Chapter 2

Linear Relations and Functions

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
2-1 Relations and Functions *(pp. 56–62)* • Analyze and graph relations. • Find functional values.	1	optional	0.5	optional
2-2 Linear Equations *(pp. 63–67)* • Identify linear equations and functions. • Write linear equations in standard form and graph them.	1	optional	0.5	optional
2-3 Slope *(pp. 68–74)* • Find and use the slope of a line. • Graph parallel and perpendicular lines.	1	optional	0.5	optional
2-4 Writing Linear Equations *(pp. 75–80)* • Write an equation of a line given the slope and a point on the line. • Write an equation of a line parallel or perpendicular to a given line.	1	optional	0.5	optional
2-5 Modeling Real-World Data: Using Scatter Plots *(pp. 81–88)* • Draw scatter plots. • Find and use prediction equations. ***Follow-Up:*** Lines of Regression	2 (with 2-5 Follow-Up)	optional	1	optional
2-6 Special Functions *(pp. 89–95)* • Identify and graph step, constant, and identity functions. • Identify and graph absolute value and piecewise functions.	1	optional	0.5	optional
2-7 Graphing Inequalities *(pp. 96–99)* • Graph linear inequalities. • Graph absolute value inequalities.	1	optional	0.5	optional
Study Guide and **Practice Test** *(pp. 100–105)* **Standardized Test Practice** *(pp. 106–107)*	1	2	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**10**	**3**	**5**	**1**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter Resource Manager

CHAPTER 2 RESOURCE MASTERS									
Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
57–58	59–60	61	62			2-1	2-1		
63–64	65–66	67	68	113		2-2	2-2	3	spaghetti
69–70	71–72	73	74			2-3	2-3		graphing calculator, spaghetti
75–76	77–78	79	80	113, 115	GCS 30, SC 3	2-4	2-4		
81–82	83–84	85	86		SC 4 SM 97–102	2-5	2-5		tape measure, graph paper (*Follow-Up:* graphing calculator)
87–88	89–90	91	92	114	GCS 29	2-6	2-6		graphing calculator, toothpicks
93–94	95–96	97	98	114		2-7	2-7	4	
				99–112, 116–118					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 2

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

In prior years students worked with coordinate systems, ordered pairs, and linear equations and functions. They manipulated and solved linear equations and inequalities algebraically. Also, they used graphs to represent two-variable data sets.

This Chapter

Students explore algebraic descriptions of linear functions, graphs of lines, and how to go back and forth between linear equations and graphed lines. They find the slope of a line containing two given points, relate the slope and *y*-intercept of a line to the values *m* and *b* in the slope-intercept form of an equation, and write equations for lines given two points or given one point and the slope. They find lines of fit for data and graph inequalities and special functions such as the greatest integer function and the absolute value function.

Future Connections

Students will continue to learn how algebraic expressions and the coordinate plane are related. At a simple level, they will learn how the "horizontal line test" identifies a one-to-one function. As activities become more complex, they will use graphs to explore quadratic and other non-linear equations and inequalities and use graphs to represent and solve systems of equations and inequalities.

2-1 Relations and Functions

This lesson begins an exploration of two important themes of algebra. One theme is how algebraic equations and the Cartesian coordinate system are related. The other theme is the relationships between equations that represent entire lines and numbers that represent points on or properties of that line.

For this lesson the central idea is ordered pairs. First, ordered pairs are explored as names for points in a coordinate plane. Second, several ordered pairs are used to describe how the set of the first elements (the domain) can be related to the set of the second elements (the range). Third, for equations that represent a line or a curve, ordered pairs are used to determine the graph that represents that equation in the coordinate plane.

In the lesson the relation between functions and relations is explored in two ways. Mappings of domain elements to range elements are used to identify functions that are one to one, functions that are not one to one, and relations that are not functions. In the coordinate plane, the vertical line test is used to distinguish a relation from a function. Relationships between ordered pairs and functions are explored in two ways. First, students are given an equation (for a curve or for a line) and make a table of ordered pairs for the equation. Second, students are given a function and a domain value, and evaluate the function to find the range value.

2-2 Linear Equations

In this lesson students deal with linear functions and intercepts. Linear functions and equations can be written in slope-intercept form, $f(x) = mx + b$ or $y = mx + b$, or in standard form, $Ax + By = C$. The graph of a linear function or equation is always a line.

2-3 Slope

Slope is a fundamental concept in algebra and higher mathematics. In this lesson, students calculate the slope of a line given two points on the line and explore the slopes of families or pairs of lines that are parallel and the slopes of pairs of lines that are perpendicular.

Students graph a line given two points or given one point and the slope. In the coordinate plane, students associate lines with slopes that are positive, negative, zero, or undefined.

2-4 Writing Linear Equations

This lesson focuses on the slope and y-intercept of a linear equation. In the slope-intercept form of a linear equation, $y = mx + b$, m represents the slope of the line and b is the y-intercept.

Students use two forms of a linear equation, the slope-intercept form and the point-slope form, to write an equation given two points, given a point and the slope, or given a point and the equation of a parallel or a perpendicular line.

2-5 Modeling Real-World Data: Using Scatter Plots

This lesson explores equations that approximate the relation between domain values and range values, extending the idea of using an algebraic equation to represent a set of points in a plane. Starting with a scatter plot of data, students mentally picture a line through the data. After selecting two points on that line, they calculate the slope and y-intercept of that line. The equation, called a line of fit or a prediction equation, may be used to calculate the value of one variable given a value of the other.

Activities in this lesson require three steps: given a set of ordered pairs, students identify a line that represents a set of ordered pairs; then they select two ordered pairs that lie on the line; and finally they calculate the slope and y-intercept for that line.

2-6 Special Functions

In this lesson, students explore special functions. The identity and constant functions are special linear functions. The graph of a step function is a series of line segments. An absolute value function has a V-shaped graph made up of portions of two lines. A piecewise function is a function written using two or more algebraic expressions.

2-7 Graphing Inequalities

In this lesson the graph of an equation is seen as the boundary between two regions of the coordinate plane. An inequality is a description of one of the two regions, and whether the boundary is part of that region depends on the inequality symbol that is used. Students explore how inequalities, including absolute value inequalities, are modeled by points in the coordinate plane, and vice versa.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Linear Relations and Functions (Lesson 5)
- Graphing Linear Equations (Lessons 6 and 12)
- Slope (Lesson 7)
- Writing Linear Equations in Point-Slope and Standard Forms (Lesson 8)
- Writing Linear Equations in Slope-Intercept Form (Lesson 10)
- Integration: Geometry/Parallel and Perpendicular Lines (Lesson 13)
- Statistics: Scatter Plots and Best-Line Fits (Lesson 9)
- Graphing Inequalities in Two Variables (Lesson 17)

Chapter 2

DAILY INTERVENTION and Assessment

INTERVENTION

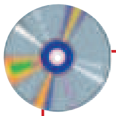
ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 55, 62, 67, 74, 80, 86, 95 Practice Quiz 1, p. 74 Practice Quiz 2, p. 95	5-Minute Check Transparencies Quizzes, *CRM* pp. 113–114 Mid-Chapter Test, *CRM* p. 115 Study Guide and Intervention, *CRM* pp. 57–58, 63–64, 69–70, 75–76, 81–82, 87–88, 93–94	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
Mixed Review	pp. 62, 67, 74, 80, 86, 95, 99	Cumulative Review, *CRM* p. 116	
Error Analysis	Find the Error, pp. 60, 71	Find the Error, *TWE* pp. 60, 71 Unlocking Misconceptions, *TWE* p. 58 Tips for New Teachers, *TWE* pp. 62, 74, 90	
Standardized Test Practice	pp. 62, 67, 74, 76, 78, 80, 86, 95, 99, 105, 106–107	*TWE* p. 76 Standardized Test Practice, *CRM* pp. 117–118	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 62, 67, 73, 80, 86, 94, 99 Open Ended, pp. 60, 65, 71, 78, 83, 92, 98	Modeling: *TWE* pp. 67, 74, 95 Speaking: *TWE* pp. 62, 98 Writing: *TWE* pp. 80, 86 Open-Ended Assessment, *CRM* p. 111	
Chapter Assessment	Study Guide, pp. 100–104 Practice Test, p. 105	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 99–104 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 105–110 Vocabulary Test/Review, *CRM* p. 112	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS



TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
2-2	**3** *Graphing Linear Equations on the Coordinate Plane*
2-7	**4** *Graphing Linear Inequalities on the Coordinate Plane*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 55
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 60, 65, 71, 78, 83, 92, 98, 100)
- Writing in Math questions in every lesson, pp. 62, 67, 73, 80, 86, 94, 99
- Reading Study Tip, pp. 56, 59, 71, 82
- WebQuest, p. 84

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 55, 100
- Study Notebook suggestions, pp. 60, 65, 71, 78, 83, 93, 97
- Modeling activities, pp. 67, 74, 95
- Speaking activities, pp. 62, 98
- Writing activities, pp. 80, 86
- Differentiated Instruction, (Verbal/Linguistic), p. 92
- ELL Resources, pp. 54, 61, 66, 73, 79, 85, 92, 94, 99, 100

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 2 Resource Masters,* pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 2 Resource Masters*, pp. 61, 67, 73, 79, 85, 91, 97)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 2 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

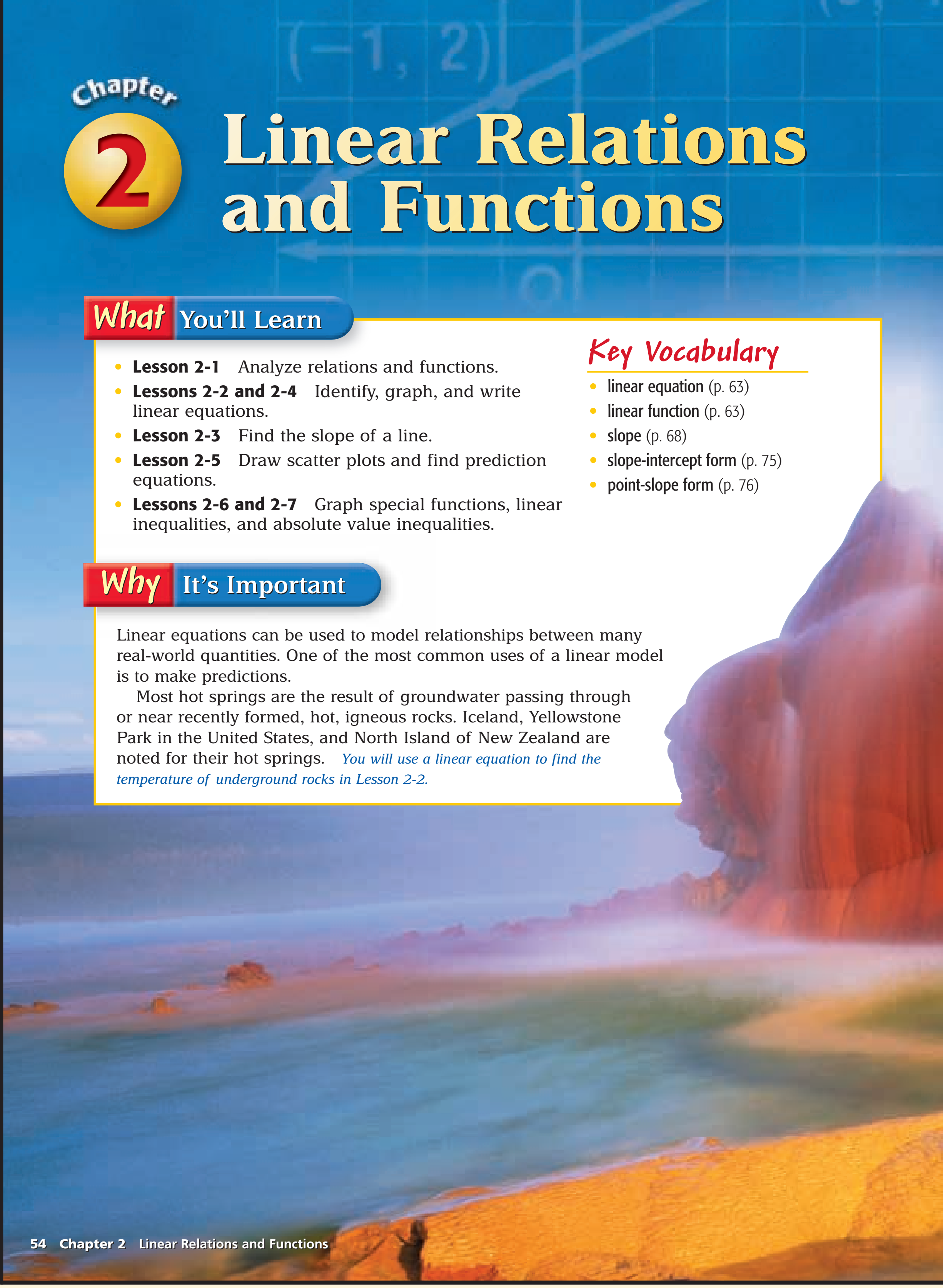
Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
2-1	1, 2, 7, 9, 10	
2-2	1, 2, 4, 6, 8, 9	
2-3	1, 2, 4, 6, 7, 8, 9, 10	
2-4	1, 2, 6, 8, 9, 10	
2-5	1, 2, 5, 6, 8, 9, 10	
2-5 Follow-Up	1, 2, 4, 5, 6, 9, 10	
2-6	1, 2, 5, 6, 8, 9, 10	
2-7	1, 2, 6, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation



Chapter 2

Linear Relations and Functions

What You'll Learn

- **Lesson 2-1** Analyze relations and functions.
- **Lessons 2-2 and 2-4** Identify, graph, and write linear equations.
- **Lesson 2-3** Find the slope of a line.
- **Lesson 2-5** Draw scatter plots and find prediction equations.
- **Lessons 2-6 and 2-7** Graph special functions, linear inequalities, and absolute value inequalities.

Key Vocabulary

- linear equation (p. 63)
- linear function (p. 63)
- slope (p. 68)
- slope-intercept form (p. 75)
- point-slope form (p. 76)

Why It's Important

Linear equations can be used to model relationships between many real-world quantities. One of the most common uses of a linear model is to make predictions.

Most hot springs are the result of groundwater passing through or near recently formed, hot, igneous rocks. Iceland, Yellowstone Park in the United States, and North Island of New Zealand are noted for their hot springs. *You will use a linear equation to find the temperature of underground rocks in Lesson 2-2.*

Vocabulary Builder

ELL

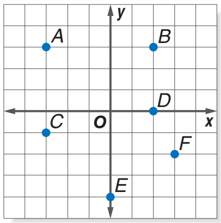
The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 2 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 2 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

For Lesson 2-1 — Identify Points on a Coordinate Plane

Write the ordered pair for each point.



1. A **(−3, 3)**
2. B **(2, 3)**
3. C **(−3, −1)**
4. D **(2, 0)**
5. E **(0, −4)**
6. F **(3, −2)**

For Lesson 2-1 — Evaluate Expressions

Evaluate each expression if $a = -1$, $b = 3$, $c = -2$, and $d = 0$. *(For review, see Lesson 1-1.)*

7. $c + d$ **−2**
8. $4c - b$ **−11**
9. $a^2 - 5a + 3$ **9**
10. $2b^2 + b + 7$ **28**
11. $\frac{a-b}{c-d}$ **−2**
12. $\frac{a+c}{b+c}$ **−3**

For Lesson 2-4 — Simplify Expressions

Simplify each expression. *(For review, see Lesson 1-2.)*

13. $x - (-1)$ **$x + 1$**
14. $x - (-5)$ **$x + 5$**
15. $2[x - (-3)]$ **$2x + 6$**
16. $4[x - (-2)]$ **$4x + 8$**
17. $\frac{1}{2}[x - (-4)]$ **$\frac{1}{2}x + 2$**
18. $\frac{1}{3}[x - (-6)]$ **$\frac{1}{3}x + 2$**

For Lessons 2-6 and 2-7 — Evaluate Expressions with Absolute Value

Evaluate each expression if $x = -3$, $y = 4$, and $z = -4.5$. *(For review, see Lesson 1-4.)*

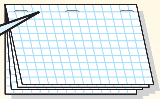
19. $|x|$ **3**
20. $|y|$ **4**
21. $|5x|$ **15**
22. $-|2z|$ **−9**
23. $5|y + z|$ **2.5**
24. $-3|x + y| - |x + z|$ **−10.5**

FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about relations and functions. Begin with two sheets of grid paper.

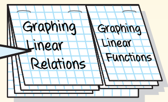
Step 1 Fold

Fold in half along the width and staple along the fold.



Step 2 Cut and Label

Cut the top three sheets and label as shown.



Reading and Writing As you read and study the chapter, write notes, examples, and graphs under the tabs.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 2. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
2-2	Solving Equations (p. 62)
2-4	Solving Equations (p. 74)
2-5	Finding a Median (p. 80)
2-6	Absolute Value (p. 86)
2-7	Inequalities (p. 95)

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Organization of Data: Annotating As students read and work their way through the chapter, have them make annotations under the appropriate tabs of their Foldable. Explain to them that annotations are usually notes taken in the margins of books, which we own, to organize the text for review or studying. Annotations often include questions that arise, reader comments and reactions, short summaries, steps or data numbered by the reader, and key points highlighted or underlined.

2-1 Lesson Notes

1 Focus

5-Minute Check Transparency 2-1 Use as a quiz or review of Chapter 1.

Mathematical Background notes are available for this lesson on p. 54C.

Building on Prior Knowledge

In Chapter 1, students solved equations and inequalities. In this lesson, students relate equations to functions and relations, as well as to their graphs.

How do relations and functions apply to biology?

Ask students:

- What is the difference between average lifetime and maximum lifetime? **The average lifetime is a representative number of years for any animal of that type, while the maximum lifetime is the greatest age ever attained by an animal of that type.**
- Why can you be sure that the second number in the ordered pairs for this data is always greater than or equal to the first? **For each animal, the maximum age will always equal or exceed the average age.**

2-1 Relations and Functions

What You'll Learn

- Analyze and graph relations.
- Find functional values.

Vocabulary

- ordered pair
- Cartesian coordinate plane
- quadrant
- relation
- domain
- range
- function
- mapping
- one-to-one function
- vertical line test
- independent variable
- dependent variable
- functional notation

How do relations and functions apply to biology?

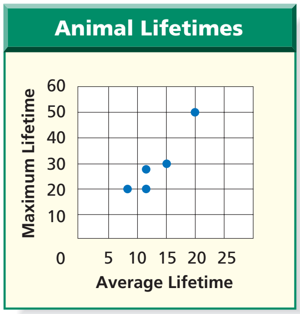
The table shows the average lifetime and maximum lifetime for some animals. The data can also be represented as **ordered pairs**. The ordered pairs for the data are (12, 28), (15, 30), (8, 20), (12, 20), and (20, 50). The first number in each ordered pair is the average lifetime, and the second number is the maximum lifetime.

(12, 28)
average lifetime ↑ ↑ maximum lifetime

Animal	Average Lifetime (years)	Maximum Lifetime (years)
Cat	12	28
Cow	15	30
Deer	8	20
Dog	12	20
Horse	20	50

Source: *The World Almanac*

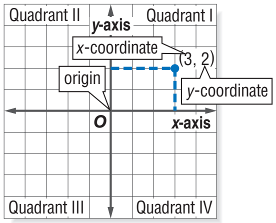
GRAPH RELATIONS You can graph the ordered pairs above by creating a *coordinate system* with two axes. Each point represents one of the ordered pairs above. Remember that each point in the coordinate plane can be named by exactly one ordered pair and that every ordered pair names exactly one point in the coordinate plane.



The vertical axis represents the maximum lifetime. The horizontal axis represents the average lifetime.

The graph of the animal lifetime data lies in only one part of the Cartesian coordinate plane—the part with all positive numbers. The **Cartesian coordinate plane** is composed of the *x-axis* (horizontal) and the *y-axis* (vertical), which meet at the *origin* (0, 0) and divide the plane into four **quadrants**. *The points on the two axes do not lie in any quadrant.*

In general, any ordered pair in the coordinate plane can be written in the form (x, y).



Assume that each square on a graph represents 1 unit unless otherwise labeled.

A **relation** is a set of ordered pairs, such as the one for the longevity of animals. The **domain** of a relation is the set of all first coordinates (x-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (y-coordinates) from the ordered pairs. The *graph* of a relation is the set of points in the coordinate plane corresponding to the ordered pairs in the relation.

Study Tip

Reading Math
An x-coordinate is sometimes called an *abscissa*, and a y-coordinate is sometimes called an *ordinate*.

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 57–58
- Skills Practice, p. 59
- Practice, p. 60
- Reading to Learn Mathematics, p. 61
- Enrichment, p. 62

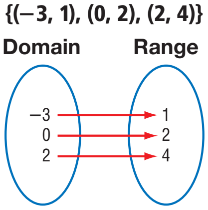
Transparencies

5-Minute Check Transparency 2-1
Answer Key Transparencies

Technology

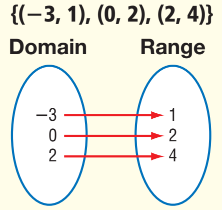
Interactive Chalkboard

A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element of the range. A **mapping** shows how each member of the domain is paired with each member of the range.

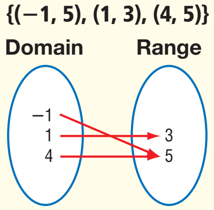


The first two relations shown below are functions. The third relation is not a function because the −3 in the domain is paired with both 0 and 6 in the range. A function like the first one below, where each element of the range is paired with exactly one element of the domain, is called a **one-to-one function**.

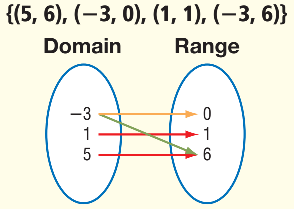
Concept Summary — *Functions*



one-to-one function



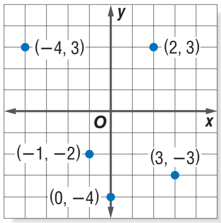
function,
not one-to-one



not a function

Example 1 *Domain and Range*

State the domain and range of the relation shown in the graph. Is the relation a function?



The relation is {(−4, 3), (−1, −2), (0, −4), (2, 3), (3, −3)}.
The domain is {−4, −1, 0, 2, 3}.
The range is {−4, −3, −2, 3}.

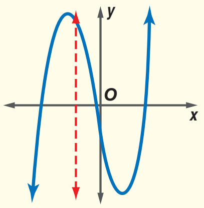
Each member of the domain is paired with exactly one member of the range, so this relation is a function.

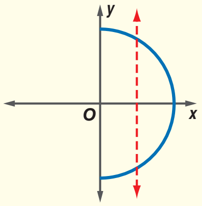
You can use the **vertical line test** to determine whether a relation is a function.

Key Concept — *Vertical Line Test*

• **Words**	If no vertical line intersects a graph in more than one point, the graph represents a function.	If some vertical line intersects a graph in two or more points, the graph does not represent a function.

• **Models**





In Example 1, there is no vertical line that contains more than one of the points. Therefore, the relation is a function.

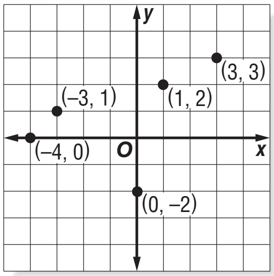
2 Teach

GRAPH RELATIONS

In-Class Example

Power Point®

1 State the domain and range of the relation shown in the graph. Is the relation a function?



The domain is {−4, −3, 0, 1, 3}. The range is {−2, 0, 1, 2, 3}. Each member of the domain is paired with exactly one member of the range, so this relation is a function.

Teaching Tip Ask students to explain why the set of ordered pairs (9, 3), (9, −3), (4, 2), (4, −2) is not a function. **Sample answer: For at least one of the *x* values, there are two different *y* values.**

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

In-Class Example

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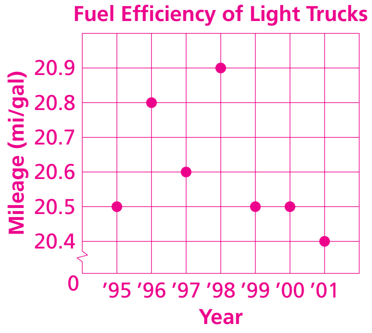
2 **TRANSPORTATION** The table shows the average fuel efficiency in miles per gallon for light trucks for several years. Graph this information and determine whether it represents a function.

Year	Fuel Efficiency (mi/gal)
1995	20.5
1996	20.8
1997	20.6
1998	20.9
1999	20.5
2000	20.5
2001	20.4

Source: U.S. Environmental Protection Agency



Yes, this relation is a function.

EQUATIONS OF FUNCTIONS AND RELATIONS

In-Class Example

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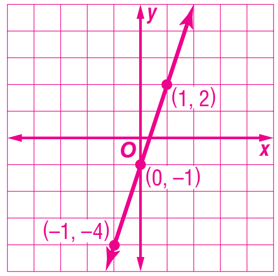
3 a. Graph the relation represented by $y = 3x - 1$.



b. Find the domain and range. **The domain and range are both all real numbers.**

c. Determine whether the relation is a function. **Yes, the equation $y = 3x - 1$ represents a function.**

Study Tip

Vertical Line Test

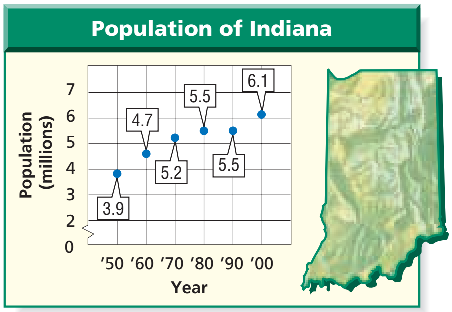
You can use a pencil to represent a vertical line. Slowly move the pencil to the right across the graph to see if it intersects the graph at more than one point.

Example 2 Vertical Line Test

GEOGRAPHY **The table shows the population of the state of Indiana over the last several decades. Graph this information and determine whether it represents a function.**

Year	Population (millions)
1950	3.9
1960	4.7
1970	5.2
1980	5.5
1990	5.5
2000	6.1

Source: U.S. Census Bureau



Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function. *Notice also that each year is paired with only one population value.*

EQUATIONS OF FUNCTIONS AND RELATIONS

Relations and functions can also be represented by equations. The solutions of an equation in x and y are the set of ordered pairs (x, y) that make the equation true.

Consider the equation $y = 2x - 6$. Since x can be any real number, the domain has an infinite number of elements. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.

Example 3 Graph Is a Line

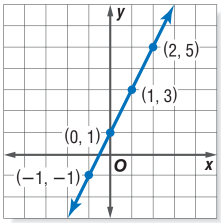
a. Graph the relation represented by $y = 2x + 1$.

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

x	y
–1	
0	
1	
2	

→

x	y
–1	–1
0	1
1	3
2	5



b. Find the domain and range.

Since x can be any real number, there is an infinite number of ordered pairs that can be graphed. All of them lie on the line shown. Notice that every real number is the x-coordinate of some point on the line. Also, every real number is the y-coordinate of some point on the line. So the domain and range are both all real numbers.

c. Determine whether the relation is a function.

This graph passes the vertical line test. For each x value, there is exactly one y value, so the equation $y = 2x + 1$ represents a function.

DAILY INTERVENTION

Unlocking Misconceptions

- **Relations and Functions** Some students may not realize that all functions are relations. Explain that a function is a special type of relation, analogous to how a square is a special type of rectangle.
- **Vertical Line Test** Make sure students understand that when two points on the graph of a relation are intersected by a vertical line, this means those two points have the same x value but different y values. That is, one domain value is paired with more than one range value.

Example 4 Graph Is a Curve

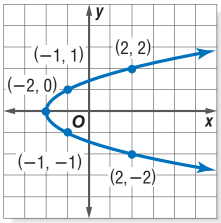
a. Graph the relation represented by $x = y^2 - 2$.

Make a table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a smooth curve.

x	y
	−2
	−1
	0
	1
	2

⟶

x	y
2	−2
−1	−1
−2	0
−1	1
2	2



b. Find the domain and range.

Every real number is the y-coordinate of some point on the graph, so the range is all real numbers. But, only real numbers greater than or equal to −2 are x-coordinates of points on the graph. So the domain is $\{x \mid x \geq -2\}$.

c. Determine whether the relation is a function.

You can see from the table and the vertical line test that there are two y values for each x value except $x = -2$. Therefore, the equation $x = y^2 - 2$ does not represent a function.

When an equation represents a function, the variable, usually x, whose values make up the domain is called the **independent variable**. The other variable, usually y, is called the **dependent variable** because its values depend on x.

Equations that represent functions are often written in **functional notation**. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$. The symbol $f(x)$ replaces the y and is read "f of x." The f is just the name of the function. It is not a variable that is multiplied by x. Suppose you want to find the value in the range that corresponds to the element 4 in the domain of the function. This is written as $f(4)$ and is read "f of 4." The value $f(4)$ is found by substituting 4 for each x in the equation. Therefore, $f(4) = 2(4) + 1$ or 9. *Letters other than f can be used to represent a function. For example, $g(x) = 2x + 1$.*

Example 5 Evaluate a Function

Given $f(x) = x^2 + 2$ and $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

a. $f(-3)$

$f(x) = x^2 + 2$ Original function

$f(-3) = (-3)^2 + 2$ Substitute.

$= 9 + 2$ or 11 Simplify.

b. $g(2.8)$

$g(x) = 0.5x^2 - 5x + 3.5$ Original function

$g(2.8) = 0.5(2.8)^2 - 5(2.8) + 3.5$ **Estimate:** $g(3) = 0.5(3)^2 - 5(3) + 3.5$ or -7

$= 3.92 - 14 + 3.5$ Multiply.

$= -6.58$ Compare with the estimate.

c. $f(3z)$

$f(x) = x^2 + 2$ Original function

$f(3z) = (3z)^2 + 2$ Substitute.

$= 9z^2 + 2$ $(ab)^2 = a^2b^2$

Study Tip

Reading Math

Suppose you have a job that pays by the hour. Since your pay *depends* on the number of hours you work, you might say that your pay is a *function* of the number of hours you work.

TEACHING TIP

Your pay is the dependent variable, and the number of hours you work is the independent variable.

www.algebra2.com/extra_examples

In-Class Examples

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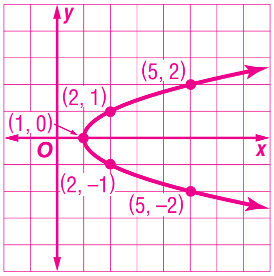
4 **a.** Graph the relation represented by $x = y^2 + 1$.



b. Find the domain and range. The domain is $\{x \mid x \geq 1\}$ and the range is all real numbers.

c. Determine whether the relation is a function. No, the equation $x = y^2 + 1$ does not represent a function.

Teaching Tip Some students may recognize this curve as a parabola. It is the same shape as the more familiar graph of the equation $y = x^2 + 1$.

5 Given $f(x) = x^3 - 3$ and $h(x) = 0.3x^2 - 3x - 2.7$, find each value.

a. $f(-2)$ -11

b. $h(1.6)$ -6.732

c. $f(2t)$ $8t^3 - 3$

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Encourage students to relate to the mathematics in this lesson by asking those who are familiar with reading musical notation to explain to the class how graphing points on a coordinate plane compares to writing musical notes on a staff.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- draw their own diagrams, similar to those in the Concept Summary on p. 57.
- make a sketch illustrating how to use the vertical line test, both for a function and a non-function.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Graph Relations:** 17–28, 35–45, 55
- **Equations of Functions and Relations:** 29–34, 46–54, 56

Odd/Even Assignments
Exercises 17–34 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–31 odd, 35–37, 47–53 odd, 55–58, 63–73

Average: 17–33 odd, 35–41, 47–53 odd, 55–58, 63–73 (optional: 59–62)

Advanced: 18–34 even, 42–45, 46–54 even, 55–69 (optional: 70–73)

DAILY INTERVENTION

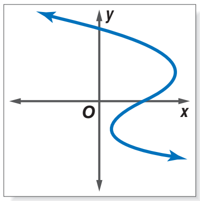
FIND THE ERROR Suggest that students rewrite the original function by substituting the expression (2*a*) for each variable *x* before they begin simplifying.

Check for Understanding

Concept Check

1. Sample answer: {(−4, 3), (−2, 3), (1, 5), (−2, 1)}
2. See pp. 107A–107H.

1. **OPEN ENDED** Write a relation of four ordered pairs that is *not* a function.
2. **Copy** the graph at the right. Then draw a vertical line that shows that the graph does not represent a function.



3. **FIND THE ERROR** Teisha and Molly are finding $g(2a)$ for the function $g(x) = x^2 + x - 1$.

Teisha

$g(2a) = 2(a^2 + a - 1)$
$= 2a^2 + 2a - 2$

Molly

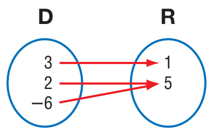
$g(2a) = (2a)^2 + 2a - 1$
$= 4a^2 + 2a - 1$

Who is correct? Explain your reasoning. See margin.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–8	1, 2
9	3
10	4
11–14	2
15, 16	5

Determine whether each relation is a function. Write *yes* or *no*.

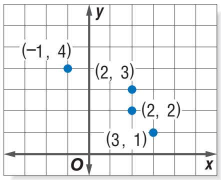
4.


yes

5.

x	y
5	−2
10	−2
15	−2
20	−2

yes

6.


no

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function. 7–10. pp. 107A–107H.

7. {(7, 8), (7, 5), (7, 2), (7, −1)}
8. {(6, 2.5), (3, 2.5), (4, 2.5)}
9. $y = -2x + 1$
10. $x = y^2$
11. Find $f(5)$ if $f(x) = x^2 - 3x$. 10
12. Find $h(-2)$ if $h(x) = x^3 + 1$. −7

Application

13. D = {70, 72, 88}, R = {95, 97, 105, 114}
14–16. See margin.

WEATHER **For Exercises 13–16, use the table of record high temperatures (°F) for January and July.**

City	Jan.	July
Los Angeles	88	97
Sacramento	70	114
San Diego	88	95
San Francisco	72	105

Source: U.S. National Oceanic and Atmospheric Administration

13. Identify the domain and range. Assume that the January temperatures are the domain.
14. Write a relation of ordered pairs for the data.
15. Graph the relation.
16. Is this relation a function? Explain.

★ indicates increased difficulty

Practice and Apply

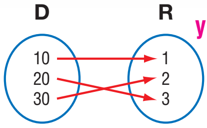
Homework Help

For Exercises	See Examples
17–28	1, 2
29–32	3
33, 34	4
35–45, 55	2
46–54, 56	5

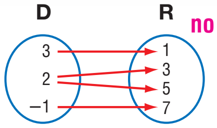
Extra Practice

See page 830.

Determine whether each relation is a function. Write *yes* or *no*.

17.


yes

18.


no

19.

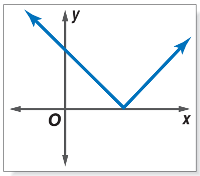
x	y
0.5	−3
2	0.8
0.5	8

no

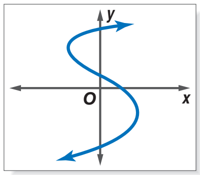
20.

x	y
2000	$4000
2001	$4300
2002	$4000
2003	$4500

yes

21.


yes

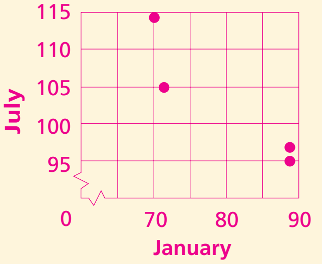
22.


no

Answers

3. Molly; to find $g(2a)$, replace x with $2a$. Teisha found $2g(a)$, not $g(2a)$.
14. {(88, 97), (70, 114), (88, 95), (72, 105)}
15. See graph at right.
16. No; the domain value 88 is paired with two range values.

Record High Temperatures



23. D = {−3, 1, 2}, R = {0, 1, 5}; yes

24. D = {3, 4, 6}, R = {5}; yes

25. D = {−2, 3}, R = {5, 7, 8}; no

26. D = {3, 4, 5, 6}, R = {3, 4, 5, 6}; yes

27. D = {−3.6, 0, 1.4, 2}, R = {−3, −1.1, 2, 8}; yes

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function. 23–34. See pp. 107A–107H for graphs.

23. $\{(2, 1), (-3, 0), (1, 5)\}$
24. $\{(4, 5), (6, 5), (3, 5)\}$
25. $\{(-2, 5), (3, 7), (-2, 8)\}$
26. $\{(3, 4), (4, 3), (6, 5), (5, 6)\}$
27. $\{(0, -1.1), (2, -3), (1.4, 2), (-3.6, 8)\}$
28. $\{(-2.5, 1), (-1, -1), (0, 1), (-1, 1)\}$
29. $y = -5x$
30. $y = 3x$
31. $y = 3x - 4$
32. $y = 7x - 6$
★33. $y = x^2$ D = all reals, R = $\{y \mid y \geq 0\}$; yes
★34. $x = 2y^2 - 3$ D = $\{x \mid x \geq -3\}$, R = all reals; no

SPORTS For Exercises 35–37, use the table that shows the leading home run and runs batted in totals in the American League for 1996–2000.

Year	1996	1997	1998	1999	2000
HR	52	56	56	48	47
RBI	148	147	157	165	145

Source: *The World Almanac*

35. Make a graph of the data with home runs on the horizontal axis and runs batted in on the vertical axis. See pp. 107A–107H.
36. Identify the domain and range.
37. Does the graph represent a function? Explain your reasoning. See margin.

36. D = {47, 48, 52, 56}, R = {145, 147, 148, 157, 165}

FINANCE For Exercises 38–41, use the table that shows a company's stock price in recent years. 38, 40. See margin.

Year	Price
1997	\$39
1998	\$43
1999	\$48
2000	\$55
2001	\$61
2002	\$52

38. Write a relation to represent the data.
39. Graph the relation. See pp. 107A–107H.
40. Identify the domain and range.
41. Is the relation a function? Explain your reasoning.
Yes; each domain value is paired with only one range value.

GOVERNMENT For Exercises 42–45, use the table below that shows the number of members of the U.S. House of Representatives with 30 or more consecutive years of service in Congress from 1987 to 1999.

Year	1987	1989	1991	1993	1995	1997	1999
Representatives	12	13	11	12	9	6	3

Source: *Congressional Directory*

42. Write a relation to represent the data. See margin.
43. Graph the relation. See pp.107A–107H.
44. Identify the domain and range. See pp. 107A–107H.
45. Is the relation a function? If so, is it a one-to-one function? Explain.
Yes; no; see pp. 107A–107H for explanation.

Find each value if $f(x) = 3x - 5$ and $g(x) = x^2 - x$.

46. $f(-3)$ −14
47. $g(3)$ 6
48. $g\left(\frac{1}{3}\right)$ $-\frac{2}{9}$
49. $f\left(\frac{2}{3}\right)$ −3
50. $f(a)$ $3a - 5$
51. $g(5n)$ $25n^2 - 5n$

52. Find the value of $f(x) = -3x + 2$ when $x = 2$. −4

53. What is $g(4)$ if $g(x) = x^2 - 5$? 11

More About. . .

Sports

The major league record for runs batted in (RBIs) is 191 by Hack Wilson.

Source: www.baseball-almanac.com

28. D = {−2.5, −1, 0}, R = {−1, 1}; no

29. D = all reals, R = all reals; yes

30. D = all reals, R = all reals, yes

31. D = all reals, R = all reals, yes

32. D = all reals, R = all reals; yes

www.algebra2.com/self_check_quiz

Answers

37. No; the domain value 56 is paired with two different range values.

38. {(1997, 39), (1998, 43), (1999, 48), (2000, 55), (2001, 61), (2002, 52)}

40. D = {1997, 1998, 1999, 2000, 2001, 2002}, R = {39, 43, 48, 52, 55, 61}

42. {(1987, 12), (1989, 13), (1991, 11), (1993, 12), (1995, 9), (1997, 6), (1999, 3)}

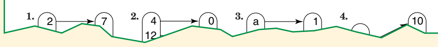
Enrichment, p. 62

Mappings

There are three special ways in which one set can be mapped to another. A set can be mapped *into* another set, *onto* another set, or can have a *one-to-one correspondence* with another set.

Into mapping	A mapping from set *A* to set *B* where every element of *A* is mapped to one or more elements of set *B*, but never to an element not in *B*.
Onto mapping	A mapping from set *A* to set *B* where each element of set *B* has at least one element of set *A* mapped to it.
One-to-one correspondence	A mapping from set *A* onto set *B* where each element of set *A* is mapped to exactly one element of set *B* and different elements of *A* are never mapped to the same element of *B*.

State whether each set is mapped into the second set, onto the second set, or has a one-to-one correspondence with the second set.



Study Guide and Intervention, p. 57 (shown) and p. 58

Graph Relations A **relation** can be represented as a set of ordered pairs or as an equation; the relation is then the set of all ordered pairs (x, y) that make the equation true. The **domain** of a relation is the set of all first coordinates of the ordered pairs, and the **range** is the set of all second coordinates.

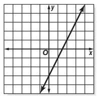
A **function** is a relation in which each element of the domain is paired with exactly one element of the range. You can tell if a relation is a function by graphing, then using the **vertical line test**. If a vertical line intersects the graph at more than one point, the relation is not a function.

Example **Graph the equation $y = 2x - 3$ and find the domain and range. Does the equation represent a function?**

Make a table of values to find ordered pairs that satisfy the equation. Then graph the ordered pairs.

The domain and range are both all real numbers. The graph passes the vertical line test, so it is function.

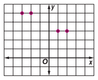
x	*y*
−1	−5
0	−3
1	−1
2	1
3	3



Exercises

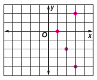
Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

1. $\{(1, 3), (-3, 5), (-2, 5), (2, 3)\}$



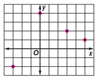
D = {−3, −2, 1, 2}, R = {3, 5}; yes

2. $\{(3, -4), (1, 0), (2, -2), (3, 2)\}$



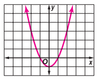
D = {1, 2, 3}, R = {−4, −2, 0, 2}; no

3. $\{(0, 4), (-3, -2), (3, 2), (5, 1)\}$



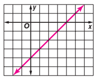
D = {−3, 0, 3, 5}, R = {−2, 1, 2, 4}; yes

4. $y = x^2 - 1$



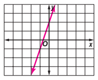
D = all reals, R = $\{y \mid y \geq -1\}$; yes

5. $y = x - 4$



D = all reals, R = all reals; yes

6. $y = 3x + 2$

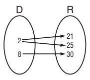


D = all reals, R = all reals; yes

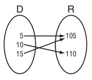
Skills Practice, p. 59 and Practice, p. 60 (shown)

Determine whether each relation is a function. Write *yes* or *no*.

1. no



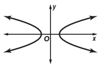
2. yes



3. yes

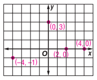
x	*y*
−3	0
−1	−1
0	0
2	−2
3	4

4. no



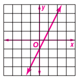
Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.

5. $\{(-4, -1), (4, 0), (0, 3), (2, 0)\}$



D = {−4, 0, 2, 4}, R = {−1, 0, 3}; yes

6. $y = 2x - 1$



D = all reals, R = all reals; yes

Find each value if $f(x) = \frac{5}{x + 2}$ and $g(x) = -2x + 3$.

7. $f(3)$ 1
8. $f(-4)$ $-\frac{5}{2}$
9. $g\left(\frac{1}{2}\right)$ 2
10. $f(-2)$ undefined
11. $g(-6)$ 15
12. $f(m - 2)$ $\frac{5}{m}$

13. **MUSIC** The ordered pairs (1, 16), (2, 16), (3, 32), (4, 32), and (5, 48) represent the cost of buying various numbers of CDs through a music club. Identify the domain and range of the relation. Is the relation a function? D = {1, 2, 3, 4, 5}, R = {16, 32, 48}; yes

14. **COMPUTING** If a computer can do one calculation in 0.0000000015 second, then the function $T(n) = 0.0000000015n$ gives the time required for the computer to do n

Reading to Learn Mathematics, p. 61

ELL

Pre-Activity **How do relations and functions apply to biology?**

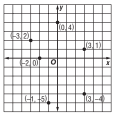
Read the introduction to Lesson 2-1 at the top of page 56 in your textbook.

- Refer to the table. What does the ordered pair (8, 20) tell you? For a deer, the average longevity is 8 years and the maximum longevity is 20 years.
- Suppose that this table is extended to include more animals. Is it possible to have an ordered pair for the data in which the first number is larger than the second? Sample answer: No, the maximum longevity must always be greater than the average longevity.

Reading the Lesson

1. a. Explain the difference between a relation and a function. Sample answer: A relation is any set of ordered pairs. A function is a special kind of relation in which each element of the domain is paired with exactly one element in the range.
   b. Explain the difference between domain and range. Sample answer: The domain of a relation is the set of all first coordinates of the ordered pairs. The range is the set of all second coordinates.

2. a. Write the domain and range of the relation shown in the graph.



D: {−3, −2, −1, 0, 3}; R: {−5, −4, 0, 1, 2, 4}

   b. Is this relation a function? Explain. Sample answer: No, it is not a function because one of the elements of the domain, 3, is paired with two elements of the range.

Helping You Remember

3. Look up the words *dependent* and *independent* in a dictionary. How can the meaning of these words help you distinguish between independent and dependent variables in a function? Sample answer: The variable whose values depend on, or are determined by, the values of the other variable is the dependent variable.

4 Assess

Open-Ended Assessment

Speaking Ask students to discuss function notation, including what they may find confusing and how to read it out loud. Make sure they understand and can correctly explain the difference between f times x and f of x.

Tips for New Teachers

Intervention This lesson has a number of vocabulary words that may be new or challenging for some students. Make sure that all students are comfortable with the mathematical language of this lesson before they go on.

Getting Ready for Lesson 2-2

PREREQUISITE SKILL Lesson 2-2 presents identifying and graphing linear equations in two variables. The skills needed to write linear equations in standard form are similar to solving equations in one variable. Exercises 70–73 should be used to determine your students' familiarity with solving single-variable equations.

Answer

56. Relations and functions can be used to represent biological data. Answers should include the following.
    - If the data are written as ordered pairs, then those ordered pairs are a relation.
    - The maximum lifetime of an animal is not a function of its average lifetime.

54. **HOBBIES** Chaz has a collection of 15 CDs. After he gets a part-time job, he decides to buy 3 more CDs every time he goes to the music store. The function $C(t) = 15 + 3t$ counts the number of CDs, $C(t)$, he has after t trips to the music store. How many CDs will he have after he has been to the music store 8 times? **39**

55. **CRITICAL THINKING** If $f(3a - 1) = 12a - 7$, find $f(x)$. $f(x) = 4x - 3$

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How do relations and functions apply to biology?

Include the following in your answer:
- an explanation of how a relation can be used to represent data, and
- a sentence that includes the words *average lifetime*, *maximum lifetime*, and *function*.

Standardized Test Practice

57. If $f(x) = 2x - 5$, then $f(0) =$ **B**

Ⓐ 0. Ⓑ −5. Ⓒ −3. Ⓓ $\frac{5}{2}$.

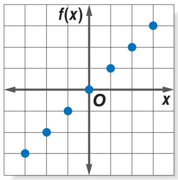
58. If $g(x) = x^2$, then $g(x + 1) =$ **C**

Ⓐ 1. Ⓑ $x^2 + 1$. Ⓒ $x^2 + 2x + 1$. Ⓓ $x^2 - x$.

Extending the Lesson

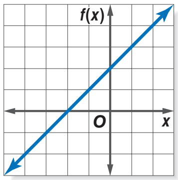
A function whose graph consists of disconnected points is called a *discrete function*. A function whose graph you can draw without lifting your pencil is called a *continuous function*. Determine whether each function is *discrete* or *continuous*.

59.



discrete

60.



continuous

61. $\{(-3, 0), (-1, 1), (1, 3)\}$ **discrete**

62. $y = -x + 4$ **continuous**

Maintain Your Skills

Mixed Review

Solve each inequality. *(Lessons 1-5 and 1-6)* **63.** $\{y \mid -8 < y < 6\}$ **64.** $\{m \mid 4 < m < 6\}$

63. $|y + 1| < 7$

64. $|5 - m| < 1$

65. $x - 5 < 0.1$ $\{x \mid x < 5.1\}$

SHOPPING **For Exercises 66 and 67, use the following information.**
Javier had \$25.04 when he went to the mall. His friend Sally had \$32.67. Javier wanted to buy a shirt for \$27.89. *(Lesson 1-3)*

66. How much money did he have to borrow from Sally to buy the shirt? **\$2.85**

67. How much money did that leave Sally? **\$29.82**

Simplify each expression. *(Lessons 1-1 and 1-2)*

68. $3^2(2^2 - 1^2) + 4^2$ **43**

69. $3(5a + 6b) + 8(2a - b)$ $31a + 10b$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solution.**
*(To review **solving equations**, see Lesson 1-3.)*

70. $x + 3 = 2$ **−1**

71. $-4 + 2y = 0$ **2**

72. $0 = \frac{1}{2}x - 3$ **6**

73. $\frac{1}{3}x - 4 = 1$ **15**

2-2 Linear Equations

What You'll Learn

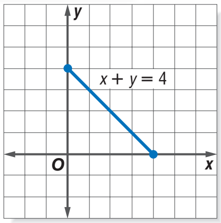
- Identify linear equations and functions.
- Write linear equations in standard form and graph them.

Vocabulary

- linear equation
- linear function
- standard form
- y-intercept
- x-intercept

How do linear equations relate to time spent studying?

Lolita has 4 hours after dinner to study and do homework. She has brought home math and chemistry. If she spends x hours on math and y hours on chemistry, a portion of the graph of the equation $x + y = 4$ can be used to relate how much time she spends on each.



IDENTIFY LINEAR EQUATIONS AND FUNCTIONS An equation such as $x + y = 4$ is called a linear equation. A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is always a line.

Linear equations	Not linear equations
$5x - 3y = 7$	$7a + 4b^2 = -8$
$x = 9$	$y = \sqrt{x + 5}$
$6s = -3t - 15$	$x + xy = 1$
$y = \frac{1}{2}x$	$y = \frac{1}{x}$

TEACHING TIP

When variables other than x and y are used, the letter coming first in the alphabet usually represents the domain variable or horizontal coordinate.

A **linear function** is a function whose ordered pairs satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers.

Example 1 Identify Linear Functions

State whether each function is a linear function. Explain.

a. $f(x) = 10 - 5x$ This is a linear function because it can be written as $f(x) = -5x + 10$. $m = -5$, $b = 10$

b. $g(x) = x^4 - 5$ This is not a linear function because x has an exponent other than 1.

c. $h(x, y) = 2xy$ This is not a linear function because the two variables are multiplied together.

2-2 Lesson Notes

1 Focus

5-Minute Check Transparency 2-2 Use as a quiz or review of Lesson 2-1.

Mathematical Background notes are available for this lesson on p. 54C.

Building on Prior Knowledge

In Lesson 2-1, students graphed functions and equations by using a table of points. In this lesson, they generalize these skills to write equations in standard form before graphing them.

How do linear equations relate to time spent studying?

Ask students:

- What does a value of zero for x mean in this situation? **Lolita spends 0 hours studying math.**
- What does a negative value for x or for y mean in this situation? **No meaning; she cannot spend negative hours studying.**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 63–64
- Skills Practice, p. 65
- Practice, p. 66
- Reading to Learn Mathematics, p. 67
- Enrichment, p. 68
- Assessment, p. 113

Transparencies

5-Minute Check Transparency 2-2
Real-World Transparency 2
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 3
Interactive Chalkboard

2 Teach

IDENTIFY LINEAR EQUATIONS AND FUNCTIONS

In-Class Examples Power Point®

1 State whether each function is a linear function. Explain.

a. $g(x) = 2x - 5$ **yes; $m = 2$; $b = -5$**

b. $p(x) = x^3 + 2$ **No; x has an exponent other than 1.**

c. $t(x) = 4 + 7x$ **yes; $m = 7$; $b = 4$**

2 **METEOROLOGY** The linear function $f(C) = 1.8C + 32$ can be used to find the number of degrees Fahrenheit, f, that are equivalent to a given number of degrees Celsius, C.

a. On the Celsius scale, normal body temperature is 37°C. What is normal body temperature in degrees Fahrenheit? **98.6°F**

b. There are 100 Celsius degrees between the freezing and boiling points of water and 180 Fahrenheit degrees between these two points. How many Fahrenheit degrees equal 1 Celsius degree? **1.8°F = 1°C**

STANDARD FORM

In-Class Example Power Point®

3 Write each equation in standard form. Identify A, B, and C.

a. $y = 3x - 9$ **$3x - y = 9$; $A = 3$, $B = -1$, $C = 9$**

b. $-\frac{2}{3}x = 2y - 1$ **$2x + 6y = 3$; $A = 2$, $B = 6$, $C = 3$**

c. $8x - 6y + 4 = 0$ **$4x - 3y = -2$; $A = 4$, $B = -3$, $C = -2$**

More About. . .

Military

To avoid decompression sickness, it is recommended that divers ascend no faster than 30 feet per minute.

Source: www.emedicine.com

Example 2 Evaluate a Linear Function

MILITARY **In August 2000, the Russian submarine *Kursk* sank to a depth of 350 feet in the Barents Sea. The linear function $P(d) = 62.5d + 2117$ can be used to find the pressure (lb/ft^2) at a depth of d feet below the surface of the water.**

a. Find the pressure at a depth of 350 feet.

$P(d) = 62.5d + 2117$ Original function

$P(350) = 62.5(350) + 2117$ Substitute.

$= 23{,}992$ Simplify.

The pressure at a depth of 350 feet is about 24,000 lb/ft^2.

b. The term 2117 in the function represents the atmospheric pressure at the surface of the water. How many times as great is the pressure at a depth of 350 feet as the pressure at the surface?

Divide the pressure 350 feet below the surface by the pressure at the surface.

$\frac{23{,}992}{2117} \approx 11.33$ Use a calculator.

The pressure at that depth is more than 11 times as great as the pressure at the surface.

STANDARD FORM Any linear equation can be written in **standard form**, $Ax + By = C$, where A, B, and C are integers whose greatest common factor is 1.

Key Concept *Standard Form of a Linear Equation*

The standard form of a linear equation is $Ax + By = C$, where $A \geq 0$, A and B are not both zero.

Example 3 Standard Form

Write each equation in standard form. Identify A, B, and C.

a. $y = -2x + 3$

$y = -2x + 3$ Original equation

$2x + y = 3$ Add $2x$ to each side.

So, $A = 2$, $B = 1$, and $C = 3$.

b. $-\frac{3}{5}x = 3y - 2$

$-\frac{3}{5}x = 3y - 2$ Original equation

$-\frac{3}{5}x - 3y = -2$ Subtract $3y$ from each side.

$3x + 15y = 10$ Multiply each side by -5 so that the coefficients are integers and $A \geq 0$.

So, $A = 3$, $B = 15$, and $C = 10$.

c. $3x - 6y - 9 = 0$

$3x - 6y - 9 = 0$ Original equation

$3x - 6y = 9$ Add 9 to each side.

$x - 2y = 3$ Divide each side by 3 so that the coefficients have a GCF of 1.

So, $A = 1$, $B = -2$, and $C = 3$.

Reading Tip Make sure that students understand the difference between the x- and y-intercepts. Some students may use the word *intersect* instead of the correct term *intercept*. Help them see that an intercept is the nonzero coordinate of the point where the graph intersects either axis.

Answer (page 65)

1. The function can be written as $f(x) = \frac{1}{2}x + 1$, so it is of the form $f(x) = mx + b$, where $m = \frac{1}{2}$ and $b = 1$.

Study Tip

Vertical and Horizontal Lines

An equation of the form $x = C$ represents a vertical line, which has only an x-intercept. $y = C$ represents a horizontal line, which has only a y-intercept.

TEACHING TIP

Some students may find it helpful to remember that if there is no y in the equation, the graph cannot cross the y-axis. Similarly, if there is no x in the equation, the graph cannot cross the x-axis.

In Lesson 2-1, you graphed an equation or function by making a table of values, graphing enough ordered pairs to see a pattern, and connecting the points with a line or smooth curve. Since two points determine a line, there are quicker ways to graph a linear equation or function. One way is to find the points at which the graph intersects each axis and connect them with a line. The y-coordinate of the point at which a graph crosses the y-axis is called the **y-intercept**. Likewise, the x-coordinate of the point at which it crosses the x-axis is the **x-intercept**.

Example 4 Use Intercepts to Graph a Line

Find the x-intercept and the y-intercept of the graph of $3x - 4y + 12 = 0$. Then graph the equation.

The x-intercept is the value of x when $y = 0$.

$3x - 4y + 12 = 0$ Original equation

$3x - 4(0) + 12 = 0$ Substitute 0 for y.

$3x = -12$ Subtract 12 from each side.

$x = -4$ Divide each side by 3.

The x-intercept is -4. The graph crosses the x-axis at $(-4, 0)$.

Likewise, the y-intercept is the value of y when $x = 0$.

$3x - 4y + 12 = 0$ Original equation

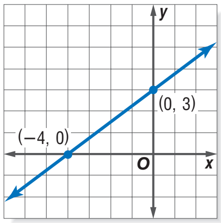
$3(0) - 4y + 12 = 0$ Substitute 0 for x.

$-4y = -12$ Subtract 12 from each side.

$y = 3$ Divide each side by -4.

The y-intercept is 3. The graph crosses the y-axis at $(0, 3)$.

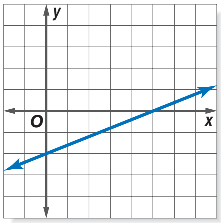
Use these ordered pairs to graph the equation.



Check for Understanding

Concept Check

1. **Explain** why $f(x) = \frac{x+2}{2}$ is a linear function. **See margin.**
2. **Name** the x- and y-intercepts of the graph shown at the right. **5, −2**
3. **OPEN ENDED** Write an equation of a line with an x-intercept of 2. **Sample answer: $x + y = 2$**



Guided Practice

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

4. $x^2 + y^2 = 4$ **No, the variables have an exponent other than 1.**
5. $h(x) = 1.1 - 2x$ **yes**

Write each equation in standard form. Identify A, B, and C.

6. $y = 3x - 5$ **$3x - y = 5$; 3, −1, 5**
7. $4x = 10y + 6$ **$2x - 5y = 3$; 2, −5, 3**
8. $y = \frac{2}{3}x + 1$ **$2x - 3y = -3$; 2, −3, −3**

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation. **9–12. See pp. 107A–107H for graphs.**

9. $y = -3x - 5$ **$-\frac{5}{3}$, −5**
10. $x - y - 2 = 0$ **2, −2**
11. $3x + 2y = 6$ **2, 3**
12. $4x + 8y = 12$ **3, $\frac{3}{2}$**

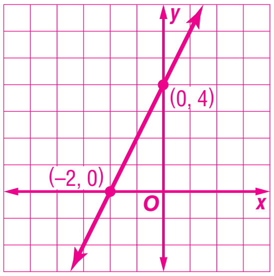
GUIDED PRACTICE KEY

Exercises	Examples
4, 5	1
6–8	3
9–12	4
13, 14	2

www.algebra2.com/extra_examples

In-Class Example PowerPoint®

4 Find the x-intercept and the y-intercept of the graph of $-2x + y - 4 = 0$. Then graph the equation. **x-intercept: −2; y-intercept: 4**



3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- include any other items(s) that students find helpful in mastering the skills in the lesson.

About the Exercises...

Organization by Objective
- **Identify Linear Equations and Functions:** 15–26
- **Standard Form:** 27–52

Odd/Even Assignments

Exercises 15–24 and 27–50 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–23 odd, 25, 26, 27–35 odd, 39–47 odd, 51–55, 61–78

Average: 15–23 odd, 25, 26, 27–49 odd, 51–55, 61–78

Advanced: 16–24 even, 28–50 even, 51–70 (optional: 71–78)

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Encourage students to relate the intercepts and the graph of the equation to the standard form of the equation, $Ax + By = C$. Point out that the ratio $-\frac{A}{B}$ is equivalent to the ratio of the graph's intercepts: $-\frac{y\text{-intercept}}{x\text{-intercept}}$. These ratios can be visualized on the graph by counting grid squares vertically and horizontally from one intercept to the other. Recognition of the ratios leads to the discussion of slope in Lesson 2-3.

Study Guide and Intervention, p. 63 (shown) and p. 64

Identify Linear Equations and Functions A **linear equation** has no operations other than addition, subtraction, and multiplication of a variable by a constant. The variables may not be multiplied together or appear in a denominator. A linear equation does not contain variables with exponents other than 1. The graph of a linear equation is a line.

A **linear function** is a function whose ordered pairs satisfy a linear equation. Any linear function can be written in the form $f(x) = mx + b$, where m and b are real numbers.

If an equation is linear, you need only two points that satisfy the equation in order to graph the equation. One way is to find the x-intercept and the y-intercept and connect these two points with a line.

Example 1 **Is $f(x) = 0.2 - \frac{x}{5}$ a linear function? Explain.**

Yes; it is a linear function because it can be written in the form $f(x) = -\frac{1}{5}x + 0.2$.

Example 2 **Is $2x + xy - 3y = 0$ a linear function? Explain.**

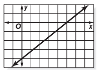
No; it is not a linear function because the variables x and y are multiplied together in the middle term.

Example 3 **Find the x-intercept and the y-intercept of the graph of $4x - 5y = 20$. Then graph the equation.**

The x-intercept is the value of x when $y = 0$.

$4x - 5y = 20$ Original equation
$4x - 5(0) = 20$ Substitute 0 for y.
$x = 5$ Simplify.

So the x-intercept is 5. Similarly, the y-intercept is -4.



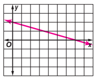
Exercises

State whether each equation or function is linear. Write *yes* or *no*. If no, explain.

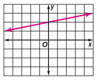
1. $6y - x = 7$ **yes**
2. $9x = \frac{18}{y}$ **No; the variable y appears in the denominator.**
3. $f(x) = 2 - \frac{x}{11}$ **yes**

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

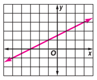
4. $2x + 7y = 14$ **x-int: 7; y-int: 2**



5. $5y - x = 10$ **x-int: -10; y-int: 2**



6. $2.5x - 5y + 7.5 = 0$ **x-int: -3; y-int: 1.5**



Skills Practice, p. 65 and Practice, p. 66 (shown)

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

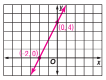
1. $h(x) = 23$ **yes**
2. $y = \frac{2}{3}x$ **yes**
3. $y = \frac{5}{x}$ **No; x is a denominator.**
4. $9 - 5xy = 2$ **No; x and y are multiplied.**

Write each equation in standard form. Identify A, B, and C.

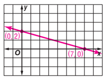
5. $y = 7x - 5$ **$7x - y = 5; 7, -1, 5$**
6. $y = \frac{3}{8}x + 5$ **$3x - 8y = -40; 3, -8, -40$**
7. $3y - 5 = 0$ **$3y = 5; 0, 3, 5$**
8. $x = -\frac{2}{7}y + \frac{3}{4}$ **$28x + 8y = 21; 28, 8, 21$**

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation.

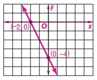
9. $y = 2x + 4$ **$-2, 4$**



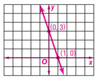
10. $2x + 7y = 14$ **7, 2**



11. $y = -2x - 4$ **$-2, -4$**



12. $6x + 2y = 6$ **1, 3**



13. **MEASURE** The equation $y = 2.54x$ gives the length in centimeters corresponding to a length x in inches. What is the length in centimeters of a 1-foot ruler? **30.48 cm**

LONG DISTANCE **For Exercises 14 and 15, use the following information.**

For Meg's long-distance calling plan, the monthly cost C in dollars is given by the linear function $C(t) = 6 + 0.05t$, where t is the number of minutes talked.

14. What is the total cost of talking 8 hours? of talking 20 hours? **$30; $66**
15. What is the effective cost per minute (the total cost divided by the number of minutes talked) of talking 8 hours? of talking 20 hours? **$0.0625; $0.055**

Reading to Learn Mathematics, p. 67

ELL

Pre-Activity **How do linear equations relate to time spent studying?**

Read the introduction to Lesson 2-2 at the top of page 63 in your textbook.

- If Lolita spends $2\frac{1}{2}$ hours studying math, how many hours will she have to study chemistry? **$1\frac{1}{2}$ hours**
- Suppose that Lolita decides to stay up one hour later so that she now has 5 hours to study and do homework. Write a linear equation that describes this situation. **$x + y = 5$**

Reading the Lesson

1. Write *yes* or *no* to tell whether each linear equation is in standard form. If it is not, explain why it is not.
   a. $-x + 2y = 5$ **No; A is negative.**
   b. $9x - 12y = -5$ **yes**
   c. $5x - 7y = 3$ **yes**
   d. $2x - \frac{4}{7}y = 1$ **No; B is not an integer.**
   e. $0x + 0y = 0$ **No; A and B are both 0.**
   f. $2x + 4y = 8$ **No; The greatest common factor of 2, 4, and 8 is 2, not 1.**
2. How can you use the standard form of a linear equation to tell whether the graph is a horizontal line or a vertical line? **If $A = 0$, then the graph is a horizontal line. If $B = 0$, then the graph is a vertical line.**

Helping You Remember

3. One way to remember something is to explain it to another person. Suppose that you are studying this lesson with a friend who thinks that she should let $x = 0$ to find the x-intercept and let $y = 0$ to find the y-intercept. How would you explain to her how to remember the correct way to find intercepts of a line? **Sample answer: The x-intercept is the x-coordinate of a point on the x-axis. Every point on the x-axis has y-coordinate 0, so let $y = 0$ to find an x-intercept. The y-intercept is the y-coordinate of a point on the y-axis. Every point on the y-axis has x-coordinate 0, so let $x = 0$ to find a y-intercept.**

Application **ECONOMICS** **For Exercises 13 and 14, use the following information.**
On January 1, 1999, the euro became legal tender in 11 participating countries in Europe. Based on the exchange rate on March 22, 2001, the linear function $d(x) = 0.8881x$ could be used to convert x euros to U.S. dollars.

13. On that date, what was the value in U.S. dollars of 200 euros? **$177.62**
14. On that date, what was the value in euros of 500 U.S. dollars? **563.00 euros**

Online Research **Data Update** How do the dollar and the euro compare today? Visit www.algebra2.com/data_update to convert among currencies.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–24	1
25, 26	2
27–38	3
39–52	4
53–60	2, 4

Extra Practice

See page 830.

23. $x^2 + 5y = 0$

27. $3x + y = 4; 3, 1, 4$

28. $12x - y = 0; 12, -1, 0$

29. $x - 4y = -5; 1, -4, -5$

30. $x - 7y = 2; 1, -7, 2$

31. $2x - y = 5; 2, -1, 5$

32. $x - 2y = -3; 1, -2, -3$

33. $x + y = 12; 1, 1, 12$

34. $x - y = -6; 1, -1, -6$

41. $\frac{10}{3}, -\frac{5}{2}$

51. See margin for graph. The lines are parallel but have different y-intercepts.

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning. **16–19, 21–22. See margin for explanations.**

15. $x + y = 5$ **yes**
16. $\frac{1}{x} + 3y = -5$ **no**
17. $x + \sqrt{y} = 4$ **no**
18. $h(x) = 2x^3 - 4x^2 + 5$ **no**
19. $g(x) = 10 + \frac{2}{x^2}$ **no**
20. $f(x) = 6x - 19$ **yes**
21. $f(x) = 7x^5 + x - 1$ **no**
22. $y = \sqrt{2x - 5}$ **no**
23. Which of the equations $x + 9y = 7$, $x^2 + 5y = 0$, and $y = 3x - 1$ is not linear?
24. Which of the functions $f(x) = 2x + 4$, $g(x) = 7$, and $h(x) = x^3 - x^2 + 3x$ is not linear? **$h(x) = x^3 - x^2 + 3x$**

PHYSICS **For Exercises 25 and 26, use the following information.**
When a sound travels through water, the distance y in meters that the sound travels in x seconds is given by the equation $y = 1440x$.

25. How far does a sound travel underwater in 5 seconds? **7200 m**
26. In air, the equation is $y = 343x$. Does sound travel faster in air or water? Explain. **Sound travels only 1715 m in 5 seconds in air, so it travels faster underwater.**

Write each equation in standard form. Identify A, B, and C.

27. $y = -3x + 4$
28. $y = 12x$
29. $x = 4y - 5$
30. $x = 7y + 2$
31. $5y = 10x - 25$
32. $4x = 8y - 12$
33. $\frac{1}{2}x + \frac{1}{2}y = 6$
34. $\frac{1}{3}x - \frac{1}{3}y = -2$
35. $0.5x = 3$ **$x = 6; 1, 0, 6$**
36. $0.25y = 10$ **$y = 40; 0, 1, 40$**
★ 37. $\frac{5}{6}x + \frac{1}{15}y = \frac{3}{10}$ **$25x + 2y = 9; 25, 2, 9$**
★ 38. $0.25x = 0.1 + 0.2y$ **$5x - 4y = 2; 5, -4, 2$**

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation. **39–50. See pp. 107A–107H for graphs.**

39. $5x + 3y = 15$ **3, 5**
40. $2x - 6y = 12$ **6, -2**
41. $3x - 4y - 10 = 0$
42. $2x + 5y - 10 = 0$ **5, 2**
43. $y = x$ **0, 0**
44. $y = 4x - 2$ **$\frac{1}{2}, -2$**
45. $y = -2$ **none, -2**
46. $y = 4$ **none, 4**
47. $x = 8$ **8, none**
48. $x = 1$ **1, none**
★ 49. $f(x) = 4x - 1$ **$\frac{1}{4}, -1$**
★ 50. $g(x) = 0.5x - 3$ **6, -3**

CRITICAL THINKING **For Exercises 51 and 52, use $x + y = 0$, $x + y = 5$, and $x + y = -5$.**

51. Graph the equations on a coordinate plane. Compare and contrast the graphs.
52. Write a linear equation whose graph is between the graphs of $x + y = 0$ and $x + y = 5$. **Sample answer: $x + y = 2$**

Enrichment, p. 68

Greatest Common Factor

Suppose we are given a linear equation $ax + by = c$ where a, b, and c are nonzero integers, and we want to know if there exist *integers* x and y that satisfy the equation. We could try guessing a few times, but this process would be time consuming for an equation such as $588x + 432y = 72$. By using the Euclidean Algorithm, we can determine not only if such integers x and y exist, but also find them. The following example shows how this algorithm works.

Example **Find integers x and y that satisfy $588x + 432y = 72$.**

Divide the greater of the two coefficients by the lesser to get a quotient and remainder. Then, repeat the process by dividing the divisor by the remainder until you get a remainder of 0. The process can be written as follows.

$588 = 432(1) + 156$ (1)
$432 = 156(2) + 120$ (2)
$156 = 120(1) + 36$ (3)
$120 = 36(3) + 12$ (4)

Answers

16. No; x appears in a denominator.
17. No; y is inside a square root.
18. No; x has exponents other than 1.
19. No; x appears in a denominator.
20. Yes; this is a linear function because it is written as $f(x) = 6x - 19$, $m = 6$, $b = -19$.
21. No; x has an exponent other than 1.
22. No; x is inside a square root.

More About. . .

Geology

Geothermal energy from hot springs is being used for electricity in California, Italy, and Iceland.

GEOLOGY **For Exercises 53–55, use the following information.**
Suppose the temperature T (°C) below Earth's surface is given by $T(d) = 35d + 20$, where d is the depth (km).

53. Find the temperature at a depth of 2 kilometers. **90°C**

54. Find the depth if the temperature is 160°C. **4 km**

55. Graph the linear function. **See margin.**

FUND-RAISING **For Exercises 56–59, use the following information.**
The Jackson Band Boosters sell beverages for \$1.75 and candy for \$1.50 at home games. Their goal is to have total sales of \$525 for each game.

56. Write an equation that is a model for the different numbers of beverages and candy that can be sold to meet the goal. **$1.75b + 1.5c = 525$**

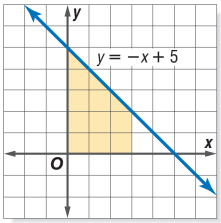
57. Graph the equation. **See margin.**

58. Does this equation represent a function? Explain.

59. If they sell 100 beverages and 200 pieces of candy, will the Band Boosters meet their goal? **no**

58. Yes; the graph passes the vertical line test.

60. **GEOMETRY** Find the area of the shaded region in the graph. (*Hint:* The area of a trapezoid is given by $A = \frac{1}{2}h(b_1 + b_2)$.) **$\frac{21}{2}$ units^2**



61. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How do linear equations relate to time spent studying?

Include the following in your answer: **See margin.**

- why only the part of the graph in the first quadrant is shown, and
- an interpretation of the graph's intercepts in terms of the amount of time Lolita spends on each subject.

Standardized Test Practice

62. Which function is linear? **B**

Ⓐ $f(x) = x^2$ Ⓑ $g(x) = 2.7$ Ⓒ $g(x) = \sqrt{x - 1}$ Ⓓ $f(x) = \sqrt{9 - x^2}$

63. What is the y-intercept of the graph of $10 - x = 2y$? **B**

Ⓐ 2 Ⓑ 5 Ⓒ 6 Ⓓ 10

Maintain Your Skills

Mixed Review

State the domain and range of each relation. Then graph the relation and determine whether it is a function. *(Lesson 2-1)* **64–65. See pp. 107A–107H for graphs.**

64. $\{(-1, 5), (1, 3), (2, -4), (4, 3)\}$

65. $\{(0, 2), (1, 3), (2, -1), (1, 0)\}$

64. D = {−1, 1, 2, 4}, R = {−4, 3, 5}; yes

65. D = {0, 1, 2}, R = {−1, 0, 2, 3}; no

Solve each inequality. *(Lesson 1-6)*

66. $-2 < 3x + 1 < 7$ **$\{x \mid -1 < x < 2\}$**

67. $|x + 4| > 2$ **$\{x \mid x < -6 \text{ or } x > -2\}$**

68. **TAX** Including a 6% sales tax, a paperback book costs \$8.43. What is the price before tax? *(Lesson 1-3)* **\$7.95**

Simplify each expression. *(Lesson 1-1)*

69. $(9s - 4) - 3(2s - 6)$ **$3s + 14$**

70. $[19 - (8 - 1)] \div 3$ **4**

Getting Ready for the Next Lesson

BASIC SKILL **Find the reciprocal of each number.**

71. 3 **$\frac{1}{3}$**

72. -4 **$-\frac{1}{4}$**

73. $\frac{1}{2}$ **2**

74. $-\frac{2}{3}$ **$-\frac{3}{2}$**

75. $-\frac{1}{5}$ **-5**

76. $3\frac{3}{4}$ **$\frac{4}{15}$**

77. 2.5 **0.4**

78. -1.25 **-0.8**

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Modeling Have students place a piece of spaghetti or a pencil on a large coordinate plane to model the graphs of these equations: $x = 4$, $x = -2$, $y = 0$, $y = -3$, $x = y$, and $x = -y$.

Getting Ready for Lesson 2-3

BASIC SKILL Lesson 2-3 presents the fact that perpendicular lines have slopes that are negative reciprocals. Exercises 71–78 should be used to determine your students' familiarity with finding reciprocals.

Assessment Options

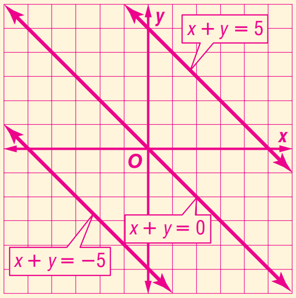
Quiz (Lessons 2-1 and 2-2) is available on p. 113 of the *Chapter 2 Resource Masters*.

Answers

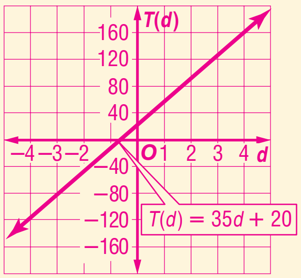
61. A linear equation can be used to relate the amounts of time that a student spends on each of two subjects if the total amount of time is fixed. Answers should include the following.

- **x and y must be nonnegative because Lolita cannot spend a negative amount of time studying a subject.**
- **The intercepts represent Lolita spending all of her time on one subject. The x-intercept represents her spending all of her time on math, and the y-intercept represents her spending all of her time on chemistry.**

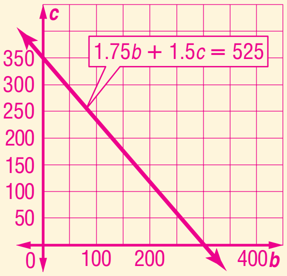
51.



55.



57.



2-3 Lesson Notes

1 Focus

5-Minute Check Transparency 2-3 Use as a quiz or review of Lesson 2-2.

Mathematical Background notes are available for this lesson on p. 54C.

Building on Prior Knowledge

In Lesson 2-2, students wrote linear equations in standard form and graphed them using intercepts. In this lesson, they apply this skill to finding the slope of a line and to graphing parallel and perpendicular lines.

How does slope apply to the steepness of roads?

Ask students:

- An engineer designed a road with a rise of 4 feet for each horizontal distance of 100 feet. What is the grade of this road? **4%**
- If a road has a grade of 3%, what is its rise for each horizontal distance of 50 feet? **1.5 ft**

2-3 Slope

What You'll Learn

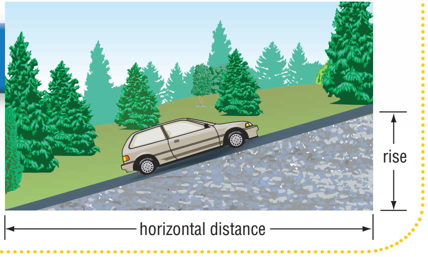
- Find and use the slope of a line.
- Graph parallel and perpendicular lines.

Vocabulary

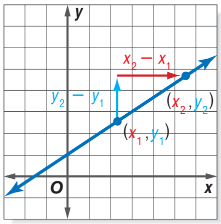
- slope
- rate of change
- family of graphs
- parent graph
- oblique

How does slope apply to the steepness of roads?

The grade of a road is a percent that measures the steepness of the road. It is found by dividing the amount the road rises by the corresponding horizontal distance.



SLOPE The **slope** of a line is the ratio of the change in y-coordinates to the corresponding change in x-coordinates. The slope measures how steep a line is.



Suppose a line passes through points (x_1, y_1) and (x_2, y_2). The change in y-coordinates is $y_2 - y_1$. The change in x-coordinates is $x_2 - x_1$.

$$\text{slope} = \frac{\text{change in } y\text{-coordinates}}{\text{change in } x\text{-coordinates}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of a line is the same, no matter what two points on the line are used.

Study Tip

Slope
The formula for slope is often remembered as *rise over run*, where the rise is the difference in y-coordinates and the run is the difference in x-coordinates.

Key Concept *Slope of a Line*

- **Words** The slope of a line is the ratio of the change in y-coordinates to the change in x-coordinates.
- **Symbols** The slope m of the line passing through (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

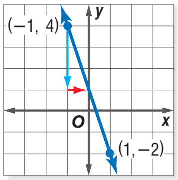
Example 1 Find Slope

Find the slope of the line that passes through (−1, 4) and (1, −2). Then graph the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2 - 4}{1 - (-1)}$ $(x_1, y_1) = (-1, 4), (x_2, y_2) = (1, -2)$

$= \frac{-6}{2}$ or −3 Simplify.



The slope of the line is −3.

Graph the two ordered pairs and draw the line. Use the slope to check your graph by selecting any point on the line. Then go down 3 units and right 1 unit or go up 3 units and left 1 unit. This point should also be on the line.

TEACHING TIP

Make sure students realize that y_2 must come from the same ordered pair as x_2.

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 69–70
- Skills Practice, p. 71
- Practice, p. 72
- Reading to Learn Mathematics, p. 73
- Enrichment, p. 74

Transparencies

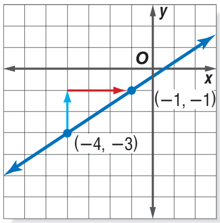
5-Minute Check Transparency 2-3
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 2 Use Slope to Graph a Line

Graph the line passing through (−4, −3) with a slope of $\frac{2}{3}$.



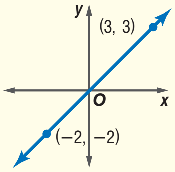
Graph the ordered pair (−4, −3). Then, according to the slope, go up 2 units and right 3 units. Plot the new point at (−1, −1). *You can also go right 3 units and then up 2 units to plot the new point.*

Draw the line containing the points.

The slope of a line tells the direction in which it rises or falls.

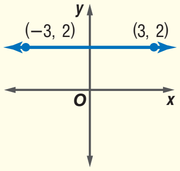
Concept Summary

If the line rises to the right, then the slope is *positive*.



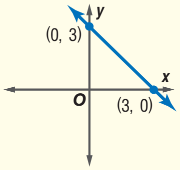
$m = \frac{3-(-2)}{3-(-2)}$
$= 1$

If the line is horizontal, then the slope is *zero*.



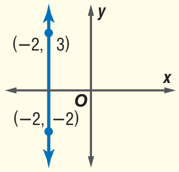
$m = \frac{2-2}{3-(-3)}$
$= 0$

If the line falls to the right, then the slope is *negative*.



$m = \frac{0-3}{3-0}$
$= -1$

If the line is vertical, then the slope is *undefined*.

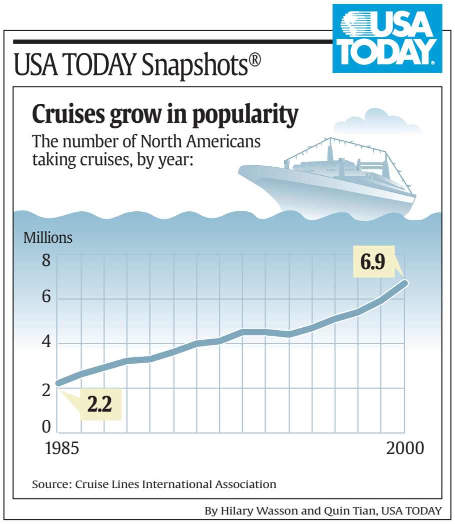


$x_1 = x_2$, so m is undefined.

Slope is often referred to as **rate of change**. It measures how much a quantity changes, on average, relative to the change in another quantity, often time.

Example 3 Rate of Change

TRAVEL Refer to the graph at the right. Find the rate of change of the number of people taking cruises from 1985 to 2000.



Source: Cruise Lines International Association

By Hilary Wasson and Quin Tian, USA TODAY

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{6.9 - 2.2}{2000 - 1985}$ Substitute.

≈ 0.31 Simplify.

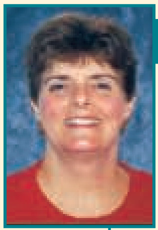
Between 1985 and 2000, the number of people taking cruises increased at an average rate of about 0.31(100,000) or 310,000 people per year.

Log on for:
- Updated data
- More activities on rate of change

www.algebra2.com/usa_today

www.algebra2.com/extra_examples

Teacher to Teacher



Judy Buchholtz **Dublin Scioto H.S., Dublin, OH**

"A match it, graph it CBL-motion detector lab is a fun way for students to 'experience' slope."

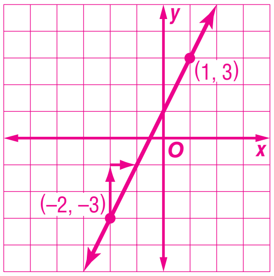
2 Teach

SLOPE

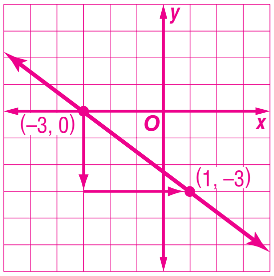
In-Class Examples

Power Point®

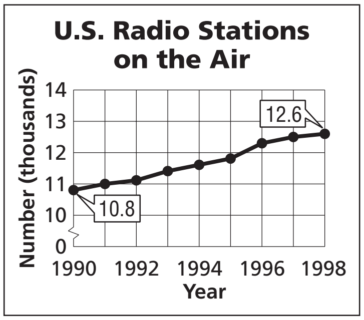
1 Find the slope of the line that passes through (1, 3) and (−2, −3). Then graph the line. **2**



2 Graph the line passing through (1, −3) with a slope of $-\frac{3}{4}$.



3 **COMMUNICATION** Refer to the graph below. Find the rate of change of the number of radio stations on the air in the United States from 1990 to 1998.



Source: *The New York Times Almanac*

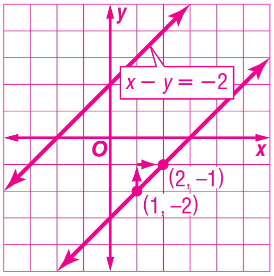
Between 1990 and 1998, the number of radio stations on the air in the United States increased at an average rate of 0.225(1000) or 225 stations per year.

PARALLEL AND PERPENDICULAR LINES

In-Class Example

Power Point®

4 Graph the line through (1, −2) that is parallel to the line with equation $x - y = -2$.



Answer

1. $y = 3x$; The graphs are parallel lines, but they have different y-intercepts.

Study Tip

Horizontal Lines

All horizontal lines are parallel because they all have a slope of 0.

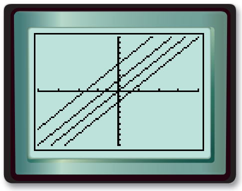
PARALLEL AND PERPENDICULAR LINES

A **family of graphs** is a group of graphs that displays one or more similar characteristics. The **parent graph** is the simplest of the graphs in a family. A graphing calculator can be used to graph several graphs in a family on the same screen.

Graphing Calculator Investigation

Lines with the Same Slope

The calculator screen shows the graphs of $y = 3x$, $y = 3x + 2$, $y = 3x - 2$, and $y = 3x + 5$.



[−4, 4] scl: 1 by [−10, 10] scl: 1

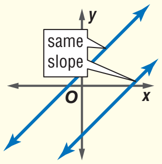
Think and Discuss 1. See margin.

1. Identify the parent function and describe the family of graphs. What is similar about the graphs? What is different about the graphs?
2. Find the slope of each line. 3
3. Write another function that has the same characteristics as this family of graphs. Check by graphing. Sample answer: $y = 3x - 4$

In the Investigation, you saw that lines that have the same slope are parallel. These and other similar examples suggest the following rule.

Key Concept — *Parallel Lines*

- **Words** In a plane, nonvertical lines with the same slope are parallel. All vertical lines are parallel.
- **Model**



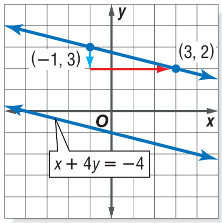
Example 4 Parallel Lines

Graph the line through (−1, 3) that is parallel to the line with equation $x + 4y = -4$.

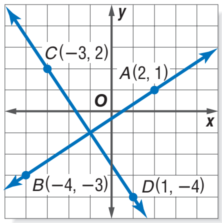
The x-intercept is −4, and the y-intercept is −1. Use the intercepts to graph $x + 4y = -4$.

The line falls 1 unit for every 4 units it moves to the right, so the slope is $-\frac{1}{4}$.

Now use the slope and the point at (−1, 3) to graph the line parallel to the graph of $x + 4y = -4$.



The figure at the right shows the graphs of two lines that are perpendicular. You know that parallel lines have the same slope. What is the relationship between the slopes of two perpendicular lines?



slope of line AB

$$\frac{-3-1}{-4-2} = \frac{-4}{-6} \text{ or } \frac{2}{3}$$

slope of line CD

$$\frac{-4-2}{1-(-3)} = \frac{-6}{4} \text{ or } -\frac{3}{2}$$

The slopes are opposite reciprocals of each other. This relationship is true in general. When you multiply the slopes of two perpendicular lines, the product is always −1.

Graphing Calculator Investigation

Lines with the Same Slope Point out that the simplest of the graphs in a family is often the one that passes through the origin, where the values of x and y are both zero. Suggest that students substitute 0 for x in each equation to find a point that will help them identify which graph goes with each equation.

Study Tip

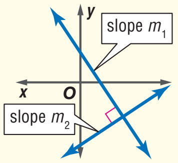
Reading Math
An *oblique* line is a line that is neither horizontal nor vertical.

Key Concept — Perpendicular Lines

- **Words** In a plane, two oblique lines are perpendicular if and only if the product of their slopes is −1.
- **Symbols** Suppose m_1 and m_2 are the slopes of two oblique lines. Then the lines are perpendicular if and only if

$$m_1 m_2 = -1, \text{ or } m_2 = -\frac{1}{m_2}.$$

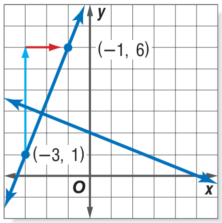
- **Model**



Any vertical line is perpendicular to any horizontal line.

Example 5 Perpendicular Line

Graph the line through (−3, 1) that is perpendicular to the line with equation $2x + 5y = 10$.



The x-intercept is 5, and the y-intercept is 2. Use the intercepts to graph $2x + 5y = 10$.

The line falls 2 units for every 5 units it moves to the right, so the slope is $-\frac{2}{5}$. The slope of the perpendicular line is the opposite reciprocal of $-\frac{2}{5}$, or $\frac{5}{2}$.

Start at (−3, 1) and go up 5 units and right 2 units. Use this point and (−3, 1) to graph the line.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an equation of a line with slope 0. **Sample answer: $y = 1$**
2. **Decide** whether the statement below is *sometimes*, *always*, or *never* true. Explain.
The slope of a line is a real number.
3. **FIND THE ERROR** Mark and Luisa are finding the slope of the line through (2, 4) and (−1, 5).

Mark	Luisa
$m = \frac{5-4}{2-(-1)}$ or $\frac{1}{3}$	$m = \frac{4-5}{2-(-1)}$ or $-\frac{1}{3}$

Who is correct? Explain your reasoning. **See margin.**

2. Sometimes; the slope of a vertical line is undefined.

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–6	1
7, 8	2
9	4
10, 11	5
12–14	3

Find the slope of the line that passes through each pair of points.

4. (1, 1), (3, 1) **0**
5. (−1, 0), (3, −2) **$-\frac{1}{2}$**
6. (3, 4), (1, 2) **1**

Graph the line passing through the given point with the given slope.

7. (2, −1), −3
8. (−3, −4), $\frac{3}{2}$ **7–8. See pp. 107A–107H.**

Graph the line that satisfies each set of conditions. 9–11. See pp. 107A–107H.

9. passes through (0, 3), parallel to graph of $6y - 10x = 30$
10. passes through (4, −2), perpendicular to graph of $3x - 2y = 6$
11. passes through (−1, 5), perpendicular to graph of $5x - 3y - 3 = 0$

DAILY INTERVENTION

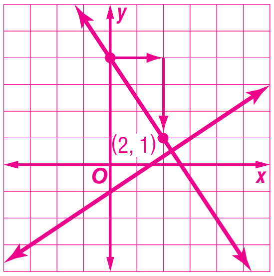
Differentiated Instruction

Naturalist Ask students to apply what they have learned about slope, grade, or steepness as they describe or sketch what they see around them in nature, such as the shapes of hills, branches of trees, and roof lines of houses.

In-Class Example

Power Point®

5 Graph the line through (2, 1) that is perpendicular to the line with equation $2x - 3y = 3$.



3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- draw sample graphs to compare and contrast lines with positive slope, negative slope, zero slope, and a slope that is undefined.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Point out that when finding a slope, if you use $y_1 - y_2$ as the numerator, you must use $x_1 - x_2$ as the denominator. To find the slope, you can move from point A to point B, or from point B to point A, but you must move in a consistent direction for both the rise and the run.

Answer

3. Luisa; Mark did not subtract in a consistent manner when using the slope formula. If $y_2 = 5$ and $y_1 = 4$, then x_2 must be −1 and x_1 must be 2, not vice versa.

About the Exercises…

Organization by Objective

- **Slope:** 15–42
- **Parallel and Perpendicular Lines:** 43–52

Odd/Even Assignments

Exercises 15–36 and 43–50 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 56 and 57 require a graphing calculator.

Assignment Guide

Basic: 15–21 odd, 31–35 odd, 37–39, 43–47 odd, 51–55, 58–75

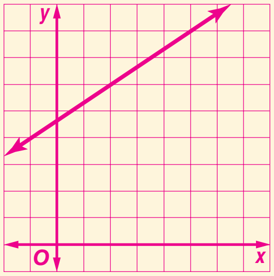
Average: 15–35 odd, 37–39, 43–51 odd, 52–55, 58–75 (optional: 56, 57)

Advanced: 16–36 even, 40–42, 44–50 even, 52–69 (optional: 70–75)

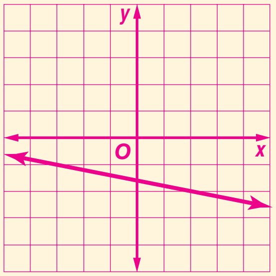
All: Practice Quiz 1 (1–5)

Answers

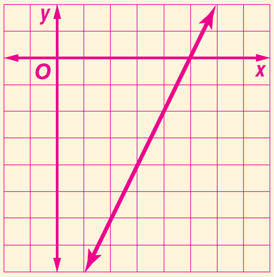
31.



32.



33.



Application

WEATHER For Exercises 12–14, use the table that shows the temperatures at different times on March 23, 2002.

Time	8:00 A.M.	10:00 A.M.	12:00 P.M.	2:00 P.M.	4:00 P.M.
Temp (°F)	36	47	55	58	60

12. What was the average rate of change of the temperature from 8:00 A.M. to 10:00 A.M.? **5.5°/h**

13. What was the average rate of change of the temperature from 12:00 P.M. to 4:00 P.M.? **1.25°/h**

14. During what 2-hour period was the average rate of change of the temperature the least? **2:00 P.M.–4:00 P.M.**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–30	1
31–36	2
37–42	3
43–52	4, 5

Extra Practice

See page 830.

Find the slope of the line that passes through each pair of points. **15.** $-\frac{5}{2}$

15. (6, 1), (8, −4)

16. (6, 8), (5, −5) **13**

17. (−6, −5), (4, 1) $\frac{3}{5}$

18. (2, −7), (4, 1) **4**

19. (7, 8), (1, 8) **0**

20. (−2, −3), (0, −5) **−1**

21. (2.5, 3), (1, −9) **8**

22. (4, −1.5), (4, 4.5) **undefined**

★**23.** $\left(\frac{1}{2}, -\frac{1}{3}\right), \left(\frac{1}{4}, \frac{2}{3}\right)$ **−4**

★**24.** $\left(\frac{1}{2}, \frac{2}{3}\right), \left(\frac{5}{6}, \frac{1}{4}\right)$ $-\frac{5}{4}$

★**25.** $(a, 2), (a, -2)$ **undefined**

★**26.** $(3, b), (-5, b)$ **0**

★**27.** Determine the value of r so that the line through $(6, r)$ and $(9, 2)$ has slope $\frac{1}{3}$. **1**

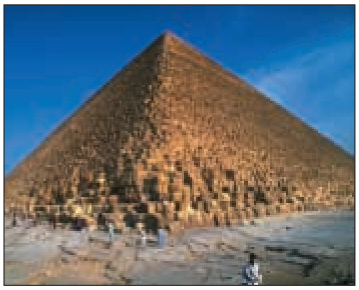
★**28.** Determine the value of r so that the line through $(5, r)$ and $(2, 3)$ has slope 2. **9**

ANCIENT CULTURES Mayan Indians of Mexico and Central America built pyramids that were used as their temples. Ancient Egyptians built pyramids to use as tombs for the pharohs. Estimate the slope that a face of each pyramid makes with its base.

★**29.**

The Pyramid of the Sun in Teotihuacán, Mexico, measures about 700 feet on each side of its square base and is about 210 feet high. **about 0.6**

★**30.**



The Great Pyramid in Egypt measures 756 feet on each side of its square base and was originally 481 feet high. **about 1.3**

Graph the line passing through the given point with the given slope.

31–36. See margin.

31. $(2, 6), m = \frac{2}{3}$

32. $(-3, -1), m = -\frac{1}{5}$

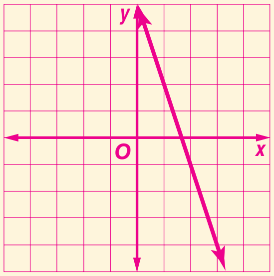
33. $(3, -4), m = 2$

34. $(1, 2), m = -3$

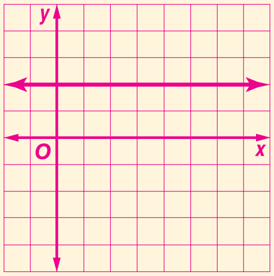
35. $(6, 2), m = 0$

36. $(-2, -3)$, undefined

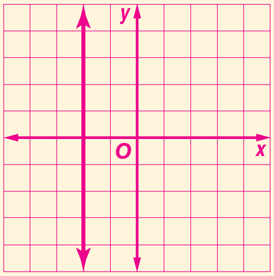
34.



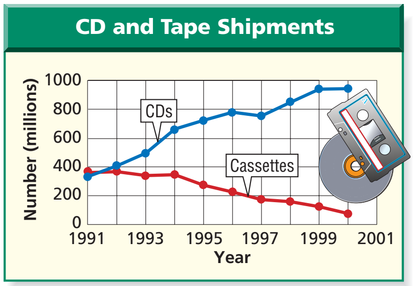
35.



36.



ENTERTAINMENT For Exercises 37–39, refer to the graph that shows the number of CDs and cassette tapes shipped by manufacturers to retailers in recent years.



Source: Recording Industry Association of America

37. Find the average rate of change of the number of CDs shipped from 1991 to 2000. **37. about 68 million per year**

38. Find the average rate of change of the number of cassette tapes shipped from 1991 to 2000. **38. about −32 million per year**

39. Interpret the sign of your answer to Exercise 38.
The number of cassette tapes shipped has been decreasing.

TRAVEL For Exercises 40–42, use the following information.
Mr. and Mrs. Wellman are taking their daughter to college. The table shows their distance from home after various amounts of time.

Time (h)	Distance (mi)
0	0
1	55
2	110
3	165
4	165
5	225

40. Find the average rate of change of their distance from home between 1 and 3 hours after leaving home. **55 mph**

41. Find the average rate of change of their distance from home between 0 and 5 hours after leaving home. **45 mph**

42. What is another word for *rate of change* in this situation? **speed or velocity**

Graph the line that satisfies each set of conditions. **43–50. See pp. 107A–107H.**

43. passes through (−2, 2), parallel to a line whose slope is −1

44. passes through (−4, 1), perpendicular to a line whose slope is $-\frac{3}{2}$

45. passes through (3, 3), perpendicular to graph of $y = 3$

46. passes through (2, −5), parallel to graph of $x = 4$

47. passes through (2, −1), parallel to graph of $2x + 3y = 6$

48. passes through origin, parallel to graph of $x + y = 10$

★**49.** perpendicular to graph of $3x - 2y = 24$, intersects that graph at its x-intercept

★**50.** perpendicular to graph of $2x + 5y = 10$, intersects that graph at its y-intercept

51. GEOMETRY Determine whether quadrilateral *ABCD* with vertices *A*(−2, −1), *B*(1, 1), *C*(3, −2), and *D*(0, −4) is a rectangle. Explain. **Yes; slopes show that adjacent sides are perpendicular.**

52. CRITICAL THINKING If the graph of the equation $ax + 3y = 9$ is perpendicular to the graph of the equation $3x + y = -4$, find the value of a. **−1**

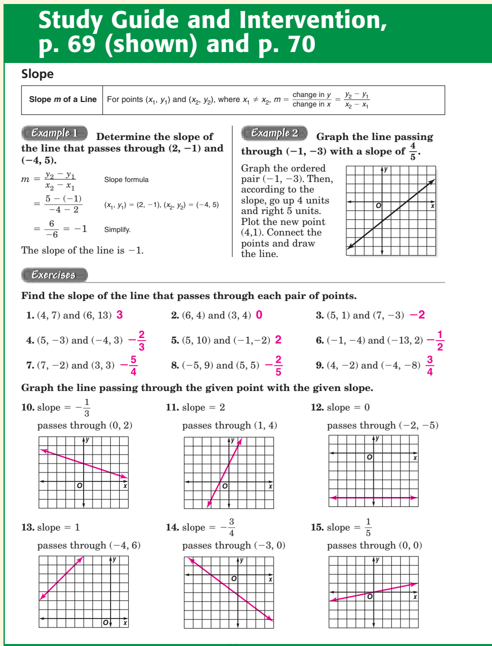
53. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson. **See pp. 107A–107H.**

How does slope apply to the steepness of roads?

Include the following in your answer:

- a few sentences explaining the relationship between the grade of a road and the slope of a line, and
- a graph of $y = 0.08x$, which corresponds to a grade of 8%. (A road with a grade of 6% to 8% is considered to be fairly steep. The scales on your x- and y-axes should be the same.)

www.algebra2.com/self_check_quiz



Study Guide and Intervention, p. 69 (shown) and p. 70

Slope

Slope m of a Line	For points (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$, $m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example 1 **Determine the slope of the line that passes through (2, −1) and (−4, 5).**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{5 - (-1)}{-4 - 2}$ $(x_1, y_1) = (2, -1), (x_2, y_2) = (-4, 5)$

$= \frac{6}{-6} = -1$ Simplify.

The slope of the line is −1.

Example 2 **Graph the line passing through (−1, −3) with a slope of $\frac{4}{5}$.**

Graph the ordered pair (−1, −3). Then, according to the slope, go up 4 units and right 5 units. Plot the new point (4,1). Connect the points and draw the line.

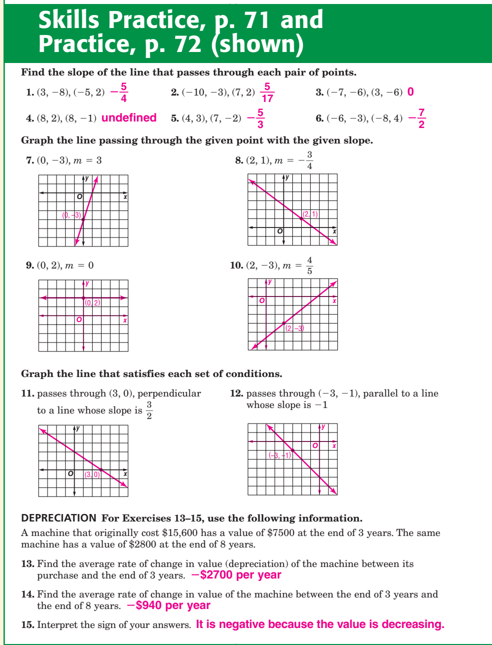
Exercises

Find the slope of the line that passes through each pair of points.

1. (4, 7) and (6, 13) **3**
2. (6, 4) and (3, 4) **0**
3. (5, 1) and (7, −3) **−2**
4. (5, −3) and (−4, 3) $-\frac{2}{3}$
5. (5, 10) and (−1,−2) **2**
6. (−1, −4) and (−13, 2) $-\frac{1}{2}$
7. (7, −2) and (3, 3) $-\frac{5}{4}$
8. (−5, 9) and (5, 5) $-\frac{2}{5}$
9. (4, −2) and (−4, −8) $\frac{3}{4}$

Graph the line passing through the given point with the given slope.

10. slope $= -\frac{1}{3}$ passes through (0, 2)
11. slope = 2 passes through (1, 4)
12. slope = 0 passes through (−2, −5)
13. slope = 1 passes through (−4, 6)
14. slope $= -\frac{3}{4}$ passes through (−3, 0)
15. slope $= \frac{1}{5}$ passes through (0, 0)



Skills Practice, p. 71 and Practice, p. 72 (shown)

Find the slope of the line that passes through each pair of points.

1. (3, −8), (−5, 2) $-\frac{5}{4}$
2. (−10, −3), (7, 2) $\frac{5}{17}$
3. (−7, −6), (3, −6) **0**
4. (8, 2), (8, −1) **undefined**
5. (4, 3), (7, −2) $-\frac{5}{3}$
6. (−6, −3), (−8, 4) $-\frac{7}{2}$

Graph the line passing through the given point with the given slope.

7. (0, −3), $m = 3$
8. (2, 1), $m = -\frac{3}{4}$
9. (0, 2), $m = 0$
10. (2, −3), $m = \frac{4}{5}$

Graph the line that satisfies each set of conditions.

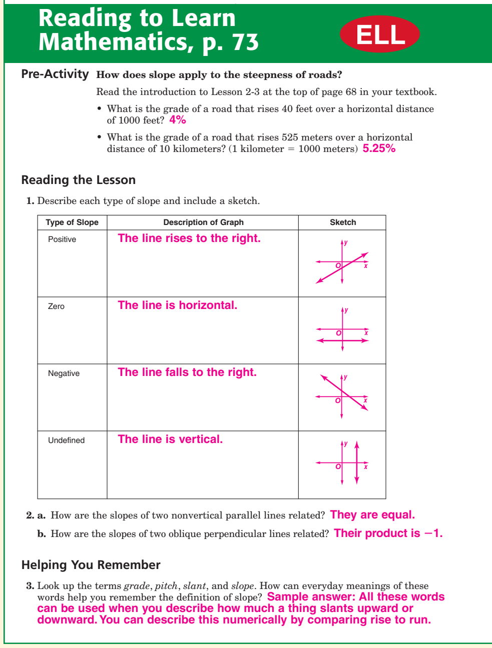
11. passes through (3, 0), perpendicular to a line whose slope is $\frac{3}{2}$
12. passes through (−3, −1), parallel to a line whose slope is −1

DEPRECIATION **For Exercises 13–15, use the following information.**
A machine that originally cost $15,600 has a value of $7500 at the end of 3 years. The same machine has a value of $2800 at the end of 8 years.

13. Find the average rate of change in value (depreciation) of the machine between its purchase and the end of 3 years. **−$2700 per year**

14. Find the average rate of change in value of the machine between the end of 3 years and the end of 8 years. **−$940 per year**

15. Interpret the sign of your answers. **It is negative because the value is decreasing.**



Reading to Learn Mathematics, p. 73

ELL

Pre-Activity **How does slope apply to the steepness of roads?**

Read the introduction to Lesson 2-3 at the top of page 68 in your textbook.

- What is the grade of a road that rises 40 feet over a horizontal distance of 1000 feet? **4%**
- What is the grade of a road that rises 525 meters over a horizontal distance of 10 kilometers? (1 kilometer = 1000 meters) **5.25%**

Reading the Lesson

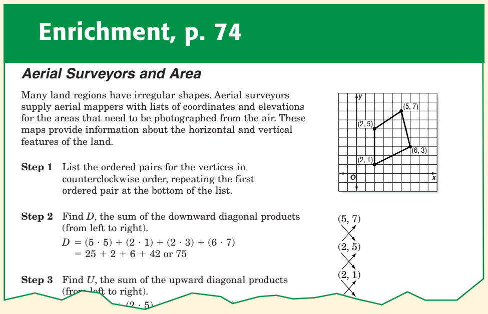
1. Describe each type of slope and include a sketch.

Type of Slope	Description of Graph	Sketch
Positive	The line rises to the right.	
Zero	The line is horizontal.	
Negative	The line falls to the right.	
Undefined	The line is vertical.	

2. a. How are the slopes of two nonvertical parallel lines related? **They are equal.**
b. How are the slopes of two oblique perpendicular lines related? **Their product is −1.**

Helping You Remember

3. Look up the terms *grade*, *pitch*, *slant*, and *slope*. How can everyday meanings of these words help you remember the definition of slope? **Sample answer: All these words can be used when you describe how much a thing slants upward or downward. You can describe this numerically by comparing rise to run.**



Enrichment, p. 74

Aerial Surveyors and Area

Many land regions have irregular shapes. Aerial surveyors supply aerial mappers with lists of coordinates and elevations for the areas that need to be photographed from the air. These maps provide information about the horizontal and vertical features of the land.

Step 1 List the ordered pairs for the vertices in counterclockwise order, repeating the first ordered pair at the bottom of the list.

Step 2 Find D, the sum of the downward diagonal products (from left to right).

$D = (5 \cdot 5) + (2 \cdot 1) + (2 \cdot 3) + (6 \cdot 7)$
$= 25 + 2 + 6 + 42$ or 75

Step 3 Find U, the sum of the upward diagonal products (from left to right).

4 Assess

Open-Ended Assessment

Modeling Have students use a piece of spaghetti or a pencil on a coordinate plane to model lines with slopes of 1, −1, and 0, as well as a line with undefined slope. Then have them use a second piece of spaghetti or a second pencil to model a pair of parallel lines and then a pair of perpendicular lines.

Tips for New Teachers

Intervention If there is any doubt whether your students thoroughly understand the concept of slope, consider spending an extra day on this lesson. Use the Extra Practice on p. 830, the Study Guide and Intervention Masters, or the Practice Masters to reinforce this concept.

Getting Ready for Lesson 2-4

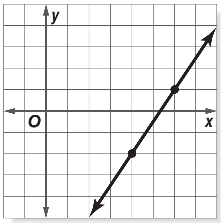
PREREQUISITE SKILL Lesson 2-4 presents writing the equation of a line given the slope and a point on the line. Exercises 70–75 should be used to determine your students' familiarity with solving equations for a given variable.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 2-1 through 2-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Standardized Test Practice

54. What is the slope of the line shown in the graph at the right? **D**



Ⓐ $-\frac{3}{2}$ Ⓑ $-\frac{2}{3}$ Ⓒ $\frac{2}{3}$ Ⓓ $\frac{3}{2}$

55. What is the slope of a line perpendicular to a line with slope $-\frac{1}{2}$? **D**

Ⓐ −2 Ⓑ $-\frac{1}{2}$ Ⓒ $\frac{1}{2}$ Ⓓ 2

Graphing Calculator

FAMILY OF GRAPHS **Use a graphing calculator to investigate each family of graphs. Explain how changing the slope affects the graph of the line.**

56. $y = 2x + 3, y = 4x + 3, y = 8x + 3, y = x + 3$ **56–57. See margin.**

57. $y = -3x + 1, y = -x + 1, y = -5x + 1, y = -7x + 1$

Maintain Your Skills

Mixed Review

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation. *(Lesson 2-2)* **58–60. See pp. 107A–107H for graphs.**

58. $-2x + 5y = 20$ **−10, 4** 59. $4x - 3y + 8 = 0$ **$-2, \frac{8}{3}$** 60. $y = 7x$ **0, 0**

Find each value if $f(x) = 3x - 4$. *(Lesson 2-1)*

61. $f(-1)$ **−7** 62. $f(3)$ **5** 63. $f\left(\frac{1}{2}\right)$ **$-\frac{5}{2}$** 64. $f(a)$ **3a − 4**

Solve each inequality. *(Lessons 1-5 and 1-6)*

65. $5 < 2x + 7 < 13$ **$\{x \mid -1 < x < 3\}$** 66. $2z + 5 \geq 1475$ **$\{z \mid z \geq 735\}$**

67. **SCHOOL** A test has multiple-choice questions worth 4 points each and true-false questions worth 3 points each. Marco answers 14 multiple-choice questions correctly. How many true-false questions must he answer correctly to get at least 80 points total? *(Lesson 1-5)* **at least 8**

Simplify. *(Lessons 1-1 and 1-2)*

68. $\frac{1}{3}(15a + 9b) - \frac{1}{7}(28b - 84a)$ **17a − b** 69. $3 + (21 \div 7) \times 8 \div 4$ **9**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation for y.**
*(To review **solving equations**, see Lesson 1-2.)* **70–75. See margin.**

70. $x + y = 9$ 71. $4x + y = 2$ 72. $-3x - y + 7 = 0$

73. $5x - 2y - 1 = 0$ 74. $3x - 5y + 4 = 0$ 75. $2x + 3y - 11 = 0$

Practice Quiz 1

Lessons 2-1 through 2-3

1. State the domain and range of the relation {(2, 5), (−3, 2), (2, 1), (−7, 4), (0, −2)}. *(Lesson 2-1)* **D = {−7, −3, 0, 2}, R = {−2, 1, 2, 4, 5}**
2. Find the value of $f(15)$ if $f(x) = 100x - 5x^2$. *(Lesson 2-1)* **375**
3. Write $y = -6x + 4$ in standard form. *(Lesson 2-2)* **$6x + y = 4$**
4. Find the x-intercept and the y-intercept of the graph of $3x + 5y = 30$. Then graph the equation. *(Lesson 2-2)* **10, 6; See margin for graph.**
5. Graph the line that goes through (4, −3) and is parallel to the line whose equation is $2x + 5y = 10$. *(Lesson 2-3)* **See margin.**

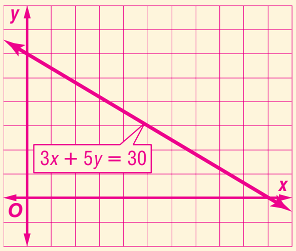
Answers

56. The graphs have the same y-intercept. As the slopes increase, the lines get steeper.
57. The graphs have the same y-intercept. As the slopes become more negative, the lines get steeper.

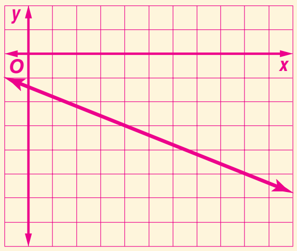
70. $y = 9 - x$
71. $y = -4x + 2$
72. $y = -3x + 7$
73. $y = \frac{5}{2}x - \frac{1}{2}$
74. $y = \frac{3}{5}x + \frac{4}{5}$
75. $y = -\frac{2}{3}x + \frac{11}{3}$

Answers (Practice Quiz 1)

4.



5.



2-4 Writing Linear Equations

What You'll Learn

- Write an equation of a line given the slope and a point on the line.
- Write an equation of a line parallel or perpendicular to a given line.

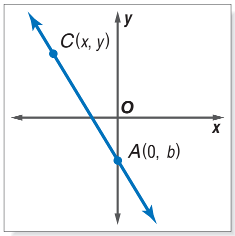
Vocabulary

- slope-intercept form
- point-slope form

How do linear equations apply to business?

When a company manufactures a product, they must consider two types of cost. There is the *fixed cost*, which they must pay no matter how many of the product they produce, and there is *variable cost*, which depends on how many of the product they produce. In some cases, the total cost can be found using a linear equation such as $y = 5400 + 1.37x$.

FORMS OF EQUATIONS Consider the graph at the right. The line passes through $A(0, b)$ and $C(x, y)$. Notice that b is the y-intercept of $\overleftrightarrow{AC}$. You can use these two points to find the slope of $\overleftrightarrow{AC}$. Substitute the coordinates of points A and C into the slope formula.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{y - b}{x - 0}$ $(x_1, y_1) = (0, b)$, $(x_2, y_2) = (x, y)$

$m = \frac{y - b}{x}$ Simplify.

Now solve the equation for y.

$mx = y - b$ Multiply each side by x.

$mx + b = y$ Add b to each side.

$y = mx + b$ Symmetric Property of Equality

When an equation is written in this form, it is in **slope-intercept form**.

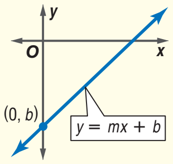
Study Tip

Slope-intercept Form

The equation of a vertical line cannot be written in slope-intercept form because its slope is undefined.

Key Concept — Slope-Intercept Form of a Linear Equation

- **Words** The slope-intercept form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.
- **Symbols** $y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$
- **Model**



If you are given the slope and y-intercept of a line, you can find an equation of the line by substituting the values of m and b into the slope-intercept form. For example, if you know that the slope of a line is -3 and the y-intercept is 4, the equation of the line is $y = -3x + 4$, or, in standard form, $3x + y = 4$.

You can also use the slope-intercept form to find an equation of a line if you know the slope and the coordinates of any point on the line.

2-4 Lesson Notes

1 Focus

5-Minute Check Transparency 2-4 Use as a quiz or review of Lesson 2-3.

Mathematical Background notes are available for this lesson on p. 54D.

Building on Prior Knowledge

In Lesson 2-3, students studied slope and the graphs of parallel and perpendicular lines. In this lesson, students apply these ideas to writing the equations of lines.

How do linear equations apply to business?

Ask students:

- What are some examples of costs that might be included in the fixed cost? **utilities, salaries, rent**
- What are some examples of costs that might be included in the variable cost? **raw materials, packaging, shipping**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 75–76
- Skills Practice, p. 77
- Practice, p. 78
- Reading to Learn Mathematics, p. 79
- Enrichment, p. 80
- Assessment, pp. 113, 115

Graphing Calculator and Spreadsheet Masters, p. 30

School-to-Career Masters, p. 3

Transparencies

5-Minute Check Transparency 2-4
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

FORMS OF EQUATIONS

In-Class Examples

Power Point®

1 Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{5}$ and passes through (5, −2). $y = -\frac{3}{5}x + 1$

Teaching Tip Make sure that students understand that the letter m is always used for slope, and b for the y-intercept, in $y = mx + b$.

2 What is an equation of the line through (2, −3) and (−3, 7)? **D**

A $y = -2x - 1$

B $y = -\frac{1}{2}x + 1$

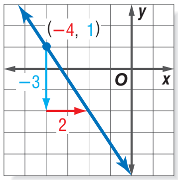
C $y = \frac{1}{2}x + 1$

D $y = -2x + 1$

Example 1 Write an Equation Given Slope and a Point

Write an equation in slope-intercept form for the line that has a slope of $-\frac{3}{2}$ and passes through (−4, 1).

Substitute for m, x, and y in the slope-intercept form.



$y = mx + b$	Slope-intercept form
$1 = \left(-\frac{3}{2}\right)(-4) + b$	$(x, y) = (-4, 1)$, $m = -\frac{3}{2}$
$1 = 6 + b$	Simplify.
$-5 = b$	Subtract 6 from each side.

The y-intercept is −5. So, the equation in slope-intercept form is $y = -\frac{3}{2}x - 5$.

If you are given the coordinates of two points on a line, you can use the **point-slope form** to find an equation of the line that passes through them.

Key Concept Point-Slope Form of a Linear Equation

- **Words** The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.
- **Symbols**

slope

$$y - y_1 = m(x - x_1)$$

coordinates of point on line

Standardized Test Practice

Example 2 Write an Equation Given Two Points

Multiple-Choice Test Item

What is an equation of the line through (−1, 4) and (−4, 5)?

Ⓐ $y = -\frac{1}{3}x + \frac{11}{3}$ Ⓑ $y = \frac{1}{3}x + \frac{13}{3}$ Ⓒ $y = -\frac{1}{3}x + \frac{13}{3}$ Ⓓ $y = -3x + 1$

Read the Test Item

You are given the coordinates of two points on the line. Notice that the answer choices are in slope-intercept form.

Solve the Test Item

- First, find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Slope formula
$= \frac{5 - 4}{-4 - (-1)}$	$(x_1, y_1) = (-1, 4)$, $(x_2, y_2) = (-4, 5)$
$= \frac{1}{-3}$ or $-\frac{1}{3}$	Simplify.

The slope is $-\frac{1}{3}$. That eliminates choices B and D.

- Then use the point-slope form to find an equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = -\frac{1}{3}[x - (-1)]$	$m = -\frac{1}{3}$; you can use either point for (x_1, y_1).
$y - 4 = -\frac{1}{3}x - \frac{1}{3}$	Distributive Property
$y = -\frac{1}{3}x + \frac{11}{3}$	The answer is A.

Test-Taking Tip

To check your answer, substitute each ordered pair into your answer. Each should satisfy the equation.

Standardized Test Practice

Example 2 Point out that finding the slope eliminated two of the choices. Emphasize that eliminating some of the answer choices helps you to use your time efficiently when taking a timed test.

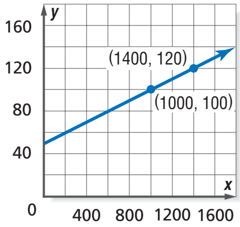
When changes in real-world situations occur at a linear rate, a linear equation can be used as a model for describing the situation.

Example 3 Write an Equation for a Real-World Situation

SALES **As a salesperson, Eric Fu is paid a daily salary plus commission. When his sales are $1000, he makes $100. When his sales are $1400, he makes $120.**

a. Write a linear equation to model this situation.

Let x be his sales and let y be the amount of money he makes. Use the points (1000, 100) and (1400, 120) to make a graph to represent the situation.



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{120 - 100}{1400 - 1000}$ $(x_1, y_1) = (1000, 100)$, $(x_2, y_2) = (1400, 120)$

$= 0.05$ Simplify.

Now use the slope and either of the given points with the point-slope form to write the equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 100 = 0.05(x - 1000)$ $m = 0.05$, $(x_1, y_1) = (1000, 100)$

$y - 100 = 0.05x - 50$ Distributive Property

$y = 0.05x + 50$ Add 100 to each side.

The slope-intercept form of the equation is $y = 0.05x + 50$.

b. What are Mr. Fu's daily salary and commission rate?

The y-intercept of the line is 50. The y-intercept represents the money Eric would make if he had no sales. In other words, $50 is his daily salary.

The slope of the line is 0.05. Since the slope is the coefficient of x, which is his sales, he makes 5% commission.

c. How much would he make in a day if Mr. Fu's sales were $2000?

Find the value of y when $x = 2000$.

$y = 0.05x + 50$ Use the equation you found in part **a**.

$= 0.05(2000) + 50$ Replace x with 2000.

$= 100 + 50$ or 150 Simplify.

Mr. Fu would make $150 if his sales were $2000.

Study Tip

Alternative Method

You could also find Mr. Fu's salary in part **c** by extending the graph. Then find the y value when x is 2000.

PARALLEL AND PERPENDICULAR LINES The slope-intercept and point-slope forms can be used to find equations of lines that are parallel or perpendicular to given lines.

Example 4 Write an Equation of a Perpendicular Line

Write an equation for the line that passes through (−4, 3) and is perpendicular to the line whose equation is $y = -4x - 1$.

The slope of the given line is −4. Since the slopes of perpendicular lines are opposite reciprocals, the slope of the perpendicular line is $\frac{1}{4}$.

(continued on the next page)

www.algebra2.com/extra_examples

In-Class Example Power Point®

3 **SALES** As a part-time salesperson, Jean Stock is paid a daily salary plus commission. When her sales are $100, she makes $58. When her sales are $300, she make $78.

a. Write a linear equation to model this situation. $y = 0.1x + 48$

b. What are Ms. Stock's daily salary and commission rate? **$48; 10%**

c. How much would Jean make in a day if her sales were $500? **$98**

Teaching Tip Point out that when the units differ on the two axes, you cannot estimate the slope of the graphed line by comparing it to the slope of $x = y$ as the 45-degree line with a slope of 1.

PARALLEL AND PERPENDICULAR LINES

In-Class Example Power Point®

4 Write an equation for the line that passes through (3, −2) and is perpendicular to the line whose equation is $y = -5x + 1$. $y = \frac{1}{5}x - \frac{13}{5}$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- add the slope-intercept and point-slope forms of a linear equation to their notebooks, with examples of both types of equations.
- add items from Example 2 to their list of test-taking tips, which they can review as they prepare for standardized tests.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Forms of Equations:** 13–34
- **Parallel and Perpendicular Lines:** 35–38

Odd/Even Assignments

Exercises 13–40 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13, 15, 19–35 odd, 39, 41–43, 45, 49–54, 57–67

Average: 13–39 odd, 41–43, 45–54, 57–67 (optional: 55, 56)

Advanced: 14–40 even, 44, 46–63 (optional: 64–67)

Answer

3. Solve the equation for y to get $y = \frac{3}{5}x - \frac{2}{5}$. The slope of this line is $\frac{3}{5}$. The slope of a parallel line is the same.

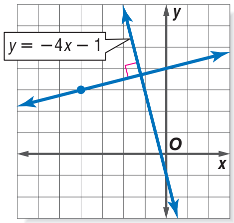
TEACHING TIP

You could also use slope-intercept form.

Use the point-slope form and the ordered pair $(-4, 3)$ to write the equation.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 3 = \frac{1}{4}[x - (-4)]$	$(x, y) = (-4, 3)$, $m = \frac{1}{4}$
$y - 3 = \frac{1}{4}x + 1$	Distributive Property
$y = \frac{1}{4}x + 4$	Add 3 to each side.

An equation of the line is $y = \frac{1}{4}x + 4$.



Check for Understanding

Concept Check

1. **OPEN ENDED** Write an equation of a line in slope-intercept form.
2. **Identify** the slope and y-intercept of the line with equation $y = 6x$. **6, 0**
3. **Explain** how to find the slope of a line parallel to the graph of $3x - 5y = 2$. **See margin.**

1. Sample answer: $y = 3x + 2$

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–7	1
8, 9, 11, 12	2, 3
10	4

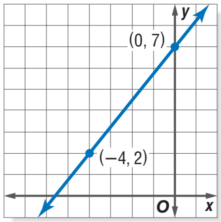
State the slope and y-intercept of the graph of each equation.

4. $y = 2x - 5$ **2, −5**
5. $3x + 2y - 10 = 0$ $-\frac{3}{2}, 5$

Write an equation in slope-intercept form for the line that satisfies each set of conditions. **6.** $y = 0.5x + 1$ **7.** $y = -\frac{3}{4}x + 2$ **8.** $y = -\frac{5}{2}x + 16$

6. slope 0.5, passes through $(6, 4)$
7. slope $-\frac{3}{4}$, passes through $\left(2, \frac{1}{2}\right)$
8. passes through $(6, 1)$ and $(8, -4)$
9. passes through $(-3, 5)$ and $(2, 2)$
10. passes through $(0, -2)$, perpendicular to the graph of $y = x - 2$ $y = -x - 2$
11. Write an equation in slope-intercept form for the graph at the right. $y = \frac{5}{4}x + 7$

9. $y = -\frac{3}{5}x + \frac{16}{5}$



Standardized Test Practice

12. What is an equation of the line through $(2, -4)$ and $(-3, -1)$? **B**

Ⓐ $y = -\frac{3}{5}x + \frac{26}{5}$
Ⓑ $y = -\frac{3}{5}x - \frac{14}{5}$
Ⓒ $y = \frac{3}{5}x - \frac{26}{5}$
Ⓓ $y = \frac{3}{5}x + \frac{14}{5}$

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
13–18, 21–28	1
19, 20, 29–34, 39, 40	2, 3
35–38	4
41–52	1–3

Extra Practice

See page 831.

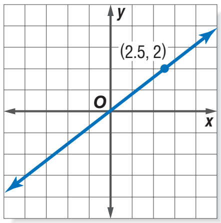
16. $-\frac{3}{5}, 6$

State the slope and y-intercept of the graph of each equation.

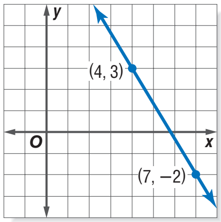
13. $y = -\frac{2}{3}x - 4$ $-\frac{2}{3}, -4$
14. $y = \frac{3}{4}x$ $\frac{3}{4}, 0$
15. $2x - 4y = 10$ $\frac{1}{2}, -\frac{5}{2}$
16. $3x + 5y - 30 = 0$
★17. $x = 7$ **undefined, none**
★18. $cx + y = d$ $-c, d$

Write an equation in slope-intercept form for each graph.

19. $y = 0.8x$



20. $y = -\frac{5}{3}x + \frac{29}{3}$

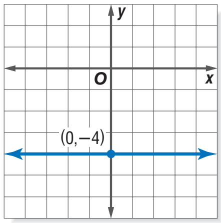


DAILY INTERVENTION

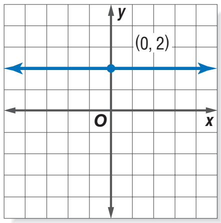
Differentiated Instruction

Intrapersonal Have students write a paragraph summarizing any problems they have with relating graphs to their equations. Ask them to include specific techniques they find useful to help them with this task.

Write an equation in slope-intercept form for each graph.

21. 

$y = -4$

22. 

$y = 2$

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

23. slope 3, passes through $(0, -6)$
24. slope 0.25, passes through $(0, 4)$
25. slope $-\frac{1}{2}$, passes through $(1, 3)$
26. slope $\frac{3}{2}$, passes through $-5, 1)$
27. slope -0.5, passes through $(2, -3)$
28. slope 4, passes through the origin
29. passes through $(-2, 5)$ and $(3, 1)$
30. passes through $(7, 1)$ and $(7, 8)$
31. passes through $(-4, 0)$ and $(3, 0)$
32. passes through $(-2, -3)$ and $(0, 0)$
33. x-intercept -4, y-intercept 4 $y = x + 4$
34. x-intercept $\frac{1}{3}$, y-intercept $-\frac{1}{4}$
35. passes through $(4, 6)$, parallel to the graph of $y = \frac{2}{3}x + 5$ $y = \frac{2}{3}x + \frac{10}{3}$
36. passes through $(2, -5)$, perpendicular to the graph of $y = \frac{1}{4}x + 7$ $y = -4x + 3$
★37. passes through $(6, -5)$, perpendicular to the line whose equation is $3x - \frac{1}{5}y = 3$
★38. passes through $(-3, -1)$, parallel to the line that passes through $(3, 3)$ and $(0, 6)$
$y = -x - 4$

23. $y = 3x - 6$
24. $y = 0.25x + 4$
25. $y = -\frac{1}{2}x + \frac{7}{2}$
26. $y = \frac{3}{2}x + \frac{17}{2}$
27. $y = -0.5x - 2$
28. $y = 4x$
29. $y = -\frac{4}{5}x + \frac{17}{5}$
30. no slope-intercept form for $x = 7$
31. $y = 0$
32. $y = \frac{3}{2}x$
34. $y = \frac{3}{4}x - \frac{1}{4}$
37. $y = -\frac{1}{15}x - \frac{23}{5}$

39. Write an equation in slope-intercept form of the line that passes through the points indicated in the table. $y = 3x - 2$

40. Write an equation in slope-intercept form of the line that passes through $(-2, 10)$, $(2, 2)$, and $(4, -2)$. $y = -2x + 6$

x	y
-1	-5
1	1
3	7

GEOMETRY **For Exercises 41–43, use the equation $d = 180(c - 2)$ that gives the total number of degrees d in any convex polygon with c sides.**

41. Write this equation in slope-intercept form. $d = 180c - 360$
42. Identify the slope and d-intercept. 180, -360
43. Find the number of degrees in a pentagon. 540°

44. **ECOLOGY** A park ranger at Blendon Woods estimates there are 6000 deer in the park. She also estimates that the population will increase by 75 deer each year thereafter. Write an equation that represents how many deer will be in the park in x years. $y = 75x + 6000$

45. **BUSINESS** Refer to the signs below. At what distance do the two stores charge the same amount for a balloon arrangement? 10 mi

Conrad's Balloon Bouquets
$20 balloon arrangements
Delivery: $3 per mile

The Balloon House
$30 Balloon Arrangements
$2 per mile delivery

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 75 (shown) and p. 76

Forms of Equations

Slope-Intercept Form of a Linear Equation	$y = mx + b$, where m is the slope and b is the y-intercept
Point-Slope Form of a Linear Equation	$y - y_1 = m(x - x_1)$, where (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line

Example 1 **Write an equation in slope-intercept form for the line that has slope -2 and passes through the point $(3, 7)$.**

Substitute for m, x, and y in the slope-intercept form.

$y = mx + b$ Slope-intercept form
$7 = (-2)(3) + b$ $(x, y) = (3, 7)$, $m = -2$
$7 = -6 + b$ Simplify.
$13 = b$ Add 6 to both sides.

The y-intercept is 13. The equation in slope-intercept form is $y = -2x + 13$.

Example 2 **Write an equation in slope-intercept form for the line that has slope $\frac{1}{3}$ and x-intercept 5.**

$y = mx + b$ Slope-intercept form
$0 = \left(\frac{1}{3}\right)(5) + b$ $(x, y) = (5, 0)$, $m = \frac{1}{3}$
$0 = \frac{5}{3} + b$ Simplify.
$-\frac{5}{3} = b$ Subtract $\frac{5}{3}$ from both sides.

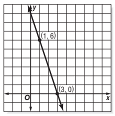
The y-intercept is $-\frac{5}{3}$. The slope-intercept form is $y = \frac{1}{3}x - \frac{5}{3}$.

Exercises

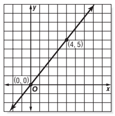
Write an equation in slope-intercept form for the line that satisfies each set of conditions.

1. slope -2, passes through $(-4, 6)$ $y = -2x - 2$
2. slope $\frac{3}{2}$, y-intercept 4 $y = \frac{3}{2}x + 4$
3. slope 1, passes through $(2, 5)$ $y = x + 3$
4. slope $-\frac{13}{5}$, passes through $(5, -7)$ $y = -\frac{13}{5}x + 6$

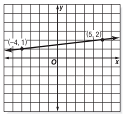
Write an equation in slope-intercept form for each graph.

5. 

$y = -3x + 9$

6. 

$y = \frac{5}{4}x$

7. 

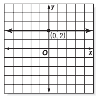
$y = \frac{1}{9}x + 1\frac{4}{9}$

Skills Practice, p. 77 and Practice, p. 78 (shown)

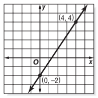
State the slope and y-intercept of the graph of each equation.

1. $y = 8x + 12$ 8, 12
2. $y = 0.25x - 1$ 0.25, -1
3. $y = -\frac{3}{5}x$ $-\frac{3}{5}$, 0
4. $3y = 7$ 0, $\frac{7}{3}$
5. $3x = -15 + 5y$ $\frac{3}{5}$, 3
6. $2x - 3y = 10$ $\frac{2}{3}$, $-\frac{10}{3}$

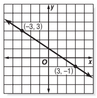
Write an equation in slope-intercept form for each graph.

7. 

$y = 2$

8. 

$y = \frac{3}{2}x - 2$

9. 

$y = -\frac{2}{3}x + 1$

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

10. slope -5, passes through $(-3, -8)$ $y = -5x - 23$
11. slope $\frac{4}{5}$, passes through $(10, -3)$ $y = \frac{4}{5}x - 11$
12. slope 0, passes through $(0, -10)$ $y = -10$
13. slope $-\frac{2}{3}$, passes through $(6, -8)$ $y = -\frac{2}{3}x - 4$
14. passes through $(3, 11)$ and $(-6, 5)$ $y = \frac{2}{3}x + 9$
15. passes through $(7, -2)$ and $(3, -1)$ $y = -\frac{1}{4}x - \frac{1}{4}$
16. x-intercept 3, y-intercept 2 $y = -\frac{2}{3}x + 2$
17. x-intercept -5, y-intercept 7 $y = \frac{7}{5}x + 7$
18. passes through $(-8, -7)$, perpendicular to the graph of $y = 4x - 3$ $y = -\frac{1}{4}x - 9$
19. **RESERVOIRS** The surface of Grand Lake is at an elevation of 648 feet. During the current drought, the water level is dropping at a rate of 3 inches per day. If this trend continues, write an equation that gives the elevation in feet of the surface of Grand Lake after x days. $y = -0.25x + 648$
20. **BUSINESS** Tony Marconi's company manufactures CD-ROM drives. The company will make $150,000 profit if it manufactures 100,000 drives, and $1,750,000 profit if it manufactures 500,000 drives. The relationship between the number of drives manufactured and the profit is linear. Write an equation that gives the profit P when n drives are manufactured. $P = 4n - 250{,}000$

Reading to Learn Mathematics, p. 79

ELL

Pre-Activity **How do linear equations apply to business?**

Read the introduction to Lesson 2-4 at the top of page 75 in your textbook.

- If the total cost of producing a product is given by the equation $y = 5400 + 1.37x$, what is the fixed cost? What is the variable cost (for each item produced)? $5400; $1.37
- Write a linear equation that describes the following situation: A company that manufactures computers has a fixed cost of $228,750 and a variable cost of $852 to produce each computer. $y = 228{,}750 + 852x$

Reading the Lesson

1. a. Write the slope-intercept form of the equation of a line. Then explain the meaning of each of the variables in the equation. $y = mx + b$; m is the slope and b is the y-intercept. The variables x and y are the coordinates of any point on the line.

   b. Write the point-slope form of the equation of a line. Then explain the meaning of each of the variables in the equation. $y - y_1 = m(x - x_1)$; m is the slope. x and y are the coordinates of any point on the line. x_1 and y_1 are the coordinates of one specific point on the line.

2. Suppose that your algebra teacher asks you to write the point-slope form of the equation of the line through the points $(-6, 7)$ and $(-3, -2)$. You write $y + 2 = -3(x + 3)$ and your classmate writes $y - 7 = -3(x + 6)$. Which of you is correct? Explain. You are both correct. Either point may be used as (x_1, y_1) in the point-slope form. You used $(-3, -2)$, and your classmate used $(-6, 7)$.

3. You are asked to write an equation of two lines that pass through $(3, -5)$, one of them parallel to and one of them perpendicular to the line whose equation is $y = -3x + 4$. The first step in finding these equations is to find their slopes. What is the slope of the parallel line? What is the slope of the perpendicular line? -3; $\frac{1}{3}$

Helping You Remember

4. Many students have trouble remembering the point-slope form for a linear equation. How can you use the definition of slope to remember this form? Sample answer: Write the definition of slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$. Multiply both sides of this equation by $x_2 - x_1$. Drop the subscripts in y_2 and x_2. This gives the point-slope form of the equation of a line.

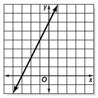
Enrichment, p. 80

Two-Intercept Form of a Linear Equation

You are already familiar with the slope-intercept form of a linear equation, $y = mx + b$. Linear equations can also be written in the form $\frac{x}{a} + \frac{y}{b} = 1$ with x-intercept a and y-intercept b. This is called two-intercept form.

Example 1 **Draw the graph of $\frac{x}{-3} + \frac{y}{6} = 1$.**

The graph crosses the x-axis at -3 and the y-axis at 6. Graph $(-3, 0)$ and $(0, 6)$, then draw a straight line through them.



Example 2 **Write $3x + 4y = 12$ in two-intercept form.**

Divide by 12 to obtain 1 on the right side.

4 Assess

Open-Ended Assessment

Writing Have students write their own summary of how to relate graphs to their equations, including slope, intercepts, and parallel and perpendicular lines.

Getting Ready for Lesson 2-5

PREREQUISITE SKILL Lesson 2-5 presents modeling real-world data using scatter plots. Exercises 64–67 should be used to determine your students' familiarity with finding the median of a set of numbers.

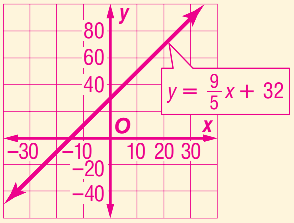
Assessment Options

Quiz (Lessons 2-3 and 2-4) is available on p. 113 of the *Chapter 2 Resource Masters*.

Mid-Chapter Test (Lessons 2-1 through 2-4) is available on p. 115 of the *Chapter 2 Resource Masters*.

Answers

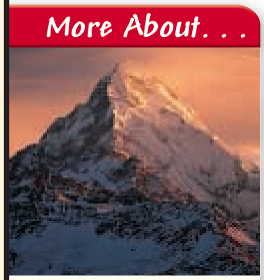
46.



52. A linear equation can sometimes be used to relate a company's cost to the number they produce of a product. Answers should include the following.

- The *y*-intercept, 5400, is the cost the company must pay if they produce 0 units, so it is the fixed cost. The slope, 1.37, means that it costs $1.37 to produce each unit. The variable cost is 1.37*x*.
- $6770

More About. . .



Science

Ice forms at a temperature of 0°C, which corresponds to a temperature of 32°F. A temperature of 100°C corresponds to a temperature of 212°F.

SCIENCE **For Exercises 46–48, use the information on temperatures at the left.**

46. Write and graph the linear equation that gives the number y of degrees Fahrenheit in terms of the number x of degrees Celsius. $y = \frac{9}{5}x + 32$; **See margin for graph.**

47. What temperature corresponds to 20°C? **68°F**

48. What temperature is the same on both scales? **−40°**

TELEPHONES **For Exercises 49 and 50, use the following information.**
Namid is examining the calling card portion of his phone bill. A 4-minute call at the night rate cost \$2.65. A 10-minute call at the night rate cost \$4.75.

49. Write a linear equation to model this situation. $y = 0.35x + 1.25$

50. How much would it cost to talk for half an hour at the night rate? **\$11.75**

51. **CRITICAL THINKING** Given $\triangle ABC$ with vertices $A(-6, -8)$, $B(6, 4)$, and $C(-6, 10)$, write an equation of the line containing the altitude from A. (*Hint:* The altitude from A is a segment that is perpendicular to $\overline{BC}$.) $y = 2x + 4$

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do linear equations apply to business?

Include the following in your answer: **See margin.**

- the *fixed cost* and the *variable cost* in the equation $y = 5400 + 1.37x$, where y is the cost for a company to produce x units of its product, and
- the cost for the company to produce 1000 units of its product.

Standardized Test Practice

53. Find an equation of the line through $(0, -3)$ and $(4, 1)$. **C**

Ⓐ $y = -x + 3$ Ⓑ $y = -x - 3$ Ⓒ $y = x - 3$ Ⓓ $y = -x + 3$

54. Choose the equation of the line through $\left(\frac{1}{2}, -\frac{3}{2}\right)$ and $\left(-\frac{1}{2}, \frac{1}{2}\right)$. **A**

Ⓐ $y = -2x - \frac{1}{2}$ Ⓑ $y = -3x$ Ⓒ $y = 2x - \frac{5}{2}$ Ⓓ $y = \frac{1}{2}x + 1$

Extending the Lesson

For Exercises 55 and 56, use the following information.

The form $\frac{x}{a} + \frac{y}{b} = 1$ is known as the **intercept form** of the equation of a line because a is the x-intercept and b is the y-intercept.

55. Write the equation $2x - y - 5 = 0$ in intercept form. $\frac{x}{\frac{5}{2}} - \frac{y}{5} = 1$

56. Identify the x- and y-intercepts of the graph of $2x - y - 5 = 0$. $\frac{5}{2}, -5$

Maintain Your Skills

Mixed Review

Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

57. (7, 2), (5, 6) **−2**
58. (1, −3), (3, 3) **3**
59. (−5, 0), (4, 0) **0**

60. **INTERNET** A Webmaster estimates that the time (seconds) required to connect to the server when n people are connecting is given by $t(n) = 0.005n + 0.3$. Estimate the time required to connect when 50 people are connecting. *(Lesson 2-2)* **0.55 s**

Solve each inequality. *(Lessons 1-5 and 1-6)*

61. $|x - 2| \leq -99$ $\varnothing$
62. $-4x + 7 \leq 31$ $\{x \mid x \geq -6\}$
63. $2(r - 4) + 5 \geq 9$ $\{r \mid r \geq 6\}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the median of each set of numbers.**
*(To review **finding a median**, see pages 822 and 823.)*

64. {3, 2, 1, 3, 4, 8, 4} **3**
65. {9, 3, 7, 5, 6, 3, 7, 9} **6.5**
66. {138, 235, 976, 230, 412, 466} **323.5**
67. {2.5, 7.8, 5.5, 2.3, 6.2, 7.8} **5.85**

2-5 Modeling Real-World Data: Using Scatter Plots

What You'll Learn

- Draw scatter plots.
- Find and use prediction equations.

Vocabulary

- scatter plot
- line of fit
- prediction equation

How can a linear equation model the number of Calories you burn exercising?

The table shows the number of Calories burned per hour by a 140-pound person running at various speeds. A linear function can be used to model these data.

Speed (mph)	Calories
5	508
6	636
7	731
8	858

SCATTER PLOTS To model data with a function, it is helpful to graph the data. A set of data graphed as ordered pairs in a coordinate plane is called a **scatter plot**. A scatter plot can show whether there is a relationship between the data.

Example 1 Draw a Scatter Plot

HOUSING **The table below shows the median selling price of new, privately-owned, one-family houses for some recent years. Make a scatter plot of the data.**

Year	1990	1992	1994	1996	1998	2000
Price ($1000)	122.9	121.5	130.0	140.0	152.5	169.0

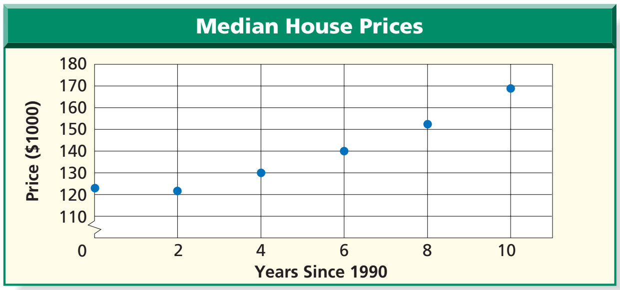
Source: U.S. Census Bureau and U.S. Department of Housing and Urban Development

Study Tip

Choosing the Independent Variable

Letting *x* be the number of years since the first year in the data set sometimes simplifies the calculations involved in finding a function to model the data.

Graph the data as ordered pairs, with the number of years since 1990 on the horizontal axis and the price on the vertical axis.



PREDICTION EQUATIONS Except for (0, 122.9), the data in Example 1 appear to lie nearly on a straight line. When you find a line that closely approximates a set of data, you are finding a **line of fit** for the data. An equation of such a line is often called a **prediction equation** because it can be used to predict one of the variables given the other variable.

2-5 Lesson Notes

1 Focus

5-Minute Check Transparency 2-5 Use as a quiz or review of Lesson 2-4.

Mathematical Background notes are available for this lesson on p. 54D.

Building on Prior Knowledge

In Lesson 2-4, students wrote linear equations based on information provided about their graphs. In this lesson, students apply those skills to modeling real-world data with scatter plots and writing their prediction equations.

How can a linear equation model the number of Calories you burn exercising?

Ask students:

- Will a person that weighs less than 140 pounds burn more or less Calories than shown in the table at the given speeds? **less**
- What is a reasonable estimate of the number of Calories a 140-pound person burns running at a speed of 4 miles per hour? **about 400 Calories**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters
- Study Guide and Intervention, pp. 81–82
- Skills Practice, p. 83
- Practice, p. 84
- Reading to Learn Mathematics, p. 85
- Enrichment, p. 86

School-to-Career Masters, p. 4
Science and Mathematics Lab Manual, pp. 97–102
Teaching Algebra With Manipulatives Masters, p. 218

Transparencies

5-Minute Check Transparency 2-5
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

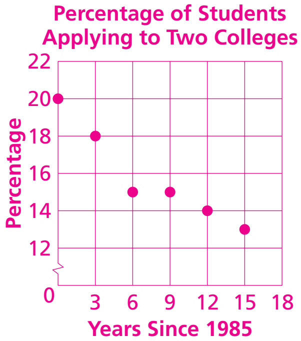
SCATTER PLOTS

In-Class Example

1 **EDUCATION** The table below shows the approximate percent of students who sent applications to two colleges in various years since 1985. Make a scatter plot of the data.

Years Since 1985	0	3	6	9	12	15
Percentage	20	18	15	15	14	13

Source: U.S. News & World Report

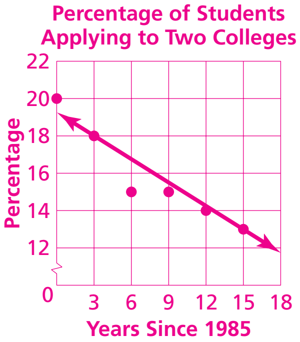


PREDICTION EQUATIONS

In-Class Example

2 **EDUCATION** Refer to the data in In-Class Example 1.

a. Draw a line of fit for the data. How well does the line fit the data?



Except for (6, 15), the line fits the data fairly well.

(continued on the next page)

To find a line of fit and a prediction equation for a set of data, select two points that appear to represent the data well. This is a matter of personal judgment, so your line and prediction equation may be different from someone else's.

Study Tip

Reading Math

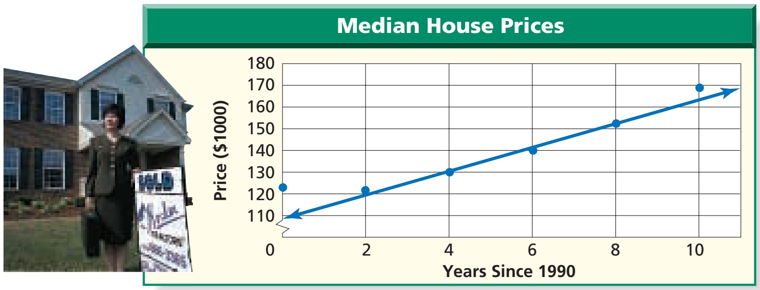
A data point that does not appear to belong with the rest of the set is called an **outlier**.

Example 2 Find and Use a Prediction Equation

HOUSING **Refer to the data in Example 1.**

a. Draw a line of fit for the data. How well does the line fit the data?

Ignore the point (0, 122.9) since it would not be close to a line that represents the rest of the data points. The points (4, 130.0) and (8, 152.5) appear to represent the data well. Draw a line through these two points. Except for (0, 122.9), this line fits the data very well.



b. Find a prediction equation. What do the slope and y-intercept indicate?

Find an equation of the line through (4, 130.0) and (8, 152.5). Begin by finding the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$= \frac{152.5 - 130.0}{8 - 4} \quad \text{Substitute.}$$

$$\approx 5.63 \quad \text{Simplify.}$$

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 130.0 = 5.63(x - 4) \quad m = 5.63,\ (x_1, y_1) = (4, 130.0)$$

$$y - 130.0 = 5.63x - 22.52 \quad \text{Distributive Property}$$

$$y = 5.63x + 107.48 \quad \text{Add 130.0 to each side.}$$

One prediction equation is $y = 5.63x + 107.48$. The slope indicates that the median price is increasing at a rate of about $5630 per year. The y-intercept indicates that, according to the trend of the rest of the data, the median price in 1990 should have been about $107,480.

Study Tip

Reading Math

When you are predicting for an x value greater than any in the data set, the process is known as **extrapolation**. When you are predicting for an x value between the least and greatest in the data set, the process is known as **interpolation**.

c. Predict the median price in 2010.

The year 2010 is 20 years after 1990, so use the prediction equation to find the value of y when $x = 20$.

$$y = 5.63x + 107.48 \quad \text{Prediction equation}$$

$$= 5.63(20) + 107.48 \quad x = 20$$

$$= 220.08 \quad \text{Simplify.}$$

The model predicts that the median price in 2010 will be about $220,000.

d. How accurate is the prediction?

Except for the outlier, the line fits the data very well, so the predicted value should be fairly accurate.

TEACHING TIP

On pages 87 and 88, students will learn about a number that measures how well a line fits a set of data.

DAILY INTERVENTION

Differentiated Instruction

Naturalist Ask students how scatter plots and prediction equations might be used to relate local insect and animal populations to food supplies, temperature, and rainfall. Have interested students devise a plan for conducting the research necessary for developing such a prediction equation.

Study Tip

Outliers

If your scatter plot includes points that are far from the others on the graph, check your data before deciding it is an outlier. You may have made a graphing or recording mistake.

Algebra Activity

Head versus Height

Collect the Data

- Collect data from several of your classmates. Use a tape measure to measure the circumference of each person's head and his or her height. Record the data as ordered pairs of the form (height, circumference).

Analyze the Data 1–5. See students' work.

1. Graph the data in a scatter plot.
2. Choose two ordered pairs and write a prediction equation.
3. Explain the meaning of the slope in the prediction equation.

Make a Conjecture

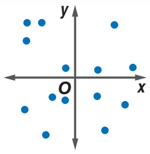
4. Predict the head circumference of a person who is 66 inches tall.
5. Predict the height of an individual whose head circumference is 18 inches.

Check for Understanding

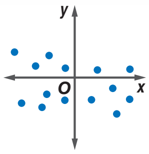
Concept Check

1. **Choose** the scatter plot with data that could best be modeled by a linear function. d

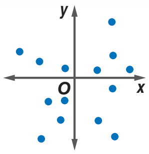
a.



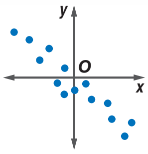
b.



c.

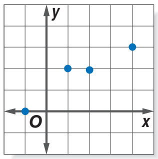


d.



2. **Identify** the domain and range of the relation in the graph at the right. **Predict** the value of y when $x = 5$.

2. D = {–1, 1, 2, 4}, R = {0, 2, 3}; Sample answer using (–1, 0) and (2, 2): 4



3. **OPEN ENDED** Write a different prediction equation for the data in Examples 1 and 2 on pages 81 and 82.
Sample answer using (4, 130.0) and (6, 140.0): $y = 5x + 110$

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1, 2

Complete parts a–c for each set of data in Exercises 4 and 5.

a. **Draw a scatter plot.**

b. **Use two ordered pairs to write a prediction equation.**

c. **Use your prediction equation to predict the missing value.**

4. **SCIENCE** Whether you are climbing a mountain or flying in an airplane, the higher you go, the colder the air gets. The table shows the temperature in the atmosphere at various altitudes. 4–5. See pp. 107A–107H.

Altitude (ft)	0	1000	2000	3000	4000	5000
Temp (°C)	15.0	13.0	11.0	9.1	7.1	?

Source: NASA

5. **TELEVISION** As more channels have been added, cable television has become attractive to more viewers. The table shows the number of U.S. households with cable service in some recent years.

Year	1990	1992	1994	1996	1998	2010
Households (millions)	55	57	59	65	67	?

Source: Nielsen Media Research

b. Find a prediction equation. What do the slope and y-intercept indicate? Using (3, 18) and (15, 13): $y = -0.42x + 19.25$; The slope indicates that the percent is falling at about 0.4% each year. The y-intercept indicates the percent in 1985 should have been about 19%.

c. Predict the percent in 2010. about 9%

d. How accurate is the prediction? The fit is only approximate, so the prediction may not be very accurate.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- write a summary of the procedure for placing a line of fit on a scatter plot and how to find a prediction equation using this line.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Algebra Activity

Materials: tape measure, grid paper

- You can use string and a ruler as an alternative way to measure the circumference of a person's head as well as their height.
- Measurements can be made in inches or in centimeters. You may want to have half the class use one system and the other half the alternate system.

About the Exercises...

Organization by Objective

- **Scatter Plots:** 6a, 7a, 8a, 9a, 15
- **Prediction Equations:** 6b, 6c, 7b, 7c, 8b, 8c, 9b, 9c, 10–14, 16–21

Odd/Even Assignments

Exercises 6–9 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 21 involves research on the Internet or other reference materials.

Assignment Guide

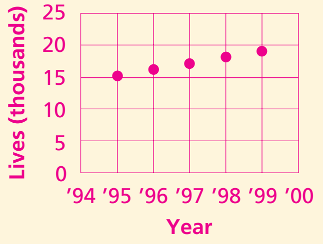
Basic: 7, 10–12, 19–24, 31–42

Average: 7, 9, 10–12, 15–24, 31–42 (optional: 25–30)

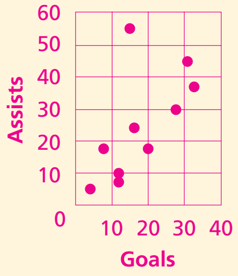
Advanced: 6, 8, 13–37 (optional: 38–42)

Answers

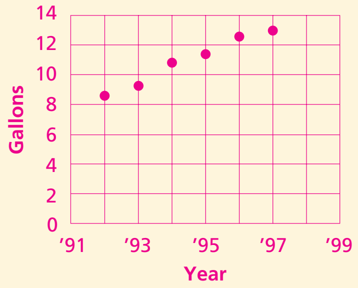
6a. Lives Saved by Minimum Drinking Age



7a. 2000–2001 Detroit Red Wings



8a. Bottled Water Consumption



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
6–21	1, 2

Extra Practice

See page 831.

6a. See margin.

6b. Sample answer using (1996, 16.5) and (1998, 18.2): $y = 0.85x - 1680.1$

6c. Sample answer: 28,400

7a. See margin.

7b. Sample answer using (4,5) and (32,37): $y = 1.14x + 0.44$

7c. Sample answer: about 13

WebQuest

A scatter plot of loan payments can help you analyze home loans. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

8a. See margin.

8b. Sample answer using (1993, 9.4) and (1996, 12.5): $y = 1.03x - 2043.39$

8c. Sample answer: about 26.9 gal

9a. See pp. 107A–107H.

9b. Sample answer using (1, 499) and (3, 588): $y = 44.5x + 454.5$, where x is the number of seasons since 1995–1996

9c. Sample answer: about $1078 million or $1.1 billion

Complete parts a–c for each set of data in Exercises 6–9.

a. **Draw a scatter plot.**

b. **Use two ordered pairs to write a prediction equation.**

c. **Use your prediction equation to predict the missing value.**

6. **SAFETY** All states and the District of Columbia have enacted laws setting 21 as the minimum drinking age. The table shows the estimated cumulative number of lives these laws have saved by reducing traffic fatalities.

Year	1995	1996	1997	1998	1999	2010
Lives (1000s)	15.7	16.5	17.4	18.2	19.1	?

Source: National Highway Traffic Safety Administration

7. **HOCKEY** Each time a hockey player scores a goal, up to two teammates may be credited with assists. The table shows the number of goals and assists for some of the members of the Detroit Red Wings in the 2000–2001 NHL season.

Goals	31	15	32	27	16	20	8	4	12	12	?
Assists	45	56	37	30	24	18	17	5	10	7	15

Source: www.detroitredwings.com

8. **HEALTH** Bottled water has become very popular. The table shows the number of gallons of bottled water consumed per person in some recent years.

Year	1992	1993	1994	1995	1996	1997	2010
Gallons	8.2	9.4	10.7	11.6	12.5	13.1	?

Source: U.S. Department of Agriculture

★ 9. **THEATER** Broadway, in New York City, is the center of American theater. The table shows the total revenue of all Broadway plays for some recent seasons.

Season	'95–'96	'96–'97	'97–'98	'98–'99	'99–'00	'09–'10
Revenue ($ millions)	436	499	558	588	603	?

Source: The League of American Theatres and Producers, Inc.

MEDICINE **For Exercises 10–12, use the graph that shows how much Americans spent on doctors' visits in some recent years.**

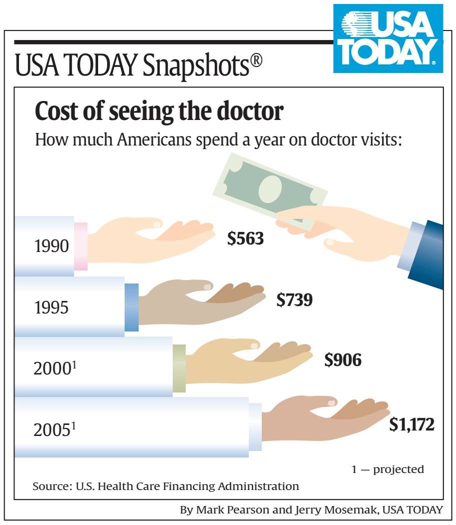
10. Write a prediction equation from the data for 1990, 1995, and 2000.

11. Use your equation to predict the amount for 2005.

12. Compare your prediction to the one given in the graph. **The value predicted by the equation is somewhat lower than the one given in the graph.**

10. Sample answer using (1990, 583) and (1995, 739): $y = 31.2x - 61{,}505$

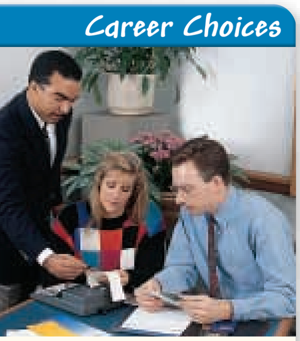
11. Sample answer: $1051



Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

Career Choices



Finance

A financial analyst can advise people about how to invest their money and plan for retirement.

Online Research For information about a career as a financial analyst, visit: www.algebra2.com/careers

FINANCE **For Exercises 13 and 14, use the following information.** Della has $1000 that she wants to invest in the stock market. She is considering buying stock in either Company 1 or Company 2. The values of the stocks at the ends of the last four months are shown in the tables below.

Company 1	
Month	**Share Price ($)**
Aug.	25.13
Sept.	22.94
Oct.	24.19
Nov.	22.56

Company 2	
Month	**Share Price ($)**
Aug.	31.25
Sept.	32.38
Oct.	32.06
Nov.	32.44

★13. Based only on these data, which stock should Della buy? Explain.

★14. Do you think investment decisions should be based on this type of reasoning? If not, what other factors should be considered?

13–14. See pp. 107A–107H.

GEOGRAPHY **For Exercises 15–18, use the table below that shows the elevation and average precipitation for selected cities.**

City	Elevation (feet)	Average Precip. (inches)	City	Elevation (feet)	Average Precip. (inches)
Rome, Italy	79	33	London, England	203	30
Algiers, Algeria	82	27	Paris, France	213	26
Istanbul, Turkey	108	27	Bucharest, Romania	298	23
Montreal, Canada	118	37	Budapest, Hungary	456	20
Stockholm, Sweden	171	21	Toronto, Canada	567	31
Berlin, Germany	190	23			

Source: World Meteorological Association

★15. Draw a scatter plot with elevation as the independent variable. **See pp. 107A–107H.**

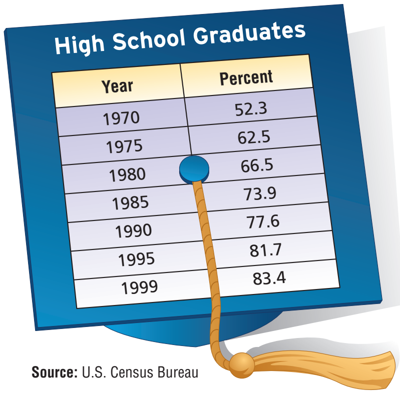
★16. Write a prediction equation.

16. Sample answer using (213, 26) and (298, 23): $y = -0.04x + 34.52$

★17. Predict the average annual precipitation for Dublin, Ireland, which has an elevation of 279 feet. **Sample answer: about 23 in.**

★18. Compare your prediction to the actual value of 29 inches. **See margin.**

CRITICAL THINKING **For Exercises 19 and 20, use the table that shows the percent of people ages 25 and over with a high school diploma over the last few decades.**



High School Graduates

Year	Percent
1970	52.3
1975	62.5
1980	66.5
1985	73.9
1990	77.6
1995	81.7
1999	83.4

Source: U.S. Census Bureau

19. Use a prediction equation to predict the percent in 2010.

19. Sample answer using (1975, 62.5) and (1995, 81.7): 96.1%

20. Do you think your prediction is accurate? Explain. **See margin.**

21. **RESEARCH** Use the Internet or other resource to look up the population of your community or state in several past years. Use a prediction equation to predict the population in some future year. **See students' work.**

www.algebra2.com/self_check_quiz

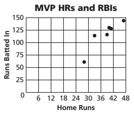
Study Guide and Intervention, p. 81 (shown) and p. 82

Scatter Plots When a set of data points is graphed as ordered pairs in a coordinate plane, the graph is called a **scatter plot**. A scatter plot can be used to determine if there is a relationship among the data.

Example **BASEBALL The table below shows the number of home runs and runs batted in for various baseball players who won the Most Valuable Player Award during the 1990s. Make a scatter plot of the data.**

Home Runs	Runs Batted In
33	114
39	116
40	130
28	61
41	128
47	144

Source: *New York Times Almanac*



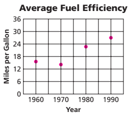
Exercises

Make a scatter plot for the data in each table below.

1. **FUEL EFFICIENCY** The table below shows the average fuel efficiency in miles per gallon of new cars manufactured during the years listed.

Year	Fuel Efficiency (mpg)
1960	15.5
1970	14.1
1980	22.6
1990	26.9

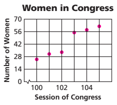
Source: *New York Times Almanac*



2. **CONGRESS** The table below shows the number of women serving in the United States Congress during the years 1987–1999.

Congressional Session	Number of Women
100	25
101	31
102	33
103	55
104	58
105	62

Source: *Wall Street Journal Almanac*



Skills Practice, p. 83 and Practice, p. 84 (shown)

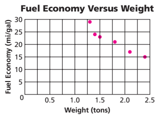
For Exercises 1–3, complete parts a–c for each set of data.
a. Draw a scatter plot.
b. Use two ordered pairs to write a prediction equation.
c. Use your prediction equation to predict the missing value.

1. **FUEL ECONOMY** The table gives the approximate weights in tons and estimates for overall fuel economy in miles per gallon for several cars.

Weight (tons)	1.3	1.4	1.5	1.8	2	2.1	2.4
Miles per Gallon	29	24	23	21	?	17	15

1b. Sample answer using (1.4, 24) and (2.4, 15): $y = -9x + 36.6$

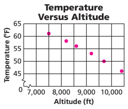
1c. Sample answer: 18.6 mi/gal



2. **ALTITUDE** In most cases, temperature decreases with increasing altitude. As Anchara drives into the mountains, her car thermometer registers the temperatures (°F) shown in the table at the given altitudes (feet).

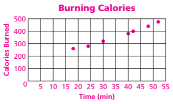
Altitude (ft)	7500	8200	8600	9200	9700	10,400	12,000
Temperature (°F)	61	58	56	53	50	46	?

2b. Sample answer using (7500, 61) and (9700, 50): $y = -0.005x + 98.5$
2c. Sample answer: 38.5°F



3. **HEALTH** Alton has a treadmill that uses the time on the treadmill and the speed of walking or running to estimate the number of Calories he burns during a workout. The table gives workout times and Calories burned for several workouts.

Time (min)	18	24	30	40	42	48	52	60
Calories Burned	260	280	320	380	400	440	475	?



3b. Sample answer using (24, 280) and (48, 440): $y = 6.67x + 119.92$
3c. Sample answer: about 520 calories

Reading to Learn Mathematics, p. 85

ELL

Pre-Activity **How can a linear equation model the number of Calories you burn exercising?**

Read the introduction to Lesson 2-5 at the top of page 81 in your textbook.

- If a woman runs 5.5 miles per hour, about how many Calories will she burn in an hour? **Sample answer: 572 Calories**
- If a man runs 7.5 miles per hour, about how many Calories will he burn in half an hour? **Sample answer: 397 Calories**

Reading the Lesson

1. Suppose that a set of data can be modeled by a linear equation. Explain the difference between a scatter plot of the data and a graph of the linear equation that models that data.
Sample answer: The scatter plot is a discrete graph. It is made up just of the individual points that represent the data points. The linear equation has a continuous graph that is the line that best fits the data points.

2. Suppose that tuition at a state college was $3500 per year in 1995 and has been increasing at a rate of $225 per year.
   a. Write a prediction equation that expresses this information.
   $y = 3500 + 225x$
   b. Explain the meaning of each variable in your prediction equation.
   x represents the number of year since 1995 and y represents the tuition in that year.

3. Use this model to predict the tuition at this college in 2007. **$6200**

Helping You Remember

4. Look up the word *scatter* in a dictionary. How can its definition help you to remember the meaning of the difference between a scatter plot and the graph of a linear equation?
Sample answer: To *scatter* means to break up and go in many directions. The points on a scatter plot are broken up. In a scatter plot, the points are scattered or broken up. In the graph of a linear equation, the points are connected to form a continuous line.

Answers

18. Sample answer: The predicted value differs from the actual value by more than 20%, possibly because no line fits the data very well.

20. Sample answer: The predicted percent is almost certainly too high. Since the percent cannot exceed 100%, it cannot continue to increase indefinitely at a linear rate.

Enrichment, p. 86

Median-Fit Lines

A **median-fit line** is a particular type of line of fit. Follow the steps below to find the equation of the median-fit line for the data.

Approximate Percentage of Violent Crimes Committed by Juveniles That Victims Reported to Law Enforcement

Year	1980	1982	1984	1986	1988	1990	1992	1994	1996
Offenders	36	35	33	32	31	30	29	29	30

Source: U.S. Bureau of Justice Statistics

1. Divide the data into three approximately equal groups. There should always be the same number of points in the first and third groups. In this case, there will be three data points in each group.

Group 1		Group 2		Group 3	
Year	Offenders	Year	Offenders	Year	Offenders

4 Assess

Open-Ended Assessment

Writing Ask students to write instructions that could be used to teach a friend how to read a graph such as the one shown in Example 2 on p. 82. The importance of reading titles, captions, and text, as well as the scales should be included as part of the instructions.

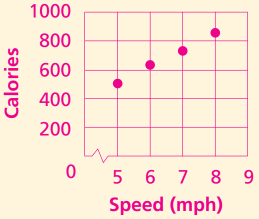
Getting Ready for Lesson 2-6

PREREQUISITE SKILL Lesson 2-6 presents the graphing of special functions, including absolute value functions. Exercises 38–42 should be used to determine your students' familiarity with finding absolute values.

Answer

22. Data can be used to write a linear equation that approximates the number of Calories burned per hour in terms of the speed that a person runs. Answers should include the following.

- **Calories Burned While Running**



Sample answer using (5, 508) and (8, 858): $y = 116.67x - 75.35$

- about 975 calories; Sample answer: The predicted value differs from the actual value by only about 2%.

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22. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a linear equation model the number of Calories you burn exercising?

Include the following in your answer: **See margin.**

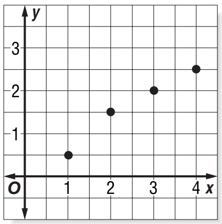
- a scatter plot and a prediction equation for the data, and
- a prediction of the number of Calories burned in an hour by a 140-pound person running at 9 miles per hour, with a comparison of your predicted value with the actual value of 953.

Standardized Test Practice

23. Which line best fits the data in the graph at the right? **D**

Ⓐ $y = x$ Ⓑ $y = -0.5x + 4$

Ⓒ $y = -0.5x - 4$ Ⓓ $y = 0.5 + 0.5x$



24. A prediction equation for a set of data is $y = 0.63x + 4.51$. For which x value is the predicted y value 6.4? **A**

Ⓐ 3 Ⓑ 4.5

Ⓒ 6 Ⓓ 8.54

Extending the Lesson

For Exercises 25–30, use the following information.

A **median-fit line** is a particular type of line of fit. Follow the steps below to find the equation of the median-fit line for the data.

Federal and State Prisoners (per 100,000 U.S. citizens)

Year	1986	1988	1990	1992	1994	1996	1998	1999
Prisoners	217	247	297	332	389	427	461	476

Source: U.S. Bureau of Justice Statistics

25. 1988, 1993, 1998; 247, 360.5, 461

25. Divide the data into three approximately equal groups. There should always be the same number of points in the first and third groups. Find x_1, x_2, and x_3, the medians of the x values in Groups 1, 2, and 3, respectively. Find y_1, y_2, and y_3, the medians of the y values in Groups 1, 2, and 3, respectively.

26. Find an equation of the line through (x_1, y_1) and (x_3, y_3). $y = 21.4x - 42{,}296.2$

27. Find Y, the y-coordinate of the point on the line in Exercise 26 with an x-coordinate of x_2. **354**

28. about (1993, 356.17)

28. The median-fit line is parallel to the line in Exercise 26, but is one-third closer to (x_2, y_2). This means it passes through $\left(x_2, \frac{2}{3}Y + \frac{1}{3}y_2\right)$. Find this ordered pair.

29. Write an equation of the median-fit line. $y = 21.4x - 42{,}294.03$

30. Predict the number of prisoners per 100,000 citizens in 2005 and 2010. **about 613, about 720**

Maintain Your Skills

Mixed Review

Write an equation in slope-intercept form that satisfies each set of conditions. *(Lesson 2-4)* **31.** $y = 4x + 6$

31. slope 4, passes through (0, 6)
32. passes through (5, −3) and (−2, 0)

32. $y = -\frac{3}{7}x - \frac{6}{7}$

Find each value if $g(x) = -\frac{4x}{3} + 7$. *(Lesson 2-1)*

33. $g(3)$ **3**
34. $g(0)$ **7**
35. $g(-2)$ $\frac{29}{3}$
36. $g(-4)$ $\frac{37}{3}$

37. Solve $|x + 4| > 3$. *(Lesson 1-6)* $\{x \mid x < -7 \text{ or } x > -1\}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each absolute value.** *(To review **absolute value**, see Lesson 1-4.)*

38. $|-3|$ **3**
39. $|11|$ **11**
40. $|0|$ **0**
41. $\left|-\frac{2}{3}\right|$ $\frac{2}{3}$
42. $|-1.5|$ **1.5**

Teacher to Teacher

Susan Nelson Spring H.S., Spring, TX

"I have my students take measurements of their height and arm span and record them. We enter the entire class' data into a graphing calculator and find the linear regression. Then we use the regression equation to make predictions."

Graphing Calculator Investigation

A Follow-Up of Lesson 2-5

Lines of Regression

You can use a TI-83 Plus graphing calculator to find a line that best fits a set of data. This line is called a **regression line** or **line of best fit**. You can also use the calculator to draw scatter plots and make predictions.

INCOME **The table shows the median income of U.S. families for the period 1970–1998.**

Year	1970	1980	1985	1990	1995	1998
Income ($)	9867	21,023	27,735	35,353	40,611	46,737

Source: U.S. Census Bureau

Find and graph a regression equation. Then predict the median income in 2010.

Step 1 *Find a regression equation.*

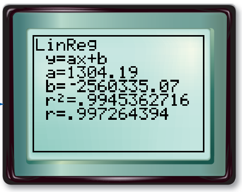
- Enter the years in L1 and the incomes in L2.

  KEYSTROKES: STAT ENTER 1970 ENTER

- Find the regression equation by selecting LinReg(ax+b) on the STAT CALC menu.

  KEYSTROKES: STAT ▶ 4 ENTER

The regression equation is about $y = 1304.19x - 2{,}560335.07$.



```
LinReg
 y=ax+b
 a=1304.19
 b=-2560335.07
 r²=.9945362716
 r=.997264394
```

The slope indicates that family incomes were increasing at a rate of about $1300 per year.

The number r is called the **linear correlation coefficient**. The closer the value of r is to 1 or -1, the closer the data points are to the line. *If the values of r^2 and r are not displayed, use DiagnosticOn from the CATALOG menu.*

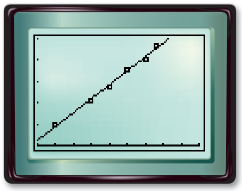
Step 2 *Graph the regression equation.*

- Use STAT PLOT to graph a scatter plot.

  KEYSTROKES: 2nd [STAT PLOT] ENTER ENTER

- Select the scatter plot, L1 as the Xlist, and L2 as the Ylist.
- Copy the equation to the Y= list and graph.

  KEYSTROKES: Y= VARS 5 ▶ ▶ 1 GRAPH



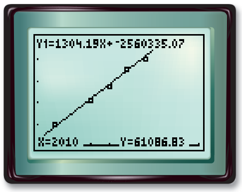
[1965, 2010] scl: 5 by [0, 50,000] scl: 10,000

Notice that the regression line does not pass through any of the data points, but comes close to all of them. The line fits the data very well.

Step 3 *Predict using the regression equation.*

- Find y when $x = 2010$. Use value on the CALC menu.

  KEYSTROKES: 2nd CALC 1 2010 ENTER



```
Y1=1304.19X+-2560335.07
X=2010          Y=61086.83
```

According to the regression equation, the median family income in 2010 will be about $61,087.

www.algebra2.com/other_calculator_keystrokes

Graphing Calculator Investigation

A Follow-Up of Lesson 2-5

Getting Started

Know Your Calculator The TI-83 Plus graphing calculator has a linear regression function, LinReg($ax + b$), that uses a least-squares fit method to determine the values for a and b. This involves calculus and finding the distance from each point to the line of best fit.

Correlation Coefficient With DiagnosticOn, the calculator also displays values for r^2 and r. The closer the value of $|r|$ is to 1, the better the equation fits the data.

Teach

- Make sure students have cleared the L1 and L2 lists before they enter the new data.
- After students have entered data, have them work in pairs to verify that the data display shows the correct numbers before proceeding.
- Have students complete Exercises 1–15.

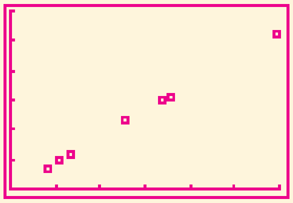
Assess

Refer students to the calculator display shown in Step 1 on p. 87, and ask the following questions.

- What is the regression equation for the income data? $y = 1304.19x - 2{,}560{,}335.07$
- What does the value of r tell you about the regression equation? The value of r is very close to 1, so the data points are very close to the graph of that equation.

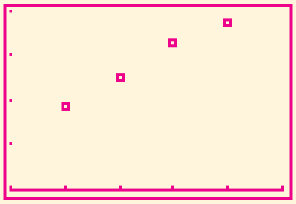
Answers

1.



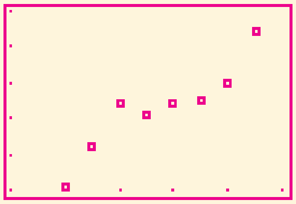
[0, 30] scl: 5 by [0, 60] scl: 10

4.



[1980, 2005] scl: 5 by [0, 40] scl: 10

7.



[1990, 2000] scl: 2 by [13,000, 18,000] scl: 1000

11. The prediction may not be accurate because different parts of the data could be represented by lines with different slopes. The sales could drop, as they did in 1995, or they could level out, as they did in 1996 and 1997.

15. The correlation coefficient, 0.9761660092, is closer to 1. The new regression line fits the data better.

Graphing Calculator Investigation

Exercises

GOVERNMENT **For Exercises 1–3, use the table below that shows the population and the number of representatives in Congress for selected states.**

State	CA	NY	TX	FL	NC	IN	AL
Population (millions)	29.8	18.0	17.0	12.9	6.6	5.5	4.0
Representatives	52	31	30	23	12	10	7

Source: *The World Almanac*

1. Make a scatter plot of the data. See margin.
2. Find a regression equation for the data. $y = 1.73x + 0.39$
3. Predict the number of representatives for Oregon, which has a population of about 2.8 million. 5

BASEBALL **For Exercises 4–6, use the table at the right that shows the total attendance for minor league baseball in some recent years.**

Year	Attendance (millions)
1985	18.4
1990	25.2
1995	33.1
2000	37.6

Source: National Association of Professional Baseball Leagues

4. Make a scatter plot of the data. See margin.
5. Find a regression equation for the data. $y = 1.31x - 2581.6$
6. Predict the attendance in 2010. 51,500,000

TRANSPORTATION **For Exercises 7–11, use the table below that shows the retail sales of motor vehicles in the United States for the period 1992–1999.**

Motor Vehicle Sales								
Year	1992	1993	1994	1995	1996	1997	1998	1999
Vehicles (thousands)	13,118	14,199	15,413	15,118	15,456	15,498	15,963	17,414

Source: American Automobile Manufacturers Association

9. about 470,000 vehicles more per year

7. Make a scatter plot of the data. See margin.
8. Find a regression equation for the data. $y = 470.06x - 922{,}731.40$
9. According to the regression equation, what was the average rate of change of vehicle sales during the period?
10. Predict the sales in 2010. about 22,088,000
11. How accurate do you think your prediction is? Explain. See margin.

RECREATION **For Exercises 12–15, use the table at the right that shows the amount of money spent on skin diving and scuba equipment in some recent years.** 14. about $440,000,000

Skin Diving and Scuba Equipment	
Year	Sales ($ millions)
1993	315
1994	322
1995	328
1996	340
1997	332
1998	345
1999	363

Source: National Sporting Goods Association

12. Find a regression equation for the data. $y = 6.93x - 13{,}494.43$
13. Delete the outlier (1997, 332) from the data set. Then find a new regression equation for the data. $y = 7.36x - 14{,}354.33$
14. Use the new regression equation to predict the sales in 2010.
15. Compare the new correlation coefficient to the old value and state whether the regression line fits the data better. See margin.

2-6 Special Functions

What You'll Learn

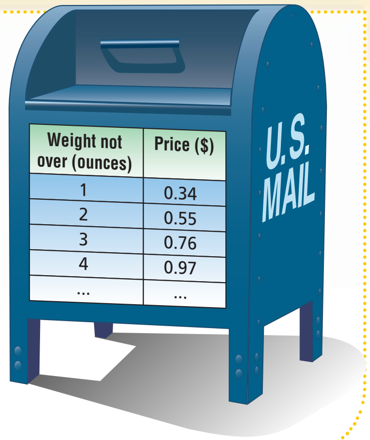
- Identify and graph step, constant, and identity functions.
- Identify and graph absolute value and piecewise functions.

Vocabulary

- step function
- greatest integer function
- constant function
- identity function
- absolute value function
- piecewise function

How do step functions apply to postage rates?

The cost of the postage to mail a letter is a function of the weight of the letter. But the function is not linear. It is a special function called a **step function**.



Weight not over (ounces)	Price ($)
1	0.34
2	0.55
3	0.76
4	0.97
...	...

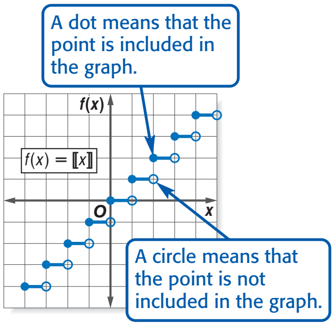
STEP FUNCTIONS, CONSTANT FUNCTIONS, AND THE IDENTITY FUNCTION The graph of a step function is not linear. It consists of line segments or rays. The **greatest integer function**, written $f(x) = [\![x]\!]$, is an example of a step function. The symbol $[\![x]\!]$ means *the greatest integer less than or equal to x*. For example, $[\![7.3]\!] = 7$ and $[\![-1.5]\!] = -2$ because $-1 > -1.5$. Study the table and graph below.

Study Tip

Greatest Integer Function

Notice that the domain of this step function is all real numbers and the range is all integers.

$f(x) = [\![x]\!]$	
x	$f(x)$
$-3 \le x < -2$	-3
$-2 \le x < -1$	-2
$-1 \le x < 0$	-1
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2
$3 \le x < 4$	3



Example 1 Step Function

BUSINESS **Labor costs at the Fix-It Auto Repair Shop are $60 per hour or any fraction thereof. Draw a graph that represents this situation.**

Explore The total labor charge must be a multiple of $60, so the graph will be the graph of a step function.

Plan If the time spent on labor is greater than 0 hours, but less than or equal to 1 hour, then the labor cost is $60. If the time is greater than 1 hour but less than or equal to 2 hours, then the labor cost is $120, and so on.

Solve Use the pattern of times and costs to make a table, where x is the number of hours of labor and $C(x)$ is the total labor cost. Then draw the graph.

(continued on the next page)

2-6 Lesson Notes

1 Focus

5-Minute Check Transparency 2-6 Use as a quiz or review of Lesson 2-5.

Mathematical Background notes are available for this lesson on p. 54D.

Building on Prior Knowledge

In Lesson 2-5, students drew scatter plots. In this lesson they draw graphs of several special functions: step, constant, identity, absolute value, and piecewise.

How do step functions apply to postage rates?

Ask students:

- What is the cost of mailing a letter when the weight is 0.9 ounces? **$0.34** when it is 1.1 ounces? **$0.55** What is the ratio of the change in price over the change in weight from 0.9 ounce to 1.1 ounces? **1.05** from 1.8 ounces to 2.0 ounces? **0**
- How can you tell that this is not a linear function? **Sample answer: The slope between some pairs of points is not the same as the slope between other pairs of points.**

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 87–88
- Skills Practice, p. 89
- Practice, p. 90
- Reading to Learn Mathematics, p. 91
- Enrichment, p. 92
- Assessment, p. 114

Graphing Calculator and Spreadsheet Masters, p. 29

Teaching Algebra With Manipulatives Masters, p. 219

Transparencies

5-Minute Check Transparency 2-6
Answer Key Transparencies

Technology

Interactive Chalkboard

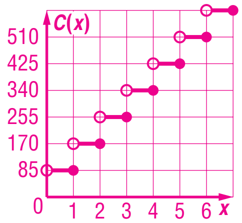
2 Teach

STEP FUNCTIONS, CONSTANT FUNCTIONS, AND THE IDENTITY FUNCTION

In-Class Examples

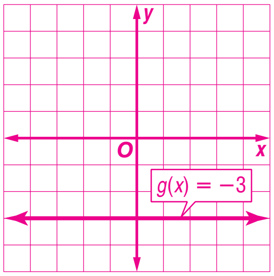
Power Point®

1 **PSYCHOLOGY** One psychologist charges for counseling sessions at the rate of $85 per hour or any fraction thereof. Draw a graph that represents this situation.



Teaching Tip Help clear up confusions about step functions by asking students to choose several sample times, in hours and minutes, and find the associated costs. Lead them to see that two different times (x values) can have the same cost (C value).

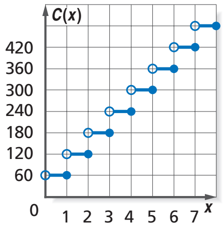
2 Graph $g(x) = -3$.



Tips for New Teachers

Intervention Students who are having trouble with the absolute value graphs may see more clearly what is happening if they draw their own graph of the parent function $y = |x|$.

x	$C(x)$
$0 < x \leq 1$	$60
$1 < x \leq 2$	$120
$2 < x \leq 3$	$180
$3 < x \leq 4$	$240
$4 < x \leq 5$	$300



Examine Since the shop rounds any fraction of an hour up to the next whole number, each segment on the graph has a circle at the left endpoint and a dot at the right endpoint.

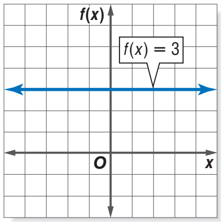
You learned in Lesson 2-4 that the slope-intercept form of a linear function is $y = mx + b$, or in functional notation, $f(x) = mx + b$. When $m = 0$, the value of the function is $f(x) = b$ for every x value. So, $f(x) = b$ is called a **constant function**. *The function $f(x) = 0$ is called the zero function.*

Example 2 Constant Function

Graph $f(x) = 3$.

For every value of x, $f(x) = 3$. The graph is a horizontal line.

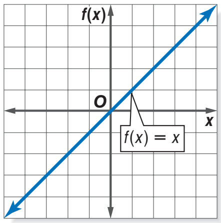
$f(x) = 3$	
x	$f(x)$
−2	3
−0.5	3
0	3
$\frac{1}{3}$	3



Another special case of slope-intercept form is $m = 1$, $b = 0$. This is the function $f(x) = x$. The graph is the line through the origin with slope 1.

Since the function does not change the input value, $f(x) = x$ is called the **identity function**.

$f(x) = x$	
x	$f(x)$
−2	−2
−0.5	−0.5
0	0
$\frac{1}{3}$	$\frac{1}{3}$



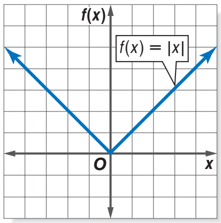
Study Tip

Absolute Value Function

Notice that the domain is all real numbers and the range is all nonnegative real numbers.

ABSOLUTE VALUE AND PIECEWISE FUNCTIONS Another special function is the **absolute value function**, $f(x) = |x|$.

$f(x) = \|x\|$	
x	$f(x)$
−3	3
−2	2
−1	1
0	0
1	1
2	2
3	3



The absolute value function can be written as $f(x) = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$. A function that is written using two or more expressions is called a **piecewise function**.

Recall that a family of graphs is a group of graphs that displays one or more similar characteristics. The parent graph of most absolute value functions is $y = |x|$.

Example 3 Absolute Value Functions

Graph $f(x) = |x| + 1$ and $g(x) = |x| - 2$ on the same coordinate plane. Determine the similarities and differences in the two graphs.

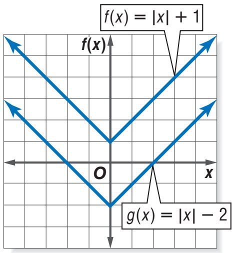
Find several ordered pairs for each function.

x	$\lvert x\rvert + 1$
−2	3
−1	2
0	1
1	2
2	3

x	$\lvert x\rvert - 2$
−2	0
−1	−1
0	−2
1	−1
2	0

Graph the points and connect them.

- The domain of each function is all real numbers.
- The range of $f(x) = |x| + 1$ is $\{y \mid y \geq 1\}$.
  The range of $g(x) = |x| - 2$ is $\{y \mid y \geq -2\}$.
- The graphs have the same shape, but different y-intercepts.
- The graph of $g(x) = |x| - 2$ is the graph of $f(x) = |x| + 1$ translated down 3 units.

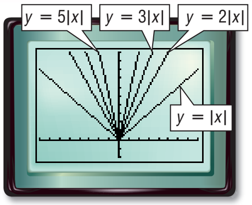


You can also use a graphing calculator to investigate families of absolute value graphs.

Graphing Calculator Investigation

Families of Absolute Value Graphs

The calculator screen shows the graphs of $y = |x|$, $y = 2|x|$, $y = 3|x|$, and $y = 5|x|$.



[−8, 8] scl: 1 by [−2, 10] scl: 1

Think and Discuss

1. What do these graphs have in common?
2. Describe how the graph of $y = a|x|$ changes as a increases. Assume $a > 0$.
3. Write an absolute value function whose graph is between the graphs of $y = 2|x|$ and $y = 3|x|$. **Sample answer: $y = 2.5|x|$**
4. Graph $y = |x|$ and $y = -|x|$ on the same screen. Then graph $y = 2|x|$ and $y = -2|x|$ on the same screen. What is true in each case?
5. In general, what is true about the graph of $y = a|x|$ when $a < 0$?

Study Tip

Look Back

To review **families of graphs**, see Lesson 2-3.

TEACHING TIP

abs(is located on the MATH NUM menu.

1. All of the graphs have a corner point at the origin.
2. The graph becomes narrower.
4. The graphs are reflections of each other about the x-axis.
5. The graph opens downward.

www.algebra2.com/extra_examples

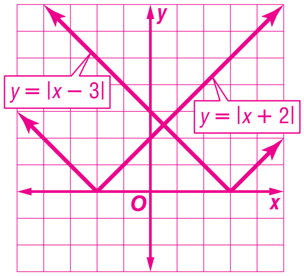
ABSOLUTE VALUE AND PIECEWISE FUNCTIONS

In-Class Example

Power Point®

Teaching Tip Lead students to see that the y-intercept, b, of the graph of an absolute value function of the form $y = |x| + b$ indicates how the parent graph of $y = |x|$ is translated.

3 Graph $f(x) = |x - 3|$ and $g(x) = |x + 2|$ on the same coordinate plane. Determine the similarities and differences in the two graphs.



The domain of both graphs is all real numbers. The range of $f(x) = |x - 3|$ is $\{y|y \geq 0\}$. The range of $g(x) = |x + 2|$ is $\{y|y \geq 0\}$. The graphs have the same shape, but different x-intercepts. The graph of $g(x) = |x + 2|$ is the graph of $f(x) = |x - 3|$ translated 5 units to the left.

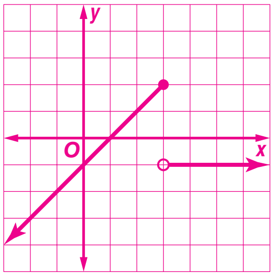
Graphing Calculator Investigation

Families of Absolute Value Graphs It is important that students arrive at the conclusion in Exercise 2 that the graph of the function narrows as the coefficient a increases. As an extension, ask students to compare the graph of $y = |x|$ with the graph of $y = |x| + 2$ and with $y = 2|x|$. Make sure they see the difference between adding 2 and having 2 as a coefficient.

In-Class Examples

Power Point®

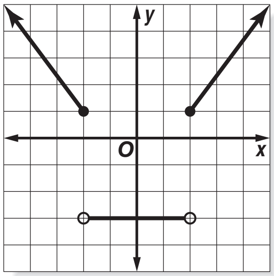
4 Graph $f(x) = \begin{cases} x - 1 \text{ if } x \leq 3 \\ -1 \text{ if } x > 3 \end{cases}$. Identify the domain and range.



The domain is all real numbers. The range is $\{y | y \leq 2\}$.

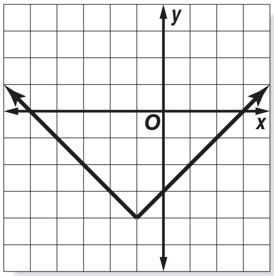
5 Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.



piecewise function

b.



absolute value function

To graph other piecewise functions, examine the inequalities in the definition of the function to determine how much of each piece to include.

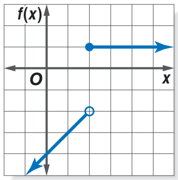
Study Tip

Graphs of Piecewise Functions

The graphs of each part of a piecewise function may or may not connect. A graph may stop at a given x value and then begin again at a different y value for the same x value.

Example 4 Piecewise Function

Graph $f(x) = \begin{cases} x - 4 \text{ if } x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$. Identify the domain and range.

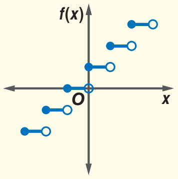
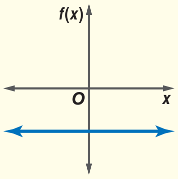
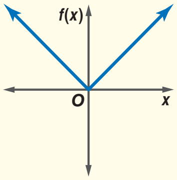
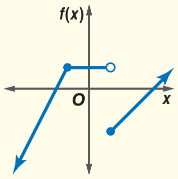


Step 1 Graph the linear function $f(x) = x - 4$ for $x < 2$. Since 2 does not satisfy this inequality, stop with an open circle at $(2, -2)$.

Step 2 Graph the constant function $f(x) = 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a closed circle at $(2, 1)$ and draw a horizontal ray to the right.

The function is defined for all values of x, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 1 and all real numbers less than -2, so the range is $\{y \mid y < -2 \text{ or } y = 1\}$.

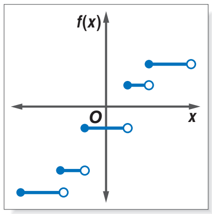
Concept Summary — Special Functions

Step Function	Constant Function	Absolute Value Function	Piecewise Function
			
horizontal segments and rays	horizontal line	V-shape	different rays, segments, and curves

Example 5 Identify Functions

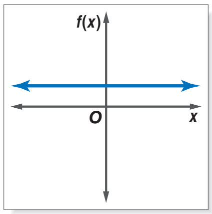
Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.



Since this graph consists of multiple horizontal segments, it represents a step function.

b.



Since this graph is a horizontal line, it represents a constant function.

Check for Understanding

Concept Check

1. **Find a counterexample** to the statement *To find the greatest integer function of x when x is not an integer, round x to the nearest integer.* Sample answer: $[\![1.9]\!] = 1$
2. **Evaluate** $g(4.3)$ if $g(x) = [\![x - 5]\!]$. -1
3. **OPEN ENDED** Write a function involving absolute value for which $f(-2) = 3$.

3. Sample answer: $f(x) = |x - 1|$

DAILY INTERVENTION

Differentiated Instruction ELL

Verbal/Linguistic Have students explain why step functions, constant functions, and piecewise functions are so named.

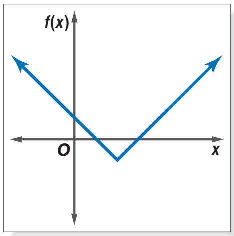
Guided Practice

GUIDED PRACTICE KEY

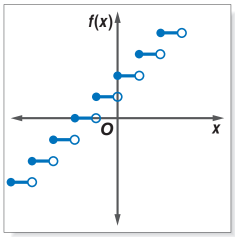
Exercises	Examples
4, 5	5
6–11	1–4
12–14	1

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

4. A



5. S



Graph each function. Identify the domain and range. 6–11. See pp. 107A–107H for graphs.

6. $f(x) = -[\![x]\!]$

7. $g(x) = [\![2x]\!]$

8. $h(x) = |x - 4|$

9. $f(x) = |3x - 2|$

10. $g(x) = \begin{cases} -1 \text{ if } x < 0 \\ -x + 2 \text{ if } x \geq 0 \end{cases}$ D = all reals, R = {$y|y \leq 2$}

11. $h(x) = \begin{cases} x + 3 \text{ if } x \leq -1 \\ 2x \text{ if } x > -1 \end{cases}$ D = all reals, R = all reals

6. D = all reals, R = all integers
7. D = all reals, R = all integers
8. D = all reals, R = all nonnegative reals
9. D = all reals, R = all nonnegative reals

Application

PARKING For Exercises 12–14, use the following information.

A downtown parking lot charges \$2 for the first hour and \$1 for each additional hour or part of an hour.

12. What type of special function models this situation? step function
13. Draw a graph of a function that represents this situation. See margin.
14. Use the graph to find the cost of parking there for $4\frac{1}{2}$ hours. \$6

★ indicates increased difficulty

Practice and Apply

Homework Help

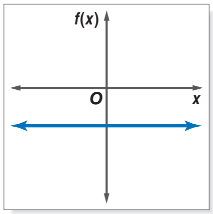
For Exercises	See Examples
15–20	5
21–29	1
30–37, 45–47, 49	3
38–41, 44, 48	2, 4
42, 43	1, 3

Extra Practice

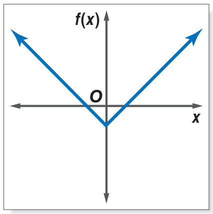
See page 831.

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

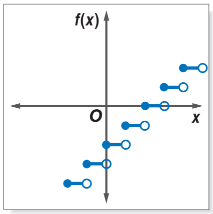
15. C



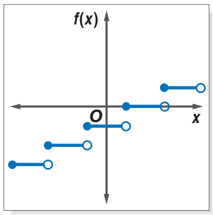
16. A



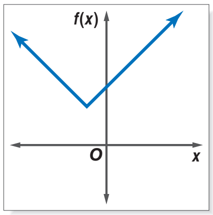
17. S



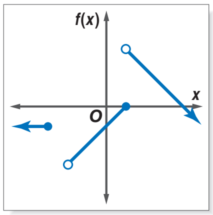
18. S



19. A



20. P



21. **TRANSPORTATION** Bluffton High School chartered buses so the student body could attend the girls' basketball state tournament games. Each bus held a maximum of 60 students. Draw a graph of a step function that shows the relationship between the number of students x who went to the game and the number of buses y that were needed. See margin.

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 2.
- draw their own graphs to compare and contrast step, constant, absolute value, and piecewise functions.
- Include any other item(s) that they find helpful in mastering the skills in the lesson.

About the Exercises...

Organization by Objective
- **Step Functions, Constant Functions, and the Identity Function:** 24–29
- **Absolute Value and Piecewise Functions:** 30–41

Odd/Even Assignments

Exercises 15–20 and 24–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–21 odd, 22, 23, 25–41 odd, 49–65

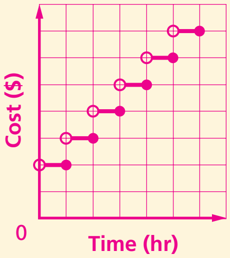
Average: 15–21 odd, 22, 23, 25–43 odd, 49–65

Advanced: 16–20 even, 22, 23, 24–44 even, 45–59 (optional: 60–65)

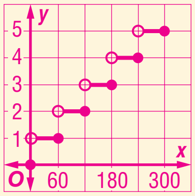
All: Practice Quiz 2 (1–5)

Answers

13.



21.



Study Guide and Intervention, p. 87 (shown) and p. 88

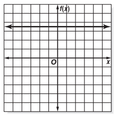
Step Functions, Constant Functions, and the Identity Function The chart below lists some special functions you should be familiar with.

Function	Written as	Graph
Constant	$f(x) = c$	horizontal line
Identity	$f(x) = x$	line through the origin with slope 1
Greatest Integer Function	$f(x) = \llbracket x \rrbracket$	one-unit horizontal segments, with right endpoints missing, arranged like steps

The greatest integer function is an example of a **step function**, a function with a graph that consists of horizontal segments.

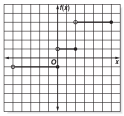
Example **Identify each function as a constant function, the identity function, or a step function.**

a.



a constant function

b.

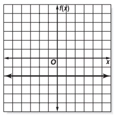


a step function

Exercises

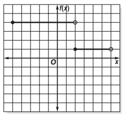
Identify each function as a constant function, the identity function, a greatest integer function, or a step function.

1.



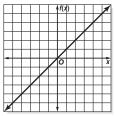
a constant function

2.



a step function

3.

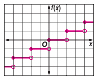


the identity function

Skills Practice, p. 89 and Practice, p. 90 (shown)

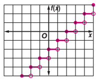
Graph each function. Identify the domain and range.

1. $f(x) = \llbracket 0.5x \rrbracket$



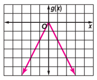
D = all reals, R = all integers

2. $f(x) = \llbracket x \rrbracket - 2$



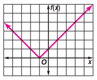
D = all reals, R = all integers

3. $g(x) = -2|x|$



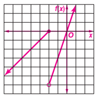
D = all reals,
R = nonpositive reals

4. $f(x) = |x + 1|$



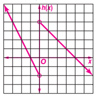
D = all reals,
R = nonnegative reals

5. $f(x) = \begin{cases} x + 2 \text{ if } x \leq -2 \\ 3x \text{ if } x > -2 \end{cases}$



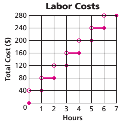
D = all reals, R = all reals

6. $h(x) = \begin{cases} 4 - x \text{ if } x > 0 \\ -2x - 2 \text{ if } x < 0 \end{cases}$

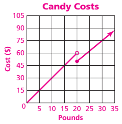


D = all nonzero reals, R = all reals

7. **BUSINESS** *A Stitch in Time* charges \$40 per hour or any fraction thereof for labor. Draw a graph of the step function that represents this situation.



8. **BUSINESS** A wholesaler charges a store \$3.00 per pound for less than 20 pounds of candy and \$2.50 per pound for 20 or more pounds. Draw a graph of the function that represents this situation.



Reading to Learn Mathematics, p. 91

ELL

Pre-Activity **How do step functions apply to postage rates?**

Read the introduction to Lesson 2-6 at the top of page 89 in your textbook.

- What is the cost of mailing a letter that weighs 0.5 ounce? **\$0.34 or 34 cents**
- Give three different weights of letters that would each cost 55 cents to mail. **Answers will vary. Sample answer: 1.1 ounces, 1.9 ounces, 2.0 ounces**

Reading the Lesson

1. Find the value of each expression.

a. $|-3| =$ **3** $\llbracket -3 \rrbracket =$ **−3**

b. $|6.2| =$ **6.2** $\llbracket 6.2 \rrbracket =$ **6**

c. $|-4.01| =$ **4.01** $\llbracket -4.01 \rrbracket =$ **−5**

2. Tell how the name of each kind of function can help you remember what the graph looks like.

a. constant function **Sample answer: Something is constant if it does not change. The *y*-values of a constant function do not change, so the graph is a horizontal line.**

b. absolute value function **Sample answer: The absolute value of a number tells you how far it is from 0 on the number line. It makes no difference whether you go to the left or right so long as you go the same distance each time.**

c. step function **Sample answer: A step function's graph looks like steps that go up or down.**

d. identity function **Sample answer: The *x*- and *y*-values are always identically the same for any point on the graph. So the graph is a line through the origin that has slope 1.**

Helping You Remember

3. Many students find the greatest integer function confusing. Explain how you can use a number line to find the value of this function for any real number. **Answers will vary. Sample answer: Draw a number line that shows the integers. To find the value of the greatest integer function for any real number, place that number on the number line. If it is an integer, the value of the function is the number itself. If not, move to the integer directly to the left of the number you chose. This integer will give the value you need.**

TELEPHONE RATES **For Exercises 22 and 23, use the following information.**
Sarah has a long-distance telephone plan where she pays 10¢ for each minute or part of a minute that she talks, regardless of the time of day.

22. Graph a step function that represents this situation. **See pp. 107A–107H.**

23. Sarah made a call to her brother that lasted 9 minutes and 40 seconds. How much did the call cost? **\$1.00**

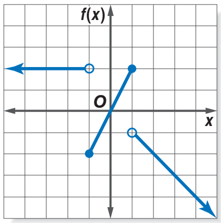
Graph each function. Identify the domain and range. **24–43. See pp. 107A–107H.**

24. $f(x) = \llbracket x + 3 \rrbracket$
25. $g(x) = \llbracket x - 2 \rrbracket$
26. $f(x) = 2\llbracket x \rrbracket$
27. $h(x) = -3\llbracket x \rrbracket$
28. $g(x) = \llbracket x \rrbracket + 3$
29. $f(x) = \llbracket x \rrbracket - 1$
30. $f(x) = |2x|$
31. $h(x) = |-x|$
32. $g(x) = |x| + 3$
33. $g(x) = |x| - 4$
34. $h(x) = |x + 3|$
35. $f(x) = |x + 2|$
36. $f(x) = \left|x - \frac{1}{4}\right|$
37. $f(x) = \left|x + \frac{1}{2}\right|$
38. $f(x) = \begin{cases} -x \text{ if } x \leq 3 \\ 2 \text{ if } x > 3 \end{cases}$
39. $h(x) = \begin{cases} -1 \text{ if } x < -2 \\ 1 \text{ if } x > 2 \end{cases}$
40. $f(x) = \begin{cases} x \text{ if } x < -3 \\ 2 \text{ if } -3 \leq x < 1 \\ -2x + 2 \text{ if } x \geq 1 \end{cases}$
41. $g(x) = \begin{cases} -1 \text{ if } x \leq -2 \\ x \text{ if } -2 < x < 2 \\ -x + 1 \text{ if } x \geq 2 \end{cases}$

★ 42. $f(x) = \llbracket |x| \rrbracket$
★ 43. $g(x) = |\llbracket x \rrbracket|$

★ 44. Write the function shown in the graph.

$$f(x) = \begin{cases} 2 \text{ if } x < -1 \\ 2x \text{ if } -1 \leq x < 1 \\ -x \text{ if } x > 1 \end{cases}$$



More About. . .

Nutrition

Good sources of vitamin C include citrus fruits and juices, cantaloupe, broccoli, brussels sprouts, potatoes, sweet potatoes, tomatoes, and cabbage.

Source: *The World Almanac*

NUTRITION **For Exercises 45–47, use the following information.**
The recommended dietary allowance for vitamin C is 2 micrograms per day.

45. $f(x) = |x - 2|$

45. Write an absolute value function for the difference between the number of micrograms of vitamin C you ate today x and the recommended amount.

46. What is an appropriate domain for the function? $\{x \mid x \geq 0\}$

47. Use the domain to graph the function. **See pp. 107A–107H.**

48. **INSURANCE** According to the terms of Lavon's insurance plan, he must pay the first \$300 of his annual medical expenses. The insurance company pays 80% of the rest of his medical expenses. Write a function for how much the insurance company pays if x represents Lavon's annual medical expenses. **See margin.**

49. **CRITICAL THINKING** Graph $|x| + |y| = 3$. **See pp. 107A–107H.**

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do step functions apply to postage rates?

Include the following in your answer: **See pp. 107A–107H.**

- an explanation of why a step function is the best model for this situation, while your gas mileage as a function of time as you drive to the post office cannot be modeled with a step function, and
- a graph of a function that represents the cost of a first-class letter.

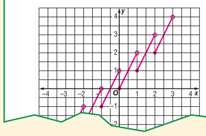
Enrichment, p. 92

Greatest Integer Functions

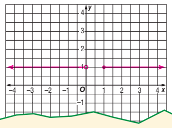
Use the greatest integer function $\llbracket x \rrbracket$ to explore some unusual graphs. It will be helpful to make a chart of values for each functions and to use a colored pen or pencil.

Graph each function.

1. $y = 2x - \llbracket x \rrbracket$



2. $y = \frac{\llbracket x \rrbracket}{\llbracket x \rrbracket}$



Answer

$$48.\ f(x) = \begin{cases} 0 \text{ if } 0 \leq x \leq 300 \\ 0.8(x - 300) \text{ if } x > 300 \end{cases}$$

Standardized Test Practice

51. For which function does $f\left(-\frac{1}{2}\right) \neq -1$? **B**

Ⓐ $f(x) = 2x$ Ⓑ $f(x) = |-2x|$ Ⓒ $f(x) = [\![x]\!]$ Ⓓ $f(x) = [\![2x]\!]$

52. For which function is the range $\{y \mid y \leq 0\}$? **D**

Ⓐ $f(x) = -x$ Ⓑ $f(x) = [\![x]\!]$ Ⓒ $f(x) = |x|$ Ⓓ $f(x) = -|x|$

Maintain Your Skills

Mixed Review

HEALTH **For Exercises 53–55, use the table that shows the life expectancy for people born in various years.** *(Lesson 2-5)*

Year	1950	1960	1970	1980	1990	1997
Expectancy	68.2	69.7	70.8	73.7	75.4	76.5

Source: National Center for Health Statistics

53. Draw a scatter plot in which x is the number of years since 1950. **See margin.**

54. Find a prediction equation.

55. Predict the life expectancy of a person born in 2010. **Sample answer: 78.7 yr**

54. Sample answer using (10, 69.7) and (47, 76.5): $y = 0.18x + 67.9$

Write an equation in slope-intercept form that satisfies each set of conditions. *(Lesson 2-4)*

56. slope 3, passes through $(-2, 4)$ **$y = 3x + 10$**

57. passes through $(0, -2)$ and $(4, 2)$ **$y = x - 2$**

Solve each inequality. Graph the solution set. *(Lesson 1-5)*

58. $3x - 5 \geq 4$ **$\{x \mid x \geq 3\}$**

59. $28 - 6y < 23$ **$\left\{y \mid y > \frac{5}{6}\right\}$**

58–59. See margin for graphs.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Determine whether (0, 0) satisfies each inequality. Write *yes* or *no*.** *(To review **inequalities**, see Lesson 1-5.)*

60. $y < 2x + 3$ **yes**

61. $y \geq -x + 1$ **no**

62. $y \leq \frac{3}{4}x - 5$ **no**

63. $2x + 6y + 3 > 0$ **yes**

64. $y > |x|$ **no**

65. $|x| + y \leq 3$ **yes**

Practice Quiz 2

Lessons 2-4 through 2-6

1. Write an equation in slope-intercept form of the line with slope $-\frac{2}{3}$ that passes through $(-2, 5)$. *(Lesson 2-4)* **$y = -\frac{2}{3}x + \frac{11}{3}$**

BASKETBALL **For Exercises 2–4, use the following information.**
On August 26, 2000, the Houston Comets beat the New York Liberty to win their fourth straight WNBA championship. The table shows the heights and weights of the Comets who played in that final game. *(Lesson 2-5)*

Height (in.)	74	71	76	70	66	74	72
Weight (lb)	178	147	195	150	138	190	?

Source: WNBA

2. Draw a scatter plot. **See margin.**

3. Use two ordered pairs to write a prediction equation. **Sample answer using (66, 138) and (74, 178): $y = 5x - 192$**

4. Use your prediction equation to predict the missing value. **Sample answer: 168 lb**

5. Graph $f(x) = |x - 1|$. Identify the domain and range. *(Lesson 2-6)* **See margin.**

4 Assess

Open-Ended Assessment

Modeling Have students draw a large coordinate plane on a sheet of paper. Then have them use toothpicks (or other similar objects) to model the general shapes of step, constant, and absolute value functions. Students should identify each type of graph as they model it.

Getting Ready for Lesson 2-7

PREREQUISITE SKILL Lesson 2-7 presents graphing inequalities. It is sometimes necessary to solve an inequality for y in order to determine whether to shade above or below the boundary. Exercises 60–65 should be used to determine your students' familiarity with inequalities.

Assessment Options

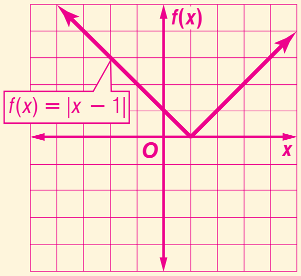
Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 2-4 through 2-6. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 2-5 and 2-6) is available on p. 114 of the *Chapter 2 Resource Masters*.

Answers (Practice Quiz 2)

2. Houston Comets

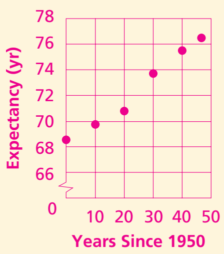
5.



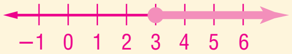
D = all reals
R = nonnegative reals

Answers

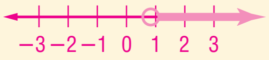
53. Life Expectancy



58.



59.



2-7 Lesson Notes

1 Focus

5-Minute Check Transparency 2-7 Use as a quiz or review of Lesson 2-6.

Mathematical Background notes are available for this lesson on p. 54D.

How do inequalities apply to fantasy football?

Ask students:

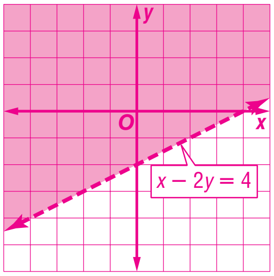
- What is the meaning of "receiving yards?" **the number of yards that a team advances down the field when the receiver of a pass catches the ball**

2 Teach

GRAPH LINEAR INEQUALITIES

In-Class Example Power Point®

1 Graph $x - 2y < 4$.



2-7 Graphing Inequalities

What You'll Learn

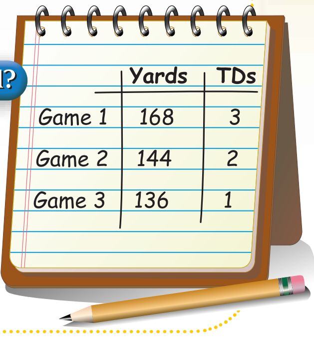
- Graph linear inequalities.
- Graph absolute value inequalities.

Vocabulary

- boundary

How do inequalities apply to fantasy football?

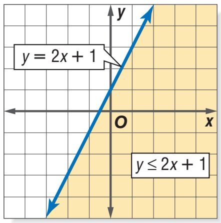
Dana has Vikings receiver Randy Moss as a player on his online fantasy football team. Dana gets 5 points per receiving yard that Moss gets and 100 points per touchdown that Moss scores. He considers 1000 points or more to be a good game. Dana can use a linear inequality to check whether certain combinations of yardage and touchdowns, such as those in the table, result in 1000 points or more.



	Yards	TDs
Game 1	168	3
Game 2	144	2
Game 3	136	1

GRAPH LINEAR INEQUALITIES A linear inequality resembles a linear equation, but with an inequality symbol instead of an equals symbol. For example, $y \leq 2x + 1$ is a linear inequality and $y = 2x + 1$ is the related linear equation.

The graph of $y = 2x + 1$ separates the coordinate plane into two regions. The line is the **boundary** of each region. The graph of the inequality $y \leq 2x + 1$ is the shaded region. Every point in the shaded region satisfies the inequality. The graph of $y = 2x + 1$ is drawn as a solid line to show that points on the line satisfy the inequality. If the inequality symbol were $<$ or $>$, then points on the boundary would not satisfy the inequality, so the boundary would be drawn as a dashed line.



You can graph an inequality by following these steps.

Step 1 Determine whether the boundary should be solid or dashed. Graph the boundary.

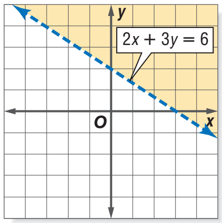
Step 2 Choose a point not on the boundary and test it in the inequality.

Step 3 If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

Example 1 *Dashed Boundary*

Graph $2x + 3y > 6$.

The boundary is the graph of $2x + 3y = 6$. Since the inequality symbol is $>$, the boundary will be dashed. Use the slope-intercept form, $y = -\frac{2}{3}x + 2$.



Now test the point (0, 0). *The point (0, 0) is usually a good point to test because it results in easy calculations.*

$2x + 3y > 6$ Original inequality

$2(0) + 3(0) > 6$ $(x, y) = (0, 0)$

$0 > 6$ false

Shade the region that does *not* contain (0, 0).

TEACHING TIP

Students may wish to test a point in the shaded region as a check of their work.

Resource Manager

Workbook and Reproducible Masters

Chapter 2 Resource Masters

- Study Guide and Intervention, pp. 93–94
- Skills Practice, p. 95
- Practice, p. 96
- Reading to Learn Mathematics, p. 97
- Enrichment, p. 98
- Assessment, p. 114

Transparencies

5-Minute Check Transparency 2-7
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 4
Interactive Chalkboard

Inequalities can sometimes be used to model real-world situations.

Example 2 Solid Boundary

BUSINESS **A mail-order company is hiring temporary employees to help in their packing and shipping departments during their peak season.**

a. Write an inequality to describe the number of employees that can be assigned to each department if the company has 20 temporary employees available.

Let *p* be the number of employees assigned to packing and let *s* be the number assigned to shipping. Since the company can assign *at most* 20 employees total to the two departments, use a ≤ symbol.

$$\underbrace{\text{The number of employees for packing}}_{p}\ \underbrace{\text{and}}_{+}\ \underbrace{\text{the number of employees for shipping}}_{s}\ \underbrace{\text{is at most}}_{\le}\ \underbrace{\text{twenty.}}_{20}$$

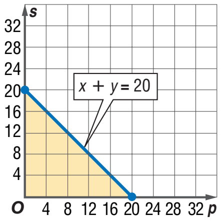
b. Graph the inequality.

Since the inequality symbol is ≤, the graph of the related linear equation $p + s = 20$ is solid. This is the boundary of the inequality.

Test (0, 0).

$p + s \le 20$	Original inequality
$0 + 0 \le 20$	$(p, s) = (0, 0)$
$0 \le 20$	true

Shade the region that contains (0, 0). *Since the variables cannot be negative, shade only the part in the first quadrant.*



c. Can the company assign 8 employees to packing and 10 employees to shipping?

The point (8, 10) is in the shaded region, so it satisfies the inequality. The company can assign 8 employees to packing and 10 to shipping.

GRAPH ABSOLUTE VALUE INEQUALITIES

Graphing absolute value inequalities is similar to graphing linear inequalities. The inequality symbol determines whether the boundary is solid or dashed, and you can test a point to determine which region to shade.

Example 3 Absolute Value Inequality

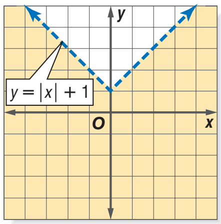
Graph $y < |x| + 1$.

Since the inequality symbol is <, the graph of the related equation $y = |x| + 1$ is dashed. Graph the equation.

Test (0, 0).

$y < \|x\| + 1$	Original inequality
$0 < \|0\| + 1$	$(x, y) = (0, 0)$
$0 < 0 + 1$	$\|0\| = 0$
$0 < 1$	true

Shade the region that includes (0, 0).



Study Tip

Look Back

To review **translating verbal expressions to inequalities,** see Lesson 1-5.

www.algebra2.com/extra_examples

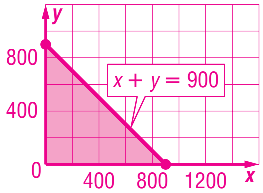
In-Class Example

Power Point®

2 **EDUCATION** The SAT has two parts. One tutoring company advertises that it specializes in helping students who have a combined score on the SAT that is 900 or less.

a. Write an inequality to describe the combined scores of students who are prospective tutoring clients. $x + y \le 900$

b. Graph the inequality.



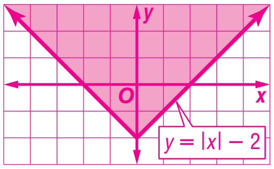
c. Does a student with a verbal score of 480 and a math score of 410 fit the tutoring company's guidelines? **yes**

GRAPH ABSOLUTE VALUE INEQUALITIES

In-Class Example

Power Point®

3 Graph $y \ge |x| - 2$.



3 Practice/Apply

Study Notebook

Have students—

- *complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 2.*
- *include any other items(s) that they find helpful in mastering the skills in this lesson.*

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students work in pairs as they graph the inequalities, so they can check and correct each other's procedures and results. When graphing, one partner can work to determine the location of the boundary while the other partner determines whether the shaded region is above or below the boundary.

About the Exercises...

Organization by Objective

- **Graph Linear Inequalities:** 13–24, 33–39
- **Graph Absolute Value Inequalities:** 25–30

Odd/Even Assignments

Exercises 13–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 44–47 require a graphing calculator.

Assignment Guide

Basic: 13–27 odd, 31, 33, 34, 40–43, 48–56

Average: 13–31 odd, 33–37, 40–43, 48–56 (optional: 44–47)

Advanced: 14–32 even, 38–56

4 Assess

Open-Ended Assessment

Speaking Have students explain how to tell just from looking at an inequality with y alone on the left whether the shaded area will be below or above the boundary, as well as whether the boundary is solid or dashed.

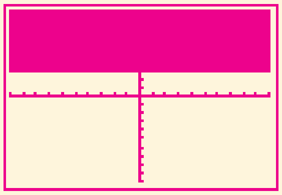
Assessment Options

Quiz (Lesson 2-7) is available on p. 114 of the *Chapter 2 Resource Masters*.

Answers

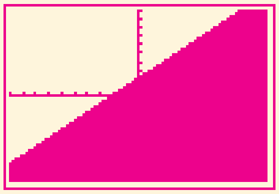
2. Substitute the coordinates of a point not on the boundary into the inequality. If the inequality is satisfied, shade the region containing the point. If the inequality is not satisfied, shade the region that does not contain the point.

44.



[−10, 10] scl: 1 by [−10, 10] scl: 1

45.



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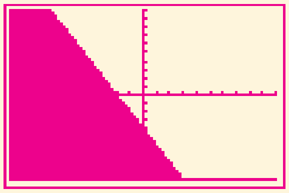
46.



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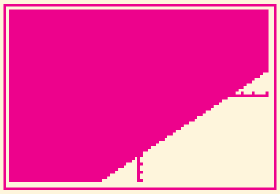
47.

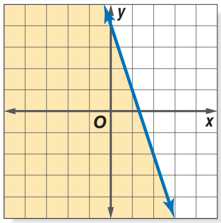


[−10, 10] scl: 1 by [−10, 10] scl: 1

Check for Understanding

Concept Check

1. **Write** an inequality for the graph at the right. **1.** $y \le -3x + 4$



2. **Explain** how to determine which region to shade when graphing an inequality. **See margin.**
3. **OPEN ENDED** Write an absolute value inequality for which the boundary is solid and the solution is the region above the graph of the related equation. **Sample answer:** $y \ge |x|$

Guided Practice

Graph each inequality. **4–9. See pp. 107A–107H.**

4. $y < 2$
5. $y > 2x - 3$
6. $x - y \ge 0$
7. $x - 2y \le 5$
8. $y > |2x|$
9. $y \le 3|x| - 1$

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1, 2
8, 9	3
10–12	2

Application

SHOPPING For Exercises 10–12, use the following information.
Gwen wants to buy some cassettes that cost \$10 each and some CDs that cost \$13 each. She has \$40 to spend.

10. Write an inequality to represent the situation, where c is the number of cassettes she buys and d is the number of CDs. $10c + 13d \le 40$
11. Graph the inequality. **See pp. 107A–107H.**
12. Can she buy 3 cassettes and 2 CDs? Explain. **No; (3, 2) is not in the shaded region.**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
13–24, 31, 32	1, 2
25–30, 41	3
33–40	2

Extra Practice

See page 832.

Graph each inequality. **13–30. See pp. 107A–107H.**

13. $x + y > -5$
14. $3 \ge x - 3y$
15. $y > 6x - 2$
16. $x - 5 \le y$
17. $y \ge -4x + 3$
18. $y - 2 < 3x$
19. $y \ge 1$
20. $y + 1 < 4$
21. $4x - 5y - 10 \le 0$
22. $x - 6y + 3 > 0$
23. $y > \frac{1}{3}x + 5$
24. $y \ge \frac{1}{2}x - 5$
25. $y \le |x|$
26. $y > |4x|$
27. $y + |x| < 3$
28. $y \ge |x - 1| - 2$
29. ★ $|x + y| > 1$
30. ★ $|x| \le |y|$

31. Graph all the points on the coordinate plane to the left of the graph of $x = -2$. Write an inequality to describe these points. $x < -2$
32. Graph all the points on the coordinate plane below the graph of $y = 3x - 5$. Write an inequality to describe these points. $y < 3x - 5$

31–32. See pp. 107A–107H for graphs.

SCHOOL For Exercises 33 and 34, use the following information.
Rosa's professor says that the midterm exam will count for 40% of each student's grade and the final exam will count for 60%. A score of at least 90 is required for an A.

33. The inequality $0.4x + 0.6y \ge 90$ represents this situation, where x is the midterm score and y is the final exam score. Graph this inequality. **See pp. 107A–107H.**
34. If Rosa scores 85 on the midterm and 95 on the final, will she get an A? **yes**

DRAMA For Exercises 35–37, use the following information.
Tickets for the Prestonville High School Drama Club's spring play cost \$4 for adults and \$3 for students. In order to cover expenses, at least \$2000 worth of tickets must be sold.

35. Write an inequality that describes this situation. $4a + 3s \ge 2000$
36. Graph the inequality. **See pp. 107A–107H.**
37. If 180 adult and 465 student tickets are sold, will the club cover its expenses? **yes**

More About. . .

Finance

A *dividend* is a payment from a company to an investor. It is a way to make money on a stock without selling it.

FINANCE **For Exercises 38–40, use the following information.**
Carl Talbert estimates that he will need to earn at least $9000 per year combined in dividend income from the two stocks he owns to supplement his retirement plan.

Company	Dividend per Share
Able Rentals	$1.20
Best Bikes	$1.80

38. Write and graph an inequality for this situation. **See pp. 107A–107H.**

39. Will he make enough from 3000 shares of each company? **yes**

40. **CRITICAL THINKING** Graph $|y| < x$. **See pp. 107A–107H.**

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do inequalities apply to fantasy football?

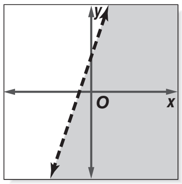
Include the following in your answer: **See pp. 107A–107H.**

- an inequality, and an explanation of how you obtained it, to represent a good game for Randy Moss in Dana's fantasy football league,
- a graph of your inequality (remember that the number of touchdowns cannot be negative, but receiving yardage can be), and
- which of the games with statistics in the table qualify as good games.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

42. Which could be the inequality for the graph? **A**



Ⓐ $y < 3x + 2$ Ⓑ $y \leq 3x + 2$
Ⓒ $y > 3x + 2$ Ⓓ $y \geq 3x + 2$

43. Which point satisfies $y > 5|x| - 3$? **B**

Ⓐ (2, 2) Ⓑ (−1, 3)
Ⓒ (3, 7) Ⓓ (−2, 4)

Graphing Calculator

SHADE(COMMAND You can graph inequalities with a graphing calculator by using the Shade(command located in the DRAW menu. You must enter two functions.

- The first function defines the lower boundary of the region to be shaded.
- The second function defines the upper boundary of the region.
- If the inequality is "$y \leq$," use the Ymin window value as the lower boundary.
- If the inequality is "$y \geq$," use the Ymax window value as the upper boundary.

Graph each inequality. **44–47. See margin.**

44. $y \geq 3$ **45.** $y \leq x + 2$ **46.** $y \leq -2x - 4$ **47.** $x - 7 \leq y$

Maintain Your Skills

Mixed Review

Graph each function. Identify the domain and range. *(Lesson 2-6)*

48. $f(x) = [\![x]\!] - 4$ **49.** $g(x) = |x| - 1$ **50.** $h(x) = |x - 3|$

48–50. See pp. 107A–107H for graphs.

48. D = all reals, R = all integers

49. D = all reals, R = $\{y \mid y \geq -1\}$

50. D = all reals, R = all nonnegative reals

SALES **For Exercises 51–53, use the table that shows the years of experience for eight sales representatives and their sales during a given period of time.** *(Lesson 2-5)*

Years	6	5	3	1	4	3	6	2
Sales ($)	9000	6000	4000	3000	6000	5000	8000	2000

51. Draw a scatter plot. **See margin.**

52. Find a prediction equation.

52. Sample answer using (4, 6000) and (6, 8000): $y = 1000x + 2000$

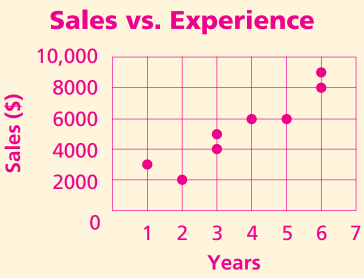
53. Predict the sales for a representative with 8 years of experience.
Sample answer: $10,000

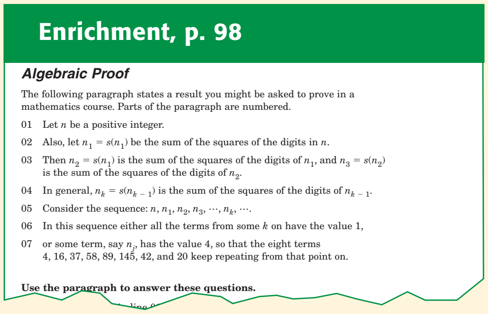
Solve each equation. Check your solution. *(Lesson 1-3)*

54. $4x - 9 = 23$ **8** **55.** $11 - 2y = 5$ **3** **56.** $2z - 3 = -6z + 1$ $\frac{1}{2}$

www.algebra2.com/self_check_quiz

51.





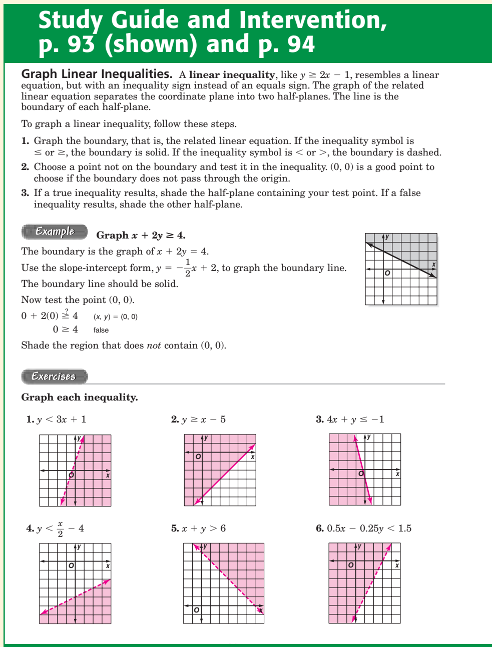
Enrichment, p. 98

Algebraic Proof

The following paragraph states a result you might be asked to prove in a mathematics course. Parts of the paragraph are numbered.

01 Let n be a positive integer.
02 Also, let $n_1 = s(n_1)$ be the sum of the squares of the digits in n.
03 Then $n_2 = s(n_1)$ is the sum of the squares of the digits of n_1, and $n_3 = s(n_2)$ is the sum of the squares of the digits of n_2.
04 In general, $n_k = s(n_{k-1})$ is the sum of the squares of the digits of n_{k-1}.
05 Consider the sequence: $n, n_1, n_2, n_3, \cdots, n_k, \cdots$.
06 In this sequence either all the terms from some k on have the value 1,
07 or some term, say n_j, has the value 4, so that the eight terms 4, 16, 37, 58, 89, 145, 42, and 20 keep repeating from that point on.

Use the paragraph to answer these questions.



Study Guide and Intervention, p. 93 (shown) and p. 94

Graph Linear Inequalities. A **linear inequality**, like $y \geq 2x - 1$, resembles a linear equation, but with an inequality sign instead of an equals sign. The graph of the related linear equation separates the coordinate plane into two half-planes. The line is the boundary of each half-plane.

To graph a linear inequality, follow these steps.

1. Graph the boundary, that is, the related linear equation. If the inequality symbol is $\leq$ or $\geq$, the boundary is solid. If the inequality symbol is $<$ or $>$, the boundary is dashed.
2. Choose a point not on the boundary and test it in the inequality. (0, 0) is a good point to choose if the boundary does not pass through the origin.
3. If a true inequality results, shade the half-plane containing your test point. If a false inequality results, shade the other half-plane.

Example **Graph $x + 2y \geq 4$.**

The boundary is the graph of $x + 2y = 4$.

Use the slope-intercept form, $y = -\frac{1}{2}x + 2$, to graph the boundary line.

The boundary line should be solid.

Now test the point (0, 0).

$0 + 2(0) \overset{?}{\geq} 4$ $(x, y) = (0, 0)$
$0 \geq 4$ false

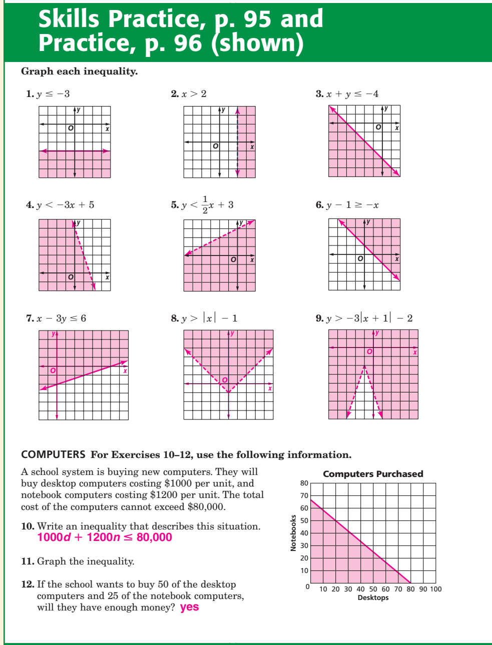
Shade the region that does *not* contain (0, 0).

Exercises

Graph each inequality.

1. $y < 3x + 1$ **2.** $y \geq x - 5$ **3.** $4x + y \leq -1$

4. $y < \frac{x}{2} - 4$ **5.** $x + y > 6$ **6.** $0.5x - 0.25y < 1.5$



Skills Practice, p. 95 and Practice, p. 96 (shown)

Graph each inequality.

1. $y \leq -3$ **2.** $x > 2$ **3.** $x + y \leq -4$

4. $y < -3x + 5$ **5.** $y < \frac{1}{2}x + 3$ **6.** $y - 1 \geq -x$

7. $x - 3y \leq 6$ **8.** $y > |x| - 1$ **9.** $y > -3|x + 1| - 2$

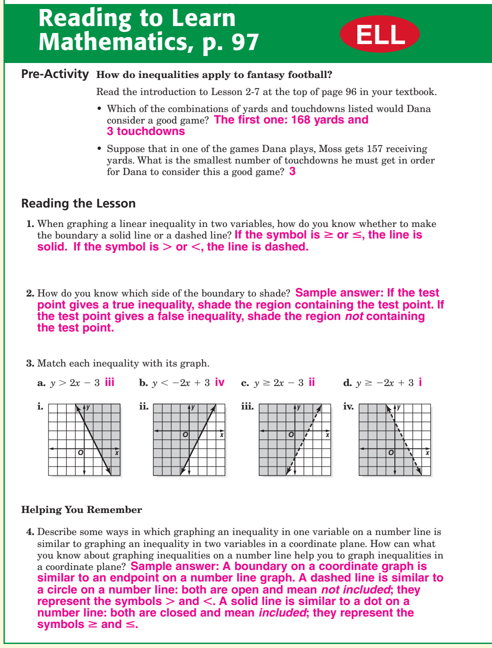
COMPUTERS **For Exercises 10–12, use the following information.**

A school system is buying new computers. They will buy desktop computers costing $1000 per unit, and notebook computers costing $1200 per unit. The total cost of the computers cannot exceed $80,000.

10. Write an inequality that describes this situation. **$1000d + 1200n \leq 80{,}000$**

11. Graph the inequality.

12. If the school wants to buy 50 of the desktop computers and 25 of the notebook computers, will they have enough money? **yes**



Reading to Learn Mathematics, p. 97

ELL

Pre-Activity **How do inequalities apply to fantasy football?**

Read the introduction to Lesson 2-7 at the top of page 96 in your textbook.

- Which of the combinations of yards and touchdowns listed would Dana consider a good game? **The first one: 168 yards and 3 touchdowns**
- Suppose that in one of the games Dana plays, Moss gets 157 receiving yards. What is the smallest number of touchdowns he must get in order for Dana to consider this a good game? **3**

Reading the Lesson

1. When graphing a linear inequality in two variables, how do you know whether to make the boundary a solid line or a dashed line? **If the symbol is ≥ or ≤, the line is solid. If the symbol is > or <, the line is dashed.**
2. How do you know which side of the boundary to shade? **Sample answer: If the test point gives a true inequality, shade the region containing the test point. If the test point gives a false inequality, shade the region *not* containing the test point.**
3. Match each inequality with its graph.

a. $y > 2x - 3$ **iii** **b.** $y < -2x + 3$ **iv** **c.** $y \geq 2x - 3$ **ii** **d.** $y \geq -2x + 3$ **i**

i. ii. iii. iv.

Helping You Remember

4. Describe some ways in which graphing an inequality in one variable on a number line is similar to graphing an inequality in two variables in a coordinate plane. How can what you know about graphing inequalities on a number line help you to graph inequalities in a coordinate plane? **Sample answer: A boundary on a coordinate graph is similar to an endpoint on a number line graph. A dashed line is similar to a circle on a number line: both are open and mean *not included*; they represent the symbols > and <. A solid line is similar to a dot on a number line: both are closed and mean *included*; they represent the symbols ≥ and ≤.**

Chapter 2

Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 2 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 2 is available on p. 112 of the *Chapter 2 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 2

Study Guide and Review

Vocabulary and Concept Check

absolute value function (p. 90)
boundary (p. 96)
Cartesian coordinate plane (p. 56)
constant function (p. 90)
dependent variable (p. 59)
domain (p. 56)
family of graphs (p. 70)
function (p. 57)
functional notation (p. 59)
greatest integer function (p. 89)
identity function (p. 90)
independent variable (p. 59)
linear equation (p. 63)
linear function (p. 63)
line of fit (p. 81)
mapping (p. 57)
one-to-one function (p. 57)
ordered pair (p. 56)
parent graph (p. 70)
piecewise function (p. 91)
point-slope form (p. 76)
prediction equation (p. 81)
quadrant (p. 56)
range (p. 56)
rate of change (p. 69)
relation (p. 56)
scatter plot (p. 81)
slope (p. 68)
slope-intercept form (p. 75)
standard form (p. 64)
step function (p. 89)
vertical line test (p. 57)
x-intercept (p. 65)
y-intercept (p. 65)

Choose the correct term to complete each sentence.

1. The (*constant*, *identity*) function is a linear function described by $f(x) = x$.
2. The graph of the (*absolute value*, *greatest integer*) function forms a V-shape and is described by $f(x) = |x|$.
3. The (*slope-intercept*, *standard*) form of the equation of a line is $Ax + By = C$, where A and B are not both zero.
4. Two lines in the same plane having the same slope are (*parallel*, *perpendicular*).
5. The (*domain*, *range*) is the set of all x-coordinates of the ordered pairs of a relation.
6. The set of all y-coordinates of the ordered pairs of a relation is the (*domain*, *range*).
7. The ratio of the change in y-coordinates to the corresponding change in x-coordinates is called the (*slope*, *y-intercept*) of a line.
8. The (*line of fit*, *vertical line test*) can be used to determine if a relation is a function.

Lesson-by-Lesson Review

2-1 Relations and Functions

See pages 56–62.

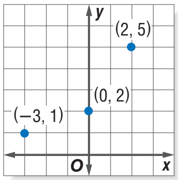
Concept Summary

- A relation is a set of ordered pairs. The domain is the set of all x-coordinates, and the range is the set of all y-coordinates.
- A function is a relation where each member of the domain is paired with exactly one member of the range.

Example **Graph the relation {(−3, 1), (0, 2), (2, 5)} and find the domain and range. Then determine whether the relation is a function.**

The domain is {−3, 0, 2}, and the range is {1, 2, 5}.

Graph the ordered pairs. Since each x value is paired with exactly one y value, the relation is a function.



www.algebra2.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables*.

Some students may need help in deciding which tabs to use. Ask student volunteers to share the thinking process they used to decide where various material should go. Remind students that their notations should be complete sentences that will make sense when they are reviewed weeks later.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Exercises Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function.
See Examples 1 and 2 on pages 57 and 58. **9–12. See margin for graphs.**

9. $\{(6, 3), (2, 1), (-2, 3)\}$

10. $\{(-5, 2), (2, 4), (1, 1), (-5, -2)\}$

11. $y = 0.5x$ **D = all reals, R = all reals; yes**

12. $y = 2x + 1$ **D = all reals, R = all reals; yes**

Find each value if $f(x) = 5x - 9$. *See Example 5 on page 59.*

13. $f(6)$ **21**

14. $f(-2)$ **−19**

15. $f(y)$ **$5y - 9$**

16. $f(-2v)$ **$-10v - 9$**

9. D = {−2, 2, 6}, R = {1, 3}; yes **10. D = {−5, 1 2}, R = {−2, 1, 2, 4}; no**

2-2 Linear Equations

See pages 63–67.

Concept Summary

- A linear equation is an equation whose graph is a line. A linear function can be written in the form $f(x) = mx + b$.
- The standard form of a linear equation is $Ax + By = C$.

Example **Write $2x - 6 = y + 8$ in standard form. Identify A, B, and C.**

$2x - 6 = y + 8$ Original equation

$2x - y - 6 = 8$ Subtract y from each side.

$2x - y = 14$ Add 6 to each side.

The standard form is $2x - y = 14$. So, $A = 2$, $B = -1$, and $C = 14$.

Exercises State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning. *See Example 1 on page 63.*

17. $3x^2 - y = 6$ **No; x has an exponent other than 1.**

18. $2x + y = 11$ **yes**

19. $h(x) = \sqrt{2x + 1}$ **No; x is inside a square root.**

Write each equation in standard form. Identify A, B, and C. *See Example 3 on page 64.*

20. $y = 7x + 15$ **$7x - y = -15$; 7, −1, −15**

21. $0.5x = -0.2y - 0.4$ **$5x + 2y = -4$; 5, 2, −4**

22. $\frac{2}{3}x - \frac{3}{4}y = 6$ **$8x - 9y = 72$; 8, −9, 72**

Find the x-intercept and the y-intercept of the graph of each equation. Then graph the equation. *See Example 4 on page 65.* **23–25. See margin for graphs.**

23. $-\frac{1}{5}y = x + 4$ **−4, −20**

24. $6x = -12y + 48$ **8, 4**

25. $y - x = -9$ **9, −9**

2-3 Slope

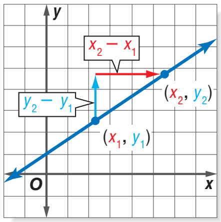
See pages 68–74.

Concept Summary

- The slope of a line is the ratio of the change in y-coordinates to the corresponding change in x-coordinates.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Lines with the same slope are parallel. Lines with slopes that are opposite reciprocals are perpendicular.



Answers

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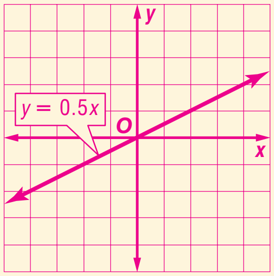

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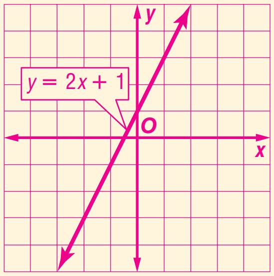

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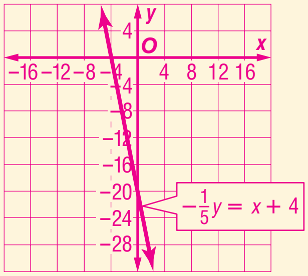

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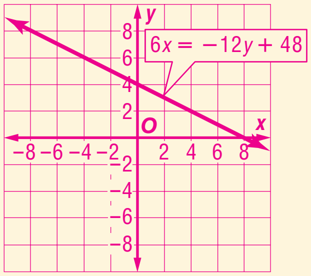

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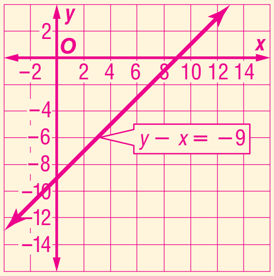

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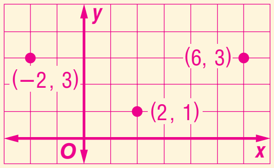

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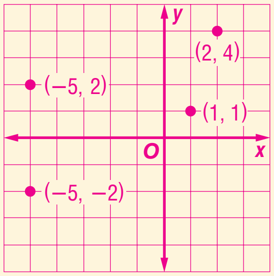


Example Find the slope of the line that passes through (−5, 3) and (7, 9).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{9 - 3}{7 - (-5)}$ $(x_1, y_1) = (-5, 3), (x_2, y_2) = (7, 9)$

$= \frac{6}{12}$ or $\frac{1}{2}$ Simplify.

Exercises **Find the slope of the line that passes through each pair of points.** *See Example 1 on page 68.*

26. (−6, −3), (6, 7) $\frac{5}{6}$ **27.** (5.5, −5.5), (11, −7) $-\frac{3}{11}$ **28.** (−3, 24), (10, −41) -5

Graph the line passing through the given point with the given slope. *See Example 2 on page 69.* **29–31. See margin.**

29. (0, 1), $m = 2$ **30.** (3, −2), $m = \frac{5}{2}$ **31.** (−5, 2), $m = -\frac{1}{4}$

Graph the line that satisfies each set of conditions. *See Examples 4 and 5 on pages 70 and 71.* **32–35. See margin.**

32. passes through (2, 0), parallel to a line whose slope is 3

33. passes through (−1, −2), perpendicular to a line whose slope is $\frac{1}{2}$

34. passes through (4, 1), perpendicular to graph of $2x + 3y = 1$

35. passes through (−2, 2), parallel to graph of $-2x + y = 4$

2-4 Writing Linear Equations

See pages 75–80.

Concept Summary

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$

Example **Write an equation in slope-intercept form for the line through (4, 5) that is parallel to the line through (−1, −3) and (2, −1).**

First, find the slope of the given line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-1 - (-3)}{2 - (-1)}$ $(x_1, y_1) = (-1, -3),$ $(x_2, y_2) = (2, -1)$

$= \frac{2}{3}$ Simplify.

The parallel line will also have slope $\frac{2}{3}$.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 5 = \frac{2}{3}(x - 4)$ $(x_1, y_1) = (4, 5), m = \frac{2}{3}$

$y = \frac{2}{3}x + \frac{7}{3}$ Slope-intercept form

Exercises **Write an equation in slope-intercept form for the line that satisfies each set of conditions.** *See Examples 1, 2, and 4 on pages 76–78.*

36. slope $\frac{3}{4}$, passes through (−6, 9) $y = \frac{3}{4}x + \frac{27}{2}$

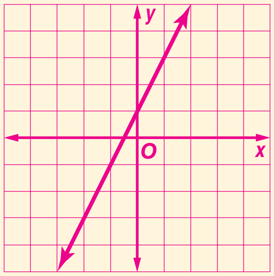
37. passes through (3, −8) and (−3, 2) $y = -\frac{5}{3}x - 3$

38. passes through (−1, 2), parallel to the graph of $x - 3y = 14$ $y = \frac{1}{3}x + \frac{7}{3}$

39. passes through (3, 2), perpendicular to the graph of $4x - 3y = 12$ $y = -\frac{3}{4}x + \frac{17}{4}$

Answers

29.



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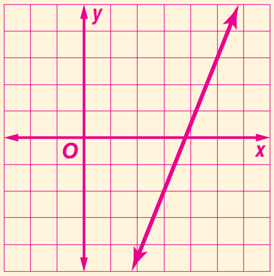

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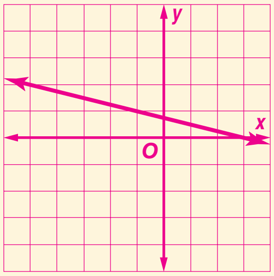

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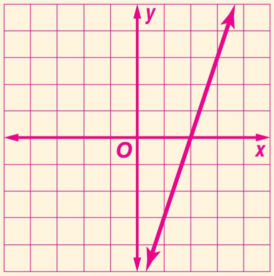

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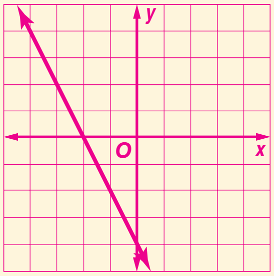

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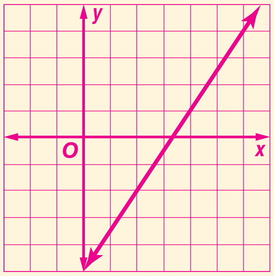

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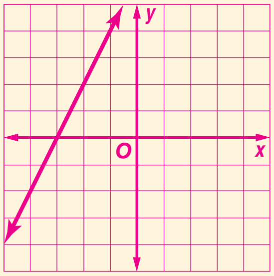


2-5 Modeling Real-World Data: Using Scatter Plots

See pages 81–86.

Concept Summary

- A scatter plot is a graph of ordered pairs of data.
- A prediction equation can be used to predict one of the variables given the other variable.

Example **WEEKLY PAY** **The table below shows the median weekly earnings for American workers for the period 1985–1999. Predict the median weekly earnings for 2010.**

Year	1985	1990	1995	1999	2010
Earnings ($)	343	412	479	549	?

Source: U.S. Bureau of Labor Statistics

A scatter plot suggests that any two points could be used to find a prediction equation. Use (1985, 343) and (1990, 412).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

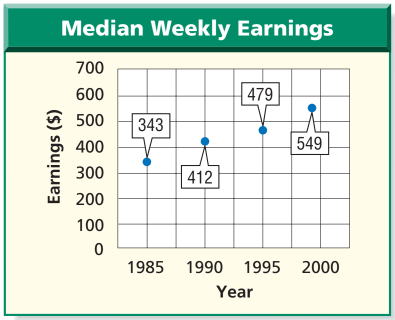
$= \frac{412 - 343}{1990 - 1985}$ $(x_1, y_1) = (1985, 343)$, $(x_2, y_2) = (1990, 412)$

$= \frac{69}{5}$ or 13.8 Simplify.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 343 = 13.8(x - 1985)$ Substitute.

$y = 13.8x - 27{,}050$ Add 343 to each side.



Source: U.S. Bureau of Labor Statistics

To predict the earnings for 2010, substitute 2010 for x.

$y = 13.8(2010) - 27{,}050$ $x = 2010$

$= 688$ Simplify.

The model predicts median weekly earnings of $688 in 2010.

Exercises **For Exercises 40–42, use the table that shows the number of people below the poverty level for the period 1980–1998.** *See Examples 1 and 2 on pages 81 and 82.*

Year	People (millions)
1980	29.3
1985	33.1
1990	33.6
1995	36.4
1998	34.5
2010	?

Source: U.S. Census Bureau

40. Draw a scatter plot. **See margin.**

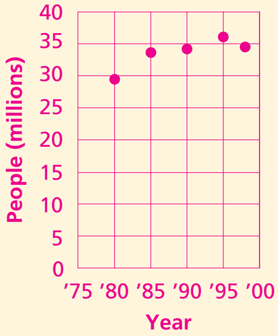
41. Use two ordered pairs to write a prediction equation.

42. Use your prediction equation to predict the number for 2010. **Sample answer: 42.2 million**

41. Sample answer using (1980, 29.3) and (1990, 33.6): $y = 0.43x - 822.1$

Answer

40. **People Below Poverty Level**



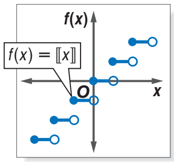
Chapter 2 For More ...
- Extra Practice, see pages 830–832.
- Mixed Problem Solving, see page 863.

2-6 Special Functions

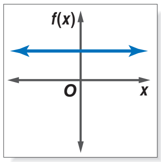
See pages 89–95.

Concept Summary

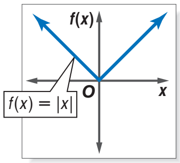
Greatest Integer



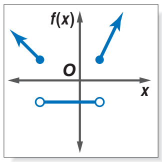
Constant



Absolute Value

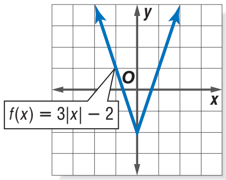


Piecewise



Example **Graph the function $f(x) = 3|x| - 2$. Identify the domain and range.**

The domain is all real numbers. The range is all real numbers greater than or equal to -2.



Exercises **Graph each function. Identify the domain and range.**
See Examples 1–3 on pages 89–91. **43–48. See pp. 107A–107H.**

43. $f(x) = [\![x]\!] - 2$

44. $h(x) = [\![2x - 1]\!]$

45. $g(x) = |x| + 4$

46. $h(x) = |x - 1| - 7$

47. $f(x) = \begin{cases} 2 & \text{if } x < -1 \\ -x - 1 & \text{if } x \geq -1 \end{cases}$

48. $g(x) = \begin{cases} -2x - 3 & \text{if } x < 1 \\ x - 4 & \text{if } x > 1 \end{cases}$

2-7 Graphing Inequalities

See pages 96–99.

Concept Summary

You can graph an inequality by following these steps.

Step 1 Determine whether the boundary is solid or dashed. Graph the boundary.

Step 2 Choose a point not on the boundary and test it in the inequality.

Step 3 If a true inequality results, shade the region containing your test point. If a false inequality results, shade the other region.

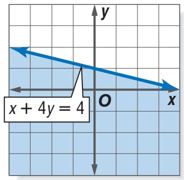
Example **Graph $x + 4y \leq 4$.**

Since the inequality symbol is ≤, the graph of the boundary should be solid. Graph the equation. Test (0, 0).

$x + 4y \leq 4$ Original inequality

$0 + 4(0) \leq 4$ $(x, y) = (0, 0)$

$0 \leq 4$ Shade the region that contains (0, 0).



Exercises **Graph each inequality.** *See Examples 1–3 on pages 96 and 97.* **49–54. See margin.**

49. $y \leq 3x - 5$

50. $x > y - 1$

51. $y + 0.5x < 4$

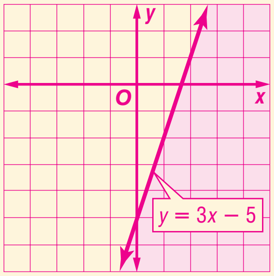
52. $2x + y \geq 3$

53. $y \geq |x| + 2$

54. $y > |x - 3|$

Answers

49.



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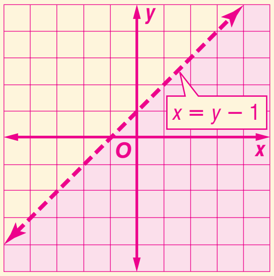

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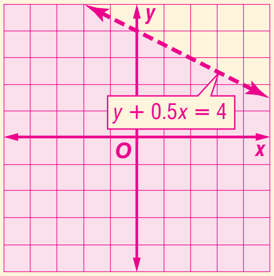

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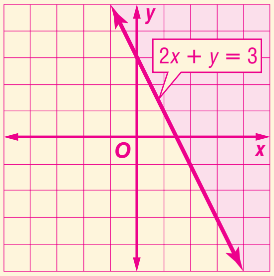

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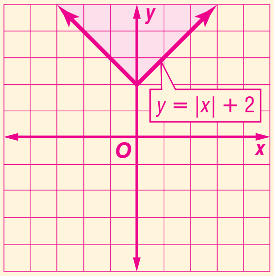

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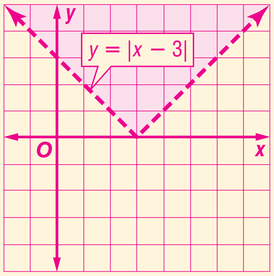


Chapter 2 Practice Test

Vocabulary and Concepts

Choose the correct term to complete each sentence.

1. The variable whose values make up the domain of a function is called the (*independent*, *dependent*) variable.
2. To find the (*x-intercept*, *y-intercept*) of the graph of a linear equation, let $y = 0$.
3. An equation of the form ($Ax + By = C$, $y = mx + b$) is in slope-intercept form.

Skills and Applications

Graph each relation and find the domain and range. Then determine whether the relation is a function. **4–5. See pp. 107A–107H for graphs.**

4. $\{(-4, -8), (-2, 2), (0, 5), (2, 3), (4, -9)\}$ **D = {−4, −2, 0, 2, 4), R = {−9, −8, 2, 3, 5}; yes**

5. $y = 3x - 3$ **D = all reals, R = all reals; yes**

Find each value.

6. $f(3)$ if $f(x) = 7 - x^2$ **−2**

7. $f(0)$ if $f(x) = x - 3x^2$ **0**

Graph each equation or inequality. **8–19. See pp. 107A–107H.**

8. $y = \frac{3}{5}x - 4$

9. $4x - y = 2$

10. $x = -4$

11. $y = 2x - 5$

12. $f(x) = 3x - 1$

13. $f(x) = [\![3x]\!] + 3$

14. $g(x) = |x + 2|$

15. $h(x) = \begin{cases} x + 2 \text{ if } x < -2 \\ 2x - 1 \text{ if } x \geq -2 \end{cases}$

16. $y \leq 10$

17. $x > 6$

18. $-2x + 5 \leq 3y$

19. $y < 4|x - 1|$

Find the slope of the line that passes through each pair of points.

20. $(8, -4), (6, 1)$ $-\frac{5}{2}$

21. $(-2, 5), (4, 5)$ **0**

22. $(5, 7), (4, -6)$ **13**

Graph the line passing through the given point with the given slope. **23–25. See pp. 107A–107H.**

23. $(1, -3), 2$

24. $(-2, 2), -\frac{1}{3}$

25. $(3, -2)$, undefined

Write an equation in slope-intercept form for the line that satisfies each set of conditions.

26. slope −5, *y*-intercept 11 $y = -5x + 11$

27. *x*-intercept 9, *y*-intercept −4 $y = \frac{4}{9}x - 4$

28. passes through (−6, 15), parallel to the graph of $2x + 3y = 1$ $y = -\frac{2}{3}x + 11$

29. passes through (5, 2), perpendicular to the graph of $x + 3y = 7$ $y = 3x - 13$

RECREATION **For Exercises 30–32, use the table that shows the amount Americans spent on recreation in recent years.**

Year	1995	1996	1997	1998
Amount ($ billions)	401.6	429.6	457.8	494.7

Source: U.S. Bureau of Economic Analysis

30. Draw a scatter plot, where *x* represents the number of years since 1995. **See margin.**

31. Write a prediction equation. **Sample answer using (0, 401.6) and (1, 429.6): $y = 28x + 401.6$**

32. Predict the amount that will be spent on recreation in 2010. **Sample answer: $821.6 billion**

33. **STANDARDIZED TEST PRACTICE** What is the slope of a line parallel to $y - 2 = 4(x + 1)$? **D**

Ⓐ -4 Ⓑ $-\frac{1}{4}$ Ⓒ $\frac{1}{4}$ Ⓓ 4

www.algebra2.com/chapter_test

Chapter 2 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 2 can be found on p. 112 of the *Chapter 2 Resource Masters*.

Chapter Tests There are six Chapter 2 Tests and an Open-Ended Assessment task available in the *Chapter 2 Resource Masters*.

Chapter 2 Tests			
Form	Type	Level	Pages
1	MC	basic	99–100
2A	MC	average	101–102
2B	MC	average	103–104
2C	FR	average	105–106
2D	FR	average	107–108
3	FR	advanced	109–110

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 2 can be found on p. 111 of the *Chapter 2 Resource Masters*. A sample scoring rubric for these tasks appears on p. A28.

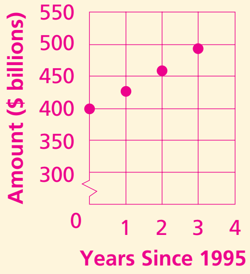
TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Answer

30. **Money Spent on Recreation**



Portfolio Suggestion

Introduction In this chapter, you have graphed many different kinds of functions. The appearances of these graphs were also very different from one another.

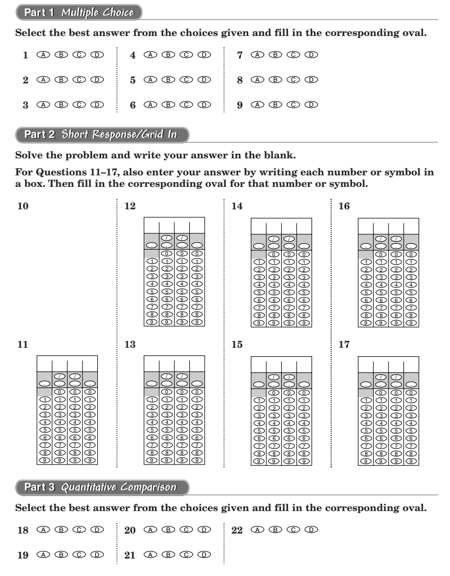
Ask Students Select one kind of graph that you found difficult to master and explain why you felt this to be the case. Suggest ways that this topic might be presented in a different way to help other students who have the same difficulty.

Chapter 2 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 2 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ
2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ
3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 11–17, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 12 14 16
11 13 15 17

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

18 Ⓐ Ⓑ Ⓒ Ⓓ 20 Ⓐ Ⓑ Ⓒ Ⓓ 22 Ⓐ Ⓑ Ⓒ Ⓓ
19 Ⓐ Ⓑ Ⓒ Ⓓ 21 Ⓐ Ⓑ Ⓒ Ⓓ

Additional Practice

See pp. 117–118 in the *Chapter 2 Resource Masters* for additional standardized test practice.

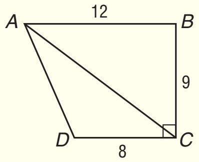
Chapter 2 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In the figure, $\angle B$ and $\angle BCD$ are right angles. $\overline{BC}$ is 9 units, $\overline{AB}$ is 12 units, and $\overline{CD}$ is 8 units. What is the area, in square units, of $\triangle ACD$? **A**

Ⓐ 36
Ⓑ 60
Ⓒ 72
Ⓓ 135



2. If $x + 3$ is an even integer, then x could be which of the following? **B**

Ⓐ -2 Ⓑ -1
Ⓒ 0 Ⓓ 2

3. What is the slope of the line that contains the points (15, 7) and (6, 4)? **B**

Ⓐ $\frac{1}{4}$ Ⓑ $\frac{1}{3}$
Ⓒ $\frac{3}{8}$ Ⓓ $\frac{2}{3}$

4. In 2000, Matt had a collection of 30 music CDs. Since then he has given away 2 CDs, purchased 6 new CDs, and traded 3 of his CDs to Kashan for 4 of Kashan's CDs. Since 2000, what has been the percent of increase in the number of CDs in Matt's collection? **D**

Ⓐ $3\frac{1}{3}\%$ Ⓑ 10%
Ⓒ $14\frac{2}{7}\%$ Ⓓ $16\frac{2}{3}\%$

5. If the product of (2 + 3), (3 + 4), and (4 + 5) is equal to three times the sum of 40 and x, then $x =$ ______. **B**

Ⓐ 43 Ⓑ 65
Ⓒ 105 Ⓓ 195

6. If one side of a triangle is three times as long as a second side and the second side is s units long, then the length of the third side of the triangle can be **A**

Ⓐ $3s$. Ⓑ $4s$.
Ⓒ $5s$. Ⓓ $6s$.

7. Which of the following sets of numbers has the property that the *product* of any two numbers is also a number in the set? **D**

I the set of positive numbers
II the set of prime numbers
III the set of even integers

Ⓐ I only
Ⓑ II only
Ⓒ III only
Ⓓ I and III only

8. If $\frac{3+x}{7+x} = \frac{3}{7} + \frac{3}{7}$, then $x =$ ______. **D**

Ⓐ $\frac{3}{7}$ Ⓑ 3
Ⓒ 7 Ⓓ 21

9. The average (arithmetic mean) of r, s, x, and y is 8, and the average of x and y is 4. What is the average of r and s? **D**

Ⓐ 4 Ⓑ 6
Ⓒ 8 Ⓓ 12

The Princeton Review **Test-Taking Tip**

Questions 1–9 On multiple-choice questions, try to compute the answer first. Then compare your answer to the given answer choices. If you don't find your answer among the choices, check your calculations.

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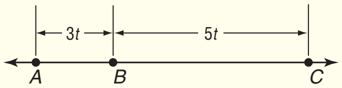
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

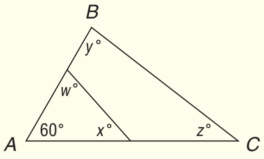
10. If n is a prime integer such that $2n > 19 \geq \frac{7}{8}n$ what is one possible value of n?
11, 13, 17, or 19

11. If $\overline{AC}$ is 2 units, what is the value of t? 1/4 or .25



12. If $0.85x = 8.5$, what is the value of $\frac{1}{x}$?
1/10 or .1

13. In $\triangle ABC$, what is the value of $w + x + y + z$? 240



14. In an election, a total of 4000 votes were cast for three candidates, A, B, and C. Candidate C received 800 votes. If candidate B received more votes than candidate C and candidate A received more votes than candidate B, what is the least number of votes that candidate A could have received? 1601

15. If the points $P(-2, 3)$, $Q(2, 5)$, and $R(2, 3)$ are vertices of a triangle, what is the area of the triangle? 4

16. How many of the first one hundred positive integers contain the digit 7? 19

17. A triangle has a base of length 17, and the other two sides are equal in length. If the lengths of the sides of the triangle are integers, what is the shortest possible length of a side? 9

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Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

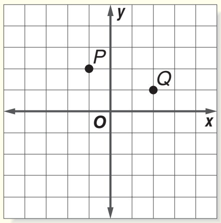
- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B

18. m is an integer greater than 3. A

$\frac{1}{4}$	$\frac{1}{m} - \frac{1}{4}$

19. C



the x-coordinate of point Q	the y-coordinate of point P

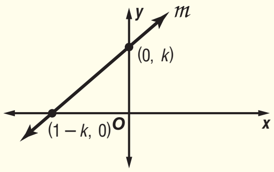
20. The cost of 3 bananas and 2 apples is \$1.50. D

cost of one apple	cost of one banana

21. The average (arithmetic mean) of three integers, x, y, and z is 30. B

the average (arithmetic mean) of x, y, z, and 29	30

22. A



the slope of line m	1

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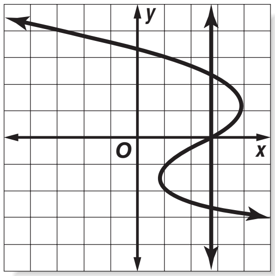
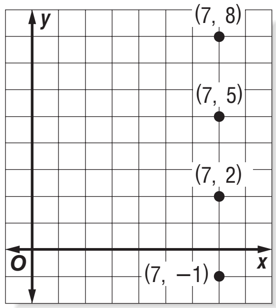
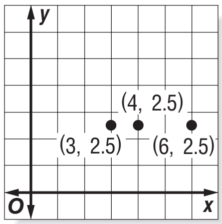
2. Sample answer:



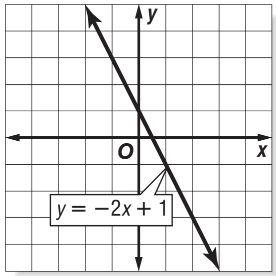
7. D = {7},
R = {−1, 2, 5, 8}, no



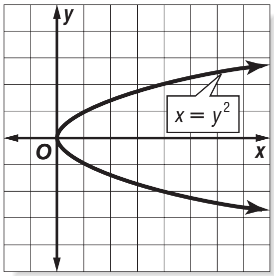
8. D = {3, 4, 6},
R = {2.5}, yes



9. D = all reals,
R = all reals, yes



10. D = {x|x ≥ 0},
R = all reals, no



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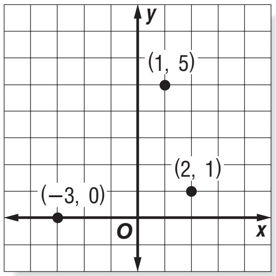

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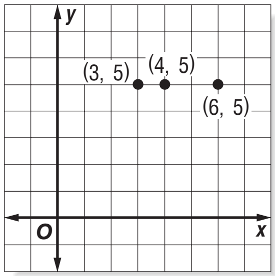

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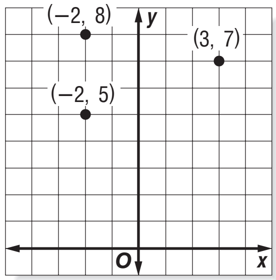

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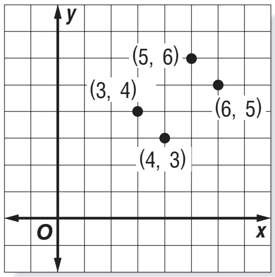

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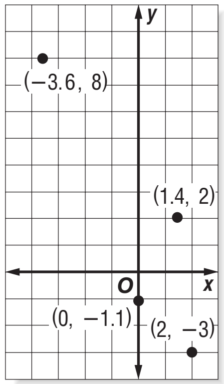

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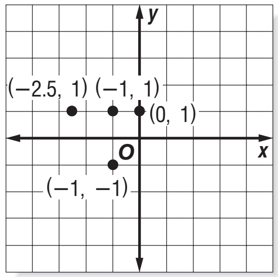

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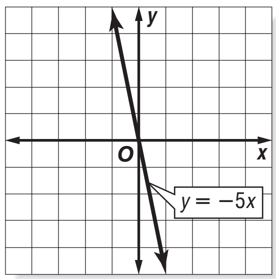

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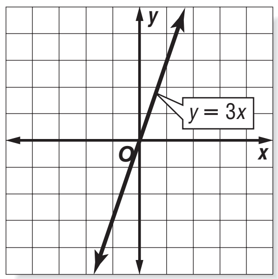

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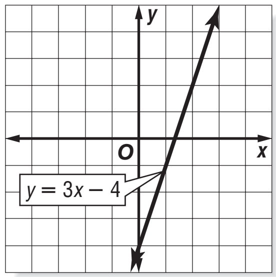

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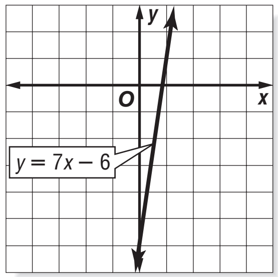

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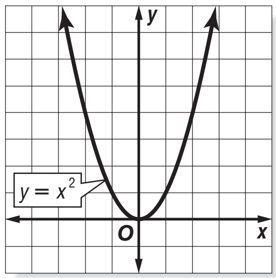

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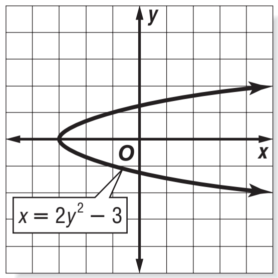

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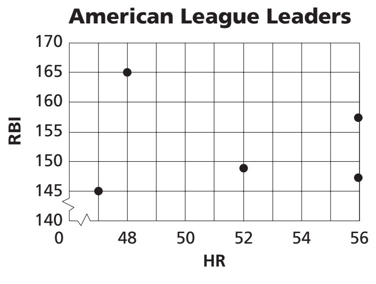

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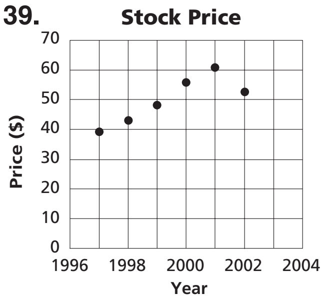

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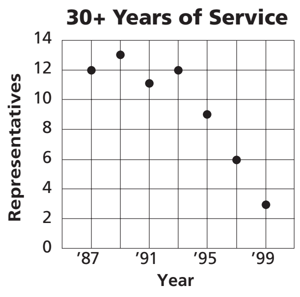

44. D = {1987, 1989, 1991, 1993, 1995, 1997, 1999}, R = {3, 6, 9, 11, 12, 13}

45. Each domain value is paired with only one range value so the relation is a function, but the range value 12 is paired with two domain values so the function is not one-to-one.

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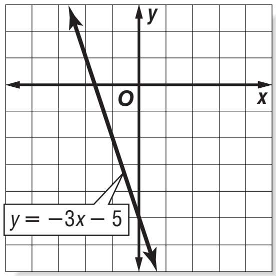
9.



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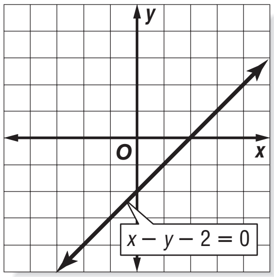

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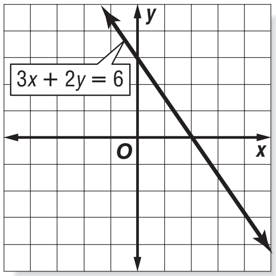

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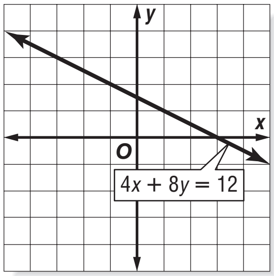

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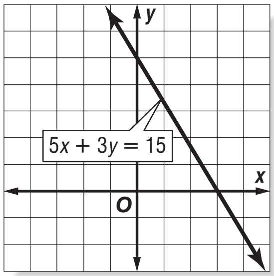

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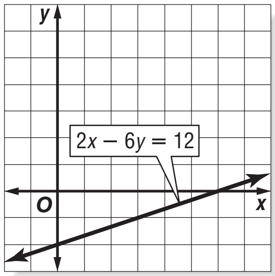

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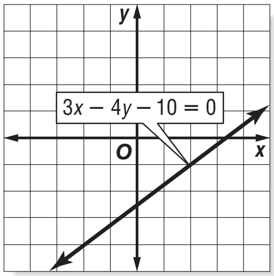

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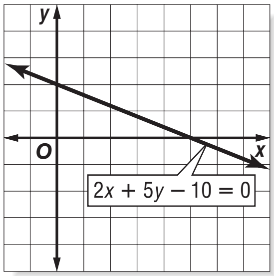

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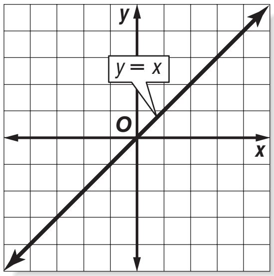

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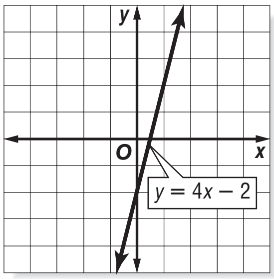

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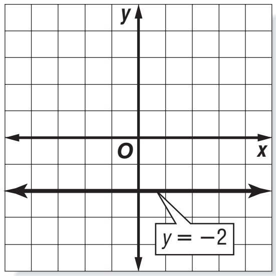

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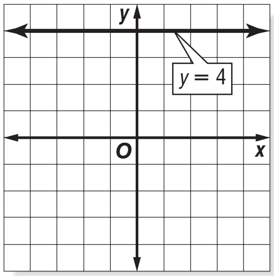

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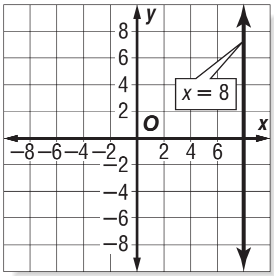

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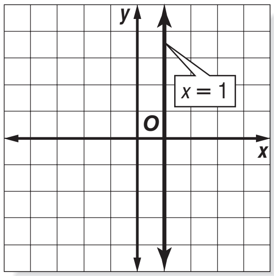

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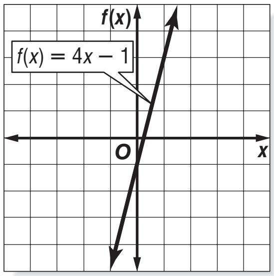

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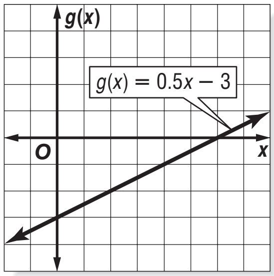

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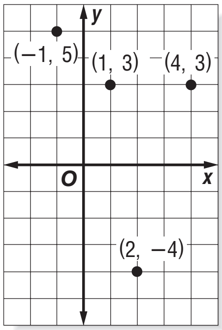

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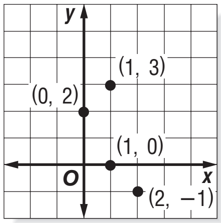

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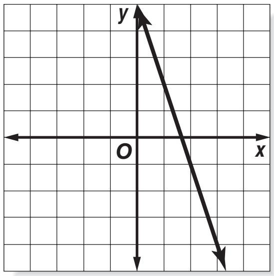
7.



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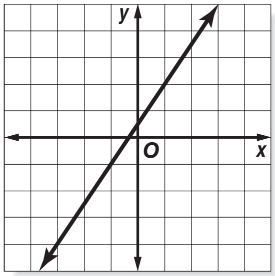

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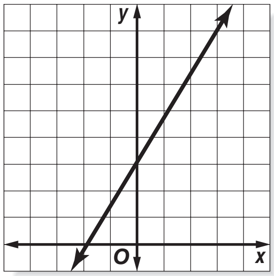

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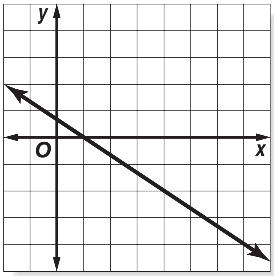

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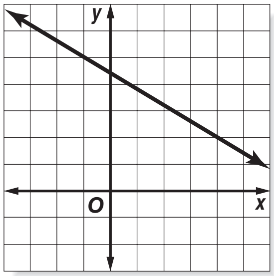

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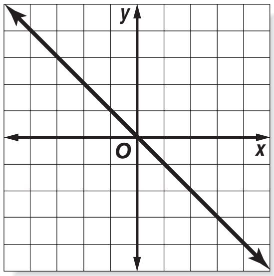

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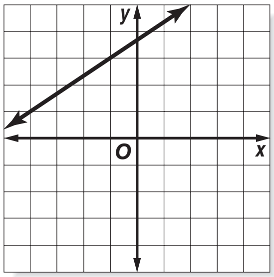

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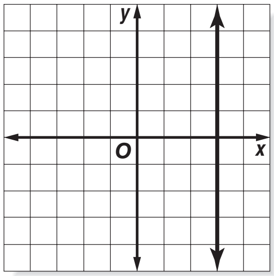

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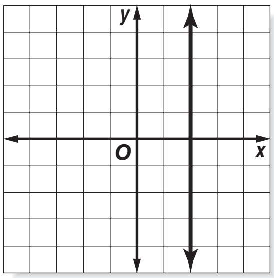

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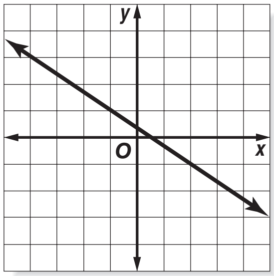

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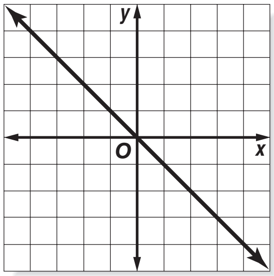

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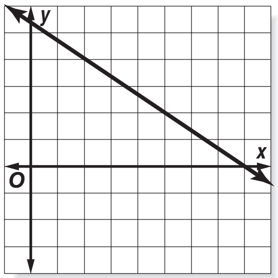

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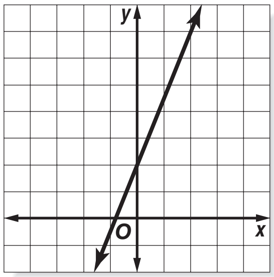
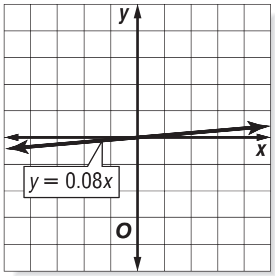


53. The grade or steepness of a road can be interpreted mathematically as a slope. Answers should include the following.

- Think of the diagram at the beginning of the lesson as being in a coordinate plane. Then the rise is a change in *y*-coordinates and the horizontal distance is a change in *x*-coordinates. Thus, the grade is a slope expressed as a percent.
- 

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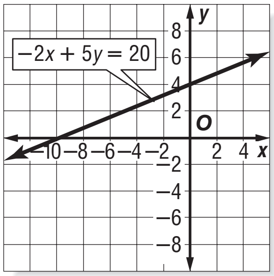

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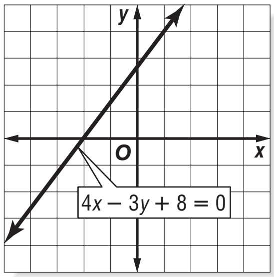

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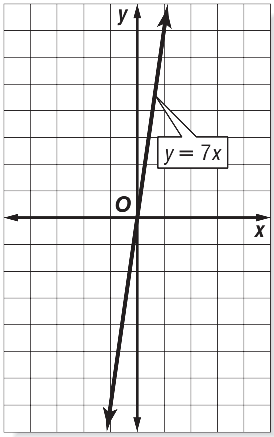

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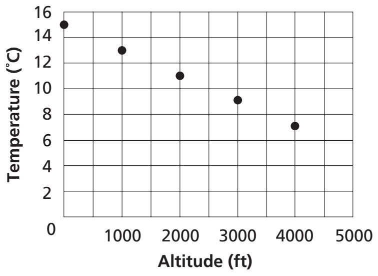
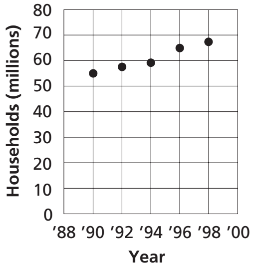
4a. **Atmospheric Temperature**



4b. Sample answer using (2000, 11.0) and (3000, 9.1): $y = -0.0019x + 14.8$

4c. Sample answer: 5.3°C

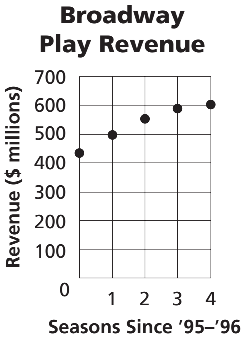
5a. **Cable Television**



5b. Sample answer using (1992, 57) and (1998, 67): $y = 1.67x - 3269.64$

5c. Sample answer: about 87 million

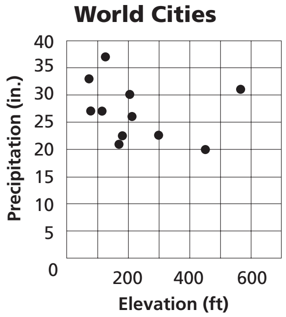
9a.



13. Sample answer: Using the data for August and November, a prediction equation for Company 1 is $y = -0.86x + 25.13$, where x is the number of months since August. The negative slope suggests that the value of Company 1's stock is going down. Using the data for October and November, a prediction equation for Company 2 is $y = 0.38x + 31.3$, where x is the number of months since August. The positive slope suggests that the value of Company 2's stock is going up. Since the value of Company 1's stock appears to be going down, and the value of Company 2's stock appears to be going up, Della should buy Company 2.

14. No. Past performance is no guarantee of the future performance of a stock. Other factors that should be considered include the companies' earnings data and how much debt they have.

15.



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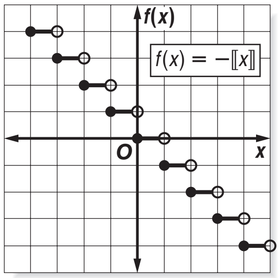
6.



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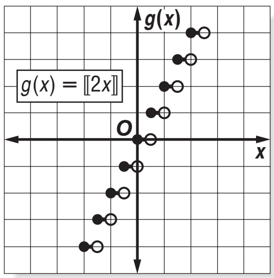

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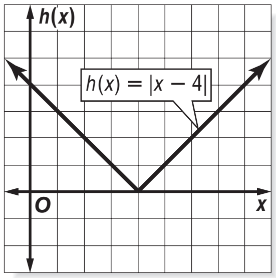

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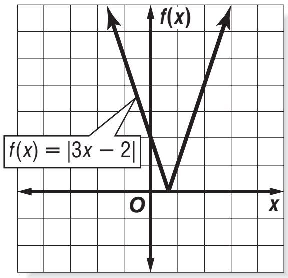

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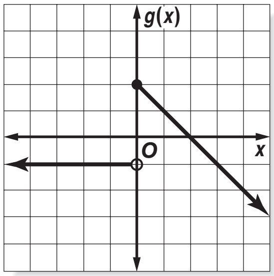

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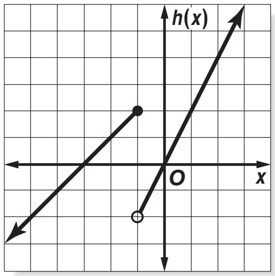

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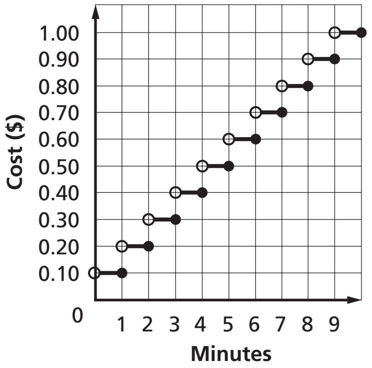
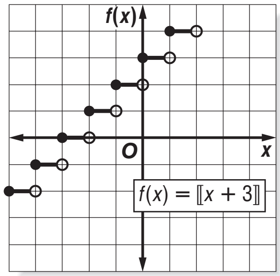
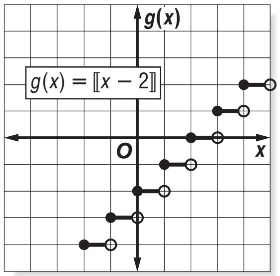


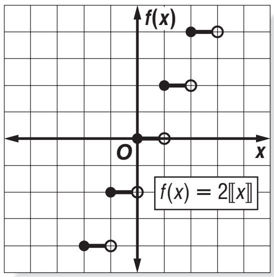
24. D = all reals,
R = all integers



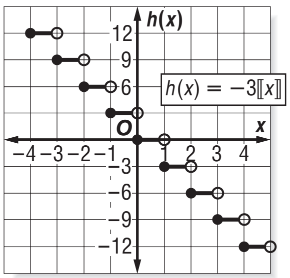
25. D = all reals,
R = all integers



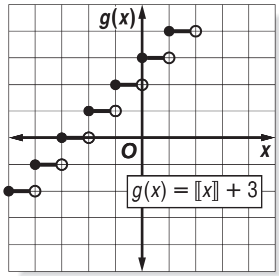
26. D = all reals,
R = all even integers



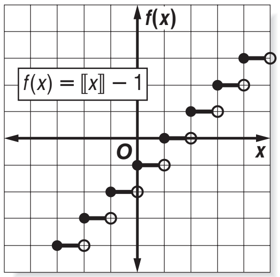
27. D = all reals,
R = {3a|a is an integer}



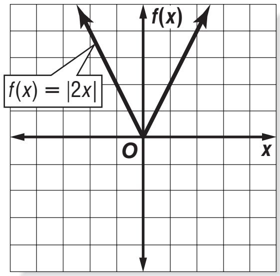
28. D = all reals,
R = all integers



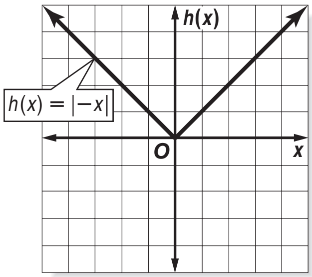
29. D = all reals,
R = all integers



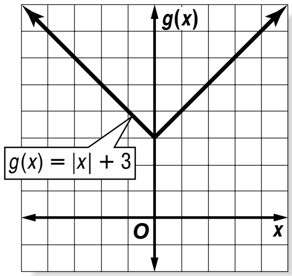
30. D = all reals, R = all nonnegative reals



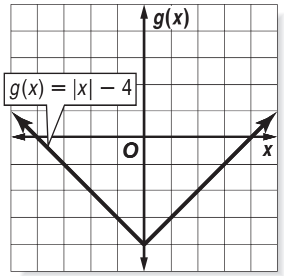
31. D = all reals, R = all nonnegative reals



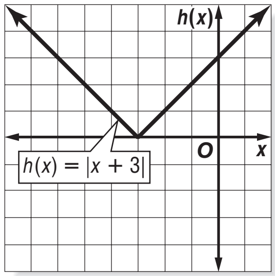
32. D = all reals, R = $\{y \mid y \geq 3\}$



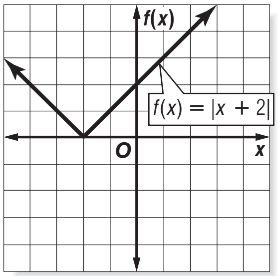
33. D = all reals, R = $\{y \mid y \geq -4\}$



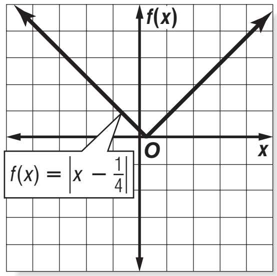
34. D = all reals, R = all nonnegative reals



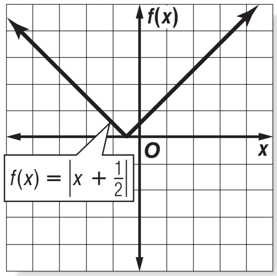
35. D = all reals, R = all nonnegative reals



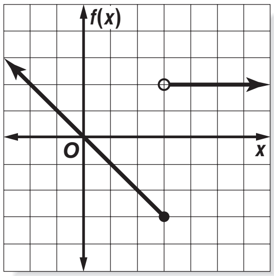
36. D = all reals, R = all nonnegative reals



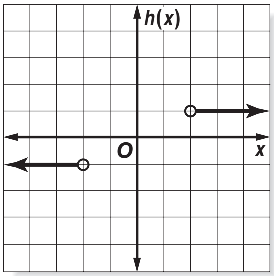
37. D = all reals, R = all nonnegative reals



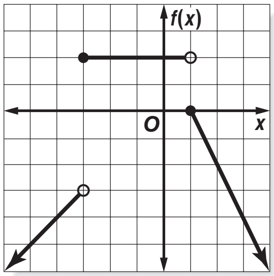
38. D = all reals, R = $\{y \mid y \geq -3\}$



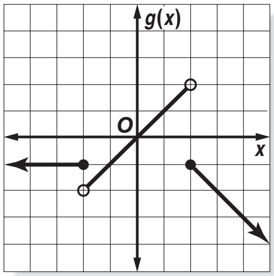
39. D = $\{x \mid x < -2 \text{ or } x > 2\}$, R = $\{-1, 1\}$



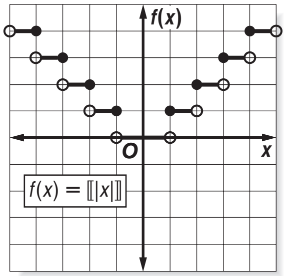
40. D = all reals, R = $\{y \mid y \leq 0 \text{ or } y = 2\}$



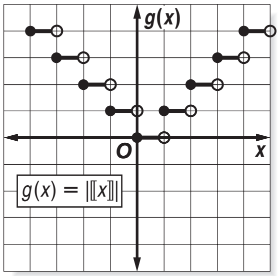
41. D = all reals, R = $\{y \mid y < 2\}$



42. D = all reals, R = all nonnegative whole numbers



43. D = all reals, R = all nonnegative whole numbers



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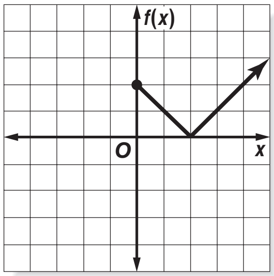

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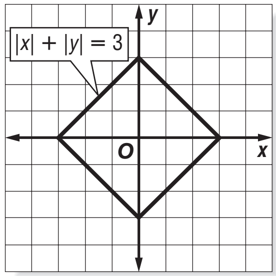
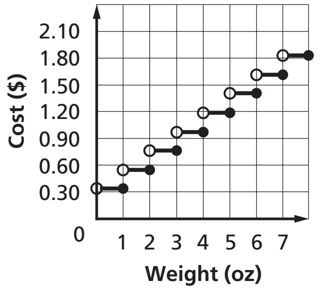


50. A step function can be used to model the cost of a letter in terms of its weight. Answers should include the following.

- Since the cost of a letter must be one of the values \$0.34, \$0.55, \$0.76, \$0.97, and so on, a step function is the best model for the cost of mailing a letter. The gas mileage of a car can be any real number in an interval of real numbers, so it cannot be modeled by a step function. In other words, gas mileage is a continuous function of time.
- 

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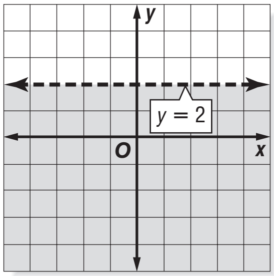
4.



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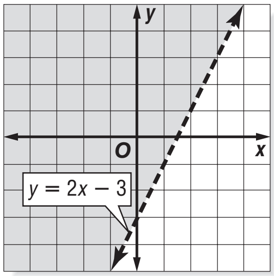

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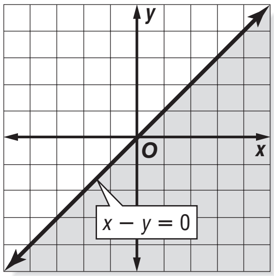

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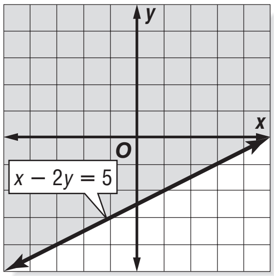

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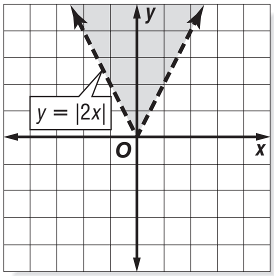

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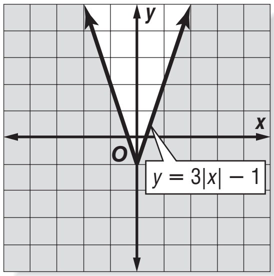

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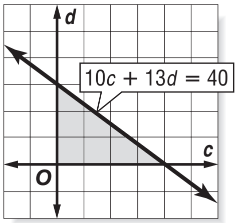

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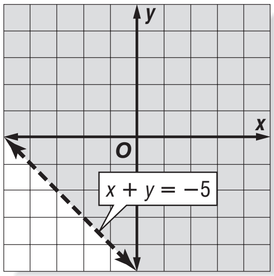

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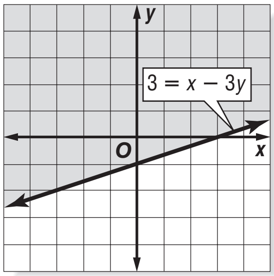

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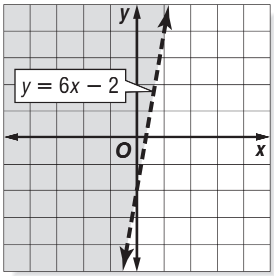

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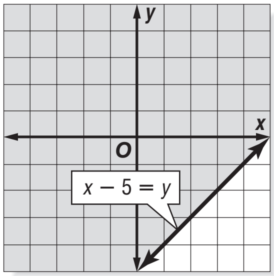

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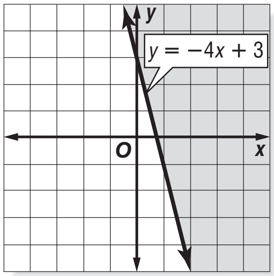

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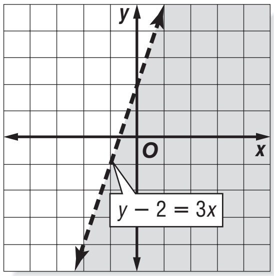

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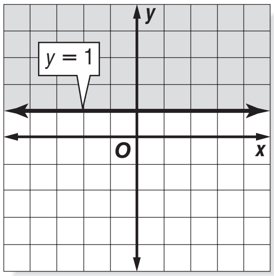

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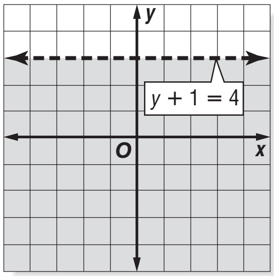

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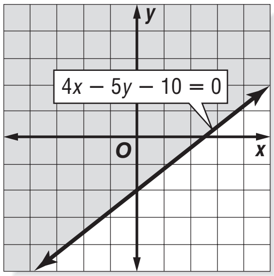

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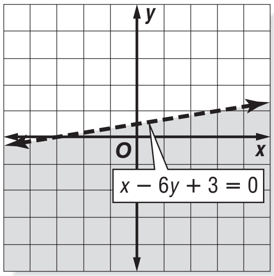

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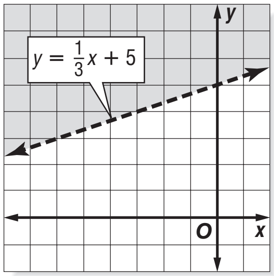

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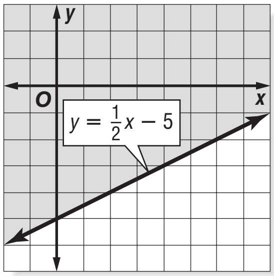

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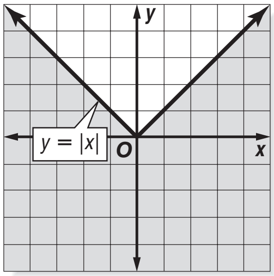

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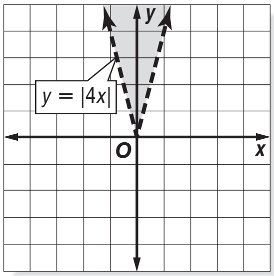

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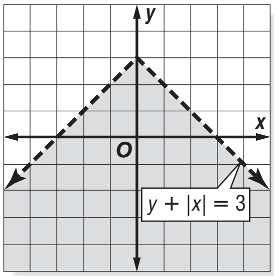

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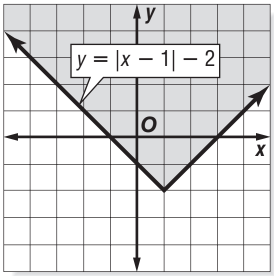

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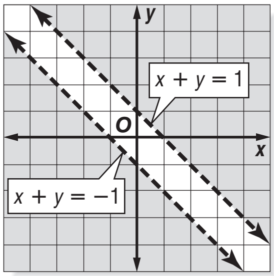

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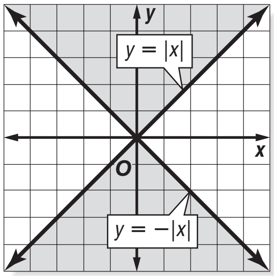

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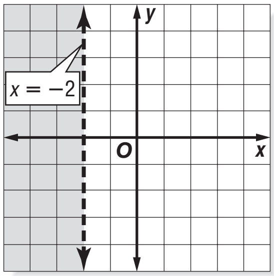

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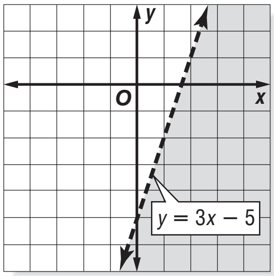

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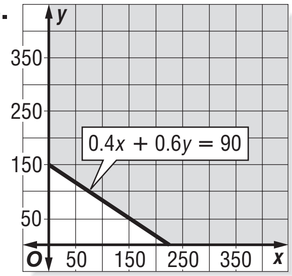

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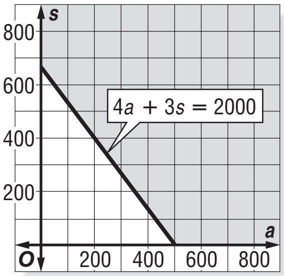
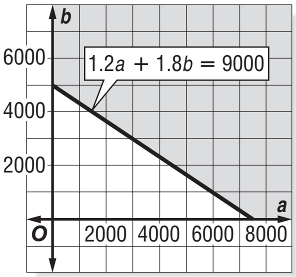


38. $1.2a + 1.8b \geq 9000$



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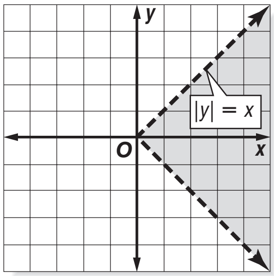
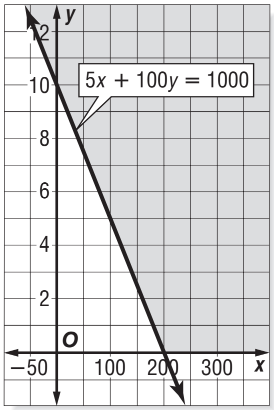
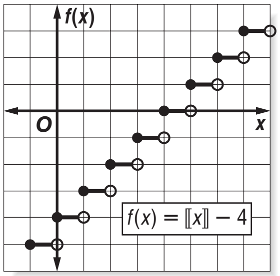


41. Linear inequalities can be used to track the performance of players in fantasy football leagues. Answers should include the following.

- Let x be the number of receiving yards and let y be the number of touchdowns. The number of points Dana gets from receiving yards is $5x$ and the number of points he gets from touchdowns is $100y$. His total number of points is $5x + 100y$. He wants at least 1000 points, so the inequality $5x + 100y \geq 1000$ represents the situation.
- 

- the first one

48.



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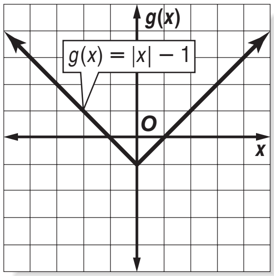
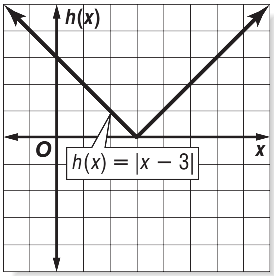


50.



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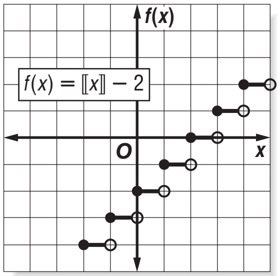
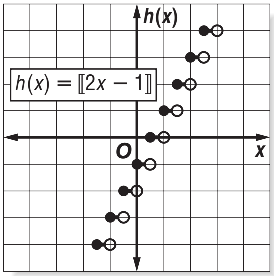
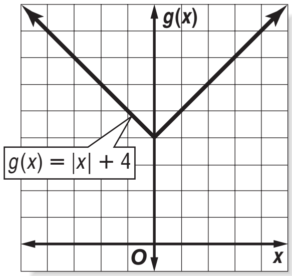
43. D = all reals,
R = all integers



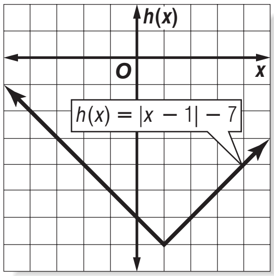
44. D = all reals,
R = all integers



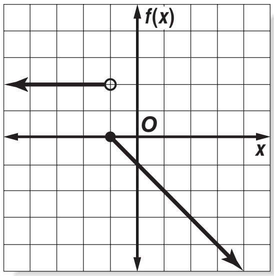
45. D = all reals,
R = $\{y \mid y \geq 4\}$



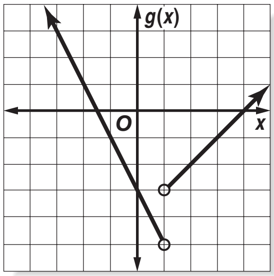
46. D = all reals,
R = $\{y \mid y \geq -7\}$



47. D = all reals,
R = $\{y \mid y \leq 0 \text{ or } y = 2\}$



48. D = $\{x \mid x \neq 1\}$,
R = $\{y \mid y > -5\}$



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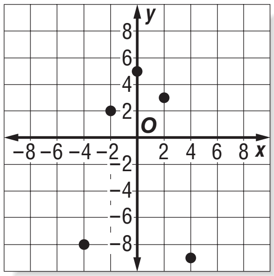
4.



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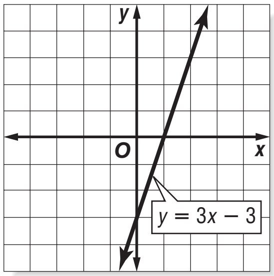

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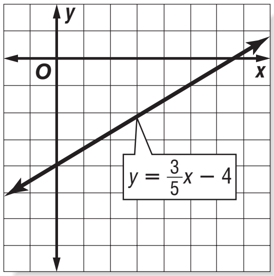

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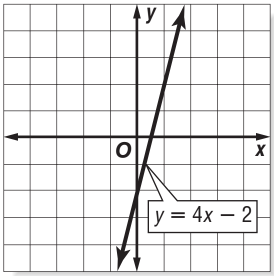

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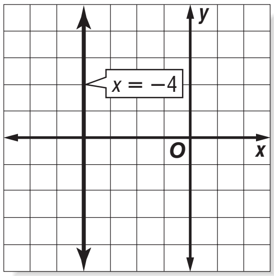

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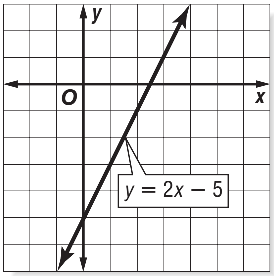

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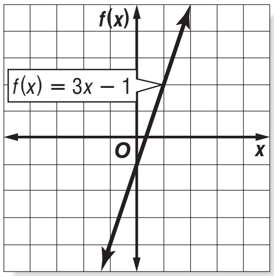

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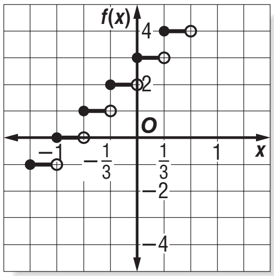

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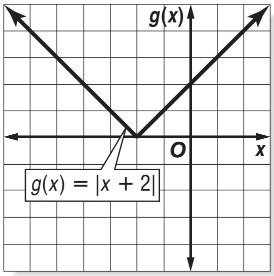

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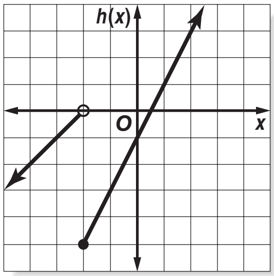

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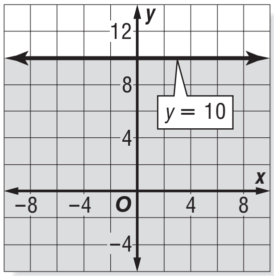

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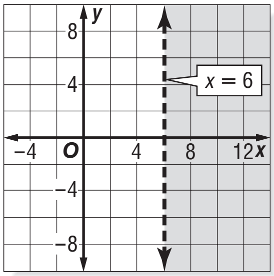

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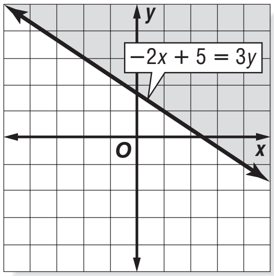

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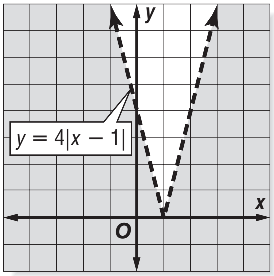

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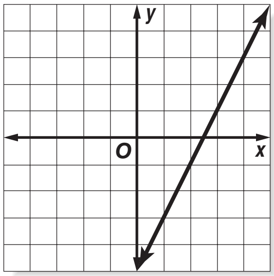

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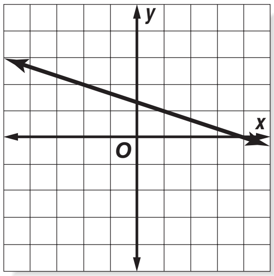

25.