 1000$ **about 3%**
19. $p = 33\%$, $n = 1000$ **about 3%**
20. $p = 67\%$, $n = 1500$ **about 2%**

21. A poll asked people to name the most serious problem facing the country. Forty-six percent of the 800 randomly selected people said crime. **about 4%**

22. Although skim milk has as much calcium as whole milk, only 33% of 2406 adults surveyed in *Shape* magazine said skim milk is a good calcium source.

23. Three hundred sixty-seven of 425 high school students said pizza was their favorite food in the school cafeteria. **about 3%**

24. Nine hundred thirty-four of 2150 subscribers to a particular newspaper said their favorite sport was football. **about 2%**

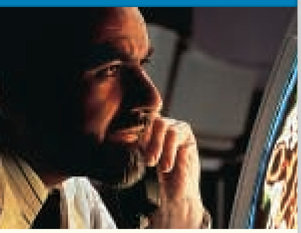
25. **ECONOMICS** In a poll conducted by ABC News, 83% of the 1020 people surveyed said they supported raising the minimum wage. What was the margin of error? **about 2%**

15. about 8%
16. about 4%
17. about 4%
22. about 2%

Answers

9. The probability is 0.95 that the percent of Americans ages 12 and older who listen to the radio every day is between 72% and 82%.
11. You would tend to point toward the middle of the page.
12. All seniors would have the same chance of being selected.
13. A wide variety of people would be called since almost everyone has a phone.
14. Freshmen are more likely than older students to be still growing, so a sample of freshmen would not give representative heights for the whole school.

Career Choices



Physican

Physicians diagnose illnesses and prescribe and administer treatment.

Online Research

For information about a career as a physician, visit: www.algebra2.com/careers

26. **PHYSICIANS** In a recent Harris Poll, 61% of the 1010 people surveyed said they considered being a physician to be a very prestigious occupation. What was the margin of error? **about 3%**

27. **SHOPPING** According to a Gallup Poll, 33% of shoppers planned to spend $1000 or more during a recent holiday season. The margin of error was 3%. How many people were surveyed? **about 983**

28. **CRITICAL THINKING** One hundred people were asked a yes-or-no question in an opinion poll. How many said "yes" if the margin of error was 9.6%? **36 or 64**

29. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 695A–695B.**

How are opinion polls used in political campaigns?

Include the following in your answer:
- a description of how a candidate could use statistics from opinion polls to determine where to make campaign stops,
- the margin of error for Bush if 807 people were surveyed, and
- an explanation of how to use the margin of error to determine the range of percent of Florida voters who favored Bush.

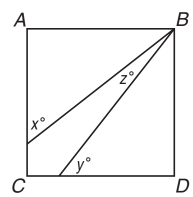
Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

30. In rectangle $ABCD$, what is $x + y$ in terms of z? **A**

Ⓐ $90 + z$ Ⓑ $190 - z$

Ⓒ $180 + z$ Ⓓ $270 - z$



31. If $xy^{-2} + y^{-1} = y^{-2}$, then the value of x *cannot* equal which of the following? **C**

Ⓐ -1 Ⓑ 0 Ⓒ 1 Ⓓ 2

Maintain Your Skills

Mixed Review

A student guesses at all 5 questions on a true-false quiz. Find each probability. *(Lesson 12-8)*

32. P(all 5 correct) $\frac{1}{32}$ 33. P(exactly 4 correct) $\frac{5}{32}$ 34. P(at least 3 correct) $\frac{1}{2}$

A set of 250 data values is normally distributed with a mean of 50 and a standard deviation of 5.5. *(Lesson 12-7)*

35. What percent of the data lies between 39 and 61? **95%**

36. How many data values are less than 55.5? **210**

37. What is the probability that a data value selected at random is greater than 39? **37. 97.5%**

38. Given $x^3 - 3x^2 - 4x + 12$ and one of its factors $x + 2$, find the remaining factors of the polynomial. *(Lesson 7-4)* **$x - 2$, $x - 3$**

WebQuest Internet Project

'Minesweeper': Secret to Age-Old Puzzle?

It is time to complete your project. Use the information and data you have gathered about the history of mathematics to prepare a presentation or web page. Be sure to include transparencies and a sample mathematics problem or idea in the presentation.

www.algebra2.com/webquest

www.algebra2.com/extra_examples

Study Guide and Intervention, p. 747 (shown) and p. 748

Bias A sample of size n is random (or **unbiased**) when every possible sample of size n has an equal chance of being selected. If a sample is biased, then information obtained from it may not be reliable.

Example **To find out how people in the U.S. feel about mass transit, people at a commuter train station are asked their opinion. Does this situation represent a random sample?**

No; the sample includes only people who actually use a mass-transit facility. The sample does not include people who ride bikes, drive cars, or walk.

Exercises

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. asking people in Phoenix, Arizona, about rainfall to determine the average rainfall for the United States **No; it rains less in Phoenix than most places in the U.S.**
2. obtaining the names of tree types in North America by surveying all of the U.S. National Forests **Yes; there are National Forests in about every state in the U.S.**
3. surveying every tenth person who enters the mall to find out about music preferences in that part of the country **Yes; mall customers should be fairly representative in terms of music tastes.**
4. interviewing country club members to determine the average number of televisions per household in the community **No; country club members would tend to be more affluent and thus not a representative sample of the community.**
5. surveying all students whose ID numbers end in 4 about their grades and career counseling needs **Yes; ID numbers are probably assigned alphabetically or by some other method not connected to students' grades or counseling needs.**
6. surveying parents at a day care facility about their preferences for brands of baby food for a marketing campaign **Yes; choice of a daycare facility would probably not influence baby food preferences.**
7. asking people in a library about the number of magazines to which they subscribe in order to describe the reading habits of a town **No; library visitors tend to read more than most citizens.**

Skills Practice, p. 749 and Practice, p. 750 (shown)

Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.

1. calling every twentieth registered voter to determine whether people own or rent their homes in your community **No; registered voters may be more likely to be homeowners, causing the survey to underrepresent renters.**
2. predicting local election results by polling people in every twentieth residence in all the different neighborhoods of your community **Yes; since all neighborhoods are represented proportionally, the views of the community should as a whole should be well represented.**
3. to find out why not many students are using the library, a school's librarian gives a questionnaire to every tenth student entering the library **No; she is polling only the students who are coming to the library, and will obtain no input from those who aren't using the library.**
4. testing overall performance of tires on interstate highways only **No; for overall performance, tires should be tested on many kinds of surfaces, and under many types of conditions.**
5. selecting every 50th hamburger from a fast-food restaurant chain and determining its fat content to assess the fat content of hamburgers served in fast-food restaurant chains throughout the country **No; the selected hamburgers are a random sample of the hamburgers served in one chain, and may represent the fat content for that chain, but will not necessarily represent the fat content of hamburgers served in other fast-food restaurant chains.**
6. assigning all shift workers in a manufacturing plant a unique identification number, and then placing the numbers in a hat and drawing 30 at random to determine the annual average salary of the workers **Yes; because the numbers are randomly chosen from among all shift workers, all workers have the same chance of being selected.**

Find the margin of sampling error to the nearest percent.

7. $p = 26\%, n = 100$ **about 9%** 8. $p = 55\%, n = 100$ **about 10%** 9. $p = 75\%, n = 500$ **about 4%**

10. $p = 14\%, n = 500$ **about 3%** 11. $p = 96\%, n = 1000$ **about 1%** 12. $p = 21\%, n = 1000$ **about 3%**

13. $p = 34\%, n = 1000$ **about 3%** 14. $p = 49\%, n = 1500$ **about 3%** 15. $p = 65\%, n = 1500$ **about 2%**

16. **COMPUTING** According to a poll of 500 teenagers, 43% said that they use a personal computer at home. What is the margin of sampling error? **about 4%**
17. **TRUST** A survey of 605 people, ages 13–33, shows that 68% trust their parents more than their best friends to tell them the truth. What is the margin of sampling error? **about 4%**
18. **PRODUCTIVITY** A study by the University of Illinois in 1995 showed an increase in productivity by 10% of the employees who wore headsets and listened to music of their choice while they were working. The margin of sampling error for the study was about 7%. How many employees participated in the study? **about 76**

Reading to Learn Mathematics, p. 751 ELL

Pre-Activity **How are opinion polls used in political campaigns?**

Read the introduction to Lesson 12-9 at the top of page 682 in your textbook.

Do you think the results of the survey about the presidential preferences demonstrates that Bush was actually ahead in Florida a month before the election? If there is not enough information given to determine this, list at least two questions you would ask about the survey that would help you determine the significance of the survey. **Sample answer: There is not enough information to tell. 1. How many people were surveyed? 2. How was the sample for the survey selected? 3. What is the margin of error for this survey?**

Reading the Lesson

1. Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.
   a. asking all the customers at five restaurants on the same evening how many times a month they eat dinner in restaurants to determine how often the average American eats dinner in a restaurants **No; people surveyed at a restaurant might be likely to eat dinner in restaurants more often than other people.**
   b. putting the names of all seniors at your high school in a hat and then drawing 20 names for a survey to find out where seniors would like to hold their prom **Yes; every senior would have an equal chance of being chosen for the survey.**
2. A survey determined that 58% of registered voters in the United States support increased federal spending for education. The margin of error for this survey is 4%. Explain in your own words what this tells you about the actual percentage of registered voters who support increased spending for education. **Sample answer: There is a 95% chance that the actual percentage of voters supporting increased federal spending for education is between 54% and 62%.**

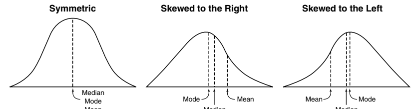
Helping You Remember

3. The formula for margin of sampling error may be tricky to remember. A good way to start is to think about the variables that must be included in the formula. What are these variables, and what do they represent? What is an easy way to remember which variable goes in the denominator in the formula? **Sample answer: *p* is the probability of a certain response and *n* is the sample size. The larger the sample size, the smaller the margin of error, so *n* must go in the denominator since dividing by a larger number gives a smaller number. The square root of a smaller number is a smaller number, and twice the square root of a smaller number is a smaller number.**

Enrichment, p. 752

Shapes of Distribution Curves

Graphs of frequency distributions can be described as either symmetric or skewed.



In a distribution skewed to the right, there are a larger number of high values. The long "tail" extends to the right.

In a distribution skewed to the left, there are a larger number of low values. The "tail" extends to the left.

Algebra Activity

A Follow-Up of Lesson 12-9

Getting Started

Objective State hypotheses for conjectures and design an experiment to test a hypothesis.

Materials
ruler
stopwatch

Teach

- Ask students why the tested hypothesis is called the null hypothesis. **Because it is often stated in the form "there is no (or null) difference."**
- Make sure students know how to use the stopwatches before beginning the experiment.
- Have students complete the simulation to collect data and then complete Exercises 1–4.

Assess

In **Exercises 1–3**, students should

- state the null hypothesis saying that there is no difference.
- state the alternative hypothesis saying that there is a difference.

In **Exercise 4**, students should

- design an experiment that they could carry out.
- restate the hypothesis so that it is in the form of a null hypothesis.

Algebra Activity

A Follow-Up of Lesson 12-9

Testing Hypotheses

A **hypothesis** is a statement to be tested. Testing a hypothesis to determine whether it is supported by the data involves five steps.

Step 1 State the hypothesis. The statement should include a *null hypothesis*, which is the hypothesis to be tested, and an *alternative hypothesis*.

Step 2 Design the experiment.

Step 3 Conduct the experiment and collect the data.

Step 4 Evaluate the data. Decide whether to reject the null hypothesis.

Step 5 Summarize the results.

Test the following hypothesis.

People react to sound and touch at the same rate.

You can measure reaction time by having someone drop a ruler and then having someone else catch it between their fingers. The distance the ruler falls will depend on their reaction time. Half of the class will investigate the time it takes to react when someone is told the ruler has dropped. The other half will measure the time it takes to react when the catcher is alerted by touch.

Step 1 The null hypothesis H_0 and alternative hypothesis H_1 are as follows.
These statements often use $=, \neq, <, >, \geq$, and $\leq$.

- H_0: reaction time to sound = reaction time to touch
- H_1: reaction time to sound ≠ reaction time to touch

Step 2 You will need to decide the height from which the ruler is dropped, the position of the person catching the ruler, the number of practice runs, and whether to use one try or the average of several tries.

Step 3 Conduct the experiment in each group and record the results.

Step 4 Organize the results so that they can be compared.

Step 5 Based on the results of your experiment, do you think the hypothesis is true? Explain.

Analyze

State the null and alternative hypotheses for each conjecture. **1–3. See pp. 695A–695B.**

1. A teacher feels that playing classical music during a math test will cause the test scores to change (either up or down). In the past, the average test score was 73.
2. An engineer thinks that the mean number of defects can be decreased by using robots on an assembly line. Currently, there are 18 defects for every 1000 items.
3. A researcher is concerned that a new medicine will cause pulse rates to rise dangerously. The mean pulse rate for the population is 82 beats per minute.
4. **MAKE A CONJECTURE** Design an experiment to test the following hypothesis. *Pulse rates increase 20% after moderate exercise.* **See students' work.**

Resource Manager

Teaching Algebra with Manipulatives
- p. 24 (master for rulers)
- p. 296 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit
- rulers
- stopwatches

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

Chapter 12

Study Guide and Review

Vocabulary and Concept Check

- area diagram (p. 651)
- binomial experiment (p. 677)
- combination (p. 640)
- compound event (p. 658)
- continuous probability distribution (p. 671)
- dependent events (p. 633)
- discrete probability distributions (p. 671)
- event (p. 632)
- failure (p. 644)
- Fundamental Counting Principle (p. 633)
- inclusive events (p. 659)
- independent events (p. 632)
- linear permutation (p. 638)
- margin of sampling error (p. 682)
- measure of central tendency (p. 664)
- measure of variation (p. 665)
- mutually exclusive events (p. 658)
- normal distribution (p. 671)
- odds (p. 645)
- outcome (p. 632)
- permutation (p. 638)
- probability (p. 644)
- probability distribution (p. 646)
- random (p. 646)
- random variable (p. 645)
- relative-frequency histogram (p. 646)
- sample space (p. 632)
- simple event (p. 658)
- skewed distribution (p. 671)
- standard deviation (p. 665)
- success (p. 644)
- unbiased sample (p. 682)
- variance (p. 665)

Choose the letter of the term that best matches each statement or phrase.

1. the ratio of the number of ways an event can succeed to the number of possible outcomes **c**
2. an arrangement of objects in which order does not matter **b**
3. two or more events in which the outcome of one event affects the outcome of another event **a**
4. a sample in which every member of the population has an equal chance to be selected **g**
5. an arrangement of objects in which order matters **d**
6. two events in which the outcome can never be the same **e**
7. the ratio of the number of ways an event can succeed to the number of ways it can fail **f**

a. dependent events
b. combination
c. probability
d. permutation
e. mutually exclusive events
f. odds
g. unbiased sample

Lesson-by-Lesson Review

12-1 The Counting Principle

See pages 632–637.

Concept Summary

- Fundamental Counting Principle: If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by the event N can occur in $m \cdot n$ ways.
- Independent Events: The outcome of one event does *not* affect the outcome of another.
- Dependent Events: The outcome of one event *does* affect the outcome of another.

Example **How many different license plates are possible with two letters followed by three digits?**

There are 26 possibilities for each letter. There are 10 possibilities, the digits 0–9, for each number. Thus, the number of possible license plates is as follows.

$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^2 \cdot 10^3$ or 676,000

www.algebra2.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Remind students to review the Foldable and make sure that the lists of terms, concepts, and examples are complete. Have student volunteers share some of the printed examples of statistics that they found. Ask them to check over their notes and examples about probability and statistics to see if they wish to add any further information about the uses and misuses of statistics in the world around them.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Chapter 12

Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 12 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 12 is available on p. 766 of the *Chapter 12 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Exercises Solve each problem. *See Examples 2 and 3 on page 633.*

8. The letters a, c, e, g, i, and k are used to form 6-letter passwords for a movie theater security system. How many passwords can be formed if the letters can be used more than once in any given password? **46,656 passwords**

9. How many 4-digit personal identification codes can be formed if each numeral can only be used once? **5040 codes**

12-2 Permutations and Combinations

See pages 638–643.

Concept Summary

- In a permutation, the order of objects is important.
- In a combination, the order of objects is not important.

Example

A basket contains 3 apples, 6 oranges, 7 pears, and 9 peaches. How many ways can 1 apple, 2 oranges, 6 pears, and 2 peaches be selected?

This involves the product of four combinations, one for each type of fruit.

$$C(3, 1) \cdot C(6, 2) \cdot C(7, 6) \cdot C(9, 2) = \frac{3!}{(3-1)!1!} \cdot \frac{6!}{(6-2)!2!} \cdot \frac{7!}{(7-6)!6!} \cdot \frac{9!}{(9-2)!2!}$$

$$= 3 \cdot 15 \cdot 7 \cdot 36 \text{ or } 11{,}340$$

There are 11,340 different ways to choose the fruit from the basket.

Exercises Solve each problem. *See Example 4 on page 640.*

10. A committee of 3 is selected from Jillian, Miles, Mark, and Nikia. How many committees contain 2 boys and 1 girl? **2**

11. Five cards are drawn from a standard deck of cards. How many different hands consist of four queens and one king? **4**

12. A box of pencils contains 4 red, 2 white, and 3 blue pencils. How many different ways can 2 red, 1 white, and 1 blue pencil be selected? **36**

12-3 Probability

See pages 644–650.

Concept Summary

- $P(\text{success}) = \frac{s}{s+f}$; $P(\text{failure}) = \frac{f}{s+f}$
- odds of success $= s{:}f$; odds of failure $= f{:}s$

Example

A bag of golf tees contains 23 red, 19 blue, 16 yellow, 21 green, 11 orange, 19 white, and 17 black tees. What is the probability that if you choose a tee from the bag at random, you will choose a green tee?

There are 21 ways to choose a green tee and 23 + 19 + 16 + 11 + 19 + 17 or 105 ways not to choose a green tee. So, s is 21 and f is 105.

$$P(\text{green tee}) = \frac{s}{s+f}$$

$$= \frac{21}{21+105} \text{ or } \frac{1}{6}$$

The probability is 1 out of 6 or about 16.7%.

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Exercises **Find the odds of an event occurring, given the probability of the event.**
See Example 3 on pages 645 and 646.

13. $\frac{1}{4}$ 1:3
14. $\frac{5}{8}$ 5:3
15. $\frac{7}{12}$ 7:5
16. $\frac{3}{7}$ 3:4
17. $\frac{2}{5}$ 2:3

18. The table shows the distribution of the number of heads occurring when four coins are tossed. Find $P(H = 3)$. $\frac{1}{4}$
See Example 4 on page 646.

H = Heads	0	1	2	3	4
Probability	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

12-4 Multiplying Probabilities

See pages 651–657.

Concept Summary

- Probability of two independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
- Probability of two dependent events: $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Example

There are 3 dimes, 2 quarters, and 5 nickels in Langston's pocket. If he reaches in and selects three coins at random without replacing any of them, what is the probability that he will choose a dime d, then a quarter q, then a nickel n?

Because the outcomes of the first and second choices affect the later choices, these are dependent events.

$P(d, \text{ then } q, \text{ then } n) = \frac{3}{10} \cdot \frac{2}{9} \cdot \frac{5}{8}$ or $\frac{1}{24}$ The probability is $\frac{1}{24}$ or about 4.2%.

Exercises **Determine whether the events are** ***independent*** **or** ***dependent*****. Then find the probability.** *See Examples 1–4 on pages 652 and 654.* 19. independent; $\frac{1}{36}$

19. Two dice are rolled. What is the probability that each die shows a 4?
20. Two cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a heart and a club, in that order. 20. dependent; $\frac{13}{204}$
21. Luz has 2 red, 2 white, and 3 blue marbles in a cup. If she draws two marbles at random and does not replace the first one, find the probability of a white marble and then a blue marble. dependent; $\frac{1}{7}$

12-5 Adding Probabilities

See pages 658–663.

Concept Summary

- Probability of mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$
- Probability of inclusive events: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example

Trish has four \$1 bills and six \$5 bills. She takes three bills from her wallet at random. What is the probability that Trish will select at least two \$1 bills?

$$P(\text{at least two \$1 bills}) = P(\text{two \$1, one \$5}) + P(\text{three \$1, no \$5})$$
$$= \frac{C(4, 2) \cdot C(6, 1)}{C(10, 3)} + \frac{C(4, 3) \cdot C(6, 0)}{C(10, 3)}$$
$$= \frac{\frac{4! \cdot 6!}{(4-2)!2!(6-1)!1!}}{} + \frac{4! \cdot 6!}{(4-3)!3!(6-0)!0!}$$
$$= \frac{36}{120} + \frac{4}{120} \text{ or } \frac{1}{3}$$

The probability is $\frac{1}{3}$ or about 0.333.

Chapter 12 Study Guide and Review 689

Exercises **Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.** *See Examples 1–3 on pages 659 and 660.*

22. There are 5 English, 2 math, and 3 chemistry books on a shelf. If a book is randomly selected, what is the probability of selecting a math book or a chemistry book? **mutually exclusive; $\frac{1}{2}$** **23. mutually exclusive; $\frac{2}{3}$**

23. A die is rolled. What is the probability of rolling a 6 or a number less than 4?

24. A die is rolled. What is the probability of rolling a 6 or a number greater than 4?

25. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a red card? **inclusive; $\frac{7}{13}$** **24. inclusive; $\frac{1}{3}$**

12-6 Statistical Measures

See pages 664–670.

Concept Summary

- To represent a set of data, use the mean if the data are spread out and you want an average of the values, the median when the data contain outliers, or the mode when the data are tightly clustered around one or two values.
- Standard deviation for n values:

$$\sigma = \sqrt{\frac{(x^1 - \bar{x})^2 + (x^2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}},\ \bar{x} \text{ is the mean}$$

Example **Find the variance and standard deviation for {100, 156, 158, 159, 162, 165, 170, 190}.**

Step 1 Find the mean.

$$\bar{x} = \frac{100 + 156 + 158 + 159 + 162 + 165 + 170 + 190}{8}$$ Add the data and divide by the number of items.

$$= \frac{1260}{8}$$

$$= 157.5$$

Step 2 Find the variance.

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n}$$

$$= \frac{(100 - 157.5)^2 + (156 - 157.5)^2 + \cdots + (170 - 157.5)^2 + (190 - 157.5)^2}{8}$$

$$= \frac{4600}{8}$$ Simplify.

$$= 575$$ Use a calculator.

Step 3 Find the standard deviation.

$\sigma^2 = 575$ Take the square root of each side.

$\sigma \approx 23.98$ Use a calculator.

Exercises **Find the variance and standard deviation of each set of data to the nearest tenth.** *See Examples 1 and 2 on pages 664 and 665.*

26. {56, 56, 57, 58, 58, 58, 59, 61} **2.4, 1.5**

27. {302, 310, 331, 298, 348, 305, 314, 284, 321, 337} **341.0, 18.5**

28. {3.4, 4.2, 8.6, 5.1, 3.6, 2.8, 7.1, 4.4, 5.2, 5.6} **2.8, 1.7**

12-7 The Normal Distribution

See pages 671–675.

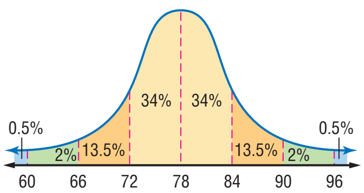
Concept Summary

Normal distributions have these properties.

- The graph is maximized and the data are symmetric at the mean.
- The mean, median, and mode are about equal.
- About 68% of the values are within one standard deviation of the mean.
- About 95% of the values are within two standard deviations of the mean.
- About 99% of the values are within three standard deviations of the mean.

Example

Mr. Byrum gave an exam to his 30 Algebra 2 students at the end of the first semester. The scores were normally distributed with a mean score of 78 and a standard deviation of 6.



a. What percent of the class would you expect to have scored between 72 and 84?

Since 72 and 84 are 1 standard deviation to the left and right of the mean, respectively, 34% + 34% or 68% of the students scored within this range.

b. What percent of the class would you expect to have scored between 90 and 96?

90 to 96 on the test includes 2% of the students.

c. Approximately how many students scored between 84 and 90?

84 to 90 on the test includes 13.5% of the students. $0.135 \times 30 = 4$ students

d. Approximately how many students scored between 72 and 84?

34% + 34% or 68% of the students scored between 72 and 84.

$0.68 \times 30 = 20$ students

Exercises **For Exercises 29–32, use the following information.**

The utility bills in a city of 5000 households are normally distributed with a mean of $180 and a standard deviation of $16. *See Example 2 on pages 672 and 673.*

29. About how many utility bills were between $164 and $196? **3400**

30. About how many bills were more than $212? **125**

31. About how many bills were less than $164? **800**

32. What is the probability that a household selected at random will have a utility bill between $164 and $180? **34%**

12-8 Binomial Experiments

See pages 676–680.

Concept Summary

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The possibilities for each trial are the same.

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Chapter 12 For More ...
• Extra Practice, see pages 854–856.
• Mixed Problem Solving, see page 837.

Example **To practice for a jigsaw puzzle competition, Laura and Julian completed four jigsaw puzzles. The probability that Laura places the last piece is $\frac{3}{5}$, and the probability that Julian places the last piece is $\frac{2}{5}$. What is the probability that Laura will place the last piece of at least two puzzles?**

$P = L^4 + 4L^3J + 6L^2J^2$ — *P*(last piece in 4) + *P*(last piece in 3) + *P*(last piece in 2)

$= \left(\frac{3}{5}\right)^4 + 4\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right) + 6\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$ — $L = \frac{3}{5}, J = \frac{2}{5}$

$= \frac{81}{625} + \frac{216}{625} + \frac{216}{625}$ or 0.8208 — The probability is 82.08%.

Exercises *See Example 2 on pages 677 and 678.*

33. Find the probability of getting 7 heads in 8 tosses of a coin. $\frac{1}{32}$

34. Find the probability that a family with seven children has exactly five boys. $\frac{21}{128}$

Find each probability if a die is rolled twelve times.

35. *P*(twelve 3s) $\frac{1}{2,176,782,336}$ **36.** *P*(exactly one 3) $\frac{48,828,125}{181,398,528}$ **37.** *P*(six 3s) $\frac{14,437,500}{2,176,782,336}$

12-9 Sampling and Error

See pages 682–685.

Concept Summary

- Margin of sampling error: $ME = 2\sqrt{\frac{p(1-p)}{n}}$ if the percent of people in a sample responding in a certain way is p and the size of the sample is n

Example **In a survey taken at a local high school, 75% of the student body stated that they thought school lunches should be free. This survey had a margin of error of 2%. How many people were surveyed?**

$ME = 2\sqrt{\frac{p(1-p)}{n}}$ — Formula for margin of sampling error

$0.02 = 2\sqrt{\frac{0.75(1-0.75)}{n}}$ — $ME = 0.02$, $p = 0.75$

$0.01 = \sqrt{\frac{0.75(1-0.75)}{n}}$ — Divide each side by 2.

$0.0001 = \frac{0.75(0.25)}{n}$ — Square each side of the equation.

$n = \frac{0.75(0.25)}{0.0001}$ — Multiply each side by n and divide each side by 0.0001.

$n = 1875$ — There were about 1875 people in the survey.

Exercises

38. In a poll asking people to name their most valued freedom, 51% of the randomly selected people said it was the freedom of speech. Find the margin of sampling error if 625 people were randomly selected. *See Example 2 on page 683.* **about 4%**

39. According to a recent survey of mothers with children who play sports, 63% of them would prefer that their children not play football. Suppose the margin of error is 4.5%. How many mothers were surveyed? *See Example 3 on page 683.* **460 mothers**

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Chapter 12 Practice Test

Vocabulary and Concepts

Match the following terms and descriptions.

1. data are symmetric about the mean **c**
2. variance and standard deviation **b**
3. mode, median, mean **a**

a.	measures of central tendency
b.	measures of variation
c.	normal distribution

Skills and Applications

Evaluate each expression.

4. $P(7, 3)$ **210**
5. $C(7, 3)$ **35**
6. $P(13, 5)$ **154,440**

Solve each problem. **10. 31,824**

7. How many ways can 9 bowling balls be arranged on the upper rack of a bowling ball rack? **362,880 ways**
8. How many different outfits can be made if you choose 1 each from 11 skirts, 9 blouses, 3 belts, and 7 pairs of shoes? **2079 outfits**
9. How many ways can the letters of the word *probability* be arranged? **9,979,200 ways**
10. How many different soccer teams consisting of 11 players can be formed from 18 players?
11. In a row of 10 parking spaces in a parking lot, how many ways can 4 cars park? **5040 ways**
12. Eleven points are equally spaced on a circle. How many ways can 5 of these points be chosen as the vertices of a pentagon? **462 pentagons**
13. A number is drawn at random from a hat that contains all the numbers from 1 to 100. What is the probability that the number is less than sixteen? $\frac{3}{20}$
14. Two cards are drawn in succession from a standard deck of cards without replacement. What is the probability that both cards are greater than 2 and less than 9? $\frac{46}{221}$
15. A shipment of ten television sets contains 3 defective sets. How many ways can a hospital purchase 4 of these sets and receive at least 2 of the defective sets? **70 ways**
16. While shooting arrows, William Tell can hit an apple 9 out of 10 times. What is the probability that he will hit it exactly 4 out of 7 times? $\frac{45,927}{2,000,000}$
17. Ten people are going on a camping trip in 3 cars that hold 5, 2, and 4 passengers, respectively. How many ways is it possible to transport the 10 people to their campsite? **6930 ways**
18. From a box containing 5 white golf balls and 3 red golf balls, 3 golf balls are drawn in succession, each being replaced in the box before the next draw is made. What is the probability that all 3 golf balls are the same color? $\frac{19}{64}$

For Exercises 19–21, use the following information.
In a ten-question multiple-choice test with four choices for each question, a student who was not prepared guesses on each item. Find each probability.

19. six questions correct $\frac{8505}{524,288}$
20. at least eight questions correct $\frac{109}{262,144}$
21. fewer than eight questions correct $\frac{262,035}{262,144}$

22. **STANDARDIZED TEST PRACTICE** Lila throws a die and writes down the number showing. If she throws the number cube again, what is the probability that the second throw will have the same number showing as the first throw? **D**

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$

www.algebra2.com/chapter_test

Portfolio Suggestion

Introduction The Fundamental Counting Principle, permutations and combinations, probability, and statistics may have been new topics for many students. These topics are used in many ways in almost all fields of employment.

Ask Students Which was your favorite word problem from this chapter? Put the problem in your portfolio and write a note that explains why it is your favorite. Add a brief conjecture about how you might be able to use the topics of this chapter in a future career that you might like to have.

Chapter 12 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 12 can be found on p. 766 of the *Chapter 12 Resource Masters*.

Chapter Tests There are six Chapter 12 Tests and an Open-Ended Assessment task available in the *Chapter 12 Resource Masters*.

Chapter 12 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	753–754
2A	MC	average	755–756
2B	MC	average	757–758
2C	FR	average	759–760
2D	FR	average	761–762
3	FR	advanced	763–764

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment Performance tasks for Chapter 12 can be found on p. 765 of the *Chapter 12 Resource Masters*. A sample scoring rubric for these tasks appears on p. A34.

Unit 4 Test A unit test/review can be found on pp. 773–774 of the *Chapter 12 Resource Masters*.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

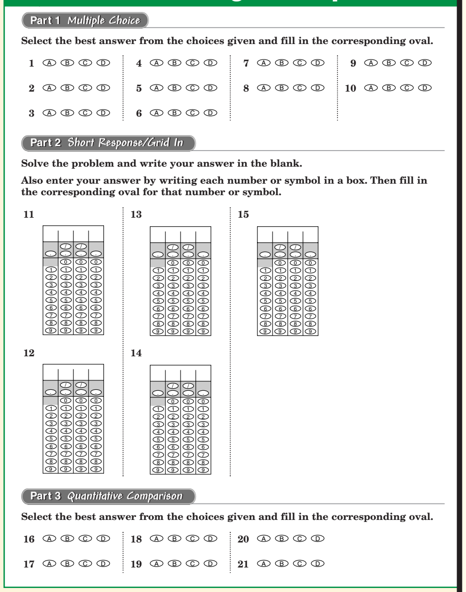
- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Chapter 12 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 12 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice
Select the best answer from the choices given and fill in the corresponding oval.
1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ 10 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ
Part 2 Short Response/Grid In
Solve the problem and write your answer in the blank.
Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.
11 13 15
12 14
Part 3 Quantitative Comparison
Select the best answer from the choices given and fill in the corresponding oval.
16 Ⓐ Ⓑ Ⓒ Ⓓ 18 Ⓐ Ⓑ Ⓒ Ⓓ 20 Ⓐ Ⓑ Ⓒ Ⓓ
17 Ⓐ Ⓑ Ⓒ Ⓓ 19 Ⓐ Ⓑ Ⓒ Ⓓ 21 Ⓐ Ⓑ Ⓒ Ⓓ

Teaching Tip In Question 8, students may want to write a list of the odd numbers in the set that are divisible by 3.

Additional Practice

See pp. 771–772 in the *Chapter 12 Resource Masters* for additional standardized test practice.

Chapter 12 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

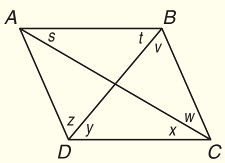
1. In a jar of red and green gumdrops, the ratio of red gumdrops to green gumdrops is 7 to 3. If the jar contains a total of 150 gumdrops, how many gumdrops are green? **C**

Ⓐ 21 Ⓑ 30
Ⓒ 45 Ⓓ 105

2. $\langle x \rangle = \frac{1}{2}x$ if x is composite and $\langle x \rangle = 2x$ if x is prime. What is the value of $\langle 16 \rangle + \langle 11 \rangle$? **B**

Ⓐ 10 Ⓑ 30
Ⓒ 54 Ⓓ 60

3. In rhombus *ABCD*, which of the following are true? **D**



I. $\angle s$ and $\angle x$ are congruent.
II. $\angle t$ and $\angle v$ are congruent.
III. $\angle z$ and $\angle t$ are congruent.

Ⓐ I only
Ⓑ II only
Ⓒ I and II only
Ⓓ I, II, and III

4. What is the area of an isosceles right triangle with hypotenuse $3\sqrt{2}$ units? **B**

Ⓐ $1.5\sqrt{2}$ units^2
Ⓑ 4.5 units^2
Ⓒ 9 units^2
Ⓓ $6 + 3\sqrt{2}$ units^2

5. What is the solution set for $t(t + 7) = 18$? **D**

Ⓐ $\{-2, 9\}$
Ⓑ $\{-3, 6\}$
Ⓒ $\{0, 18\}$
Ⓓ $\{-9, 2\}$

6. The equation $3x - 8 = 5x^2 - y$ represents which of the following conic sections? **B**

Ⓐ hyperbola
Ⓑ parabola
Ⓒ circle
Ⓓ ellipse

7. If the equations $x^2 + y^2 = 16$ and $y = x^2 + 4$ are graphed on the same coordinate plane, how many points of intersection exist? **B**

Ⓐ none
Ⓑ one
Ⓒ two
Ⓓ three

8. A number is chosen at random from the set $\{1, 2, 3, \ldots 20\}$. What is the probability that the number is odd and divisible by 3? **A**

Ⓐ $\frac{3}{20}$ Ⓑ $\frac{3}{10}$
Ⓒ $\frac{7}{20}$ Ⓓ $\frac{13}{20}$

9. What is the least positive integer that is divisible by 3, 4, 5, and 6? **A**

Ⓐ 60 Ⓑ 180
Ⓒ 240 Ⓓ 360

10. If $4y - 5x + 6xy - 50 = 0$ and $x + 7 = 13$, then what is $y + 5$? **C**

Ⓐ 2 Ⓑ 6
Ⓒ 7 Ⓓ 11

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Log On **for Test Practice**

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

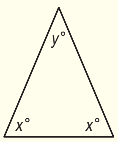
Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

11. In a high school, 250 students take math and 50 students take art. If there are 280 students enrolled in the school and they all take at least one of these courses, how many students take both math and art? **20**

12. If $20 < y < 30$ and x and y are both integers, what is the greatest possible value for x? **79**



13. Four numbers are selected at random. Their average (arithmetic mean) is 45. The fourth number selected is 34. What is the sum of the other three numbers? **146**

14. If one half of an even positive integer and three fourths of the next greater even integer have a sum of 24, what is the mean of the two integers? **19**

15. Shane has six tiles, each of which has one of the letters A, B, C, D, E, or F on it. If one of the letters must be A and the last letter must be F, how many different arrangements of three letters (such as ADF) can Shane create with these titles? **8**

The Princeton Review Test-Taking Tip

Question 14 When answering short-response questions, read carefully and make sure that you know exactly what the question is asking you to find. For example, if you only find the value of y in Question 14, you have not solved the problem. You need to find the value of $y + 5$.

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Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

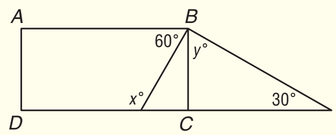
- **(A) the quantity in Column A is greater,**
- **(B) the quantity in Column B is greater,**
- **(C) the two quantities are equal, or**
- **(D) the relationship cannot be determined from the information given.**

Column A	Column B

16. $x < 0$ **B**

Column A	Column B
$x - 2$	$2 - x$

17. $ABCD$ is a rectangle. **C**



Column A	Column B
$(x + y)°$	180°

18. $x > y$, $w < z$

w, x, y, and z are positive integers. **D**

Column A	Column B
$\frac{y}{z}$	$\frac{x}{w}$

19. For $t \neq 0$, $\langle t \rangle = \frac{t^2 - 1}{t}$. **A**

Column A	Column B
$\langle 2 \rangle$	$\langle -2 \rangle$

20. For $t \neq 0$, $\langle t \rangle = \frac{t^2 - 1}{t}$. **C**

Column A	Column B
$\langle 1 \rangle$	$\langle -1 \rangle$

21. $y = -3$ **A**

Column A	Column B
y^2	y^{-2}

Page 637, Lesson 12-1

34. Step 1: When $n = 1$, the left side of the given equation is 4. The right side is $\frac{1[3(1) + 5]}{2}$ or 4, so the equation is true for $n = 1$.

Step 2: Assume $4 + 7 + 10 + \ldots + (3k + 1) = \frac{k(3k + 5)}{2}$ for some positive integer k.

Step 3: $4 + 7 + 10 + \ldots + (3k + 1) + [3(k + 1) + 1]$

$$= \frac{k(3k + 5)}{2} + [3(k + 1) + 1]$$
$$= \frac{k(3k + 5) + 2[3(k + 1) + 1]}{2}$$
$$= \frac{3k^2 + 5k + 6k + 6 + 2}{2}$$
$$= \frac{3k^2 + 11k + 8}{2}$$
$$= \frac{(k + 1)(3k + 8)}{2}$$
$$= \frac{(k + 1)[3(k + 1) + 5]}{2}$$

The last expression is the right side of the equation to be proved, where $n = k + 1$. Thus, the equation is true for $n = k + 1$.

Therefore, $4 + 7 + 10 + \ldots + (3n + 1) = \frac{n(3n + 5)}{2}$ for all positive integers n.

Page 642, Lesson 12-2

37. $C(n - 1, r) + C(n - 1, r - 1)$

$$= \frac{(n-1)!}{(n-1-r)!r!} + \frac{(n-1)!}{[n-1-(r-1)]!(r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!r!} + \frac{(n-1)!}{(n-r)!(r-1)!}$$
$$= \frac{(n-1)!}{(n-r-1)!r!} \cdot \frac{n-r}{n-r} + \frac{(n-1)!}{(n-r)!(r-1)!} \cdot \frac{r}{r}$$
$$= \frac{(n-1)!(n-r)}{(n-r)!r!} + \frac{(n-1)!r}{(n-r)!r!}$$
$$= \frac{(n-1)!(n-r+r)}{(n-r)!r!}$$
$$= \frac{(n-1)!n}{(n-r)!r!}$$
$$= \frac{n!}{(n-r)!r!}$$
$$= C(n, r)$$

38. Permutations and combinations can be used to find the number of different lineups. Answers should include the following.

- There are 9! different 9-person lineups available: 9 choices for the first player, 8 choices for the second player, 7 for the third player, and so on. So, there are 362,880 different lineups.
- There are $C(16, 9)$ ways to choose 9 players from 16: $C(16, 9) = \frac{16!}{7!9!}$ or 11,440.

Page 651, Algebra Activity

1. The area of rectangle A is $\frac{1}{2}$; it represents the probability of drawing a silver clip and a blue clip. The area of rectangle B is $\frac{1}{6}$; it represents the probability of drawing a silver clip and a red clip. The area of rectangle C is $\frac{1}{4}$; it represents the probability of drawing a gold clip and a blue clip. The area of rectangle D is $\frac{1}{12}$; it represents the probability of drawing a gold clip and a red clip.

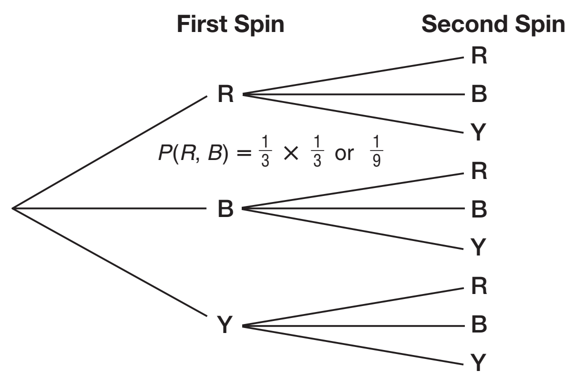
4.

		Spinner 2: blue $\frac{3}{4}$	Spinner 2: red $\frac{1}{4}$
Spinner 1	green $\frac{1}{2}$	*A*	*B*
	yellow $\frac{1}{3}$	*C*	*D*
	purple $\frac{1}{6}$	*E*	*F*

The area of rectangle *A* represents the probability of spinning green and blue. The area of rectangle *B* represents the probability of spinning green and red. The area of rectangle *C* represents the probability of spinning yellow and blue. The area of rectangle *D* represents the probability of spinning yellow and red. The area of rectangle *E* represents the probability of spinning purple and blue. The area of rectangle *F* represents the probability of spinning purple and red.

Page 656, Lesson 12-4

36.



37.

		First Spin: blue $\frac{1}{3}$	First Spin: yellow $\frac{1}{3}$	First Spin: red $\frac{1}{3}$
Second Spin	blue $\frac{1}{3}$	BB $\frac{1}{9}$	BY $\frac{1}{9}$	BR $\frac{1}{9}$
	yellow $\frac{1}{3}$	YB $\frac{1}{9}$	YY $\frac{1}{9}$	YR $\frac{1}{9}$
	red $\frac{1}{3}$	RB $\frac{1}{9}$	RY $\frac{1}{9}$	RR $\frac{1}{9}$

51. Probability can be used to analyze the chances of a player making 0, 1, or 2 free throws when he or she goes to the foul line to shoot 2 free throws. Answers should include the following.

- One of the decimals in the table could be used as the value of p, the probability that a player makes a given free throw. The probability that a player misses both free throws is $(1 - p)(1 - p)$ or $(1 - p)^2$. The probability that a player makes both free throws is $p \cdot p$ or p^2. Since the sum of the probabilities of all the possible outcomes is 1, the probability that a player makes exactly 1 of the 2 free throws is $1 - (1 - p)^2 - p^2$ or $2p(1 - p)$.
- The result of the first free throw could affect the player's confidence on the second free throw. For example, if the player makes the first free throw, the probability of he or she making the second free throw might increase. Or, if the player misses the first free throw, the probability that he or she makes the second free throw might decrease.

Page 662, Lesson 12-5

48. Probability can be used to estimate the percents of people who do the same things before going to bed. Answers should include the following.

- The events are inclusive because some people brush their teeth and set their alarm. Also, you know that the events are inclusive because the sum of the percents is not 100%.
- According to the information in the text and the table, $P(\text{read book}) = \frac{38}{100}$ and $P(\text{brush teeth}) = \frac{81}{100}$.
  Since the events are inclusive,
  $P(\text{read book and brush teeth}) =$
  $P(\text{read book}) + P(\text{brush teeth}) -$
  $P(\text{read book and brush teeth}) =$
  $\frac{38}{100} + \frac{81}{100} - \frac{600}{2000} = \frac{89}{100}$.

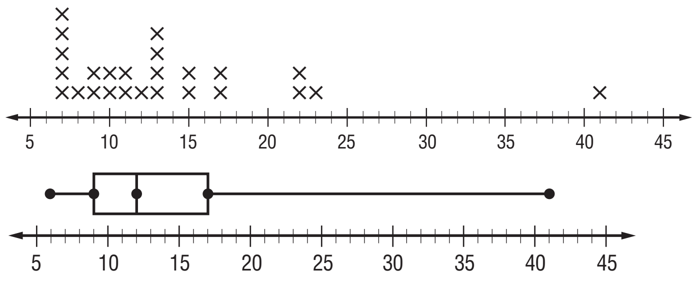
Page 679, Lesson 12-8

39. Getting a right answer and a wrong answer are the outcomes of a binomial experiment. The probability is far greater that guessing will result in a low grade than in a high grade. Answers should include the following.

- Use $(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$ and the chart on page 676 to determine the probabilities of each combination of right and wrong.
- $P(\text{5 right})$: $r^5 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ or about 0.098%;
  $P(\text{4 right, 1 wrong})$: $\frac{15}{1024}$ or about 1.5%;
  $P(\text{3 right, 2 wrong})$: $10r^3w^2 = 10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{45}{512}$ or about 8.8%;
  $P(\text{3 wrong, 2 right})$: $10r^2w^3 = 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = \frac{135}{512}$ or about 26.4%;
  $P(\text{4 wrong, 1 right})$: $5rw^4 = 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 = \frac{405}{1024}$ or about 39.6%;
  $P(\text{5 wrong})$: $w^5 = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ or about 23.7%.

Page 681, Follow-Up of Lesson 12-8
Algebra Activity

1. Sample answer:



2. Sample answer: mean = 13.56; median = 12; maximum = 41; minimum = 7; standard deviation ≈ 7.3.

Pages 683–685, Lesson 12-9

1. Sample answer: If a sample is not random, the results of a survey may not be valid.
2. Sample answer for good sample: doing a random telephone poll to rate the mayor's performance; sample answer for bad sample: conducting a survey on how much the average person reads at a bookstore
3. The margin of sampling error decreases when the size of the sample n increases. As n increases, $\frac{p(1 - p)}{n}$ decreases.

29. A political candidate can use the statistics from an opinion poll to analyze his or her standing and to help plan the rest of the campaign. Answers should include the following.
   - The candidate could decide to skip areas where he or she is way ahead or way behind, and concentrate on areas where the polls indicate the race is close.
   - about 3.5%
   - The margin of error indicates that with a probability of 0.95 the percent of the Florida population that favored Bush was between 43.5% and 50.5%. The margin of error for Gore was also about 3.5%, so with probability 0.95 the percent that favored Gore was between 40.5% and 47.5%. Therefore, it was possible that the percent of the Florida population that favored Bush was less than the percent that favored Gore.

Page 686, Follow-Up of Lesson 12-9
Algebra Activity

1. H_0: playing classical music during a math test, average test score $\neq 73$
   H_1: playing classical music during a math test, average test score $= 73$
2. H_0: using robots on an assembly line, mean number of defects per 1000 items < 18
   H_1: using robots on an assembly line, mean number of defects per 1000 items ≥ 18
3. H_0: taking medication, mean pulse rate for the population > 82 beats per minute
   H_1: taking medication, mean pulse rate for the population ≤ 82 beats per minute