UNIT 2 Notes

Introduction

In this unit, students extend their knowledge of first-degree equations and their graphs to radical equations and inequalities. Then they graph quadratic functions and solve quadratic equations and inequalities by various methods, including completing the square and using the Quadratic Formula.

The unit concludes with methods for evaluating polynomial functions, including the Remainder and Factor Theorems. Students graph polynomial functions and investigate their roots and zeros. Finally, they study the composition of two functions, and then find the inverse of a function.

Assessment Options

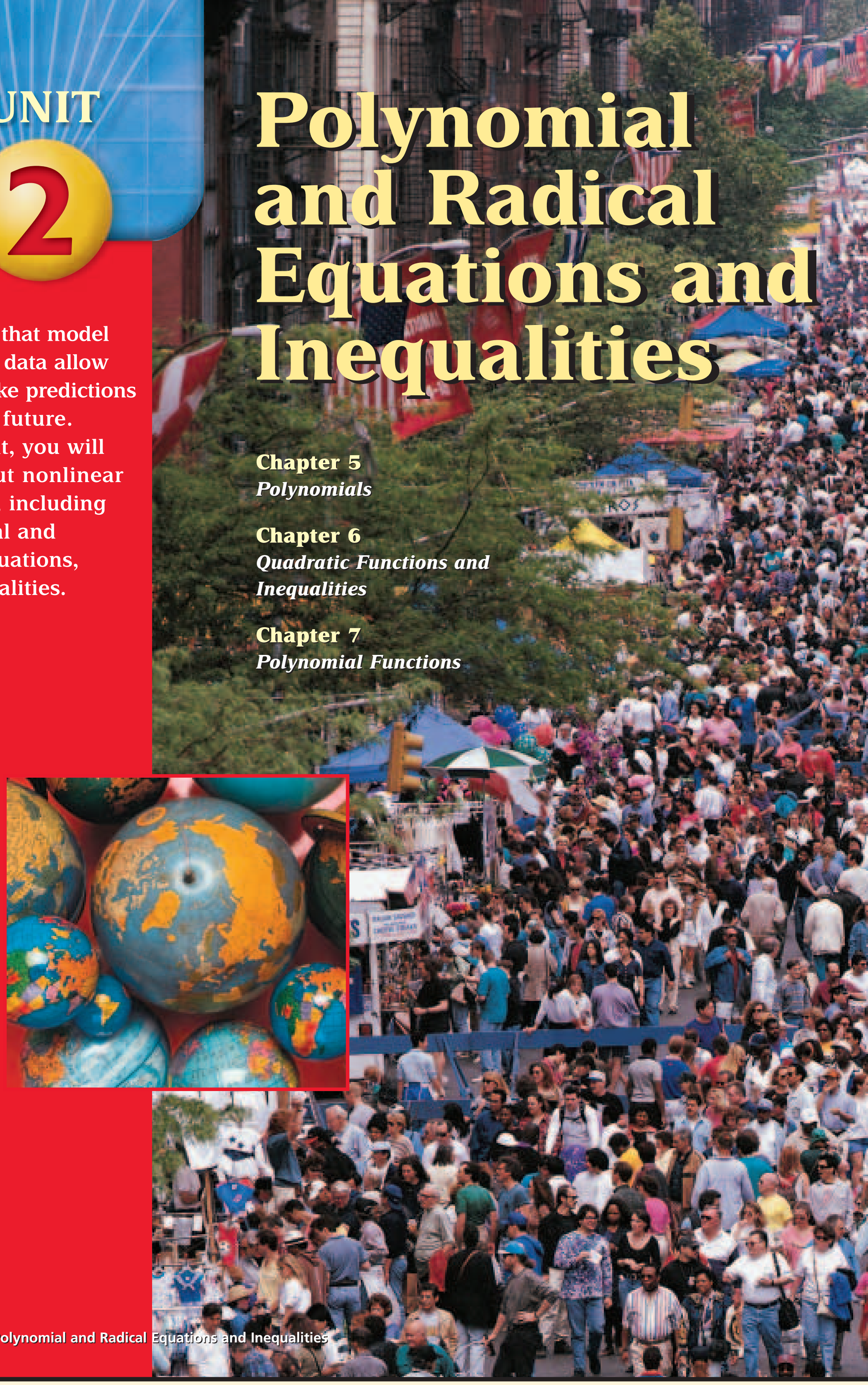
Unit 2 Test Pages 449–450 of the *Chapter 7 Resource Masters* may be used as a test or review for Unit 2. This assessment contains both multiple-choice and short answer items.

TestCheck and Worksheet Builder

This CD-ROM can be used to create additional unit tests and review worksheets.

UNIT 2

Polynomial and Radical Equations and Inequalities



Equations that model real-world data allow you to make predictions about the future. In this unit, you will learn about nonlinear equations, including polynomial and radical equations, and inequalities.

Teaching Suggestions

Have students study the USA TODAY Snapshot®.

- Have students make conjectures about why a quadratic or polynomial model may be better than a linear one for modeling population data.
- According to the given data, what urban area has a population nearly as great as that of New York and Los Angeles combined? **Tokyo**

Additional USA TODAY Snapshots® appearing in Unit 2:

Chapter 5 Hanging on to the old buggy (p. 228)

Chapter 6 More Americans study abroad (p. 328)

Chapter 7 Digital book sales expected to grow (p. 368)

WebQuest Internet Project

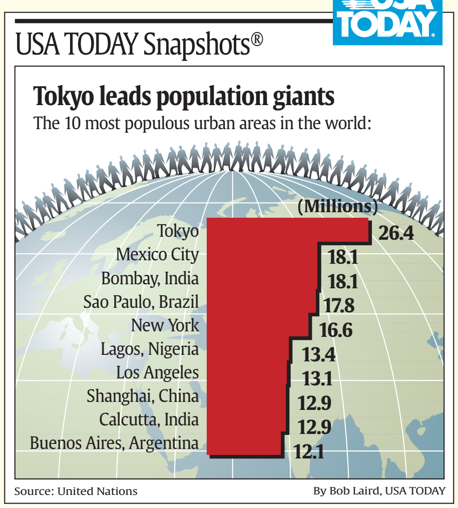
Population Explosion

The United Nations estimated that the world's population reached 6 billion in 1999. The population had doubled in about 40 years and gained 1 billion people in just 12 years. Assuming middle-range birth and death trends, world population is expected to exceed 9 billion by 2050, with most of the increase in countries that are less economically developed. In this project, you will use quadratic and polynomial mathematical models that will help you to project future populations.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 2.

Lesson	5-1	6-6	7-4
Page	227	326	369



WebQuest Internet Project

A WebQuest is an online project in which students do research on the Internet, gather data, and make presentations using word processing, graphing, page-making, or presentation software. In each chapter, students advance to the next step in their WebQuest. At the end of Chapter 7, the project culminates with a presentation of their findings.

Teaching suggestions and sample answers are available in the *WebQuest and Project Resources.*

Chapter 5

Polynomials

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
5-1 Monomials *(pp. 222–228)* • Multiply and divide monomials. • Use expressions written in scientific notation.	1	1	0.5	0.5
5-2 Polynomials *(pp. 229–232)* • Add and subtract polynomials. • Multiply polynomials.	1	1	0.5	0.5
5-3 Dividing Polynomials *(pp. 233–238)* • Divide polynomials using long division. • Divide polynomials using synthetic division.	1	1	0.5	0.5
5-4 Factoring Polynomials *(pp. 239–244)* • Factor polynomials. • Simplify polynomial quotients by factoring.	2	2	1	1
5-5 Roots of Real Numbers *(pp. 245–249)* • Simplify radicals. • Use a calculator to approximate radicals.	1	1	0.5	0.5
5-6 Radical Expressions *(pp. 250–256)* • Simplify radical expressions. • Add, subtract, multiply, and divide radical expressions.	2	2	1	1
5-7 Rational Exponents *(pp. 257–262)* • Write expressions with rational exponents in radical form, and vice versa. • Simplify expressions in exponential or radical form.	2	2	1	1
5-8 Radical Equations and Inequalities *(pp. 263–269)* • Solve equations containing radicals. • Solve inequalities containing radicals. ***Follow-Up:*** Solving Radical Equations and Inequalities by Graphing	2	2 (with 4-8 Follow-Up)	1	1
5-9 Complex Numbers *(pp. 270–275)* • Add and subtract complex numbers. • Multiply and divide complex numbers.	2	2	1	1
Study Guide and **Practice Test** *(pp. 276–281)* **Standardized Test Practice** *(pp. 282–283)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**16**	**16**	**8**	**8**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools

TeacherWorks™

All-In-One Planner and Resource Center

See pages T12–T13.

Chapter Resource Manager

Chapter 5 Resource Masters: Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
239–240	241–242	243	244		SM 109–114	5-1	5-1		
245–246	247–248	249	250		SC 9	5-2	5-2		algebra tiles
251–252	253–254	255	256	307		5-3	5-3		
257–258	259–260	261	262		GCS 35	5-4	5-4	8	algebra tiles, graphing calculator
263–264	265–266	267	268	307, 309		5-5	5-5		index cards, string, small weights
269–270	271–272	273	274			5-6	5-6		
275–276	277–278	279	280	308	GCS 36	5-7	5-7		
281–282	283–284	285	286			5-8	5-8	9	(*Follow-Up:* graphing calculator)
287–288	289–290	291	292	308	SC 10	5-9	5-9		grid paper
				293–306, 310–312					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 5 Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students are familiar with the coefficients, variables, and positive integer exponents that make up monomials, and they have worked with radicals as square roots. They are familiar with the basic arithmetic operations (addition, subtraction, multiplication, division) that will be applied in new situations in this chapter.

This Chapter

Students learn how to apply the basic arithmetic operations to polynomials, radical expressions, and complex numbers. They explore factoring polynomials and solving radical equations and inequalities. They learn how to write equivalent statements by using the Distributive Property, properties of exponents, or properties of radicals. They solve equations and inequalities involving polynomials, radicals, or complex numbers.

Future Connections

As early as the next two chapters, students will factor polynomials to solve quadratic equations, use complex numbers to express the solution to equations or inequalities, and relate complex numbers and roots of polynomials. In their exploration of complex numbers, they will greatly expand on this chapter's brief introduction to the complex coordinate plane.

5-1 Monomials

Throughout this chapter students are introduced to new symbols and new notation. Each time, the new symbols are related to each other and to familiar ideas. In this lesson the new symbol is a negative exponent. The familiar idea is simplifying an expression, and to simplify a monomial means to write an equivalent expression without negative exponents and without parentheses. The rules for writing equivalent expressions include the definition of negative exponents and properties of exponents.

The lesson introduces the terms *coefficient* and *degree* when discussing monomials. It also explores writing and operating on numbers written in scientific notation, and using dimensional analysis to compute with units of measure.

5-2 Polynomials

In this lesson students find the sum or difference of several monomials, which is called a *polynomial*. The familiar ideas in this lesson are the operations of addition, subtraction, and multiplication as applied to polynomials. To add polynomials means to rewrite an indicated sum of polynomials as a sum of terms and then, by combining like terms, to rewrite that sum as a single polynomial. To subtract polynomials means to use the Distributive Property to rewrite the subtraction as a sum of terms, and then to combine like terms. To multiply a polynomial by a monomial means to use the Distributive Property to rewrite the product as a single polynomial.

To multiply two binomials, the lesson shows how two applications of the Distributive Property result in finding the products of the First, Outer, Inner, and Last pairs of terms of the binomials. The two-time application of the Distributive Property is called the FOIL method.

5-3 Dividing Polynomials

This lesson explores polynomial division; with the previous lesson, the four basic arithmetic operations are interpreted for polynomials. Dividing by a monomial uses the Distributive Property. Dividing a polynomial by a binomial (or by any polynomial) uses a process and format analogous to the long division algorithm for whole numbers. The student follows the four steps of "divide, multiply, subtract, bring down"; the steps are repeated until there are no more terms in the dividend.

The lesson also introduces an abbreviated form, called synthetic division, that records and manipulates just the coefficients of the polynomial terms. The divisor must be a binomial of degree 1, the terms of the dividend must be in descending order, using zeros to represent any missing terms, and the polynomial quotient must be written so that the leading coefficient of the divisor is 1.

5-4 Factoring Polynomials

In factoring, a polynomial of degree two or more is rewritten as a product of polynomials each having a lesser degree. Taking out a common factor uses the Distributive Property; factoring by grouping uses two applications of the Distributive Property.

Binomials written in the form of the difference of two squares or as the sum or difference of two cubes can be rewritten as a product of two factors, and a perfect square trinomial can be rewritten as the square of a binomial. Some other trinomials can be factored as the product of two binomials.

Students reduce quotients of polynomials by removing common factors in the numerator and denominator. A record is kept of values of variables that would imply division by zero.

5-5 Roots of Real Numbers

This lesson begins with the familiar idea of a square root: a is a square root of b if $a^2 = b$. Then two new ideas are introduced. One idea is to use the same kind of definition to introduce the nth root of a number: a is an nth root of b if $a^n = b$. The second new idea is to introduce the symbols for principal roots. The principal square root is always a nonnegative number. The value of a principal nth root depends on the sign of the radicand and whether its index is even or odd. The lesson explores how to use absolute value symbols to simplify nth roots.

5-6 Radical Expressions

This lesson explains that "simplifying," as it pertains to radical expressions, takes into account the index of the radical and the form of the radicand. The lesson also presents some rules for writing equivalent radical expressions, and students apply the rules as they add, subtract, multiply, and divide radicals.

5-7 Rational Exponents

The new symbol introduced in this lesson is a fraction used as an exponent. The rules for writing equivalent expressions for rational exponents include properties that describe how to translate between radical form and exponential form. Those rules include dealing with rational exponents that are unit fractions, either positive or negative. The rules for dealing with a base raised to a fractional exponent require that the denominator of the fraction is a positive integer and take into account the sign of the radicand and whether its index is even or odd.

5-8 Radical Equations and Inequalities

This lesson deals with the familiar skills of solving equations and inequalities; the new concept is that the equations contain a variable inside a radical. No new properties are needed to solve these equations or inequalities; equivalent equations or inequalities are written until the variable is isolated on one side. At least once in the solution, both sides of the equation or inequality are raised to a power in order to remove a radical symbol. A most-important idea in the lesson is that sometimes this process of raising both sides to a power does not produce an equivalent statement. For example, it is clear that $\sqrt{x} = -5$ has no real number solution. Squaring both sides results in $x = 25$; the two statements are not equivalent and $x = 25$ is not a solution to the original equation. Raising both sides to a power can introduce an extraneous solution, which is an apparent solution that will not satisfy the original equation or inequality.

5-9 Complex Numbers

This lesson introduces not simply a new symbol but a new set of numbers that are not part of the real number system. The new symbol is $\boldsymbol{i}$, and a new rule is that an expression such as $\sqrt{-5}$ can be rewritten as the equivalent expression $\boldsymbol{i}\sqrt{5}$. The complex number $a + b\boldsymbol{i}$ can be treated as if it is a binomial, and operations on complex numbers follow the properties for adding, subtracting, multiplying, and dividing binomials, with one exception. That exception is to replace $\boldsymbol{i}^2$ with -1 whenever $\boldsymbol{i}^2$ appears in an expression.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Multiplying Monomials (Lesson 22)
- Dividing Monomials (Lesson 23)
- Adding and Subtracting Polynomials (Lesson 24)
- Multiplying a Polynomial by a Monomial (Lesson 25)
- Multiplying Polynomials (Lesson 26)

Chapter 5

DAILY INTERVENTION and Assessment

	Type	Student Edition	Teacher Resources	Technology/Internet
INTERVENTION	Ongoing	Prerequisite Skills, pp. 221, 228, 232, 238, 244, 249, 256, 262, 267 Practice Quiz 1, p. 238 Practice Quiz 2, p. 256	5-Minute Check Transparencies Quizzes, *CRM* pp. 307–308 Mid-Chapter Test, *CRM* p. 309 Study Guide and Intervention, *CRM* pp. 239–240, 245–246, 251–252, 257–258, 263–264, 269–270, 275–276, 281–282, 287–288	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
	Mixed Review	pp. 228, 232, 238, 244, 249, 256, 262, 267, 275	Cumulative Review, *CRM* p. 310	
	Error Analysis	Find the Error, pp. 226, 236	Find the Error, *TWE* pp. 226, 236 Unlocking Misconceptions, *TWE* pp. 223, 235, 244, 246, 253, 258 Tips for New Teachers, *TWE* pp. 228, 238, 244, 246, 256, 262, 267, 275	
ASSESSMENT	Standardized Test Practice	pp. 228, 232, 234, 236, 238, 244, 249, 255, 262, 267, 275, 281, 282–283	*TWE* p. 234 Standardized Test Practice, *CRM* pp. 311–312	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
	Open-Ended Assessment	Writing in Math, pp. 227, 232, 238, 243, 249, 255, 262, 267, 275 Open Ended, pp. 226, 231, 236, 242, 247, 254, 260, 265, 273	Modeling: *TWE* pp. 244, 249 Speaking: *TWE* pp. 228, 256, 262, 275 Writing: *TWE* pp. 232, 238, 267 Open-Ended Assessment, *CRM* p. 305	
	Chapter Assessment	Study Guide, pp. 276–280 Practice Test, p. 281	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 293–298 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 299–304 Vocabulary Test/Review, *CRM* p. 306	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson	
5-4	**8**	*Factoring Expressions II*
5-8	**9**	*Solving Radical Equations*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 221
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 226, 231, 236, 242, 247, 254, 260, 265, 273, 280)
- Writing in Math questions in every lesson, pp. 227, 232, 238, 243, 249, 255, 262, 267, 275
- Reading Study Tip, pp. 229, 246, 252, 270, 271, 273
- WebQuest, p. 227

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 221, 276
- Study Notebook suggestions, pp. 226, 230, 236, 242, 247, 254, 260, 265, 273
- Modeling activities, pp. 244, 249
- Speaking activities, pp. 228, 256, 262, 275
- Writing activities, pp. 232, 238, 267
- Differentiated Instruction, (Verbal/Linguistic), p. 271
- ELL Resources, pp. 220, 227, 231, 237, 243, 248, 255, 261, 266, 271, 274, 276

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 5 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 5 Resource Masters*, pp. 243, 249, 255, 261, 267, 273, 279, 285, 291)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 5 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
5-1	1, 2, 6, 7, 8, 9, 10	
5-2	1, 2, 6, 8, 9, 10	
5-3	1, 2, 6, 7, 8, 9	
5-4	1, 2, 3, 6, 8, 9, 10	
5-5	1, 2, 6, 7, 8, 9	
5-6	1, 2, 6, 7, 8, 9, 10	
5-7	1, 2, 6, 8, 9	
5-8	1, 2, 6, 8, 9, 10	
5-8 Follow-Up	1, 2, 10	
5-9	1, 2, 3, 6, 7, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 5 Polynomials

What You'll Learn

- **Lessons 5-1 through 5-4** Add, subtract, multiply, divide, and factor polynomials.
- **Lessons 5-5 through 5-8** Simplify and solve equations involving roots, radicals, and rational exponents.
- **Lesson 5-9** Perform operations with complex numbers.

Key Vocabulary

- scientific notation (p. 225)
- polynomial (p. 229)
- FOIL method (p. 230)
- synthetic division (p. 234)
- complex number (p. 271)

Why It's Important

Many formulas involve polynomials and/or square roots. For example, equations involving speeds or velocities of objects are often written with square roots. You can use such an equation to find the velocity of a roller coaster. *You will use an equation relating the velocity of a roller coaster and the height of a hill in Lesson 5-6.*

220 Chapter 5 Polynomials

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 5 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 5 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lessons 5-2 and 5-9 — Rewrite Differences as Sums

Rewrite each difference as a sum.

1. $2 - 7$ $2 + (-7)$ **2.** $-6 - 11$ $-6 + (-11)$ **3.** $x - y$ $x + (-y)$

4. $8 - 2x$ $8 + (-2x)$ **5.** $2xy - 6yz$ $2xy + (-6yz)$ **6.** $6a^2b - 12b^2c$ $6a^2b + (-12b^2c)$

For Lesson 5-2 — Distributive Property

Use the Distributive Property to rewrite each expression without parentheses. *(For review, see Lesson 1-2.)* 7. $-8x^3 - 2x + 6$

7. $-2(4x^3 + x - 3)$ **8.** $-1(x + 2)$ $-x - 2$ **9.** $-1(x - 3)$ $-x + 3$

10. $-3(2x^4 - 5x^2 - 2)$ $-6x^4 + 15x^2 + 6$ **11.** $-\frac{1}{2}(3a + 2)$ $-\frac{3}{2}a - 1$ **12.** $-\frac{2}{3}(2 + 6z)$ $-\frac{4}{3} - 4z$

For Lessons 5-5 and 5-9 — Classify Numbers

Find the value of each expression. Then name the sets of numbers to which each value belongs. *(For review, see Lesson 1-2.)* **13–18. See margin.**

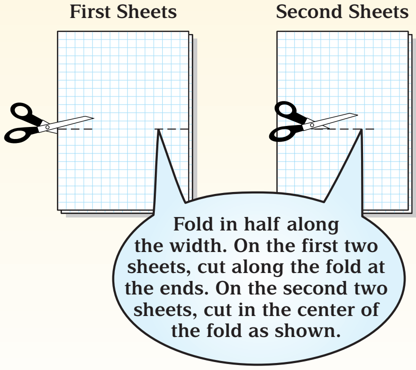
13. $2.6 + 3.7$ **14.** $18 \div (-3)$ **15.** $2^3 + 3^2$

16. $\sqrt{4 + 1}$ **17.** $\frac{18 + 14}{8}$ **18.** $3\sqrt{4}$

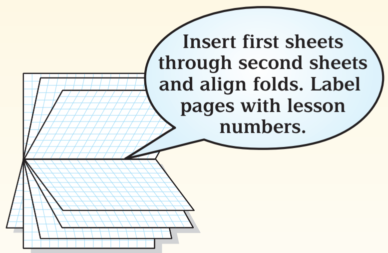
FOLDABLES™ Study Organizer

Make this Foldable to record information about polynomials. Begin with four sheets of grid paper.

Step 1 Fold and Cut



Step 2 Fold and Label



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for polynomials.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 5. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
5-2	Distributive Property (p. 228)
5-3	Properties of Exponents (p. 232)
5-5	Rational and Irrational Numbers (p. 244)
5-6	Multiplying Binomials (p. 249)
5-8	Multiplying Radicals (p. 262)
5-9	Binomials (p. 267)

Answers

13. 6.3; reals, rationals
14. −6; reals, rationals, integers
15. 17; reals, rationals, integers, whole numbers, natural numbers
16. $\sqrt{5}$; reals, irrationals
17. 4; reals, rationals, integers, whole numbers, natural numbers
18. 6; reals, rationals, integers, whole numbers, natural numbers

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Organization of Data and Journal Writing When labeling the pages for the lessons, combine Lessons 5-1 and 5-2 on the same page and Lessons 5-8 and 5-9 on the same page. Use extra pages for vocabulary lists and applications. Writer's journals can also be used to record the direction and progress of learning, to describe positive and negative experiences during learning, to write about personal associations and experiences while learning, and to list examples of ways in which new knowledge has or will be used in their daily life.

5-1 Lesson Notes

1 Focus

5-Minute Check Transparency 5-1 Use as a quiz or a review of Chapter 4.

Mathematical Background notes are available for this lesson on p. 220C.

Why is scientific notation useful in economics?

Ask students:

- What are the powers of ten? **…, 10^{-3}, 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, 10^{3}, …**
- What are some other fields that use scientific notation for very large or very small numbers? **astronomy, biology, computer science**

2 Teach

MONOMIALS

In-Class Example Power Point®

Teaching Tip Help students think carefully about the meaning of exponents by asking them to read this expression aloud correctly. If students read x^3 as "x three," instead of correctly saying "x cubed" or "x to the third (power)," they are apt to confuse x^3 with $3x$.

1 Simplify $(-2a^3b)(-5ab^4)$. **$10a^4b^5$**

5-1 Monomials

What You'll Learn

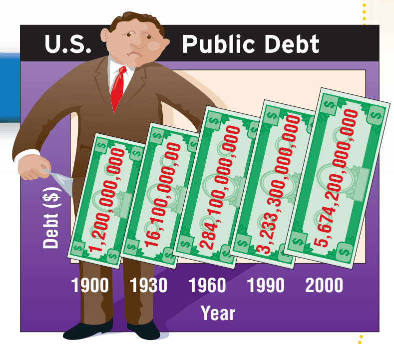
- Multiply and divide monomials.
- Use expressions written in scientific notation.

Vocabulary

- monomial
- constant
- coefficient
- degree
- power
- simplify
- standard notation
- scientific notation
- dimensional analysis

Why is scientific notation useful in economics?

Economists often deal with very large numbers. For example, the table shows the U.S. public debt for several years in the last century. Such numbers, written in standard notation, are difficult to work with because they contain so many digits. Scientific notation uses powers of ten to make very large or very small numbers more manageable.



Source: U.S. Department of the Treasury

MONOMIALS A **monomial** is an expression that is a number, a variable, or the product of a number and one or more variables. Monomials cannot contain variables in denominators, variables with exponents that are negative, or variables under radicals.

Monomials	Not Monomials
$5b, -w, 23, x^2, \frac{1}{3}x^3y^4$	$\frac{1}{n^4}, \sqrt[3]{x}, x + 8, a^{-1}$

Constants are monomials that contain no variables, like 23 or -1. The numerical factor of a monomial is the **coefficient** of the variable(s). For example, the coefficient of m in $-6m$ is -6. The **degree** of a monomial is the sum of the exponents of its variables. For example, the degree of $12g^7h^4$ is $7 + 4$ or 11. The degree of a constant is 0.

A **power** is an expression of the form x^n. The word *power* is also used to refer to the exponent itself. Negative exponents are a way of expressing the multiplicative inverse of a number. For example, $\frac{1}{x^2}$ can be written as x^{-2}. Note that an expression such as x^{-2} is not a monomial. *Why?*

Key Concept — *Negative Exponents*

- **Words** For any real number $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$.
- **Examples** $2^{-3} = \frac{1}{2^3}$ and $\frac{1}{b^{-8}} = b^8$

To **simplify** an expression containing powers means to rewrite the expression without parentheses or negative exponents.

Example 1 *Simplify Expressions with Multiplication*

Simplify $(3x^3y^2)(-4x^2y^4)$.

$(3x^3y^2)(-4x^2y^4) = (3 \cdot x \cdot x \cdot x \cdot y \cdot y)(-4 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y)$ Definition of exponents

$= 3(-4) \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$ Commutative Property

$= -12x^5y^6$ Definition of exponents

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 239–240
- Skills Practice, p. 241
- Practice, p. 242
- Reading to Learn Mathematics, p. 243
- Enrichment, p. 244

Science and Mathematics Lab Manual, pp. 109–114

Transparencies

5-Minute Check Transparency 5-1
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 1 suggests the following property of exponents.

Key Concept — *Product of Powers*

- **Words** For any real number a and integers m and n, $a^m \cdot a^n = a^{m+n}$.
- **Examples** $4^2 \cdot 4^9 = 4^{11}$ and $b^3 \cdot b^5 = b^8$

To multiply powers of the same variable, add the exponents. Knowing this, it seems reasonable to expect that when dividing powers, you would subtract exponents. Consider $\frac{x^9}{x^5}$.

$$\frac{x^9}{x^5} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot x \cdot x \cdot x}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}}} \quad \text{Remember that } x \neq 0.$$

$$= x \cdot x \cdot x \cdot x \quad \text{Simplify.}$$

$$= x^4 \quad \text{Definition of exponents}$$

It appears that our conjecture is true. To divide powers of the same base, you subtract exponents.

Key Concept — *Quotient of Powers*

- **Words** For any real number $a \neq 0$, and integers m and n, $\frac{a^m}{a^n} = a^{m-n}$.
- **Examples** $\frac{5^3}{5} = 5^{3-1}$ or 5^2 and $\frac{x^7}{x^3} = x^{7-3}$ or x^4

Example 2 *Simplify Expressions with Division*

Simplify $\frac{p^3}{p^8}$. Assume that $p \neq 0$.

$$\frac{p^3}{p^8} = p^{3-8} \quad \text{Subtract exponents.}$$

$$= p^{-5} \text{ or } \frac{1}{p^5} \quad \text{Remember that a simplified expression cannot contain negative exponents.}$$

CHECK

$$\frac{p^3}{p^8} = \frac{\overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}} \cdot \overset{1}{\cancel{p}}}{\underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot \underset{1}{\cancel{p}} \cdot p \cdot p \cdot p \cdot p \cdot p} \quad \text{Definition of exponents}$$

$$= \frac{1}{p^5} \quad \text{Simplify.}$$

You can use the Quotient of Powers property and the definition of exponents to simplify $\frac{y^4}{y^4}$, if $y \neq 0$.

Method 1

$$\frac{y^4}{y^4} = y^{4-4} \quad \text{Quotient of Powers}$$

$$= y^0 \quad \text{Subtract.}$$

Method 2

$$\frac{y^4}{y^4} = \frac{\overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{y}}}{\underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{y}}} \quad \text{Definition of exponents}$$

$$= 1 \quad \text{Divide.}$$

In order to make the results of these two methods consistent, we define $y^0 = 1$, where $y \neq 0$. In other words, any nonzero number raised to the zero power is equal to 1. *Notice that 0^0 is undefined.*

www.algebra2.com/extra_examples

In-Class Example — Power Point®

2 Simplify $\frac{s^2}{s^{10}}$. Assume that $s \neq 0$. $\frac{1}{s^8}$

Teaching Tip When discussing In-Class Example 2, if any students get the incorrect answer $\frac{1}{s^5}$, lead them to understand that they divided the exponents instead of subtracting them as a method for dividing the two expressions.

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

DAILY INTERVENTION — Unlocking Misconceptions

- **Correcting Errors** Encourage students to analyze the error or errors they made when they get an incorrect answer. Stress that students should use errors as an opportunity to clarify their thinking.
- **Using Definitions** Suggest that students return to the basic definitions for exponents rather than just memorizing rules. For example, they can derive the rule for multiplying quantities such as $x^2 \cdot x^3$ by rewriting the problem as $x \cdot x \cdot x \cdot x \cdot x$.

In-Class Examples

Power Point®

3 Simplify each expression.

a. $(b^2)^4$ b^8

b. $(-3c^2d^5)^3$ $-27c^6d^{15}$

c. $\left(\frac{-2a}{b^2}\right)^5$ $-\frac{32a^5}{b^{10}}$

d. $\left(\frac{x}{3}\right)^{-4}$ $\frac{81}{x^4}$

Teaching Tip Encourage students to begin each step of a simplification by naming the operation they are about to perform, such as finding the power of a power.

4 Simplify $\left(\frac{-3a^{5y}}{a^{6y}b^4}\right)^5$. $-\frac{243}{a^{5y}b^{20}}$

Concept Check

Monomials Have students write their own summary of the properties of exponents, such as "to multiply expressions with exponents, you add the exponents; to divide, you subtract the exponents" and so on.

The properties we have presented can be used to verify the properties of powers that are listed below.

Key Concept — *Properties of Powers*

- **Words** Suppose a and b are real numbers and m and n are integers. Then the following properties hold.
- **Examples**

Power of a Power: $(a^m)^n = a^{mn}$	$(a^2)^3 = a^6$
Power of a Product: $(ab)^m = a^m b^m$	$(xy)^2 = x^2y^2$
Power of a Quotient: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$ and	$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$
$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}$, $a \neq 0$, $b \neq 0$	$\left(\frac{x}{y}\right)^{-4} = \frac{y^4}{x^4}$

Example 3 Simplify Expressions with Powers

Simplify each expression.

a. $(a^3)^6$

$(a^3)^6 = a^{3(6)}$ Power of a power

$= a^{18}$

b. $(-2p^3s^2)^5$

$(-2p^3s^2)^5 = (-2)^5 \cdot (p^3)^5 \cdot (s^2)^5$

$= -32p^{15}s^{10}$ Power of a power

c. $\left(\frac{-3x}{y}\right)^4$

$\left(\frac{-3x}{y}\right)^4 = \frac{(-3x)^4}{y^4}$ Power of a quotient

$= \frac{(-3)^4x^4}{y^4}$ Power of a product

$= \frac{81x^4}{y^4}$ $(-3)^4 = 81$

d. $\left(\frac{a}{4}\right)^{-3}$

$\left(\frac{a}{4}\right)^{-3} = \left(\frac{4}{a}\right)^3$ Power of a quotient

$= \frac{4^3}{a^3}$ Power of a quotient

$= \frac{64}{a^3}$ $4^3 = 64$

Study Tip

Simplified Expressions

A monomial expression is in simplified form when:

- there are no powers of powers,
- each base appears exactly once,
- all fractions are in simplest form, and
- there are no negative exponents.

With complicated expressions, you often have a choice of which way to start simplifying.

Example 4 Simplify Expressions Using Several Properties

Simplify $\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4$.

Method 1

Raise the numerator and denominator to the fourth power before simplifying.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \frac{(-2x^{3n})^4}{(x^{2n}y^3)^4}$$
$$= \frac{(-2)^4(x^{3n})^4}{(x^{2n})^4(y^3)^4}$$
$$= \frac{16x^{12n}}{x^{8n}y^{12}}$$
$$= \frac{16x^{12n-8n}}{y^{12}}$$
$$= \frac{16x^{4n}}{y^{12}}$$

Method 2

Simplify the fraction before raising to the fourth power.

$$\left(\frac{-2x^{3n}}{x^{2n}y^3}\right)^4 = \left(\frac{-2x^{3n-2n}}{y^3}\right)^4$$
$$= \left(\frac{-2x^n}{y^3}\right)^4$$
$$= \frac{16x^{4n}}{y^{12}}$$

224 Chapter 5 Polynomials

SCIENTIFIC NOTATION The form that you usually write numbers in is **standard notation**. A number is in **scientific notation** when it is in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer. Scientific notation is used to express very large or very small numbers.

Study Tip

Graphing Calculators

To solve scientific notation problems on a graphing calculator, use the EE function. Enter 6.38×10^6 as 6.38 [2nd] [EE] 6.

Example 5 Express Numbers in Scientific Notation

Express each number in scientific notation.

a. 6,380,000

$6{,}380{,}000 = 6.38 \times 1{,}000{,}000$ — $1 \le 6.38 < 10$

$= 6.38 \times 10^6$ — Write 1,000,000 as a power of 10.

b. 0.000047

$0.000047 = 4.7 \times 0.00001$ — $1 \le 4.7 < 10$

$= 4.7 \times \frac{1}{10^5}$ — $0.00001 = \frac{1}{100{,}000}$ or $\frac{1}{10^5}$

$= 4.7 \times 10^{-5}$ — Use a negative exponent.

You can use properties of powers to multiply and divide numbers in scientific notation.

Example 6 Multiply Numbers in Scientific Notation

Evaluate. Express the result in scientific notation.

a. $(4 \times 10^5)(2 \times 10^7)$

$(4 \times 10^5)(2 \times 10^7) = (4 \cdot 2) \times (10^5 \cdot 10^7)$ — Associative and Commutative Properties

$= 8 \times 10^{12}$ — $4 \cdot 2 = 8$, $10^5 \cdot 10^7 = 10^{5+7}$ or 10^{12}

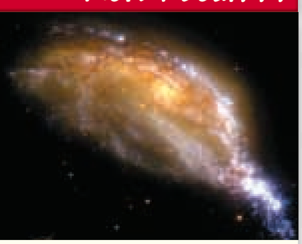
b. $(2.7 \times 10^{-2})(3 \times 10^6)$

$(2.7 \times 10^{-2})(3 \times 10^6) = (2.7 \cdot 3) \times (10^{-2} \cdot 10^6)$ — Associative and Commutative Properties

$= 8.1 \times 10^4$ — $2.7 \cdot 3 = 8.1$, $10^{-2} \cdot 10^6 = 10^{-2+6}$ or 10^4

Real-world problems often involve units of measure. Performing operations with units is known as **dimensional analysis**.

More About. . .



Astronomy

Light travels at a speed of about 3.00×10^8 m/s. The distance that light travels in a year is called a *light-year*.

Source: www.britannica.com

Example 7 Divide Numbers in Scientific Notation

ASTRONOMY **After the Sun, the next-closest star to Earth is Alpha Centauri C, which is about 4×10^{16} meters away. How long does it take light from Alpha Centauri C to reach Earth? Use the information at the left.**

Begin with the formula $d = rt$, where d is distance, r is rate, and t is time.

$t = \frac{d}{r}$ — Solve the formula for time.

$= \frac{4 \times 10^{16} \text{ m}}{3.00 \times 10^8 \text{ m/s}}$ — ← Distance from Alpha Centauri C to Earth; ← speed of light

$= \frac{4}{3.00} \cdot \frac{10^{16}}{10^8 \text{ 1/s}}$ — **Estimate:** The result should be slightly greater than $\frac{10^{16}}{10^8}$ or 10^8.

$\approx 1.33 \times 10^8$ s — $\frac{4}{3.00} \approx 1.33$, $\frac{10^{16}}{10^8} = 10^{16-8}$ or 10^8

It takes about 1.33×10^8 seconds or 4.2 years for light from Alpha Centauri C to reach Earth.

SCIENTIFIC NOTATION

In-Class Examples Power Point®

5 Express each number in scientific notation.

a. 4,560,000 **4.56×10^6**

b. 0.000092 **9.2×10^{-5}**

6 Evaluate. Express the result in scientific notation.

a. $(5 \times 10^3)(7 \times 10^8)$ **3.5×10^{12}**

b. $(1.8 \times 10^{-4})(4 \times 10^7)$ **7.2×10^3**

7 **BIOLOGY** There are about 5×10^6 red blood cells in one milliliter of blood. A certain blood sample contains 8.32×10^6 red blood cells. About how many milliliters of blood are in the sample? **about 1.66 mL**

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students discuss with a partner or in a small group the methods for multiplying and dividing monomial expressions with exponents, and also numbers written in scientific notation. Ask them to work together to develop a list of common errors for such problems, and to suggest ways to correct and avoid these errors.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- add the information about the meaning of dimensional analysis to their notebook.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Monomials:** 18–43
- **Scientific Notation:** 44–60

Odd/Even Assignments

Exercises 18–55 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 59 involves research on the Internet or other reference materials.

Assignment Guide

Basic: 19–39 odd, 45–57 odd, 59–84

Average: 19–57 odd, 59–84

Advanced: 18–58 even, 60–78 (optional: 79–84)

DAILY INTERVENTION **FIND THE ERROR** Suggest that students use two steps to simplify expressions such as $\frac{1}{(-2)^{-2}}$ by first rewriting with the reciprocal and then squaring.

Answers

1. Sample answer: $(2x^2)^3 = 8x^6$ since $(2x^2)^3 = (2x^2) \cdot (2x^2) \cdot (2x^2) = 2x^2 \cdot 2x^2 \cdot 2x^2 = 2x \cdot x \cdot 2x \cdot x \cdot 2x \cdot x = 8x^6$

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an example that illustrates a property of powers. Then use multiplication or division to explain why it is true. **See margin.**
2. **Determine** whether $x^y \cdot x^z = x^{yz}$ is *sometimes, always,* or *never* true. Explain.
3. **FIND THE ERROR** Alejandra and Kyle both simplified $\frac{2a^2b}{(-2ab^3)^{-2}}$.

Alejandra

$$\frac{2a^2b}{(-2ab^3)^{-2}} = (2a^2b)(-2ab^3)^2 = (2a^2b)(-2)^2a^2(b^3)^2 = (2a^2b)2^2a^2b^6 = 8a^4b^7$$

Kyle

$$\frac{2a^2b}{(-2ab^3)^{-2}} = \frac{2a^2b}{(-2)^{-2}a(b^3)^{-2}} = \frac{2a^2b}{4ab^{-6}} = \frac{2a^2bb^6}{4a} = \frac{ab^7}{2}$$

Who is correct? Explain your reasoning.

2. Sometimes; in general $x^y \cdot x^z = x^{y+z}$, so $x^y \cdot x^z = x^{yz}$ when $y + z = yz$, such as when $y = 2$ and $z = 2$.

3. Alejandra; when Kyle used the Power of a Product property in his first step, he forgot to put an exponent of -2 on a. Also, in his second step, $(-2)^{-2}$ should be $\frac{1}{4}$, not 4.

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–9	1–3
10–12	4
13, 14	5
15	6
16, 17	7

Simplify. Assume that no variable equals 0.

4. $x^2 \cdot x^8$ **x^{10}**
5. $(2b)^4$ **$16b^4$**
6. $(n^3)^3(n^{-3})^3$ **1**
7. $\frac{30y^4}{-5y^2}$ **$-6y^2$**
8. $\frac{-2a^3b^6}{18a^2b^2}$ **$-\frac{ab^4}{9}$**
9. $\frac{81p^6q^5}{(3p^2q)^2}$ **$9p^2q^3$**
10. $\left(\frac{1}{w^4z^2}\right)^3$ **$\frac{1}{w^{12}z^6}$**
11. $\left(\frac{cd}{3}\right)^{-2}$ **$\frac{9}{c^2d^2}$**
12. $\left(\frac{-6x^6}{3x^3}\right)^{-2}$ **$\frac{1}{4x^6}$**

Express each number in scientific notation.

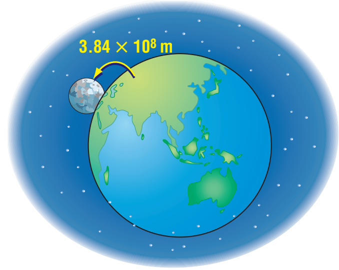
13. 421,000 **4.21×10^5**
14. 0.000862 **8.62×10^{-4}**

Evaluate. Express the result in scientific notation.

15. $(3.42 \times 10^8)(1.1 \times 10^{-5})$ **3.762×10^3**
16. $\frac{8 \times 10^{-1}}{16 \times 10^{-2}}$ **5×10^0**

Application

17. **ASTRONOMY** Refer to Example 7 on page 225. The average distance from Earth to the Moon is about 3.84×10^8 meters. How long would it take a radio signal traveling at the speed of light to cover that distance? **about 1.28 s**



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
18–35, 60	1–3
36–39	4
40–43	1, 2
44–49, 56, 57	5
50–55, 58, 59	6, 7

Extra Practice

See page 836.

Simplify. Assume that no variable equals 0.

18. $a^2 \cdot a^6$ **a^8**
19. $b^{-3} \cdot b^7$ **b^4**
20. $(n^4)^4$ **n^{16}**
21. $(z^2)^5$ **z^{10}**
22. $(2x)^4$ **$16x^4$**
23. $(-2c)^3$ **$-8c^3$**
24. $\frac{a^2n^6}{an^5}$ **an**
25. $\frac{-y^5z^7}{y^2z^5}$ **$-y^3z^2$**
26. $(7x^3y^{-5})(4xy^3)$ **$\frac{28x^4}{y^2}$**
27. $(-3b^3c)(7b^2c^2)$ **$-21b^5c^3$**
28. $(a^3b^3)(ab)^{-2}$ **ab**
29. $(-2r^2s)^3(3rs^2)$ **$-24r^7s^5$**
30. $2x^2(6y^3)(2x^2y)$ **$24x^4y^4$**
31. $3a(5a^2b)(6ab^3)$ **$90a^4b^4$**
32. $\frac{-5x^3y^3z^4}{20x^3y^7z^4}$ **$-\frac{1}{4y^4}$**

62. $(ab)^m = \underbrace{ab \cdot ab \cdot \ldots \cdot ab}_{m \text{ factors}} = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}} \cdot \underbrace{b \cdot b \cdot \ldots \cdot b}_{m \text{ factors}} = a^mb^m$

63. Economics often involves large amounts of money. Answers should include the following.
- **The national debt in 2000 was five trillion, six hundred seventy-four billion, two hundred million or 5.6742×10^{12} dollars. The population was two hundred eighty-one million or 2.81×10^8.**
- **Divide the national debt by the population: $\frac{5.6742 \times 10^{12}}{2.81 \times 10^8} \approx \$2.0193 \times 10^4$ or about \$20,193 per person.**

33. $\frac{3a^5b^3c^3}{9a^3b^7c}$ $\frac{a^2c^2}{3b^4}$

34. $\frac{2c^3d(3c^2d^5)}{30c^4d^2}$ $\frac{cd^4}{5}$

35. $\frac{-12m^4n^8(m^3n^2)}{36m^3n}$ $-\frac{m^4n^9}{3}$

36. $\left(\frac{8a^3b^2}{16a^2b^3}\right)^4$ $\frac{a^4}{16b^4}$

37. $\left(\frac{6x^2y^4}{3x^4y^3}\right)^3$ $\frac{8y^3}{x^6}$

38. $\left(\frac{x}{y^{-1}}\right)^{-2}$ $\frac{1}{x^2y^2}$

39. $\left(\frac{v}{w^{-2}}\right)^{-3}$ $\frac{1}{v^3w^6}$

★ 40. $\frac{30a^{-2}b^{-6}}{60a^{-6}b^{-8}}$ $\frac{a^4b^2}{2}$

★ 41. $\frac{12x^{-3}y^{-2}z^{-8}}{30x^{-6}y^{-4}z^{-1}}$ $\frac{2x^3y^2}{5z^7}$

★ 42. If $2^{r+5} = 2^{2r-1}$, what is the value of r? **6**

★ 43. What value of r makes $y^{28} = y^{3r} \cdot y^7$ true? **7**

Express each number in scientific notation.

44. 462.3 4.623×10^2

45. 43,200 4.32×10^4

46. 0.0001843 1.843×10^{-4}

47. 0.006810 6.81×10^{-3}

48. 502,020,000 5.0202×10^8

49. 675,400,000 6.754×10^8

Evaluate. Express the result in scientific notation.

50. $(4.15 \times 10^3)(3.0 \times 10^6)$ 1.245×10^{10}

51. $(3.01 \times 10^{-2})(2 \times 10^{-3})$ 6.02×10^{-5}

52. $\frac{6.3 \times 10^5}{1.4 \times 10^3}$ 4.5×10^2

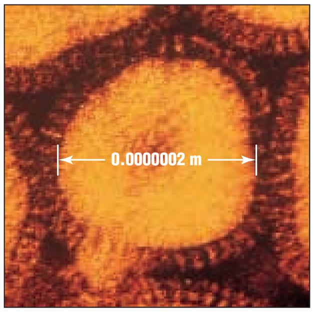
53. $\frac{9.3 \times 10^7}{1.5 \times 10^{-3}}$ 6.2×10^{10}

54. $(6.5 \times 10^4)^2$ 4.225×10^9

55. $(4.1 \times 10^{-4})^2$ 1.681×10^{-7}

56. **POPULATION** The population of Earth is about 6,080,000,000. Write this number in scientific notation. 6.08×10^9

57. **BIOLOGY** Use the diagram at the right to write the diameter of a typical flu virus in scientific notation. 2×10^{-7} m



58. **CHEMISTRY** One gram of water contains about 3.34×10^{22} molecules. About how many molecules are in 500 grams of water? 1.67×10^{25}

59. **RESEARCH** Use the Internet or other source to find the masses of Earth and the Sun. About how many times as large as Earth is the Sun? **about 330,000 times**

60. **CRITICAL THINKING** Determine which is greater, 100^{10} or 10^{100}. Explain. $100^{10} = (10^2)^{10}$ or 10^{20}, and $10^{100} > 10^{20}$, so $10^{100} > 100^{10}$.

CRITICAL THINKING For Exercises 61 and 62, use the following proof of the Power of a Power Property. **61. Definition of an exponent**

$$a^m a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ factors}} \cdot \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ factors}}$$
$$= \underbrace{a \cdot a \cdot \ldots \cdot a}_{m + n \text{ factors}}$$
$$= a^{m+n}$$

61. What definition or property allows you to make each step of the proof?

62. Prove the Power of a Product Property, $(ab)^m = a^m b^m$. **See margin.**

63. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. **See margin.**

Why is scientific notation useful in economics?

Include the following in your answer:
- the 2000 national debt of \$5,674,200,000,000 and the U.S. population of 281,000,000, both written in words and in scientific notation, and
- an explanation of how to find the amount of debt per person, with the result written in scientific notation and in standard notation.

WebQuest

A scatter plot of populations will help you make a model for the data. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 239 (shown) and p. 240

Monomials A **monomial** is a number, a variable, or the product of a number and one or more variables. Constants are monomials that contain no variables.

Negative Exponent	$a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ for any real number $a \neq 0$ and any integer n.

When you **simplify an expression**, you rewrite it without parentheses or negative exponents. The following properties are useful when simplifying expressions.

Product of Powers	$a^m \cdot a^n = a^{m+n}$ for any real number a and integers m and n.
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$ for any real number $a \neq 0$ and integers m and n.
Properties of Powers	For a, b real numbers and m, n integers: $(a^m)^n = a^{mn}$; $(ab)^m = a^m b^m$; $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$; $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ or $\frac{b^n}{a^n}, a \neq 0, b \neq 0$

Example **Simplify. Assume that no variable equals 0.**

a. $(3m^4n^{-2})(-5mn)^2$

$(3m^4n^{-2})(-5mn)^2 = 3m^4n^{-2} \cdot 25m^2n^2 = 75m^4m^2n^{-2}n^2 = 75m^{4+2}n^{-2+2} = 75m^6$

b. $\frac{(-m^4)^3}{(2m^2)^{-2}}$

$\frac{(-m^4)^3}{(2m^2)^{-2}} = \frac{-m^{12}}{\frac{1}{4m^4}} = -m^{12} \cdot 4m^4 = -4m^{16}$

Exercises

Simplify. Assume that no variable equals 0.

1. $c^{12} \cdot c^{-4} \cdot c^6$ c^{14}
2. $\frac{b^8}{b^2}$ b^6
3. $(a^4)^5$ a^{20}
4. $\frac{x^{-2}y}{x^4y^{-1}}$ $\frac{y^2}{x^6}$
5. $\left(\frac{a^2b}{a^{-3}b^2}\right)^{-1}$ $\frac{b}{a^5}$
6. $\left(\frac{x^2y}{xy^3}\right)^2$ $\frac{x^2}{y^4}$
7. $\frac{1}{5}(-5a^2b^3)^2(abc)^2$ $5a^6b^8c^2$
8. $m^7 \cdot m^8$ m^{15}
9. $\frac{8m^3n^2}{4mn^3}$ $\frac{2m^2}{n}$
10. $\frac{2^3c^4t^2}{2^2c^4t^2}$ 2
11. $4j(2j^{-2}k^2)(3j^3k^{-7})$ $\frac{24j^2}{k^5}$
12. $\frac{2mn^2(3m^2n)^2}{12m^3n^4}$ $\frac{3}{2}m^2$

Skills Practice, p. 241 and Practice, p. 242 (shown)

Simplify. Assume that no variable equals 0.

1. $n^5 \cdot n^2$ n^7
2. $y^7 \cdot y^3 \cdot y^2$ y^{12}
3. $t^9 \cdot t^{-8}$ t
4. $x^{-4} \cdot x^{-4} \cdot x^4$ $\frac{1}{x^4}$
5. $(2f^4)^6$ $64f^{24}$
6. $(-2b^{-2}c^3)^3$ $-\frac{8c^9}{b^6}$
7. $(4d^2t^5v^{-4})(-5dt^{-3}v^{-1})$ $-\frac{20d^3t^2}{v^5}$
8. $8u(2z)^3$ $64uz^3$
9. $\frac{12m^8y^6}{-9my^4}$ $-\frac{4m^7y^2}{3}$
10. $\frac{-6s^5x^3}{18sx^7}$ $-\frac{s^4}{3x^4}$
11. $\frac{-27x^3(-x^7)}{16x^4}$ $\frac{27x^6}{16}$
12. $\left(\frac{2}{3r^2s^3z^6}\right)^2$ $\frac{4}{9r^4s^6z^{12}}$
13. $-(4w^{-3}z^{-5})(8w)^2$ $-\frac{256}{wz^5}$
14. $(m^4n^6)^4(m^3n^2p^5)^6$ $m^{34}n^{36}p^{30}$
15. $\left(\frac{3}{2}d^2f^4\right)^4\left(-\frac{4}{3}d^5f\right)^3$ $-12d^{23}f^{19}$
16. $\left(\frac{2x^3y^2}{-x^2y^5}\right)^{-2}$ $\frac{y^6}{4x^2}$
17. $\frac{(3x^{-2}y^3)(5xy^{-8})}{(x^{-3})^4y^{-2}}$ $\frac{15x^{11}}{y^3}$
18. $\frac{-20(m^2v)(-v)^3}{5(-v)^2(-m^4)}$ $-\frac{4v^2}{m^2}$

Express each number in scientific notation.

19. 896,000 8.96×10^5
20. 0.000056 5.6×10^{-5}
21. 433.7×10^8 4.337×10^{10}

Evaluate. Express the result in scientific notation.

22. $(4.8 \times 10^2)(6.9 \times 10^4)$ 3.312×10^7
23. $(3.7 \times 10^9)(8.7 \times 10^2)$ 3.219×10^{12}
24. $\frac{2.7 \times 10^6}{9 \times 10^{10}}$ 3×10^{-5}

25. **COMPUTING** The term *bit*, short for *binary digit*, was first used in 1946 by John Tukey. A single bit holds a zero or a one. Some computers use 32-bit numbers, or strings of 32 consecutive bits, to identify each address in their memories. Each 32-bit number corresponds to a number in our base-ten system. The largest 32-bit number is nearly 4,295,000,000. Write this number in scientific notation. 4.295×10^9

26. **LIGHT** When light passes through water, its velocity is reduced by 25%. If the speed of light in a vacuum is 1.86×10^5 miles per second, at what velocity does it travel through water? Write your answer in scientific notation. 1.395×10^5 mi/s

27. **TREES** Deciduous and coniferous trees are hard to distinguish in a black-and-white photo. But because deciduous trees reflect infrared energy better than coniferous trees, the two types of trees are more distinguishable in an infrared photo. If an infrared wavelength measures about 8×10^{-7} meters and a blue wavelength measures about 4.5×10^{-7} meters, about how many times longer is the infrared wavelength than the blue wavelength? **about 1.8 times**

Reading to Learn Mathematics, p. 243 ELL

Pre-Activity **Why is scientific notation useful in economics?**

Read the introduction to Lesson 5-1 at the top of page 222 in your textbook.

Your textbook gives the U.S. public debt as an example from economics that involves large numbers that are difficult to work with when written in standard notation. Give an example from science that involves very large numbers and one that involves very small numbers. **Sample answer: distances between Earth and the stars, sizes of molecules and atoms**

Reading the Lesson

1. Tell whether each expression is a monomial or not a monomial. If it is a monomial, tell whether it is a constant or not a constant.

   a. $3x^2$ **monomial; not a constant**
   b. $y^2 + 5y - 6$ **not a monomial**
   c. -73 **monomial; constant**
   d. $\frac{1}{z}$ **not a monomial**

2. Complete the following definitions of a negative exponent and a zero exponent.

   For any real number $a \neq 0$ and any integer n, $a^{-n} = \underline{\frac{1}{a^n}}$.

   For any real number $a \neq 0$, $a^0 = \underline{1}$.

3. Name the property or properties of exponents that you would use to simplify each expression. (Do not actually simplify.)

   a. $\frac{m^8}{m^3}$ **quotient of powers**
   b. $y^6 \cdot y^9$ **product of powers**
   c. $(3r^2s)^4$ **power of a product and power of a power**

Helping You Remember

4. When writing a number in scientific notation, some students have trouble remembering when to use positive exponents and when to use negative ones. What is an easy way to remember this? **Sample answer: Use a positive exponent if the number is 10 or greater. Use a negative number if the number is less than 1.**

Enrichment, p. 244

Properties of Exponents

The rules about powers and exponents are usually given with letters such as m, n, and k to represent exponents. For example, one rule states that $a^m \cdot a^n = a^{m+n}$.

In practice, such exponents are handled as algebraic expressions and the rules of algebra apply.

Example 1 **Simplify $2a^2(a^{n+1} + a^{4n})$.**

$2a^2(a^{n+1} + a^{4n}) = 2a^2 \cdot a^{n+1} + 2a^2 \cdot a^{4n}$ Use the Distributive Law.

$= 2a^{2+n+1} + 2a^{2+4n}$ Recall $a^m \cdot a^n = a^{m+n}$.

$= 2a^{n+3} + 2a^{2+4n}$ Simplify the exponent $2 + n + 1$ as $n + 3$.

It is important always to collect *like* terms only.

Example 2 **Simplify $(a^n + b^m)^2$.**

$(a^n + b^m)^2 = (a^n + b^m)(a^n + b^m)$

4 Assess

Open-Ended Assessment

Speaking Have students explain in informal language how to tell whether a number is written in scientific notation. Then have them explain how to simplify monomial expressions involving negative exponents.

Tips for New Teachers

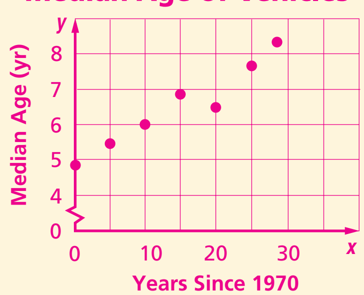
Intervention
Simplifying expressions with exponents is a skill that is needed frequently in algebra. Take time to clear up any misconceptions at this point and to help students develop an understanding of the properties so that they remember the procedures correctly for later use.

Getting Ready for Lesson 5-2

PREREQUISITE SKILL Lesson 5-2 presents multiplying polynomials. This multiplication involves the use of the Distributive Property. Exercises 79–84 should be used to determine your students' familiarity with the Distributive Property.

Answer

74.



Standardized Test Practice

64. Simplify $\frac{(2x^2)^3}{12x^4}$. **D**

(A) $\frac{x}{2}$ (B) $\frac{2x}{3}$ (C) $\frac{1}{2x^2}$ (D) $\frac{2x^2}{3}$

65. $7.3 \times 10^5 = ?$ **B**

(A) 73,000 (B) 730,000 (C) 7,300,000 (D) 73,000,000

Maintain Your Skills

Mixed Review Solve each system of equations by using inverse matrices. *(Lesson 4-8)*

66. $2x + 3y = 8$, $x - 2y = -3$ **(1, 2)**

67. $x + 4y = 9$, $3x + 2y = -3$ **(−3, 3)**

Find the inverse of each matrix, if it exists. *(Lesson 4-7)*

68. $\begin{bmatrix} 2 & 5 \\ -1 & -2 \end{bmatrix}$ $\begin{bmatrix} -2 & -5 \\ 1 & 2 \end{bmatrix}$

69. $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$ $\begin{bmatrix} -\frac{1}{2} & \frac{3}{2} \\ 1 & -2 \end{bmatrix}$

Evaluate each determinant. *(Lesson 4-3)*

70. $\begin{vmatrix} 3 & 0 \\ 2 & -2 \end{vmatrix}$ **−6**

71. $\begin{vmatrix} 1 & 0 & -3 \\ 2 & -1 & 4 \\ -3 & 0 & 2 \end{vmatrix}$ **7**

Solve each system of equations. *(Lesson 3-5)*

72. $x + y = 5$, $x + y + z = 4$, $2x - y + 2z = -1$ **(2, 3, −1)**

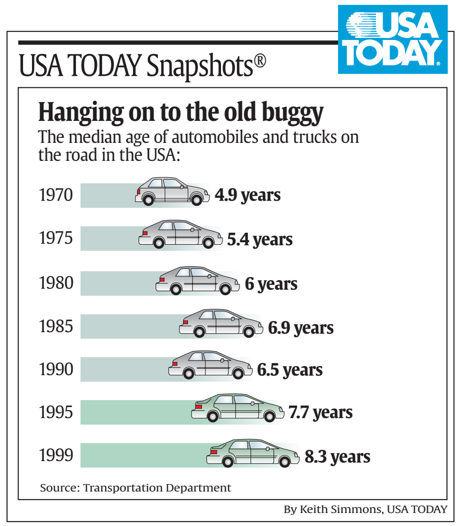
73. $a + b + c = 6$, $2a - b + 3c = 16$, $a + 3b - 2c = -6$ **(2, 0, 4)**

TRANSPORTATION For Exercises 74–76, refer to the graph at the right. *(Lesson 2-5)*

74. Make a scatter plot of the data, where the horizontal axis is the number of years since 1970. **74. See margin.**

75. Write a prediction equation. **75. Sample answer using (0, 4.9) and (28, 8.3): $y = 0.12x + 4.9$**

76. Predict the median age of vehicles on the road in 2010. **Sample answer: 9.7 yr**



Solve each equation. *(Lesson 1-3)*

77. $2x + 11 = 25$ **7**

78. $-12 - 5x = 3$ **−3**

Getting Ready for the Next Lesson Use the Distributive Property to find each product.
*(To review the **Distributive Property**, see Lesson 1-2.)*

79. $2(x + y)$ **$2x + 2y$**

80. $3(x - z)$ **$3x - 3z$**

81. $4(x + 2)$ **$4x + 8$**

82. $-2(3x - 5)$ **$-6x + 10$**

83. $-5(x - 2y)$ **$-5x + 10y$**

84. $-3(-y + 5)$ **$3y - 15$**

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

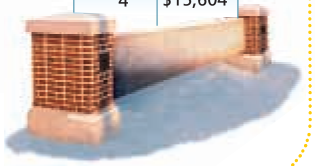
5-2 Polynomials

What You'll Learn

- Add and subtract polynomials.
- Multiply polynomials.

How can polynomials be applied to financial situations?

Shenequa wants to attend Purdue University in Indiana, where the out-of-state tuition is \$13,872. Suppose the tuition increases at a rate of 4% per year. You can use polynomials to represent the increasing tuition costs.



Year	Tuition
1	\$13,872
2	\$14,427
3	\$15,004
4	\$15,604

Vocabulary

- polynomial
- terms
- like terms
- trinomial
- binomial
- FOIL method

ADD AND SUBTRACT POLYNOMIALS

If r represents the rate of increase of tuition, then the tuition for the second year will be $13{,}872(1 + r)$. For the third year, it will be $13{,}872(1 + r)^2$, or $13{,}872r^2 + 27{,}744r + 13{,}872$ in expanded form. The expression $13{,}872r^2 + 27{,}744r + 13{,}872$ is called a polynomial. A **polynomial** is a monomial or a sum of monomials.

The monomials that make up a polynomial are called the **terms** of the polynomial. In a polynomial such as $x^2 + 2x + x + 1$, the two monomials $2x$ and x can be combined because they are **like terms**. The result is $x^2 + 3x + 1$. The polynomial $x^2 + 3x + 1$ is a **trinomial** because it has three unlike terms. A polynomial such as $xy + z^3$ is a **binomial** because it has two unlike terms. The *degree* of a polynomial is the degree of the monomial with the greatest degree. For example, the degree of $x^2 + 3x + 1$ is 2, and the degree of $xy + z^3$ is 3.

Study Tip

Reading Math
The prefix *bi-* means *two*, and the prefix *tri-* means *three*.

Example 1 Degree of a Polynomial

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $\frac{1}{6}x^3y^5 - 9x^4$

This expression is a polynomial because each term is a monomial.
The degree of the first term is 3 + 5 or 8, and the degree of the second term is 4. The degree of the polynomial is 8.

b. $x + \sqrt{x} + 5$

This expression is not a polynomial because $\sqrt{x}$ is not a monomial.

To *simplify* a polynomial means to perform the operations indicated and combine like terms.

Example 2 Subtract and Simplify

Simplify $(3x^2 - 2x + 3) - (x^2 + 4x - 2)$.

$(3x^2 - 2x + 3) - (x^2 + 4x - 2) = 3x^2 - 2x + 3 - x^2 - 4x + 2$ — Distribute the −1.

$= (3x^2 - x^2) + (-2x - 4x) + (3 + 2)$ — Group like terms.

$= 2x^2 - 6x + 5$ — Combine like terms.

www.algebra2.com/extra_examples

5-2 Lesson Notes

1 Focus

5-Minute Check Transparency 5-2 Use as a quiz or review of Lesson 5-1.

Mathematical Background notes are available for this lesson on p. 220C.

How can polynomials be applied to financial situations?

Ask students:

- What is meant by "tuition increases at a rate of 4% per year?" **Each year the tuition is 4% higher than it was the year before.**
- Will the amount of the tuition increase be the same each year? **no**

2 Teach

ADD AND SUBTRACT POLYNOMIALS

In-Class Examples Power Point®

1 Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

a. $c^4 - 4\sqrt{c} + 18$ **no**

b. $-16p^5 + \frac{3}{4}p^2q^7$ **yes, 9**

2 Simplify $(2a^3 + 5a - 7) - (a^3 - 3a + 2)$. $a^3 + 8a - 9$

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 245–246
- Skills Practice, p. 247
- Practice, p. 248
- Reading to Learn Mathematics, p. 249
- Enrichment, p. 250

School-to-Career Masters, p. 9

Teaching Algebra With Manipulatives Masters, p. 234

Transparencies

5-Minute Check Transparency 5-2
Answer Key Transparencies

Technology

Interactive Chalkboard

MULTIPLY POLYNOMIALS

In-Class Examples Power Point®

3 Find $-y(4y^2 + 2y - 3)$.
$-4y^3 - 2y^2 + 3y$

4 Find $(2p + 3)(4p + 1)$.
$8p^2 + 14p + 3$

5 Find $(a^2 + 3a - 4)(a + 2)$.
$a^3 + 5a^2 + 2a - 8$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Add and Subtract Polynomials:** 16–27
- **Multiply Polynomials:** 28–33, 37–50

Odd/Even Assignments
Exercises 16–33 and 37–50 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–33 odd, 35, 36, 37–45 odd, 51, 53–69

Average: 17–33 odd, 35, 36, 37–53 odd, 54–69

Advanced: 16–34 even, 35, 36, 38–52 even, 54–65 (optional: 66–69)

MULTIPLY POLYNOMIALS You can use the Distributive Property to multiply polynomials.

Example 3 Multiply and Simplify

Find $2x(7x^2 - 3x + 5)$.

$2x(7x^2 - 3x + 5) = 2x(7x^2) + 2x(-3x) + 2x(5)$ Distributive Property

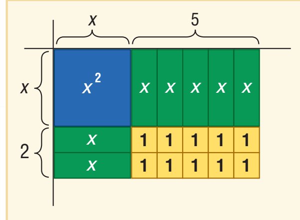
$= 14x^3 - 6x^2 + 10x$ Multiply the monomials.

You can use algebra tiles to model the product of two binomials.

Algebra Activity

Multiplying Binomials

Use algebra tiles to find the product of $x + 5$ and $x + 2$.
- Draw a 90° angle on your paper.
- Use an x tile and a 1 tile to mark off a length equal to $x + 5$ along the top.
- Use the tiles to mark off a length equal to $x + 2$ along the side.
- Draw lines to show the grid formed.
- Fill in the lines with the appropriate tiles to show the area product. The model shows the polynomial $x^2 + 7x + 10$.



The area of the rectangle is the product of its length and width. Substituting for the length, width, and area with the corresponding polynomials, we find that $(x + 5)(x + 2) = x^2 + 7x + 10$.

In Example 4, the **FOIL method** is used to multiply binomials. The FOIL method is an application of the Distributive Property that makes the multiplication easier.

Key Concept *FOIL Method for Multiplying Binomials*

The product of two binomials is the sum of the products of **F** the *first* terms, **O** the *outer* terms, **I** the *inner* terms, and **L** the *last* terms.

Study Tip

Vertical Method
You may also want to use the vertical method to multiply polynomials.

$$\begin{array}{r} 3y + 2 \\ (\times)\ 5y + 4 \\ \hline 12y + 8 \\ 15y^2 + 10y \quad \\ \hline 15y^2 + 22y + 8 \end{array}$$

Example 4 Multiply Two Binomials

Find $(3y + 2)(5y + 4)$.

$(3y + 2)(5y + 4) = \underbrace{3y \cdot 5y}_{\text{First terms}} + \underbrace{3y \cdot 4}_{\text{Outer terms}} + \underbrace{2 \cdot 5y}_{\text{Inner terms}} + \underbrace{2 \cdot 4}_{\text{Last terms}}$

$= 15y^2 + 22y + 8$ Multiply monomials and add like terms.

Example 5 Multiply Polynomials

Find $(n^2 + 6n - 2)(n + 4)$.

$(n^2 + 6n - 2)(n + 4)$

$= n^2(n + 4) + 6n(n + 4) + (-2)(n + 4)$ Distributive Property

$= n^2 \cdot n + n^2 \cdot 4 + 6n \cdot n + 6n \cdot 4 + (-2) \cdot n + (-2) \cdot 4$ Distributive Property

$= n^3 + 4n^2 + 6n^2 + 24n - 2n - 8$ Multiply monomials.

$= n^3 + 10n^2 + 22n - 8$ Combine like terms.

Answers

22. $4x^2 + 3x - 7$
23. $-3y - 3y^2$
24. $r^2 - r + 6$
25. $10m^2 + 5m - 15$
26. $4x^2 - 3xy - 6y^2$

Algebra Activity

Materials: protractor, ruler/straightedge, algebra tiles
- Remind students that the length of an x tile is *not* a multiple of the length of a side of a unit tile.
- Point out to students that the width of an x tile is exactly one unit (the same as the length of a side of a unit tile).

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a polynomial of degree 5 that has three terms.
   1. Sample answer: $x^5 + x^4 + x^3$
2. **Identify** the degree of the polynomial $2x^3 - x^2 + 3x^4 - 7$. **4**
3. **Model** $3x(x + 2)$ using algebra tiles. **See pp. 283A–283B.**

Guided Practice

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

4. $2a + 5b$ **yes, 1**
5. $\frac{1}{3}x^3 - 9y$ **yes, 3**
6. $\frac{mw^2 - 3}{nz^3 + 1}$ **no**

GUIDED PRACTICE KEY

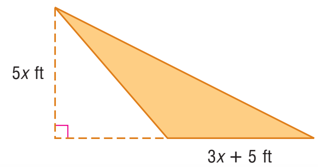
Exercises	Examples
4–6	1
7, 8	2
9, 10	3
11–14	4
15	5

Simplify. **8. $-3x^2 - 7x + 8$ 10. $10p^3q^2 - 6p^5q^3 + 8p^3q^5$**

7. $(2a + 3b) + (8a - 5b)$ **$10a - 2b$**
8. $(x^2 - 4x + 3) - (4x^2 + 3x - 5)$
9. $2x(3y + 9)$ **$6xy + 18x$**
10. $2p^2q(5pq - 3p^3q^2 + 4pq^4)$
11. $(y - 10)(y + 7)$ **$y^2 - 3y - 70$**
12. $(x + 6)(x + 3)$ **$x^2 + 9x + 18$**
13. $(2z - 1)(2z + 1)$ **$4z^2 - 1$**
14. $(2m - 3n)^2$ **$4m^2 - 12mn + 9n^2$**

Application

15. **GEOMETRY** Find the area of the triangle.
    $7.5x^2 + 12.5x$ ft²



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–27, 35, 36, 51	2
28–33, 47, 48	3
34	2, 3
37–46, 52, 53	4
49, 50, 54	5

Extra Practice
See page 837.

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

16. $3z^2 - 5z + 11$ **yes, 2**
17. $x^3 - 9$ **yes, 3**
18. $\frac{6xy}{z} - \frac{3c}{d}$ **no**
19. $\sqrt{m - 5}$ **no**
20. $5x^2y^4 + x\sqrt{3}$ **yes, 6**
21. $\frac{4}{3}y^2 + \frac{5}{6}y^7$ **yes, 7**

Simplify. **22–33. See margin.**

22. $(3x^2 - x + 2) + (x^2 + 4x - 9)$
23. $(5y + 3y^2) + (-8y - 6y^2)$
24. $(9r^2 + 6r + 16) - (8r^2 + 7r + 10)$
25. $(7m^2 + 5m - 9) + (3m^2 - 6)$
26. $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$
27. $(10x^2 - 3xy + 4y^2) - (3x^2 + 5xy)$
28. $4b(cb - zd)$
29. $4a(3a^2 + b)$
30. $-5ab^2(-3a^2b + 6a^3b - 3a^4b^4)$
31. $2xy(3xy^3 - 4xy + 2y^4)$
32. $\frac{3}{4}x^2(8x + 12y - 16xy^2)$
33. $\frac{1}{2}a^3(4a - 6b + 8ab^4)$

34. **PERSONAL FINANCE** Toshiro wants to know how to invest the $850 he has saved. He can invest in a savings account that has an annual interest rate of 3.7%, and he can invest in a money market account that pays about 5.5% per year. Write a polynomial to represent the amount of interest he will earn in 1 year if he invests x dollars in the savings account and the rest in the money market account. **$46.75 - 0.018x$**

E-SALES **For Exercises 35 and 36, use the following information.**
A small online retailer estimates that the cost, in dollars, associated with selling x units of a particular product is given by the expression $0.001x^2 + 5x + 500$. The revenue from selling x units is given by $10x$. **35. $-0.001x^2 + 5x - 500$**

35. Write a polynomial to represent the profit generated by the product.
36. Find the profit from sales of 1850 units. **$5327.50**

www.algebra2.com/self_check_quiz

Answers

27. $7x^2 - 8xy + 4y^2$
28. $4b^2c - 4bdz$
29. $12a^3 + 4ab$
30. $15a^3b^3 - 30a^4b^3 + 15a^5b^6$
31. $6x^2y^4 - 8x^2y^2 + 4xy^5$
32. $6x^3 + 9x^2y - 12x^3y^2$
33. $2a^4 - 3a^3b + 4a^4b^4$

Enrichment, p. 250

Polynomials with Fractional Coefficients

Polynomials may have fractional coefficients as long as there are no variables in the denominators. Computing with fractional coefficients is performed in the same way as computing with whole-number coefficients.

Simpliply. Write all coefficients as fractions.

1. $\left(\frac{3}{5}m - \frac{2}{7}p - \frac{1}{3}n\right) - \left(\frac{7}{3}p - \frac{5}{2}m - \frac{3}{4}n\right)$ **$\frac{31}{10}m + \frac{5}{12}n - \frac{55}{21}p$**
2. $\left(\frac{3}{2}x - \frac{4}{3}y - \frac{5}{4}z\right) + \left(-\frac{1}{4}x + y + \frac{2}{5}z\right) + \left(-\frac{7}{8}x - \frac{6}{7}y + \frac{1}{2}z\right)$ **$\frac{3}{8}x - \frac{25}{21}y - \frac{7}{20}z$**
3. $\left(\frac{1}{2}a^2 - \frac{1}{3}ab + \frac{1}{4}b^2\right) + \left(\frac{5}{6}a^2 + \frac{2}{3}ab - \frac{3}{4}b^2\right)$ **$\frac{4}{3}a^2 + \frac{1}{3}ab - \frac{1}{2}b^2$**

Study Guide and Intervention, p. 245 (shown) and p. 246

Add and Subtract Polynomials

Polynomial	a monomial or a sum of monomials
Like Terms	terms that have the same variable(s) raised to the same power(s)

To add or subtract polynomials, perform the indicated operations and combine like terms.

Example 1 **Simplify $-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$.**

$-6rs + 18r^2 - 5s^2 - 14r^2 + 8rs - 6s^2$
$= (18r^2 - 14r^2) + (-6rs + 8rs) + (-5s^2 - 6s^2)$ Group like terms.
$= 4r^2 + 2rs - 11s^2$ Combine like terms.

Example 2 **Simplify $4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$.**

$4xy^2 + 12xy - 7x^2y - (20xy + 5xy^2 - 8x^2y)$
$= 4xy^2 + 12xy - 7x^2y - 20xy - 5xy^2 + 8x^2y$ Distribute the minus sign.
$= (-7x^2y + 8x^2y) + (4xy^2 - 5xy^2) + (12xy - 20xy)$ Group like terms.
$= x^2y - xy^2 - 8xy$ Combine like terms.

Exercises

Simplify.

1. $(6x^2 - 3x + 2) - (4x^2 + x - 3)$ **$2x^2 - 4x + 5$**
2. $(7y^2 + 12xy - 5x^2) + (6xy - 4y^2 - 3x^2)$ **$3y^2 + 18xy - 8x^2$**
3. $(-4m^2 - 6m) - (6m + 4m^2)$ **$-8m^2 - 12m$**
4. $27x^2 - 5y^2 + 12y^2 - 14x^2$ **$13x^2 + 7y^2$**
5. $(18p^2 + 11pq - 6q^2) - (15p^2 - 3pq + 4q^2)$ **$3p^2 + 14pq - 10q^2$**
6. $17j^2 - 12k^2 + 3j^2 - 15j^2 + 14k^2$ **$5j^2 + 2k^2$**
7. $(8m^2 - 7n^2) - (n^2 - 12m^2)$ **$20m^2 - 8n^2$**
8. $14bc + 6b - 4c + 8b - 8c + 8bc$ **$14b + 22bc - 12c$**
9. $6r^2s + 11rs^2 + 3r^2s - 7rs^2 + 15r^2s - 9rs^2$ **$24r^2s - 5rs^2$**
10. $-9xy + 11x^2 - 14y^2 - (6y^2 - 5xy - 3x^2)$ **$14x^2 - 4xy - 20y^2$**
11. $(12xy - 8x + 3y) + (15x - 7y - 8xy)$ **$7x + 4xy - 4y$**
12. $10.8b^2 - 5.7b + 7.2 - (2.9b^2 - 4.6b - 3.1)$ **$7.9b^2 - 1.1b + 10.3$**
13. $(3bc - 9b^2 - 6c^2) + (4c^2 - b^2 + 5bc)$ **$-10b^2 + 8bc - 2c^2$**
14. $11x^2 + 4y^2 + 6xy + 3y^2 - 5xy - 10x^2$ **$x^2 + xy + 7y^2$**
15. $\frac{1}{4}x^2 - \frac{3}{8}xy + \frac{1}{2}y^2 - \frac{1}{2}xy + \frac{1}{4}y^2 - \frac{3}{8}x^2$ **$-\frac{1}{8}x^2 - \frac{7}{8}xy + \frac{3}{4}y^2$**
16. $24p^3 - 15p^2 + 3p - 15p^3 + 13p^2 - 7p$ **$9p^3 - 2p^2 - 4p$**

Skills Practice, p. 247 and Practice, p. 248 (shown)

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1. $5x^3 + 2xy^4 + 6xy$ **yes; 5**
2. $-\frac{4}{3}ac - a^5d^3$ **yes; 8**
3. $\frac{12m^8n^9}{(m - n)^2}$ **no**
4. $25x^3z - x\sqrt{78}$ **yes; 4**
5. $6c^{-2} + c - 1$ **no**
6. $\frac{5}{r} + \frac{6}{s}$ **no**

Simplify.

7. $(3n^2 + 1) + (8n^2 - 8)$ **$11n^2 - 7$**
8. $(6w - 11w^2) - (4 + 7w^2)$ **$-18w^2 + 6w - 4$**
9. $(-6n - 13n^2) + (-3n + 9n^2)$ **$-9n - 4n^2$**
10. $(8x^2 - 3x) - (4x^2 + 5x - 3)$ **$4x^2 - 8x + 3$**
11. $(5m^2 - 2mp - 6p^2) - (-3m^2 + 5mp + p^2)$ **$8m^2 - 7mp - 7p^2$**
12. $(2x^2 - xy + y^2) + (-3x^2 + 4xy + 3y^2)$ **$-x^2 + 3xy + 4y^2$**
13. $(5t - 7) + (2t^2 + 3t + 12)$ **$2t^2 + 8t + 5$**
14. $(u - 4) - (6 + 3u^2 - 4u)$ **$-3u^2 + 5u - 10$**
15. $-9(y^2 - 7w)$ **$-9y^2 + 63w$**
16. $-9r^4y^2(-3ry^7 + 2r^3y^4 - 8r^{10})$ **$27r^5y^9 - 18r^7y^6 + 72r^{14}y^2$**
17. $-6a^2w(a^3w - aw^4)$ **$-6a^5w^2 + 6a^3w^5$**
18. $5a^2w^3(a^2w^6 - 3a^4w^2 + 9aw^6)$ **$5a^4w^9 - 15a^6w^5 + 45a^3w^9$**
19. $2x^2(x^2 + xy - 2y^2)$ **$2x^4 + 2x^3y - 4x^2y^2$**
20. $-\frac{3}{5}ab^3d^2(-5ab^2d^5 - 5ab)$ **$3a^2b^5d^7 + 3a^2b^4d^2$**
21. $(v^2 - 6)(v^2 + 4)$ **$v^4 - 2v^2 - 24$**
22. $(7a + 9y)(2a - y)$ **$14a^2 + 11ay - 9y^2$**
23. $(y - 8)^2$ **$y^2 - 16y + 64$**
24. $(x^2 + 5y)^2$ **$x^4 + 10x^2y + 25y^2$**
25. $(5x + 4w)(5x - 4w)$ **$25x^2 - 16w^2$**
26. $(2n^4 - 3)(2n^4 + 3)$ **$4n^8 - 9$**
27. $(w + 2s)(w^2 - 2ws + 4s^2)$ **$w^3 + 8s^3$**
28. $(x + y)(x^2 - 3xy + 2y^2)$ **$x^3 - 2x^2y - xy^2 + 2y^3$**
29. **BANKING** Terry invests $1500 in two mutual funds. The first year, one fund grows 3.8% and the other grows 6%. Write a polynomial to represent the amount Terry's $1500 grows to in that year if x represents the amount he invested in the fund with the lesser growth rate. **$-0.022x + 1590$**
30. **GEOMETRY** The area of the base of a rectangular box measures $2x^2 + 4x - 3$ square units. The height of the box measures x units. Find a polynomial expression for the volume of the box. **$2x^3 + 4x^2 - 3x$ units³**

Reading to Learn Mathematics, p. 249 ELL

Pre-Activity **How can polynomials be applied to financial situations?**

Read the introduction to Lesson 5-2 at the top of page 229 in your textbook.

Suppose that Shenequa decides to enroll in a five-year engineering program rather than a four-year program. Using the model given in your textbook, how could she estimate the tuition for the fifth year of her program? (Do not actually calculate, but describe the calculation that would be necessary.) **Multiply $15,604 by 1.04.**

Reading the Lesson

1. State whether the terms in each of the following pairs are *like terms* or *unlike terms*.
   a. $3x^2, 3y^2$ **unlike terms**
   b. $-m^4, 5m^4$ **like terms**
   c. $8r^3, 8s^3$ **unlike terms**
   d. $-6, 6$ **like terms**
2. State whether each of the following expressions is a *monomial*, *binomial*, *trinomial*, or *not a polynomial*. If the expression is a polynomial, give its degree.
   a. $4r^4 - 2r + 1$ **trinomial; degree 4**
   b. $\sqrt{3x}$ **not a polynomial**
   c. $5x + 4y$ **binomial; degree 1**
   d. $2ab + 4ab^2 - 6ab^3$ **trinomial; degree 4**
3. a. What is the FOIL method used for in algebra? **to multiply binomials**
   b. The FOIL method is an application of what property of real numbers? **Distributive Property**
   c. In the FOIL method, what do the letters F, O, I, and L mean? **first, outer, inner, last**
   d. Suppose you want to use the FOIL method to multiply $(2x + 3)(4x + 1)$. Show the terms you would multiply, but do not actually multiply them.
   F **$(2x)(4x)$**
   O **$(2x)(1)$**
   I **$(3)(4x)$**
   L **$(3)(1)$**

Helping You Remember

4. You can remember the difference between *monomials*, *binomials*, and *trinomials* by thinking of common English words that begin with the same prefixes. Give two words unrelated to mathematics that start with *mono-*, two that begin with *bi-*, and two that begin with *tri-*. **Sample answer: monotonous, monogram; bicycle, bifocal; tricycle, tripod**

4 Assess

Open-Ended Assessment

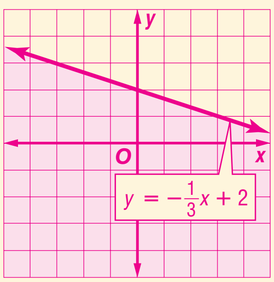
Writing Have students write an explanation, including an example, showing why the FOIL method is a valid alternative to applying the Distributive Property when multiplying two binomials.

Getting Ready for Lesson 5-3

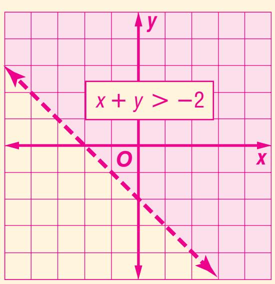
PREREQUISITE SKILL Lesson 5-3 presents dividing polynomials. Dividing polynomials requires the use of the properties of exponents. Exercises 66–69 should be used to determine your students' familiarity with the properties of exponents.

Answers

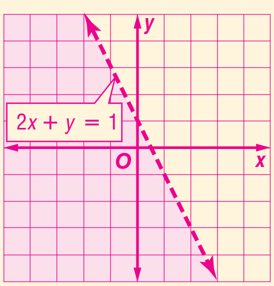
63.



64.



65.



48. $xy^3 + y + \frac{1}{x}$

51. $9c^2 - 12cd + 7d^2$

Simplify.

37. $(p + 6)(p - 4)$ $p^2 + 2p - 24$

38. $(a + 6)(a + 3)$ $a^2 + 9a + 18$

39. $(b + 5)(b - 5)$ $b^2 - 25$

40. $(6 - z)(6 + z)$ $36 - z^2$

41. $(3x + 8)(2x + 6)$ $6x^2 + 34x + 48$

42. $(4y - 6)(2y + 7)$ $8y^2 + 16y - 42$

43. $(a^3 - b)(a^3 + b)$ $a^6 - b^2$

44. $(m^2 - 5)(2m^2 + 3)$ $2m^4 - 7m^2 - 15$

45. $(x - 3y)^2$ $x^2 - 6xy + 9y^2$

46. $(1 + 4c)^2$ $1 + 8c + 16c^2$

★ 47. $d^{-3}(d^5 - 2d^3 + d^{-1})$ $d^2 - 2 + \frac{1}{d^4}$

★ 48. $x^{-3}y^2(yx^4 + y^{-1}x^3 + y^{-2}x^2)$

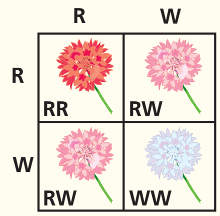
★ 49. $(3b - c)^3$ $27b^3 - 27b^2c + 9bc^2 - c^3$

★ 50. $(x^2 + xy + y^2)(x - y)$ $x^3 - y^3$

51. Simplify $(c^2 - 6cd - 2d^2) + (7c^2 - cd + 8d^2) - (-c^2 + 5cd - d^2)$.

52. Find the product of $6x - 5$ and $-3x + 2$. $-18x^2 + 27x - 10$

More About. . .



Genetics

The possible genes of parents and offspring can be summarized in a *Punnett square*, such as the one above.

Source: *Biology: The Dynamics of Life*

53. **GENETICS** Suppose R and W represent two genes that a plant can inherit from its parents. The terms of the expansion of $(R + W)^2$ represent the possible pairings of the genes in the offspring. Write $(R + W)^2$ as a polynomial. $R^2 + 2RW + W^2$

54. **CRITICAL THINKING** What is the degree of the product of a polynomial of degree 8 and a polynomial of degree 6? Include an example in support of your answer. 14; Sample answer: $(x^8 + 1)(x^6 + 1) = x^{14} + x^8 + x^6 + 1$

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 283A–283B.

How can polynomials be applied to financial situations?

Include the following in your answer:

- an explanation of how a polynomial can be applied to a situation with a fixed percent rate of increase,
- two expressions in terms of r for the tuition in the fourth year, and
- an explanation of how to use one of the expressions and the 4% rate of increase to estimate Shenequa's tuition in the fourth year, and a comparison of the value you found to the value given in the table.

Standardized Test Practice

56. Which polynomial has degree 3? D

(A) $x^3 + x^2 - 2x^4$　(B) $-2x^2 - 3x + 4$

(C) $x^2 + x + 12^3$　(D) $1 + x + x^3$

57. $(x + y) - (y + z) - (x + z) = ?$ B

(A) $2x + 2y + 2z$　(B) $-2z$

(C) $2y$　(D) $x - y - z$

Maintain Your Skills

Mixed Review

Simplify. Assume that no variable equals 0. *(Lesson 5-1)*

58. $(-4d^2)^3$ $-64d^6$

59. $5rt^2(2rt)^2$ $20r^3t^4$

60. $\frac{x^2yz^4}{xy^3z^2}$ $\frac{xz^2}{y^2}$

61. $\left(\frac{3ab^2}{6a^2b}\right)^2$ $\frac{b^2}{4a^2}$

62. Solve the system $4x - y = 0$, $2x + 3y = 14$ by using inverse matrices. *(Lesson 4-8)* (1, 4)

Graph each inequality. *(Lesson 2-7)* 63–65. See margin.

63. $y \le -\frac{1}{3}x + 2$

64. $x + y > -2$

65. $2x + y < 1$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify. Assume that no variable equals 0.**

*(To review **properties of exponents**, see Lesson 5-1.)*

66. $\frac{x^3}{x}$ x^2

67. $\frac{4y^5}{2y^2}$ $2y^3$

68. $\frac{x^2y^3}{xy}$ xy^2

69. $\frac{9a^3b}{3ab}$ $3a^2$

DAILY INTERVENTION

Differentiated Instruction

Logical Have students demonstrate how to use algebra tiles to multiply two binomials that contain at least one negative coefficient.

5-3 Dividing Polynomials

What You'll Learn

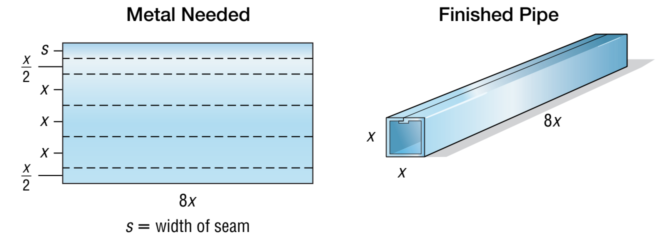
- Divide polynomials using long division.
- Divide polynomials using synthetic division.

Vocabulary

- synthetic division

How can you use division of polynomials in manufacturing?

A machinist needed $32x^2 + x$ square inches of metal to make a square pipe $8x$ inches long. In figuring the area needed, she allowed a fixed amount of metal for overlap of the seam. If the width of the finished pipe will be x inches, how wide is the seam? You can use a quotient of polynomials to help find the answer.



USE LONG DIVISION In Lesson 5-1, you learned to divide monomials. You can divide a polynomial by a monomial by using those same skills.

Example 1 Divide a Polynomial by a Monomial

Simplify $\frac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy}$.

$$\frac{4x^3y^2 + 8xy^2 - 12x^2y^3}{4xy} = \frac{4x^3y^2}{4xy} + \frac{8xy^2}{4xy} - \frac{12x^2y^3}{4xy} \quad \text{Sum of quotients}$$

$$= \frac{4}{4}\cdot x^{3-1}y^{2-1} + \frac{8}{4}\cdot x^{1-1}y^{2-1} - \frac{12}{4}\cdot x^{2-1}y^{3-1} \quad \text{Divide.}$$

$$= x^2y + 2y - 3xy^2 \quad x^{1-1} = x^0 \text{ or } 1$$

You can use a process similar to long division to divide a polynomial by a polynomial with more than one term. The process is known as the *division algorithm*. When doing the division, remember that you can only add or subtract like terms.

Example 2 Division Algorithm

Use long division to find $(z^2 + 2z - 24) \div (z - 4)$.

$$\begin{array}{r} z \phantom{{}-24} \\ z-4\overline{)z^2+2z-24} \\ \underline{(-)z^2-4z} \phantom{{}-24} \\ 6z-24 \end{array} \qquad \begin{array}{l} \\ \\ z(z-4) = z^2-4z \\ 2z-(-4z) = 6z \end{array}$$

$$\begin{array}{r} z+6 \\ z-4\overline{)z^2+2z-24} \\ \underline{(-)z^2-4z} \phantom{{}-24} \\ 6z-24 \\ \underline{(-)6z-24} \\ 0 \end{array}$$

The quotient is $z + 6$. The remainder is 0.

5-3 Lesson Notes

1 Focus

5-Minute Check Transparency 5-3 Use as a quiz or review of Lesson 5-2.

Mathematical Background notes are available for this lesson on p. 220C.

How can you use division of polynomials in manufacturing?

Ask students:

- What does the expression $\frac{x}{2}$ shown in the figure represent? **one half of the side length of the pipe opening**
- What happens to the width of the pipe opening as the length of the pipe increases? **The width of the pipe opening, x, increases also.**

2 Teach

USE LONG DIVISION

In-Class Example Power Point®

1 Simplify $\frac{5a^2b - 15ab^3 + 10a^3b^4}{5ab}$. $a - 3b^2 + 2a^2b^3$

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 251–252
- Skills Practice, p. 253
- Practice, p. 254
- Reading to Learn Mathematics, p. 255
- Enrichment, p. 256
- Assessment, p. 307

Transparencies

5-Minute Check Transparency 5-3
Answer Key Transparencies

Technology

Interactive Chalkboard

In-Class Examples (Power Point®)

2 Use long division to find $(x^2 - 2x - 15) \div (x - 5)$. $x + 3$

Teaching Tip If students are having difficulty with the use of the division algorithm, review the algorithm as it is used for long division of numbers.

3 Which expression is equal to $(a^2 - 5a + 3)(2 - a)^{-1}$? **D**

A $a + 3$

B $-a + 3 + \frac{3}{2 - a}$

C $-a - 3 + \frac{3}{2 - a}$

D $-a + 3 - \frac{3}{2 - a}$

USE SYNTHETIC DIVISION

In-Class Example (Power Point®)

Teaching Tip When discussing Example 4, stress that the divisor must be of the form $x - r$ in order to use synthetic division.

4 Use synthetic division to find $(x^3 - 4x^2 + 6x - 4) \div (x - 2)$. $x^2 - 2x + 2$

Teaching Tip Ask students to discuss whether they would rather use long division or synthetic division, giving a reason for their choice.

Standardized Test Practice

Example 3 Point out that the denominator $5 - t$ is rewritten as $-t + 5$ before starting the division in order to have both numerator and denominator written in descending order of the variable. Point out that the first step in the long division eliminates choice A. Students could also quickly eliminate choice B by multiplying $-t - 8$ by $-t + 5$ and noting that the product is not $t^2 + 3t - 9$.

Just as with the division of whole numbers, the division of two polynomials may result in a quotient with a remainder. Remember that $9 \div 4 = 2 + \text{R}1$ and is often written as $2\frac{1}{4}$. The result of a division of polynomials with a remainder can be written in a similar manner.

Standardized Test Practice

Example 3 Quotient with Remainder

Multiple-Choice Test Item

Which expression is equal to $(t^2 + 3t - 9)(5 - t)^{-1}$?

Ⓐ $t + 8 - \frac{31}{5 - t}$	Ⓑ $-t - 8$
Ⓒ $-t - 8 + \frac{31}{5 - t}$	Ⓓ $-t - 8 - \frac{31}{5 - t}$

Read the Test Item

Since the second factor has an exponent of -1, this is a division problem.

$(t^2 + 3t - 9)(5 - t)^{-1} = \frac{t^2 + 3t - 9}{5 - t}$

Solve the Test Item

$$\begin{array}{r l}
-t - 8 & \\
-t + 5\overline{)t^2 + 3t - 9} & \text{For ease in dividing, rewrite } 5 - t \text{ as } -t + 5. \\
(-)t^2 - 5t & -t(-t + 5) = t^2 - 5t \\
8t - 9 & 3t - (-5t) = 8t \\
(-)8t - 40 & -8(-t + 5) = 8t - 40 \\
31 & \text{Subtract. } -9 - (-40) = 31
\end{array}$$

The quotient is $-t - 8$, and the remainder is 31. Therefore, $(t^2 + 3t - 9)(5 - t)^{-1} = -t - 8 + \frac{31}{5 - t}$. The answer is C.

The Princeton Review — Test-Taking Tip

You may be able to eliminate some of the answer choices by substituting the same value for t in the original expression and the answer choices and evaluating.

USE SYNTHETIC DIVISION

Synthetic division is a simpler process for dividing a polynomial by a binomial. Suppose you want to divide $5x^3 - 13x^2 + 10x - 8$ by $x - 2$ using long division. Compare the coefficients in this division with those in Example 4.

$$\begin{array}{r}
5x^2 - 3x + 4 \\
x - 2\overline{)5x^3 - 13x^2 + 10x - 8} \\
(-)5x^3 - 10x^2 \\
-3x^2 + 10x \\
(-)-3x^2 + 6x \\
4x - 8 \\
(-)4x - 8 \\
0
\end{array}$$

Example 4 Synthetic Division

Use synthetic division to find $(5x^3 - 13x^2 + 10x - 8) \div (x - 2)$.

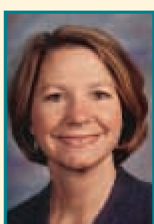
Step 1 Write the terms of the dividend so that the degrees of the terms are in descending order. Then write just the coefficients as shown at the right.

$$\begin{array}{cccc}
5x^3 & -13x^2 & +10x & -8 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
5 & -13 & 10 & -8
\end{array}$$

Step 2 Write the constant r of the divisor $x - r$ to the left. In this case, $r = 2$. Bring the first coefficient, 5, down as shown.

$$\begin{array}{c|cccc}
2 & 5 & -13 & 10 & -8 \\
\hline
 & 5 & & &
\end{array}$$

Teacher to Teacher



Christine Waddell — Albion M.S., Sandy, UT

"To help students better understand the division algorithm for polynomials, I first work through a long division problem with large whole numbers, such as $3248 \div 24$, step by step. Then I work through Example 2 and point out the similarities in each process."

Step 3 Multiply the first coefficient by r: $2 \cdot 5 = 10$. Write the product under the second coefficient. Then add the product and the second coefficient: $-13 + 10 = -3$.

2	5	−13	10	−8
		10		
	5	−3		

Step 4 Multiply the sum, −3, by r: $2(-3) = -6$. Write the product under the next coefficient and add: $10 + (-6) = 4$.

2	5	−13	10	−8
		10	−6	
	5	−3	4	

Step 5 Multiply the sum, 4, by r: $2 \cdot 4 = 8$. Write the product under the next coefficient and add: $-8 + 8 = 0$. The remainder is 0.

2	5	−13	10	−8
		10	−6	8
	5	−3	4	0

The numbers along the bottom row are the coefficients of the quotient. Start with the power of x that is one less than the degree of the dividend. Thus, the quotient is $5x^2 - 3x + 4$.

To use synthetic division, the divisor must be of the form $x - r$. If the coefficient of x in a divisor is not 1, you can rewrite the division expression so that you can use synthetic division.

Example 5 Divisor with First Coefficient Other than 1

Use synthetic division to find $(8x^4 - 4x^2 + x + 4) \div (2x + 1)$.

Use division to rewrite the divisor so it has a first coefficient of 1.

$$\frac{8x^4 - 4x^2 + x + 4}{2x + 1} = \frac{(8x^4 - 4x^2 + x + 4) \div 2}{(2x + 1) \div 2}$$ Divide numerator and denominator by 2.

$$= \frac{4x^4 - 2x^2 + \frac{1}{2}x + 2}{x + \frac{1}{2}}$$ Simplify the numerator and denominator.

Since the numerator does not have an x^3-term, use a coefficient of 0 for x^3.

$x - r = x + \frac{1}{2}$, so $r = -\frac{1}{2}$.

$-\frac{1}{2}$	4	0	−2	$\frac{1}{2}$	2
		−2	1	$\frac{1}{2}$	$-\frac{1}{2}$
	4	−2	−1	1	$\frac{3}{2}$

The result is $4x^3 - 2x^2 - x + 1 + \frac{\frac{3}{2}}{x + \frac{1}{2}}$. Now simplify the fraction.

$$\frac{\frac{3}{2}}{x + \frac{1}{2}} = \frac{3}{2} \div \left(x + \frac{1}{2}\right)$$ Rewrite as a division expression.

$$= \frac{3}{2} \div \frac{2x + 1}{2}$$ $x + \frac{1}{2} = \frac{2x}{2} + \frac{1}{2} = \frac{2x + 1}{2}$

$$= \frac{3}{2} \cdot \frac{2}{2x + 1}$$ Multiply by the reciprocal.

$$= \frac{3}{2x + 1}$$ Multiply.

The solution is $4x^3 - 2x^2 - x + 1 + \frac{3}{2x + 1}$.

(continued on the next page)

www.algebra2.com/extra_examples

In-Class Example

Power Point®

5 Use synthetic division to find $(4y^4 - 5y^2 + 2y + 4) \div (2y - 1)$. $2y^3 + y^2 - 2y + \frac{4}{2y - 1}$

Teaching Tip Remind students to include a coefficient of 0 for any missing terms of the variable in the dividend.

DAILY INTERVENTION

Unlocking Misconceptions

- **Subtracting** Have students analyze any errors they make when using long division. Verify that they are using the signs correctly.
- **Remainders** Have students do a simple numeric division example, such as $8 \div 5$, to help them remember how to write the remainder as part of the quotient.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- add the Test-Taking Tip on p. 234 to their list of tips which they can review as they prepare for standardized tests.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Use Long Division:** 15–20, 45–48
- **Use Synthetic Division:** 21–44, 49, 50

Odd/Even Assignments
Exercises 15–50 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–43 odd, 49, 51, 53, 54, 58–74

Average: 15–51 odd, 53, 54, 58–74

Advanced: 16–52 even, 55–68 (optional: 69–74)

All: Practice Quiz 1 (1–10)

DAILY INTERVENTION **FIND THE ERROR**

Suggest that students recheck their calculations immediately whenever they begin to get large numbers as the coefficients of their quotient. While nothing forbids large coefficients, this is sometimes the first indication that they have made an error in their calculations.

CHECK Divide using long division.

$$\begin{array}{r} 4x^3 - 2x^2 - x + 1 \\ 2x+1\overline{)8x^4 + 0x^3 - 4x^2 + x + 4} \\ \underline{(-)8x^4 + 4x^3} \qquad\qquad\qquad \\ -4x^3 - 4x^2 \qquad\qquad \\ \underline{(-)-4x^3 - 2x^2} \qquad\qquad \\ -2x^2 + x \qquad \\ \underline{(-)-2x^2 - x} \qquad \\ 2x + 4 \\ \underline{(-)2x + 1} \\ 3 \end{array}$$

The result is $4x^3 - 2x^2 - x + 1 + \frac{3}{2x+1}$. ✓

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a quotient of two polynomials such that the remainder is 5. **Sample answer: $(x^2 + x + 5) \div (x + 1)$**
2. **Explain** why synthetic division cannot be used to simplify $\frac{x^3 - 3x + 1}{x^2 + 1}$. **2. The divisor contains an x^2 term.**
3. **FIND THE ERROR** Shelly and Jorge are dividing $x^3 - 2x^2 + x - 3$ by $x - 4$. **3. Jorge; Shelly is subtracting in the columns instead of adding.**

Shelly

4	1	-2	1	-3
		4	-24	100
	1	-6	25	-103

Jorge

4	1	-2	1	-3
		4	8	36
	1	2	9	33

Who is correct? Explain your reasoning.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	**Examples**
4, 5	1
6–10	2, 4
11, 14	3
12, 13	5

Simplify. **7. $3a^3 - 9a^2 + 7a - 6$ 8. $z^4 + 2z^3 + 4z^2 + 5z + 10$**

4. $\frac{6xy^2 - 3xy + 2x^2y}{xy}$ **$6y - 3 + 2x$**
5. $(5ab^2 - 4ab + 7a^2b)(ab)^{-1}$ **$5b - 4 + 7a$**
6. $(x^2 - 10x - 24) \div (x + 2)$ **$x - 12$**
7. $(3a^4 - 6a^3 - 2a^2 + a - 6) \div (a + 1)$
8. $(z^5 - 3z^2 - 20) \div (z - 2)$
9. $(x^3 + y^3) \div (x + y)$ **$x^2 - xy + y^2$**
10. $\frac{x^3 + 13x^2 - 12x - 8}{x + 2}$ **10. $x^2 + 11x - 34 + \frac{60}{x+2}$**
11. $(b^4 - 2b^3 + b^2 - 3b + 2)(b - 2)^{-1}$ **11. $b^3 + b - 1$**
12. $(12y^2 + 36y + 15) \div (6y + 3)$ **$2y + 5$**
13. $\frac{9b^2 + 9b - 10}{3b - 2}$ **$3b + 5$**

Standardized Test Practice

14. Which expression is equal to $(x^2 - 4x + 6)(x - 3)^{-1}$? **B**

Ⓐ $x - 1$ Ⓑ $x - 1 + \frac{3}{x-3}$

Ⓒ $x - 1 - \frac{3}{x-3}$ Ⓓ $-x + 1 - \frac{3}{x-3}$

★ indicates increased difficulty

Practice and Apply

Simplify.

15. $\frac{9a^3b^2 - 18a^2b^3}{3a^2b}$ **$3ab - 6b^2$**
16. $\frac{5xy^2 - 6y^3 + 3x^2y^3}{xy}$ **$5y - \frac{6y^2}{x} + 3xy^2$**
17. $(28c^3d - 42cd^2 + 56cd^3) \div (14cd)$ **17. $2c^2 - 3d + 4d^2$**
18. $(12mn^3 + 9m^2n^2 - 15m^2n) \div (3mn)$ **18. $4n^2 + 3mn - 5m$**
19. $(2y^3z + 4y^2z^2 - 8y^4z^5)(yz)^{-1}$ **$2y^2 + 4yz - 8y^3z^4$**
20. $(a^3b^2 - a^2b + 2a)(-ab)^{-1}$ **$-a^2b + a - \frac{2}{b}$**

DAILY INTERVENTION **Differentiated Instruction**

Interpersonal To help discover confusions and catch careless errors, have students work in pairs as they do division problems. One person should write the solution steps, explaining each step out loud while the other person watches, listens, and checks the work. The students should then exchange roles and repeat the activity.

Homework Help

For Exercises	See Examples
15–20, 51	1
21–34, 49, 50, 52–54	2, 4
35–38	3, 4
39–48	2, 3, 5

Extra Practice

See page 837.

21–48. See pp. 283A–283B.

21. $(b^3 + 8b^2 - 20b) \div (b - 2)$
22. $(x^2 - 12x - 45) \div (x + 3)$
23. $(n^3 + 2n^2 - 5n + 12) \div (n + 4)$
24. $(2c^3 - 3c^2 + 3c - 4) \div (c - 2)$
25. $(x^4 - 3x^3 + x^2 - 5) \div (x + 2)$
26. $(6w^5 - 18w^2 - 120) \div (w - 2)$
27. $(x^3 - 4x^2) \div (x - 4)$
28. $(x^3 - 27) \div (x - 3)$
29. $\dfrac{y^3 + 3y^2 - 5y - 4}{y + 4}$
30. $\dfrac{m^3 + 3m^2 - 7m - 21}{m + 3}$
31. $\dfrac{a^4 - 5a^3 - 13a^2 + 10}{a + 1}$
32. $\dfrac{2m^4 - 5m^3 - 10m + 8}{m - 3}$
33. $\dfrac{x^5 - 7x^3 + x + 1}{x + 3}$
34. $\dfrac{3c^5 + 5c^4 + c + 5}{c + 2}$
35. $(g^2 + 8g + 15)(g + 3)^{-1}$
36. $(2b^3 + b^2 - 2b + 3)(b + 1)^{-1}$
37. $(t^5 - 3t^2 - 20)(t - 2)^{-1}$
38. $(y^5 + 32)(y + 2)^{-1}$
39. $(6t^3 + 5t^2 + 9) \div (2t + 3)$
40. $(2h^3 - 5h^2 + 22h) \div (2h + 3)$
41. $\dfrac{9d^3 + 5d - 8}{3d - 2}$
42. $\dfrac{4x^3 + 5x^2 - 3x - 1}{4x + 1}$
43. $\dfrac{2x^4 + 3x^3 - 2x^2 - 3x - 6}{2x + 3}$
44. $\dfrac{6x^4 + 5x^3 + x^2 - 3x + 1}{3x + 1}$
45. ★ $\dfrac{x^3 - 3x^2 + x - 3}{x^2 + 1}$
46. ★ $\dfrac{x^4 + x^2 - 3x + 5}{x^2 + 2}$
47. ★ $\dfrac{x^3 + 3x^2 + 3x + 2}{x^2 + x + 1}$
48. ★ $\dfrac{x^3 - 4x^2 + 5x - 6}{x^2 - x + 2}$
49. What is $x^3 - 2x^2 + 4x - 3$ divided by $x - 1$? $x^2 - x + 3$
50. Divide $2y^3 + y^2 - 5y + 2$ by $y + 2$. $2y^2 - 3y + 1$

Career Choices

Cost Analyst

Cost analysts study and write reports about the factors involved in the cost of production.

Online Research For information about a career in cost analysis, visit: www.algebra2.com/careers

51. **BUSINESS** A company estimates that it costs $0.03x^2 + 4x + 1000$ dollars to produce x units of a product. Find an expression for the average cost per unit.

52. **ENTERTAINMENT** A magician gives these instructions to a volunteer.
    - Choose a number and multiply it by 3.
    - Then add the sum of your number and 8 to the product you found.
    - Now divide by the sum of your number and 2.

    What number will the volunteer always have at the end? Explain. **4; See margin for explanation.**

MEDICINE **For Exercises 53 and 54, use the following information.**
The number of students at a large high school who will catch the flu during an outbreak can be estimated by $n = \dfrac{170t^2}{t^2 + 1}$, where t is the number of weeks from the beginning of the epidemic and n is the number of ill people.

53. Perform the division indicated by $\dfrac{170t^2}{t^2 + 1}$. $170 - \dfrac{170}{t^2 + 1}$
54. Use the formula to estimate how many people will become ill during the first week. **85 people**

51. $\$0.03x + 4 + \dfrac{1000}{x}$

PHYSICS **For Exercises 55–57, suppose an object moves in a straight line so that after t seconds, it is $t^3 + t^2 + 6t$ feet from its starting point.** **55.** $x^3 + x^2 + 6x - 24$ **ft**

55. Find the distance the object travels between the times $t = 2$ and $t = x$.
56. How much time elapses between $t = 2$ and $t = x$? $x - 2$ **s**
57. Find a simplified expression for the average speed of the object between times $t = 2$ and $t = x$. $x^2 + 3x + 12$ **ft/s**

58. Sample answer: $r^3 - 9r^2 + 27r - 28$ **and** $r - 3$

58. **CRITICAL THINKING** Suppose the result of dividing one polynomial by another is $r^2 - 6r + 9 - \dfrac{1}{r - 3}$. What two polynomials might have been divided?

www.algebra2.com/self_check_quiz

Answer

52. Let x be the number. Multiplying by 3 results in $3x$. The sum of the number, 8, and the result of the multiplication is $x + 8 + 3x$ or $4x + 8$. Dividing by the sum of the number and 2 gives $\dfrac{4x + 8}{x + 2}$ or 4. The end result is always 4.

Study Guide and Intervention, p. 251 (shown) and p. 252

Use Long Division To divide a polynomial by a monomial, use the properties of powers from Lesson 5-1.

To divide a polynomial by a polynomial, use a long division pattern. Remember that only like terms can be added or subtracted.

Example 1 **Simplify** $\dfrac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr}$.

$$\frac{12p^3t^2r - 21p^2qtr^2 - 9p^3tr}{3p^2tr} = \frac{12p^3t^2r}{3p^2tr} - \frac{21p^2qtr^2}{3p^2tr} - \frac{9p^3tr}{3p^2tr}$$
$$= \frac{12}{3}p^{3-2}t^{2-1}r^{1-1} - \frac{21}{3}p^{2-2}qt^{1-1}r^{2-1} - \frac{9}{3}p^{3-2}t^{1-1}r^{1-1}$$
$$= 4pt - 7qr - 3p$$

Example 2 **Use long division to find $(x^3 - 8x^2 + 4x - 9) \div (x - 4)$.**

```
              x² −  4x − 12
x − 4 ) x³ − 8x² +  4x −  9
    (−) x³ − 4x²
            −4x² +  4x
        (−) −4x² + 16x
                  −12x −  9
              (−) −12x + 48
                        −57
```

The quotient is $x^2 - 4x - 12$, and the remainder is -57.

Therefore $\dfrac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \dfrac{57}{x - 4}$.

Exercises

Simplify.

1. $\dfrac{18a^3 + 30a^2}{3a}$ $6a^2 + 10a$
2. $\dfrac{24mn^6 - 40m^2n^3}{4m^2n^3}$ $\dfrac{6n^3}{m} - 10$
3. $\dfrac{60a^2b^3 - 48b^4 + 84a^5b^2}{12ab^2}$ $5ab - \dfrac{4b^2}{a} + 7a^4$
4. $(2x^2 - 5x - 3) \div (x - 3)$ $2x + 1$
5. $(m^2 - 3m - 7) \div (m + 2)$ $m - 5 + \dfrac{3}{m + 2}$
6. $(p^3 - 6) \div (p - 1)$ $p^2 + p + 1 - \dfrac{5}{p - 1}$
7. $(t^3 - 6t^2 + 1) \div (t + 2)$ $t^2 - 8t + 16 - \dfrac{31}{t + 2}$
8. $(x^5 - 1) \div (x - 1)$ $x^4 + x^3 + x^2 + x + 1$
9. $(2x^3 - 5x^2 + 4x - 4) \div (x - 2)$ $2x^2 - x + 2$

Skills Practice, p. 253 and Practice, p. 254 (shown)

Simplify.

1. $\dfrac{15r^{10} - 5r^8 + 40r^2}{5r^4}$ $3r^6 - r^4 + \dfrac{8}{r^2}$
2. $\dfrac{6k^2m - 12k^3m^2 + 9m^3}{2km^2}$ $\dfrac{3k}{m} - 6k^2 + \dfrac{9m}{2k}$
3. $(-30x^3y + 12x^2y^2 - 18x^2y) \div (-6x^2y)$ $5x - 2y + 3$
4. $(-6w^3z^4 - 3w^2z^5 + 4w + 5z) \div (2w^2z)$ $-3wz^3 - \dfrac{3z^4}{2} + \dfrac{2}{wz} + \dfrac{5}{2w^2}$
5. $(4a^3 - 8a^2 + a^2)(4a)^{-1}$ $a^2 - 2a + \dfrac{a}{4}$
6. $(28d^3k^2 + d^2k^2 - 4dk^2)(4dk^2)^{-1}$ $7d^2 + \dfrac{d}{4} - 1$
7. $\dfrac{f^2 + 7f + 10}{f + 2}$ $f + 5$
8. $\dfrac{2x^2 + 3x - 14}{x - 2}$ $2x + 7$
9. $(a^3 - 64) \div (a - 4)$ $a^2 + 4a + 16$
10. $(b^3 + 27) \div (b + 3)$ $b^2 - 3b + 9$
11. $\dfrac{2x^3 + 6x + 152}{x + 4}$ $2x^2 - 8x + 38$
12. $\dfrac{2x^3 + 4x - 6}{x + 3}$ $2x^2 - 6x + 22 - \dfrac{72}{x + 3}$
13. $(3w^3 + 7w^2 - 4w + 3) \div (w + 3)$ $3w^2 - 2w + 2 - \dfrac{3}{w + 3}$
14. $(6y^4 + 15y^3 - 28y - 6) \div (y + 2)$ $6y^3 + 3y^2 - 6y - 16 + \dfrac{26}{y + 2}$
15. $(x^4 - 3x^3 - 11x^2 + 3x + 10) \div (x - 5)$ $x^3 + 2x^2 - x - 2$
16. $(3m^5 + m - 1) \div (m + 1)$ $3m^4 - 3m^3 + 3m^2 - 3m + 4 - \dfrac{5}{m + 1}$
17. $(x^4 - 3x^3 + 5x - 6)(x + 2)^{-1}$ $x^3 - 5x^2 + 10x - 15 + \dfrac{24}{x + 2}$
18. $(6y^2 - 5y - 15)(2y + 3)^{-1}$ $3y - 7 + \dfrac{6}{2y + 3}$
19. $\dfrac{4x^2 - 2x + 6}{2x - 3}$ $2x + 2 + \dfrac{12}{2x - 3}$
20. $\dfrac{6x^2 - x - 7}{3x + 1}$ $2x - 1 - \dfrac{6}{3x + 1}$
21. $(2r^3 + 5r^2 - 2r - 15) \div (2r - 3)$ $r^2 + 4r + 5$
22. $(6t^3 + 5t^2 - 2t + 1) \div (3t + 1)$ $2t^2 + t - 1 + \dfrac{2}{3t + 1}$
23. $\dfrac{4p^4 - 17p^2 + 14p - 3}{2p - 3}$ $2p^3 + 3p^2 - 4p + 1$
24. $\dfrac{2h^4 - h^3 + h^2 + h - 3}{h^2 - 1}$ $2h^2 - h + 3$
25. **GEOMETRY** The area of a rectangle is $2x^2 - 11x + 15$ square feet. The length of the rectangle is $2x - 5$ feet. What is the width of the rectangle? $x - 3$ ft
26. **GEOMETRY** The area of a triangle is $15x^4 + 3x^3 + 4x^2 - x - 3$ square meters. The length of the base of the triangle is $6x^2 - 2$ meters. What is the height of the triangle? $5x^2 + x + 3$ m

Reading to Learn Mathematics, p. 255

ELL

Pre-Activity **How can you use division of polynomials in manufacturing?**

Read the introduction to Lesson 5-3 at the top of page 233 in your textbook.

Using the division symbol (÷), write the division problem that you would use to answer the question asked in the introduction. (Do not actually divide.) $(32x^2 + x) \div (8x)$

Reading the Lesson

1. a. Explain in words how to divide a polynomial by a monomial. **Divide each term of the polynomial by the monomial.**
   b. If you divide a trinomial by a monomial and get a polynomial, what kind of polynomial will the quotient be? **trinomial**
2. Look at the following division example that uses the division algorithm for polynomials.

```
          2x  + 4
x − 4 ) 2x² − 4x + 7
        2x² − 8x
              4x + 7
              4x − 16
                   23
```

Which of the following is the correct way to write the quotient? **C**

A. $2x + 4$ B. $x - 4$ C. $2x + 4 + \dfrac{23}{x - 4}$ D. $\dfrac{23}{x - 4}$

3. If you use synthetic division to divide $x^3 + 3x^2 - 5x - 8$ by $x - 2$, the division will look like this:

```
2 |  1   3   −5   −8
         2   10   10
    ----------------
     1   5    5 |  2
```

Which of the following is the answer for this division problem? **B**

A. $x^2 + 5x + 5$ B. $x^2 + 5x + 5 + \dfrac{2}{x - 2}$
C. $x^3 + 5x^2 + 5x + \dfrac{2}{x - 2}$ D. $x^3 + 5x^2 + 5x + 2$

Helping You Remember

4. When you translate the numbers in the last row of a synthetic division into the quotient and remainder, what is an easy way to remember which exponents to use in writing the terms of the quotient? **Sample answer: Start with the power that is one less than the degree of the dividend. Decrease the power by one for each term after the first. The final number will be the remainder. Drop any term that is represented by a 0.**

Enrichment, p. 256

Oblique Asymptotes

The graph of $y = ax + b$, where $a \neq 0$, is called an oblique asymptote of $y = f(x)$ if the graph of f comes closer and closer to the line as $x \to \infty$ or $x \to -\infty$. ∞ is the mathematical symbol for **infinity**, which means *endless*.

For $f(x) = 3x + 4 + \dfrac{2}{x}$, $y = 3x + 4$ is an oblique asymptote because $f(x) - 3x - 4 = \dfrac{2}{x}$, and $\dfrac{2}{x} \to 0$ as $x \to \infty$ or $-\infty$. In other words, as $|x|$ increases, the value of $\dfrac{2}{x}$ gets smaller and smaller approaching 0.

Example **Find the oblique asymptote for $f(x) = \dfrac{x^2 + 8x + 15}{x + 2}$.**

```
−2 |  1   8    15
         −2   −12
     -------------
      1   6  |  3
```

Use synthetic division.

$y = \dfrac{x^2 + 8x + 15}{x + 2} = x + 6 + \dfrac{3}{x + 2}$

4 Assess

Open-Ended Assessment

Writing Have students write their own list of tips for how to do division problems, describing the techniques they use to help avoid making errors.

Tips for New Teachers

Intervention Some students may have trouble keeping their concentration throughout the sequence of steps required in long division. Encourage them to compare intermediate results with a partner, so that they can ask questions and catch errors before completing the entire problem.

Getting Ready for Lesson 5-4

BASIC SKILL Lesson 5-4 presents factoring polynomials. This requires a knowledge of the greatest common factor. Exercises 69–74 should be used to determine your students' familiarity with the greatest common factor of a set of numbers.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 5-1 through 5-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 5-1 through 5-3) is available on p. 307 of the *Chapter 5 Resource Masters*.

---

59. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 283A–283B.**

How can you use division of polynomials in manufacturing?

Include the following in your answer:

- the dimensions of the piece of metal that the machinist needs,
- the formula from geometry that applies to this situation, and
- an explanation of how to use division of polynomials to find the width s of the seam.

Standardized Test Practice

60. An office employs x women and 3 men. What is the ratio of the total number of employees to the number of women? **A**

(A) $1 + \frac{3}{x}$ (B) $\frac{x}{x+3}$ (C) $\frac{3}{x}$ (D) $\frac{x}{3}$

61. If $a + b = c$ and $a = b$, then all of the following are true EXCEPT **D**

(A) $a - c = b - c$. (B) $a - b = 0$.
(C) $2a + 2b = 2c$. (D) $c - b = 2a$.

Maintain Your Skills

Mixed Review

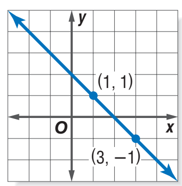
Simplify. *(Lesson 5-2)* **62.** $-x^2 - 4x + 14$ **63.** $y^4z^4 - y^3z^3 + 3y^2z$

62. $(2x^2 - 3x + 5) - (3x^2 + x - 9)$ **63.** $y^2z(y^2z^3 - yz^2 + 3)$

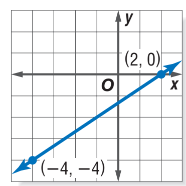
64. $(y + 5)(y - 3)$ $y^2 + 2y - 15$ **65.** $(a - b)^2$ $a^2 - 2ab + b^2$

66. ASTRONOMY Earth is an average of 1.5×10^{11} meters from the Sun. Light travels at 3×10^8 meters per second. About how long does it take sunlight to reach Earth? *(Lesson 5-1)* 5×10^2 **s or 8 min 20 s**

Write an equation in slope-intercept form for each graph. *(Lesson 2-4)*

67. 

$y = -x + 2$

68. 

$y = \frac{2}{3}x - \frac{4}{3}$

Getting Ready for the Next Lesson

BASIC SKILL **Find the greatest common factor of each set of numbers.**

69. 18, 27 **9** **70.** 24, 84 **12** **71.** 16, 28 **4**

72. 12, 27, 48 **3** **73.** 12, 30, 54 **6** **74.** 15, 30, 65 **5**

Practice Quiz 1 — *Lessons 5-1 through 5-3*

Express each number in scientific notation. *(Lesson 5-1)*

1. 653,000,000 6.53×10^8 **2.** 0.0072 7.2×10^{-3}

Simplify. *(Lessons 5-1 and 5-2)*

3. $(-3x^2y)^3(2x)^2$ $-108x^8y^3$ **4.** $\frac{a^6b^{-2}c}{a^3b^2c^4}$ $\frac{a^3}{b^4c^3}$ **5.** $\left(\frac{x^2z}{xz^4}\right)^2$ $\frac{x^2}{z^6}$

6. $(9x + 2y) - (7x - 3y)$ $2x + 5y$ **7.** $(t + 2)(3t - 4)$ $3t^2 + 2t - 8$ **8.** $(n + 2)(n^2 - 3n + 1)$ $n^3 - n^2 - 5n + 2$

Simplify. *(Lesson 5-3)* **9.** $m^2 - 3 - \frac{19}{m-4}$

9. $(m^3 - 4m^2 - 3m - 7) \div (m - 4)$ **10.** $\frac{2d^3 - d^2 - 9d + 9}{2d - 3}$ $d^2 + d - 3$

238 Chapter 5 Polynomials

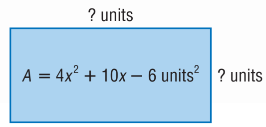
5-4 Factoring Polynomials

What You'll Learn

- Factor polynomials.
- Simplify polynomial quotients by factoring.

How does factoring apply to geometry?

Suppose the expression $4x^2 + 10x - 6$ represents the area of a rectangle. Factoring can be used to find possible dimensions of the rectangle.



FACTOR POLYNOMIALS Whole numbers are factored using prime numbers. For example, $100 = 2 \cdot 2 \cdot 5 \cdot 5$. Many polynomials can also be factored. Their factors, however, are other polynomials. Polynomials that cannot be factored are called *prime*.

The table below summarizes the most common factoring techniques used with polynomials.

Concept Summary — *Factoring Techniques*

Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$a^3b^2 + 2a^2b - 4ab^2 = ab(a^2b + 2a - 4b)$
two	Difference of Two Squares Sum of Two Cubes Difference of Two Cubes	$a^2 - b^2 = (a + b)(a - b)$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials	$acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by = x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Whenever you factor a polynomial, always look for a common factor first. Then determine whether the resulting polynomial factor can be factored again using one or more of the methods listed in the table above.

Example 1 GCF

Factor $6x^2y^2 - 2xy^2 + 6x^3y$.

$$6x^2y^2 - 2xy^2 + 6x^3y = (2 \cdot 3 \cdot x \cdot x \cdot y \cdot y) - (2 \cdot x \cdot y \cdot y) + (2 \cdot 3 \cdot x \cdot x \cdot x \cdot y)$$
$$= (2xy \cdot 3xy) - (2xy \cdot y) + (2xy \cdot 3x^2)$$
$$= 2xy(3xy - y + 3x^2)$$

The GCF is $2xy$. The remaining polynomial cannot be factored using the methods above.

Check this result by finding the product.

A GCF is also used in grouping to factor a polynomial of four or more terms.

5-4 Lesson Notes

1 Focus

5-Minute Check Transparency 5-4 Use as a quiz or review of Lesson 5-3.

Mathematical Background notes are available for this lesson on p. 220D.

Building on Prior Knowledge

In this lesson, students will need to recall how to find the area of a rectangle, and they will also need to remember the set of prime numbers.

How does factoring apply to geometry?

Ask students:

- How do Examples 1 and 2 in Lesson 5-3 relate to factoring, the topic of this lesson? **The quotient and the divisor are factors of the dividend.**

2 Teach

FACTOR POLYNOMIALS

In-Class Example Power Point®

1 Factor $10a^3b^2 + 15a^2b - 5ab^3$. **$5ab(2a^2b + 3a - b^2)$**

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters
- Study Guide and Intervention, pp. 257–258
- Skills Practice, p. 259
- Practice, p. 260
- Reading to Learn Mathematics, p. 261
- Enrichment, p. 262

Graphing Calculator and Spreadsheet Masters, p. 35

Teaching Algebra With Manipulatives Masters, p. 235

Transparencies

5-Minute Check Transparency 5-4
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 8
Interactive Chalkboard

In-Class Example

Power Point®

2 Factor $x^3 + 5x^2 - 2x - 10$. $(x + 5)(x^2 - 2)$

Teaching Tip Point out to students that it is often difficult to recognize that grouping can be used to factor a polynomial. Stress that this technique should only be considered when trying to factor a polynomial with four terms.

Example 2 Grouping

Factor $a^3 - 4a^2 + 3a - 12$.

$a^3 - 4a^2 + 3a - 12 = (a^3 - 4a^2) + (3a - 12)$ Group to find a GCF.

$= a^2(a - 4) + 3(a - 4)$ Factor the GCF of each binomial.

$= (a - 4)(a^2 + 3)$ Distributive Property

You can use algebra tiles to model factoring a polynomial.

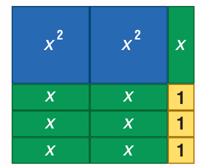
Algebra Activity

Factoring Trinomials

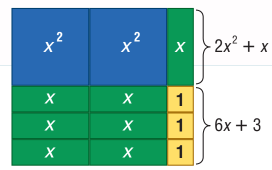
Use algebra tiles to factor $2x^2 + 7x + 3$.

Model and Analyze

- Use algebra tiles to model $2x^2 + 7x + 3$.
- To find the product that resulted in this polynomial, arrange the tiles to form a rectangle.



- Notice that the total area can be expressed as the sum of the areas of two smaller rectangles.



Use these expressions to rewrite the trinomial. Then factor.

$2x^2 + 7x + 3 = (2x^2 + x) + (6x + 3)$ total area = sum of areas of smaller rectangles

$= x(2x + 1) + 3(2x + 1)$ Factor out each GCF.

$= (2x + 1)(x + 3)$ Distributive Property

Make a Conjecture

Study the factorization of $2x^2 + 7x + 3$ above.

1. What are the coefficients of the two x terms in $(2x^2 + x) + (6x + 3)$? Find their sum and their product. **1 and 6; 7; 6**
2. Compare the sum you found in Exercise 1 to the coefficient of the x term in $2x^2 + 7x + 3$. **They are the same.**
3. Find the product of the coefficient of the x^2 term and the constant term in $2x^2 + 7x + 3$. How does it compare to the product in Exercise 1?
4. Make a conjecture about how to factor $3x^2 + 7x + 2$.

3. 6; It is the same.

4. Find two numbers with a product of $3 \cdot 2$ or 6 and a sum of 7. Use those numbers to rewrite the trinomial. Then factor.

Study Tip

Algebra Tiles When modeling a polynomial with algebra tiles, it is easiest to arrange the x^2 tiles first, then the x tiles and finally the 1 tiles to form a rectangle.

The FOIL method can help you factor a polynomial into the product of two binomials. Study the following example.

$$(ax + b)(cx + d) = \overbrace{ax \cdot cx}^{F} + \overbrace{ax \cdot d}^{O} + \overbrace{b \cdot cx}^{I} + \overbrace{b \cdot d}^{L}$$

$$= acx^2 + (ad + bc)x + bd$$

Notice that the product of the coefficient of x^2 and the constant term is $abcd$. The product of the two terms in the coefficient of x is also $abcd$.

Algebra Activity

Materials: algebra tiles

- Inform students that the two factors of the trinomial can be read directly from the completed array of tiles in another way. Point out that the length of the array is $2x + 1$, and the height is $x + 3$. The product of the length and width gives the area, $2x^2 + 7x + 3$, of the array.
- You might wish to have students experiment to see if there is another way to form a rectangle with the tiles.

Example 3 Two or Three Terms

Factor each polynomial.

a. $5x^2 - 13x + 6$

To find the coefficients of the x-terms, you must find two numbers whose product is $5 \cdot 6$ or 30, and whose sum is -13. The two coefficients must be -10 and -3 since $(-10)(-3) = 30$ and $-10 + (-3) = -13$.

Rewrite the expression using $-10x$ and $-3x$ in place of $-13x$ and factor by grouping.

$5x^2 - 13x + 6 = 5x^2 - 10x - 3x + 6$ Substitute $-10x - 3x$ for $-13x$.

$= (5x^2 - 10x) + (-3x + 6)$ Associative Property

$= 5x(x - 2) - 3(x - 2)$ Factor out the GCF of each group.

$= (5x - 3)(x - 2)$ Distributive Property

b. $3xy^2 - 48x$

$3xy^2 - 48x = 3x(y^2 - 16)$ Factor out the GCF.

$= 3x(y + 4)(y - 4)$ $y^2 - 16$ is the difference of two squares.

c. $c^3d^3 + 27$

$c^3d^3 = (cd)^3$ and $27 = 3^3$. Thus, this is the sum of two cubes.

$c^3d^3 + 27 = (cd + 3)[(cd)^2 - 3(cd) + 3^2]$ Sum of two cubes formula with $a = cd$ and $b = 3$

$= (cd + 3)(c^2d^2 - 3cd + 9)$ Simplify.

d. $m^6 - n^6$

This polynomial could be considered the difference of two squares or the difference of two cubes. The difference of two squares should always be done before the difference of two cubes. This will make the next step of the factorization easier.

$m^6 - n^6 = (m^3 + n^3)(m^3 - n^3)$ Difference of two squares

$= (m + n)(m^2 - mn + n^2)(m - n)(m^2 + mn + n^2)$ Sum and difference of two cubes

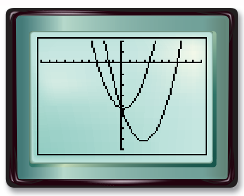
You can use a graphing calculator to check that the factored form of a polynomial is correct.

Graphing Calculator Investigation

Factoring Polynomials

Is the factored form of $2x^2 - 11x - 21$ equal to $(2x - 7)(x + 3)$? You can find out by graphing $y = 2x^2 - 11x - 21$ and $y = (2x - 7)(x + 3)$. If the two graphs coincide, the factored form is probably correct.

- Enter $y = 2x^2 - 11x - 21$ and $y = (2x - 7)(x + 3)$ on the Y= screen.
- Graph the functions. Since two different graphs appear, $2x^2 - 11x - 21 \neq (2x - 7)(x + 3)$.



[−10, 10] scl: 1 by [−40, 10] scl: 5

Think and Discuss

1. Determine if $x^2 + 5x - 6 = (x - 3)(x - 2)$ is a true statement. If not, write the correct factorization. no; $(x + 6)(x - 1)$
2. Does this method guarantee a way to check the factored form of a polynomial? Why or why not?

2. No; in some cases, the graphs might be so close in shape that they seem to coincide but do not.

www.algebra2.com/extra_examples

In-Class Example

Power Point®

3 Factor each polynomial.

a. $3y^2 - 2y - 5$ $(3y - 5)(y + 1)$

b. $5mp^2 - 45m$ $5m(p + 3)(p - 3)$

c. $x^3y^3 + 8$ $(xy + 2)(x^2y^2 - 2xy + 4)$

d. $64x^6 - y^6$ $(2x - y)(4x^2 + 2xy + y^2)(2x + y)(4x^2 - 2xy + y^2)$

Teaching Tip Emphasize the importance of checking each factor to make sure it is prime before deciding that the final group of factors has been found.

Concept Check

Ask students to write an ordered list describing what they will check for as they factor a polynomial. **Sample answer: First look for any common factors of the terms; if there is a common factor, find the GCF. After factoring out the GCF, look for the difference of two squares, a perfect square trinomial, and so on. Use the factoring techniques listed on p. 239 to factor the expression further. Examine the resulting factored form to see if there are any factors that are not prime. If so, continue the process. If not, the factoring is complete.**

Graphing Calculator Investigation

Factoring Polynomials So that students see what happens when a polynomial and a correct factorization are graphed, have students graph the functions $y = x^2 - 81$ and $y = (x - 9)(x + 9)$ in the same screen. It looks like only one graph appears on the screen because both graphs are the same.

Lesson 5-4 Factoring Polynomials 241

SIMPLIFY QUOTIENTS

In-Class Example Power Point®

4 Simplify $\frac{a^2 - a - 6}{a^2 + 7a + 10}$.

$\frac{a - 3}{a + 5}$, **if** $a \neq -5, -2$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- add a list of factoring techniques to their notebook, including the factoring of the special cases listed in the Concept Summary on p. 239 and the FOIL method described on p. 240.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Factor Polynomials:** 15–45
- **Simplify Quotients:** 46–51

Odd/Even Assignments

Exercises 15–44 and 46–51 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–37 odd, 43–49 odd, 55–58, 63–81

Average: 15–51 odd, 55–58, 63–81 (optional: 59–62)

Advanced: 16–50 even, 52–75 (optional: 76–81)

SIMPLIFY QUOTIENTS

In Lesson 5-3, you learned to simplify the quotient of two polynomials by using long division or synthetic division. Some quotients can be simplified using factoring.

Example 4 Quotient of Two Trinomials

Simplify $\frac{x^2 + 2x - 3}{x^2 + 7x + 12}$.

$\frac{x^2 + 2x - 3}{x^2 + 7x + 12} = \frac{\overset{1}{\cancel{(x + 3)}}(x - 1)}{(x + 4)\underset{1}{\cancel{(x + 3)}}}$ Factor the numerator and denominator.

$= \frac{x - 1}{x + 4}$ Divide. Assume $x \neq -3, -4$.

Therefore, $\frac{x^2 + 2x - 3}{x^2 + 7x + 12} = \frac{x - 1}{x + 4}$, if $x \neq -3, -4$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an example of a perfect square trinomial.
2. **Find a counterexample** to the statement $a^2 + b^2 = (a + b)^2$.
3. **Decide** whether the statement $\frac{x - 2}{x^2 + x - 6} = \frac{1}{x + 3}$ is *sometimes*, *always*, or *never* true. **sometimes**

1. Sample answer: $x^2 + 2x + 1$

2. Sample answer: If $a = 1$ **and** $b = 1$**, then** $a^2 + b^2 = 2$ **but** $(a + b)^2 = 4$.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–5	1
6	2
7–11, 14	3
12–13	4

Factor completely. If the polynomial is not factorable, write *prime*.

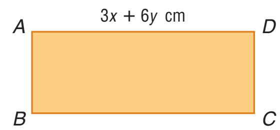
4. $-12x^2 - 6x$ $\mathbf{-6x(2x + 1)}$
5. $a^2 + 5a + ab$ $\mathbf{a(a + 5 + b)}$
6. $21 - 7y + 3x - xy$ $\mathbf{(x + 7)(3 - y)}$
7. $y^2 - 6y + 8$ $\mathbf{(y - 2)(y - 4)}$
8. $z^2 - 4z - 12$ $\mathbf{(z - 6)(z + 2)}$
9. $3b^2 - 48$ $\mathbf{3(b - 4)(b + 4)}$
10. $16w^2 - 169$ $\mathbf{(4w + 13)(4w - 13)}$
11. $h^3 + 8000$ $\mathbf{(h + 20)(h^2 - 20h + 400)}$

Simplify. Assume that no denominator is equal to 0.

12. $\frac{x^2 - 2x - 8}{x^2 - 5x - 14}$ $\mathbf{\frac{x - 4}{x - 7}}$
13. $\frac{2y^2 + 8y}{y^2 - 16}$ $\mathbf{\frac{2y}{y - 4}}$

Application

14. **GEOMETRY** Find the width of rectangle *ABCD* if its area is $3x^2 + 9xy + 6y^2$ square centimeters. $\mathbf{x + y}$ **cm**



★ indicates increased difficulty

Practice and Apply

Factor completely. If the polynomial is not factorable, write *prime*.

15. $2xy^3 - 10x$ $\mathbf{2x(y^3 - 5)}$
16. $6a^2b^2 + 18ab^3$ $\mathbf{6ab^2(a + 3b)}$
17. $12cd^3 - 8c^2d^2 + 10c^5d^3$
18. $3a^2bx + 15cx^2y + 25ad^3y$ **prime**
19. $8yz - 6z - 12y + 9$ $\mathbf{(2z - 3)(4y - 3)}$
20. $3ax - 15a + x - 5$ $\mathbf{(3a + 1)(x - 5)}$
21. $x^2 + 7x + 6$ $\mathbf{(x + 1)(x + 6)}$
22. $y^2 - 5y + 4$ $\mathbf{(y - 1)(y - 4)}$
23. $2a^2 + 3a + 1$ $\mathbf{(2a + 1)(a + 1)}$
24. $2b^2 + 13b - 7$ $\mathbf{(2b - 1)(b + 7)}$
25. $6c^2 + 13c + 6$ $\mathbf{(2c + 3)(3c + 2)}$
26. $12m^2 - m - 6$ $\mathbf{(3m + 2)(4m - 3)}$
27. $3n^2 + 21n - 24$ $\mathbf{3(n + 8)(n - 1)}$
28. $3z^2 + 24z + 45$ $\mathbf{3(z + 3)(z + 5)}$

17. $\mathbf{2cd^2(6d - 4c + 5c^4d)}$

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Ask students to create songs or raps to help them remember the factoring techniques for the difference of two squares, for the sum or difference of two cubes, or for one of the two perfect square trinomial types.

Homework Help

For Exercises	See Examples
15–18	1
19, 20	2
21–38, 43–45, 55	3
39–42	2, 3
46–54	4

Extra Practice

See page 837.

29. $x^2 + 12x + 36$ $(x + 6)^2$
30. $x^2 - 6x + 9$ $(x - 3)^2$
31. $16a^2 + 25b^2$ prime
32. $3m^2 - 3n^2$ $3(m + n)(m - n)$
33. $y^4 - z^2$ $(y^2 + z)(y^2 - z)$
34. $3x^2 - 27y^2$ $3(x + 3y)(x - 3y)$
35. $z^3 + 125$ $(z + 5)(z^2 - 5z + 25)$
36. $t^3 - 8$ $(t - 2)(t^2 + 2t + 4)$
37. $p^4 - 1$ $(p^2 + 1)(p + 1)(p - 1)$
38. $x^4 - 81$ $(x^2 + 9)(x + 3)(x - 3)$

★ 39. $7ac^2 + 2bc^2 - 7ad^2 - 2bd^2$ $(7a + 2b)(c + d)(c - d)$

★ 40. $8x^2 + 8xy + 8xz + 3x + 3y + 3z$ $(8x + 3)(x + y + z)$

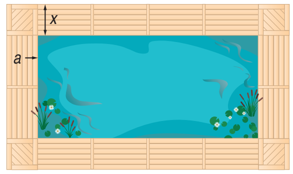
★ 41. $5a^2x + 4aby + 3acz - 5abx - 4b^2y - 3bcz$ $(a - b)(5ax + 4by + 3cz)$

★ 42. $3a^3 + 2a^2 - 5a + 9a^2b + 6ab - 15b$ $(a + 3b)(3a + 5)(a - 1)$

43. Find the factorization of $3x^2 + x - 2$. $(3x - 2)(x + 1)$

44. What are the factors of $2y^2 + 9y + 4$? $(2y + 1)(y + 4)$

45. **LANDSCAPING** A boardwalk that is x feet wide is built around a rectangular pond. The combined area of the pond and the boardwalk is $4x^2 + 140x + 1200$ square feet. What are the dimensions of the pond? 30 ft by 40 ft



Simplify. Assume that no denominator is equal to 0.

46. $\dfrac{x^2 + 4x + 3}{x^2 - x - 12}$ $\dfrac{x + 1}{x - 4}$

47. $\dfrac{x^2 + 4x - 5}{x^2 - 7x + 6}$ $\dfrac{x + 5}{x - 6}$

48. $\dfrac{x^2 - 25}{x^2 + 3x - 10}$ $\dfrac{x - 5}{x - 2}$

49. $\dfrac{x^2 - 6x + 8}{x^3 - 8}$ $\dfrac{x - 4}{x^2 + 2x + 4}$

★ 50. $\dfrac{x^2}{(x^2 - x)(x - 1)^{-1}}$ x

★ 51. $\dfrac{x + 1}{(x^2 + 3x + 2)(x + 2)^{-2}}$ $x + 2$

BUILDINGS **For Exercises 52 and 53, use the following information.**
When an object is dropped from a tall building, the distance it falls between 1 second after it is dropped and x seconds after it is dropped is $16x^2 - 16$ feet.

52. How much time elapses between 1 second after it is dropped and x seconds after it is dropped? $x - 1$ s

53. What is the average speed of the object during that time period? $16x + 16$ ft/s

54. **GEOMETRY** The length of one leg of a right triangle is $x - 6$ centimeters, and the area is $\frac{1}{2}x^2 - 7x + 24$ square centimeters. What is the length of the other leg? $x - 8$ cm

55. **CRITICAL THINKING** Factor $64p^{2n} + 16p^n + 1$. $(8p^n + 1)^2$

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 283A–283B.

How does factoring apply to geometry?

Include the following in your answer:

- an explanation of how to use factoring to find possible dimensions for the rectangle described at the beginning of the lesson, and
- why your dimensions are not the only ones possible, even if you assume that the dimensions are binomials with integer coefficients.

More About. . .

Buildings

The tallest buildings in the world are the Petronas Towers in Kuala Lumpur, Malaysia. Each is 1483 feet tall.

Source: www.worldstallest.com

www.algebra2.com/self_check_quiz

Enrichment, p. 262

Using Patterns to Factor

Study the patterns below for factoring the sum and the difference of cubes.

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

This pattern can be extended to other odd powers. Study these examples.

Example 1 **Factor $a^5 + b^5$.**

Extend the first pattern to obtain $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$.

Check: $(a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4) = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4$
$+ a^4b - a^3b^2 + a^2b^3 - ab^4 + b^5$
$= a^5 + b^5$

Example 2 **Factor $a^5 - b^5$.**

Extend the second pattern to obtain $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$.

Check: $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4) = a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4$

Study Guide and Intervention, p. 257 (shown) and p. 258

Factor Polynomials

Techniques for Factoring Polynomials	For any number of terms, check for: greatest common factor
	For two terms, check for: Difference of two squares $a^2 - b^2 = (a + b)(a - b)$ Sum of two cubes $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ Difference of two cubes $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
	For three terms, check for: Perfect square trinomials $a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$ General trinomials $acx^2 + (ad + bc)x + bd = (ax + b)(cx + d)$
	For four terms, check for: Grouping $ax + bx + ay + by = x(a + b) + y(a + b) = (a + b)(x + y)$

Example **Factor $24x^2 - 42x - 45$.**

First factor out the GCF to get $24x^2 - 42x - 45 = 3(8x^2 - 14x - 15)$. To find the coefficients of the x terms, you must find two numbers whose product is $8 \cdot (-15) = -120$ and whose sum is -14. The two coefficients must be -20 and 6. Rewrite the expression using $-20x$ and $6x$ and factor by grouping.

$8x^2 - 14x - 15 = 8x^2 - 20x + 6x - 15$ Group to find a GCF.
$= 4x(2x - 5) + 3(2x - 5)$ Factor the GCF of each binomial.
$= (4x + 3)(2x - 5)$ Distributive Property

Thus, $24x^2 - 42x - 45 = 3(4x + 3)(2x - 5)$.

Exercises

Factor completely. If the polynomial is not factorable, write *prime*.

1. $14x^2y^2 + 42xy^3$ $14xy^2(x + 3y)$
2. $6mn + 18m - n - 3$ $(6m - 1)(n + 3)$
3. $2x^2 + 18x + 16$ $2(x + 8)(x + 1)$
4. $x^4 - 1$ $(x^2 + 1)(x + 1)(x - 1)$
5. $35x^3y^4 - 60x^4y$ $5x^3y(7y^3 - 12x)$
6. $2r^3 + 250$ $2(r + 5)(r^2 - 5r + 25)$
7. $100m^8 - 9$ $(10m^4 - 3)(10m^4 + 3)$
8. $x^2 + x + 1$ prime
9. $c^4 + c^3 - c^2 - c$ $c(c + 1)^2(c - 1)$

Skills Practice, p. 259 and Practice, p. 260 (shown)

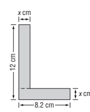
Factor completely. If the polynomial is not factorable, write *prime*.

1. $15a^2b - 10ab^2$ $5ab(3a - 2b)$
2. $3st^2 - 9s^3t + 6s^2t^2$ $3st(t - 3s^2 + 2st)$
3. $3x^3y^2 - 2x^2y + 5xy$ $xy(3x^2y - 2x + 5)$
4. $2x^3y - x^2y + 5xy^2 + xy^3$ $xy(2x^2 - x + 5y + y^2)$
5. $21 - 7t + 3r - rt$ $(7 + r)(3 - t)$
6. $x^2 - xy + 2x - 2y$ $(x + 2)(x - y)$
7. $y^2 + 20y + 96$ $(y + 8)(y + 12)$
8. $4ab + 2a + 6b + 3$ $(2a + 3)(2b + 1)$
9. $6n^2 - 11n - 2$ $(6n + 1)(n - 2)$
10. $6x^2 + 7x - 3$ $(3x - 1)(2x + 3)$
11. $x^2 - 8x - 8$ prime
12. $6p^2 - 17p - 45$ $(2p - 9)(3p + 5)$
13. $r^3 + 3r^2 - 54r$ $r(r + 9)(r - 6)$
14. $8a^2 + 2a - 6$ $2(4a - 3)(a + 1)$
15. $c^2 - 49$ $(c - 7)(c + 7)$
16. $x^3 + 8$ $(x + 2)(x^2 - 2x + 4)$
17. $16r^2 - 169$ $(4r + 13)(4r - 13)$
18. $b^4 - 81$ $(b^2 + 9)(b + 3)(b - 3)$
19. $8m^3 - 25$ prime
20. $2t^3 + 32t^2 + 128t$ $2t(t + 8)^2$
21. $5y^5 + 135y^2$ $5y^2(y + 3)(y^2 - 3y + 9)$
22. $81x^4 - 16$ $(9x^2 + 4)(3x + 2)(3x - 2)$

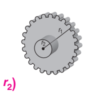
Simplify. Assume that no denominator is equal to 0.

23. $\dfrac{x^2 - 16}{x^2 + x - 20}$ $\dfrac{x + 4}{x + 5}$
24. $\dfrac{x^2 - 16x + 64}{x^2 + x - 72}$ $\dfrac{x - 8}{x + 9}$
25. $\dfrac{3x^2 - 27}{x^3 - 27}$ $\dfrac{3(x + 3)}{x^2 + 3x + 9}$

26. **DESIGN** Bobbi Jo is using a software package to create a drawing of a cross section of a brace as shown at the right. Write a simplified, factored expression that represents the area of the cross section of the brace. $x(20.2 - x)$ cm^2



27. **COMBUSTION ENGINES** In an internal combustion engine, the up and down motion of the pistons is converted into the rotary motion of the crankshaft, which drives the flywheel. Let r_1 represent the radius of the flywheel at the right and let r_2 represent the radius of the crankshaft passing through it. If the formula for the area of a circle is $A = \pi r^2$, write a simplified, factored expression for the area of the cross section of the flywheel outside the crankshaft. $\pi(r_1 - r_2)(r_1 + r_2)$



Reading to Learn Mathematics, p. 261 ELL

Pre-Activity **How does factoring apply to geometry?**

Read the introduction to Lesson 5-4 at the top of page 239 in your textbook.

If a trinomial that represents the area of a rectangle is factored into two binomials, what might the two binomials represent? the length and width of the rectangle

Reading the Lesson

1. Name three types of binomials that it is always possible to factor. difference of two squares, sum of two cubes, difference of two cubes

2. Name a type of trinomial that it is always possible to factor. perfect square trinomial

3. Complete: Since $x^2 + y^2$ cannot be factored, it is an example of a ___prime___ polynomial.

4. On an algebra quiz, Marlene needed to factor $2x^2 - 4x - 70$. She wrote the following answer: $(x + 5)(2x - 14)$. When she got her quiz back, Marlene found that she did not get full credit for her answer. She thought she should have gotten full credit because she checked her work by multiplication and showed that $(x + 5)(2x - 14) = 2x^2 - 4x - 70$.

   a. If you were Marlene's teacher, how would you explain to her that her answer was not entirely correct? Sample answer: When you are asked to factor a polynomial, you must factor it *completely*. The factorization was not complete, because $2x - 14$ can be factored further as $2(x - 7)$.

   b. What advice could Marlene's teacher give her to avoid making the same kind of error in factoring in the future? Sample answer: Always look for a common factor first. If there is a common factor, factor it out first, and then see if you can factor further.

Helping You Remember

5. Some students have trouble remembering the correct signs in the formulas for the sum and difference of two cubes. What is an easy way to remember the correct signs? Sample answer: In the binomial factor, the operation sign is the same as in the expression that is being factored. In the trinomial factor, the operation sign before the middle term is the opposite of the sign in the expression that is being factored. The sign before the last term is always a plus.

4 Assess

Open-Ended Assessment

Modeling Have students use algebra tiles to model and determine the factored form of the polynomial $3x^2 + 11x + 6$. $(3x + 2)(x + 3)$

Tips for New Teachers

Intervention Point out that factoring involving $a^2 - b^2$, $a^3 - b^3$, $a^3 + b^3$, $(a - b)^2$, and $(a + b)^2$ occurs so frequently in algebra that students should memorize these forms. Then students can easily recognize and use them whenever the need arises.

Getting Ready for Lesson 5-5

PREREQUISITE SKILL Lesson 5-5 discusses radicals. Many radicals are irrational numbers whose approximate value can be found using a calculator. Exercises 76–81 should be used to determine your students' familiarity with rational and irrational numbers.

Standardized Test Practice

57. Which of the following is the factorization of $2x - 15 + x^2$? **B**
   (A) $(x - 3)(x - 5)$ (B) $(x - 3)(x + 5)$
   (C) $(x + 3)(x - 5)$ (D) $(x + 3)(x + 5)$

58. Which is not a factor of $x^3 - x^2 - 2x$? **C**
   (A) x (B) $x + 1$ (C) $x - 1$ (D) $x - 2$

Graphing Calculator

CHECK FACTORING Use a graphing calculator to determine if each polynomial is factored correctly. Write *yes* or *no*. If the polynomial is not factored correctly, find the correct factorization. **60. no; $(x + 2)(x^2 - 2x + 4)$**

59. $3x^2 + 5x + 2 \stackrel{?}{=} (3x + 2)(x + 1)$ **yes**
60. $x^3 + 8 \stackrel{?}{=} (x + 2)(x^2 - x + 4)$
61. $2x^2 - 5x - 3 \stackrel{?}{=} (x - 1)(2x + 3)$ **no; $(2x + 1)(x - 3)$**
62. $3x^2 - 48 \stackrel{?}{=} 3(x + 4)(x - 4)$ **yes**

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 5-3)*

63. $(t^3 - 3t + 2) \div (t + 2)$ **$t^2 - 2t + 1$**
64. $(y^2 + 4y + 3)(y + 1)^{-1}$ **$y + 3$**
65. $\dfrac{x^3 - 3x^2 + 2x - 6}{x - 3}$ **$x^2 + 2$**
66. $\dfrac{3x^4 + x^3 - 8x^2 + 10x - 3}{3x - 2}$ **$x^3 + x^2 - 2x + 2 + \dfrac{1}{3x - 2}$**

Simplify. *(Lesson 5-2)*

67. $(3x^2 - 2xy + y^2) + (x^2 + 5xy - 4y^2)$ **$4x^2 + 3xy - 3y^2$**
68. $(2x + 4)(7x - 1)$ **$14x^2 + 26x - 4$**

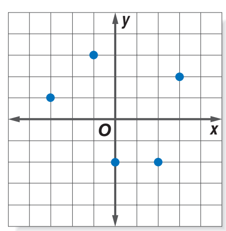
Perform the indicated operations, if possible. *(Lesson 4-5)*

69. $[3 \quad -1] \cdot \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ **$[-2]$**
70. $\begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 9 & -1 \end{bmatrix}$ **$\begin{bmatrix} -36 & 7 \\ 18 & 4 \end{bmatrix}$**

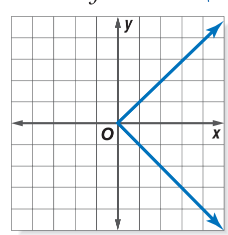
71. **PHOTOGRAPHY** The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture? *(Lesson 3-2)* **15 in. by 28 in.**

Determine whether each relation is a function. Write *yes* or *no*. *(Lesson 2-1)*

72. **yes**



73. **no**



State the property illustrated by each equation. *(Lesson 1-2)*

74. $(3 + 8)5 = 3(5) + 8(5)$ **Distributive**
75. $1 + (7 + 4) = (1 + 7) + 4$ **Associative (+)**

Getting Ready for the Next Lesson

PREREQUISITE SKILL Determine whether each number is *rational* or *irrational*.
*(To review **rational and irrational numbers**, see Lesson 1-2.)* **80. irrational**

76. 4.63 **rational**
77. π **irrational**
78. $\frac{16}{3}$ **rational**
79. 8.333… **rational**
80. 7.323223222…
81. $9.7\overline{1}$ **rational**

DAILY INTERVENTION

Unlocking Misconceptions

- **Difference of Two Squares** Many people think that the expressions $a^2 - b^2$ and $(a - b)^2$ are the same. Have students choose values for a and b, such as $a = 5$ and $b = 3$, to see that this is not true.
- **Sum of Two Squares** Students may need to be convinced that $a^2 + b^2$ cannot be factored after seeing that $a^3 + b^3$ can be factored. Have them substitute values for a and b to test possible factored forms, such as $(a + b)(a + b)$, to verify they do not equal $a^2 + b^2$.

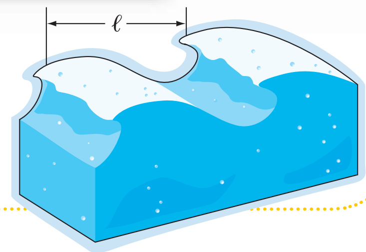
5-5 Roots of Real Numbers

What You'll Learn

- Simplify radicals.
- Use a calculator to approximate radicals.

How do square roots apply to oceanography?

The speed s in knots of a wave can be estimated using the formula $s = 1.34\sqrt{\ell}$, where ℓ is the length of the wave in feet. This is an example of an equation that contains a square root.



Vocabulary

- square root
- *n*th root
- principal root

SIMPLIFY RADICALS Finding the square root of a number and squaring a number are inverse operations. To find the **square root** of a number n, you must find a number whose square is n. For example, 7 is a square root of 49 since $7^2 = 49$. Since $(-7)^2 = 49$, -7 is also a square root of 49.

Key Concept — *Definition of Square Root*

- **Words** For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
- **Example** Since $5^2 = 25$, 5 is a square root of 25.

Since finding the square root of a number and squaring a number are inverse operations, it makes sense that the inverse of raising a number to the nth power is finding the **nth root** of a number. The table below shows the relationship between raising a number to a power and taking that root of a number.

Powers	Factors	Roots
$a^3 = 125$	$5 \cdot 5 \cdot 5 = 125$	5 is a cube root of 125.
$a^4 = 81$	$3 \cdot 3 \cdot 3 \cdot 3 = 81$	3 is a fourth root of 81.
$a^5 = 32$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$	2 is a fifth root of 32.
$a^n = b$	$\underbrace{a \cdot a \cdot a \cdot a \cdot \ldots \cdot a}_{n \text{ factors of } a} = b$	a is an *n*th root of b.

This pattern suggests the following formal definition of an nth root.

Key Concept — *Definition of nth Root*

- **Words** For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
- **Example** Since $2^5 = 32$, 2 is a fifth root of 32.

5-5 Lesson Notes

1 Focus

5-Minute Check Transparency 5-5 Use as a quiz or review of Lesson 5-4.

Mathematical Background notes are available for this lesson on p. 220D.

How do square roots apply to oceanography?

Ask students:

- One *knot* means one nautical mile per hour and one nautical mile is about 6076 feet. One mile on land (called a statute mile) is 5280 feet. Which is faster, 1 knot or 1 statute mile per hour? **1 knot**
- As the length of a wave (represented by ℓ in the diagram) increases, does the speed of the wave increase or decrease? **increases**

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 263–264
- Skills Practice, p. 265
- Practice, p. 266
- Reading to Learn Mathematics, p. 267
- Enrichment, p. 268
- Assessment, pp. 307, 309

Teaching Algebra With Manipulatives Masters, pp. 236–237

Transparencies

5-Minute Check Transparency 5-5
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

SIMPLIFY RADICALS

In-Class Example Power Point®

1 Simplify.

a. $\pm\sqrt{16x^6}$ $\pm 4x^3$

b. $-\sqrt{(q^3+5)^4}$ $-(q^3+5)^2$

c. $\sqrt[5]{243a^{10}b^{15}}$ $3a^2b^3$

d. $\sqrt{-4}$ not a real number

Teaching Tip Be sure students understand that since $3^2 = 9$ and $(-3)^2 = 9$, then the equation $x^2 = 9$ has two roots, 3 and -3. However, the value of the expression $\sqrt{9}$ is 3 only. To indicate *both* square roots and not just the principal root, the expression must be given as $\pm\sqrt{9}$.

Teaching Tip When discussing the information following Example 1, offer this alternative. Another way to simplify radicals that involve only numbers and no variables, is to simplifying the expression under the radical sign first. For example, $\sqrt{(-5)^2}$ could be rewritten by *first* simplifying under the radical sign to get $\sqrt{25}$, and then taking the principal root to get 5. Similarly, $\sqrt{(-2)^6}$ simplifies to $\sqrt{64}$, whose principal square root is 8.

Tips for New Teachers

Reading Tip Make sure that students understand what it means to say that the radical sign $\sqrt{\ }$ designates the principal root.

Study Tip

Reading Math
$\sqrt[n]{50}$ is read *the nth root of 50.*

The symbol $\sqrt[n]{\ }$ indicates an nth root.

index → $\sqrt[n]{50}$ ← radical sign; ← radicand

Some numbers have more than one real nth root. For example, 36 has two square roots, 6 and -6. When there is more than one real root, the nonnegative root is called the **principal root**. When no index is given, as in $\sqrt{36}$, the radical sign indicates the principal square root. The symbol $\sqrt[n]{b}$ stands for the principal nth root of b. If n is odd and b is negative, there will be no nonnegative root. In this case, the principal root is negative.

$\sqrt{16} = 4$ — $\sqrt{16}$ indicates the principal square root of 16.

$-\sqrt{16} = -4$ — $-\sqrt{16}$ indicates the opposite of the principal square root of 16.

$\pm\sqrt{16} = \pm 4$ — $\pm\sqrt{16}$ indicates both square roots of 16. ± means positive or negative.

$\sqrt[3]{-125} = -5$ — $\sqrt[3]{-125}$ indicates the principal cube root of -125.

$-\sqrt[4]{81} = -3$ — $-\sqrt[4]{81}$ indicates the opposite of the principal fourth root of 81.

The chart below gives a summary of the real nth roots of a number b.

Concept Summary — *Real nth roots of b, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$*

n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$b = 0$
even	one positive root, one negative root $\pm\sqrt[4]{625} = \pm 5$	no real roots $\sqrt{-4}$ is not a real number.	one real root, 0 $\sqrt[n]{0} = 0$
odd	one positive root, no negative roots $\sqrt[3]{8} = 2$	no positive roots, one negative root $\sqrt[5]{-32} = -2$	

Example 1 *Find Roots*

Simplify.

a. $\pm\sqrt{25x^4}$

$\pm\sqrt{25x^4} = \pm\sqrt{(5x^2)^2}$
$= \pm 5x^2$

The square roots of $25x^4$ are $\pm 5x^2$.

b. $-\sqrt{(y^2+2)^8}$

$-\sqrt{(y^2+2)^8} = -\sqrt{[(y^2+2)^4]^2}$
$= -(y^2+2)^4$

The opposite of the principal square root of $(y^2+2)^8$ is $-(y^2+2)^4$.

c. $\sqrt[5]{32x^{15}y^{20}}$

$\sqrt[5]{32x^{15}y^{20}} = \sqrt[5]{(2x^3y^4)^5}$
$= 2x^3y^4$

The principal fifth root of $32x^{15}y^{20}$ is $2x^3y^4$.

d. $\sqrt{-9}$

$\sqrt{-9} = \sqrt[2]{-9}$ (n is even.) (b is negative.)

Thus, $\sqrt{-9}$ is not a real number.

When you find the nth root of an even power and the result is an odd power, you must take the absolute value of the result to ensure that the answer is nonnegative.

$\sqrt{(-5)^2} = |-5|$ or 5 $\quad\sqrt{(-2)^6} = |(-2)^3|$ or 8

If the result is an even power or you find the nth root of an odd power, there is no need to take the absolute value. *Why?*

DAILY INTERVENTION — **Unlocking Misconceptions**

- **Variables** Some students tend to think that x must represent a positive number and $-x$ must represent a negative number. Reading $-x$ as "the opposite of x" should help them understand that $-x$ is 5 if $x = -5$.
- **Square Roots of Negative Numbers** Explain that -9 has no square root that is a real number. That is, no real number can be squared to give -9. However, inform students that $\sqrt{-9}$ *does* represent a number, called an *imaginary number*. Lesson 5-9 discusses such numbers.

246 Chapter 5 Polynomials

Example 2 Simplify Using Absolute Value

Simplify.

a. $\sqrt[8]{x^8}$

Note that x is an eighth root of x^8. The index is even, so the principal root is nonnegative. Since x could be negative, you must take the absolute value of x to identify the principal root.

$\sqrt[8]{x^8} = |x|$

b. $\sqrt[4]{81(a+1)^{12}}$

$\sqrt[4]{81(a+1)^{12}} = \sqrt[4]{[3(a+1)^3]^4}$

Since the index 4 is even and the exponent 3 is odd, you must use the absolute value of $(a+1)^3$.

$\sqrt[4]{81(a+1)^{12}} = 3|(a+1)^3|$

APPROXIMATE RADICALS WITH A CALCULATOR Recall that real numbers that cannot be expressed as terminating or repeating decimals are *irrational numbers*. $\sqrt{2}$ and $\sqrt{3}$ are examples of irrational numbers. Decimal approximations for irrational numbers are often used in applications.

Example 3 Approximate a Square Root

PHYSICS **The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared. Find the value of T for a 3-foot-long pendulum in a grandfather clock.**

Explore You are given the values of L and g and must find the value of T. Since the units on g are feet per second squared, the units on the time T should be seconds.

Plan Substitute the values for L and g into the formula. Use a calculator to evaluate.

Solve

$T = 2\pi\sqrt{\frac{L}{g}}$ Original formula

$= 2\pi\sqrt{\frac{3}{32}}$ $L = 3, g = 32$

≈ 1.92 Use a calculator.

It takes the pendulum about 1.92 seconds to make a complete swing.

Examine The closest square to $\frac{3}{32}$ is $\frac{1}{9}$, and π is approximately 3, so the answer should be close to $2(3)\sqrt{\frac{1}{9}} = 2(3)\left(\frac{1}{3}\right)$ or 2. The answer is reasonable.

Study Tip

Graphing Calculators

To find a root of index greater than 2, first type the index. Then select $\sqrt[x]{\ }$ from the MATH MATH menu. Finally, enter the radicand.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a number whose principal square root and cube root are both integers. **Sample answer: 64**
2. **Explain** why it is not always necessary to take the absolute value of a result to indicate the principal root. **See margin.**
3. **Determine** whether the statement $\sqrt[4]{(-x)^4} = x$ is *sometimes*, *always*, or *never* true. Explain your reasoning. **Sometimes; it is true when $x > 0$.**

www.algebra2.com/extra_examples

In-Class Example

Power Point®

2 Simplify.

a. $\sqrt[6]{t^6}$ $|t|$

b. $\sqrt[5]{243(x+2)^{15}}$ $3(x+2)^3$

APPROXIMATE RADICALS WITH A CALCULATOR

In-Class Example

Power Point®

3 **PHYSICS** Use the formula given in Example 3. Find the value of T for a 1.5-foot-long pendulum. **about 1.36 s**

Teaching Tip Help students see how time and the length of a pendulum are related by having them experiment using weights with different lengths of string.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- keep a list of study tips for the graphing calculator, including the one in this lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answer

2. If all of the powers in the result of an even root have even exponents, the result is nonnegative without taking absolute value.

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Have students create various rectangles using index cards or cardboard, or using masking tape on the classroom floor. Have them use the formula $d = \sqrt{\ell^2 + w^2}$ to find the length of the diagonal of each rectangle. After creating several rectangles, have students experiment with using the diagonal measures of two rectangles to create another rectangle whose length and width are irrational numbers. Students should then find the length of the diagonal of this new rectangle.

Study Guide and Intervention, p. 263 (shown) and p. 264

Simplify Radicals

Square Root	For any real numbers a and b, if $a^2 = b$, then a is a square root of b.
nth Root	For any real numbers a and b, and any positive integer n, if $a^n = b$, then a is an nth root of b.
Real nth Roots of b, $\sqrt[n]{b}, -\sqrt[n]{b}$	1. If n is even and $b > 0$, then b has one positive root and one negative root. 2. If n is odd and $b > 0$, then b has one positive root. 3. If n is even and $b < 0$, then b has no real roots. 4. If n is odd and $b < 0$, then b has one negative root.

Example 1 Simplify $\sqrt{49z^8}$.

$\sqrt{49z^8} = \sqrt{(7z^4)^2} = 7z^4$

z^4 must be positive, so there is no need to take the absolute value.

Example 2 Simplify $-\sqrt[3]{(2a-1)^6}$

$-\sqrt[3]{(2a-1)^6} = -\sqrt[3]{[(2a-1)^2]^3} = (2a-1)^2$

Exercises

Simplify.

1. $\sqrt{81}$ **9**
2. $\sqrt[3]{-343}$ **−7**
3. $\sqrt{144p^6}$ **$12|p^3|$**
4. $\pm\sqrt{4a^{10}}$ **$\pm 2a^5$**
5. $\sqrt[5]{243p^{10}}$ **$3p^2$**
6. $-\sqrt[3]{m^6n^9}$ **$-m^2n^3$**
7. $\sqrt[3]{-b^{12}}$ **$-b^4$**
8. $\sqrt{16a^{10}b^8}$ **$4|a^5|b^4$**
9. $\sqrt{121x^6}$ **$11|x^3|$**
10. $\sqrt{(4k)^4}$ **$16k^2$**
11. $\pm\sqrt{169r^4}$ **$\pm 13r^2$**
12. $-\sqrt[3]{-27p^6}$ **$3p^2$**
13. $-\sqrt{625y^2z^4}$ **$-25|y|z^2$**
14. $\sqrt{36q^{34}}$ **$6|q^{17}|$**
15. $\sqrt{100x^2y^4z^6}$ **$10|x|y^2|z^3|$**
16. $\sqrt[3]{-0.027}$ **−0.3**
17. $-\sqrt{-0.36}$ **not a real number**
18. $\sqrt{0.64p^{10}}$ **$0.8|p^5|$**
19. $\sqrt[4]{(2x)^8}$ **$4x^2$**
20. $\sqrt{(11y^2)^4}$ **$121y^4$**
21. $\sqrt[3]{(5a^2b)^6}$ **$25a^4b^2$**
22. $\sqrt{(3x-1)^2}$ **$|3x-1|$**
23. $\sqrt[3]{(m-5)^6}$ **$(m-5)^2$**
24. $\sqrt{36x^2-12x+1}$ **$|6x-1|$**

Skills Practice, p. 265 and Practice, p. 266 (shown)

Use a calculator to approximate each value to three decimal places.

1. $\sqrt{7.8}$ **2.793**
2. $-\sqrt{89}$ **−9.434**
3. $\sqrt[3]{25}$ **2.924**
4. $\sqrt[3]{-4}$ **−1.587**
5. $\sqrt[4]{1.1}$ **1.024**
6. $\sqrt[5]{-0.1}$ **−0.631**
7. $\sqrt[6]{5555}$ **4.208**
8. $\sqrt[4]{(0.94)^2}$ **0.970**

Simplify.

9. $\sqrt{0.81}$ **0.9**
10. $-\sqrt{324}$ **−18**
11. $-\sqrt[4]{256}$ **−4**
12. $\sqrt[6]{64}$ **2**
13. $\sqrt[3]{-64}$ **−4**
14. $\sqrt[3]{0.512}$ **0.8**
15. $\sqrt[5]{-243}$ **−3**
16. $-\sqrt[4]{1296}$ **−6**
17. $\sqrt[5]{\frac{-1024}{243}}$ **$-\frac{4}{3}$**
18. $\sqrt[5]{243x^{10}}$ **$3x^2$**
19. $\sqrt{(14a)^2}$ **$14|a|$**
20. $\sqrt{-(14a)^2}$ **not a real number**
21. $\sqrt{49m^2t^8}$ **$7|m|t^4$**
22. $\sqrt{\frac{16m^2}{25}}$ **$\frac{4|m|}{5}$**
23. $\sqrt[3]{-64r^6w^{15}}$ **$-4r^2w^5$**
24. $\sqrt{(2x)^8}$ **$16x^4$**
25. $-\sqrt[4]{625s^8}$ **$-5s^2$**
26. $\sqrt[3]{216p^3q^9}$ **$6pq^3$**
27. $\sqrt{676x^4y^6}$ **$26x^2|y^3|$**
28. $\sqrt[3]{-27x^9y^{12}}$ **$-3x^3y^4$**
29. $-\sqrt{144m^8n^6}$ **$-12m^4|n^3|$**
30. $\sqrt[5]{-32x^5y^{10}}$ **$-2xy^2$**
31. $\sqrt[6]{(m+4)^6}$ **$|m+4|$**
32. $\sqrt[3]{(2x+1)^3}$ **$2x+1$**
33. $-\sqrt{49a^{10}b^{16}}$ **$-7|a^5|b^8$**
34. $\sqrt[4]{(x-5)^8}$ **$(x-5)^2$**
35. $\sqrt[3]{343d^6}$ **$7d^2$**
36. $\sqrt{x^2+10x+25}$ **$|x+5|$**

37. **RADIANT TEMPERATURE** Thermal sensors measure an object's *radiant* temperature, which is the amount of energy radiated by the object. The *internal* temperature of an object is called its *kinetic* temperature. The formula $T_r = T_k\sqrt[4]{e}$ relates an object's radiant temperature T_r to its kinetic temperature T_k. The variable e in the formula is a measure of how well the object radiates energy. If an object's kinetic temperature is 30°C and $e = 0.94$, what is the object's radiant temperature to the nearest tenth of a degree? **29.5°C**

38. **HERO'S FORMULA** Salvatore is buying fertilizer for his triangular garden. He knows the lengths of all three sides, so he is using Hero's formula to find the area. Hero's formula states that the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the lengths of the sides of the triangle and s is half the perimeter of the triangle. If the lengths of the sides of Salvatore's garden are 15 feet, 17 feet, and 20 feet, what is the area of the garden? Round your answer to the nearest whole number. **124 ft²**

Reading to Learn Mathematics, p. 267 ELL

Pre-Activity **How do square roots apply to oceanography?**

Read the introduction to Lesson 5-5 at the top of page 245 in your textbook.

Suppose the length of a wave is 5 feet. Explain how you would estimate the speed of the wave to the nearest tenth of a knot using a calculator. (Do not actually calculate the speed.) **Sample answer: Using a calculator, find the positive square root of 5. Multiply this number by 1.34. Then round the answer to the nearest tenth.**

Reading the Lesson

1. For each radical below, identify the radicand and the index.
   a. $\sqrt[3]{23}$ radicand: **23** index: **3**
   b. $\sqrt{15x^2}$ radicand: **$15x^2$** index: **2**
   c. $\sqrt[5]{-343}$ radicand: **−343** index: **5**
2. Complete the following table. (Do not actually find any of the indicated roots.)

Number	Number of Positive Square Roots	Number of Negative Square Roots	Number of Positive Cube Roots	Number of Negative Cube Roots
27	1	1	1	0
−16	0	0	0	1

3. State whether each of the following is *true* or *false*.
   a. A negative number has no real fourth roots. **true**
   b. $\pm\sqrt{121}$ represents both square roots of 121. **true**
   c. When you take the fifth root of x^5, you must take the absolute value of x to identify the principal fifth root. **false**

Helping You Remember

4. What is an easy way to remember that a negative number has no real square roots but has one real cube root? **Sample answer: The square of a positive or negative number is positive, so there is no real number whose square is negative. However, the cube of a negative number is negative, so a negative number has one real cube root, which is a negative number.**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–6	3
7–14	1, 2
15	3

Use a calculator to approximate each value to three decimal places.

4. $\sqrt{77}$ **8.775**
5. $-\sqrt[3]{19}$ **−2.668**
6. $\sqrt[4]{48}$ **2.632**

Simplify. **10. not a real number 13. $6|a|b^2$ 14. $|4x+3y|$**

7. $\sqrt[3]{64}$ **4**
8. $\sqrt{(-2)^2}$ **2**
9. $\sqrt[5]{-243}$ **−3**
10. $\sqrt[4]{-4096}$
11. $\sqrt[3]{x^3}$ **x**
12. $\sqrt[4]{y^4}$ **$|y|$**
13. $\sqrt{36a^2b^4}$
14. $\sqrt{(4x+3y)^2}$

Application

15. **OPTICS** The distance D in miles from an observer to the horizon over flat land or water can be estimated using the formula $D = 1.23\sqrt{h}$, where h is the height in feet of the point of observation. How far is the horizon for a person whose eyes are 6 feet above the ground? **about 3.01 mi**

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
16–27, 60–62	3
28–59	1, 2

Extra Practice

See page 838.

Use a calculator to approximate each value to three decimal places.

16. $\sqrt{129}$ **11.358**
17. $-\sqrt{147}$ **−12.124**
18. $\sqrt{0.87}$ **0.933**
19. $\sqrt{4.27}$ **2.066**
20. $\sqrt[3]{59}$ **3.893**
21. $\sqrt[3]{-480}$ **−7.830**
22. $\sqrt[4]{602}$ **4.953**
23. $\sqrt[5]{891}$ **3.890**
24. $\sqrt[6]{4123}$ **4.004**
25. $\sqrt[7]{46{,}815}$ **4.647**
26. $\sqrt[6]{(723)^3}$ **26.889**
27. $\sqrt[4]{(3500)^2}$ **59.161**

Simplify.

30. not a real number

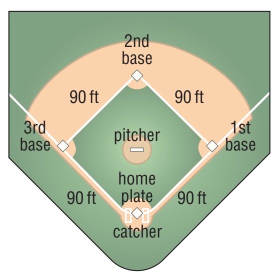
28. $\sqrt{225}$ **15**
29. $\pm\sqrt{169}$ **±13**
30. $\sqrt{-(-7)^2}$
31. $\sqrt{(-18)^2}$ **18**
32. $\sqrt[3]{-27}$ **−3**
33. $\sqrt[3]{-128}$ **−2**
34. $\sqrt{\frac{1}{16}}$ **$\frac{1}{4}$**
35. $\sqrt[3]{\frac{1}{125}}$ **$\frac{1}{5}$**
36. $\sqrt{0.25}$ **0.5**
37. $\sqrt[3]{-0.064}$ **−0.4**
38. $\sqrt[4]{z^8}$ **z^2**
39. $-\sqrt[6]{x^6}$ **$-|x|$**
40. $\sqrt{49m^6}$ **$7|m^3|$**
41. $\sqrt{64a^8}$ **$8a^4$**
42. $\sqrt[3]{27r^3}$ **$3r$**
43. $\sqrt[3]{-c^6}$ **$-c^2$**
44. $\sqrt{(5g)^4}$ **$25g^2$**
45. $\sqrt[3]{(2z)^6}$ **$4z^2$**
46. $\sqrt{25x^4y^6}$ **$5x^2|y^3|$**
47. $\sqrt{36x^4z^4}$ **$6x^2z^2$**
48. $\sqrt{169x^8y^4}$ **$13x^4y^2$**
49. $\sqrt{9p^{12}q^6}$ **$3p^6|q^3|$**
50. $\sqrt[3]{8a^3b^3}$ **$2ab$**
51. $\sqrt[3]{-27c^9d^{12}}$ **$-3c^3d^4$**
52. $\sqrt{(4x-y)^2}$ **$|4x-y|$**
53. $\sqrt[3]{(p+q)^3}$ **$p+q$**

54. $-|x+2|$

★54. $-\sqrt{x^2+4x+4}$
★55. $\sqrt{z^2+8z+16}$ **$|z+4|$**
★56. $\sqrt{4a^2+4a+1}$ **$|2a+1|$**
★57. $\sqrt{-9x^2-12x-4}$ **not a real number**

58. Find the principal fifth root of 32. **2**
59. What is the third root of −125? **−5**

60. **SPORTS** Refer to the drawing at the right. How far does the catcher have to throw a ball from home plate to second base? **about 127.28 ft**



61. **FISH** The relationship between the length and mass of Pacific halibut can be approximated by the equation $L = 0.46\sqrt[3]{M}$, where L is the length in meters and M is the mass in kilograms. Use this equation to predict the length of a 25-kilogram Pacific halibut. **about 1.35 m**

Enrichment, p. 268

Approximating Square Roots

Consider the following expansion.

$$\left(a + \frac{b}{2a}\right)^2 = a^2 + \frac{2ab}{2a} + \frac{b^2}{4a^2} = a^2 + b + \frac{b^2}{4a^2}$$

Think what happens if a is very great in comparison to b. The term $\frac{b^2}{4a^2}$ is very small and can be disregarded in an approximation.

$$\left(a + \frac{b}{2a}\right)^2 \approx a^2 + b$$

$$a + \frac{b}{2a} \approx \sqrt{a^2 + b}$$

Suppose a number can be expressed as $a^2 + b$, $a > b$. Then an approximate value of the square root is $a + \frac{b}{2a}$. You should also see that $a - \frac{b}{2a} \approx \sqrt{a^2 - b}$.

More About. . .

Space Science

The escape velocity for the Moon is about 2400 m/s. For the Sun, it is about 618,000 m/s.

Source: NASA

62. SPACE SCIENCE The velocity v required for an object to escape the gravity of a planet or other body is given by the formula $v = \sqrt{\frac{2GM}{R}}$, where M is the mass of the body, R is the radius of the body, and G is Newton's gravitational constant. Use $M = 5.98 \times 10^{24}$ kg, $R = 6.37 \times 10^{6}$ m, and $G = 6.67 \times 10^{-11}$ N · m^2/kg^2 to find the escape velocity for Earth. **about 11,200 m/s**

63. CRITICAL THINKING Under what conditions does $\sqrt{x^2 + y^2} = x + y$?
$x = 0$ and $y \geq 0$, or $y = 0$ and $x \geq 0$

64. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How do square roots apply to oceanography?

Include the following in your answer:
- the values of s for $\ell = 2$, 5, and 10 feet, and
- an observation of what happens to the value of s as the value of ℓ increases.

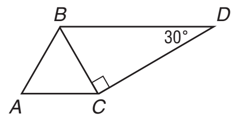
Standardized Test Practice

65. Which of the following is closest to $\sqrt{7.32}$? **B**

Ⓐ 2.6 Ⓑ 2.7 Ⓒ 2.8 Ⓓ 2.9

66. In the figure, $\triangle ABC$ is an equilateral triangle with sides 9 units long. What is the length of $\overline{BD}$ in units? **D**

Ⓐ 3 Ⓑ 9
Ⓒ $9\sqrt{2}$ Ⓓ 18



Maintain Your Skills

Mixed Review

Factor completely. If the polynomial is not factorable, write *prime*. *(Lesson 5-4)*

67. $7xy^3 - 14x^2y^5 + 28x^3y^2$ **68.** $ab - 5a + 3b - 15$ **$(a + 3)(b - 5)$**

69. $2x^2 + 15x + 25$ **$(2x + 5)(x + 5)$** **70.** $c^3 - 216$ **$(c - 6)(c^2 + 6c + 36)$**

67. $7xy^2(y - 2xy^3 + 4x^2)$

Simplify. *(Lesson 5-3)* **71. $4x^2 + x + 5 + \frac{8}{x-2}$**

71. $(4x^3 - 7x^2 + 3x - 2) \div (x - 2)$ **72.** $\frac{x^4 + 4x^3 - 4x^2 + 5x}{x + 5}$ **$x^3 - x^2 + x$**

73. TRAVEL The matrix at the right shows the costs of airline flights between some cities. Write a matrix that shows the costs of two tickets for these flights. *(Lesson 4-2)* **$\begin{bmatrix} 810 & 2320 \\ 1418 & 2504 \end{bmatrix}$**

	New York	LA
Atlanta	405	1160
Chicago	709	1252

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

74. $a + 4b = 6$
$3a + 2b = -2$ **$(-2, 2)$**

75. $10x - y = 13$
$3x - 4y = 15$ **$(1, -3)$**

76. $3c - 7d = -1$
$2c - 6d = -6$ **$(9, 4)$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**
*(To review **multiplying binomials**, see Lesson 5-2.)*

77. $(x + 3)(x + 8)$ **$x^2 + 11x + 24$** **78.** $(y - 2)(y + 5)$ **$y^2 + 3y - 10$**

79. $(a + 2)(a - 9)$ **$a^2 - 7a - 18$** **80.** $(a + b)(a + 2b)$ **$a^2 + 3ab + 2b^2$**

81. $(x - 3y)(x + 3y)$ **$x^2 - 9y^2$** **82.** $(2w + z)(3w - 5z)$ **$6w^2 - 7wz - 5z^2$**

www.algebra2.com/self_check_quiz

Answer

64. The speed and length of a wave are related by an expression containing a square root. Answers should include the following.
- **about 1.90 knots, about 3.00 knots, and 4.24 knots**
- **As the value of ℓ increases, the value of s increases.**

About the Exercises...

Organization by Objective
- **Simplify Radicals:** 28–59
- **Approximate Radicals with a Calculator:** 16–27

Odd/Even Assignments
Exercises 16–59 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–53 odd, 59, 61, 63–82

Average: 17–61 odd, 63–82

Advanced: 16–62 even, 63–76 (optional: 77–82)

4 Assess

Open-Ended Assessment

Modeling Have students use the hypotenuse of a right triangle with both legs one unit long to demonstrate and explain that $\sqrt{2}$ is a number on the real number line.

Getting Ready for Lesson 5-6

PREREQUISITE SKILL Lesson 5-6 presents operations with radical expressions. In Example 5 on p. 253, they will encounter the multiplication of two binomials involving radical expressions. Exercises 77–82 should be used to determine your students' familiarity with multiplying binomials.

Assessment Options

Quiz (Lessons 5-4 and 5-5) is available on p. 307 of the *Chapter 5 Resource Masters.*

Mid-Chapter Test (Lessons 5-1 through 5-5) is available on p. 309 of the *Chapter 5 Resource Masters.*

5-6 Lesson Notes

1 Focus

5-Minute Check Transparency 5-6 Use as a quiz or a review of Lesson 5-5.

Mathematical Background notes are available for this lesson on p. 220D.

Building on Prior Knowledge

In Lesson 5-5, students simplified radicals. In this lesson, students build on the skills they learned in that lesson to simplify and combine radical expressions.

How do radical expressions apply to falling objects?

Ask students:

- If the value of d in the formula doubles, will the value of t also double? **no**
- Is the relationship between time t and distance d for a falling object *linear* or *nonlinear*? **nonlinear**

5-6 Radical Expressions

What You'll Learn

- Simplify radical expressions.
- Add, subtract, multiply, and divide radical expressions.

Vocabulary

- rationalizing the denominator
- like radical expressions
- conjugates

How do radical expressions apply to falling objects?

The amount of time t in seconds that it takes for an object to drop d feet is given by $t = \sqrt{\frac{2d}{g}}$, where $g = 32$ ft/s^2 is the acceleration due to gravity. In this lesson, you will learn how to simplify radical expressions like $\sqrt{\frac{2d}{g}}$.

SIMPLIFY RADICAL EXPRESSIONS You can use the Commutative Property and the definition of square root to find an equivalent expression for a product of radicals such as $\sqrt{3} \cdot \sqrt{5}$. Begin by squaring the product.

$$(\sqrt{3} \cdot \sqrt{5})^2 = \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{3} \cdot \sqrt{5}$$
$$= \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{5} \quad \text{Commutative Property of Multiplication}$$
$$= 3 \cdot 5 \text{ or } 15 \quad \text{Definition of square root}$$

Since $\sqrt{3} \cdot \sqrt{5} > 0$ and $(\sqrt{3} \cdot \sqrt{5})^2 = 15$, $\sqrt{3} \cdot \sqrt{5}$ is the principal square root of 15. That is, $\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$. This illustrates the following property of radicals.

Key Concept — *Product Property of Radicals*

For any real numbers a and b and any integer $n > 1$,
1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$, and
2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

Follow these steps to simplify a square root.

Step 1 Factor the radicand into as many squares as possible.

Step 2 Use the Product Property to isolate the perfect squares.

Step 3 Simplify each radical.

Example 1 *Square Root of a Product*

Simplify $\sqrt{16p^8q^7}$.

$$\sqrt{16p^8q^7} = \sqrt{4^2 \cdot (p^4)^2 \cdot (q^3)^2 \cdot q} \quad \text{Factor into squares where possible.}$$
$$= \sqrt{4^2} \cdot \sqrt{(p^4)^2} \cdot \sqrt{(q^3)^2} \cdot \sqrt{q} \quad \text{Product Property of Radicals}$$
$$= 4p^4|q^3|\sqrt{q} \quad \text{Simplify.}$$

However, for $\sqrt{16p^8q^7}$ to be defined, $16p^8q^7$ must be nonnegative. If that is true, q must be nonnegative, since it is raised to an odd power. Thus, the absolute value is unnecessary, and $\sqrt{16p^8q^7} = 4p^4q^3\sqrt{q}$.

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters
- Study Guide and Intervention, pp. 269–270
- Skills Practice, p. 271
- Practice, p. 272
- Reading to Learn Mathematics, p. 273
- Enrichment, p. 274

Teaching Algebra With Manipulatives Masters, p. 238

Transparencies

5-Minute Check Transparency 5-6
Real-World Transparency 5
Answer Key Transparencies

Technology

Interactive Chalkboard

Look at a radical that involves division to see if there is a quotient property for radicals that is similar to the Product Property. Consider $\frac{49}{9}$. The radicand is a perfect square, so $\sqrt{\frac{49}{9}} = \sqrt{\left(\frac{7}{3}\right)^2}$ or $\frac{7}{3}$. Notice that $\frac{7}{3} = \frac{\sqrt{49}}{\sqrt{9}}$. This suggests the following property.

Key Concept — Quotient Property of Radicals

- **Words** For any real numbers a and $b \neq 0$, and any integer $n > 1$,
  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.
- **Example** $\frac{\sqrt{27}}{\sqrt{3}} = \sqrt{9}$ or 3

You can use the properties of radicals to write expressions in simplified form.

Concept Summary — Simplifying Radical Expressions

A radical expression is in simplified form when the following conditions are met.

- The index n is as small as possible.
- The radicand contains no factors (other than 1) that are nth powers of an integer or polynomial.
- The radicand contains no fractions.
- No radicals appear in a denominator.

Study Tip

Rationalizing the Denominator
You may want to think of rationalizing the denominator as making the denominator a rational number.

To eliminate radicals from a denominator or fractions from a radicand, you can use a process called **rationalizing the denominator**. To rationalize a denominator, multiply the numerator and denominator by a quantity so that the radicand has an exact root. Study the examples below.

Example 2 Simplify Quotients

Simplify each expression.

a. $\sqrt{\frac{x^4}{y^5}}$

$\sqrt{\frac{x^4}{y^5}} = \frac{\sqrt{x^4}}{\sqrt{y^5}}$ Quotient Property

$= \frac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2 \cdot y}}$ Factor into squares.

$= \frac{\sqrt{(x^2)^2}}{\sqrt{(y^2)^2} \cdot \sqrt{y}}$ Product Property

$= \frac{x^2}{y^2\sqrt{y}}$ $\sqrt{(x^2)^2} = x^2$

$= \frac{x^2}{y^2\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}}$ Rationalize the denominator.

$= \frac{x^2\sqrt{y}}{y^3}$ $\sqrt{y} \cdot \sqrt{y} = y$

b. $\sqrt[5]{\frac{5}{4a}}$

$\sqrt[5]{\frac{5}{4a}} = \frac{\sqrt[5]{5}}{\sqrt[5]{4a}}$ Quotient Property

$= \frac{\sqrt[5]{5}}{\sqrt[5]{4a}} \cdot \frac{\sqrt[5]{8a^4}}{\sqrt[5]{8a^4}}$ Rationalize the denominator.

$= \frac{\sqrt[5]{5 \cdot 8a^4}}{\sqrt[5]{4a \cdot 8a^4}}$ Product Property

$= \frac{\sqrt[5]{40a^4}}{\sqrt[5]{32a^5}}$ Multiply.

$= \frac{\sqrt[5]{40a^4}}{2a}$ $\sqrt[5]{32a^5} = 2a$

www.algebra2.com/extra_examples

2 Teach

SIMPLIFY RADICAL EXPRESSIONS

In-Class Examples Power Point®

Teaching Tip When discussing the Product Property of Radicals, stress the fact that a and b must both be nonnegative if n is even. This means that $\sqrt{-2}$ times $\sqrt{-8}$ may *not* be written as $\sqrt{16}$. This condition is necessary because $\sqrt{-2}$ and $\sqrt{-8}$ are *not* real numbers.

1 Simplify $\sqrt{25a^4b^9}$. $5a^2b^4\sqrt{b}$

2 Simplify each expression.

a. $\sqrt{\frac{y^8}{x^7}}$ $\frac{y^4\sqrt{x}}{x^4}$

b. $\sqrt[3]{\frac{2}{9x}}$ $\frac{\sqrt[3]{6x^2}}{3x}$

Teaching Tip Urge students to verify that each of their final answers is in simplified form by testing it against the four conditions listed in the Concept Summary for Simplifying Radical Expressions.

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students think about irrational numbers, radicals, and the rules for operations with radicals. Ask them to write down what puzzles them most about these concepts, including a list of the definitions and operations about which they feel some confusion. Invite students to share these concerns with you so that they can be cleared up.

Lesson 5-6 Radical Expressions 251

OPERATIONS WITH RADICALS

In-Class Example

Power Point®

3 Simplify $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$. **50a**

Teaching Tip After discussing the information presented in the Algebra Activity, make sure students also understand that $\sqrt{a} + \sqrt{b}$ is not equivalent to $\sqrt{a + b}$. Suggest they use the values $a = 16$ and $b = 9$ to verify this fact.

OPERATIONS WITH RADICALS You can use the Product and Quotient Properties to multiply and divide some radicals, respectively.

Example 3 Multiply Radicals

Simplify $6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n}$.

$6\sqrt[3]{9n^2} \cdot 3\sqrt[3]{24n} = 6 \cdot 3 \cdot \sqrt[3]{9n^2 \cdot 24n}$ Product Property of Radicals

$= 18 \cdot \sqrt[3]{2^3 \cdot 3^3 \cdot n^3}$ Factor into cubes where possible.

$= 18 \cdot \sqrt[3]{2^3} \cdot \sqrt[3]{3^3} \cdot \sqrt[3]{n^3}$ Product Property of Radicals

$= 18 \cdot 2 \cdot 3 \cdot n$ or $108n$ Multiply.

Can you add radicals in the same way that you multiply them? In other words, if $\sqrt{a} \cdot \sqrt{a} = \sqrt{a \cdot a}$, does $\sqrt{a} + \sqrt{a} = \sqrt{a + a}$?

Algebra Activity

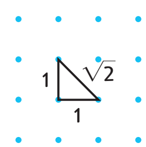
Adding Radicals

You can use dot paper to show the sum of two like radicals, such as $\sqrt{2} + \sqrt{2}$.

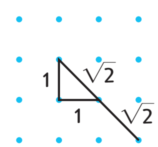
Model and Analyze

Step 1 First, find a segment of length $\sqrt{2}$ units by using the Pythagorean Theorem with the dot paper.

$a^2 + b^2 = c^2$
$1^2 + 1^2 = c^2$
$2 = c^2$



Step 2 Extend the segment to twice its length to represent $\sqrt{2} + \sqrt{2}$.



Make a Conjecture

1. Is $\sqrt{2} + \sqrt{2} = \sqrt{2 + 2}$ or 2? Justify your answer using the geometric models above.
2. Use this method to model other irrational numbers. Do these models support your conjecture? **See students' work.**

1. No; $\sqrt{2} + \sqrt{2}$ units is the length of the hypotenuse of an isosceles right triangle whose legs have length 2 units. Therefore, $\sqrt{2} + \sqrt{2} > 2$.

In the activity, you discovered that you cannot add radicals in the same manner as you multiply them. You add radicals in the same manner as adding monomials. That is, you can add only the like terms or like radicals.

Two radical expressions are called **like radical expressions** if both the indices and the radicands are alike. Some examples of like and unlike radical expressions are given below.

Study Tip

Reading Math
Indices is the plural of *index*.

$\sqrt{3}$ and $\sqrt[3]{3}$ are not like expressions. Different indices

$\sqrt[4]{5x}$ and $\sqrt[4]{5}$ are not like expressions. Different radicands

$2\sqrt[4]{3a}$ and $5\sqrt[4]{3a}$ are like expressions. Radicands are $3a$; indices are 4.

Algebra Activity

Materials: rectangular dot paper, ruler/straightedge

- Ask students what leg lengths they could use on a right triangle to find a line whose length is $\sqrt{5}$. **lengths of 1 and 2 units**
- Point out that there are other instances where you can perform a multiplication but not an addition. For example, you can multiply fractions by multiplying the numerators and denominators separately, but you do not add fractions this way.

Example 4 Add and Subtract Radicals

Simplify $2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$.

$2\sqrt{12} - 3\sqrt{27} + 2\sqrt{48}$

$= 2\sqrt{2^2 \cdot 3} - 3\sqrt{3^2 \cdot 3} + 2\sqrt{2^2 \cdot 2^2 \cdot 3}$ — Factor using squares.

$= 2\sqrt{2^2} \cdot \sqrt{3} - 3\sqrt{3^2} \cdot \sqrt{3} + 2\sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3}$ — Product Property

$= 2 \cdot 2 \cdot \sqrt{3} - 3 \cdot 3 \cdot \sqrt{3} + 2 \cdot 2 \cdot 2 \cdot \sqrt{3}$ — $\sqrt{2^2} = 2$, $\sqrt{3^2} = 3$

$= 4\sqrt{3} - 9\sqrt{3} + 8\sqrt{3}$ — Multiply.

$= 3\sqrt{3}$ — Combine like radicals.

Just as you can add and subtract radicals like monomials, you can multiply radicals using the FOIL method as you do when multiplying binomials.

Example 5 Multiply Radicals

a. $(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3})$

$(3\sqrt{5} - 2\sqrt{3})(2 + \sqrt{3}) = \overset{F}{3\sqrt{5} \cdot 2} + \overset{O}{3\sqrt{5} \cdot \sqrt{3}} - \overset{I}{2\sqrt{3} \cdot 2} - \overset{L}{2\sqrt{3} \cdot \sqrt{3}}$

$= 6\sqrt{5} + 3\sqrt{5 \cdot 3} - 4\sqrt{3} - 2\sqrt{3^2}$ — Product Property

$= 6\sqrt{5} + 3\sqrt{15} - 4\sqrt{3} - 6$ — $2\sqrt{3^2} = 2 \cdot 3$ or 6

b. $(5\sqrt{3} - 6)(5\sqrt{3} + 6)$

$(5\sqrt{3} - 6)(5\sqrt{3} + 6) = 5\sqrt{3} \cdot 5\sqrt{3} + 5\sqrt{3} \cdot 6 - 6 \cdot 5\sqrt{3} - 6 \cdot 6$ — FOIL

$= 25\sqrt{3^2} + 30\sqrt{3} - 30\sqrt{3} - 36$ — Multiply.

$= 75 - 36$ — $25\sqrt{3^2} = 25 \cdot 3$ or 75

$= 39$ — Subtract.

Binomials like those in Example 5b, of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ where a, b, c, and d are rational numbers, are called **conjugates** of each other. The product of conjugates is always a rational number. You can use conjugates to rationalize denominators.

Example 6 Use a Conjugate to Rationalize a Denominator

Simplify $\frac{1 - \sqrt{3}}{5 + \sqrt{3}}$.

$\frac{1 - \sqrt{3}}{5 + \sqrt{3}} = \frac{(1 - \sqrt{3})(5 - \sqrt{3})}{(5 + \sqrt{3})(5 - \sqrt{3})}$ — Multiply by $\frac{5 - \sqrt{3}}{5 - \sqrt{3}}$ because $5 - \sqrt{3}$ is the conjugate of $5 + \sqrt{3}$.

$= \frac{1 \cdot 5 - 1 \cdot \sqrt{3} - \sqrt{3} \cdot 5 + (\sqrt{3})^2}{5^2 - (\sqrt{3})^2}$ — FOIL; Difference of squares

$= \frac{5 - \sqrt{3} - 5\sqrt{3} + 3}{25 - 3}$ — Multiply.

$= \frac{8 - 6\sqrt{3}}{22}$ — Combine like terms.

$= \frac{4 - 3\sqrt{3}}{11}$ — Divide numerator and denominator by 2.

In-Class Examples

Power Point®

4 Simplify $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$. $-3\sqrt{5}$

5 Simplify each expression.

a. $(2\sqrt{3} + 3\sqrt{5})(3 - \sqrt{3})$ $6\sqrt{3} - 6 + 9\sqrt{5} - 3\sqrt{15}$

b. $(4\sqrt{2} + 7)(4\sqrt{2} - 7)$ -17

6 Simplify $\frac{2 + \sqrt{3}}{4 - \sqrt{3}}$. $\frac{11 + 6\sqrt{3}}{13}$

DAILY INTERVENTION — Unlocking Misconceptions

Radical Expressions When presented with a radical expression such as $11 + 6\sqrt{3}$, some students may persist in trying to add the 11 and the 6. Help them understand why this cannot be done by comparing this radical expression $11 + 6\sqrt{3}$ to the expression $11 + 6x$. Stress that the radical $6\sqrt{3}$ is a multiplication expression just like $6x$. Remind students that the order of operations requires that multiplication be performed before addition. Students may find it helpful to rewrite $11 + 6\sqrt{3}$, as $11 + 6 \cdot \sqrt{3}$.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- add the information from the Key Concept and Concept Summary features to their notebook.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective

- **Simplify Radical Expressions:** 15–30
- **Operations with Radicals:** 31–48

Odd/Even Assignments

Exercises 15–48 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–45 odd, 49–51, 55–82

Average: 15–49 odd, 50–53, 55–82

Advanced: 16–48 even, 50–74 (optional: 75–82)

All: Practice Quiz 2 (1–10)

Check for Understanding

Concept Check

1. Sometimes; $\frac{1}{\sqrt[n]{a}} = \sqrt[n]{a}$ only when $a = 1$.

1. **Determine** whether the statement $\frac{1}{\sqrt[n]{a}} = \sqrt[n]{a}$ is *sometimes*, *always*, or *never* true. Explain.
2. **OPEN ENDED** **Write** a sum of three radicals that contains two like terms.
3. **Explain** why the product of two conjugates is always a rational number.

2–3. See margin.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–5, 14	1
6	2
7–9	3
10, 11	4
12	5
13	6

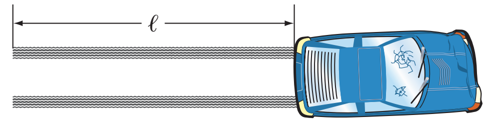
Simplify.

4. $5\sqrt{63}$ $15\sqrt{7}$
5. $\sqrt[4]{16x^5y^4}$ $2x|y|\sqrt[4]{x}$
6. $\sqrt{\frac{7}{8y}}$ $\frac{\sqrt{14y}}{4y}$
7. $(-2\sqrt{15})(4\sqrt{21})$
8. $\frac{\sqrt[3]{625}}{\sqrt[3]{25}}$ $\sqrt[3]{25}$
9. $\sqrt{2ab^2} \cdot \sqrt{6a^3b^2}$
10. $\sqrt{3} - 2\sqrt[4]{3} + 4\sqrt{3} + 5\sqrt[4]{3}$
11. $3\sqrt[3]{128} + 5\sqrt[3]{16}$ $22\sqrt[3]{2}$
12. $(3 - \sqrt{5})(1 + \sqrt{3})$ $3 + 3\sqrt{3} - \sqrt{5} - \sqrt{15}$
13. $\frac{1 + \sqrt{5}}{3 - \sqrt{5}}$ $2 + \sqrt{5}$

Application

7. $-24\sqrt{35}$
9. $2a^2b^2\sqrt{3}$
10. $5\sqrt{3} + 3\sqrt[4]{3}$

14. **LAW ENFORCEMENT** A police accident investigator can use the formula $s = 2\sqrt{5\ell}$ to estimate the speed s of a car in miles per hour based on the length ℓ in feet of the skid marks it left. How fast was a car traveling that left skid marks 120 feet long? about 49 mph



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–26	1
27–30	2
31–34	3
35–38	4
39–42	5
43–48	6

Extra Practice

See page 838.

25. $\frac{1}{3}c|d|\sqrt[4]{c}$

26. $\frac{1}{2}wz\sqrt[5]{wz^2}$

40. $6 + 3\sqrt{6} + 2\sqrt{7} + \sqrt{42}$

Simplify. 21. $3|x|y\sqrt{2y}$ 22. $2ab^2\sqrt{10a}$ 23. $6y^2z\sqrt[3]{7}$ 24. $4mn\sqrt[3]{3mn^2}$

15. $\sqrt{243}$ $9\sqrt{3}$
16. $\sqrt{72}$ $6\sqrt{2}$
17. $\sqrt[3]{54}$ $3\sqrt[3]{2}$
18. $\sqrt[4]{96}$ $2\sqrt[4]{6}$
19. $\sqrt{50x^4}$ $5x^2\sqrt{2}$
20. $\sqrt[3]{16y^3}$ $2y\sqrt[3]{2}$
21. $\sqrt{18x^2y^3}$
22. $\sqrt{40a^3b^4}$
23. $3\sqrt[3]{56y^6z^3}$
24. $2\sqrt[3]{24m^4n^5}$
25. $\sqrt[4]{\frac{1}{81}c^5d^4}$
26. $\sqrt[5]{\frac{1}{32}w^6z^7}$
27. $\sqrt[3]{\frac{3}{4}}$ $\frac{\sqrt[3]{6}}{2}$
28. $\sqrt[4]{\frac{2}{3}}$ $\frac{\sqrt[4]{54}}{3}$
29. $\sqrt{\frac{a^4}{b^3}}$ $\frac{a^2\sqrt{b}}{b^2}$
30. $\sqrt{\frac{4r^8}{t^9}}$ $\frac{2r^4\sqrt{t}}{t^5}$
31. $(3\sqrt{12})(2\sqrt{21})$ $36\sqrt{7}$
32. $(-3\sqrt{24})(5\sqrt{20})$ $-60\sqrt{30}$
33. What is $\sqrt{39}$ divided by $\sqrt{26}$? $\frac{\sqrt{6}}{2}$
34. Divide $\sqrt{14}$ by $\sqrt{35}$. $\frac{\sqrt{10}}{5}$

Simplify. 37. $7\sqrt{3} - 2\sqrt{2}$ 38. $4\sqrt{5} + 23\sqrt{6}$ 39. $25 - 5\sqrt{2} + 5\sqrt{6} - 2\sqrt{3}$

35. $\sqrt{12} + \sqrt{48} - \sqrt{27}$ $3\sqrt{3}$
36. $\sqrt{98} - \sqrt{72} + \sqrt{32}$ $5\sqrt{2}$
37. $\sqrt{3} + \sqrt{72} - \sqrt{128} + \sqrt{108}$
38. $5\sqrt{20} + \sqrt{24} - \sqrt{180} + 7\sqrt{54}$
39. $(5 + \sqrt{6})(5 - \sqrt{2})$
40. $(3 + \sqrt{7})(2 + \sqrt{6})$
41. $(\sqrt{11} - \sqrt{2})^2$ $13 - 2\sqrt{22}$
42. $(\sqrt{3} - \sqrt{5})^2$ $8 - 2\sqrt{15}$
43. $\frac{7}{4 - \sqrt{3}}$ $\frac{28 + 7\sqrt{3}}{13}$
44. $\frac{\sqrt{6}}{5 + \sqrt{3}}$ $\frac{5\sqrt{6} - 3\sqrt{2}}{22}$
45. $\frac{-2 - \sqrt{3}}{1 + \sqrt{3}}$ $\frac{-1 - \sqrt{3}}{2}$
46. $\frac{2 + \sqrt{2}}{5 - \sqrt{2}}$ $\frac{12 + 7\sqrt{2}}{23}$

★ 47. $\frac{x + 1}{\sqrt{x^2 - 1}}$ $\frac{\sqrt{x^2 - 1}}{x - 1}$

★ 48. $\frac{x - 1}{\sqrt{x} - 1}$ $\sqrt{x} + 1$

Answers

2. Sample answer: $\sqrt{2} + \sqrt{3} + \sqrt{2}$

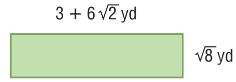
3. The product of two conjugates yields a difference of two squares. Each square produces a rational number and the difference of two rational numbers is a rational number.

50. The square root of a difference is not the difference of the square roots.

56. The formula for the time it takes an object to fall a certain distance can be written in various forms involving radicals. Answers should include the following.

- By the Quotient Property of Radicals, $t = \frac{\sqrt{2d}}{\sqrt{g}}$. Multiply by $\frac{\sqrt{g}}{\sqrt{g}}$ to rationalize the denominator. The result is $\frac{\sqrt{2dg}}{g}$.
- about 1.12 s

49. **GEOMETRY** Find the perimeter and area of the rectangle. $6 + 16\sqrt{2}$ yd, $24 + 6\sqrt{2}$ yd^2



More About. . .

Amusement Parks

Attendance at the top 50 theme parks in North America increased to 175.1 million in 2000.

Source: *Amusement Business*

AMUSEMENT PARKS **For Exercises 50 and 51, use the following information.**
The velocity v in feet per second of a roller coaster at the bottom of a hill is related to the vertical drop h in feet and the velocity v_0 in feet per second of the coaster at the top of the hill by the formula $v_0 = \sqrt{v^2 - 64h}$.

50. Explain why $v_0 = v - 8\sqrt{h}$ is not equivalent to the given formula. **See margin.**

51. What velocity must a coaster have at the top of a 225-foot hill to achieve a velocity of 120 feet per second at the bottom? **0 ft/s**

Online Research **Data Update** What are the values of v and h for some of the world's highest and fastest roller coasters? Visit www.algebra2.com/data_update to learn more.

SPORTS **For Exercises 52 and 53, use the following information.**
A ball that is hit or thrown horizontally with a velocity of v meters per second will travel a distance of d meters before hitting the ground, where $d = v\sqrt{\frac{h}{4.9}}$ and h is the height in meters from which the ball is hit or thrown.

52. Use the properties of radicals to rewrite the formula. $d = v\frac{\sqrt{4.9h}}{4.9}$

53. How far will a ball that is hit horizontally with a velocity of 45 meters per second at a height of 0.8 meter above the ground travel before hitting the ground? **about 18.18 m**

54. **AUTOMOTIVE ENGINEERING** An automotive engineer is trying to design a safer car. The maximum force a road can exert on the tires of the car being redesigned is 2000 pounds. What is the maximum velocity v in ft/s at which this car can safely round a turn of radius 320 feet? Use the formula $v = \sqrt{\frac{F_c r}{100}}$, where F_c is the force the road exerts on the car and r is the radius of the turn. **80 ft/s or about 55 mph**

55. **CRITICAL THINKING** Under what conditions is the equation $\sqrt{x^3y^2} = xy\sqrt{x}$ true? ***x* and *y* are nonnegative.**

56. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

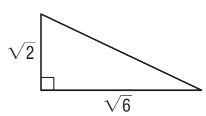
How do radical expressions apply to falling objects?

Include the following in your answer:
- an explanation of how you can use the properties in this lesson to rewrite the formula $t = \sqrt{\frac{2d}{g}}$, and
- the amount of time a 5-foot tall student has to get out of the way after a balloon is dropped from a window 25 feet above.

Standardized Test Practice

57. The expression $\sqrt{180}$ is equivalent to which of the following? **B**
(A) $5\sqrt{6}$ (B) $6\sqrt{5}$ (C) $3\sqrt{10}$ (D) $36\sqrt{5}$

58. Which of the following is *not* a length of a side of the triangle? **D**
(A) $\sqrt{8}$ (B) $2\sqrt{2}$
(C) $\sqrt{4 + 2}$ (D) $\sqrt{4} + \sqrt{2}$



www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 269 (shown) and p. 270

Simplify Radical Expressions

Product Property of Radicals	For any real numbers a and b, and any integer $n > 1$: 1. if n is even and a and b are both nonnegative, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. 2. if n is odd, then $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

To simplify a square root, follow these steps:
1. Factor the radicand into as many squares as possible.
2. Use the Product Property to isolate the perfect squares.
3. Simplify each radical.

Quotient Property of Radicals	For any real numbers a and $b \neq 0$, and any integer $n > 1$, $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, if all roots are defined.

To eliminate radicals from a denominator or fractions from a radicand, multiply the numerator and denominator by a quantity so that the radicand has an exact root.

Example 1 **Simplify $\sqrt[3]{-16a^5b^7}$.**

$$\sqrt[3]{-16a^5b^7} = \sqrt[3]{(-2)^3 \cdot 2 \cdot a^3 \cdot a^2 \cdot (b^2)^3 \cdot b} = -2ab^2\sqrt[3]{2a^2b}$$

Example 2 **Simplify $\sqrt{\frac{8x^3}{45y^5}}$.**

$\sqrt{\frac{8x^3}{45y^5}} = \frac{\sqrt{8x^3}}{\sqrt{45y^5}}$ Quotient Property

$= \frac{\sqrt{(2x)^2 \cdot 2x}}{\sqrt{(3y^2)^2 \cdot 5y}}$ Factor into squares.

$= \frac{\sqrt{(2x)^2} \cdot \sqrt{2x}}{\sqrt{(3y^2)^2} \cdot \sqrt{5y}}$ Product Property

$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}}$ Simplify.

$= \frac{2|x|\sqrt{2x}}{3y^2\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}}$ Rationalize the denominator.

$= \frac{2|x|\sqrt{10xy}}{15y^3}$ Simplify.

Exercises

Simplify.

1. $5\sqrt{54}$ $15\sqrt{6}$
2. $\sqrt[4]{32a^9b^{20}}$ $2a^2|b^5|\sqrt[4]{2a}$
3. $\sqrt{75x^4y^7}$ $5x^2y^3\sqrt{5y}$
4. $\sqrt{\frac{36}{125}}$ $\frac{6\sqrt{5}}{25}$
5. $\sqrt{\frac{a^6b^3}{98}}$ $\frac{|a^3|b\sqrt{2b}}{14}$
6. $\sqrt[3]{\frac{p^5q^3}{40}}$ $\frac{pq\sqrt[3]{5p^2}}{10}$

Skills Practice, p. 271 and Practice, p. 272 (shown)

Simplify.

1. $\sqrt{540}$ $6\sqrt{15}$
2. $\sqrt[3]{-432}$ $-6\sqrt[3]{2}$
3. $\sqrt[3]{128}$ $4\sqrt[3]{2}$
4. $-\sqrt[4]{405}$ $-3\sqrt[4]{5}$
5. $\sqrt[3]{-5000}$ $-10\sqrt[3]{5}$
6. $\sqrt[5]{-1215}$ $-3\sqrt[5]{5}$
7. $\sqrt[3]{125t^6w^2}$ $5t^2\sqrt[3]{w^2}$
8. $\sqrt[4]{48v^8z^{13}}$ $2v^2z^3\sqrt[4]{3z}$
9. $\sqrt[3]{8g^3k^8}$ $2gk^2\sqrt[3]{k^2}$
10. $\sqrt{45x^3y^8}$ $3xy^4\sqrt{5x}$
11. $\sqrt{\frac{11}{9}}$ $\frac{\sqrt{11}}{3}$
12. $\sqrt[3]{\frac{216}{24}}$ $\sqrt[3]{9}$
13. $\sqrt{\frac{1}{128}c^4d^7}$ $\frac{1}{16}c^2d^3\sqrt{2d}$
14. $\sqrt{\frac{9a^5}{64b^4}}$ $\frac{3a^2\sqrt{a}}{8b^2}$
15. $\sqrt[4]{\frac{8}{9a^3}}$ $\frac{\sqrt[4]{72a}}{3a}$
16. $(3\sqrt{15})(-4\sqrt{45})$ $-180\sqrt{3}$
17. $(2\sqrt{24})(7\sqrt{18})$ $168\sqrt{3}$
18. $\sqrt{810} + \sqrt{240} - \sqrt{250}$ $4\sqrt{10} + 4\sqrt{15}$
19. $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$ $5\sqrt{5}$
20. $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$ $2\sqrt{3} + 28\sqrt{5}$
21. $(3\sqrt{2} + 2\sqrt{3})^2$ $30 + 12\sqrt{6}$
22. $(3 - \sqrt{7})^2$ $16 - 6\sqrt{7}$
23. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$ $5 + \sqrt{10} - \sqrt{30} - 2\sqrt{3}$
24. $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$ -8
25. $(1 + \sqrt{6})(5 - \sqrt{7})$ $5 - \sqrt{7} + 5\sqrt{6} - \sqrt{42}$
26. $(\sqrt{3} + 4\sqrt{7})^2$ $115 + 8\sqrt{21}$
27. $(\sqrt{108} - 6\sqrt{3})^2$ 0
28. $\frac{\sqrt{3}}{\sqrt{5} - 2}$ $\sqrt{15} + 2\sqrt{3}$
29. $\frac{6}{\sqrt{2} - 1}$ $6\sqrt{2} + 6$
30. $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$ $\frac{17 - \sqrt{3}}{13}$
31. $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$ $\frac{8 + 5\sqrt{2}}{2}$
32. $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$ $27 + 11\sqrt{6}$
33. $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$ $\frac{6 + 5\sqrt{x} + x}{4 - x}$

34. **BRAKING** The formula $s = 2\sqrt{5\ell}$ estimates the speed s in miles per hour of a car when it leaves skid marks ℓ feet long. Use the formula to write a simplified expression for s if $\ell = 85$. Then evaluate s to the nearest mile per hour. $10\sqrt{17}$; 41 mi/h

35. **PYTHAGOREAN THEOREM** The measures of the legs of a right triangle can be represented by the expressions $6x^2y$ and $9x^2y$. Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse. $3x^2|y|\sqrt{13}$

Reading to Learn Mathematics, p. 273 ELL

Pre-Activity **How do radical expressions apply to falling objects?**

Read the introduction to Lesson 5-6 at the top of page 250 in your textbook.

Describe how you could use the formula given in your textbook and a calculator to find the time, to the nearest tenth of a second, that it would take for the water balloons to drop 22 feet. (Do not actually calculate the time.) **Sample answer: Multiply 22 by 2 (giving 44) and divide by 32. Use the calculator to find the square root of the result. Round this square root to the nearest tenth.**

Reading the Lesson

1. Complete the conditions that must be met for a radical expression to be in simplified form.
- The **index** n is as **small** as possible.
- The **radicand** contains no **factors** (other than 1) that are nth powers of a(n) **integer** or polynomial.
- The radicand contains no **fractions**.
- No **radicals** appear in the **denominator**.

2. a. What are conjugates of radical expressions used for? **to rationalize binomial denominators**

b. How would you use a conjugate to simplify the radical expression $\frac{1 + \sqrt{2}}{3 - \sqrt{2}}$? **Multiply numerator and denominator by $3 + \sqrt{2}$.**

c. In order to simplify the radical expression in part b, two multiplications are necessary. The multiplication in the numerator would be done by the **FOIL** method, and the multiplication in the denominator would be done by finding the **difference** of **two** **squares**.

Helping You Remember

3. One way to remember something is to explain it to another person. When rationalizing the denominator in the expression $\frac{1}{\sqrt[3]{2}}$, many students think they should multiply numerator and denominator by $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$. How would you explain to a classmate why this is incorrect and what he should do instead. **Sample answer: Because you are working with cube roots, not square roots, you need to make the radicand in the denominator a perfect cube, not a perfect square. Multiply numerator and denominator by $\frac{\sqrt[3]{4}}{\sqrt[3]{4}}$ to make the denominator $\sqrt[3]{8}$, which equals 2.**

Enrichment, p. 274

Special Products with Radicals

Notice that $(\sqrt{3})(\sqrt{3}) = 3$, or $(\sqrt{3})^2 = 3$.

In general, $(\sqrt{x})^2 = x$ when $x \geq 0$.

Also, notice that $(\sqrt{9})(\sqrt{4}) = \sqrt{36}$.

In general, $(\sqrt{x})(\sqrt{y}) = \sqrt{xy}$ when x and y are not negative.

You can use these ideas to find the special products below.

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$

$(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a})^2 + 2\sqrt{ab} + (\sqrt{b})^2 = a + 2\sqrt{ab} + b$

$(\sqrt{a} - \sqrt{b})^2 = (\sqrt{a})^2 - 2\sqrt{ab} + (\sqrt{b})^2 = a - 2\sqrt{ab} + b$

Example 1 **Find the product: $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$.**

$(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5}) = (\sqrt{2})^2 - (\sqrt{5})^2 = 2 - 5 = -3$

Example 2 **Evaluate $(\sqrt{2} + \sqrt{\ }$**

4 Assess

Open-Ended Assessment

Speaking Ask students to describe how combining radicals is the same as combining expressions with variables, and how it differs from working with variables.

Tips for New Teachers

Intervention Students will need to simplify expressions involving radicals in much of their further work in algebra. Take time to help students uncover and correct their misconceptions by analyzing the errors they make.

Getting Ready for Lesson 5-7

BASIC SKILL Lesson 5-7 presents working with rational exponents. This often involves adding, subtracting, or multiplying fractions. Exercises 75–82 should be used to determine your students' familiarity with rational numbers.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 5-4 through 5-6. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 5-5)*

59. $\sqrt{144z^8}$ **$12z^4$** 60. $\sqrt[3]{216a^3b^9}$ **$6ab^3$** 61. $\sqrt{(y+2)^2}$ **$|y+2|$**

Simplify. Assume that no denominator is equal to 0. *(Lesson 5-4)*

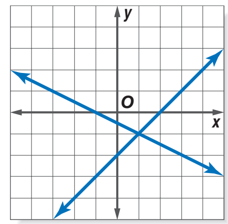
62. $\frac{x^2+5x-14}{x^2-6x+8}$ **$\frac{x+7}{x-4}$** 63. $\frac{x^2-3x-4}{x^2-16}$ **$\frac{x+1}{x+4}$**

Perform the indicated operations. *(Lesson 4-2)*

64. $\begin{bmatrix}3 & -4\\2 & 8\\0 & 1\end{bmatrix}+\begin{bmatrix}-5 & 0\\7 & 7\\3 & -6\end{bmatrix}$ **$\begin{bmatrix}-2 & -4\\9 & 15\\3 & -5\end{bmatrix}$** 65. $\begin{bmatrix}3 & 3\\0 & -2\end{bmatrix}-\begin{bmatrix}2 & -1\\5 & 2\end{bmatrix}$ **$\begin{bmatrix}1 & 4\\-5 & -4\end{bmatrix}$**

66. Find the maximum and minimum values of the function $f(x, y) = 2x + 3y$ for the region with vertices at (2, 4), (−1, 3), (−3, −3), and (2, −5). *(Lesson 3-4)* **16, −15**

67. State whether the system of equations shown at the right is *consistent and independent*, *consistent and dependent*, or *inconsistent*. *(Lesson 3-1)* **consistent and independent**



68. **BUSINESS** The amount that a mail-order company charges for shipping and handling is given by the function $c(x) = 3 + 0.15x$, where x is the weight in pounds. Find the charge for an 8-pound order. *(Lesson 2-2)* **$4.20**

Solve. *(Lessons 1-3, 1-4, and 1-5)*

69. $2x + 7 = -3$ **−5** 70. $-5x + 6 = -4$ **2**

71. $|x - 1| = 3$ **−2, 4** 72. $|3x + 2| = 5$ **$-\frac{7}{3}$, 1**

73. $2x - 4 > 8$ **$\{x \mid x > 6\}$** 74. $-x - 3 \le 4$ **$\{x \mid x \ge -7\}$**

Getting Ready for the Next Lesson

BASIC SKILL Evaluate each expression.

75. $2\left(\frac{1}{8}\right)$ **$\frac{1}{4}$** 76. $3\left(\frac{1}{6}\right)$ **$\frac{1}{2}$** 77. $\frac{1}{2}+\frac{1}{3}$ **$\frac{5}{6}$** 78. $\frac{1}{3}+\frac{3}{4}$ **$\frac{13}{12}$**

79. $\frac{1}{8}+\frac{5}{12}$ **$\frac{13}{24}$** 80. $\frac{5}{6}-\frac{1}{5}$ **$\frac{19}{30}$** 81. $\frac{5}{8}-\frac{1}{4}$ **$\frac{3}{8}$** 82. $\frac{1}{4}-\frac{2}{3}$ **$-\frac{5}{12}$**

Practice Quiz 2 — Lessons 5-4 through 5-6

Factor completely. If the polynomial is not factorable, write prime. *(Lesson 5-4)*

1. $3x^3y + x^2y^2 + x^2y$ **$x^2y(3x + y + 1)$** 2. $3x^2 - 2x - 2$ **prime**

3. $ax^2 + 6ax + 9a$ **$a(x + 3)^2$** 4. $8r^3 - 64s^6$ **$8(r - 2s^2)(r^2 + 2rs^2 + 4s^4)$**

Simplify. *(Lessons 5-5 and 5-6)*

5. $\sqrt{36x^2y^6}$ **$6|x||y^3|$** 6. $\sqrt[3]{-64a^6b^9}$ **$-4a^2b^3$** 7. $\sqrt{4n^2+12n+9}$ **$|2n+3|$**

8. $\sqrt{\frac{x^4}{y^3}}$ **$\frac{x^2\sqrt{y}}{y^2}$** 9. $(3+\sqrt{7})(2-\sqrt{7})$ **$-1-\sqrt{7}$** 10. $\frac{5+\sqrt{2}}{2+\sqrt{2}}$ **$\frac{8-3\sqrt{2}}{2}$**

5-7 Rational Exponents

What You'll Learn

- Write expressions with rational exponents in radical form, and vice versa.
- Simplify expressions in exponential or radical form.

How do rational exponents apply to astronomy?

Astronomers refer to the space around a planet where the planet's gravity is stronger than the Sun's as the *sphere of influence* of the planet. The radius r of the sphere of influence is given by the formula $r = D\left(\frac{M_p}{M_S}\right)^{\frac{2}{5}}$, where M_p is the mass of the planet, M_S is the mass of the Sun, and D is the distance between the planet and the Sun.

RATIONAL EXPONENTS AND RADICALS You know that squaring a number and taking the square root of a number are inverse operations. But how would you evaluate an expression that contains a fractional exponent such as the one above? You can investigate such an expression by assuming that fractional exponents behave as integral exponents.

$$\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}}$$ Write the square as multiplication.

$$= b^{\frac{1}{2}+\frac{1}{2}}$$ Add the exponents.

$$= b^1 \text{ or } b$$ Simplify.

Thus, $b^{\frac{1}{2}}$ is a number whose square equals b. So it makes sense to define $b^{\frac{1}{2}} = \sqrt{b}$.

Key Concept $b^{\frac{1}{n}}$

- **Words** For any real number b and for any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
- **Example** $8^{\frac{1}{3}} = \sqrt[3]{8}$ or 2

Example 1 *Radical Form*

Write each expression in radical form.

a. $a^{\frac{1}{4}}$

$a^{\frac{1}{4}} = \sqrt[4]{a}$ Definition of $b^{\frac{1}{n}}$

b. $x^{\frac{1}{5}}$

$x^{\frac{1}{5}} = \sqrt[5]{x}$ Definition of $b^{\frac{1}{n}}$

5-7 Lesson Notes

1 Focus

5-Minute Check Transparency 5-7 Use as a quiz or review of Lesson 5-6.

Mathematical Background notes are available for this lesson on p. 220D.

How do rational exponents apply to astronomy?

Ask students:

- Is the p in M_p an exponent? Is it a variable? **No, it is neither an exponent nor a variable; it is a subscript.**
- Would you expect the radius of the sphere of influence for one of the larger planets in our solar system to be greater than the radius of the sphere of influence for Earth? Use the formula to justify your answer. **Yes; for the planets larger than Earth, the value of M_p would be greater than the value of M_p for Earth while the value of M_S is the same. So the value of the ratio $\frac{M_p}{M_S}$ is greater for the larger planets.**

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters
- Study Guide and Intervention, pp. 275–276
- Skills Practice, p. 277
- Practice, p. 278
- Reading to Learn Mathematics, p. 279
- Enrichment, p. 280
- Assessment, p. 308

Graphing Calculator and Spreadsheet Masters, p. 36

Transparencies

5-Minute Check Transparency 5-7
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

RATIONAL EXPONENTS AND RADICALS

In-Class Examples

Power Point®

1 Write each expression in radical form.

a. $a^{\frac{1}{6}}$ $\sqrt[6]{a}$

b. $m^{\frac{1}{2}}$ $\sqrt{m}$

2 Write each radical using rational exponents.

a. $\sqrt[5]{b}$ $b^{\frac{1}{5}}$

b. $\sqrt{w}$ $w^{\frac{1}{2}}$

3 Evaluate each expression.

a. $49^{-\frac{1}{2}}$ $\frac{1}{7}$

b. $32^{\frac{2}{5}}$ 4

Teaching Tip If students are having difficulty remembering which part of the fractional exponent is the index, suggest that they recall the basic definition $b^{\frac{1}{2}} = \sqrt{b}$.

Example 2 Exponential Form

Write each radical using rational exponents.

a. $\sqrt[3]{y}$

$\sqrt[3]{y} = y^{\frac{1}{3}}$ Definition of $b^{\frac{1}{n}}$

b. $\sqrt[8]{c}$

$\sqrt[8]{c} = c^{\frac{1}{8}}$ Definition of $b^{\frac{1}{n}}$

Many expressions with fractional exponents can be evaluated using the definition of $b^{\frac{1}{n}}$ or the properties of powers.

Study Tip

Negative Base
Suppose the base of a monomial is negative such as $(-9)^2$ or $(-9)^3$. The expression is undefined if the exponent is even because there is no number that, when multiplied an even number of times, results in a negative number. However, the expression is defined for an odd exponent.

Example 3 Evaluate Expressions with Rational Exponents

Evaluate each expression.

a. $16^{-\frac{1}{4}}$

Method 1

$16^{-\frac{1}{4}} = \frac{1}{16^{\frac{1}{4}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{\sqrt[4]{16}}$ $16^{\frac{1}{4}} = \sqrt[4]{16}$

$= \frac{1}{\sqrt[4]{2^4}}$ $16 = 2^4$

$= \frac{1}{2}$ Simplify.

Method 2

$16^{-\frac{1}{4}} = (2^4)^{-\frac{1}{4}}$ $16 = 2^4$

$= 2^{4\left(-\frac{1}{4}\right)}$ Power of a Power

$= 2^{-1}$ Multiply exponents.

$= \frac{1}{2}$ $2^{-1} = \frac{1}{2^1}$

b. $243^{\frac{3}{5}}$

Method 1

$243^{\frac{3}{5}} = 243^{3\left(\frac{1}{5}\right)}$ Factor.

$= (243^3)^{\frac{1}{5}}$ Power of a Power

$= \sqrt[5]{243^3}$ $b^{\frac{1}{5}} = \sqrt[5]{b}$

$= \sqrt[5]{(3^5)^3}$ $243 = 3^5$

$= \sqrt[5]{3^5 \cdot 3^5 \cdot 3^5}$ Expand the cube.

$= 3 \cdot 3 \cdot 3$ or 27 Find the fifth root.

Method 2

$243^{\frac{3}{5}} = (3^5)^{\frac{3}{5}}$ $243 = 3^5$

$= 3^{5\left(\frac{3}{5}\right)}$ Power of a Power

$= 3^3$ Multiply exponents.

$= 27$ $3^3 = 3 \cdot 3 \cdot 3$

In Example 3b, Method 1 uses a combination of the definition of $b^{\frac{1}{n}}$ and the properties of powers. This example suggests the following general definition of rational exponents.

Key Concept *Rational Exponents*

- **Words** For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even.
- **Example** $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2$ or 4

In general, we define $b^{\frac{m}{n}}$ as $\left(b^{\frac{1}{n}}\right)^m$ or $(b^m)^{\frac{1}{n}}$. Now apply the definition of $b^{\frac{1}{n}}$ to $\left(b^{\frac{1}{n}}\right)^m$ and $(b^m)^{\frac{1}{n}}$.

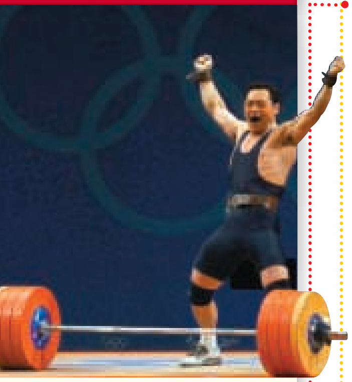
$$\left(b^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{b}\right)^m \qquad (b^m)^{\frac{1}{n}} = \sqrt[n]{b^m}$$

DAILY INTERVENTION

Unlocking Misconceptions

- **Exponents** Students may be confused because they are not perceiving and reading the exponent in a way that distinguishes it from a coefficient or multiplier. Ask them to practice reading the exponent correctly, for example, reading x^3 as "x to the third power" or as "x cubed.".
- **Radicals** Ask students to practice reading radical expressions correctly, for example, reading $\sqrt{y^3}$ as "the square root of y cubed."

More About. . .



Weight Lifting With origins in both the ancient Egyptian and Greek societies, weightlifting was among the sports on the first program of the first Modern Olympic Games, in 1896, in Athens, Greece.

Source: International Weightlifting Association

Example 4 Rational Exponent with Numerator Other Than 1

WEIGHT LIFTING **The formula $M = 512 - 146{,}230B^{-\frac{8}{5}}$ can be used to estimate the maximum total mass that a weight lifter of mass B kilograms can lift in two lifts, the snatch and the clean and jerk, combined.**

a. According to the formula, what is the maximum amount that 2000 Olympic champion Xugang Zhan of China can lift if he weighs 72 kilograms?

$M = 512 - 146{,}230B^{-\frac{8}{5}}$ Original formula

$= 512 - 146{,}230(72)^{-\frac{8}{5}}$ $B = 72$

≈ 356 kg Use a calculator.

The formula predicts that he can lift at most 356 kilograms.

b. Xugang Zhan's winning total in the 2000 Olympics was 367.50 kg. Compare this to the value predicted by the formula.

The formula prediction is close to the actual weight, but slightly lower.

In-Class Example (Power Point®)

Teaching Tip Be sure students notice that the fractional exponent is negative in the formula for the maximum total mass M.

4 **WEIGHT LIFTING** Use the formula given in Example 4.

a. U.S. weightlifter Oscar Chaplin III competed in the same weight class as Xugang Zhan, finishing in 7th place. According to the formula, what is the maximum that Chaplin can lift if he weighs 77 kilograms? Source: cnnsi.com **The formula predicts that he can lift at most 372 kilograms.**

b. Oscar Chaplin's total in the 2000 Olympics was 335 kg. Compare this to the value predicted by the formula. **The formula prediction is somewhat higher than his actual total.**

SIMPLIFY EXPRESSIONS All of the properties of powers you learned in Lesson 5-1 apply to rational exponents. When simplifying expressions containing rational exponents, leave the exponent in rational form rather than writing the expression as a radical. To simplify such an expression, you must write the expression with all positive exponents. Furthermore, any exponents in the denominator of a fraction must be positive *integers*. So, it may be necessary to rationalize a denominator.

Example 5 Simplify Expressions with Rational Exponents

Simplify each expression.

a. $x^{\frac{1}{5}} \cdot x^{\frac{7}{5}}$

$x^{\frac{1}{5}} \cdot x^{\frac{7}{5}} = x^{\frac{1}{5}+\frac{7}{5}}$ Multiply powers.

$= x^{\frac{8}{5}}$ Add exponents.

b. $y^{-\frac{3}{4}}$

$y^{-\frac{3}{4}} = \frac{1}{y^{\frac{3}{4}}}$ $b^{-n} = \frac{1}{b^n}$

$= \frac{1}{y^{\frac{3}{4}}} \cdot \frac{y^{\frac{1}{4}}}{y^{\frac{1}{4}}}$ Why use $\frac{y^{\frac{1}{4}}}{y^{\frac{1}{4}}}$?

$= \frac{y^{\frac{1}{4}}}{y^{\frac{4}{4}}}$ $y^{\frac{3}{4}} \cdot y^{\frac{1}{4}} = y^{\frac{3}{4}+\frac{1}{4}}$

$= \frac{y^{\frac{1}{4}}}{y}$ $y^{\frac{4}{4}} = y^1$ or y

SIMPLIFY EXPRESSIONS

In-Class Example (Power Point®)

5 Simplify each expression.

a. $y^{\frac{1}{7}} \cdot y^{\frac{4}{7}}$ $y^{\frac{5}{7}}$

b. $x^{-\frac{2}{3}}$ $\frac{x^{\frac{1}{3}}}{x}$

TEACHING TIP

Tell students that if they are simplifying an expression that was originally written with radicals, they should write the answer with radicals. If the expression was originally written with rational exponents, they should write the answer with rational exponents.

When simplifying a radical expression, always use the smallest index possible. Using rational exponents makes this process easier, but the answer should be written in radical form.

www.algebra2.com/extra_examples

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Have students name and demonstrate on a keyboard, guitar, or other instrument, the sounds of the notes described in Exercises 67 and 68. Other students can work to associate the sound of the note with the number of vibrations per second given by the formula.

In-Class Example — Power Point®

6 Simplify each expression.

a. $\dfrac{\sqrt[6]{16}}{\sqrt[3]{2}}$ $2^{\frac{1}{3}}$ or $\sqrt[3]{2}$

b. $\sqrt[6]{4x^4}$ $\sqrt[3]{2x^2}$

c. $\dfrac{y^{\frac{1}{2}} + 1}{y^{\frac{1}{2}} - 1}$ $\dfrac{y + 2y^{\frac{1}{2}} + 1}{y - 1}$

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.
- add the information listed in the Concept Summary below Example 6 to their notebook.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Rational Exponents and Radicals:** 21–40
- **Simplify Expressions:** 41–64

Odd/Even Assignments

Exercises 21–66 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 21–61 odd, 65, 69–84

Average: 21–65 odd, 69–84

Advanced: 22–66 even, 67, 68, 70–80 (optional: 81–84)

Example 6 Simplify Radical Expressions

Simplify each expression.

a. $\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}}$

$\dfrac{\sqrt[8]{81}}{\sqrt[6]{3}} = \dfrac{81^{\frac{1}{8}}}{3^{\frac{1}{6}}}$ Rational exponents

$= \dfrac{(3^4)^{\frac{1}{8}}}{3^{\frac{1}{6}}}$ $81 = 3^4$

$= \dfrac{3^{\frac{1}{2}}}{3^{\frac{1}{6}}}$ Power of a Power

$= 3^{\frac{1}{2} - \frac{1}{6}}$ Quotient of Powers

$= 3^{\frac{1}{3}}$ or $\sqrt[3]{3}$ Simplify.

b. $\sqrt[4]{9z^2}$

$\sqrt[4]{9z^2} = (9z^2)^{\frac{1}{4}}$ Rational exponents

$= (3^2 \cdot z^2)^{\frac{1}{4}}$ $9 = 3^2$

$= 3^{2\left(\frac{1}{4}\right)} \cdot z^{2\left(\frac{1}{4}\right)}$ Power of a Power

$= 3^{\frac{1}{2}} \cdot z^{\frac{1}{2}}$ Multiply.

$= \sqrt{3} \cdot \sqrt{z}$ $3^{\frac{1}{2}} = \sqrt{3}, z^{\frac{1}{2}} = \sqrt{z}$

$= \sqrt{3z}$ Simplify.

c. $\dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1}$

$\dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1} = \dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} + 1} \cdot \dfrac{m^{\frac{1}{2}} - 1}{m^{\frac{1}{2}} - 1}$ $m^{\frac{1}{2}} - 1$ is the conjugate of $m^{\frac{1}{2}} + 1$.

$= \dfrac{m - 2m^{\frac{1}{2}} + 1}{m - 1}$ Multiply.

Concept Summary — Expressions with Rational Exponents

An expression with rational exponents is simplified when all of the following conditions are met.

- It has no negative exponents.
- It has no fractional exponents in the denominator.
- It is not a complex fraction.
- The index of any remaining radical is the least number possible.

Check for Understanding

Concept Check

1. **OPEN ENDED** Determine a value of b for which $b^{\frac{1}{6}}$ is an integer.
2. **Explain** why $(-16)^{\frac{1}{2}}$ is not a real number. **See margin.**
3. **Explain** why $\sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$. **See margin.**

1. Sample answer: 64

Answers

2. In radical form, the expression would be $\sqrt{-16}$, which is not a real number because the index is even and the radicand is negative.

3. In exponential form $\sqrt[n]{b^m}$ is equal to $(b^m)^{\frac{1}{n}}$. By the Power of a Power Property, $(b^m)^{\frac{1}{n}} = b^{\frac{m}{n}}$. But, $b^{\frac{m}{n}}$ is also equal to $\left(b^{\frac{1}{n}}\right)^m$ by the Power of a Power Property. This last expression is equal to $\left(\sqrt[n]{b}\right)^m$. Thus, $\sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6, 7	2
8–11	3
12–17	5
18, 19	6
20	4

Write each expression in radical form.

4. $7^{\frac{1}{3}}$ $\sqrt[3]{7}$

5. $x^{\frac{2}{3}}$ $\sqrt[3]{x^2}$ or $(\sqrt[3]{x})^2$

Write each radical using rational exponents.

6. $\sqrt[4]{26}$ $26^{\frac{1}{4}}$

7. $\sqrt[3]{6x^5y^7}$ $6^{\frac{1}{3}}x^{\frac{5}{3}}y^{\frac{7}{3}}$

Evaluate each expression.

8. $125^{\frac{1}{3}}$ 5

9. $81^{-\frac{1}{4}}$ $\frac{1}{3}$

10. $27^{\frac{2}{3}}$ 9

11. $\dfrac{54}{9^{\frac{3}{2}}}$ 2

Simplify each expression.

12. $a^{\frac{2}{3}} \cdot a^{\frac{1}{4}}$ $a^{\frac{11}{12}}$

13. $\dfrac{x^{\frac{5}{6}}}{x^{\frac{1}{6}}}$ $x^{\frac{2}{3}}$

14. $\dfrac{1}{2z^{\frac{1}{3}}}$ $\dfrac{z^{\frac{2}{3}}}{2z}$

15. $\dfrac{a^2}{b^{\frac{1}{3}}} \cdot \dfrac{b}{a^{\frac{1}{2}}}$ $a^{\frac{3}{2}}b^{\frac{2}{3}}$

16. $(mn^2)^{-\frac{1}{3}}$ $\dfrac{m^{\frac{2}{3}}n^{\frac{1}{3}}}{mn}$

17. $z(x-2y)^{-\frac{1}{2}}$

17. $\dfrac{z(x-2y)^{\frac{1}{2}}}{x-2y}$

18. $\sqrt[6]{27x^3}$ $\sqrt{3x}$

19. $\dfrac{\sqrt[4]{27}}{\sqrt[4]{3}}$ $\sqrt{3}$

Application

20. **ECONOMICS** When inflation causes the price of an item to increase, the new cost C and the original cost c are related by the formula $C = c(1 + r)^n$, where r is the rate of inflation per year as a decimal and n is the number of years. What would be the price of a \$4.99 item after six months of 5% inflation? **\$5.11**

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
21–24	1
25–28	2
29–40	3
41–52, 64–66	5
53–63	6

Extra Practice

See page 838.

Write each expression in radical form.

21. $6^{\frac{1}{5}}$ $\sqrt[5]{6}$

22. $4^{\frac{1}{3}}$ $\sqrt[3]{4}$

23. $c^{\frac{2}{5}}$ $\sqrt[5]{c^2}$ or $(\sqrt[5]{c})^2$

24. $(x^2)^{\frac{4}{3}}$ $x^2\sqrt[3]{x^2}$

Write each radical using rational exponents.

25. $\sqrt{23}$ $23^{\frac{1}{2}}$

26. $\sqrt[3]{62}$ $62^{\frac{1}{3}}$

27. $\sqrt[4]{16z^2}$ $2z^{\frac{1}{2}}$

28. $\sqrt[3]{5x^2y}$ $5^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{1}{3}}$

Evaluate each expression.

29. $16^{\frac{1}{4}}$ 2

30. $216^{\frac{1}{3}}$ 6

31. $25^{-\frac{1}{2}}$ $\frac{1}{5}$

32. $81^{-\frac{3}{4}}$ $\frac{1}{27}$

33. $(-27)^{-\frac{2}{3}}$ $\frac{1}{9}$

34. $(-32)^{-\frac{3}{5}}$ $-\frac{1}{8}$

35. $81^{-\frac{1}{2}} \cdot 81^{\frac{3}{2}}$ 81

36. $8^{\frac{3}{2}} \cdot 8^{\frac{5}{2}}$ 4096

37. $\left(\frac{8}{27}\right)^{\frac{1}{3}}$ $\frac{2}{3}$

38. $\left(\frac{1}{243}\right)^{-\frac{3}{5}}$ 27

39. $\dfrac{16^{\frac{1}{2}}}{9^{\frac{1}{2}}}$ $\frac{4}{3}$

40. $\dfrac{8^{\frac{1}{3}}}{64^{\frac{1}{3}}}$ $\frac{1}{2}$

Simplify each expression.

41. $y^{\frac{5}{3}} \cdot y^{\frac{7}{3}}$ y^4

42. $x^{\frac{3}{4}} \cdot x^{\frac{9}{4}}$ x^3

43. $\left(b^{\frac{1}{3}}\right)^{\frac{3}{5}}$ $b^{\frac{1}{5}}$

44. $\left(a^{-\frac{2}{3}}\right)^{-\frac{1}{6}}$ $a^{\frac{1}{9}}$

45. $w^{-\frac{4}{5}}$ $\dfrac{w^{\frac{1}{5}}}{w}$

46. $x^{-\frac{1}{6}}$ $\dfrac{x^{\frac{5}{6}}}{x}$

47. $\dfrac{t^{\frac{3}{4}}}{t^{\frac{1}{2}}}$ $t^{\frac{1}{4}}$

48. $\dfrac{r^{\frac{2}{3}}}{r^{\frac{1}{6}}}$ $r^{\frac{1}{2}}$

49. $\dfrac{a^{-\frac{1}{2}}}{6a^{\frac{1}{3}} \cdot a^{-\frac{1}{4}}}$ $\dfrac{a^{\frac{5}{12}}}{6a}$

50. $\dfrac{2c^{\frac{1}{8}}}{c^{-\frac{1}{16}} \cdot c^{\frac{1}{4}}}$ $\dfrac{2c^{\frac{15}{16}}}{c}$

51. $\dfrac{y^{\frac{3}{2}}}{y^{\frac{1}{2}}+2}$ $\dfrac{y^2 - 2y^{\frac{3}{2}}}{y-4}$

52. $\dfrac{x^{\frac{1}{2}}+2}{x^{\frac{1}{2}}-1}$ $\dfrac{x + 3x^{\frac{1}{2}} + 2}{x-1}$

53. $\sqrt[4]{25}$ $\sqrt{5}$

54. $\sqrt[6]{27}$ $\sqrt{3}$

55. $\sqrt{17} \cdot \sqrt[3]{17^2}$ $17\sqrt[6]{17}$

56. $\sqrt[3]{5} \cdot \sqrt{5^3}$ $5\sqrt[6]{5^5}$

57. $\sqrt[8]{25x^4y^4}$ $\sqrt[4]{5x^2y^2}$

58. $\sqrt[6]{81a^4b^8}$ $b\sqrt[3]{9a^2b}$

59. $\dfrac{xy}{\sqrt{z}}$ $\dfrac{xy\sqrt{z}}{z}$

60. $\dfrac{ab}{\sqrt[3]{c}}$ $\dfrac{ab\sqrt[3]{c^2}}{c}$

61. $\sqrt[3]{\sqrt{8}}$ $\sqrt{2}$

62. $\sqrt{\sqrt[3]{36}}$ $\sqrt[3]{6}$

★ 63. $\dfrac{8^{\frac{1}{6}} - 9^{\frac{1}{4}}}{\sqrt{3} + \sqrt{2}}$ $2\sqrt{6} - 5$

★ 64. $\dfrac{x^{\frac{5}{3}} - x^{\frac{1}{3}}z^{\frac{4}{3}}}{x^{\frac{2}{3}} + z^{\frac{2}{3}}}$ $x - x^{\frac{1}{3}}z^{\frac{2}{3}}$

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 275 (shown) and p. 276

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$, except when $b < 0$ and n is even.

Example 1 **Write $28^{\frac{1}{2}}$ in radical form.**

Notice that $28 > 0$.

$28^{\frac{1}{2}} = \sqrt{28}$
$= \sqrt{2^2 \cdot 7}$
$= \sqrt{2^2} \cdot \sqrt{7}$
$= 2\sqrt{7}$

Example 2 **Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.**

Notice that $-8 < 0$, $-125 < 0$, and 3 is odd.

$\left(\frac{-8}{-125}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}}$
$= \frac{-2}{-5}$
$= \frac{2}{5}$

Exercises

Write each expression in radical form.

1. $11^{\frac{1}{7}}$ $\sqrt[7]{11}$
2. $15^{\frac{1}{3}}$ $\sqrt[3]{15}$
3. $300^{\frac{3}{2}}$ $\sqrt{300^3}$

Write each radical using rational exponents.

4. $\sqrt{47}$ $47^{\frac{1}{2}}$
5. $\sqrt[3]{3a^5b^2}$ $3^{\frac{1}{3}}a^{\frac{5}{3}}b^{\frac{2}{3}}$
6. $\sqrt[4]{162p^5}$ $3 \cdot 2^{\frac{1}{4}} \cdot p^{\frac{5}{4}}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$ 9
8. $\dfrac{5^{-\frac{1}{2}}}{2\sqrt{5}}$ $\frac{1}{10}$
9. $(0.0004)^{\frac{1}{2}}$ 0.02
10. $8^{\frac{2}{3}} \cdot 4^{\frac{3}{2}}$ 32
11. $\dfrac{144^{-\frac{1}{2}}}{27^{-\frac{1}{3}}}$ $\frac{1}{4}$
12. $\dfrac{16^{-\frac{1}{2}}}{(0.25)^{\frac{1}{2}}}$ $\frac{1}{2}$

Skills Practice, p. 277 and Practice, p. 278 (shown)

Write each expression in radical form.

1. $5^{\frac{1}{3}}$ $\sqrt[3]{5}$
2. $6^{\frac{2}{5}}$ $\sqrt[5]{6^2}$ or $(\sqrt[5]{6})^2$
3. $m^{\frac{4}{7}}$ $\sqrt[7]{m^4}$ or $(\sqrt[7]{m})^4$
4. $(n^3)^{\frac{2}{5}}$ $n\sqrt[5]{n}$

Write each radical using rational exponents.

5. $\sqrt{79}$ $79^{\frac{1}{2}}$
6. $\sqrt[4]{153}$ $153^{\frac{1}{4}}$
7. $\sqrt[3]{27m^6n^4}$ $3m^2n^{\frac{4}{3}}$
8. $5\sqrt{2a^{10}b}$ $5 \cdot 2^{\frac{1}{2}}|a^5|b^{\frac{1}{2}}$

Evaluate each expression.

9. $81^{\frac{1}{4}}$ 3
10. $1024^{-\frac{1}{5}}$ $\frac{1}{4}$
11. $8^{-\frac{5}{3}}$ $\frac{1}{32}$
12. $-256^{-\frac{3}{4}}$ $-\frac{1}{64}$
13. $(-64)^{-\frac{2}{3}}$ $\frac{1}{16}$
14. $27^{\frac{1}{3}} \cdot 27^{\frac{4}{3}}$ 243
15. $\left(\frac{125}{216}\right)^{\frac{2}{3}}$ $\frac{25}{36}$
16. $\dfrac{64^{\frac{2}{3}}}{343^{\frac{2}{3}}}$ $\frac{16}{49}$
17. $\left(25^{\frac{1}{2}}\right)\left(-64^{-\frac{1}{3}}\right)$ $-\frac{5}{4}$

Simplify each expression.

18. $g^{\frac{4}{7}} \cdot g^{\frac{3}{7}}$ g
19. $s^{\frac{3}{4}} \cdot s^{\frac{13}{4}}$ s^4
20. $\left(u^{-\frac{1}{3}}\right)^{-\frac{4}{5}}$ $u^{\frac{4}{15}}$
21. $y^{-\frac{1}{2}}$ $\dfrac{y^{\frac{1}{2}}}{y}$
22. $b^{-\frac{3}{5}}$ $\dfrac{b^{\frac{2}{5}}}{b}$
23. $\dfrac{q^{\frac{3}{5}}}{q^{\frac{2}{5}}}$ $q^{\frac{1}{5}}$
24. $\dfrac{t^{\frac{2}{3}}}{5t^{\frac{1}{2}} \cdot t^{-\frac{3}{4}}}$ $\dfrac{t^{\frac{11}{12}}}{5}$
25. $\dfrac{2z^{\frac{1}{2}}}{z^{\frac{1}{2}} - 1}$ $\dfrac{2z + 2z^{\frac{1}{2}}}{z-1}$
26. $\sqrt[10]{8^5}$ $2\sqrt{2}$
27. $\sqrt{12} \cdot \sqrt[5]{12^3}$ $12\sqrt[10]{12}$
28. $\sqrt[4]{6} \cdot 3\sqrt[4]{6}$ $3\sqrt{6}$
29. $\dfrac{a}{\sqrt{3b}}$ $\dfrac{a\sqrt{3b}}{3b}$

30. **ELECTRICITY** The amount of current in amperes I that an appliance uses can be calculated using the formula $I = \left(\frac{P}{R}\right)^{\frac{1}{2}}$, where P is the power in watts and R is the resistance in ohms. How much current does an appliance use if $P = 500$ watts and $R = 10$ ohms? Round your answer to the nearest tenth. **7.1 amps**

31. **BUSINESS** A company that produces DVDs uses the formula $C = 88n^{\frac{1}{3}} + 330$ to calculate the cost C in dollars of producing n DVDs per day. What is the company's cost to produce 150 DVDs per day? Round your answer to the nearest dollar. **\$798**

Reading to Learn Mathematics, p. 279 ELL

Pre-Activity **How do rational exponents apply to astronomy?**

Read the introduction to Lesson 5-7 at the top of page 257 in your textbook. The formula in the introduction contains the exponent $\frac{2}{5}$. What do you think it might mean to raise a number to the $\frac{2}{5}$ power?

Sample answer: Take the fifth root of the number and square it.

Reading the Lesson

1. Complete the following definitions of rational exponents.
   - For any real number b and for any positive integer n, $b^{\frac{1}{n}} =$ $\sqrt[n]{b}$ except when b < 0 and n is even.
   - For any nonzero real number b, and any integers m and n, with n > 1, $b^{\frac{m}{n}} =$ $\sqrt[n]{b^m}$ $=$ $(\sqrt[n]{b})^m$, except when b < 0 and n is even.
2. Complete the conditions that must be met in order for an expression with rational exponents to be simplified.
   - It has no negative exponents.
   - It has no fractional exponents in the denominator.
   - It is not a complex fraction.
   - The index of any remaining radical is the least number possible.
3. Margarita and Pierre were working together on their algebra homework. One exercise asked them to evaluate the expression $27^{\frac{4}{3}}$. Margarita thought that they should raise 27 to the fourth power first and then take the cube root of the result. Pierre thought that they should take the cube root of 27 first and then raise the result to the fourth power. Whose method is correct? **Both methods are correct.**

Helping You Remember

4. Some students have trouble remembering which part of the fraction in a rational exponent gives the power and which part gives the root. How can your knowledge of integer exponents help you to keep this straight? **Sample answer: An integer exponent can be written as a rational exponent. For example, $2^3 = 2^{\frac{3}{1}}$. You know that this means that 2 is raised to the third power, so the numerator must give the power, and, therefore, the denominator must give the root.**

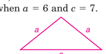
Enrichment, p. 280

Lesser-Known Geometric Formulas

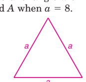
Many geometric formulas involve radical expressions.

Make a drawing to illustrate each of the formulas given on this page. Then evaluate the formula for the given value of the variable. Round answers to the nearest hundredth.

1. The area of an isosceles triangle. Two sides have length a; the other side has length c. Find A when $a = 6$ and $c = 7$.
   $A = \frac{c}{4}\sqrt{4a^2 - c^2}$
   $A \approx 17.06$



2. The area of an equilateral triangle with a side of length a. Find A when $a = 8$.
   $A = \frac{a^2}{4}\sqrt{3}$
   $A \approx 27.71$



3. The area of a regular pentagon with a side of length a. Find A when $a = 4$.

4. The area of a regular hexagon with a side of length a. Find A when $a = 9$.

4 Assess

Open-Ended Assessment

Speaking Have students write two expressions with rational exponents, one that is in simplified form and another that is not. Ask them to explain the difference between them, using the four conditions listed in the Concept Summary on p. 260.

Tips for New Teachers

Intervention In order to help students see why the exception "except when $b < 0$ and n is even" is necessary when defining rational exponents, ask them to choose values for b and n that violate these constraints, and see what results when applying the definition.

Getting Ready for Lesson 5-8

PREREQUISITE SKILL Lesson 5-8 presents solving equations and inequalities that contain radicals. Solving such equations and inequalities involves finding the power of an expression involving a radical. Exercises 81–84 should be used to determine your students' familiarity with multiplying radicals.

Assessment Options

Quiz (Lessons 5-6 and 5-7) is available on p. 308 of the *Chapter 5 Resource Masters*.

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65. Find the simplified form of $32^{\frac{1}{2}} + 3^{\frac{1}{2}} - 8^{\frac{1}{2}}$. $2^{\frac{3}{2}} + 3^{\frac{1}{2}}$

66. What is the simplified form of $81^{\frac{1}{3}} - 24^{\frac{1}{3}} + 3^{\frac{1}{3}}$? $2 \cdot 3^{\frac{1}{3}}$

More About. . .

Music

The first piano was made in about 1709 by Bartolomeo Cristofori, a maker of harpsichords in Florence, Italy.

Source: www.infoplease.com

MUSIC **For Exercises 67 and 68, use the following information.**
On a piano, the frequency of the A note above middle C should be set at 440 vibrations per second. The frequency f_n of a note that is n notes above that A should be $f_n = 440 \cdot 2^{\frac{n}{12}}$.

67. At what frequency should a piano tuner set the A that is one octave, or 12 notes, above the A above middle C? **880 vibrations per second**

68. Middle C is nine notes below the A that has a frequency of 440 vibrations per second. What is the frequency of middle C? **about 262 vibrations per second**

69. **BIOLOGY** Suppose a culture has 100 bacteria to begin with and the number of bacteria doubles every 2 hours. Then the number N of bacteria after t hours is given by $N = 100 \cdot 2^{\frac{t}{2}}$. How many bacteria will be present after 3 and a half hours? **about 336**

70. **CRITICAL THINKING** Explain how to solve $9^x = 3^{x + \frac{1}{2}}$ for x. **See margin.**

71. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 283A–283B.**

How do rational exponents apply to astronomy?

Include the following in your answer:

- an explanation of how to write the formula $r = D\left(\frac{M_p}{M_S}\right)^{\frac{2}{5}}$ in radical form and simplify it, and
- an explanation of what happens to the value of r as the value of D increases assuming that M_p and M_S are constant.

Standardized Test Practice

72. Which is the value of $4^{\frac{1}{2}} + \left(\frac{1}{2}\right)^4$? **C**

    (A) 1　(B) 2　(C) $2\frac{1}{16}$　(D) $2\frac{1}{2}$

73. If $4x + 2y = 5$ and $x - y = 1$, then what is the value of $3x + 3y$? **C**

    (A) 1　(B) 2　(C) 4　(D) 6

Maintain Your Skills

Mixed Review

Simplify. *(Lessons 5-5 and 5-6)*

74. $\sqrt{4x^3y^2}$ $2x|y|\sqrt{x}$

75. $(2\sqrt{6})(3\sqrt{12})$ $36\sqrt{2}$

76. $\sqrt{32} + \sqrt{18} - \sqrt{50}$ $2\sqrt{2}$

77. $\sqrt[4]{(-8)^4}$ 8

78. $4\sqrt{(x-5)^2}$ $4|x-5|$

79. $\sqrt{\frac{9}{36}x^4}$ $\frac{1}{2}x^2$

80. **BIOLOGY** Humans blink their eyes about once every 5 seconds. How many times do humans blink their eyes in two hours? *(Lesson 1-1)* **1440**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each power.** *(To review **multiplying radicals**, see Lesson 5-6.)*

81. $(\sqrt{x} - 2)^2$ $x - 2$

82. $(\sqrt[3]{2x} - 3)^3$ $2x - 3$

83. $(\sqrt{x} + 1)^2$ $x + 2\sqrt{x} + 1$

84. $(2\sqrt{x} - 3)^2$ $4x - 12\sqrt{x} + 9$

Answer

70. Rewrite the equation so that the bases are the same on each side.

$$9^x = 3^{x + \frac{1}{2}}$$
$$(3^2)^x = 3^{x + \frac{1}{2}}$$
$$3^{2x} = 3^{x + \frac{1}{2}}$$

Since the bases are the same and this is an equation, the exponents must be equal. Solve $2x = x + \frac{1}{2}$. The result is $x = \frac{1}{2}$.

5-8 Radical Equations and Inequalities

What You'll Learn

- Solve equations containing radicals.
- Solve inequalities containing radicals.

Vocabulary

- radical equation
- extraneous solution
- radical inequality

How do radical equations apply to manufacturing?

Computer chips are made from the element silicon, which is found in sand. Suppose a company that manufactures computer chips uses the formula $C = 10n^{\frac{2}{3}} + 1500$ to estimate the cost C in dollars of producing n chips. This equation can be rewritten as a radical equation.

SOLVE RADICAL EQUATIONS Equations with radicals that have variables in the radicands are called **radical equations**. To solve this type of equation, raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Example 1 Solve a Radical Equation

Solve $\sqrt{x+1} + 2 = 4$.

$\sqrt{x+1} + 2 = 4$	Original equation
$\sqrt{x+1} = 2$	Subtract 2 from each side to isolate the radical.
$(\sqrt{x+1})^2 = 2^2$	Square each side to eliminate the radical.
$x + 1 = 4$	Find the squares.
$x = 3$	Subtract 1 from each side.

CHECK

$\sqrt{x+1} + 2 = 4$	Original equation
$\sqrt{3+1} + 2 \stackrel{?}{=} 4$	Replace x with 3.
$4 = 4$ ✓	Simplify.

The solution checks. The solution is 3.

When you solve a radical equation, it is very important that you check your solution. Sometimes you will obtain a number that does not satisfy the original equation. Such a number is called an **extraneous solution**. You can use a graphing calculator to predict the number of solutions of an equation or to determine whether the solution you obtain is reasonable.

Example 2 Extraneous Solution

Solve $\sqrt{x-15} = 3 - \sqrt{x}$.

$\sqrt{x-15} = 3 - \sqrt{x}$	Original equation
$(\sqrt{x-15})^2 = (3 - \sqrt{x})^2$	Square each side.
$x - 15 = 9 - 6\sqrt{x} + x$	Find the squares.
$-24 = -6\sqrt{x}$	Isolate the radical.
$4 = \sqrt{x}$	Divide each side by -6.
$4^2 = (\sqrt{x})^2$	Square each side again.
$16 = x$	Evaluate the squares.

(continued on the next page)

5-8 Lesson Notes

1 Focus

5-Minute Check Transparency 5-8 Use as a quiz or review of Lesson 5-7.

Mathematical Background notes are available for this lesson on p. 220D.

How do radical equations apply to manufacturing?

Ask students:

- Why can the equation be rewritten as a radical equation? **because the variable *n* has a rational exponent**
- **Manufacturing** There are production costs associated with manufactured goods that occur even before the first item is made. That is, there is still a cost even if no items have been produced yet. How much are these costs for the production of this company's computer chips? **$1500**

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 281–282
- Skills Practice, p. 283
- Practice, p. 284
- Reading to Learn Mathematics, p. 285
- Enrichment, p. 286

Transparencies

5-Minute Check Transparency 5-8
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 9
Interactive Chalkboard

2 Teach

SOLVE RADICAL EQUATIONS

In-Class Examples

Power Point®

1 Solve $\sqrt{y - 2} - 1 = 5$. **38**

2 Solve $\sqrt{x - 12} = 2 - \sqrt{x}$. **no solution**

Teaching Tip Remind students that the square root sign in an equation means the principal root.

3 Solve $(3y + 1)^{\frac{1}{3}} + 5 = 0$. **−42**

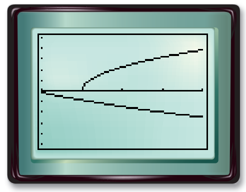
Teaching Tip Have a discussion with students about which operations may introduce extraneous solutions when solving a radical equation.

CHECK

$$\sqrt{x - 15} = 3 - \sqrt{x}$$
$$\sqrt{16 - 15} \stackrel{?}{=} 3 - \sqrt{16}$$
$$\sqrt{1} \stackrel{?}{=} 3 - 4$$
$$1 \neq -1$$

The solution does not check, so the equation has no real solution.

The graphing calculator screen shows the graphs of $y = \sqrt{x - 15}$ and $y = 3 - \sqrt{x}$. The graphs do not intersect, which confirms that there is no solution.



[10, 30] scl: 5 by [−5, 5] scl: 1

You can apply the same methods used in solving square root equations to solving equations with roots of any index. Remember that to undo a square root, you square the expression. To undo an nth root, you must raise the expression to the nth power.

Study Tip

Alternative Method

To solve a radical equation, you can substitute a variable for the radical expression. In Example 3, let $A = 5n - 1$.

$$3A^{\frac{1}{3}} - 2 = 0$$
$$3A^{\frac{1}{3}} = 2$$
$$A^{\frac{1}{3}} = \frac{2}{3}$$
$$A = \frac{8}{27}$$
$$5n - 1 = \frac{8}{27}$$
$$n = \frac{7}{27}$$

Example 3 Cube Root Equation

Solve $3(5n - 1)^{\frac{1}{3}} - 2 = 0$.

In order to remove the $\frac{1}{3}$ power, or cube root, you must first isolate it and then raise each side of the equation to the third power.

$3(5n - 1)^{\frac{1}{3}} - 2 = 0$	Original equation
$3(5n - 1)^{\frac{1}{3}} = 2$	Add 2 to each side.
$(5n - 1)^{\frac{1}{3}} = \frac{2}{3}$	Divide each side by 3.
$\left[(5n - 1)^{\frac{1}{3}}\right]^3 = \left(\frac{2}{3}\right)^3$	Cube each side.
$5n - 1 = \frac{8}{27}$	Evaluate the cubes.
$5n = \frac{35}{27}$	Add 1 to each side.
$n = \frac{7}{27}$	Divide each side by 5.

CHECK

$3(5n - 1)^{\frac{1}{3}} - 2 = 0$	Original equation
$3\left(5 \cdot \frac{7}{27} - 1\right)^{\frac{1}{3}} - 2 \stackrel{?}{=} 0$	Replace n with $\frac{7}{27}$.
$3\left(\frac{8}{27}\right)^{\frac{1}{3}} - 2 \stackrel{?}{=} 0$	Simplify.
$3\left(\frac{2}{3}\right) - 2 \stackrel{?}{=} 0$	The cube root of $\frac{8}{27}$ is $\frac{2}{3}$.
$0 = 0$ ✓	Subtract.

The solution is $\frac{7}{27}$.

SOLVE RADICAL INEQUALITIES You can use what you know about radical equations to help solve radical inequalities. A **radical inequality** is an inequality that has a variable in a radicand.

264 Chapter 5 Polynomials

Example 4 Radical Inequality

Solve $2 + \sqrt{4x - 4} \le 6$.

Since the radicand of a square root must be greater than or equal to zero, first solve $4x - 4 \ge 0$ to identify the values of x for which the left side of the given inequality is defined.

$$4x - 4 \ge 0$$
$$4x \ge 4$$
$$x \ge 1$$

Now solve $2 + \sqrt{4x - 4} \le 6$.

$2 + \sqrt{4x - 4} \le 6$	Original inequality
$\sqrt{4x - 4} \le 4$	Isolate the radical.
$4x - 4 \le 16$	Eliminate the radical.
$4x \le 20$	Add 4 to each side.
$x \le 5$	Divide each side by 4.

It appears that $1 \le x \le 5$. You can test some x values to confirm the solution. Let $f(x) = 2 + \sqrt{4x - 4}$. Use three test values: one less than 1, one between 1 and 5, and one greater than 5. Organize the test values in a table.

$x = 0$	$x = 2$	$x = 7$
$f(0) = 2 + \sqrt{4(0) - 4}$ $= 2 + \sqrt{-4}$ Since $\sqrt{-4}$ is not a real number, the inequality is not satisfied.	$f(2) = 2 + \sqrt{4(2) - 4}$ $= 4$ Since $4 \le 6$, the inequality is satisfied.	$f(7) = 2 + \sqrt{4(7) - 4}$ ≈ 6.90 Since $6.90 \not\le 6$, the inequality is not satisfied.

The solution checks. Only values in the interval $1 \le x \le 5$ satisfy the inequality. You can summarize the solution with a number line.

Study Tip

Check Your Solution
You may also want to use a graphing calculator to check. Graph each side of the original inequality and examine the intersection.

Concept Summary — Solving Radical Inequalities

To solve radical inequalities, complete the following steps.

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.

Step 2 Solve the inequality algebraically.

Step 3 Test values to check your solution.

Check for Understanding

Concept Check

1. **Explain** why you do not have to square each side to solve $2x + 1 = \sqrt{3}$. Then solve the equation. **See margin.**
2. **Show** how to solve $x - 6\sqrt{x} + 9 = 0$ by factoring. Name the properties of equality that you use. **See margin.**
3. **OPEN ENDED** Write an equation containing two radicals for which 1 is a solution. **Sample answer: $\sqrt{x} + \sqrt{x + 3} = 3$**

www.algebra2.com/extra_examples

SOLVE RADICAL INEQUALITIES

In-Class Example PowerPoint®

4 Solve $\sqrt{3x - 6} + 4 \le 7$. **$2 \le x \le 5$**

Teaching Tip Emphasize the importance of checking key test values in the appropriate ranges.

3 Practice/Apply

Study Notebook

Have students—

- *add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 5.*
- *write a list of the steps for solving radical equations and copy the list of steps for solving radical inequalities given in the Concept Summary on p. 265.*
- *include any other item(s) that they find helpful in mastering the skills in this lesson.*

Answers

1. Since x is not under the radical, the equation is a linear equation, not a radical equation. The solution is $\frac{\sqrt{3} - 1}{2}$.
2. The trinomial is a perfect square in terms of $\sqrt{x}$. $x - 6\sqrt{x} + 9 = (\sqrt{x} - 3)^2$, so the equation can be written as $(\sqrt{x} - 3)^2 = 0$. Take the square root of each side to get $\sqrt{x} - 3 = 0$. Use the Addition Property of Equality to add 3 to each side, then square each side to get $x = 9$.

DAILY INTERVENTION — Differentiated Instruction

Logical Have students compare solving radical equations and inequalities to solving other types of equations and inequalities. Have them write or give a short presentation about the similarities and differences between the procedures used in the solution processes.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9, 12	1–3
10, 11	4

Solve each equation or inequality.

4. $\sqrt{4x+1} = 3$ **2**

5. $4 - (7 - y)^{\frac{1}{2}} = 0$ **−9**

6. $1 + \sqrt{x+2} = 0$ **no solution**

7. $\sqrt{z-6} - 3 = 0$ **15**

8. $\frac{1}{6}(12a)^{\frac{1}{3}} = 1$ **18**

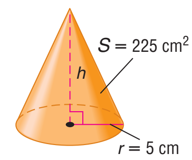
9. $\sqrt[3]{x-4} = 3$ **31**

10. $\sqrt{2x+3} - 4 \le 5$ $-\frac{3}{2} \le x \le 39$

11. $\sqrt{b+12} - \sqrt{b} > 2$ $0 \le b < 4$

Application

12. GEOMETRY The surface area S of a cone can be found by using $S = \pi r\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height of the cone. Find the height of the cone. **about 13.42 cm**



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
13–24, 29–32, 37–42	1–3
25–28, 33–36	4

Extra Practice

See page 839.

Solve each equation or inequality.

13. $\sqrt{x} = 4$ **16**

14. $\sqrt{y} - 7 = 0$ **49**

15. $a^{\frac{1}{2}} + 9 = 0$ **no solution**

16. $2 + 4z^{\frac{1}{2}} = 0$ **no solution**

17. $\sqrt[3]{c-1} = 2$ **9**

18. $\sqrt[3]{5m+2} = 3$ **5**

19. $7 + \sqrt{4x+8} = 9$ **−1**

20. $5 + \sqrt{4y-5} = 12$ $\frac{27}{2}$

21. $(6n-5)^{\frac{1}{3}} + 3 = -2$ **−20**

22. $(5x+7)^{\frac{1}{5}} + 3 = 5$ **5**

23. $\sqrt{x-5} = \sqrt{2x-4}$ **no solution**

24. $\sqrt{2t-7} = \sqrt{t+2}$ **9**

25. $1 + \sqrt{7x-3} > 3$ $x > 1$

26. $\sqrt{3x+6} + 2 \le 5$ $-2 \le x \le 1$

27. $-2 + \sqrt{9-5x} \ge 6$ $x \le -11$

28. $6 - \sqrt{2y+1} < 3$ $y > 4$

★ **29.** $\sqrt{x-6} - \sqrt{x} = 3$ **no solution**

★ **30.** $\sqrt{y+21} - 1 = \sqrt{y+12}$ **4**

★ **31.** $\sqrt{b+1} = \sqrt{b+6} - 1$ **3**

★ **32.** $\sqrt{4z+1} = 3 + \sqrt{4z-2}$ **no solution**

★ **33.** $\sqrt{2} - \sqrt{x+6} \le -\sqrt{x}$ $0 \le x \le 2$

★ **34.** $\sqrt{a+9} - \sqrt{a} > \sqrt{3}$ $0 \le a < 3$

★ **35.** $\sqrt{b-5} - \sqrt{b+7} \le 4$ $b \ge 5$

★ **36.** $\sqrt{c+5} + \sqrt{c+10} > 2.5$ $c > -\frac{79}{16}$

37. What is the solution of $2 - \sqrt{x+6} = -1$? **3**

38. Solve $\sqrt{2x+4} - 4 = 2$. **16**

39. CONSTRUCTION The minimum depth d in inches of a beam required to support a load of s pounds is given by the formula $d = \sqrt{\frac{s\ell}{576w}}$, where ℓ is the length of the beam in feet and w is the width in feet. Find the load that can be supported by a board that is 25 feet long, 2 feet wide, and 5 inches deep. **1152 lb**

40. AEROSPACE ENGINEERING The radius r of the orbit of a satellite is given by $r = \sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of the central object, and t is the time it takes the satellite to complete one orbit. Solve this formula for t. $t = \sqrt{\frac{4\pi^2 r^3}{GM}}$

Study Guide and Intervention, p. 281 (shown) and p. 282

Solve Radical Equations The following steps are used in solving equations that have variables in the radicand. Some algebraic procedures may be needed before you use these steps.

Step 1 Isolate the radical on one side of the equation.
Step 2 To eliminate the radical, raise each side of the equation to a power equal to the index of the radical.
Step 3 Solve the resulting equation.
Step 4 Check your solution in the original equation to make sure that you have not obtained any extraneous roots.

Example 1 Solve $2\sqrt{4x+8} - 4 = 8$.

$2\sqrt{4x+8} - 4 = 8$ Original equation
$2\sqrt{4x+8} = 12$ Add 4 to each side.
$\sqrt{4x+8} = 6$ Isolate the radical.
$4x + 8 = 36$ Square each side.
$4x = 28$ Subtract 8 from each side.
$x = 7$ Divide each side by 4.

Check
$2\sqrt{4(7)+8} - 4 \stackrel{?}{=} 8$
$2\sqrt{36} - 4 \stackrel{?}{=} 8$
$2(6) - 4 \stackrel{?}{=} 8$
$8 = 8$
The solution $x = 7$ checks.

Example 2 Solve $\sqrt{3x+1} = \sqrt{5x} - 1$.

$\sqrt{3x+1} = \sqrt{5x} - 1$ Original equation
$3x + 1 = 5x - 2\sqrt{5x} + 1$ Square each side.
$2\sqrt{5x} = 2x$ Simplify.
$\sqrt{5x} = x$ Isolate the radical.
$5x = x^2$ Square each side.
$x^2 - 5x = 0$ Subtract 5x from each side.
$x(x-5) = 0$ Factor.
$x = 0$ or $x = 5$

Check
$\sqrt{3(0)+1} = 1$, but $\sqrt{5(0)} - 1 = -1$, so 0 is not a solution.
$\sqrt{3(5)+1} = 4$, and $\sqrt{5(5)} - 1 = 4$, so the solution is $x = 5$.

Exercises

Solve each equation.

1. $3 + 2x\sqrt{3} = 5$ $\frac{\sqrt{3}}{3}$
2. $2\sqrt{3x+4} + 1 = 15$ **15**
3. $8 + \sqrt{x+1} = 2$ **no solution**
4. $\sqrt{5-x} - 4 = 6$ **−95**
5. $12 + \sqrt{2x-1} = 4$ **no solution**
6. $\sqrt{12-x} = 0$ **12**
7. $\sqrt{21} - \sqrt{5x-4} = 0$ **5**
8. $10 - \sqrt{2x} = 5$ **12.5**
9. $\sqrt{x^2+7x} = \sqrt{7x-9}$ **no solution**
10. $4\sqrt[3]{2x+11} - 2 = 10$ **8**
11. $2\sqrt{x+11} = \sqrt{x+2} + \sqrt{3x-6}$ **14**
12. $\sqrt{9x-11} = x + 1$ **3, 4**

Skills Practice, p. 283 and Practice, p. 284 (shown)

Solve each equation or inequality.

1. $\sqrt{x} = 8$ **64**
2. $4 - \sqrt{x} = 3$ **1**
3. $\sqrt{2p} + 3 = 10$ $\frac{49}{2}$
4. $4\sqrt{3h} - 2 = 0$ $\frac{1}{12}$
5. $c^{\frac{1}{2}} + 6 = 9$ **9**
6. $18 + 7h^{\frac{1}{2}} = 12$ **no solution**
7. $\sqrt[3]{d+2} = 7$ **341**
8. $\sqrt[5]{w-7} = 1$ **8**
9. $6 + \sqrt[3]{q-4} = 9$ **31**
10. $\sqrt[4]{y-9} + 4 = 0$ **no solution**
11. $\sqrt{2m-6} - 16 = 0$ **131**
12. $\sqrt[3]{4m+1} - 2 = 2$ $\frac{63}{4}$
13. $\sqrt{8n-5} - 1 = 2$ $\frac{7}{4}$
14. $\sqrt{1-4t} - 8 = -6$ $-\frac{3}{4}$
15. $\sqrt{2t-5} - 3 = 3$ $\frac{41}{2}$
16. $(7v-2)^{\frac{1}{4}} + 12 = 7$ **no solution**
17. $(3g+1)^{\frac{1}{2}} - 6 = 4$ **33**
18. $(6u-5)^{\frac{1}{3}} + 2 = -3$ **−20**
19. $\sqrt{2d-5} = \sqrt{d-1}$ **4**
20. $\sqrt{4r-6} = \sqrt{r}$ **2**
21. $\sqrt{6x-4} = \sqrt{2x+10}$ $\frac{7}{2}$
22. $\sqrt{2x+5} = \sqrt{2x+1}$ **no solution**
23. $3\sqrt{a} \ge 12$ $a \ge 16$
24. $\sqrt{z+5} + 4 \le 13$ $-5 \le z \le 76$
25. $8 + \sqrt{2q} \le 5$ **no solution**
26. $\sqrt{2a-3} < 5$ $\frac{3}{2} < a < 14$
27. $9 - \sqrt{c+4} \le 6$ $c \ge 5$
28. $\sqrt[3]{x-1} < -2$ $x < -7$

29. STATISTICS Statisticians use the formula $\sigma = \sqrt{v}$ to calculate a standard deviation σ, where v is the variance of a data set. Find the variance when the standard deviation is 15. **225**

30. GRAVITATION Helena drops a ball from 25 feet above a lake. The formula $t = \frac{1}{4}\sqrt{25-h}$ describes the time t in seconds that the ball is h feet above the water. How many feet above the water will the ball be after 1 second? **9 ft**

Reading to Learn Mathematics, p. 285 ELL

Pre-Activity How do radical equations apply to manufacturing?

Read the introduction to Lesson 5-8 at the top of page 263 in your textbook.

Explain how you would use the formula in your textbook to find the cost of producing 125,000 computer chips. (Describe the steps of the calculation in the order in which you would perform them, but do not actually do the calculation.) **Sample answer: Raise 125,000 to the $\frac{2}{3}$ power by taking the cube root of 125,000 and squaring the result (or raise 125,000 to the $\frac{2}{3}$ power by entering 125,000 ^ (2/3) on a calculator). Multiply the number you get by 10 and then add 1500.**

Reading the Lesson

1. a. What is an *extraneous solution* of a radical equation? **Sample answer: a number that satisfies an equation obtained by raising both sides of the original equation to a higher power but does not satisfy the original equation**

b. Describe two ways you can check the proposed solutions of a radical equation in order to determine whether any of them are extraneous solutions. **Sample answer: One way is to check each proposed solution by substituting it into the original equation. Another way is to use a graphing calculator to graph both sides of the original equation. See where the graphs intersect. This can help you identify solutions that may be extraneous.**

2. Complete the steps that should be followed in order to solve a radical inequality.

Step 1 If the **index** of the root is **even**, identify the values of the variable for which the **radicand** is **nonnegative**.

Step 2 Solve the **inequality** algebraically.

Step 3 Test **values** to check your solution.

Helping You Remember

3. One way to remember something is to explain it to another person. Suppose that your friend Leora thinks that she does not need to check her solutions to radical equations by substitution because she knows she is very careful and seldom makes mistakes in her work. How can you explain to her that she should nevertheless check every proposed solution in the original equation? **Sample answer: Squaring both sides of an equation can produce an equation that is not equivalent to the original one. For example, the only solution of $x = 5$ is 5, but the squared equation $x^2 = 25$ has two solutions, 5 and −5.**

Enrichment, p. 286

Truth Tables

In mathematics, the basic operations are addition, subtraction, multiplication, division, finding a root, and raising to a power. In logic, the basic operations are the following: *not* (~), *and* (∧), *or* (∨), and *implies* (→).

If P and Q are statements, then $\sim P$ means not P; $\sim Q$ means not Q; $P \wedge Q$ means P and Q; $P \vee Q$ means P or Q; and $P \to Q$ means P implies Q. The operations are defined by truth tables. On the left below is the truth table for the statement $\sim P$. Notice that there are two possible conditions for P, true (T) or false (F). If P is true, $\sim P$ is false; if P is false, $\sim P$ is true. Also shown are the truth tables for $P \wedge Q$, $P \vee Q$, and $P \to Q$.

P	~P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

You can use this information to find out under what conditions a complex

More About. . .

Health

A ponderal index p is a measure of a person's body based on height h in meters and mass m in kilograms. One such formula is $p = \frac{\sqrt[3]{m}}{h}$.

Source: *A Dictionary of Food and Nutrition*

41. PHYSICS When an object is dropped from the top of a 50-foot tall building, the object will be h feet above the ground after t seconds, where $\frac{\sqrt{50 - h}}{4} = t$. How far above the ground will the object be after 1 second? **34 ft**

42. HEALTH **Use the information about health at the left.** A 70-kilogram person who is 1.8 meters tall has a ponderal index of about 2.29. How much weight could such a person gain and still have an index of at most 2.5? **21.125 kg**

43. CRITICAL THINKING Explain how you know that $\sqrt{x + 2} + \sqrt{2x - 3} = -1$ has no real solution without trying to solve it. **See margin.**

44. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 283A–283B.**

How do radical equations apply to manufacturing?

Include the following in your answer:
- the equation $C = 10n^{\frac{2}{3}} + 1500$ rewritten as a radical equation, and
- a step-by-step explanation of how to determine the maximum number of chips the company could make for $10,000.

Standardized Test Practice

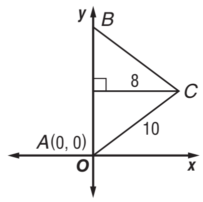
45. If $\sqrt{x + 5} + 1 = 4$, what is the value of x? **D**

(A) -4 (B) 0 (C) 2 (D) 4

46. Side $\overline{AC}$ of triangle ABC contains which of the following points? **C**

(A) (3, 4) (B) (3, 5) (C) (4, 3)

(D) (4, 5) (E) (4, 6)



Maintain Your Skills

Mixed Review

Write each radical using rational exponents. *(Lesson 5-7)*

47. $\sqrt[7]{5^3}$ $5^{\frac{3}{7}}$ **48.** $\sqrt{x + 7}$ $(x + 7)^{\frac{1}{2}}$ **49.** $(\sqrt[3]{x^2 + 1})^2$ $(x^2 + 1)^{\frac{2}{3}}$

Simplify. *(Lesson 5-6)*

50. $\sqrt{72x^6y^3}$ $6|x^3|y\sqrt{2y}$ **51.** $\frac{1}{\sqrt[3]{10}}$ $\frac{\sqrt[3]{100}}{10}$ **52.** $(5 - \sqrt{3})^2$ $28 - 10\sqrt{3}$

53. BUSINESS A dry cleaner ordered 7 drums of two different types of cleaning fluid. One type cost $30 per drum, and the other type cost $20 per drum. The total cost was $160. How much of each type of fluid did the company order? Write a system of equations and solve by graphing. *(Lesson 3-1)*
$x + y = 7$, $30x + 20y = 160$; **See margin for graph; (2, 5).**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression.**
(To review ***binomials****, see Lesson 5-2.)*

54. $(5 + 2x) + (-1 - x)$ $4 + x$ **55.** $(-3 - 2y) + (4 + y)$ $1 - y$

56. $(4 + x) - (2 - 3x)$ $2 + 4x$ **57.** $(-7 - 3x) - (4 - 3x)$ -11

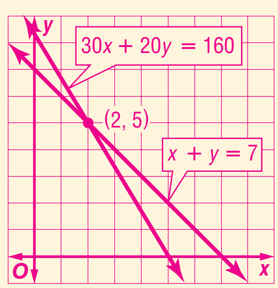
58. $(1 + z)(4 + 2z)$ $4 + 6z + 2z^2$ **59.** $(-3 - 4x)(1 + 2x)$ $-3 - 10x - 8x^2$

www.algebra2.com/self_check_quiz

Answers

43. Since $\sqrt{x + 2} \geq 0$ and $\sqrt{2x - 3} \geq 0$, the left side of the equation is nonnegative. Therefore, the left side of the equation cannot equal -1. Thus, the equation has no solution.

53.



About the Exercises...

Organization by Objective
- **Solve Radical Equations:** 13–24, 29–32, 37–42
- **Solve Radical Inequalities:** 25–28, 33–36

Odd/Even Assignments
Exercises 13–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–27 odd, 37–41 odd, 43–59

Average: 13–41 odd, 43–59

Advanced: 14–42 even, 43–53 (optional: 54–59)

4 Assess

Open-Ended Assessment

Writing Have students write a list of examples showing how to solve each of the different types of radical equations and inequalities discussed in this lesson.

Tips for New Teachers

Intervention Make sure that students know the constraints on the values of the variables in a radical equation so that the solutions are real numbers.

Getting Ready for Lesson 5-9

PREREQUISITE SKILL Lesson 5-9 presents calculating with complex numbers, often written in the form of binomials. Exercises 54–59 should be used to determine your students' familiarity with adding, subtracting, and multiplying binomials.

Graphing Calculator Investigation

A Follow-Up of Lesson 5-8

Getting Started

Know Your Calculator The TI-83 Plus automatically supplies a left parenthesis after each radical sign. When functions are entered on the Y= list, it is important to supply right parentheses as needed to ensure correct graphs and correct numerical results.

Displaying Tables In Step 2 on p. 268, students should check to be sure that the AUTO option has been selected on each of the last two lines of the TABLE SETUP screen.

Exact Solutions The approximate zero displayed on the screen shown in Step 4 on p. 268 appears to be a repeating decimal. If you go to the home screen immediately after Step 4 and use the keystrokes [X,T,θ,n] [MATH] 1 [ENTER], the calculator will display a fraction for the exact solution, $\frac{49}{36}$. The calculator can be used to verify that this is indeed the exact solution of the equation.

Teach

- After reading the sentence at the top of p. 269, have students solve the radical equation on p. 268 again treating each side as a separate function. Point out that the right side will simply be graphed as the function $y = 3$.
- After completing the discussion of the procedure on p. 269 for solving a radical inequality, have students solve it again by first subtracting $2\sqrt{x}$ from both sides and then graphing the function $y = \sqrt{x+2} + 1 - 2\sqrt{x}$. Point out that the portion of the graph below the x-axis shows the solution.
- Have students complete Exercises 1–10.

Graphing Calculator

A Follow-Up of Lesson 5-8

Solving Radical Equations and Inequalities by Graphing

You can use a TI-83 Plus to solve radical equations and inequalities. One way to do this is by rewriting the equation or inequality so that one side is 0 and then using the zero feature on the calculator.

Solve $\sqrt{x} + \sqrt{x+2} = 3$.

Step 1 *Rewrite the equation.*

- Subtract 3 from each side of the equation to obtain $\sqrt{x} + \sqrt{x+2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x+2} - 3$ in the Y= list.

KEYSTROKES: *Review entering a function on page 128.*

Step 2 *Use a table.*

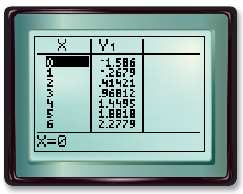
- You can use the TABLE function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

KEYSTROKES: [2nd] [TBLSET] 0 [ENTER] 1 [ENTER]

Step 3 *Estimate the solution.*

- Complete the table and estimate the solution(s).

KEYSTROKES: [2nd] [TABLE]

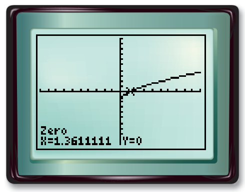


Since the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

Step 4 *Use the zero feature.*

- Graph, then select zero from the CALC menu.

KEYSTROKES: [GRAPH] [2nd] [CALC] 2



[−10, 10] scl: 1 by [−10, 10] scl: 1

Place the cursor to the left of the zero and press [ENTER] for the Left Bound. Then place the cursor to the right of the zero and press [ENTER] for the Right Bound. Press [ENTER] to solve.

The solution is about 1.36. This agrees with the estimate made by using the TABLE.

www.algebra2.com/other_calculator_keystrokes

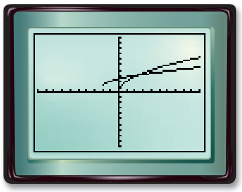
Investigation

Instead of rewriting an equation or inequality so that one side is 0, you can also treat each side of the equation or inequality as a separate function and graph both.

Solve $2\sqrt{x} > \sqrt{x+2} + 1$.

Step 1 *Graph each side of the inequality.*

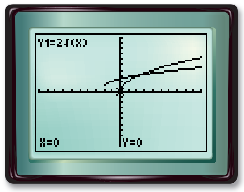
- In the Y= list, enter $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$. Then press GRAPH.



[−10, 10] scl: 1 by [−10, 10] scl: 1

Step 2 *Use the trace feature.*

- Press TRACE. You can use ▲ or ▼ to switch the cursor between the two curves.



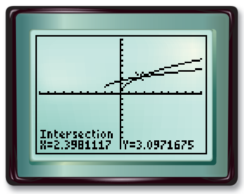
[−10, 10] scl: 1 by [−10, 10] scl: 1

The calculator screen above shows that, for points to the left of where the curves cross, $Y_1 < Y_2$ or $2\sqrt{x} < \sqrt{x+2} + 1$. To solve the original inequality, you must find points for which $Y_1 > Y_2$. These are the points to the right of where the curves cross.

Step 3 *Use the intersect feature.*

- You can use the INTERSECT feature on the CALC menu to approximate the x-coordinate of the point at which the curves cross.

  KEYSTROKES: 2nd [CALC] 5
- Press ENTER for each of First curve?, Second curve?, and Guess?.



[−10, 10] scl: 1 by [−10, 10] scl: 1

The calculator screen shows that the x-coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about $x > 2.40$. *Use the symbol > instead of ≥ in the solution because the symbol in the original inequality is >.*

Exercises 4. about 3.89 5. about 2.52 8. about $0 \le x < 1$ 9. about $1 \le x < 4.52$

Solve each equation or inequality.

1. $\sqrt{x+4} = 3$ **5**
2. $\sqrt{3x-5} = 1$ **2**
3. $\sqrt{x+5} = \sqrt{3x+4}$ **0.5**
4. $\sqrt{x+3} + \sqrt{x-2} = 4$
5. $\sqrt{3x-7} = \sqrt{2x-2} - 1$
6. $\sqrt{x+8} - 1 = \sqrt{x+2}$ **4.25**
7. $\sqrt{x-3} \ge 2$ **$x \ge 7$**
8. $\sqrt{x+3} > 2\sqrt{x}$
9. $\sqrt{x} + \sqrt{x-1} < 4$
10. Explain how you could apply the technique in the first example to solving an inequality. **See margin.**

Assess

- When examining the table of values for the equation on p. 268, why do you know a solution lies between the two x values where the graphed function changes signs? **Sample answer: Since the original equation was solved so that one side was zero, the function representing the other side has a value of 0 when the value of y is 0. That must occur for a value of x somewhere between two values of x whose corresponding values of y have different signs (because 0 lies between the positive numbers and the negative numbers).**

Answer

10. **Rewrite the inequality so that one side is 0. Then graph the other side and find the x values for which the graph is above or below the x-axis, according to the inequality symbol. Use the zero feature to approximate the x-coordinate of the point at which the graph crosses the x-axis.**

5-9 Lesson Notes

1 Focus

5-Minute Check Transparency 5-9 Use as a quiz or a review of Lesson 5-8.

Mathematical Background notes are available for this lesson on p. 220D.

How do complex numbers apply to polynomial equations?

Ask students:

- If by definition $i^2 = -1$, then what do you think is the value of i^4? Justify your answer. **Since $i^4 = (i^2)^2$, replacing i^2 with -1 gives $i^4 = (-1)^2$ or 1.**

2 Teach

ADD AND SUBTRACT COMPLEX NUMBERS

In-Class Examples Power Point®

1 Simplify.

a. $\sqrt{-28}$ $2i\sqrt{7}$

b. $\sqrt{-32y^3}$ $4i|y|\sqrt{2y}$

2 Simplify.

a. $-3i \cdot 2i$ 6

b. $\sqrt{-12} \cdot \sqrt{-2}$ $-2\sqrt{6}$

3 Simplify i^{35}. $-i$

5-9 Complex Numbers

What You'll Learn

- Add and subtract complex numbers.
- Multiply and divide complex numbers.

How do complex numbers apply to polynomial equations?

Consider the equation $2x^2 + 2 = 0$. If you solve this equation for x^2, the result is $x^2 = -1$. Since there is no real number whose square is -1, the equation has no real solutions. French mathematician René Descartes (1596–1650) proposed that a number i be defined such that $i^2 = -1$.

Vocabulary

- imaginary unit
- pure imaginary number
- complex number
- absolute value
- complex conjugates

ADD AND SUBTRACT COMPLEX NUMBERS Since i is defined to have the property that $i^2 = -1$, the number i is the principal square root of -1; that is, $i = \sqrt{-1}$. i is called the **imaginary unit**. Numbers of the form $3i$, $-5i$, and $i\sqrt{2}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2}$ or bi.

Study Tip

Reading Math

i is usually written before radical symbols to make it clear that it is not under the radical.

Example 1 Square Roots of Negative Numbers

Simplify.

a. $\sqrt{-18}$

$\sqrt{-18} = \sqrt{-1 \cdot 3^2 \cdot 2}$

$= \sqrt{-1} \cdot \sqrt{3^2} \cdot \sqrt{2}$

$= i \cdot 3 \cdot \sqrt{2}$ or $3i\sqrt{2}$

b. $\sqrt{-125x^5}$

$\sqrt{-125x^5} = \sqrt{-1 \cdot 5^2 \cdot x^4 \cdot 5x}$

$= \sqrt{-1} \cdot \sqrt{5^2} \cdot \sqrt{x^4} \cdot \sqrt{5x}$

$= i \cdot 5 \cdot x^2 \cdot \sqrt{5x}$ or $5ix^2\sqrt{5x}$

TEACHING TIP

Point out that when multiplying radicals with negative radicands, students should first take the roots, then multiply. Otherwise, their answers may be off by a factor of -1.

The Commutative and Associative Properties of Multiplication hold true for pure imaginary numbers.

Example 2 Multiply Pure Imaginary Numbers

Simplify.

a. $-2i \cdot 7i$

$-2i \cdot 7i = -14i^2$

$= -14(-1)$ $i^2 = -1$

$= 14$

b. $\sqrt{-10} \cdot \sqrt{-15}$

$\sqrt{-10} \cdot \sqrt{-15} = i\sqrt{10} \cdot i\sqrt{15}$

$= i^2\sqrt{150}$

$= -1 \cdot \sqrt{25} \cdot \sqrt{6}$

$= -5\sqrt{6}$

You can use the properties of powers to help simplify powers of i.

Example 3 Simplify a Power of i

Simplify i^{45}.

$i^{45} = i \cdot i^{44}$ Multiplying powers

$= i \cdot (i^2)^{22}$ Power of a Power

$= i \cdot (-1)^{22}$ $i^2 = -1$

$= i \cdot 1$ or i $(-1)^{22} = 1$

Resource Manager

Workbook and Reproducible Masters

Chapter 5 Resource Masters

- Study Guide and Intervention, pp. 287–288
- Skills Practice, p. 289
- Practice, p. 290
- Reading to Learn Mathematics, p. 291
- Enrichment, p. 292
- Assessment, p. 308

School-to-Career Masters, p. 10

Teaching Algebra With Manipulatives Masters, pp. 239, 240

Transparencies

5-Minute Check Transparency 5-9

Answer Key Transparencies

Technology

Interactive Chalkboard

The solutions of some equations involve pure imaginary numbers.

Example 4 Equation with Imaginary Solutions

Solve $3x^2 + 48 = 0$.

$3x^2 + 48 = 0$ — Original equation

$3x^2 = -48$ — Subtract 48 from each side.

$x^2 = -16$ — Divide each side by 3.

$x = \pm\sqrt{-16}$ — Take the square root of each side.

$x = \pm 4i$ — $\sqrt{-16} = \sqrt{16} \cdot \sqrt{-1}$

Study Tip

Quadratic Solutions

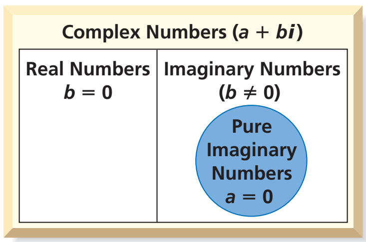
Quadratic equations always have complex solutions. If the discriminant is:
- negative, there are two imaginary roots,
- zero, there are two equal real roots, or
- positive, there are two unequal real roots.

Consider an expression such as $5 + 2i$. Since 5 is a real number and $2i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

Key Concept — Complex Numbers

- **Words** A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.
- **Examples** $7 + 4i$ and $2 - 6i = 2 + (-6)i$

The Venn diagram at the right shows the complex number system.
- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

Example 5 Equate Complex Numbers

Find the values of x and y that make the equation $2x - 3 + (y - 4)i = 3 + 2i$ true.

Set the real parts equal to each other and the imaginary parts equal to each other.

$2x - 3 = 3$ — Real parts

$2x = 6$ — Add 3 to each side.

$x = 3$ — Divide each side by 2.

$y - 4 = 2$ — Imaginary parts

$y = 6$ — Add 4 to each side.

Study Tip

Reading Math

The form $a + bi$ is sometimes called the **standard form** of a complex number.

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

4 Solve $5y^2 + 20 = 0$. $\pm 2i$

Teaching Tip Make sure students understand that when they take the square root of both sides of an equation, they must use the ± symbol in front of the radical sign.

5 Find the values of x and y that make the equation $2x + yi = -14 - 3i$ true. $x = -7$, $y = -3$

Teaching Tip Emphasize that two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

DAILY INTERVENTION — Differentiated Instruction — ELL

Verbal/Linguistic Have students write poems about the imaginary number i and the repeating values of its powers, perhaps including wordplay with the terms *real* and *imaginary*. The content of the poems should be helpful for remembering the mathematical characteristics of i.

In-Class Example

Power Point®

6 Simplify.

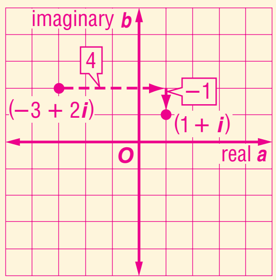
a. $(3 + 5i) + (2 - 4i)$ **$5 + i$**

b. $(4 - 6i) - (3 - 7i)$ **$1 + i$**

Answers

Algebra Activity

1.



2. Rewrite the difference as a sum, $(-3 + 2i) - (4 - i) = (-3 + 2i) + (-4 + i)$. Then apply the method discussed in this activity.

To add or subtract complex numbers, combine like terms. That is, combine the real parts and combine the imaginary parts.

Example 6 Add and Subtract Complex Numbers

Simplify.

a. $(6 - 4i) + (1 + 3i)$

$(6 - 4i) + (1 + 3i) = (6 + 1) + (-4 + 3)i$ Commutative and Associative Properties

$= 7 - i$ Simplify.

b. $(3 - 2i) - (5 - 4i)$

$(3 - 2i) - (5 - 4i) = (3 - 5) + [-2 - (-4)]i$ Commutative and Associative Properties

$= -2 + 2i$ Simplify.

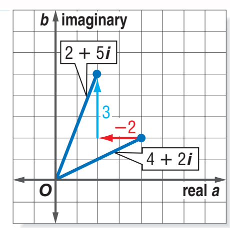
You can model the addition and subtraction of complex numbers geometrically.

Algebra Activity

Adding Complex Numbers

You can model the addition of complex numbers on a coordinate plane. The horizontal axis represents the real part a of the complex number, and the vertical axis represents the imaginary part b of the complex number.

Use a coordinate plane to find $(4 + 2i) + (-2 + 3i)$.



- Create a coordinate plane and label the axes appropriately.
- Graph $4 + 2i$ by drawing a segment from the origin to (4, 2) on the coordinate plane.
- Represent the addition of $-2 + 3i$ by moving 2 units to the left and 3 units up from (4, 2).
- You end at the point (2, 5), which represents the complex number $2 + 5i$.
  So, $(4 + 2i) + (-2 + 3i) = 2 + 5i$.

Model and Analyze

1. Model $(-3 + 2i) + (4 - i)$ on a coordinate plane. **See margin.**
2. Describe how you could model the difference $(-3 + 2i) - (4 - i)$ on a coordinate plane. **See margin.**
3. The **absolute value** of a complex number is the distance from the origin to the point representing that complex number in a coordinate plane. Refer to the graph above. Find the absolute value of $2 + 5i$. $\sqrt{29}$
4. Find an expression for the absolute value of $a + bi$. $\sqrt{a^2 + b^2}$

MULTIPLY AND DIVIDE COMPLEX NUMBERS Complex numbers are used with electricity. In a circuit with alternating current, the voltage, current, and impedance, or hindrance to current, can be represented by complex numbers.

Algebra Activity

Materials: grid paper, ruler/straightedge

- The horizontal axis is often called a real number line. What might be a corresponding name for the vertical axis? **an imaginary number line**
- Where do real numbers lie on this coordinate plane? **on the horizontal axis** Where do pure imaginary numbers lie? **on the vertical axis**
- Where do complex numbers for which neither a nor b is 0 lie on this coordinate plane? **on the regions of the plane other than the axes**

You can use the FOIL method to multiply complex numbers.

Example 7 Multiply Complex Numbers

ELECTRICITY **In an AC circuit, the voltage E, current I, and impedance Z are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 3j$ amps and impedance $7 - 5j$ ohms.**

$E = I \cdot Z$	Electricity formula
$= (1 + 3j) \cdot (7 - 5j)$	$I = 1 + 3j, Z = 7 - 5j$
$= 1(7) + 1(-5j) + (3j)7 + 3j(-5j)$	FOIL
$= 7 - 5j + 21j - 15j^2$	Multiply.
$= 7 + 16j - 15(-1)$	$j^2 = -1$
$= 22 + 16j$	Add.

The voltage is $22 + 16j$ volts.

Study Tip

Reading Math

Electrical engineers use j as the imaginary unit to avoid confusion with the I for current.

Two complex numbers of the form $a + bi$ and $a - bi$ are called **complex conjugates**. The product of complex conjugates is always a real number. For example, $(2 + 3i)(2 - 3i) = 4 - 6i + 6i + 9$ or 13. You can use this fact to simplify the quotient of two complex numbers.

Example 8 Divide Complex Numbers

Simplify.

a. $\frac{3i}{2 + 4i}$

$\frac{3i}{2+4i} = \frac{3i}{2+4i} \cdot \frac{2-4i}{2-4i}$ — $2 + 4i$ and $2 - 4i$ are conjugates.

$= \frac{6i - 12i^2}{4 - 16i^2}$ — Multiply.

$= \frac{6i + 12}{20}$ — $i^2 = -1$

$= \frac{3}{5} + \frac{3}{10}i$ — Standard form

b. $\frac{5 + i}{2i}$

$\frac{5+i}{2i} = \frac{5+i}{2i} \cdot \frac{i}{i}$ — Why multiply by $\frac{i}{i}$ intead of $\frac{-2i}{-2i}$?

$= \frac{5i + i^2}{2i^2}$ — Multiply.

$= \frac{5i - 1}{-2}$ — $i^2 = -1$

$= \frac{1}{2} - \frac{5}{2}i$ — Standard form

Check for Understanding

Concept Check

1. **Determine** if each statement is *true* or *false*. If false, find a counterexample.
   a. Every real number is a complex number. **true**
   b. Every imaginary number is a complex number. **true**
2. **Decide** which of the properties of a field and the properties of equality that the set of complex numbers satisfies. **all of them**
3. **OPEN ENDED** Write two complex numbers whose product is 10. **Sample answer: $1 + 3i$ and $1 - 3i$**

Study Tip

Look Back

Refer to Chapter 1 to review the properties of fields and the properties of equality.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6, 7	2
8	3
9	6
10, 11	7, 8

Simplify.

4. $\sqrt{-36}$ **$6i$**
5. $\sqrt{-50x^2y^2}$ **$5i|xy|\sqrt{2}$**
6. $(6i)(-2i)$ **12**
7. $5\sqrt{-24} \cdot 3\sqrt{-18}$ **$-180\sqrt{3}$**
8. i^{29} **i**
9. $(8 + 6i) - (2 + 3i)$ **$6 + 3i$**
10. $(3 - 5i)(4 + 6i)$ **$42 - 2i$**
11. $\frac{3 + i}{1 + 4i}$ **$\frac{7}{17} - \frac{11}{17}i$**

MULTIPLY AND DIVIDE COMPLEX NUMBERS

In-Class Examples Power Point®

7 **ELECTRICITY** In an AC circuit, the voltage E, current I, and impedence Z are related by the formula $E = I \cdot Z$. Find the voltage in a circuit with current $1 + 4j$ amps and impedence $3 - 6j$ ohms. **$27 + 6j$**

8 Simplify.

a. $\frac{5i}{3 + 2i}$ **$\frac{10}{13} + \frac{15}{13}i$**

b. $\frac{4 - i}{5i}$ **$-\frac{1}{5} - \frac{4}{5}i$**

3 Practice/Apply

Study Notebook

Have students—

- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 5.
- write a list of the first four powers of i: $i^1 = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Study Guide and Intervention, p. 287 (shown) and p. 288

Add and Subtract Complex Numbers

Complex Number	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit ($i^2 = -1$). a is called the real part, and b is called the imaginary part.
Addition and Subtraction of Complex Numbers	Combine like terms. $(a + bi) + (c + di) = (a + c) + (b + d)i$ $(a + bi) - (c + di) = (a - c) + (b - d)i$

Example 1 Simplify $(6 + i) + (4 - 5i)$.

$(6 + i) + (4 - 5i)$
$= (6 + 4) + (1 - 5)i$
$= 10 - 4i$

Example 2 Simplify $(8 + 3i) - (6 - 2i)$.

$(8 + 3i) - (6 - 2i)$
$= (8 - 6) + [3 - (-2)]i$
$= 2 + 5i$

To solve a quadratic equation that does not have real solutions, you can use the fact that $i^2 = -1$ to find complex solutions.

Example 3 Solve $2x^2 + 24 = 0$.

$2x^2 + 24 = 0$ Original equation
$2x^2 = -24$ Subtract 24 from each side.
$x^2 = -12$ Divide each side by 2.
$x = \pm\sqrt{-12}$ Take the square root of each side.
$x = \pm 2i\sqrt{3}$ $\sqrt{-12} = \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{3}$

Exercises

Simplify.

1. $(-4 + 2i) + (6 - 3i)$ **$2 - i$**
2. $(5 - i) - (3 - 2i)$ **$2 + i$**
3. $(6 - 3i) + (4 - 2i)$ **$10 - 5i$**
4. $(-11 + 4i) - (1 - 5i)$ **$-12 + 9i$**
5. $(8 + 4i) + (8 - 4i)$ **16**
6. $(5 + 2i) - (-6 - 3i)$ **$11 + 5i$**
7. $(12 - 5i) - (4 + 3i)$ **$8 - 8i$**
8. $(9 + 2i) + (-2 + 5i)$ **$7 + 7i$**
9. $(15 - 12i) + (11 - 13i)$ **$26 - 25i$**
10. i^4 **1**
11. i^6 **-1**
12. i^{15} **$-i$**

Solve each equation.

13. $5x^2 + 45 = 0$ **$\pm 3i$**
14. $4x^2 + 24 = 0$ **$\pm i\sqrt{6}$**
15. $-9x^2 = 9$ **$\pm i$**

Skills Practice, p. 289 and Practice, p. 290 (shown)

Simplify.

1. $\sqrt{-49}$ **$7i$**
2. $6\sqrt{-12}$ **$12i\sqrt{3}$**
3. $\sqrt{-121s^8}$ **$11s^4i$**
4. $\sqrt{-36a^3b^4}$ **$6|a|b^2i\sqrt{a}$**
5. $\sqrt{-8} \cdot \sqrt{-32}$ **-16**
6. $\sqrt{-15} \cdot \sqrt{-25}$ **$-5\sqrt{15}$**
7. $(-3i)(4i)(-5i)$ **$-60i$**
8. $(7i)^2(6i)$ **$-294i$**
9. i^{42} **-1**
10. i^{55} **$-i$**
11. i^{89} **i**
12. $(5 - 2i) + (-13 - 8i)$ **$-8 - 10i$**
13. $(7 - 6i) + (9 + 11i)$ **$16 + 5i$**
14. $(-12 + 48i) + (15 + 21i)$ **$3 + 69i$**
15. $(10 + 15i) - (48 - 30i)$ **$-38 + 45i$**
16. $(28 - 4i) - (10 - 30i)$ **$18 + 26i$**
17. $(6 - 4i)(6 + 4i)$ **52**
18. $(8 - 11i)(8 - 11i)$ **$-57 - 176i$**
19. $(4 + 3i)(2 - 5i)$ **$23 - 14i$**
20. $(7 + 2i)(9 - 6i)$ **$75 - 24i$**
21. $\frac{6 + 5i}{-2i}$ **$\frac{-5 + 6i}{2}$**
22. $\frac{2}{7 - 8i}$ **$\frac{14 + 16i}{113}$**
23. $\frac{3 - i}{2 - i}$ **$\frac{7 + i}{5}$**
24. $\frac{2 - 4i}{1 + 3i}$ **$-1 - i$**

Solve each equation.

25. $5n^2 + 35 = 0$ **$\pm i\sqrt{7}$**
26. $2m^2 + 10 = 0$ **$\pm i\sqrt{5}$**
27. $4m^2 + 76 = 0$ **$\pm i\sqrt{19}$**
28. $-2m^2 - 6 = 0$ **$\pm i\sqrt{3}$**
29. $-5m^2 - 65 = 0$ **$\pm i\sqrt{13}$**
30. $\frac{3}{4}x^2 + 12 = 0$ **$\pm 4i$**

Find the values of m and n that make each equation true.

31. $15 - 28i = 3m + 4ni$ **5, -7**
32. $(6 - m) + 3ni = -12 + 27i$ **18, 9**
33. $(3m + 4) + (3 - n)i = 16 - 3i$ **4, 6**
34. $(7 + n) + (4m - 10)i = 3 - 6i$ **1, -4**
35. **ELECTRICITY** The impedance in one part of a series circuit is $1 + 3j$ ohms and the impedance in another part of the circuit is $7 - 5j$ ohms. Add these complex numbers to find the total impedance in the circuit. **$8 - 2j$ ohms**
36. **ELECTRICITY** Using the formula $E = IZ$, find the voltage E in a circuit when the current I is $3 - j$ amps and the impedance Z is $3 + 2j$ ohms. **$11 + 3j$ volts**

Reading to Learn Mathematics, p. 291 ELL

Pre-Activity **How do complex numbers apply to polynomial equations?**

Read the introduction to Lesson 5-9 at the top of page 270 in your textbook.

Suppose the number i is defined such that $i^2 = -1$. Complete each equation.

$2i^2 =$ **-2** $(2i)^2 =$ **-4** $i^4 =$ **1**

Reading the Lesson

1. Complete each statement.
   a. The form $a + bi$ is called the **standard form** of a complex number.
   b. In the complex number $4 + 5i$, the real part is **4** and the imaginary part is **5**. This is an example of a complex number that is also a(n) **imaginary** number.
   c. In the complex number 3, the real part is **3** and the imaginary part is **0**. This is example of complex number that is also a(n) **real** number.
   d. In the complex number $7i$, the real part is **0** and the imaginary part is **7**. This is an example of a complex number that is also a(n) **pure imaginary** number.
2. Give the complex conjugate of each number.
   a. $3 + 7i$ **$3 - 7i$**
   b. $2 - i$ **$2 + i$**
3. Why are complex conjugates used in dividing complex numbers? **The product of complex conjugates is always a real number.**
4. Explain how you would use complex conjugates to find $(3 + 7i) \div (2 - i)$. **Write the division in fraction form. Then multiply numerator and denominator by $2 + i$.**

Helping You Remember

5. How can you use what you know about simplifying an expression such as $\frac{1 + \sqrt{3}}{2 - \sqrt{5}}$ to help you remember how to simplify fractions with imaginary numbers in the denominator? **Sample answer: In both cases, you can multiply the numerator and denominator by the conjugate of the denominator.**

GUIDED PRACTICE KEY	
Exercises	Examples
12–14	4
15, 16	5

Solve each equation.

12. $2x^2 + 18 = 0$ **$\pm 3i$**
13. $4x^2 + 32 = 0$ **$\pm 2i\sqrt{2}$**
14. $-5x^2 - 25 = 0$ **$\pm i\sqrt{5}$**

Find the values of m and n that make each equation true.

15. $2m + (3n + 1)i = 6 - 8i$ **3, -3**
16. $(2n - 5) + (-m - 2)i = 3 - 7i$ **5, 4**

Application

17. **ELECTRICITY** The current in one part of a series circuit is $4 - j$ amps. The current in another part of the circuit is $6 + 4j$ amps. Add these complex numbers to find the total current in the circuit. **$10 + 3j$ amps**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
18–21	1
22–25	2
26–29	3
30–33, 46, 47	6
34–37, 42, 43	7
38–41, 44, 45	8
48–55	4
56–61	5

Extra Practice

See page 839.

46. $(i + 4)x^2 + (3 - i)x + 2 - 4i$

Simplify. **22. $-13\sqrt{2}$ 30. $9 + 2i$ 33. $4 - 5i$ 35. $6 - 7i$**

18. $\sqrt{-144}$ **$12i$**
19. $\sqrt{-81}$ **$9i$**
20. $\sqrt{-64x^4}$ **$8x^2i$**
21. $\sqrt{-100a^4b^2}$ **$10a^2|b|i$**
22. $\sqrt{-13} \cdot \sqrt{-26}$
23. $\sqrt{-6} \cdot \sqrt{-24}$ **-12**
24. $(-2i)(-6i)(4i)$ **$-48i$**
25. $3i(-5i)^2$ **$-75i$**
26. i^{13} **i**
27. i^{24} **1**
28. i^{38} **-1**
29. i^{63} **$-i$**
30. $(5 - 2i) + (4 + 4i)$
31. $(3 - 5i) + (3 + 5i)$ **6**
32. $(3 - 4i) - (1 - 4i)$ **2**
33. $(7 - 4i) - (3 + i)$
34. $(3 + 4i)(3 - 4i)$ **25**
35. $(1 - 4i)(2 + i)$
36. $(6 - 2i)(1 + i)$ **$8 + 4i$**
37. $(-3 - i)(2 - 2i)$ **$-8 + 4i$**
38. $\frac{4i}{3 + i}$ **$\frac{2}{5} + \frac{6}{5}i$**
39. $\frac{4}{5 + 3i}$ **$\frac{10}{17} - \frac{6}{17}i$**
40. $\frac{10 + i}{4 - i}$ **$\frac{39}{17} + \frac{14}{17}i$**
41. $\frac{2 - i}{3 - 4i}$ **$\frac{2}{5} + \frac{1}{5}i$**
42. ★ $(-5 + 2i)(6 - i)(4 + 3i)$ **$-163 - 16i$**
43. ★ $(2 + i)(1 + 2i)(3 - 4i)$ **$20 + 15i$**
44. ★ $\frac{5 - i\sqrt{3}}{5 + i\sqrt{3}}$ **$\frac{11}{14} - \frac{5\sqrt{3}}{14}i$**
45. ★ $\frac{1 - i\sqrt{2}}{1 + i\sqrt{2}}$ **$-\frac{1}{3} - \frac{2\sqrt{2}}{3}i$**
46. Find the sum of $ix^2 - (2 + 3i)x + 2$ and $4x^2 + (5 + 2i)x - 4i$.
47. Simplify $[(3 + i)x^2 - ix + 4 + i] - [(-2 + 3i)x^2 + (1 - 2i)x - 3]$. **$(5 - 2i)x^2 + (-1 + i)x + 7 + i$**

Solve each equation.

48. $5x^2 + 5 = 0$ **$\pm i$**
49. $4x^2 + 64 = 0$ **$\pm 4i$**
50. $2x^2 + 12 = 0$ **$\pm i\sqrt{6}$**
51. $6x^2 + 72 = 0$ **$\pm 2i\sqrt{3}$**
52. $-3x^2 - 9 = 0$ **$\pm i\sqrt{3}$**
53. $-2x^2 - 80 = 0$ **$\pm 2i\sqrt{10}$**
54. $\frac{2}{3}x^2 + 30 = 0$ **$\pm 3i\sqrt{5}$**
55. $\frac{4}{5}x^2 + 1 = 0$ **$\pm\frac{\sqrt{5}}{2}i$**

Find the values of m and n that make each equation true. **61. $\frac{67}{11}, \frac{19}{11}$**

56. $8 + 15i = 2m + 3ni$ **4, 5**
57. $(m + 1) + 3ni = 5 - 9i$ **4, -3**
58. $(2m + 5) + (1 - n)i = -2 + 4i$ **$-\frac{7}{2}, -3$**
59. $(4 + n) + (3m - 7)i = 8 - 2i$ **$\frac{5}{3}, 4$**
60. ★ $(m + 2n) + (2m - n)i = 5 + 5i$ **3, 1**
61. ★ $(2m - 3n)i + (m + 4n) = 13 + 7i$
62. **ELECTRICITY** The impedance in one part of a series circuit is $3 + 4j$ ohms, and the impedance in another part of the circuit is $2 - 6j$. Add these complex numbers to find the total impedance in the circuit. **$5 - 2j$ ohms**

ELECTRICAL ENGINEERING For Exercises 63 and 64, use the formula $E = I \cdot Z$.

63. The current in a circuit is $2 + 5j$ amps, and the impedance is $4 - j$ ohms. What is the voltage? **$13 + 18j$ volts**
64. The voltage in a circuit is $14 - 8j$ volts, and the impedance is $2 - 3j$ ohms. What is the current? **$4 + 2j$ amps**

Career Choices

Electrical Engineering

The chips and circuits in computers are designed by electrical engineers.

Online Research

To learn more about electrical engineering, visit: www.algebra2.com/careers

Enrichment, p. 292

Conjugates and Absolute Value

When studying complex numbers, it is often convenient to represent a complex number by a single variable. For example, we might let $z = x + yi$. We denote the conjugate of z by $\bar{z}$. Thus, $\bar{z} = x - yi$.

We can define the absolute value of a complex number as follows.

$|z| = |x + yi| = \sqrt{x^2 + y^2}$

There are many important relationships involving conjugates and absolute values of complex numbers.

Example 1 **Show $|z|^2 = z\bar{z}$ for any complex number z.**

Let $z = x + yi$. Then,
$z = (x + yi)(x - yi)$
$= x^2 + y^2$
$= \sqrt{(x^2 + y^2)^2}$

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65. **CRITICAL THINKING** Show that the order relation "<" does not make sense for the set of complex numbers. (*Hint*: Consider the two cases $i > 0$ and $i < 0$. In each case, multiply each side by i.) **See pp. 283A–283B.**

66. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

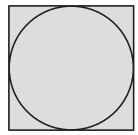
How do complex numbers apply to polynomial equations?

Include the following in your answer:
- how the a and c must be related if the equation $ax^2 + c = 0$ has complex solutions, and
- the solutions of the equation $2x^2 + 2 = 0$.

Standardized Test Practice

67. If $i^2 = -1$, then what is the value of i^{71}? **C**
   (A) -1 (B) 0 (C) $-i$ (D) i

68. The area of the square is 16 square units. What is the area of the circle? **C**
   (A) 2π units^2 (B) 12 units^2
   (C) 4π units^2 (D) 16π units^2



Extending the Lesson

PATTERN OF POWERS OF i **69. $-1, -i, 1, i, -1, -i, 1, i, -1$**

69. Find the simplified forms of $i^6, i^7, i^8, i^9, i^{10}, i^{11}, i^{12}, i^{13}$, and i^{14}.

70. Explain how to use the exponent to determine the simplified form of any power of i. **See margin.**

Maintain Your Skills

Mixed Review

Solve each equation. *(Lesson 5-8)*

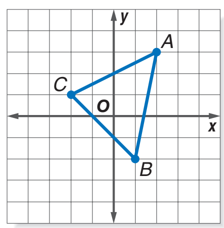
71. $\sqrt{2x+1} = 5$ **12**
72. $\sqrt[3]{x-3} + 1 = 3$ **11**
73. $\sqrt{x+5} + \sqrt{x} = 5$ **4**

Simplify each expression. *(Lesson 5-7)*

74. $x^{-\frac{1}{5}} \cdot x^{\frac{2}{3}}$ $x^{\frac{7}{15}}$
75. $\left(y^{-\frac{1}{2}}\right)^{-\frac{2}{3}}$ $y^{\frac{1}{3}}$
76. $a^{-\frac{3}{4}}$ $\frac{a^{\frac{1}{4}}}{a}$

For Exercises 77–80, triangle *ABC* is reflected over the *x*-axis. *(Lesson 4-6)*

77. Write a vertex matrix for the triangle.
78. Write the reflection matrix.
79. Write the vertex matrix for $\triangle A'B'C'$.
80. Graph $\triangle A'B'C'$. **See pp. 283A–283B.**



77. $\begin{bmatrix} 2 & 1 & -2 \\ 3 & -2 & 1 \end{bmatrix}$

78. $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

79. $\begin{bmatrix} 2 & 1 & -2 \\ -3 & 2 & -1 \end{bmatrix}$

81. **FURNITURE** A new sofa, love seat, and coffee table cost \$2050. The sofa costs twice as much as the love seat. The sofa and the coffee table together cost \$1450. How much does each piece of furniture cost? *(Lesson 3-5)* **sofa: \$1200, love seat: \$600, coffee table: \$250**

Graph each system of inequalities. *(Lesson 3-3)* **82–83. See pp. 283A–283B.**

82. $y < x + 1$
    $y > -2x - 2$

83. $x + y \geq 1$
    $x - 2y \leq 4$

Find the slope of the line that passes through each pair of points. *(Lesson 2-3)*

84. $(-2, 1), (8, 2)$ $\frac{1}{10}$
85. $(4, -3), (5, -3)$ **0**

www.algebra2.com/self_check_quiz

About the Exercises…

Organization by Objective
- **Add and Subtract Complex Numbers:** 18–33, 46–61
- **Multiply and Divide Complex Numbers:** 34–45

Odd/Even Assignments
Exercises 18–61 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 19–41 odd, 47–59 odd, 63, 65–68, 71–85

Average: 19–63 odd, 65–68, 71–85 (optional: 69, 70)

Advanced: 18–64 even, 65–85

4 Assess

Open-Ended Assessment
Speaking Have students discuss the meaning and "reality" of imaginary numbers, including their graphical representation and their usefulness in electrical engineering.

Tips for New Teachers

Intervention Suggest that students who are confused by imaginary numbers think of i as a very special kind of variable that most of the time can be treated similar to the variable x.

Assessment Options
Quiz (Lessons 5-8 and 5-9) is available on p. 308 of the *Chapter 5 Resource Masters*.

Answers

66. Some polynomial equations have complex solutions. Answers should include the following.
   - a and c must have the same sign.
   - $\pm i$

70. Examine the remainder when the exponent is divided by 4. If the remainder is 0, the result is 1. If the remainder is 1, the result is i. If the remainder is 2, the result is -1. And if the remainder is 3, the result is $-i$.

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 5 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 5 is available on p. 306 of the *Chapter 5 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 5 Study Guide and Review

Vocabulary and Concept Check

absolute value (p. 272)
binomial (p. 229)
coefficient (p. 222)
complex conjugates (p. 273)
complex number (p. 271)
conjugates (p. 253)
constant (p. 222)
degree (p. 222)
dimensional analysis (p. 225)
extraneous solution (p. 263)
FOIL method (p. 230)
imaginary unit (p. 270)
like radical expressions (p. 252)
like terms (p. 229)
monomial (p. 222)
*n*th root (p. 245)
polynomial (p. 229)
power (p. 222)
principal root (p. 246)
pure imaginary number (p. 270)
radical equation (p. 263)
radical inequality (p. 264)
rationalizing the denominator (p. 251)
scientific notation (p. 225)
simplify (p. 222)
square root (p. 245)
standard notation (p. 225)
synthetic division (p. 234)
terms (p. 229)
trinomial (p. 229)

Choose a word or term from the list above that best completes each statement or phrase.

1. A number is expressed in _______ when it is in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer. **scientific notation**
2. A shortcut method known as _______ is used to divide polynomials by binomials. **synthetic division**
3. The _______ is used to multiply two binomials. **FOIL method**
4. A(n) _______ is an expression that is a number, a variable, or the product of a number and one or more variables. **monomial**
5. A solution of a transformed equation that is not a solution of the original equation is a(n) _______. **extraneous solution**
6. _______ are imaginary numbers of the form $a + bi$ and $a - bi$. **Complex conjugates**
7. For any number a and b, if $a^2 = b$, then a is a(n) _______ of b. **square root**
8. A polynomial with three terms is known as a(n) _______. **trinomial**
9. When a number has more than one real root, the _______ is the nonnegative root. **principal root**
10. i is called the _______. **imaginary unit**

Lesson-by-Lesson Review

5-1 Monomials

See pages 222–228.

Concept Summary

- The properties of powers for real numbers a and b and integers m and n are as follows.

$a^{-n} = \frac{1}{a^n}, a \neq 0$ $\qquad (a^m)^n = a^{mn}$

$a^m \cdot a^n = a^{m+n}$ $\qquad (ab)^m = a^m b^m$

$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ $\qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

- Use scientific notation to represent very large or very small numbers.

Examples

1 **Simplify $(3x^4y^6)(-8x^3y)$.**

$(3x^4y^6)(-8x^3y) = (3)(-8)x^{4+3}y^{6+1}$ Commutative Property and products of powers

$= -24x^7y^7$ Simplify.

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FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Ask students to review their Foldable and make sure that their notes, diagrams, and examples are complete. Since journal entries are personal, remind students that these journals are shared only with their consent. Ask if anyone would like to describe one of their journal entries, perhaps something they had difficulty with but later cleared up by asking questions.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

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2 **Express each number in scientific notation.**

a. 31,000

$31{,}000 = 3.1 \times 10{,}000$

$= 3.1 \times 10^4$ $\quad 10{,}000 = 10^4$

b. 0.007

$0.007 = 7 \times 0.001$

$= 7 \times 10^{-3}$ $\quad 0.001 = \frac{1}{1000}$ or $\frac{1}{10^3}$

Exercises **Simplify. Assume that no variable equals 0.**
See Examples 1–4 on pages 222–224.

11. $f^{-7} \cdot f^4$ $\frac{1}{f^3}$ **12.** $(3x^2)^3$ $27x^6$ **13.** $(2y)(4xy^3)$ $8xy^4$ **14.** $\left(\frac{3}{5}c^2f\right)\left(\frac{4}{3}cd\right)^2$ $\frac{16}{15}c^4d^2f$

Evaluate. Express the result in scientific notation. *See Examples 5–7 on page 225.*

15. $(2000)(85{,}000)$ 1.7×10^8 **16.** $(0.0014)^2$ 1.96×10^{-6} **17.** $\frac{5{,}400{,}000}{6000}$ 9×10^2

5-2 Polynomials

See pages 229–232.

Concept Summary

- Add or subtract polynomials by combining like terms.
- Multiply polynomials by using the Distributive Property.
- Multiply binomials by using the FOIL method.

Examples

1 **Simplify $(5x^2 + 4x) - (3x^2 + 6x - 7)$.**

$5x^2 + 4x - (3x^2 + 6x - 7)$
$= 5x^2 + 4x - 3x^2 - 6x + 7$
$= (5x^2 - 3x^2) + (4x - 6x) + 7$
$= 2x^2 - 2x + 7$

2 **Find $(9k + 4)(7k - 6)$.**

$(9k + 4)(7k - 6)$
$= (9k)(7k) + (9k)(-6) + (4)(7k) + (4)(-6)$
$= 63k^2 - 54k + 28k - 24$
$= 63k^2 - 26k - 24$

18. $-3c + 1$ 19. $4x^2 + 22x - 34$ 20. $-18m^3n - 78m^3 + 30m^2n$

Exercises **Simplify.** *See Examples 2–5 on pages 229 and 230.*

18. $(4c - 5) - (c + 11) + (-6c + 17)$ **19.** $(11x^2 + 13x - 15) - (7x^2 - 9x + 19)$

20. $-6m^2(3mn + 13m - 5n)$ **21.** $x^{-8}y^{10}(x^{11}y^{-9} + x^{10}y^{-6})$ $x^3y + x^2y^4$

22. $(d - 5)(d + 3)$ $d^2 - 2d - 15$ **23.** $(2a^2 + 6)^2$ $4a^4 + 24a^2 + 36$ **24.** $(2b - 3c)^3$ $8b^3 - 36b^2c + 54bc^2 - 27c^3$

5-3 Dividing Polynomials

See pages 233–238.

Concept Summary

- Use the division algorithm or synthetic division to divide polynomials.

Example

Use synthetic division to find $(4x^4 - x^3 - 19x^2 + 11x - 2) \div (x - 2)$.

2	4	−1	−19	11	−2
		8	14	−10	2
	4	7	−5	1	0

→ The quotient is $4x^3 + 7x^2 - 5x + 1$.

26. $10x^3 - 5x^2 + 9x - 9$

Exercises **Simplify.** *See Examples 1–5 on pages 233–235.* 25. $2x^3 + x - \frac{3}{x - 3}$

25. $(2x^4 - 6x^3 + x^2 - 3x - 3) \div (x - 3)$ **26.** $(10x^4 + 5x^3 + 4x^2 - 9) \div (x + 1)$

27. $(x^2 - 5x + 4) \div (x - 1)$ $x - 4$ **28.** $(5x^4 + 18x^3 + 10x^2 + 3x) \div (x^2 + 3x)$ $5x^2 + 3x + 1$

Chapter 5 Study Guide and Review 277

5-4 Factoring Polynomials

See pages 239–244.

Concept Summary

- You can factor polynomials using the GCF, grouping, or formulas involving squares and cubes.

Examples

1 **Factor $4x^3 - 6x^2 + 10x - 15$.**

$4x^3 - 6x^2 + 10x - 15 = (4x^3 - 6x^2) + (10x - 15)$ Group to find the GCF.

$= 2x^2(2x - 3) + 5(2x - 3)$ Factor the GCF of each binomial.

$= (2x^2 + 5)(2x - 3)$ Distributive Property

2 **Factor $3m^2 + m - 4$.**

Find two numbers whose product is $3(-4)$ or 12, and whose sum is 1. The two numbers must be 4 and -3 because $4(-3) = -12$ and $4 + (-3) = 1$.

$3m^2 + m - 4 = 3m^2 + 4m - 3m - 4$

$= (3m^2 + 4m) - (3m + 4)$

$= m(3m + 4) + (-1)(3m + 4)$

$= (3m + 4)(m - 1)$

Exercises **Factor completely. If the polynomial is not factorable, write *prime*.**
See Examples 1–3 on pages 239 and 241. **30. $2(5a^2 - 1)(a - 2)$** **31. $(5w^2 + 3)(w - 4)$**

29. $200x^2 - 50$ **$50(2x + 1)(2x - 1)$**
30. $10a^3 - 20a^2 - 2a + 4$
31. $5w^3 - 20w^2 + 3w - 12$
32. $x^4 - 7x^3 + 12x^2$ **$x^2(x - 3)(x - 4)$**
33. $s^3 + 512$ **$(s + 8)(s^2 - 8s + 64)$**
34. $x^2 - 7x + 5$ **prime**

5-5 Roots of Real Numbers

See pages 245–249.

Concept Summary

Real *n*th roots of *b*, $\sqrt[n]{b}$, or $-\sqrt[n]{b}$			
n	$\sqrt[n]{b}$ if $b > 0$	$\sqrt[n]{b}$ if $b < 0$	$\sqrt[n]{b}$ if $b = 0$
even	one positive root one negative root	no real roots	one real root, 0
odd	one positive root no negative roots	no positive roots one negative root	

Examples

1 **Simplify $\sqrt{81x^6}$.**

$\sqrt{81x^6} = \sqrt{(9x^3)^2}$ $81x^6 = (9x^3)^2$

$= 9|x^3|$ Use absolute value.

2 **Simplify $\sqrt[7]{2187x^{14}y^{35}}$.**

$\sqrt[7]{2187x^{14}y^{35}} = \sqrt[7]{(3x^2y^5)^7}$ $2187x^{14}y^{35} = (3x^2y^5)^7$

$= 3x^2y^5$ Evaluate.

Exercises **Simplify.** *See Examples 1 and 2 on pages 246 and 247.*

35. $\pm\sqrt{256}$ **± 16**
36. $\sqrt[3]{-216}$ **-6**
37. $\sqrt{(-8)^2}$ **8**
38. $\sqrt[5]{c^5d^{15}}$ **cd^3**
39. $\sqrt{(x^4 - 3)^2}$ **$|x^4 - 3|$**
40. $\sqrt[3]{(512 + x^2)^3}$ **$512 + x^2$**
41. $\sqrt[4]{16m^8}$ **$2m^2$**
42. $\sqrt{a^2 - 10a + 25}$ **$|a - 5|$**

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5-6 Radical Expressions

See pages 250–256.

Concept Summary

For any real numbers a and b and any integer $n > 1$,

- Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
- Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example Simplify $6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2}$.

$6\sqrt[5]{32m^3} \cdot 5\sqrt[5]{1024m^2} = 6 \cdot 5\sqrt[5]{(32m^3 \cdot 1024m^2)}$ Product Property of Radicals

$= 30\sqrt[5]{2^5 \cdot 4^5 \cdot m^5}$ Factor into exponents of 5 if possible.

$= 30\sqrt[5]{2^5} \cdot \sqrt[5]{4^5} \cdot \sqrt[5]{m^5}$ Product Property of Radicals

$= 30 \cdot 2 \cdot 4 \cdot m$ or $240m$ Write the fifth roots.

Exercises Simplify. *See Examples 1–6 on pages 250–253.*

43. $\sqrt[6]{128}$ $2\sqrt[6]{2}$
44. $\sqrt{5} + \sqrt{20}$ $3\sqrt{5}$
45. $5\sqrt{12} - 3\sqrt{75}$ $-5\sqrt{3}$
46. $6\sqrt[5]{11} - 8\sqrt[5]{11}$ $-2\sqrt[5]{11}$
47. $(\sqrt{8} + \sqrt{12})^2$ $20 + 8\sqrt{6}$
48. $\sqrt{8} \cdot \sqrt{15} \cdot \sqrt{21}$ $6\sqrt{70}$
49. $\frac{\sqrt{243}}{\sqrt{3}}$ 9
50. $\frac{1}{3 + \sqrt{5}}$ $\frac{3 - \sqrt{5}}{4}$
51. $\frac{\sqrt{10}}{4 + \sqrt{2}}$ $\frac{2\sqrt{10} - \sqrt{5}}{7}$

5-7 Radical Exponents

See pages 257–262.

Concept Summary

- For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$

Examples

1 Write $32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}}$ in radical form.

$32^{\frac{4}{5}} \cdot 32^{\frac{2}{5}} = 32^{\frac{4}{5} + \frac{2}{5}}$ Product of powers

$= 32^{\frac{6}{5}}$ Add.

$= (2^5)^{\frac{6}{5}}$ $32 = 2^5$

$= 2^6$ or 64 Power of a power

2 Simplify $\frac{3x}{\sqrt[3]{z}}$.

$\frac{3x}{\sqrt[3]{z}} = \frac{3x}{z^{\frac{1}{3}}}$ Rational exponents

$= \frac{3x}{z^{\frac{1}{3}}} \cdot \frac{z^{\frac{2}{3}}}{z^{\frac{2}{3}}}$ Rationalize the denominator.

$= \frac{3xz^{\frac{2}{3}}}{z}$ or $\frac{3x\sqrt[3]{z^2}}{z}$ Rewrite in radical form.

Exercises Evaluate. *See Examples 3 and 5 on pages 258 and 259.*

52. $27^{-\frac{2}{3}}$ $\frac{1}{9}$
53. $9^{\frac{1}{3}} \cdot 9^{\frac{5}{3}}$ 81
54. $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$ $\frac{9}{4}$

Simplify. *See Example 5 on page 259.*

55. $\frac{1}{y^{\frac{2}{5}}}$ $\frac{y^{\frac{3}{5}}}{y}$
56. $\frac{xy}{\sqrt[3]{z}}$ $\frac{xyz^{\frac{2}{3}}}{z}$
57. $\frac{3x + 4x^2}{x^{-\frac{2}{3}}}$ $3x^{\frac{5}{3}} + 4x^{\frac{8}{3}}$

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Chapter 5 For More ...
• Extra Practice, see pages 836–839.
• Mixed Problem Solving, see page 866.

5-8 Radical Equations and Inequalities

See pages 263–267.

Concept Summary

- To solve a radical equation, isolate the radical. Then raise each side of the equation to a power equal to the index of the radical.

Example **Solve $\sqrt{3x - 8} + 1 = 3$.**

$\sqrt{3x - 8} + 1 = 3$	Original equation
$\sqrt{3x - 8} = 2$	Subtract 1 from each side.
$(\sqrt{3x - 8})^2 = 2^2$	Square each side.
$3x - 8 = 4$	Evaluate the squares.
$x = 4$	Solve for *x*.

Exercises **Solve each equation.** *See Examples 1–3 on pages 263 and 264.*

58. $\sqrt{x} = 6$ **36**
59. $y^{\frac{1}{3}} - 7 = 0$ **343**
60. $(x - 2)^{\frac{3}{2}} = -8$ **no solution**
61. $\sqrt{x + 5} - 3 = 0$ **4**
62. $\sqrt{3t - 5} - 3 = 4$ **18**
63. $\sqrt{2x - 1} = 3$ **5**
64. $\sqrt[4]{2x - 1} = 2$ **8.5**
65. $\sqrt{y + 5} = \sqrt{2y - 3}$ **8**
66. $\sqrt{y + 1} + \sqrt{y - 4} = 5$ **8**

5-9 Complex Numbers

See pages 270–275.

Concept Summary

- $i^2 = -1$ and $i = \sqrt{-1}$
- Complex conjugates can be used to simplify quotients of complex numbers.

Examples

1 **Simplify $(15 - 2i) + (-11 + 5i)$.**

$(15 - 2i) + (-11 + 5i) = [15 + (-11)] + (-2 + 5)i$	Group the real and imaginary parts.
$= 4 + 3i$	Add.

2 **Simplify $\frac{7i}{2 + 3i}$.**

$\frac{7i}{2 + 3i} = \frac{7i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i}$	$2 + 3i$ and $2 - 3i$ are conjugates.
$= \frac{14i - 21i^2}{4 - 9i^2}$	Multiply.
$= \frac{21 + 14i}{13}$ or $\frac{21}{13} + \frac{14}{13}i$	$i^2 = -1$

Exercises **Simplify.** *See Examples 1–3 and 6–8 on pages 270, 272, and 273.*

67. $\sqrt{-64m^{12}}$ **$8m^6i$**
68. $(7 - 4i) - (-3 + 6i)$ **$10 - 10i$**
69. $-6\sqrt{-9} \cdot 2\sqrt{-4}$ **72**
70. i^6 **-1**
71. $(3 + 4i)(5 - 2i)$ **$23 + 14i$**
72. $(\sqrt{6} + i)(\sqrt{6} - i)$ **7**
73. $\frac{1 + i}{1 - i}$ **i**
74. $\frac{4 - 3i}{1 + 2i}$ **$-\frac{2}{5} - \frac{11}{5}i$**
75. $\frac{3 - 9i}{4 + 2i}$ **$\frac{-3 - 21i}{10}$**

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Chapter 5

Practice Test

Vocabulary and Concepts

Choose the term that best describes the shaded part of each trinomial.

1. $\boxed{2}x^2 - 3x + 4$ **c**
2. $4x^{\boxed{2}} - 6x - 3$ **a**
3. $9x^2 + 2x + \boxed{7}$ **b**

a.	degree
b.	constant term
c.	coefficient

Skills and Applications

Simplify.

4. $(5b)^4(6c)^2$ **$22{,}500b^4c^2$**
5. $(13x - 1)(x + 3)$ **$13x^2 + 38x - 3$**
6. $(2h - 6)^3$ **$8h^3 - 72h^2 + 216h - 216$**

Evaluate. Express the result in scientific notation.

7. $(3.16 \times 10^3)(24 \times 10^2)$ **7.584×10^6**
8. $\dfrac{7{,}200{,}000 \cdot 0.0011}{0.018}$ **4.4×10^5**

Simplify.

9. $(x^4 - x^3 - 10x^2 + 4x + 24) \div (x - 2)$ **$x^3 + x^2 - 8x - 12$**
10. $(2x^3 + 9x^2 - 2x + 7) \div (x + 2)$ **$2x^2 + 5x - 12 + \dfrac{31}{x + 2}$**

Factor completely. If the polynomial is not factorable, write *prime*.

11. $x^2 - 14x + 45$ **$(x - 5)(x - 9)$**
12. $2r^2 + 3pr - 2p^2$ **$(2r - p)(r + 2p)$**
13. $x^2 + 2\sqrt{3}x + 3$ **$(x + \sqrt{3})^2$**

Simplify.

14. $\sqrt{175}$ **$5\sqrt{7}$**
15. $(5 + \sqrt{3})(7 - 2\sqrt{3})$ **$29 - 3\sqrt{3}$**
16. $3\sqrt{6} + 5\sqrt{54}$ **$18\sqrt{6}$**
17. $\dfrac{9}{5 - \sqrt{3}}$ **$\dfrac{45 + 9\sqrt{3}}{22}$**
18. $\left(9^{\frac{1}{2}} \cdot 9^{\frac{2}{3}}\right)^{\frac{1}{6}}$ **$3^{\frac{7}{18}}$**
19. $11^{\frac{1}{2}} \cdot 11^{\frac{7}{3}} \cdot 11^{\frac{1}{6}}$ **1331**
20. $\sqrt[6]{256s^{11}t^{18}}$ **$2s|t^3|\sqrt[6]{4s^5}$**
21. $v^{-\frac{7}{11}}$ **$\dfrac{v^{\frac{4}{11}}}{v}$**
22. $\dfrac{b^{\frac{1}{2}}}{b^{\frac{3}{2}} - b^{\frac{1}{2}}}$ **$\dfrac{1}{b - 1}$**

Solve each equation.

23. $\sqrt{b + 15} = \sqrt{3b + 1}$ **7**
24. $\sqrt{2x} = \sqrt{x - 4}$ **no solution**
25. $\sqrt[4]{y + 2} + 9 = 14$ **623**
26. $\sqrt[3]{2w - 1} + 11 = 18$ **172**
27. $\sqrt{4x + 28} = \sqrt{6x + 38}$ **-5**
28. $1 + \sqrt{x + 5} = \sqrt{x + 12}$ **4**

Simplify.

29. $(5 - 2i) - (8 - 11i)$ **$-3 + 9i$**
30. $(14 - 5i)^2$ **$171 - 140i$**

31. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\dfrac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time period? **1936 ft**

32. **GEOMETRY** The area of a triangle with sides of length a, b, and c is given by $\sqrt{s(s - a)(s - b)(s - c)}$, where $s = \frac{1}{2}(a + b + c)$. If the lengths of the sides of a triangle are 6, 9, and 12 feet, what is the area of the triangle expressed in radical form? **$\dfrac{27\sqrt{15}}{4}$ ft^2**

33. **STANDARDIZED TEST PRACTICE** $2 + \left(x + \dfrac{1}{x}\right)^2 =$ **D**

Ⓐ 2 Ⓑ 4 Ⓒ $x^2 + \dfrac{1}{x^2}$ Ⓓ $x^2 + \dfrac{1}{x^2} + 4$

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Chapter 5

Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 5 can be found on p. 306 of the *Chapter 5 Resource Masters*.

Chapter Tests There are six Chapter 5 Tests and an Open-Ended Assessment task available in the *Chapter 5 Resource Masters*.

Chapter 5 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	293–294
2A	MC	average	295–296
2B	MC	average	297–298
2C	FR	average	299–300
2D	FR	average	301–302
3	FR	advanced	303–304

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 5 can be found on p. 305 of the *Chapter 5 Resource Masters*. A sample scoring rubric for these tasks appears on p. A34.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction In this chapter, you have divided and simplified monomials, polynomials, radical expressions, and complex numbers, often using procedures that involved a series of steps.

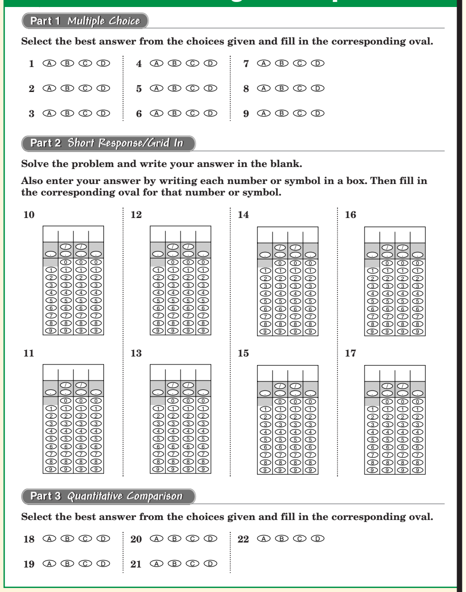
Ask Students Write a description for your portfolio comparing these various division problems. Identify which type of division problems was most challenging for you and explain why you think this is true. Be sure to include several examples of your work from this chapter.

Chapter 5 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 5 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ

2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ

3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 12 14 16

11 13 15 17

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

18 Ⓐ Ⓑ Ⓒ Ⓓ 20 Ⓐ Ⓑ Ⓒ Ⓓ 22 Ⓐ Ⓑ Ⓒ Ⓓ

19 Ⓐ Ⓑ Ⓒ Ⓓ 21 Ⓐ Ⓑ Ⓒ Ⓓ

Additional Practice

See pp. 311–312 in the *Chapter 5 Resource Masters* for additional standardized test practice.

Chapter 5 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If $x^3 = 30$ and x is a real number, then x lies between which two consecutive integers? **B**

Ⓐ 2 and 3

Ⓑ 3 and 4

Ⓒ 4 and 5

Ⓓ 5 and 6

2. If $12x + 7y = 19$ and $4x - y = 3$, then what is the value of $8x + 8y$? **C**

Ⓐ 2

Ⓑ 8

Ⓒ 16

Ⓓ 22

3. For all positive integers n,

$\boxed{n} = n - 1$, if n is even and

$\boxed{n} = \frac{1}{2}(n + 1)$, if n is odd.

What is $\boxed{8} \times \boxed{13}$? **B**

Ⓐ 42

Ⓑ 49

Ⓒ 56

Ⓓ 82

4. Let $x \circledast y = xy - y$ for all integers x and y. If $x \circledast y = 0$ and $y \neq 0$, what must x equal? **D**

Ⓐ -2

Ⓑ -1

Ⓒ 0

Ⓓ 1

5. The sum of a number and its square is three times the number. What is the number? **D**

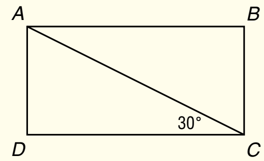
Ⓐ 0 only

Ⓑ -2 only

Ⓒ 2 only

Ⓓ 0 or 2

6. In rectangle $ABCD$, $\overline{AD}$ is 8 units long. What is the length of $\overline{AB}$ in units? **C**



Ⓐ 4

Ⓑ 8

Ⓒ $8\sqrt{3}$

Ⓓ 16

7. The sum of two positive consecutive integers is s. In terms of s, what is the value of the greater integer? **D**

Ⓐ $\frac{s}{2} - 1$ Ⓑ $\frac{s-1}{2}$

Ⓒ $\frac{s}{2}$ Ⓓ $\frac{s+1}{2}$

8. Latha, Renee, and Cindy scored a total of 30 goals for their soccer team this season. Latha scored three times as many goals as Renee. The combined number of goals scored by Latha and Cindy is four times the number scored by Renee. How many goals did Latha score? **C**

Ⓐ 5 Ⓑ 6

Ⓒ 18 Ⓓ 20

9. If $s = t + 1$ and $t \geq 1$, then which of the following must be equal to $s^2 - t^2$? **D**

Ⓐ $(s - t)^2$ Ⓑ $t^2 - 1$

Ⓒ $s^2 - 1$ Ⓓ $s + t$

The Princeton Review Test-Taking Tip

Question 9 If you simplify an expression and do not find your answer among the given answer choices, follow these steps. First, check your answer. Then, compare your answer with each of the given answer choices to determine whether it is equivalent to any of the answer choices.

Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

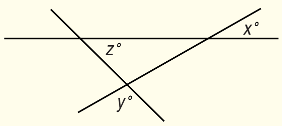
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Let a ✿ $b = a + \frac{1}{b}$, where $b \neq 0$. What is the value of 3 ✿ 4? **3.25 or 13/4**

11. If $3x^2 = 27$, what is the value of $3x^4$? **243**

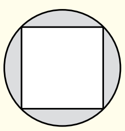
12. In the figure, if $x = 25$ and $z = 50$, what is the value of y? **105**



13. For all positive integers n, let $\textcircled{n}$ equal the greatest prime number that is a divisor of n. What does $\frac{\textcircled{70}}{\textcircled{27}}$ equal? **7/3**

14. If $3x + 2y = 36$ and $\frac{5y}{3x} = 5$, then $x =$ __?__. **4**

15. In the figure, a square with side of length $2\sqrt{2}$ is inscribed in a circle. If the area of the circle is $k\pi$, what is the exact value of k? **4**



16. For all nonnegative numbers n, let $\boxed{n}$ be defined by $\boxed{n} = \frac{\sqrt{n}}{2}$. If $\boxed{n} = 4$, what is the value of n? **64**

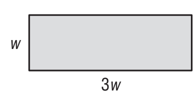
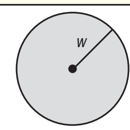
17. For the numbers a, b, and c, the average (arithmetic mean) is twice the median. If $a = 0$, and $a < b < c$, what is the value of $\frac{c}{b}$? **5**

www.algebra2.com/standardized_test

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- **(A) the quantity in Column A is greater,**
- **(B) the quantity in Column B is greater,**
- **(C) the two quantities are equal, or**
- **(D) the relationship cannot be determined from the information given.**

	Column A	Column B
18.	s and t are positive integers.	
	$\frac{s+t}{s}$	$\frac{s}{s+t}$
	A	
19.	The original price of a VCR is discounted by 20%, giving a sale price of \$108.	
	the original price of the VCR	\$130
	A	
20.	 the area of the rectangle	 the area of the circle
	B	
21.	k and n are integers. $k^n = 64$	
	k	n
	D	
22.	For all positive integers m and p, let m ✢ $p = 4(m + p) - mp$.	
	8 ✢ 3	3 ✢ 8
	C	

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Pages 231–232, Lesson 5-2

3.

	x	x	x
x	x^2	x^2	x^2
2	x	x	x
	x	x	x

55. The expression for how much an amount of money will grow to is a polynomial in terms of the interest rate. Answers should include the following.

- If an amount A grows by r percent for n years, the amount will be $A(1 + r)^n$ after n years. When this expression is expanded, a polynomial results.
- $13{,}872(1 + r)^3$, $13{,}872r^3 + 41{,}616r^2 + 41{,}616r + 13{,}872$
- Evaluate one of the expressions when $r = 0.04$. For example, $13{,}872(1 + r)^3 = 13{,}872(1.04)^3$ or $15,604.11 to the nearest cent. The value given in the table is $15,604 rounded to the nearest dollar.

Pages 237–238, Lesson 5-3

21. $b^2 + 10b$

22. $x - 15$

23. $n^2 - 2n + 3$

24. $2c^2 + c + 5 + \frac{6}{c - 2}$

25. $x^3 - 5x^2 + 11x - 22 + \frac{39}{x + 2}$

26. $6w^4 + 12w^3 + 24w^2 + 30w + 60$

27. x^2

28. $x^2 + 3x + 9$

29. $y^2 - y - 1$

30. $m^2 - 7$

31. $a^3 - 6a^2 - 7a + 7 + \frac{3}{a + 1}$

32. $2m^3 + m^2 + 3m - 1 + \frac{5}{m - 3}$

33. $x^4 - 3x^3 + 2x^2 - 6x + 19 - \frac{56}{x + 3}$

34. $3c^4 - c^3 + 2c^2 - 4c + 9 - \frac{13}{c + 2}$

35. $g + 5$

36. $2b^2 - b - 1 + \frac{4}{b + 1}$

37. $t^4 + 2t^3 + 4t^2 + 5t + 10$

38. $y^4 - 2y^3 + 4y^2 - 8y + 16$

39. $3t^2 - 2t + 3$

40. $h^2 - 4h + 17 - \frac{51}{2h + 3}$

41. $3d^2 + 2d + 3 - \frac{2}{3d - 2}$

42. $x^2 + x - 1$

43. $x^3 - x - \frac{6}{2x + 3}$

44. $2x^3 + x^2 - 1 + \frac{2}{3x + 1}$

45. $x - 3$

46. $x^2 - 1 + \frac{-3x + 7}{x^2 + 2}$

47. $x + 2$

48. $x - 3$

59. Division of polynomials can be used to solve for unknown quantities in geometric formulas that apply to manufacturing situations. Answers should include the following.

- $8x$ in. by $4x + s$ in.
- The area of a rectangle is equal to the length times the width. That is, $A = \ell w$.
- Substitute $32x^2 + x$ for A, $8x$ for ℓ, and $4x + s$ for w. Solving for s involves dividing $32x^2 + x$ by $8x$.

$$A = \ell w$$
$$32x^2 + x = 8x(4x + s)$$
$$\frac{32x^2 + x}{8x} = 4x + s$$
$$4x + \frac{1}{8} = 4x + s$$
$$\frac{1}{8} = s$$

The seam is $\frac{1}{8}$ inch.

Page 243, Lesson 5-4

56. Factoring can be used to find possible dimensions of a geometric figure, given the area. Answers should include the following.

- Since the area of the rectangle is the product of its length and its width, the length and width are factors of the area. One set of possible dimensions is $4x - 2$ by $x + 3$.
- The complete factorization of the area is $2(2x - 1)(x + 3)$, so the factor of 2 could be placed with either $2x - 1$ or $x + 3$ when assigning the dimensions.

Page 262, Lesson 5-7

71. The equation that determines the size of the region around a planet where the planet's gravity is stronger than the Sun's can be written in terms of a fractional exponent. Answers should include the following.

- The radical form of the equation is $r = D\sqrt[5]{\left(\frac{M_p}{M_S}\right)^2}$ or $r = D\sqrt[5]{\frac{M_p^2}{M_S^2}}$. Multiply the fraction under the radical by $\frac{M_S^3}{M_S^3}$.

$$r = D\sqrt[5]{\frac{M_p^2}{M_S^2} \cdot \frac{M_S^3}{M_S^3}}$$

$$= D\sqrt[5]{\frac{M_p^2 M_S^3}{M_S^5}}$$

$$= D\,\frac{\sqrt[5]{M_p^2 M_S^3}}{\sqrt[5]{M_S^5}}$$

$$= \frac{D\sqrt[5]{M_p^2 M_S^3}}{M_S}$$

The simplified radical form is $\frac{D\sqrt[5]{M_p^2 M_S^3}}{M_S}$.

- If M_p and M_S are constant, then r increases as D increases because r is a linear function of D with positive slope.

Page 267, Lesson 5-8

44. If a company's cost and number of units manufactured are related by an equation involving radicals or rational exponents, then the production level associated with a given cost can be found by solving a radical equation. Answers should include the following.

- $C = 10\sqrt[3]{n^2} + 1500$
- $10{,}000 = 10n^{\frac{2}{3}} + 1500$ — $C = 10{,}000$

$8500 = 10n^{\frac{2}{3}}$	Subtract 1500 from each side.
$850 = n^{\frac{2}{3}}$	Divide each side by 10.
$850^{\frac{3}{2}} = n$	Raise each side to the $\frac{3}{2}$ power.
$24{,}781.55 \approx n$	Use a calculator.

Round down so that the cost does not exceed $10,000. The company can make at most 24,781 chips.

Page 275, Lesson 5-9

65. Case 1: $\boldsymbol{i} > 0$

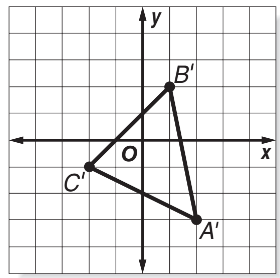
Multiply each side by $\boldsymbol{i}$ to get $\boldsymbol{i}^2 > 0 \cdot \boldsymbol{i}$ or $-1 > 0$. This is a contradiction.

Case 2: $\boldsymbol{i} < 0$

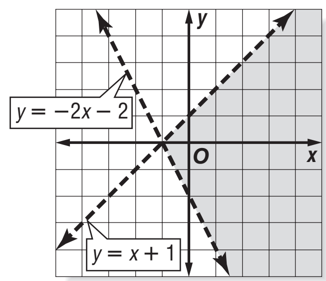
Since you are assuming $\boldsymbol{i}$ is negative in this case, you must change the inequality symbol when you multiply each side by $\boldsymbol{i}$. The result is again $\boldsymbol{i}^2 > 0 \cdot \boldsymbol{i}$ or $-1 > 0$, a contradiction.

Since both possible cases result in contradictions, the order relation "$<$" cannot be applied to the complex numbers.

80.



82.



83.