Chapter 6

Quadratic Functions and Inequalities

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
6-1 Graphing Quadratic Functions *(pp. 286–293)* - Graph quadratic functions. - Find and interpret the maximum and minimum values of a quadratic function.	1	1	0.5	0.5
6-2 Solving Quadratic Equations by Graphing *(pp. 294–300)* - Solve quadratic equations by graphing. - Estimate solutions of quadratic equations by graphing. ***Follow-Up:*** Modeling Real-World Data	1	2 (with 6-2 Follow-Up)	0.5	1
6-3 Solving Quadratic Equations by Factoring *(pp. 301–305)* - Solve quadratic equations by factoring. - Write a quadratic equation with given roots.	1	1	0.5	0.5
6-4 Completing the Square *(pp. 306–312)* - Solve quadratic equations by using the Square Root Property. - Solve quadratic equations by completing the square.	2	1	1	0.5
6-5 The Quadratic Formula and the Discriminant *(pp. 313–319)* - Solve quadratic equations by using the Quadratic Formula. - Use the discriminant to determine the number and type of roots of a quadratic equation.	1	1	0.5	0.5
6-6 Analyzing Graphs of Quadratic Functions *(pp. 320–328)* ***Preview:*** Families of Parabolas - Analyze quadratic functions of the form $y = a(x - h)^2 + k$. - Write a quadratic function in the form $y = a(x - h)^2 + k$.	2 (with 6-6 Preview)	1	1.5 (with 6-6 Preview)	0.5
6-7 Graphing and Solving Quadratic Inequalities *(pp. 329–335)* - Graph quadratic inequalities in two variables. - Solve quadratic inequalities in one variable.	1	1	0.5	0.5
Study Guide and **Practice Test** *(pp. 336–341)* **Standardized Test Practice** *(pp. 342–343)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	11	10	6	5

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools

TeacherWorks™

All-In-One Planner and Resource Center

See pages T12–T13.

Chapter Resource Manager

CHAPTER 6 RESOURCE MASTERS					Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment					
313–314	315–316	317	318			6-1	6-1	10	
319–320	321–322	323	324	369	SC 11	6-2	6-2		(*Follow-Up:* graphing calculator)
325–326	327–328	329	330			6-3	6-3		grid paper
331–332	333–334	335	336	369, 371		6-4	6-4		algebra tiles
337–338	339–340	341	342		GCS 38	6-5	6-5	11, 12	posterboard
343–344	345–346	347	348	370		6-6	6-6		(*Preview:* graphing calculator) graphing calculator, index cards
349–350	351–352	353	354	370	GCS 37, SC 12	6-7	6-7		
				355–368, 372–374					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 6

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

In the previous chapter students explored how to factor quadratic expressions, how to work with radicals, and how to perform operations on complex numbers. They have also solved linear equations and inequalities, and they are familiar with using formulas.

This Chapter

Students solve quadratic equations by graphing, by factoring, by completing the square, and by using the Quadratic Formula. They explore how values of a quadratic equation are reflected in the parabola that represents it, and use equations and graphs to explore quadratic equations that have 0, 1, or 2 roots. They relate the value of the discriminant to the number of roots and to whether the roots are rational, irrational, or complex.

Future Connections

Students will continue to explore roots and zeros of equations, examining higher-order polynomial equations in Chapter 7. Students will learn to recognize other types of equations that can be solved using the Quadratic Formula. They will explore systems of quadratic inequalities in Chapter 8.

6-1 Graphing Quadratic Functions

When graphing a quadratic function, use all the available information to produce the most accurate possible graph. This includes a table of values containing the vertex and the y-intercept. Be sure the points on the graph are connected with a smooth curve and that the graph has a U-shape and not a V-shape at the vertex of the graph. Unlike the letter U, however, the graph should become progressively wider and have arrows indicating that the graph continues to infinity.

6-2 Solving Quadratic Equations by Graphing

It is important to distinguish between finding solutions or roots of an equation and finding zeros of its related function. An equation of the form $ax^2 + bx + c = 0$ has a related function, $f(x) = ax^2 + bx + c$. The zeros of $f(x)$ are the x-coordinates of the points where the graph crosses the x-axis. These x values are the solutions of the related quadratic equation. Without the use of a graphing calculator, this method of solving quadratic equations will usually provide only an estimate of solutions. Solutions that appear to be integers should be verified by substituting them into the original equation.

6-3 Solving Quadratic Equations by Factoring

When solving a quadratic equation by factoring, it is important to review factoring techniques. These include the techniques for factoring general trinomials, perfect square trinomials, and a difference of squares. Also remember to look for a greatest common factor (GCF) that might be factored out or the possibility of factoring by grouping. Before factoring, the equation should be rewritten so that one side of the equation is 0. If the GCF of the terms of the polynomial being factored is a variable or the product of a number and a variable, such as $3x$, one solution to the equation is 0.

6-4 Complete the Square

Completing the square is a technique most often used to solve quadratic equations that are not factorable. To use this technique it is desirable to rewrite the equation so that it is equal to a constant. Then divide the coefficient of the linear term by 2 and square the result. Add this value to both sides of the equation. One side of the equation will now be a perfect square trinomial that can be rewritten as the square of a binomial. To help isolate the variable on one side of the equation, take the square root of both sides, remembering that the square root of the constant on one side of the equation will result in two values, one positive and one negative, represented by a ± sign. Finally, isolate the variable using the Addition, Subtraction, Multiplication, and/or Division Properties of Equality. Simplify any solution that involves the symbol ± by simplifying two separate expressions, one using the plus sign and the other using the minus sign. When using the technique of completing the square, be sure that the coefficient of the quadratic term is 1. If it is not 1, divide both sides of the equation by that coefficient. You can then solve the equation by completing the square.

6-5 The Quadratic Formula and the Discriminant

While the technique of completing the square can be used to solve any quadratic equation, implementing this technique can lead to operations involving unwieldy fractions. The Quadratic Formula can also be used to solve any quadratic equation. Using this formula, the variable is isolated in the very first step and the other steps involve simplifying the solutions by simplifying radicals and fractions. The discriminant is simply the portion of the Quadratic Formula that appears underneath the radical, $b^2 - 4ac$. This value alone will determine the number and type of roots (solutions) of the equation. This is because the square root of this value could result in a rational number, such as 6, an irrational number, such as $\sqrt{2}$, the value 0, or an imaginary number, such as $3\boldsymbol{i}$. By finding and examining just the value of the discriminant, you can tell very quickly what type of solutions a quadratic equation will have. This can serve as a check when solving the equation.

6-6 Analyzing Graphs of Quadratic Functions

To write a quadratic equation in the form $y = a(x - h)^2 + k$, called *vertex form*, it is important to remember that an equation is a statement of equality. When an equation is rewritten in a different form, this equality must be maintained. In previous lessons, if a value was added to one side of an equation, it was also added to the other side of the equation in order to maintain equality. Another way to maintain equality is to add a value to one side and then subtract that same value from that side. For example, if an addition of 5 is shown on the right side of an equation, a subtraction of 5 would also be shown on the right side. So in essence this would be an overall addition of $5 - 5$ or 0, which does not affect the equality of the statement. For example, $y = 3x$ is equivalent to the statement $y = 3x + 5 - 5$. Be especially careful when adding a value inside a set of parentheses, since the use of parentheses often involves multiplication. For example $y = 3(x)$ is not equivalent to $y = 3(x + 1) - 1$, but instead to $y = 3(x + 1) - 3(1)$.

6-7 Graphing and Solving Quadratic Inequalities

One way of solving a linear inequality not discussed in Chapter 1 is to first solve its related linear equation and then test values on either side of this value in the original inequality. For example, to solve $3x + 2 > -4$, you would solve the equation $3x + 2 = -4$ and find that $x = -2$. Testing a value less than -2 and a value greater than -2 in the inequality reveals that the solution to the inequality is the set of values greater than -2. The approach to solving a quadratic inequality algebraically is similar. The difference lies in the fact that many quadratic inequalities have not one but two solutions. This means that your number line is divided in three possible solution sets. Testing a value from each interval on the number line reveals which solution set or sets are correct. The solution set of a quadratic inequality will often be a compound inequality, so you will want to review this topic from Chapter 1.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Solving Quadratic Equations by Graphing (Lesson 31)
- Solving Equations by Factoring (Lesson 27)
- Solving Quadratic Equations by Completing the Square (Lesson 39)
- Solving Quadratic Equations by Using the Quadratic Formula (Lesson 32)
- Graphing Technology: Parent and Family Graphs (Lesson 29)
- Graphing Quadratic Functions (Lesson 28)
- More on Axis of Symmetry and Vertices (Lesson 30)

Chapter 6

DAILY INTERVENTION and Assessment

	Type	Student Edition	Teacher Resources	Technology/Internet
INTERVENTION	Ongoing	Prerequisite Skills, pp. 285, 293, 299, 305, 312, 319, 328 Practice Quiz 1, p. 305 Practice Quiz 2, p. 328	5-Minute Check Transparencies Quizzes, *CRM* pp. 369–370 Mid-Chapter Test, *CRM* p. 371 Study Guide and Intervention, *CRM* pp. 313–314, 319–320, 325–326, 331–332, 337–338, 343–344, 349–350	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
	Mixed Review	pp. 293, 299, 305, 312, 319, 328, 335	Cumulative Review, *CRM* p. 372	
	Error Analysis	Find the Error, pp. 303, 310, 325 Common Misconceptions, pp. 289, 308	Find the Error, *TWE* pp. 303, 310, 325 Unlocking Misconceptions, *TWE* pp. 288, 295 Tips for New Teachers, *TWE* pp. 288, 305, 312, 323	
	Standardized Test Practice	pp. 292, 293, 299, 302, 303, 305, 312, 319, 327, 335, 341, 342–343	*TWE* p. 302 Standardized Test Practice, *CRM* pp. 373–374	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
ASSESSMENT	Open-Ended Assessment	Writing in Math, pp. 292, 299, 305, 312, 319, 327, 334 Open Ended, pp. 290, 297, 303, 317, 325, 332	Modeling: *TWE* pp. 299, 319 Speaking: *TWE* pp. 293, 305, 335 Writing: *TWE* pp. 312, 328 Open-Ended Assessment, *CRM* p. 367	
	Chapter Assessment	Study Guide, pp. 336–340 Practice Test, p. 341	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 355–360 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 361–366 Vocabulary Test/Review, *CRM* p. 368	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
6-1	**10** *Graphing Quadratic Equations*
6-5	**11** *Solving Quadratic Equations Using the Quadratic Formula*
6-5	**12** *Solving Word Problems Using Quadratic Equations*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 285
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 290, 297, 303, 310, 317, 325, 332, 336)
- Writing in Math questions in every lesson, pp. 292, 299, 305, 312, 319, 327, 334
- Reading Study Tip, pp. 306, 313, 316
- WebQuest, p. 328

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 285, 336
- Study Notebook suggestions, pp. 290, 297, 303, 310, 317, 325, 332
- Modeling activities, pp. 299, 319
- Speaking activities, pp. 293, 305, 335
- Writing activities, pp. 312, 328
- Differentiated Instruction, (Verbal/Linguistic), p. 296
- ELL Resources, pp. 284, 292, 296, 298, 304, 311, 318, 327, 334, 336

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 6 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 6 Resource Masters*, pp. 317, 323, 329, 335, 341, 347, 353)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 6 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

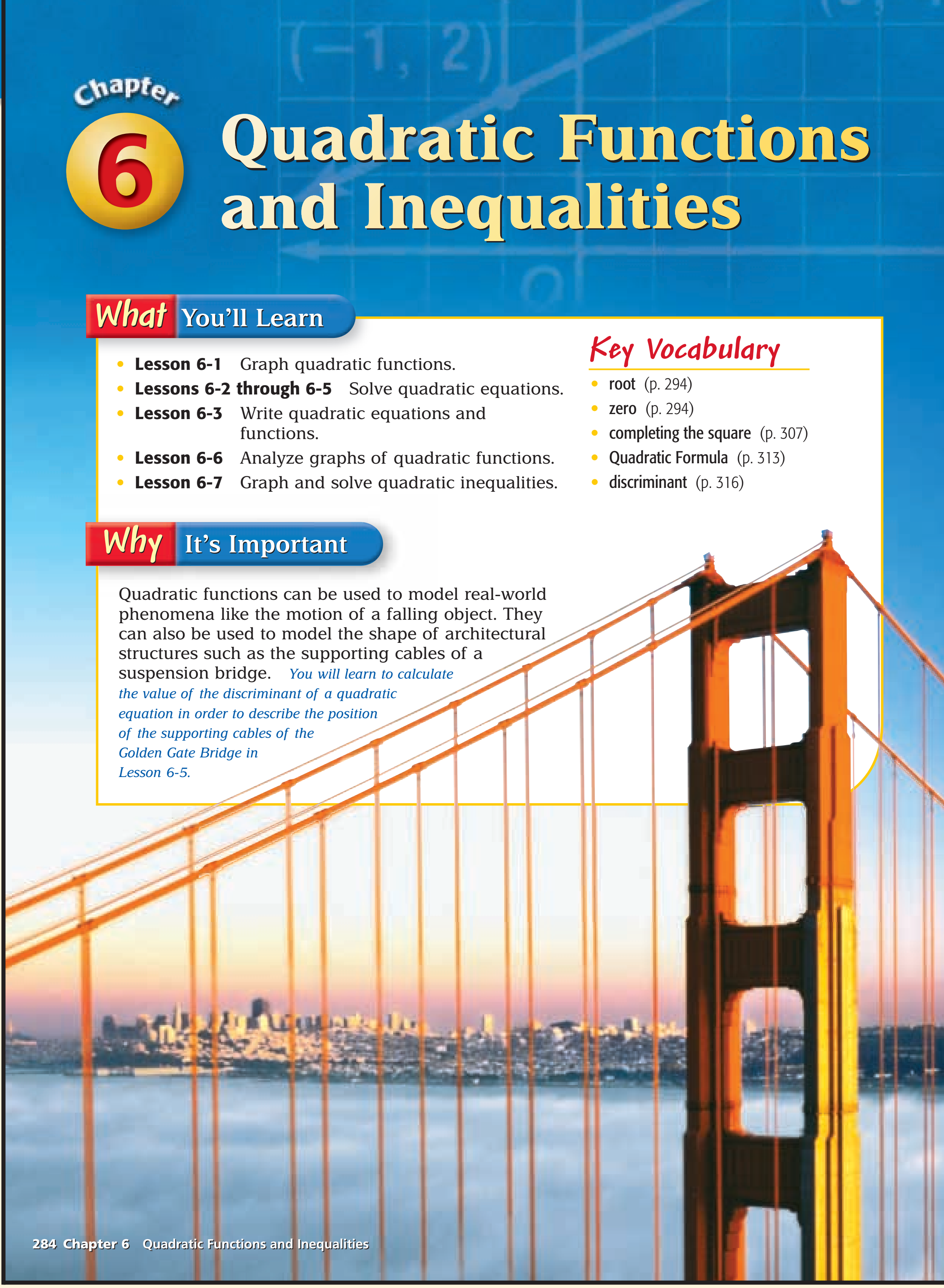
Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
6-1	2, 3, 6, 8, 9, 10	
6-2	1, 2, 6, 8, 9, 10	
6-2 Follow-Up	2, 5, 6, 8, 10	
6-3	1, 2, 3, 6, 7, 8, 9	
6-4	1, 2, 3, 6, 7, 8, 9, 10	
6-5	1, 2, 6, 8, 9	
6-6 Preview	2, 8, 10	
6-6	2, 6, 7, 8, 9, 10	
6-7	2, 3, 6, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation



Chapter 6 Quadratic Functions and Inequalities

What You'll Learn

- **Lesson 6-1** Graph quadratic functions.
- **Lessons 6-2 through 6-5** Solve quadratic equations.
- **Lesson 6-3** Write quadratic equations and functions.
- **Lesson 6-6** Analyze graphs of quadratic functions.
- **Lesson 6-7** Graph and solve quadratic inequalities.

Key Vocabulary

- root (p. 294)
- zero (p. 294)
- completing the square (p. 307)
- Quadratic Formula (p. 313)
- discriminant (p. 316)

Why It's Important

Quadratic functions can be used to model real-world phenomena like the motion of a falling object. They can also be used to model the shape of architectural structures such as the supporting cables of a suspension bridge. *You will learn to calculate the value of the discriminant of a quadratic equation in order to describe the position of the supporting cables of the Golden Gate Bridge in Lesson 6-5.*

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 6 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 6 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lessons 6-1 and 6-2 Graph Functions

Graph each equation by making a table of values. *(For review, see Lesson 2-1.)*

1. $y = 2x + 3$ **2.** $y = -x - 5$ **3.** $y = x^2 + 4$ **4.** $y = -x^2 - 2x + 1$

1–4. See pp. 343A-343F.

For Lessons 6-1, 6-2, and 6-5 Multiply Polynomials

Find each product. *(For review, see Lesson 5-2.)*

5. $(x - 4)(7x + 12)$ $7x^2 - 16x - 48$
6. $(x + 5)^2$ $x^2 + 10x + 25$
7. $(3x - 1)^2$ $9x^2 - 6x + 1$
8. $(3x - 4)(2x - 9)$ $6x^2 - 35x + 36$

For Lessons 6-3 and 6-4 Factor Polynomials

Factor completely. If the polynomial is not factorable, write *prime*. *(For review, see Lesson 5-4.)*

9. $x^2 + 11x + 30$ **10.** $x^2 - 13x + 36$ **11.** $x^2 - x - 56$ **12.** $x^2 - 5x - 14$

13. $x^2 + x + 2$ **prime**
14. $x^2 + 10x + 25$ $(x + 5)^2$
15. $x^2 - 22x + 121$ $(x - 11)^2$
16. $x^2 - 9$ $(x + 3)(x - 3)$

For Lessons 6-4 and 6-5 Simplify Radical Expressions

Simplify. *(For review, see Lessons 5-6 and 5-9.)*

17. $\sqrt{225}$ 15 **18.** $\sqrt{48}$ $4\sqrt{3}$ **19.** $\sqrt{180}$ $6\sqrt{5}$ **20.** $\sqrt{68}$ $2\sqrt{17}$

21. $\sqrt{-25}$ $5i$ **22.** $\sqrt{-32}$ $4i\sqrt{2}$ **23.** $\sqrt{-270}$ $3i\sqrt{30}$ **24.** $\sqrt{-15}$ $i\sqrt{15}$

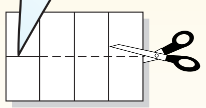
9. $(x + 6)(x + 5)$ **10.** $(x - 4)(x - 9)$ **11.** $(x - 8)(x + 7)$ **12.** $(x + 2)(x - 7)$

FOLDABLES™ Study Organizer

Make this Foldable to record information about quadratic functions and inequalities. Begin with one sheet of 11" × 17" paper.

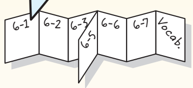
Step 1 Fold and Cut

Fold in half lengthwise. Then fold in fourths crosswise. Cut along the middle fold from the edge to the last crease as shown.



Step 2 Refold and Label

Refold along lengthwise fold and staple uncut section at top. Label the section with a lesson number and close to form a booklet.



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for each lesson.

Getting Started

This section provides a review of the basic concepts needed before beginning Chapter 6. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

For Lesson	Prerequisite Skill
6-2	Evaluating Functions (p. 293)
6-3	Factoring Trinomials (p. 299)
6-4	Simplifying Radicals (p. 305)
6-5	Evaluating Expressions (p. 312)
6-6	Perfect Square Trinomials (p. 319)
6-7	Inequalities (p. 328)

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Sequencing Information and Progression of Knowledge After students make their Foldable, have them label a section for each lesson in Chapter 6 and a section for vocabulary. As students progress through the lessons, have them summarize key concepts and note the order in which they are presented. Ask students to write about why the concepts were presented in this sequence. If they cannot see the logic in the sequence, have them reorder the key concepts and justify their reasoning.

6-1 Lesson Notes

1 Focus

5-Minute Check Transparency 6-1 Use as a quiz or a review of Chapter 5.

Mathematical Background notes are available for this lesson on p. 284C.

Building on Prior Knowledge

In Chapter 5, students wrote and solved various equations and inequalities. In this lesson, they will relate quadratic equations to their graphs.

How can income from a rock concert be maximized?

Ask students:

- How is the income represented in the given function? **by $P(x)$**
- What is significant about the value of $P(x)$ when $x = 40$ (the ticket price of \$40)? **The value of $P(x)$ is greatest when $x = 40$, or the income is at its maximum value when $x = 40$.**

6-1 Graphing Quadratic Functions

What You'll Learn

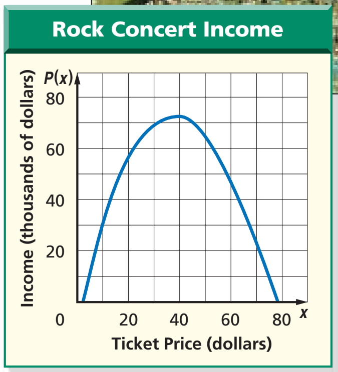
- Graph quadratic functions.
- Find and interpret the maximum and minimum values of a quadratic function.

Vocabulary

- quadratic function
- quadratic term
- linear term
- constant term
- parabola
- axis of symmetry
- vertex
- maximum value
- minimum value

How can income from a rock concert be maximized?

Rock music managers handle publicity and other business issues for the artists they manage. One group's manager has found that based on past concerts, the predicted income for a performance is $P(x) = -50x^2 + 4000x - 7500$, where x is the price per ticket in dollars. The graph of this quadratic function is shown at the right. Notice that at first the income increases as the price per ticket increases, but as the price continues to increase, the income declines.



GRAPH QUADRATIC FUNCTIONS

A **quadratic function** is described by an equation of the following form.

$$f(x) = \underbrace{ax^2}_{\text{quadratic term}} + \overbrace{bx}^{\text{linear term}} + \underbrace{c}_{\text{constant term}}, \text{ where } a \neq 0$$

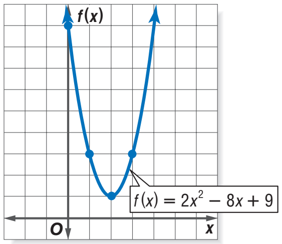
The graph of any quadratic function is called a **parabola**. One way to graph a quadratic function is to graph ordered pairs that satisfy the function.

Example 1 Graph a Quadratic Function

Graph $f(x) = 2x^2 - 8x + 9$ by making a table of values.

First, choose integer values for x. Then, evaluate the function for each x value. Graph the resulting coordinate pairs and connect the points with a smooth curve.

x	$2x^2 - 8x + 9$	$f(x)$	$(x, f(x))$
0	$2(0)^2 - 8(0) + 9$	9	(0, 9)
1	$2(1)^2 - 8(1) + 9$	3	(1, 3)
2	$2(2)^2 - 8(2) + 9$	1	(2, 1)
3	$2(3)^2 - 8(3) + 9$	3	(3, 3)
4	$2(4)^2 - 8(4) + 9$	9	(4, 9)



Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 313–314
- Skills Practice, p. 315
- Practice, p. 316
- Reading to Learn Mathematics, p. 317
- Enrichment, p. 318

Teaching Algebra With Manipulatives Masters, p. 243

Transparencies

5-Minute Check Transparency 6-1
Answer Key Transparencies

Technology

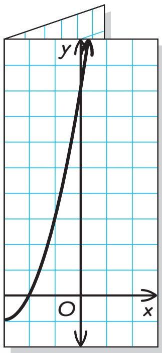
Alge2PASS: Tutorial Plus, Lesson 10
Interactive Chalkboard

TEACHING TIP

Tell students they will derive the equation for the axis of symmetry in Lesson 6-6, Exercise 53, after they have learned about a technique called completing the square.

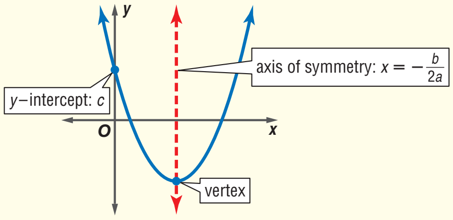
All parabolas have an **axis of symmetry**. If you were to fold a parabola along its axis of symmetry, the portions of the parabola on either side of this line would match.

The point at which the axis of symmetry intersects a parabola is called the **vertex**. The y-intercept of a quadratic function, the equation of the axis of symmetry, and the x-coordinate of the vertex are related to the equation of the function as shown below.



Key Concept *Graph of a Quadratic Function*

- **Words** Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.
  - The y-intercept is $a(0)^2 + b(0) + c$ or c.
  - The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
  - The x-coordinate of the vertex is $-\frac{b}{2a}$.
- **Model**



Knowing the location of the axis of symmetry, y-intercept, and vertex can help you graph a quadratic function.

Example 2 Axis of Symmetry, y-Intercept, and Vertex

Consider the quadratic function $f(x) = x^2 + 9 + 8x$.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

Begin by rearranging the terms of the function so that the quadratic term is first, the linear term is second, and the constant term is last. Then identify a, b, and c.

$$\begin{array}{lcl} & & f(x) = ax^2 + bx + c \\ & & \qquad\ \downarrow \qquad \downarrow \quad\ \ \downarrow \\ f(x) = x^2 + 9 + 8x & \rightarrow & f(x) = 1x^2 + 8x + 9 \end{array}$$

So, $a = 1$, $b = 8$, and $c = 9$.

The y-intercept is 9. You can find the equation of the axis of symmetry using a and b.

$x = -\frac{b}{2a}$ Equation of the axis of symmetry

$x = -\frac{8}{2(1)}$ $a = 1$, $b = 8$

$x = -4$ Simplify.

The equation of the axis of symmetry is $x = -4$. Therefore, the x-coordinate of the vertex is -4.

www.algebra2.com/extra_examples

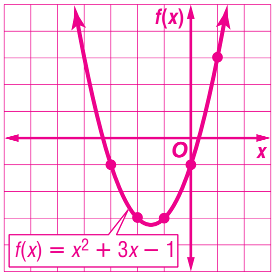
2 Teach

GRAPH QUADRATIC FUNCTIONS

In-Class Examples Power Point®

1 Graph $f(x) = x^2 + 3x - 1$ by making a table of values.

x	−3	−2	−1	0	1
$f(x)$	−1	−3	−3	−1	3



Teaching Tip Make sure students understand that the graph shows values for all the points that satisfy the function, even when the x value is not an integer. For example, the vertex is $(-1.5, -3.25)$.

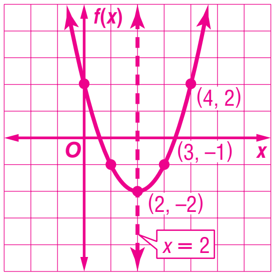
2 Consider the quadratic function $f(x) = 2 - 4x + x^2$.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex. **2; $x = 2$; 2**

b. Make a table of values that includes the vertex.

x	0	1	2	3	4
$f(x)$	2	−1	−2	−1	2

c. Use this information to graph the function.



Tips for New Teachers

Intervention When discussing Example 2, make sure students realize that $f(x)$ and y can be used interchangeably, and also that the maximum or minimum value of the function is given by the y-coordinate of the vertex of the parabola.

MAXIMUM AND MINIMUM VALUES

In-Class Example

Power Point®

3 Consider the function $f(x) = -x^2 + 2x + 3$.

a. Determine whether the function has a maximum or a minimum value. **maximum**

b. State the maximum or minimum value of the function. **4**

Teaching Tip Remind students to use the equation for the axis of symmetry to determine the x-coordinate of the vertex and then to find the y-coordinate of the vertex.

Interactive Chalkboard

PowerPoint® Presentations

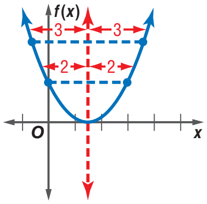
This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

Study Tip

Symmetry

Sometimes it is convenient to use symmetry to help find other points on the graph of a parabola. Each point on a parabola has a mirror image located the same distance from the axis of symmetry on the other side of the parabola.



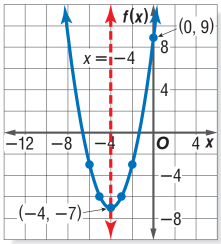
b. Make a table of values that includes the vertex.

Choose some values for x that are less than -4 and some that are greater than -4. This ensures that points on each side of the axis of symmetry are graphed.

x	$x^2 + 8x + 9$	$f(x)$	$(x, f(x))$	
-6	$(-6)^2 + 8(-6) + 9$	-3	$(-6, -3)$	
-5	$(-5)^2 + 8(-5) + 9$	-6	$(-5, -6)$	
-4	$(-4)^2 + 8(-4) + 9$	-7	$(-4, -7)$	← Vertex
-3	$(-3)^2 + 8(-3) + 9$	-6	$(-3, -6)$	
-2	$(-2)^2 + 8(-2) + 9$	-3	$(-2, -3)$	

c. Use this information to graph the function.

Graph the vertex and y-intercept. Then graph the points from your table connecting them and the y-intercept with a smooth curve. As a check, draw the axis of symmetry, $x = -4$, as a dashed line. The graph of the function should be symmetrical about this line.

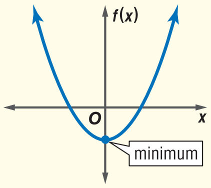


MAXIMUM AND MINIMUM VALUES The y-coordinate of the vertex of a quadratic function is the **maximum value** or **minimum value** obtained by the function.

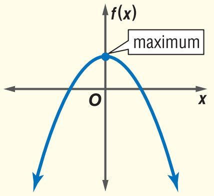
Key Concept ***Maximum and Minimum Value***

- **Words** The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,
  - opens up and has a minimum value when $a > 0$, and
  - opens down and has a maximum value when $a < 0$.
- **Models**

a is positive.



a is negative.



Example 3 ***Maximum or Minimum Value***

Consider the function $f(x) = x^2 - 4x + 9$.

a. Determine whether the function has a maximum or a minimum value.

For this function, $a = 1$, $b = -4$, and $c = 9$. Since $a > 0$, the graph opens up and the function has a minimum value.

DAILY INTERVENTION

Unlocking Misconceptions

Minimum and Maximum Values Make sure students understand that a parabola which opens upward is the graph of a function with a minimum value and that a parabola which opens downward is the graph of a function with a maximum value. Compare these parabolas to valleys (where the altitude of the valley floor is a minimum) and hills (where the peak of the hill is the maximum altitude).

Study Tip

Common Misconception

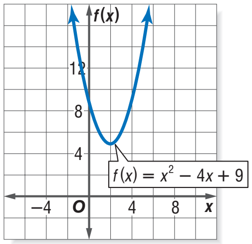
The terms minimum *point* and minimum *value* are not interchangeable. The minimum point on the graph of a quadratic function is the set of coordinates that describe the location of the vertex. The minimum value of a function is the y-coordinate of the minimum point. It is the smallest value obtained when $f(x)$ is evaluated for all values of x.

b. State the maximum or minimum value of the function.

The minimum value of the function is the y-coordinate of the vertex.

The x-coordinate of the vertex is $-\frac{-4}{2(1)}$ or 2.

Find the y-coordinate of the vertex by evaluating the function for $x = 2$.



$f(x) = x^2 - 4x + 9$ Original function

$f(2) = (2)^2 - 4(2) + 9$ or 5 $x = 2$

Therefore, the minimum value of the function is 5.

When quadratic functions are used to model real-world situations, their maximum or minimum values can have real-world meaning.

Example 4 Find a Maximum Value

FUND-RAISING **Four hundred people came to last year's winter play at Sunnybrook High School. The ticket price was $5. This year, the Drama Club is hoping to earn enough money to take a trip to a Broadway play. They estimate that for each $0.50 increase in the price, 10 fewer people will attend their play.**

a. How much should the tickets cost in order to maximize the income from this year's play?

Words The income is the number of tickets multiplied by the price per ticket.

Variables Let x = the number of $0.50 price increases.
Then $5 + 0.50x$ = the price per ticket and
$400 - 10x$ = the number of tickets sold.

Let $I(x)$ = income as a function of x.

Equation

$$\underbrace{I(x)}_{\text{The income}} \underbrace{=}_{\text{is}} \underbrace{(400 - 10x)}_{\text{the number of tickets}} \underbrace{\cdot}_{\text{multiplied by}} \underbrace{(5 + 0.50x)}_{\text{the price per ticket.}}$$

$$= 400(5) + 400(0.50x) - 10x(5) - 10x(0.50x)$$

$= 2000 + 200x - 50x - 5x^2$ Multiply.

$= 2000 + 150x - 5x^2$ Simplify.

$= -5x^2 + 150x + 2000$ Rewrite in $ax^2 + bx + c$ form.

$I(x)$ is a quadratic function with $a = -5$, $b = 150$, and $c = 2000$. Since $a < 0$, the function has a maximum value at the vertex of the graph. Use the formula to find the x-coordinate of the vertex.

x-coordinate of the vertex $= -\frac{b}{2a}$ Formula for the x-coordinate of the vertex

$= -\frac{150}{2(-5)}$ $a = -5, b = 150$

$= 15$ Simplify.

This means the Drama Club should make 15 price increases of $0.50 to maximize their income. Thus, the ticket price should be 5 + 0.50(15) or $12.50.

(continued on the next page)

More About. . .

Fund-Raising

The London Marathon, which has been run through the streets of London, England, annually since 1981, has historically raised more money than any other charity sports event. In 2000, this event raised an estimated £20 million ($31.6 million U.S. dollars).

Source: Guinness World Records

In-Class Example

Power Point®

4 **ECONOMICS** A souvenir shop sells about 200 coffee mugs each month for $6 each. The shop owner estimates that for each $0.50 increase in the price, he will sell about 10 fewer coffee mugs per month.

a. How much should the owner charge for each mug in order to maximize the monthly income from their sales? **$8**

b. What is the maximum monthly income the owner can expect to make from these items? **$1280**

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Ask students to suggest the kinds of musical sounds they might associate with a parabola that has a maximum value, and have them contrast this to a sound that might be associated with a parabola that has a minimum value. For example, an orchestra piece that rises to a crescendo and then gradually returns to the previous volume might be seen as being related to a parabola with a maximum value.

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.
- add the information in the Key Concept box on p. 287 about the graph of a quadratic function to their notebook.
- make labeled sketches similar to those on p. 288 illustrating the maximum and minimum values of the graphs of quadratic functions.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Graph Quadratic Functions:** 14–31, 44
- **Maximum and Minimum Values:** 32–43, 45–53

Odd/Even Assignments

Exercises 14–31 and 32–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 58–63 require a graphing calculator.

Assignment Guide

Basic: 15–27 odd, 33–43 odd, 44–47, 54–57, 64–78

Average: 15–43 odd, 46–50, 53–57, 64–78 (optional: 58–63)

Advanced: 14–42 even, 46–74 (optional: 75–78)

---

b. What is the maximum income the Drama Club can expect to make?

To determine maximum income, find the maximum value of the function by evaluating $I(x)$ for $x = 15$.

$I(x) = -5x^2 + 150x + 2000$ Income function

$I(15) = -5(15)^2 + 150(15) + 2000$ $x = 15$

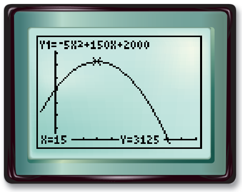
$= 3125$ Use a calculator.

Thus, the maximum income the Drama Club can expect is $3125.

CHECK Graph this function on a graphing calculator, and use the CALC menu to confirm this solution.

KEYSTROKES: 2nd [CALC] 4

0 ENTER 25 ENTER ENTER



```
Y1=-5X2+150X+2000
X=15            Y=3125
```

[−5, 50] scl: 5 by [−100, 4000] scl: 500

At the bottom of the display are the coordinates of the maximum point on the graph of $y = -5x^2 + 150x + 2000$. The y value of these coordinates is the maximum value of the function, or 3125. ✓

Check for Understanding

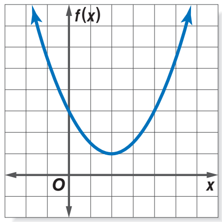
Concept Check

1. Sample answer: $f(x) = 3x^2 + 5x - 6$; $3x^2$, $5x$, -6

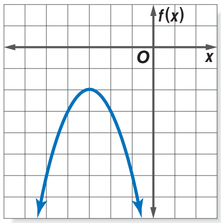
1. **OPEN ENDED** Give an example of a quadratic function. Identify its quadratic term, linear term, and constant term.

2. **Identify** the vertex and the equation of the axis of symmetry for each function graphed below. **a. (2, 1); $x = 2$ b. (−3, −2); $x = -3$**

a.



b.



3. **State** whether the graph of each quadratic function opens *up* or *down*. Then state whether the function has a *maximum* or *minimum* value.

a. $f(x) = 3x^2 + 4x - 5$ **up; min.**

b. $f(x) = -2x^2 + 9$ **down; max.**

c. $f(x) = -5x^2 - 8x + 2$ **down; max.**

d. $f(x) = 6x^2 - 5x$ **up; min.**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–9	1, 2
10–12	3
13	4

Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function. 4–9. See margin.

4. $f(x) = -4x^2$

5. $f(x) = x^2 + 2x$

6. $f(x) = -x^2 + 4x - 1$

7. $f(x) = x^2 + 8x + 3$

8. $f(x) = 2x^2 - 4x + 1$

9. $f(x) = 3x^2 + 10x$

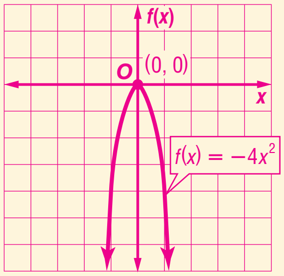
Answers

4a. 0; $x = 0$; 0

4b.

x	$f(x)$
−1	−4
0	0
1	−4

4c.

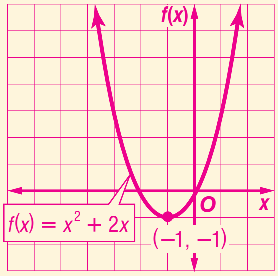


5a. 0; $x = -1$; −1

5b.

x	$f(x)$
−3	3
−2	0
−1	−1
0	0
1	3

5c.



Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

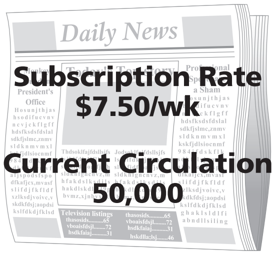
10. $f(x) = -x^2 + 7$ **max.; 7**

11. $f(x) = x^2 - x - 6$ **min.; $-\frac{25}{4}$**

12. $f(x) = 4x^2 + 12x + 9$ **min.; 0**

Application

13. **NEWSPAPERS** Due to increased production costs, the Daily News must increase its subscription rate. According to a recent survey, the number of subscriptions will decrease by about 1250 for each 25¢ increase in the subscription rate. What weekly subscription rate will maximize the newspaper's income from subscriptions? **$8.75**



★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
14–19	1
20–31	2
32–43, 54	3
44–53	4

Extra Practice

See page 839.

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**

b. **Make a table of values that includes the vertex.**

c. **Use this information to graph the function.** **14–31. See pp. 343A–343F.**

14. $f(x) = 2x^2$

15. $f(x) = -5x^2$

16. $f(x) = x^2 + 4$

17. $f(x) = x^2 - 9$

18. $f(x) = 2x^2 - 4$

19. $f(x) = 3x^2 + 1$

20. $f(x) = x^2 - 4x + 4$

21. $f(x) = x^2 - 9x + 9$

22. $f(x) = x^2 - 4x - 5$

23. $f(x) = x^2 + 12x + 36$

24. $f(x) = 3x^2 + 6x - 1$

25. $f(x) = -2x^2 + 8x - 3$

26. $f(x) = -3x^2 - 4x$

27. $f(x) = 2x^2 + 5x$

★28. $f(x) = 0.5x^2 - 1$

★29. $f(x) = -0.25x^2 - 3x$

★30. $f(x) = \frac{1}{2}x^2 + 3x + \frac{9}{2}$

★31. $f(x) = x^2 - \frac{2}{3}x - \frac{8}{9}$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

32. $f(x) = 3x^2$ **min.; 0**

33. $f(x) = -x^2 - 9$ **max.; −9**

34. $f(x) = x^2 - 8x + 2$ **min.; −14**

35. $f(x) = x^2 + 6x - 2$ **min.; −11**

36. $f(x) = 4x - x^2 + 1$ **max.; 5**

37. $f(x) = 3 - x^2 - 6x$ **max.; 12**

38. $f(x) = 2x + 2x^2 + 5$ **min.; $\frac{9}{2}$**

39. $f(x) = x - 2x^2 - 1$ **max.; $-\frac{7}{8}$**

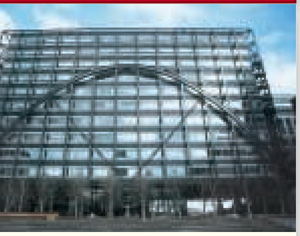
40. $f(x) = -7 - 3x^2 + 12x$ **max.; 5**

41. $f(x) = -20x + 5x^2 + 9$ **min.; −11**

42. $f(x) = -\frac{1}{2}x^2 - 2x + 3$ **max.; 5**

43. $f(x) = \frac{3}{4}x^2 - 5x - 2$ **min.; $-10\frac{1}{3}$**

More About. . .



Architecture

The Exchange House in London, England, is supported by two interior and two exterior steel arches. V-shaped braces add stability to the structure.

Source: Council on Tall Buildings and Urban Habitat

ARCHITECTURE **For Exercises 44 and 45, use the following information.**
The shape of each arch supporting the Exchange House can be modeled by $h(x) = -0.025x^2 + 2x$, where $h(x)$ represents the height of the arch and x represents the horizontal distance from one end of the base in meters.

44. Write the equation of the axis of symmetry, and find the coordinates of the vertex of the graph of $h(x)$. **$x = 40$; (40, 40)**

45. According to this model, what is the maximum height of the arch? **40 m**

www.algebra2.com/self_check_quiz

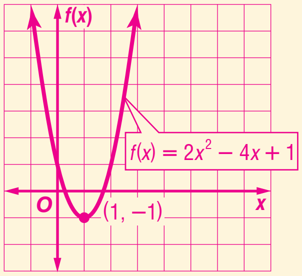
Answers

8a. 1; $x = 1$; 1

8b.

x	$f(x)$
−1	7
0	1
1	−1
2	1
3	7

8c.

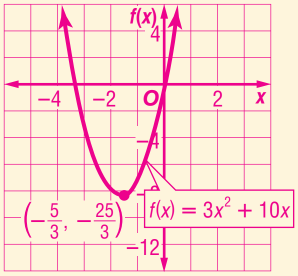


9a. 0; $x = -\frac{5}{3}$; $-\frac{5}{3}$

9b.

x	$f(x)$
−3	−3
−2	−8
$-\frac{5}{3}$	$-\frac{25}{3}$
−1	−7
0	0

9c.

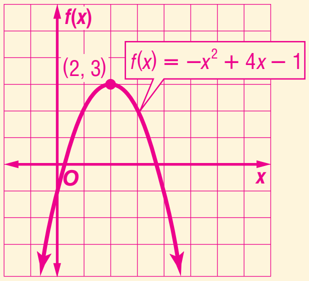


6a. −1; $x = 2$; 2

6b.

x	$f(x)$
0	−1
1	2
2	3
3	2
4	−1

6c.

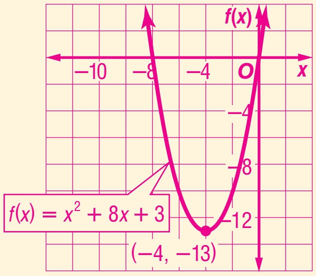


7a. 3; $x = -4$; −4

7b.

x	$f(x)$
−6	−9
−5	−12
−4	−13
−3	−12
−2	−9

7c.

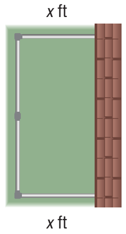


PHYSICS **For Exercises 46 and 47, use the following information.**
An object is fired straight up from the top of a 200-foot tower at a velocity of 80 feet per second. The height $h(t)$ of the object t seconds after firing is given by $h(t) = -16t^2 + 80t + 200$.

46. Find the maximum height reached by the object and the time that the height is reached. **300 ft, 2.5 s**

47. Interpret the meaning of the y-intercept in the context of this problem. **The y-intercept is the initial height of the object.**

CONSTRUCTION **For Exercises 48–50, use the following information.** **49. 60 ft by 30 ft**
Steve has 120 feet of fence to make a rectangular kennel for his dogs. He will use his house as one side.



48. Write an algebraic expression for the kennel's length. **120 – 2x**

49. What dimensions produce a kennel with the greatest area?

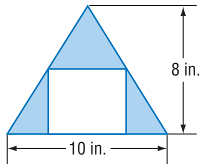
50. Find the maximum area of the kennel. **1800 ft^2**

TOURISM **For Exercises 51 and 52, use the following information.**
A tour bus in the historic district of Savannah, Georgia, serves 300 customers a day. The charge is $8 per person. The owner estimates that the company would lose 20 passengers a day for each $1 fare increase.

51. What charge would give the most income for the company? **$11.50**

52. If the company raised their fare to this price, how much daily income should they expect to bring in? **$2645**

★ 53. **GEOMETRY** A rectangle is inscribed in an isosceles triangle as shown. Find the dimensions of the inscribed rectangle with maximum area. (*Hint:* Use similar triangles.) **5 in. by 4 in.**



54. **CRITICAL THINKING** Write an expression for the minimum value of a function of the form $y = ax^2 + c$, where $a > 0$. Explain your reasoning. Then use this function to find the minimum value of $y = 8.6x^2 - 12.5$. **c; See margin for explanation; –12.5.**

55. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can income from a rock concert be maximized?

Include the following in your answer:

- an explanation of why income increases and then declines as the ticket price increases, and
- an explanation of how to algebraically and graphically determine what ticket price should be charged to achieve maximum income.

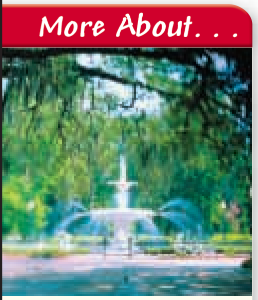
Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

56. The graph of which of the following equations is symmetrical about the y-axis? **C**

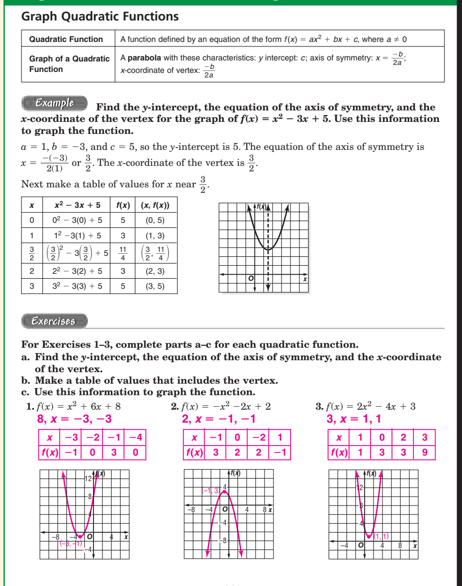
Ⓐ $y = x^2 + 3x - 1$	Ⓑ $y = -x^2 + x$
Ⓒ $y = 6x^2 + 9$	Ⓓ $y = 3x^2 - 3x + 1$

More About. . .



Tourism
Known as the Hostess City of the South, Savannah, Georgia, is a popular tourist destination. One of the first planned cities in the Americas, Savannah's Historic District is based on a grid-like pattern of streets and alleys surrounding open spaces called squares.
Source: savannah-online.com

Study Guide and Intervention, p. 313 (shown) and p. 314



Graph Quadratic Functions

Quadratic Function	A function defined by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$
Graph of a Quadratic Function	A **parabola** with these characteristics: y intercept: c; axis of symmetry: $x = \frac{-b}{2a}$; x-coordinate of vertex: $\frac{-b}{2a}$

Example **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for the graph of $f(x) = x^2 - 3x + 5$. Use this information to graph the function.**

$a = 1$, $b = -3$, and $c = 5$, so the y-intercept is 5. The equation of the axis of symmetry is $x = \frac{-(-3)}{2(1)}$ or $\frac{3}{2}$. The x-coordinate of the vertex is $\frac{3}{2}$.

Next make a table of values for x near $\frac{3}{2}$.

x	$x^2 - 3x + 5$	$f(x)$	$(x, f(x))$
0	$0^2 - 3(0) + 5$	5	(0, 5)
1	$1^2 - 3(1) + 5$	3	(1, 3)
$\frac{3}{2}$	$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 5$	$\frac{11}{4}$	$\left(\frac{3}{2}, \frac{11}{4}\right)$
2	$2^2 - 3(2) + 5$	3	(2, 3)
3	$3^2 - 3(3) + 5$	5	(3, 5)

Exercises

For Exercises 1–3, complete parts a–c for each quadratic function.
a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

1. $f(x) = x^2 + 6x + 8$
8, $x = -3$, –3

x	–3	–2	–1	–4
$f(x)$	–1	0	3	0

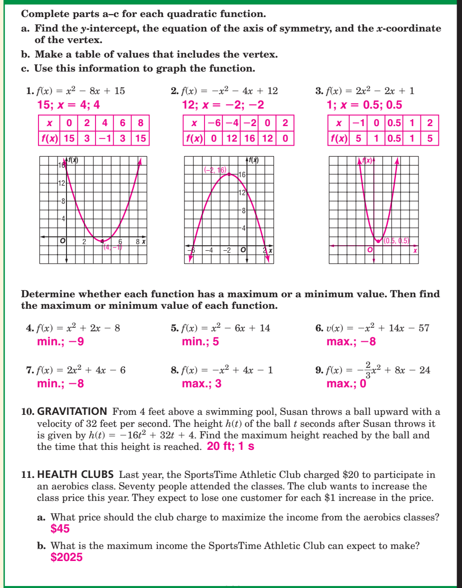
2. $f(x) = -x^2 - 2x + 2$
2, $x = -1$, –1

x	–1	0	–2	1
$f(x)$	3	2	2	–1

3. $f(x) = 2x^2 - 4x + 3$
3, $x = 1$, 1

x	1	0	2	3
$f(x)$	1	3	3	9

Skills Practice, p. 315 and Practice, p. 316 (shown)



Complete parts a–c for each quadratic function.
a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

1. $f(x) = x^2 - 8x + 15$
15; $x = 4$; 4

x	0	2	4	6	8
$f(x)$	15	3	–1	3	15

2. $f(x) = -x^2 - 4x + 12$
12; $x = -2$; –2

x	–6	–4	–2	0	2
$f(x)$	0	12	16	12	0

3. $f(x) = 2x^2 - 2x + 1$
1; $x = 0.5$; 0.5

x	–1	0	0.5	1	2
$f(x)$	5	1	0.5	1	5

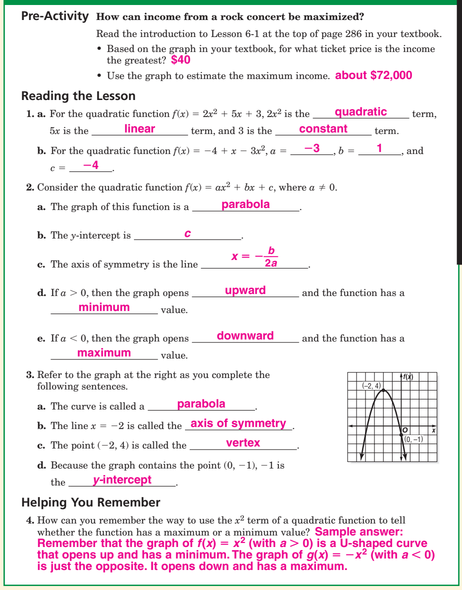
Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

4. $f(x) = x^2 + 2x - 8$ **min.; –9**
5. $f(x) = x^2 - 6x + 14$ **min.; 5**
6. $v(x) = -x^2 + 14x - 57$ **max.; –8**
7. $f(x) = 2x^2 + 4x - 6$ **min.; –8**
8. $f(x) = -x^2 + 4x - 1$ **max.; 3**
9. $f(x) = -\frac{2}{3}x^2 + 8x - 24$ **max.; 0**

10. **GRAVITATION** From 4 feet above a swimming pool, Susan throws a ball upward with a velocity of 32 feet per second. The height $h(t)$ of the ball t seconds after Susan throws it is given by $h(t) = -16t^2 + 32t + 4$. Find the maximum height reached by the ball and the time that this height is reached. **20 ft; 1 s**

11. **HEALTH CLUBS** Last year, the SportsTime Athletic Club charged $20 to participate in an aerobics class. Seventy people attended the classes. The club wants to increase the class price this year. They expect to lose one customer for each $1 increase in the price.
    a. What price should the club charge to maximize the income from the aerobics classes? **$45**
    b. What is the maximum income the SportsTime Athletic Club can expect to make? **$2025**

Reading to Learn Mathematics, p. 317 ELL



Pre-Activity **How can income from a rock concert be maximized?**

Read the introduction to Lesson 6-1 at the top of page 286 in your textbook.

- Based on the graph in your textbook, for what ticket price is the income the greatest? **$40**
- Use the graph to estimate the maximum income. **about $72,000**

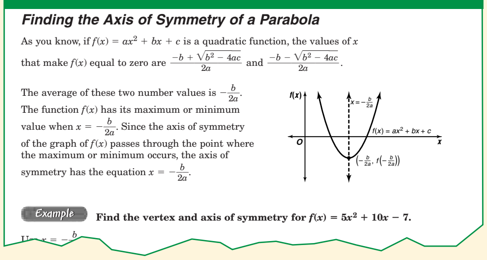
Reading the Lesson

1. a. For the quadratic function $f(x) = 2x^2 + 5x + 3$, $2x^2$ is the **quadratic** term, $5x$ is the **linear** term, and 3 is the **constant** term.
   b. For the quadratic function $f(x) = -4 + x - 3x^2$, $a =$ **–3**, $b =$ **1**, and $c =$ **–4**.
2. Consider the quadratic function $f(x) = ax^2 + bx + c$, where $a \neq 0$.
   a. The graph of this function is a **parabola**.
   b. The y-intercept is **c**.
   c. The axis of symmetry is the line **$x = -\frac{b}{2a}$**.
   d. If $a > 0$, then the graph opens **upward** and the function has a **minimum** value.
   e. If $a < 0$, then the graph opens **downward** and the function has a **maximum** value.
3. Refer to the graph at the right as you complete the following sentences.
   a. The curve is called a **parabola**.
   b. The line $x = -2$ is called the **axis of symmetry**.
   c. The point $(-2, 4)$ is called the **vertex**.
   d. Because the graph contains the point $(0, -1)$, -1 is the **y-intercept**.

Helping You Remember

4. How can you remember the way to use the x^2 term of a quadratic function to tell whether the function has a maximum or a minimum value? **Sample answer: Remember that the graph of $f(x) = x^2$ (with $a > 0$) is a U-shaped curve that opens up and has a minimum. The graph of $g(x) = -x^2$ (with $a < 0$) is just the opposite. It opens down and has a maximum.**

Enrichment, p. 318



Finding the Axis of Symmetry of a Parabola

As you know, if $f(x) = ax^2 + bx + c$ is a quadratic function, the values of x that make $f(x)$ equal to zero are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

The average of these two number values is $-\frac{b}{2a}$. The function $f(x)$ has its maximum or minimum value when $x = -\frac{b}{2a}$. Since the axis of symmetry of the graph of $f(x)$ passes through the point where the maximum or minimum occurs, the axis of symmetry has the equation $x = -\frac{b}{2a}$.

Example **Find the vertex and axis of symmetry for $f(x) = 5x^2 + 10x - 7$.**

Answer

54. The x-coordinate of the vertex of $y = ax^2 + c$ is $-\frac{0}{2a}$ or 0, so the y-coordinate of the vertex, the minimum of the function, is $a(0)^2 + c$ or c.

57. Which of the following tables represents a quadratic relationship between the two variables x and y? **C**

Ⓐ

x	1	2	3	4	5
y	3	3	3	3	3

Ⓑ

x	1	2	3	4	5
y	5	4	3	2	1

Ⓒ

x	1	2	3	4	5
y	6	3	2	3	6

Ⓓ

x	1	2	3	4	5
y	−4	−3	−4	−3	−4

Graphing Calculator

MAXIMA AND MINIMA You can use the MINIMUM or MAXIMUM feature on a graphing calculator to find the minimum or maximum value of a quadratic function. This involves defining an interval that includes the vertex of the parabola. A lower bound is an x value left of the vertex, and an upper bound is an x value right of the vertex.

Step 1 Graph the function so that the vertex of the parabola is visible.

Step 2 Select 3:minimum or 4:maximum from the CALC menu.

Step 3 Using the arrow keys, locate a left bound and press ENTER.

Step 4 Locate a right bound and press ENTER twice. The cursor appears on the maximum or minimum value of the function, and the coordinates are displayed.

Find the coordinates of the maximum or minimum value of each quadratic function to the nearest hundredth.

58. $f(x) = 3x^2 - 7x + 2$ **−2.08**

59. $f(x) = -5x^2 + 8x$ **3.2**

60. $f(x) = 2x^2 - 3x + 2$ **0.88**

61. $f(x) = -6x^2 + 9x$ **3.38**

62. $f(x) = 7x^2 + 4x + 1$ **0.43**

63. $f(x) = -4x^2 + 5x$ **1.56**

Maintain Your Skills

Mixed Review

Simplify. *(Lesson 5-9)*

64. i^{14} **−1**

65. $(4 - 3i) - (5 - 6i)$ **−1 + 3*i***

66. $(7 + 2i)(1 - i)$ **9 − 5*i***

Solve each equation. *(Lesson 5-8)*

67. $5 - \sqrt{b + 2} = 0$ **23**

68. $\sqrt[3]{x + 5} + 6 = 4$ **−13**

69. $\sqrt{n + 12} - \sqrt{n} = 2$ **4**

Perform the indicated operations. *(Lesson 4-2)*

70. $[4 \quad 1 \quad -3] + [6 \quad -5 \quad 8]$ $[10 \quad -4 \quad 5]$

71. $[2 \quad -5 \quad 7] - [-3 \quad 8 \quad -1]$ $[5 \quad -13 \quad 8]$

72. $4\begin{bmatrix} -7 & 5 & -11 \\ 2 & -4 & 9 \end{bmatrix}$ $\begin{bmatrix} -28 & 20 & -44 \\ 8 & -16 & 36 \end{bmatrix}$

73. $-2\begin{bmatrix} -3 & 0 & 12 \\ -7 & \frac{1}{3} & 4 \end{bmatrix}$ $\begin{bmatrix} 6 & 0 & -24 \\ 14 & -\frac{2}{3} & -8 \end{bmatrix}$

74. Graph the system of equations $y = -3x$ and $y - x = 4$. State the solution. Is the system of equations *consistent* and *independent*, *consistent* and *dependent*, or *inconsistent*? *(Lesson 3-1)* **See margin for graph; (−1, 3); consistent and independent.**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each function for the given value.**

*(To review **evaluating functions**, see Lesson 2-1.)*

75. $f(x) = x^2 + 2x - 3, x = 2$ **5**

76. $f(x) = -x^2 - 4x + 5, x = -3$ **8**

77. $f(x) = 3x^2 + 7x, x = -2$ **−2**

78. $f(x) = \frac{2}{3}x^2 + 2x - 1, x = -3$ **−1**

4 Assess

Open-Ended Assessment

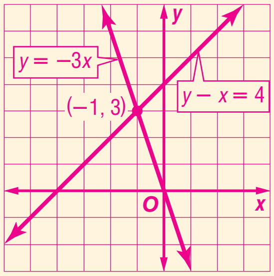
Speaking Ask students to explain how to tell by examining a quadratic function whether its graph will have a maximum or minimum value. Then ask them to give an example of what such a value might mean in a real-world problem.

Getting Ready for Lesson 6-2

PREREQUISITE SKILL Lesson 6-2 presents solving quadratic equations by graphing. Finding points on the graph of the function involves evaluating quadratic functions. Exercises 75–78 should be used to determine your students' familiarity with evaluating functions.

Answer

74.



Answer

55. If a quadratic function can be used to model ticket price versus profit, then by finding the x-coordinate of the vertex of the parabola you can determine the price per ticket that should be charged to achieve maximum profit. Answers should include the following.

- If the price of a ticket is too low, then you won't make enough money to cover your costs, but if the ticket price is too high fewer people will buy them.
- You can locate the vertex of the parabola on the graph of the function. It occurs when $x = 40$. Algebraically, this is found by calculating $x = -\frac{b}{2a}$ which, for this case, is $x = \frac{-4000}{2(-50)}$ or 40. Thus the ticket price should be set at $40 each to achieve maximum profit.

6-2 Lesson Notes

1 Focus

5-Minute Check Transparency 6-2 Use as a quiz or a review of Lesson 6-1.

Mathematical Background notes are available for this lesson on p. 284C.

How does a quadratic function model a free-fall ride?

Ask students:

- The acceleration of a free-falling object due to Earth's gravity is −32 ft/sec^2. It is given as a negative value because the acceleration is downward, toward Earth's surface. How is this fact represented in the height function? **The coefficient −16 is the one half of the acceleration due to gravity in a downward direction.**
- How far has a person fallen 1 second after beginning a free fall? after 2 seconds? after 3 seconds? **16 ft; 64 ft; 144 ft**

6-2 Solving Quadratic Equations by Graphing

What You'll Learn

- Solve quadratic equations by graphing.
- Estimate solutions of quadratic equations by graphing.

Vocabulary

- quadratic equation
- root
- zero

How does a quadratic function model a free-fall ride?

As you speed to the top of a free-fall ride, you are pressed against your seat so that you feel like you're being pushed downward. Then as you free-fall, you fall at the same rate as your seat. Without the force of your seat pressing on you, you *feel* weightless. The height above the ground (in feet) of an object in free-fall can be determined by the quadratic function $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height is h_0 feet.

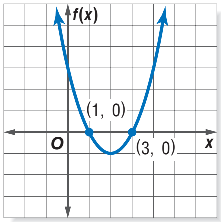
SOLVE QUADRATIC EQUATIONS When a quadratic function is set equal to a value, the result is a quadratic equation. A **quadratic equation** can be written in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Study Tip

Reading Math

In general, equations have roots, functions have zeros, and graphs of functions have x-intercepts.

The solutions of a quadratic equation are called the **roots** of the equation. One method for finding the roots of a quadratic equation is to find the **zeros** of the related quadratic function. The zeros of the function are the x-intercepts of its graph. These are the solutions of the related equation because $f(x) = 0$ at those points. The zeros of the function graphed at the right are 1 and 3.

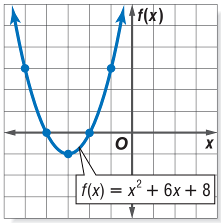


Example 1 Two Real Solutions

Solve $x^2 + 6x + 8 = 0$ by graphing.

Graph the related quadratic function $f(x) = x^2 + 6x + 8$. The equation of the axis of symmetry is $x = -\frac{6}{2(1)}$ or −3. Make a table using x values around −3. Then, graph each point.

x	−5	−4	−3	−2	−1
f(x)	3	0	−1	0	3



From the table and the graph, we can see that the zeros of the function are −4 and −2. Therefore, the solutions of the equation are −4 and −2.

CHECK Check the solutions by substituting each solution into the equation to see if it is satisfied.

$$x^2 + 6x + 8 = 0 \qquad\qquad x^2 + 6x + 8 = 0$$
$$(-4)^2 + 6(-4) + 8 \stackrel{?}{=} 0 \qquad\qquad (-2)^2 + 6(-2) + 8 \stackrel{?}{=} 0$$
$$0 = 0\ \checkmark \qquad\qquad 0 = 0\ \checkmark$$

The graph of the related function in Example 1 had two zeros; therefore, the quadratic equation had two real solutions. This is one of the three possible outcomes when solving a quadratic equation.

Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 319–320
- Skills Practice, p. 321
- Practice, p. 322
- Reading to Learn Mathematics, p. 323
- Enrichment, p. 324
- Assessment, p. 369

School-to-Career Masters, p. 11

Transparencies

5-Minute Check Transparency 6-2
Answer Key Transparencies

Technology

Interactive Chalkboard

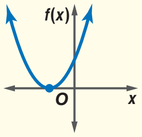
Study Tip

One Real Solution
When a quadratic equation has one real solution, it really has two solutions that are the same number.

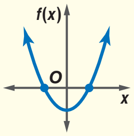
Key Concept — Solutions of a Quadratic Equation

- **Words** A quadratic equation can have one real solution, two real solutions, or no real solution.
- **Models**

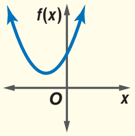
One Real Solution



Two Real Solutions



No Real Solution



Example 2 One Real Solution

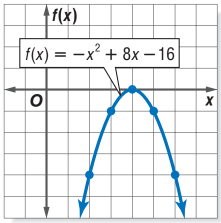
Solve $8x - x^2 = 16$ by graphing.

Write the equation in $ax^2 + bx + c = 0$ form.

$8x - x^2 = 16 \rightarrow -x^2 + 8x - 16 = 0$ Subtract 16 from each side.

Graph the related quadratic function $f(x) = -x^2 + 8x - 16$.

x	2	3	4	5	6
f(x)	−4	−1	0	−1	−4



Notice that the graph has only one x-intercept, 4. Thus, the equation's only solution is 4.

Example 3 No Real Solution

NUMBER THEORY **Find two real numbers whose sum is 6 and whose product is 10 or show that no such numbers exist.**

Explore Let x = one of the numbers. Then $6 - x$ = the other number.

Plan Since the product of the two numbers is 10, you know that $x(6 - x) = 10$.

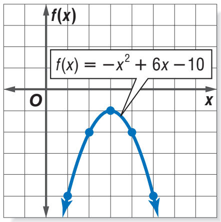
$x(6 - x) = 10$ Original equation

$6x - x^2 = 10$ Distributive Property

$-x^2 + 6x - 10 = 0$ Subtract 10 from each side.

Solve You can solve $-x^2 + 6x - 10 = 0$ by graphing the related function $f(x) = -x^2 + 6x - 10$.

x	1	2	3	4	5
f(x)	−5	−2	−1	−2	−5



Notice that the graph has no x-intercepts. This means that the original equation has no real solution. Thus, it is *not* possible for two numbers to have a sum of 6 and a product of 10.

Examine Try finding the product of several pairs of numbers whose sum is 6. Is the product of each pair less than 10 as the graph suggests?

www.algebra2.com/extra_examples

DAILY INTERVENTION

Unlocking Misconceptions

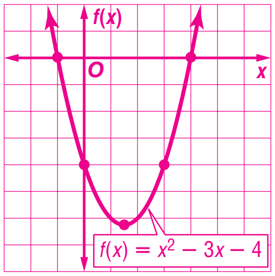
Equations and Functions Some students may notice that the equation derived in Example 2, $-x^2 + 8x - 16 = 0$, is equivalent to the equation $x^2 - 8x + 16 = 0$. Either equation can be produced from the other by multiplying each side by −1. These two equations have the same solution, 4. However, stress that the related functions, $f(x) = -x^2 + 8x - 16$ and $f(x) = x^2 - 8x + 16$ are *not* equivalent. This can be seen by looking at their graphs, which open in opposite directions.

2 Teach

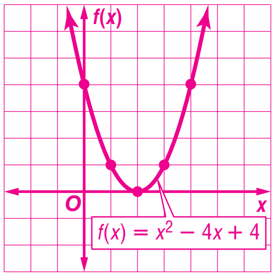
SOLVE QUADRATIC EQUATIONS

In-Class Examples Power Point®

1 Solve $x^2 - 3x - 4 = 0$ by graphing. **−1 and 4**

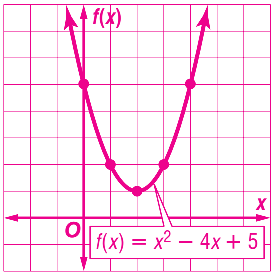


2 Solve $x^2 - 4x = -4$ by graphing. **2**



Teaching Tip In Example 3, inform students that while this equation does not have any real solutions, it does have a solution in the set of complex numbers, the topic of Lesson 5-9. Such equations will be discussed again in Lessons 6-4 and 6-5.

3 **NUMBER THEORY** Find two real numbers whose sum is 4 and whose product is 5 or show that no such numbers exist.



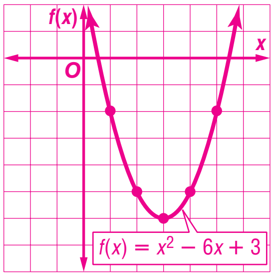
The graph of the related function does not intersect the *x*-axis. Therefore, no such numbers exist.

ESTIMATE SOLUTIONS

In-Class Examples

Power Point®

4 Solve $x^2 - 6x + 3 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. **One solution is between 0 and 1, and the other is between 5 and 6.**



5 **ROYAL GORGE BRIDGE** The highest bridge in the United States is the Royal Gorge Bridge in Colorado. The deck of the bridge is 1053 feet above the river below. Suppose a marble is dropped over the railing from a height of 3 feet above the bridge deck. How long will it take the marble to reach the surface of the water, assuming there is no air resistance? Use the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and h_0 is the initial height above the water in feet. **about 8 s**

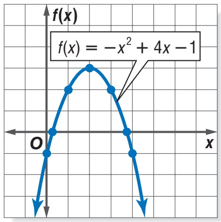
ESTIMATE SOLUTIONS Often exact roots cannot be found by graphing. In this case, you can estimate solutions by stating the consecutive integers between which the roots are located.

Example 4 Estimate Roots

Solve $-x^2 + 4x - 1 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

The equation of the axis of symmetry of the related function is $x = -\frac{4}{2(-1)}$ or 2.

x	0	1	2	3	4
f(x)	−1	2	3	2	−1



The x-intercepts of the graph are between 0 and 1 and between 3 and 4. So, one solution is between 0 and 1, and the other is between 3 and 4.

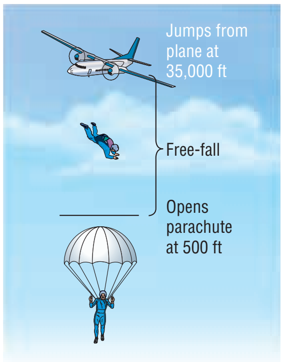
Study Tip

Location of Roots
Notice in the table of values that the value of the function changes from negative to positive between the x values of 0 and 1, and 3 and 4.

For many applications, an exact answer is not required, and approximate solutions are adequate. Another way to estimate the solutions of a quadratic equation is by using a graphing calculator.

Example 5 Write and Solve an Equation

EXTREME SPORTS **On March 12, 1999, Adrian Nicholas broke the world record for the longest human flight. He flew 10 miles from his drop point in 4 minutes 55 seconds using a specially designed, aerodynamic suit. Using the information at the right and ignoring air resistance, how long would Mr. Nicholas have been in free-fall had he not used this special suit? Use the formula $h(t) = -16t^2 + h_0$, where the time t is in seconds and the initial height h_0 is in feet.**



We need to find t when $h_0 = 35{,}000$ and $h(t) = 500$. Solve $500 = -16t^2 + 35{,}000$.

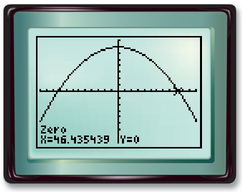
$500 = -16t^2 + 35{,}000$ Original equation

$0 = -16t^2 + 34{,}500$ Subtract 500 from each side.

Graph the related function $y = -16t^2 + 34{,}500$ using a graphing calculator. Adjust your window so that the x-intercepts of the graph are visible.

Use the ZERO feature, 2nd [CALC], to find the positive zero of the function, since time cannot be negative. Use the arrow keys to locate a left bound for the zero and press ENTER.

Then, locate a right bound and press ENTER twice. The positive zero of the function is approximately 46.4. Mr. Nicholas would have been in free-fall for about 46 seconds.



```
Zero
X=46.435439   Y=0
```

[−60, 60] scl: 5 by
[−40000, 40000] scl: 5000

DAILY INTERVENTION

Differentiated Instruction ELL

Verbal/Linguistic Have students discuss with a partner or in a small group the methods for multiplying and dividing monomial expressions with exponents, and also numbers written in scientific notation. Ask them to work together to develop a list of common errors for such problems, and to suggest ways to correct and avoid these errors.

Check for Understanding

Concept Check

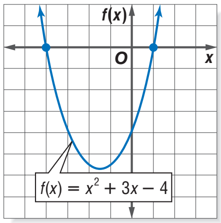
1. **Define** each term and explain how they are related. **See margin.**
   - **a.** solution
   - **b.** root
   - **c.** zero of a function
   - **d.** x-intercept
2. **OPEN ENDED** Give an example of a quadratic function and state its related quadratic equation. **Sample answer: $f(x) = 3x^2 + 2x - 1$; $3x^2 + 2x - 1 = 0$**
3. **Explain** how you can estimate the solutions of a quadratic equation by examining the graph of its related function. **See margin.**

Guided Practice

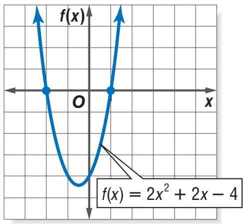
GUIDED PRACTICE KEY	
Exercises	**Examples**
4–6	1–3
7–12	1–4
13	3

Use the related graph of each equation to determine its solutions.

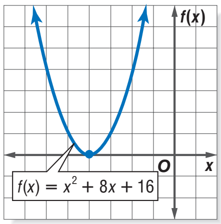
4. $x^2 + 3x - 4 = 0$ **−4, 1**



5. $2x^2 + 2x - 4 = 0$ **−2, 1**



6. $x^2 + 8x + 16 = 0$ **−4**



Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

7. $-x^2 - 7x = 0$ **−7, 0**
8. $x^2 - 2x - 24 = 0$ **−4, 6**
9. $x^2 + 3x = 28$ **−7, 4**
10. $25 + x^2 + 10x = 0$ **−5**
11. $4x^2 - 7x - 15 = 0$ **between −2 and −1; 3**
12. $2x^2 - 2x - 3 = 0$

12. between −1 and 0; between 1 and 2

Application

13. **NUMBER THEORY** Use a quadratic equation to find two real numbers whose sum is 5 and whose product is −14, or show that no such numbers exist. **−2, 7**

★ indicates increased difficulty

Practice and Apply

Homework Help

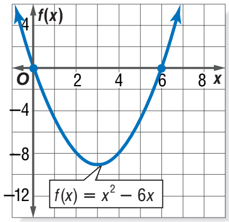
For Exercises	See Examples
14–19	1–3
20–37	1–4
38–41	3
42–46	5

Extra Practice

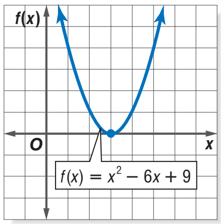
See page 840.

Use the related graph of each equation to determine its solutions.

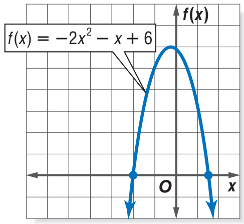
14. $x^2 - 6x = 0$ **0, 6**



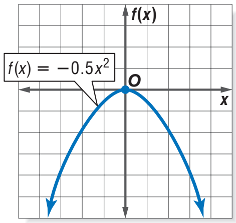
15. $x^2 - 6x + 9 = 0$ **3**



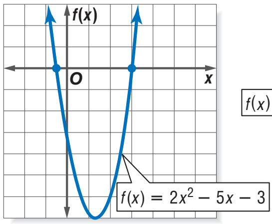
16. $-2x^2 - x + 6 = 0$ **$-2, 1\frac{1}{2}$**



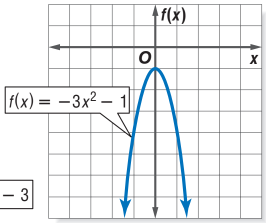
17. $-0.5x^2 = 0$ **0**



18. $2x^2 - 5x - 3 = 0$ **$-\frac{1}{2}, 3$**



19. $-3x^2 - 1 = 0$ **no real solutions**



www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.
- add the graphic representations of the three possible types of real solutions of a quadratic equation shown in the Key Concept box on p. 295.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Solve Quadratic Equations:** 14–21, 24–31, 36–41
- **Estimate Solutions:** 22, 23, 32–35, 42–46

Odd/Even Assignments

Exercises 14–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 51–56 require a graphing calculator.

Assignment Guide

Basic: 15–45 odd, 47–50, 57–72

Average: 15–45 odd, 47–50, 57–72 (optional: 51–56)

Advanced: 14–46 even, 47–66 (optional: 67–72)

Answers

1a. The solution is the value that satisfies an equation.

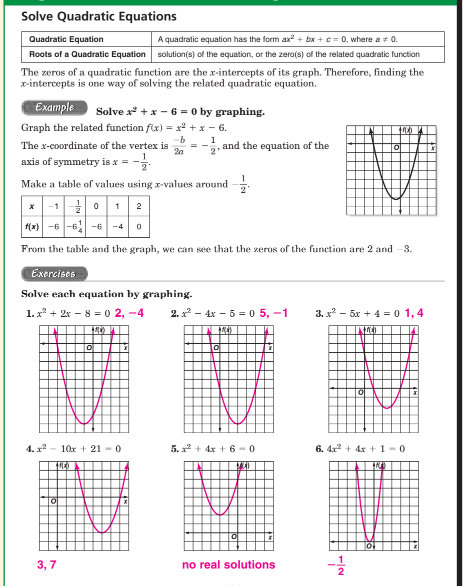
1b. A root is a solution of an equation.

1c. A zero is the x value of a function that makes the function equal to 0.

1d. An x-intercept is the point at which a graph crosses the x-axis. The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x-intercepts of its graph.

3. The x-intercepts of the related function are the solutions to the equation. You can estimate the solutions by stating the consecutive integers between which the x-intercepts are located.

Study Guide and Intervention, p. 319 (shown) and p. 320



Solve Quadratic Equations

Quadratic Equation	A quadratic equation has the form $ax^2 + bx + c = 0$, where $a \neq 0$.
Roots of a Quadratic Equation	solution(s) of the equation, or the zero(s) of the related quadratic function

The zeros of a quadratic function are the x-intercepts of its graph. Therefore, finding the x-intercepts is one way of solving the related quadratic equation.

Example **Solve $x^2 + x - 6 = 0$ by graphing.**

Graph the related function $f(x) = x^2 + x - 6$.

The x-coordinate of the vertex is $\frac{-b}{2a} = -\frac{1}{2}$, and the equation of the axis of symmetry is $x = -\frac{1}{2}$.

Make a table of values using x-values around $-\frac{1}{2}$.

x	−1	$-\frac{1}{2}$	0	1	2
f(x)	−6	$-6\frac{1}{4}$	−6	−4	0

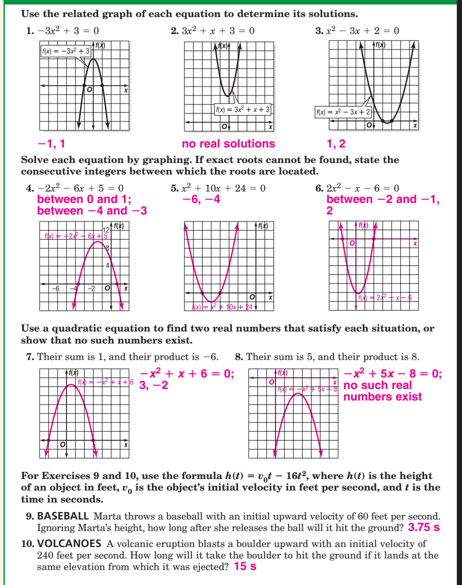
From the table and the graph, we can see that the zeros of the function are 2 and −3.

Exercises

Solve each equation by graphing.

1. $x^2 + 2x - 8 = 0$ **2, −4**
2. $x^2 - 4x - 5 = 0$ **5, −1**
3. $x^2 - 5x + 4 = 0$ **1, 4**
4. $x^2 - 10x + 21 = 0$ **3, 7**
5. $x^2 + 4x + 6 = 0$ **no real solutions**
6. $4x^2 + 4x + 1 = 0$ $-\frac{1}{2}$

Skills Practice, p. 321 and Practice, p. 322 (shown)



Use the related graph of each equation to determine its solutions.

1. $-3x^2 + 3 = 0$ **−1, 1**
2. $3x^2 + x + 3 = 0$ **no real solutions**
3. $x^2 - 3x + 2 = 0$ **1, 2**

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

4. $-2x^2 - 6x + 5 = 0$ **between 0 and 1; between −4 and −3**
5. $x^2 + 10x + 24 = 0$ **−6, −4**
6. $2x^2 - x - 6 = 0$ **between −2 and −1, 2**

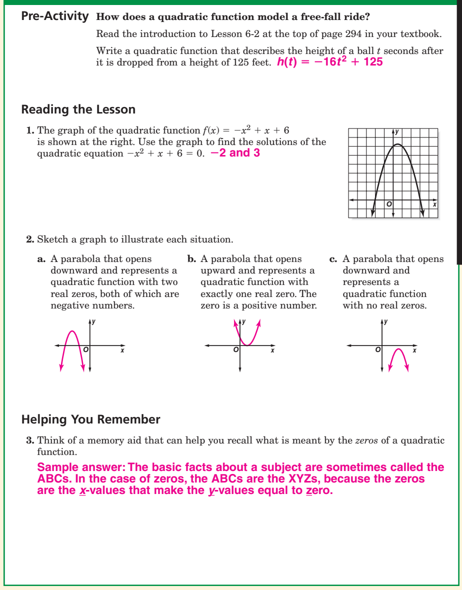
Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is 1, and their product is −6. **$-x^2 + x + 6 = 0$; 3, −2**
8. Their sum is 5, and their product is 8. **$-x^2 + 5x - 8 = 0$; no such real numbers exist**

For Exercises 9 and 10, use the formula $h(t) = v_0t - 16t^2$, where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

9. **BASEBALL** Marta throws a baseball with an initial upward velocity of 60 feet per second. Ignoring Marta's height, how long after she releases the ball will it hit the ground? **3.75 s**
10. **VOLCANOES** A volcanic eruption blasts a boulder upward with an initial velocity of 240 feet per second. How long will it take the boulder to hit the ground if it lands at the same elevation from which it was ejected? **15 s**

Reading to Learn Mathematics, p. 323 ELL



Pre-Activity **How does a quadratic function model a free-fall ride?**

Read the introduction to Lesson 6-2 at the top of page 294 in your textbook.

Write a quadratic function that describes the height of a ball t seconds after it is dropped from a height of 125 feet. **$h(t) = -16t^2 + 125$**

Reading the Lesson

1. The graph of the quadratic function $f(x) = -x^2 + x + 6$ is shown at the right. Use the graph to find the solutions of the quadratic equation $-x^2 + x + 6 = 0$. **−2 and 3**

2. Sketch a graph to illustrate each situation.
   - a. A parabola that opens downward and represents a quadratic function with two real zeros, both of which are negative numbers.
   - b. A parabola that opens upward and represents a quadratic function with exactly one real zero. The zero is a positive number.
   - c. A parabola that opens downward and represents a quadratic function with no real zeros.

Helping You Remember

3. Think of a memory aid that can help you recall what is meant by the *zeros* of a quadratic function.

Sample answer: The basic facts about a subject are sometimes called the ABCs. In the case of zeros, the ABCs are the XYZs, because the zeros are the x-values that make the y-values equal to zero.

22. between −5 and −4; between 0 and 1
23. between −1 and 0; between 2 and 3
32. between −4 and −3; between 0 and 1
33. between 0 and 1; between 3 and 4
34. between −1 and 0; between 2 and 3
35. between −3 and −2; between 2 and 3

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

20. $x^2 - 3x = 0$ **0, 3**	21. $-x^2 + 4x = 0$ **0, 4**
22. $x^2 + 4x - 4 = 0$	23. $x^2 - 2x - 1 = 0$
24. $-x^2 + x = -20$ **−4, 5**	25. $x^2 - 9x = -18$ **3, 6**
26. $14x + x^2 + 49 = 0$ **−7**	27. $-12x + x^2 = -36$ **6**
28. $2x^2 - 3x = 9$ **$-1\frac{1}{2}$, 3**	29. $4x^2 - 8x = 5$ **$-\frac{1}{2}$, $2\frac{1}{2}$**
30. $2x^2 = -5x + 12$ **−4, $1\frac{1}{2}$**	31. $2x^2 = x + 15$ **$-2\frac{1}{2}$, 3**
32. $x^2 + 3x - 2 = 0$	33. $x^2 - 4x + 2 = 0$
34. $-2x^2 + 3x + 3 = 0$	35. $0.5x^2 - 3 = 0$
36. $x^2 + 2x + 5 = 0$ **no real solutions**	37. $-x^2 + 4x - 6 = 0$ **no real solutions**

NUMBER THEORY Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

38. Their sum is −17, and their product is 72. **−8, −9**
39. Their sum is 7, and their product is 14. **See pp. 343A–343F.**
40. Their sum is −9, and their product is 24. **See pp. 343A–343F.**
41. Their sum is 12, and their product is −28. **−2, 14**

For Exercises 42–44, use the formula $h(t) = v_0t - 16t^2$ where $h(t)$ is the height of an object in feet, v_0 is the object's initial velocity in feet per second, and t is the time in seconds.

42. **ARCHERY** An arrow is shot upward with a velocity of 64 feet per second. Ignoring the height of the archer, how long after the arrow is released does it hit the ground? **4 s**

43. **TENNIS** A tennis ball is hit upward with a velocity of 48 feet per second. Ignoring the height of the tennis player, how long does it take for the ball to fall to the ground? **3 s**

44. **BOATING** A boat in distress launches a flare straight up with a velocity of 190 feet per second. Ignoring the height of the boat, how many seconds will it take for the flare to hit the water? **about 12 s**

45. **LAW ENFORCEMENT** Police officers can use the length of skid marks to help determine the speed of a vehicle before the brakes were applied. If the skid marks are on dry concrete, the formula $\frac{s^2}{24} = d$ can be used. In the formula, s represents the speed in miles per hour, and d represents the length of the skid marks in feet. If the length of the skid marks on dry concrete are 50 feet, how fast was the car traveling? **about 35 mph**

46. **EMPIRE STATE BUILDING** Suppose you could conduct an experiment by dropping a small object from the Observatory of the Empire State Building. How long would it take for the object to reach the ground, assuming there is no air resistance? Use the information at the left and the formula $h(t) = -16t^2 + h_0$, where t is the time in seconds and the initial height h_0 is in feet. **about 8 s**

47. **CRITICAL THINKING** A quadratic function has values $f(-4) = -11$, $f(-2) = 9$, and $f(0) = 5$. Between which two x values must $f(x)$ have a zero? Explain your reasoning. **−4 and −2; See margin for explanation.**

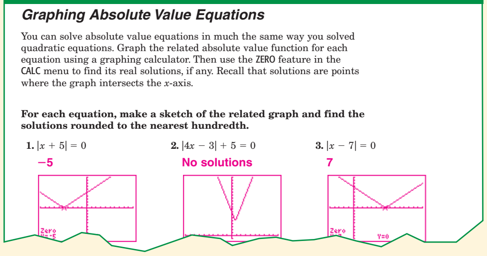
More About. . .

Empire State Building

Located on the 86th floor, 1050 feet (320 meters) above the streets of New York City, the Observatory offers panoramic views from within a glass-enclosed pavilion and from the surrounding open-air promenade.

Source: www.esbnyc.com

Enrichment, p. 324



Graphing Absolute Value Equations

You can solve absolute value equations in much the same way you solved quadratic equations. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature in the CALC menu to find its real solutions, if any. Recall that solutions are points where the graph intersects the x-axis.

For each equation, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

1. $|x + 5| = 0$ **−5**
2. $|4x - 3| + 5 = 0$ **No solutions**
3. $|x - 7| = 0$ **7**

Answer

47. The value of the function changes from negative to positive, therefore the value of the function is zero between these two numbers.

48. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 343A–343F.**

How does a quadratic function model a free-fall ride?

Include the following in your answer:

- a graph showing the height at any given time of a free-fall ride that lifts riders to a height of 185 feet, and
- an explanation of how to use this graph to estimate how long the riders would be in free-fall if the ride were allowed to hit the ground before stopping.

Standardized Test Practice

49. If one of the roots of the equation $x^2 + kx - 12 = 0$ is 4, what is the value of k? **A**

Ⓐ −1 Ⓑ 0 Ⓒ 1 Ⓓ 3

50. For what value of x does $f(x) = x^2 + 5x + 6$ reach its minimum value? **B**

Ⓐ −3 Ⓑ $-\frac{5}{2}$ Ⓒ −2 Ⓓ −5

Extending the Lesson

SOLVE ABSOLUTE VALUE EQUATIONS BY GRAPHING Similar to quadratic equations, you can solve absolute value equations by graphing. Graph the related absolute value function for each equation using a graphing calculator. Then use the ZERO feature, 2nd [CALC], to find its real solutions, if any, rounded to the nearest hundredth.

51. $|x + 1| = 0$ **−1**

52. $|x| - 3 = 0$ **±3**

53. $|x - 4| - 1 = 0$ **3, 5**

54. $-|x + 4| + 5 = 0$ **−9, 1**

55. $2|3x| - 8 = 0$ **±1.33**

56. $|2x - 3| + 1 = 0$ **no real solutions**

Maintain Your Skills

Mixed Review

Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for each quadratic function. Then graph the function by making a table of values. *(Lesson 6-1)* **57–59. See margin for graphs.**

57. $f(x) = x^2 - 6x + 4$ **4; $x = 3$; 3**

58. $f(x) = -4x^2 + 8x - 1$ **−1; $x = 1$; 1**

59. $f(x) = \frac{1}{4}x^2 + 3x + 4$ **4; $x = -6$; −6**

Simplify. *(Lesson 5-9)*

60. $\frac{2i}{3 + i}$ **$\frac{1}{5} + \frac{3}{5}i$**

61. $\frac{4}{5 - i}$ **$\frac{10}{13} + \frac{2}{13}i$**

62. $\frac{1 + i}{3 - 2i}$ **$\frac{1}{13} + \frac{5}{13}i$**

Evaluate the determinant of each matrix. *(Lesson 4-3)*

63. $\begin{bmatrix} 6 & 4 \\ -3 & 2 \end{bmatrix}$ **24**

64. $\begin{bmatrix} 2 & -1 & -6 \\ 5 & 0 & 3 \\ -3 & 2 & 11 \end{bmatrix}$ **−8**

65. $\begin{bmatrix} 6 & 5 & -2 \\ -3 & 0 & 6 \\ 1 & 4 & 2 \end{bmatrix}$ **−60**

66. **COMMUNITY SERVICE** A drug awareness program is being presented at a theater that seats 300 people. Proceeds will be donated to a local drug information center. If every two adults must bring at least one student, what is the maximum amount of money that can be raised? *(Lesson 3-4)* **$500**

68. $(x - 10)(x + 10)$
69. $(x - 7)(x - 4)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Factor completely.**
*(To review **factoring trinomials**, see Lesson 5-4.)*

67. $x^2 + 5x$ **$x(x + 5)$**

68. $x^2 - 100$

69. $x^2 - 11x + 28$

70. $x^2 - 18x + 81$ **$(x - 9)^2$**

71. $3x^2 + 8x + 4$

72. $6x^2 - 14x - 12$

71. $(3x + 2)(x + 2)$
72. $2(3x + 2)(x - 3)$

4 Assess

Open-Ended Assessment

Modeling Have students draw parabolas in various positions and label them to show how many real roots they have and approximately where those roots occur.

Getting Ready for Lesson 6-3

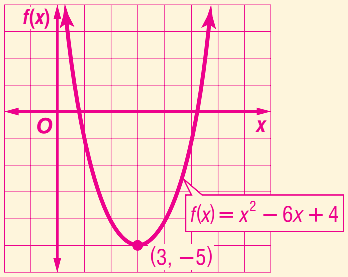
PREREQUISITE SKILL Lesson 6-3 presents solving quadratic equations by factoring. Frequently this involves factoring a trinomial expression on one side of an equation. Exercises 67–72 should be used to determine your students' familiarity with factoring trinomials.

Assessment Options

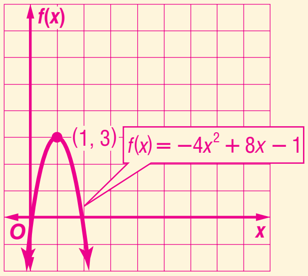
Quiz (Lessons 6-1 and 6-2) is available on p. 369 of the *Chapter 6 Resource Masters*.

Answers

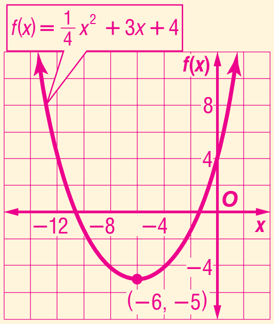
57.



58.



59.



Graphing Calculator Investigation

A Follow-Up of Lesson 6-2

Getting Started

Know Your Calculator When students use the procedure in Step 2 to copy the regression equation from Step 1 to the Y= list, the coefficients will have several more digits than the coefficients displayed on the home screen. The coefficients on the home screen are rounded versions of those in the Y= list.

Scientific Notation In Step 1, the value of the coefficient *a* is displayed as 2.1035215E−4. Point out that this is how the calculator displays the scientific notation 2.1035215×10^{-4}.

Teach

- Make sure students have cleared the L1 and L2 lists before entering new data. Also have them enter the WINDOW dimensions shown.
- For Step 1, point out that you can use the same keystrokes shown in Step 2, substituting 4 for the first 5, to select LinReg.
- If an error message appears in Step 2, have students clear the Y= list before trying Step 2 again.
- If students need to review entering data or selecting statistical plots, refer them to p. 87.
- Have students complete Exercises 1–4.

Assess

Ask students:

- What does it mean when the points on a scatter plot appear to lie along a curved path? **The equation that best models the situation may be quadratic, and is probably not linear.**

Graphing Calculator Investigation

A Follow-Up of Lesson 6-2

Modeling Real-World Data

You can use a TI-83 Plus to model data points whose curve of best fit is quadratic.

FALLING WATER **Water is allowed to drain from a hole made in a 2-liter bottle. The table shows the level of the water y measured in centimeters from the bottom of the bottle after x seconds. Find and graph a linear regression equation and a quadratic regression equation. Determine which equation is a better fit for the data.**

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220
Water level (cm)	42.6	40.7	38.9	37.2	35.8	34.3	33.3	32.3	31.5	30.8	30.4	30.1

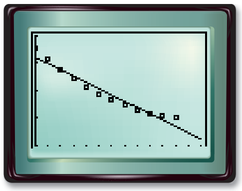
Step 1 Find a linear regression equation.

- Enter the times in L1 and the water levels in L2. Then find a linear regression equation.

  KEYSTROKES: *Review lists and finding a linear regression equation on page 87.*

- Graph a scatter plot and the regression equation.

  KEYSTROKES: *Review graphing a regression equation on page 87.*



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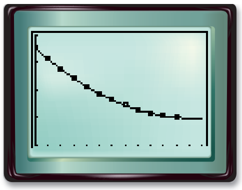
Step 2 Find a quadratic regression equation.

- Find the quadratic regression equation. Then copy the equation to the Y= list and graph.

  KEYSTROKES: STAT ▶ 5 ENTER Y= VARS 5 ▶ ▶ ENTER GRAPH



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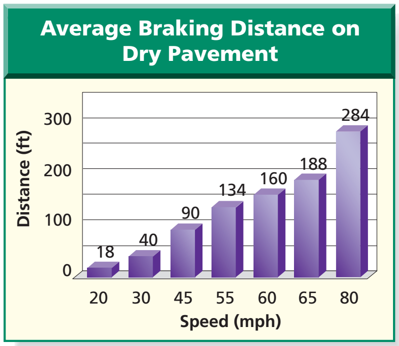
The graph of the linear regression equation appears to pass through just two data points. However, the graph of the quadratic regression equation fits the data very well.

Exercises 1–4. See margin.

For Exercises 1–4, use the graph of the braking distances for dry pavement.

1. Find and graph a linear regression equation and a quadratic regression equation for the data. Determine which equation is a better fit for the data.
2. Use the CALC menu with each regression equation to estimate the braking distance at speeds of 100 and 150 miles per hour.
3. How do the estimates found in Exercise 2 compare?
4. How might choosing a regression equation that does not fit the data well affect predictions made by using the equation?



Source: Missouri Department of Revenue

www.algebra2.com/other_calculator_keystrokes

Answers

1. See pp. 343A–343F.
2. linear: (100, 345), (150, 562); quadratic: (100, 440), (150, 990)
3. The quadratic estimates are much greater.
4. Sample answer: Choosing a model that does not fit the data well may cause inaccurate predictions when the data are very large or small.

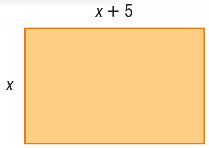
6-3 Solving Quadratic Equations by Factoring

What You'll Learn

- Solve quadratic equations by factoring.
- Write a quadratic equation with given roots.

How is the Zero Product Property used in geometry?

The length of a rectangle is 5 inches more than its width, and the area of the rectangle is 24 square inches. To find the dimensions of the rectangle you need to solve the equation $x(x + 5) = 24$ or $x^2 + 5x = 24$.



SOLVE EQUATIONS BY FACTORING In the last lesson, you learned to solve a quadratic equation like the one above by graphing. Another way to solve this equation is by factoring. Consider the following products.

$$7(0) = 0 \qquad 0(-2) = 0$$

$$(6 - 6)(0) = 0 \qquad -4(-5 + 5) = 0$$

Notice that in each case, *at least one* of the factors is zero. These examples illustrate the **Zero Product Property**.

Key Concept — *Zero Product Property*

- **Words** For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and b equal zero.
- **Example** If $(x + 5)(x - 7) = 0$, then $x + 5 = 0$ and/or $x - 7 = 0$.

Example 1 *Two Roots*

Solve each equation by factoring.

a. $x^2 = 6x$

$x^2 = 6x$ Original equation

$x^2 - 6x = 0$ Subtract $6x$ from each side.

$x(x - 6) = 0$ Factor the binomial.

$x = 0$ or $x - 6 = 0$ Zero Product Property

$x = 6$ Solve the second equation.

The solution set is {0, 6}.

CHECK Substitute 0 and 6 for x in the original equation.

$$x^2 = 6x \qquad x^2 = 6x$$

$$(0)^2 \stackrel{?}{=} 6(0) \qquad (6)^2 \stackrel{?}{=} 6(6)$$

$$0 = 0\ \checkmark \qquad 36 = 36\ \checkmark$$

6-3 Lesson Notes

1 Focus

5-Minute Check Transparency 6-3 Use as a quiz or review of Lesson 6-2.

Mathematical Background notes are available for this lesson on p. 284C.

Building on Prior Knowledge

In Lesson 6-2, students solved quadratic equations by graphing. In this lesson, they use factoring as a method for finding the roots of a quadratic equation.

How is the Zero Product Property used in geometry?

Ask students:

- What is the product of the length and the width of the rectangle? **24 in^2**
- What is the difference between the length and the width of the rectangle? **5 in.**

Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 325–326
- Skills Practice, p. 327
- Practice, p. 328
- Reading to Learn Mathematics, p. 329
- Enrichment, p. 330

Transparencies

5-Minute Check Transparency 6-3
Real-World Transparency 6
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

SOLVE EQUATIONS BY FACTORING

In-Class Examples Power Point®

Teaching Tip In Example 1a, some students may suggest solving the equation by dividing both sides by *x*. Point out that this cannot be done because the value of *x* could be zero, and division by zero is undefined.

1 Solve each equation by factoring.

a. $x^2 = -4x$ $\{0, -4\}$

b. $3x^2 = 5x + 2$ $\left\{-\frac{1}{3}, 2\right\}$

2 Solve $x^2 - 6x = -9$ by factoring. $\{3\}$

Teaching Tip Point out that the term *repeated root* is sometimes used as a substitute for the term *double root.*

3 What is the positive solution of the equation $2x^2 - 8x - 42 = 0$? D

A −3 **B** 5

C 6 **D** 7

Teaching Tip Ask students why dividing each side of the equation in this example results in an equivalent equation, without the possibility of losing a root. (The right side of the equation is 0, not *f*(*x*) or *y*, and dividing by 2 means that you can be sure that you are not dividing by zero.)

WRITE QUADRATIC EQUATIONS

In-Class Example Power Point®

4 Write a quadratic equation with $-\frac{2}{3}$ and 6 as roots. Write the equation in the form $ax^2 + bx + c = 0$, where *a*, *b*, and *c* are integers. Sample answer: $3x^2 - 16x - 12 = 0$

---

b. $2x^2 + 7x = 15$

$2x^2 + 7x = 15$	Original equation
$2x^2 + 7x - 15 = 0$	Subtract 15 from each side.
$(2x - 3)(x + 5) = 0$	Factor the trinomial.
$2x - 3 = 0$ or $x + 5 = 0$	Zero Product Property
$2x = 3$ $\quad x = -5$	Solve each equation.
$x = \frac{3}{2}$	

The solution set is $\left\{-5, \frac{3}{2}\right\}$. Check each solution.

Study Tip

Double Roots

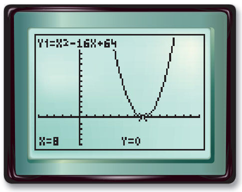
The application of the Zero Product Property produced two identical equations, $x - 8 = 0$, both of which have a root of 8. For this reason, 8 is called the *double root* of the equation.

Example 2 Double Root

Solve $x^2 - 16x + 64 = 0$ by factoring.

$x^2 - 16x + 64 = 0$	Original equation
$(x - 8)(x - 8) = 0$	Factor.
$x - 8 = 0$ or $x - 8 = 0$	Zero Product Property
$x = 8$ $\quad x = 8$	Solve each equation.

The solution set is {8}.



CHECK The graph of the related function, $f(x) = x^2 - 16x + 64$, intersects the *x*-axis only once. Since the zero of the function is 8, the solution of the related equation is 8.

Standardized Test Practice

Example 3 Greatest Common Factor

Multiple-Choice Test Item

What is the positive solution of the equation $3x^2 - 3x - 60 = 0$?

Ⓐ −4 Ⓑ 2 Ⓒ 5 Ⓓ 10

Read the Test Item

You are asked to find the *positive* solution of the given quadratic equation. This implies that the equation also has a solution that is not positive. Since a quadratic equation can either have one, two, or no solutions, we should expect to find two solutions to this equation.

Solve the Test Item

Solve this equation by factoring. But before trying to factor $3x^2 - 3x - 60$ into two binomials, look for a greatest common factor. Notice that each term is divisible by 3.

$3x^2 - 3x - 60 = 0$	Original equation
$3(x^2 - x - 20) = 0$	Factor.
$x^2 - x - 20 = 0$	Divide each side by 3.
$(x + 4)(x - 5) = 0$	Factor.
$x + 4 = 0$ or $x - 5 = 0$	Zero Product Property
$x = -4$ $\quad x = 5$	Solve each equation.

Both solutions, −4 and 5, are listed among the answer choices. Since the question asked for the positive solution, the answer is C.

The Princeton Review

Test-Taking Tip

Because the problem asked for a *positive* solution, choice A could have been eliminated even before the expression was factored.

Standardized Test Practice

Example 3 Point out to students that by reading the question carefully and noting exactly what is asked for (the *positive* solution), they can quickly eliminate answer choice A because it is negative. Choice A is an attractive (though incorrect) choice because it is indeed a solution of the equation, just not the positive one.

WRITE QUADRATIC EQUATIONS You have seen that a quadratic equation of the form $(x - p)(x - q) = 0$ has roots p and q. You can use this pattern to find a quadratic equation for a given pair of roots.

Study Tip

Writing an Equation

The pattern $(x - p)(x - q) = 0$ produces one equation with roots p and q. In fact, there are an infinite number of equations that have these same roots.

Example 4 Write an Equation Given Roots

Write a quadratic equation with $\frac{1}{2}$ and -5 as its roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

$(x - p)(x - q) = 0$ Write the pattern.

$\left(x - \frac{1}{2}\right)[x - (-5)] = 0$ Replace p with $\frac{1}{2}$ and q with -5.

$\left(x - \frac{1}{2}\right)(x + 5) = 0$ Simplify.

$x^2 + \frac{9}{2}x - \frac{5}{2} = 0$ Use FOIL.

$2x^2 + 9x - 5 = 0$ Multiply each side by 2 so that b and c are integers.

A quadratic equation with roots $\frac{1}{2}$ and -5 and integral coefficients is $2x^2 + 9x - 5 = 0$. You can check this result by graphing the related function.

Check for Understanding

Concept Check

1. **Write** the meaning of the Zero Product Property.

2. **OPEN ENDED** Choose two integers. Then, write an equation with those roots in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

3. **FIND THE ERROR** Lina and Kristin are solving $x^2 + 2x = 8$.

Lina

$x^2 + 2x = 8$

$x(x + 2) = 8$

$x = 8$ or $x + 2 = 8$

$x = 6$

Kristin

$x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $x = 2$

Who is correct? Explain your reasoning.

1. Sample answer: If the product of two factors is zero, then at least one of the factors must be zero.

2. Sample answer: roots 6 and −5; $x^2 - x - 30 = 0$

3. Kristin; the Zero Product Property applies only when one side of the equation is 0.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1, 2
10–12	4
13	3

Solve each equation by factoring.

4. $x^2 - 11x = 0$ **$\{0, 11\}$**

5. $x^2 + 6x - 16 = 0$ **$\{-8, 2\}$**

6. $x^2 = 49$ **$\{-7, 7\}$**

7. $x^2 + 9 = 6x$ **$\{3\}$**

8. $4x^2 - 13x = 12$ **$\left\{-\frac{3}{4}, 4\right\}$**

9. $5x^2 - 5x - 60 = 0$ **$\{-3, 4\}$**

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.

10. $-4, 7$ **$x^2 - 3x - 28 = 0$**

11. $\frac{1}{2}, \frac{4}{3}$ **$6x^2 - 11x + 4 = 0$**

12. $-\frac{3}{5}, -\frac{1}{3}$ **$15x^2 + 14x + 3 = 0$**

Standardized Test Practice

13. Which of the following is the sum of the solutions of $x^2 - 2x - 8 = 0$? **D**

Ⓐ −6 Ⓑ −4 Ⓒ −2 Ⓓ 2

www.algebra2.com/extra_examples

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective

- **Solve Equations by Factoring:** 14–33, 42–26
- **Write Quadratic Equations:** 34–41

Odd/Even Assignments

Exercises 14–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–31 odd, 35–43 odd, 47–68

Average: 15–43 odd, 47–68

Advanced: 14–44 even, 45–62 (optional: 63–68)

All: Practice Quiz 1 (1–5)

DAILY INTERVENTION

FIND THE ERROR To show that neither factor on the left side of Lina's second equation needs to be 8, ask students to name several pairs of numbers whose product is 8. Possible pairs are 2 and 4, 0.5 and 16, and −1 and −8.

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Provide each student with a sheet of grid paper. Have students begin by first drawing a coordinate grid with two points on the *x*-axis plotted as the roots of a quadratic equation. Then ask students to draw several different parabolas that might be the graphs of different equations having those two points as their solutions. Point out that this demonstrates that the steps demonstrated in Example 4 yield just *one* of the possible equations having the given roots.

Study Guide and Intervention, p. 325 (shown) and p. 326

Solve Equations by Factoring When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property	For any real numbers *a* and *b*, if $ab = 0$, then either $a = 0$ or $b = 0$, or both *a* and $b = 0$.

Example **Solve each equation by factoring.**

a. $3x^2 = 15x$

$3x^2 = 15x$ Original equation

$3x^2 - 15x = 0$ Subtract 15*x* from both sides.

$3x(x - 5) = 0$ Factor the binomial.

$3x = 0$ or $x - 5 = 0$ Zero Product Property

$x = 0$ or $x = 5$ Solve each equation.

The solution set is {0, 5}.

b. $4x^2 - 5x = 21$

$4x^2 - 5x = 21$ Original equation

$4x^2 - 5x - 21 = 0$ Subtract 21 from both sides.

$(4x + 7)(x - 3) = 0$ Factor the trinomial.

$4x + 7 = 0$ or $x - 3 = 0$ Zero Product Property

$x = -\frac{7}{4}$ or $x = 3$ Solve each equation.

The solution set is $\left\{-\frac{7}{4}, 3\right\}$.

Exercises

Solve each equation by factoring.

1. $6x^2 - 2x = 0$ $\left\{0, \frac{1}{3}\right\}$
2. $x^2 = 7x$ $\{0, 7\}$
3. $20x^2 = -25x$ $\left\{0, -\frac{5}{4}\right\}$
4. $6x^2 = 7x$ $\left\{0, \frac{7}{6}\right\}$
5. $6x^2 - 27x = 0$ $\left\{0, \frac{9}{2}\right\}$
6. $12x^2 - 8x = 0$ $\left\{0, \frac{2}{3}\right\}$
7. $x^2 + x - 30 = 0$ $\{5, -6\}$
8. $2x^2 - x - 3 = 0$ $\left\{\frac{3}{2}, -1\right\}$
9. $x^2 + 14x + 33 = 0$ $\{-11, -3\}$
10. $4x^2 + 27x - 7 = 0$ $\left\{\frac{1}{4}, -7\right\}$
11. $3x^2 + 29x - 10 = 0$ $\left\{-10, \frac{1}{3}\right\}$
12. $6x^2 - 5x - 4 = 0$ $\left\{-\frac{1}{2}, \frac{4}{3}\right\}$
13. $12x^2 - 8x + 1 = 0$ $\left\{\frac{1}{6}, \frac{1}{2}\right\}$
14. $5x^2 + 28x - 12 = 0$ $\left\{\frac{2}{5}, -6\right\}$
15. $2x^2 - 250x + 5000 = 0$ $\{100, 25\}$
16. $2x^2 - 11x - 40 = 0$ $\left\{8, -\frac{5}{2}\right\}$
17. $2x^2 + 21x - 11 = 0$ $\left\{-11, \frac{1}{2}\right\}$
18. $3x^2 + 2x - 21 = 0$ $\left\{\frac{7}{3}, -3\right\}$
19. $8x^2 - 14x + 3 = 0$ $\left\{\frac{3}{2}, \frac{1}{4}\right\}$
20. $6x^2 + 11x - 2 = 0$ $\left\{-2, \frac{1}{6}\right\}$
21. $5x^2 + 17x - 12 = 0$ $\left\{\frac{3}{5}, -4\right\}$
22. $12x^2 + 25x + 12 = 0$ $\left\{-\frac{4}{3}, -\frac{3}{4}\right\}$
23. $12x^2 + 18x + 6 = 0$ $\left\{-\frac{1}{2}, -1\right\}$
24. $7x^2 - 36x + 5 = 0$ $\left\{\frac{1}{7}, 5\right\}$

Skills Practice, p. 327 and Practice, p. 328 (shown)

Solve each equation by factoring.

1. $x^2 - 4x - 12 = 0$ $\{6, -2\}$
2. $x^2 - 16x + 64 = 0$ $\{8\}$
3. $x^2 - 20x + 100 = 0$ $\{10\}$
4. $x^2 - 6x + 8 = 0$ $\{2, 4\}$
5. $x^2 + 3x + 2 = 0$ $\{-2, -1\}$
6. $x^2 - 9x + 14 = 0$ $\{2, 7\}$
7. $x^2 - 4x = 0$ $\{0, 4\}$
8. $7x^2 = 4x$ $\left\{0, \frac{4}{7}\right\}$
9. $x^2 + 25 = 10x$ $\{5\}$
10. $10x^2 = 9x$ $\left\{0, \frac{9}{10}\right\}$
11. $x^2 = 2x + 99$ $\{-9, 11\}$
12. $x^2 + 12x = -36$ $\{-6\}$
13. $5x^2 - 35x + 60 = 0$ $\{3, 4\}$
14. $36x^2 = 25$ $\left\{\frac{5}{6}, -\frac{5}{6}\right\}$
15. $2x^2 - 8x - 90 = 0$ $\{9, -5\}$
16. $3x^2 + 2x - 1 = 0$ $\left\{\frac{1}{3}, -1\right\}$
17. $6x^2 = 9x$ $\left\{0, \frac{3}{2}\right\}$
18. $3x^2 + 24x + 45 = 0$ $\{-5, -3\}$
19. $15x^2 + 19x + 6 = 0$ $\left\{-\frac{3}{5}, -\frac{2}{3}\right\}$
20. $3x^2 - 8x = -4$ $\left\{2, \frac{2}{3}\right\}$
21. $6x^2 = 5x + 6$ $\left\{\frac{3}{2}, -\frac{2}{3}\right\}$

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where *a*, *b*, and *c* are integers.

22. 7, 2 $x^2 - 9x + 14 = 0$
23. 0, 3 $x^2 - 3x = 0$
24. −5, 8 $x^2 - 3x - 40 = 0$
25. −7, −8 $x^2 + 15x + 56 = 0$
26. −6, −3 $x^2 + 9x + 18 = 0$
27. 3, −4 $x^2 + x - 12 = 0$
28. $1, \frac{1}{2}$ $2x^2 - 3x + 1 = 0$
29. $\frac{1}{3}, 2$ $3x^2 - 7x + 2 = 0$
30. $0, -\frac{7}{2}$ $2x^2 + 7x = 0$
31. $\frac{1}{3}, -3$ $3x^2 + 8x - 3 = 0$
32. $4, \frac{1}{3}$ $3x^2 - 13x + 4 = 0$
33. $-\frac{2}{3}, -\frac{4}{5}$ $15x^2 + 22x + 8 = 0$

34. **NUMBER THEORY** Find two consecutive even positive integers whose product is 624. **24, 26**

35. **NUMBER THEORY** Find two consecutive odd positive integers whose product is 323. **17, 19**

36. **GEOMETRY** The length of a rectangle is 2 feet more than its width. Find the dimensions of the rectangle if its area is 63 square feet. **7 ft by 9 ft**

37. **PHOTOGRAPHY** The length and width of a 6-inch by 8-inch photograph are reduced by the same amount to make a new photograph whose area is half that of the original. By how many inches will the dimensions of the photograph have to be reduced? **2 in.**

Reading to Learn Mathematics, p. 329

ELL

Pre-Activity **How is the Zero Product Property used in geometry?**

Read the introduction to Lesson 6-3 at the top of page 301 in your textbook.

What does the expression $x(x + 5)$ mean in this situation?

It represents the area of the rectangle, since the area is the product of the width and length.

Reading the Lesson

1. The solution of a quadratic equation by factoring is shown below. Give the reason for each step of the solution.

$x^2 - 10x = -21$ Original equation

$x^2 - 10x + 21 = 0$ **Add 21 to each side.**

$(x - 3)(x - 7) = 0$ **Factor the trinomial.**

$x - 3 = 0$ or $x - 7 = 0$ **Zero Product Property**

$x = 3$ $x = 7$ **Solve each equation.**

The solution set is **{3, 7}**.

2. On an algebra quiz, students were asked to write a quadratic equation with −7 and 5 as its roots. The work that three students in the class wrote on their papers is shown below.

Marla	Rosa	Larry
$(x - 7)(x + 5) = 0$	$(x + 7)(x - 5) = 0$	$(x + 7)(x - 5) = 0$
$x^2 - 2x - 35 = 0$	$x^2 + 2x - 35 = 0$	$x^2 - 2x - 35 = 0$

Who is correct? **Rosa**

Explain the errors in the other two students' work.

Sample answer: Marla used the wrong factors. Larry used the correct factors but multiplied them incorrectly.

Helping You Remember

3. A good way to remember a concept is to represent it in more than one way. Describe an algebraic way and a graphical way to recognize a quadratic equation that has a double root.

Sample answer: Algebraic: Write the equation in the standard form $ax^2 + bx + c = 0$ and examine the trinomial. If it is a perfect square trinomial, the quadratic function has a double root. Graphical: Graph the related quadratic function. If the parabola has exactly one *x*-intercept, then the equation has a double root.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–33, 42–46	1, 2
34–41	4
51–52	3

Extra Practice

See page 840.

Solve each equation by factoring.

14. $x^2 + 5x - 24 = 0$ **{−8, 3}**
15. $x^2 - 3x - 28 = 0$ **{−4, 7}**
16. $x^2 = 25$ **{−5, 5}**
17. $x^2 = 81$ **{−9, 9}**
18. $x^2 + 3x = 18$ **{−6, 3}**
19. $x^2 - 4x = 21$ **{−3, 7}**
20. $3x^2 = 5x$ $\left\{0, \frac{5}{3}\right\}$
21. $4x^2 = -3x$ $\left\{0, -\frac{3}{4}\right\}$
22. $x^2 + 36 = 12x$ **{6}**
23. $x^2 + 64 = 16x$ **{8}**
24. $4x^2 + 7x = 2$ $\left\{-2, \frac{1}{4}\right\}$
25. $4x^2 - 17x = -4$ $\left\{\frac{1}{4}, 4\right\}$
26. $4x^2 + 8x = -3$
27. $6x^2 + 6 = -13x$
28. $9x^2 + 30x = -16$ $\left\{-\frac{8}{3}, -\frac{2}{3}\right\}$
29. $16x^2 - 48x = -27$ $\left\{\frac{3}{4}, \frac{9}{4}\right\}$
30. $-2x^2 + 12x - 16 = 0$ **{2, 4}**
31. $-3x^2 - 6x + 9 = 0$ **{−3, 1}**

26. $\left\{-\frac{1}{2}, -\frac{3}{2}\right\}$

27. $\left\{-\frac{2}{3}, -\frac{3}{2}\right\}$

★ 32. Find the roots of $x(x + 6)(x - 5) = 0$. **0, −6, 5**

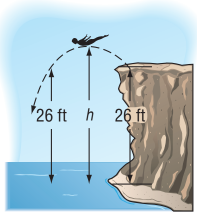
★ 33. Solve $x^3 = 9x$ by factoring. **0, −3, 3**

Write a quadratic equation with the given roots. Write the equation in the form $ax^2 + bx + c = 0$, where *a*, *b*, and *c* are integers.

34. 4, 5
35. −2, 7
36. 4, −5
37. −6, −8
38. $\frac{1}{2}, 3$ $2x^2 - 7x + 3 = 0$
39. $\frac{1}{3}, 5$ $3x^2 - 16x + 5 = 0$
40. $-\frac{2}{3}, \frac{3}{4}$ $12x^2 - x - 6 = 0$
41. $-\frac{3}{2}, -\frac{4}{5}$

34. $x^2 - 9x + 20 = 0$
35. $x^2 - 5x - 14 = 0$
36. $x^2 + x - 20 = 0$
37. $x^2 + 14x + 48 = 0$
41. $10x^2 + 23x + 12 = 0$

42. **DIVING** To avoid hitting any rocks below, a cliff diver jumps up and out. The equation $h = -16t^2 + 4t + 26$ describes her height *h* in feet *t* seconds after jumping. Find the time at which she returns to a height of 26 feet. $\frac{1}{4}$ **s**



43. **NUMBER THEORY** Find two consecutive even integers whose product is 224. **14, 16**

44. **PHOTOGRAPHY** A rectangular photograph is 8 centimeters wide and 12 centimeters long. The photograph is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new photograph? **12 cm by 16 cm**

More About. . .

Forestry

A board foot is a measure of lumber volume. One piece of lumber 1 foot long by 1 foot wide by 1 inch thick measures one board foot.

Source: www.wood-worker.com

FORESTRY **For Exercises 45 and 46, use the following information.**
Lumber companies need to be able to estimate the number of board feet that a given log will yield. One of the most commonly used formulas for estimating board feet is the *Doyle Log Rule*, $B = \frac{L}{16}(D^2 - 8D + 16)$, where *B* is the number of board feet, *D* is the diameter in inches, and *L* is the length of the log in feet.

45. Rewrite Doyle's formula for logs that are 16 feet long. $B = D^2 - 8D + 16$

★ 46. Find the root(s) of the quadratic equation you wrote in Exercise 45. What do the root(s) tell you about the kinds of logs for which Doyle's rule makes sense? **See margin.**

47. **CRITICAL THINKING** For a quadratic equation of the form $(x - p)(x - q) = 0$, show that the axis of symmetry of the related quadratic function is located halfway between the *x*-intercepts *p* and *q*. **See margin.**

CRITICAL THINKING **Find a value of *k* that makes each statement true.**

48. −3 is a root of $2x^2 + kx - 21 = 0$. **−1**

49. $\frac{1}{2}$ is a root of $2x^2 + 11x = -k$. **−6**

Enrichment, p. 330

Euler's Formula for Prime Numbers

Many mathematicians have searched for a formula that would generate prime numbers. One such formula was proposed by Euler and uses a quadratic polynomial, $x^2 + x + 41$.

Find the values of $x^2 + x + 41$ for the given values of *x*. State whether each value of the polynomial is or is not a prime number.

1. $x = 0$ **41, prime**
2. $x = 1$ **43, prime**
3. $x = 2$ **47, prime**
4. $x = 3$ **53, prime**
5. $x = 4$ **61, prime**
6. $x = 5$ **71, prime**

Answer

46. 4; The logs must have a diameter greater than 4 in. for the rule to produce positive board feet values.

50. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 343A–343F.**

How is the Zero Product Property used in geometry?

Include the following in your answer:

- an explanation of how to find the dimensions of the rectangle using the Zero Product Property, and
- why the equation $x(x + 5) = 24$ is not solved by using $x = 24$ and $x + 5 = 24$.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

51. Which quadratic equation has roots $\frac{1}{2}$ and $\frac{1}{3}$? **D**

Ⓐ $5x^2 - 5x - 2 = 0$ Ⓑ $5x^2 - 5x + 1 = 0$

Ⓒ $6x^2 + 5x - 1 = 0$ Ⓓ $6x^2 - 5x + 1 = 0$

52. If the roots of a quadratic equation are 6 and −3, what is the equation of the axis of symmetry? **B**

Ⓐ $x = 1$ Ⓑ $x = \frac{3}{2}$ Ⓒ $x = \frac{1}{2}$ Ⓓ $x = -2$

Maintain Your Skills

Mixed Review

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. *(Lesson 6-2)*

53. $f(x) = -x^2 - 4x + 5$ **54.** $f(x) = 4x^2 + 4x + 1$ **55.** $f(x) = 3x^2 - 10x - 4$

56. Determine whether $f(x) = 3x^2 - 12x - 7$ has a maximum or a minimum value. Then find the maximum or minimum value. *(Lesson 6-1)* **min.; −19**

Simplify. *(Lesson 5-6)*

57. $\sqrt{3}\left(\sqrt{6} - 2\right)$ **58.** $\sqrt{108} - \sqrt{48} + \left(\sqrt{3}\right)^3$ **59.** $\left(5 + \sqrt{8}\right)^2$ **$33 + 20\sqrt{2}$**

Solve each system of equations. *(Lesson 3-2)*

60. $4a - 3b = -4$
$3a - 2b = -4$ **(−4, −4)**

61. $2r + s = 1$
$r - s = 8$ **(3, −5)**

62. $3x - 2y = -3$
$3x + y = 3$ $\left(\frac{1}{3}, 2\right)$

53. −5, 1
54. $-\frac{1}{2}$
55. between −1 and 0; between 3 and 4
57. $3\sqrt{2} - 2\sqrt{3}$
58. $5\sqrt{3}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify.** *(To review **simplifying radicals**, see Lesson 5-5.)*

63. $\sqrt{8}$ **$2\sqrt{2}$** **64.** $\sqrt{20}$ **$2\sqrt{5}$** **65.** $\sqrt{27}$ **$3\sqrt{3}$**

66. $\sqrt{-50}$ **$5i\sqrt{2}$** **67.** $\sqrt{-12}$ **$2i\sqrt{3}$** **68.** $\sqrt{-48}$ **$4i\sqrt{3}$**

Practice Quiz 1

Lessons 6-1 through 6-3

1. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex for $f(x) = 3x^2 - 12x + 4$. Then graph the function by making a table of values. *(Lesson 6-1)* **4, $x = 2$; 2; See margin for graph.**
2. Determine whether $f(x) = 3 - x^2 + 5x$ has a maximum or minimum value. Then find this maximum or minimum value. *(Lesson 6-1)* **max.; $\frac{37}{4}$ or $9\frac{1}{4}$**
3. Solve $2x^2 - 11x + 12 = 0$ by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. *(Lesson 6-2)* **$1\frac{1}{2}$, 4**
4. Solve $2x^2 + 9x - 5 = 0$ by factoring. *(Lesson 6-3)* $\left\{-5, \frac{1}{2}\right\}$
5. Write a quadratic equation with roots −4 and $\frac{1}{3}$. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. *(Lesson 6-3)* **$3x^2 + 11x - 4 = 0$**

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Speaking Ask students to give a verbal explanation of the Zero Product Property. They should discuss why it is true and how it is used in finding the roots of a quadratic equation, demonstrating the technique using an example.

Tips for New Teachers

Intervention Suggest that students who have difficulty understanding the Zero Product Property try to find two nonzero numbers whose product is zero. Students should quickly determine that at least one of the numbers must be zero in order for their product to be zero.

Getting Ready for Lesson 6-4

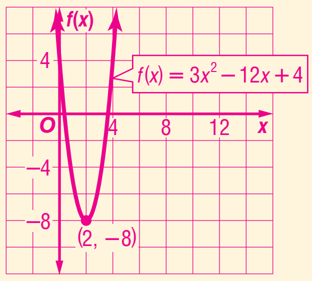
PREREQUISITE SKILL Lesson 6-4 presents solving quadratic equations by completing the square. The process involves evaluating radicals. Exercises 63–68 should be used to determine your students' familiarity with simplifying radicals.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 6-1 through 6-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Answer (Practice Quiz 1)

1.



47. $y = (x - p)(x - q)$

$y = x^2 - px - qx + pq$

$y = x^2 - (p + q)x + pq$

$a = 1,\ b = -(p + q),\ c = +pq$

axis of symmetry: $x = -\frac{b}{2a}$

$x = -\frac{-(p + q)}{2(1)}$

$x = \frac{p + q}{2}$

The axis of symmetry is the average of the x-intercepts. Therefore the axis of symmetry is located halfway between the x-intercepts.

6-4 Lesson Notes

1 Focus

5-Minute Check Transparency 6-4 Use as a quiz or review of Lesson 6-3.

Mathematical Background notes are available for this lesson on p. 284C.

Building on Prior Knowledge

In Lesson 6-3, students solved quadratic equations by factoring. In this lesson they use two other methods to solve equations: the Square Root Property and completing the square.

How can you find the time it takes an accelerating race car to reach the finish line?

Ask students:

- The number 121 is a perfect square. What is the square root of 121? **11**
- How does the number 11 relate to the coefficient of the x term? **The coefficient of x is twice 11.**

6-4 Completing the Square

What You'll Learn

- Solve quadratic equations by using the Square Root Property.
- Solve quadratic equations by completing the square.

Vocabulary

- completing the square

How can you find the time it takes an accelerating race car to reach the finish line?

Under a yellow caution flag, race car drivers slow to a speed of 60 miles per hour. When the green flag is waved, the drivers can increase their speed.

Suppose the driver of one car is 500 feet from the finish line. If the driver accelerates at a constant rate of 8 feet per second squared, the equation $t^2 + 22t + 121 = 246$ represents the time t it takes the driver to reach this line. To solve this equation, you can use the Square Root Property.

SQUARE ROOT PROPERTY You have solved equations like $x^2 - 25 = 0$ by factoring. You can also use the **Square Root Property** to solve such an equation. This method is useful with equations like the one above that describes the race car's speed. In this case, the quadratic equation contains a perfect square trinomial set equal to a constant.

Study Tip

Reading Math
$\pm\sqrt{n}$ is read *plus or minus the square root of n.*

Key Concept — *Square Root Property*

For any real number n, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Example 1 Equation with Rational Roots

Solve $x^2 + 10x + 25 = 49$ by using the Square Root Property.

$x^2 + 10x + 25 = 49$	Original equation
$(x + 5)^2 = 49$	Factor the perfect square trinomial.
$x + 5 = \pm\sqrt{49}$	Square Root Property
$x + 5 = \pm 7$	$\sqrt{49} = 7$
$x = -5 \pm 7$	Add -5 to each side.
$x = -5 + 7$ or $x = -5 - 7$	Write as two equations.
$x = 2$ $\quad$ $x = -12$	Solve each equation.

The solution set is $\{2, -12\}$. You can check this result by using factoring to solve the original equation.

TEACHING TIP

Have students solve Example 1 by factoring and compare the results. Point out that Example 2 cannot be solved by factoring.

Roots that are irrational numbers may be written as exact answers in radical form or as *approximate* answers in decimal form when a calculator is used.

Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters
- Study Guide and Intervention, pp. 331–332
- Skills Practice, p. 333
- Practice, p. 334
- Reading to Learn Mathematics, p. 335
- Enrichment, p. 336
- Assessment, pp. 369, 371

Teaching Algebra With Manipulatives Masters, pp. 244, 245–246

Transparencies

5-Minute Check Transparency 6-4
Answer Key Transparencies

Technology

Interactive Chalkboard

Example 2 Equation with Irrational Roots

Solve $x^2 - 6x + 9 = 32$ by using the Square Root Property.

$x^2 - 6x + 9 = 32$ Original equation

$(x - 3)^2 = 32$ Factor the perfect square trinomial.

$x - 3 = \pm\sqrt{32}$ Square Root Property

$x = 3 \pm 4\sqrt{2}$ Add 3 to each side; $\sqrt{32} = 4\sqrt{2}$

$x = 3 + 4\sqrt{2}$ or $x = 3 - 4\sqrt{2}$ Write as two equations.

$x \approx 8.7$ $x \approx -2.7$ Use a calculator.

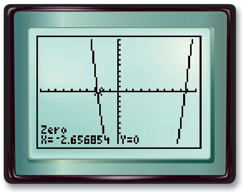
The exact solutions of this equation are $3 - 4\sqrt{2}$ and $3 + 4\sqrt{2}$. The approximate solutions are -2.7 and 8.7. Check these results by finding and graphing the related quadratic function.

$x^2 - 6x + 9 = 32$ Original equation

$x^2 - 6x - 23 = 0$ Subtract 32 from each side.

$y = x^2 - 6x - 23$ Related quadratic function

CHECK Use the ZERO function of a graphing calculator. The approximate zeros of the related function are -2.7 and 8.7.



```
Zero
X=-2.656854  Y=0
```

COMPLETE THE SQUARE

The Square Root Property can only be used to solve quadratic equations when the side containing the quadratic expression is a perfect square. However, few quadratic expressions are perfect squares. To make a quadratic expression a perfect square, a method called **completing the square** may be used.

In a perfect square trinomial, there is a relationship between the coefficient of the linear term and the constant term. Consider the pattern for squaring a sum.

$(x + 7)^2 = x^2 + 2(7)x + 7^2$ Square of a sum pattern

$= x^2 + 14x + 49$ Simplify.

$$\left(\frac{14}{2}\right)^2 \longrightarrow 7^2$$ Notice that 49 is 7^2 and 7 is one-half of 14.

You can use this pattern of coefficients to complete the square of a quadratic expression.

Key Concept Completing the Square

- **Words** To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

  Step 1 Find one half of b, the coefficient of x.

  Step 2 Square the result in Step 1.

  Step 3 Add the result of Step 2 to $x^2 + bx$.

- **Symbols** $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

2 Teach

SQUARE ROOT PROPERTY

In-Class Examples

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Teaching Tip In Example 1, point out that both constants in the equation, 25 and 49, are perfect squares.

1 Solve $x^2 + 14x + 49 = 64$ by using the Square Root Property. $\{-15, 1\}$

Teaching Tip Point out that the constant on the right side of the equation given in Example 2, is not a perfect square. Stress that this occurrence means the roots will be irrational numbers involving radicals. Also emphasize the use of the ± symbol in the step where the Square Root Property is utilized.

2 Solve $x^2 - 10x + 25 = 12$ by using the Square Root Property. $\{5 \pm 2\sqrt{3}\}$

COMPLETE THE SQUARE

Teaching Tip When discussing the steps for completing the square, emphasize that the coefficient of the quadratic term must be 1.

In-Class Examples

Power Point®

3 Find the value of c that makes $x^2 + 16x + c$ a perfect square. Then write the trinomial as a perfect square. **64; $(x + 8)^2$**

4 Solve $x^2 + 4x - 12 = 0$ by completing the square. **$\{-6, 2\}$**

Teaching Tip Remind students to think carefully about the difference between multiplying a quantity by a factor of 2 and squaring a quantity.

Example 3 Complete the Square

Find the value of c that makes $x^2 + 12x + c$ a perfect square. Then write the trinomial as a perfect square.

Step 1	Find one half of 12.	$\frac{12}{2} = 6$
Step 2	Square the result of Step 1.	$6^2 = 36$
Step 3	Add the result of Step 2 to $x^2 + 12x$.	$x^2 + 12x + 36$

The trinomial $x^2 + 12x + 36$ can be written as $(x + 6)^2$.

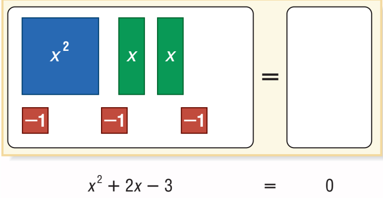
You can solve any quadratic equation by completing the square. Because you are solving an equation, add the value you use to complete the square to each side.

Algebra Activity

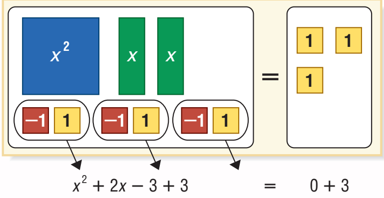
Completing the Square

Use algebra tiles to complete the square for the equation $x^2 + 2x - 3 = 0$.

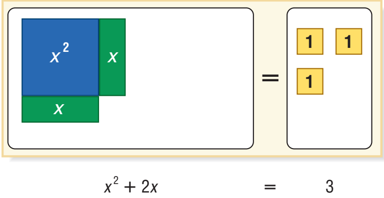
Step 1 Represent $x^2 + 2x - 3 = 0$ on an equation mat.



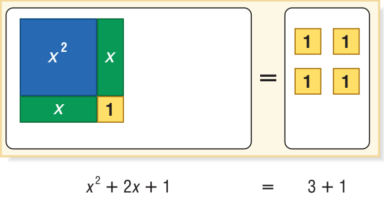
Step 2 Add 3 to each side of the mat. Remove the zero pairs.



Step 3 Begin to arrange the x^2 and x tiles into a square.



Step 4 To complete the square, add 1 yellow 1 tile to each side. The completed equation is $x^2 + 2x + 1 = 4$ or $(x + 1)^2 = 4$.



Model

Use algebra tiles to complete the square for each equation.

1. $x^2 + 2x - 4 = 0$ **$(x + 1)^2 = 5$**

2. $x^2 + 4x + 1 = 0$ **$(x + 2)^2 = 3$**

3. $x^2 - 6x = -5$ **$(x - 3)^2 = 4$**

4. $x^2 - 2x = -1$ **$(x - 1)^2 = 0$**

Study Tip

Common Misconception

When solving equations by completing the square, don't forget to add $\left(\frac{b}{2}\right)^2$ to *each* side of the equation.

Example 4 Solve an Equation by Completing the Square

Solve $x^2 + 8x - 20 = 0$ by completing the square.

$x^2 + 8x - 20 = 0$ Notice that $x^2 + 8x - 20$ is not a perfect square.

$x^2 + 8x = 20$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 8x + 16 = 20 + 16$ Since $\left(\frac{8}{2}\right)^2 = 16$, add 16 to each side.

$(x + 4)^2 = 36$ Write the left side as a perfect square by factoring.

Algebra Activity

Materials: algebra tiles, equation mat

- Ask students why the choice was made to add 3 unit tiles to each side of the equation mat in Step 2. **Sample answer: In order to simplify the work arranging the tiles into a square in Step 3.**
- Remind students that an x tile is x units long and 1 unit wide. Stress that the width is the same as the length of each side of a unit tile.

$x + 4 = \pm 6$ Square Root Property

$x = -4 \pm 6$ Add -4 to each side.

$x = -4 + 6$ or $x = -4 - 6$ Write as two equations.

$x = 2$ $x = -10$ The solution set is $\{-10, 2\}$.

You can check this result by using factoring to solve the original equation.

When the coefficient of the quadratic term is not 1, you must first divide the equation by that coefficient before completing the square.

Example 5 Equation with $a \neq 1$

Solve $2x^2 - 5x + 3 = 0$ by completing the square.

$2x^2 - 5x + 3 = 0$ Notice that $2x^2 - 5x + 3$ is not a perfect square.

$x^2 - \frac{5}{2}x + \frac{3}{2} = 0$ Divide by the coefficient of quadratic term, 2.

$x^2 - \frac{5}{2}x = -\frac{3}{2}$ Subtract $\frac{3}{2}$ from each side.

$x^2 - \frac{5}{2}x + \frac{25}{16} = -\frac{3}{2} + \frac{25}{16}$ Since $\left(-\frac{5}{2} \div 2\right)^2 = \frac{25}{16}$, add $\frac{25}{16}$ to each side.

$\left(x - \frac{5}{4}\right)^2 = \frac{1}{16}$ Write the left side as a perfect square by factoring. Simplify the right side.

$x - \frac{5}{4} = \pm\frac{1}{4}$ Square Root Property

$x = \frac{5}{4} \pm \frac{1}{4}$ Add $\frac{5}{4}$ to each side.

$x = \frac{5}{4} + \frac{1}{4}$ or $x = \frac{5}{4} - \frac{1}{4}$ Write as two equations.

$x = \frac{3}{2}$ $x = 1$ The solution set is $\left\{1, \frac{3}{2}\right\}$.

Not all solutions of quadratic equations are real numbers. In some cases, the solutions are complex numbers of the form $a + bi$, where $b \neq 0$.

Example 6 Equation with Complex Solutions

Solve $x^2 + 4x + 11 = 0$ by completing the square.

$x^2 + 4x + 11 = 0$ Notice that $x^2 + 4x + 11$ is not a perfect square.

$x^2 + 4x = -11$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 4x + 4 = -11 + 4$ Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side.

$(x + 2)^2 = -7$ Write the left side as a perfect square by factoring.

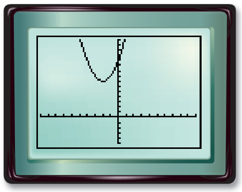
$x + 2 = \pm\sqrt{-7}$ Square Root Property

$x + 2 = \pm i\sqrt{7}$ $\sqrt{-1} = i$

$x = -2 \pm i\sqrt{7}$ Subtract 2 from each side.

The solution set is $\{-2 + i\sqrt{7}, -2 - i\sqrt{7}\}$. Notice that these are imaginary solutions.

CHECK A graph of the related function shows that the equation has no real solutions since the graph has no x-intercepts. Imaginary solutions must be checked algebraically by substituting them in the original equation.



In-Class Examples

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Teaching Tip Some students may notice that the left side of the equation in Example 5 can be factored into the product of two binomials: $(2x - 3)(x - 1)$. Then the Zero Product Property can be used to obtain the same solutions. This is a good time to point out that more than one method of solution is often possible when solving a quadratic equation.

5 Solve $3x^2 - 2x - 1 = 0$ by completing the square. $\left\{-\frac{1}{3}, 1\right\}$

6 Solve $x^2 + 2x + 3 = 0$ by completing the square. $\{-1 \pm i\sqrt{2}\}$

Teaching Tip Ask students to name three possible ways to check the solutions to an equation that they have solved by completing the square. **graphing the related function, factoring the equation, substituting the solutions into the original equation**

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Have students work with algebra tiles to help them write five equations that can be solved by completing the square. Provide each student with one x^2 tile, several x tiles, and several unit tiles. Have students begin by creating a square arrangement of their tiles and then work backwards through the steps shown in the Algebra Activity on p. 308 to find a quadratic equation. After students have written their five equations, ask them to trade their equations with another student and then use their algebra tiles to find the solutions of the equations they receive.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective

- **Square Root Property:** 14–23
- **Complete the Square:** 24–51, 53

Odd/Even Assignments

Exercises 14–47 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 51 involves research on the Internet or other reference materials.

Assignment Guide

Basic: 15–19 odd, 23–47 odd, 52, 54–72

Average: 15–47 odd, 52–72

Advanced: 14–48 even, 49–52, 54–68 (optional: 69–72)

DAILY INTERVENTION

FIND THE ERROR

Point out that, while it is possible to complete the square when the coefficient of the x^2 term is something other than 1, it is much easier to first divide each side by the coefficient and students will also be less likely to make an error like the one made by Rashid shown here.

Check for Understanding

Concept Check

1. Completing the square allows you to rewrite one side of a quadratic equation in the form of a perfect square. Once in this form, the equation is solved by using the Square Root Property.

2. Never; see margin for explanation.

1. **Explain** what it means to *complete the square.*
2. **Determine** whether the value of c that makes $ax^2 + bx + c$ a perfect square trinomial is *sometimes, always,* or *never* negative. Explain your reasoning.
3. **FIND THE ERROR** Rashid and Tia are solving $2x^2 - 8x + 10 = 0$ by completing the square.

Rashid

$$2x^2 - 8x + 10 = 0$$
$$2x^2 - 8x = -10$$
$$2x^2 - 8x + 16 = -10 + 16$$
$$(x - 4)^2 = 6$$
$$x - 4 = \pm\sqrt{6}$$
$$x = 4 \pm \sqrt{6}$$

Tia

$$2x^2 - 8x + 10 = 0$$
$$x^2 - 4x = 0 - 5$$
$$x^2 - 4x + 4 = -5 + 4$$
$$(x - 2)^2 = -1$$
$$x - 2 = \pm i$$
$$x = 2 \pm i$$

Who is correct? Explain your reasoning. **Tia; see margin for explanation.**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4, 5, 12, 13	1, 2
6, 7	3
8–11	4–6

Solve each equation by using the Square Root Property.

4. $x^2 + 14x + 49 = 9$ $\{-10, -4\}$
5. $9x^2 - 24x + 16 = 2$ $\left\{\frac{4 \pm \sqrt{2}}{3}\right\}$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

6. $x^2 - 12x + c$ $36; (x - 6)^2$
7. $x^2 - 3x + c$ $\frac{9}{4}; \left(x - \frac{3}{2}\right)^2$

Solve each equation by completing the square.

8. $x^2 + 3x - 18 = 0$ $\{-6, 3\}$
9. $x^2 - 8x + 11 = 0$ $\{4 \pm \sqrt{5}\}$
10. $x^2 + 2x + 6 = 0$ $\{-1 \pm i\sqrt{5}\}$
11. $2x^2 - 3x - 3 = 0$ $\left\{\frac{3 \pm \sqrt{33}}{4}\right\}$

Application

ASTRONOMY **For Exercises 12 and 13, use the following information.**

The height h of an object t seconds after it is dropped is given by $h = -\frac{1}{2}gt^2 + h_0$, where h_0 is the initial height and g is the acceleration due to gravity. The acceleration due to gravity near Earth's surface is 9.8 m/s^2, while on Jupiter it is 23.1 m/s^2. Suppose an object is dropped from an initial height of 100 meters from the surface of each planet.

12. On which planet should the object reach the ground first? **Jupiter**
13. Find the time it takes for the object to reach the ground on each planet to the nearest tenth of a second. **Earth: 4.5 s, Jupiter: 2.9 s**

18. $\left\{\frac{7 \pm \sqrt{5}}{2}\right\}$
19. $\left\{\frac{-5 \pm \sqrt{11}}{3}\right\}$

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–23, 48	1, 2
24–31	3
32–47, 49–50, 53	4–6

Extra Practice

See page 840.

Solve each equation by using the Square Root Property.

14. $x^2 + 4x + 4 = 25$ $\{3, -7\}$
15. $x^2 - 10x + 25 = 49$ $\{-2, 12\}$
16. $x^2 + 8x + 16 = 7$ $\{-4 \pm \sqrt{7}\}$
17. $x^2 - 6x + 9 = 8$ $\{3 \pm 2\sqrt{2}\}$
18. $4x^2 - 28x + 49 = 5$
19. $9x^2 + 30x + 25 = 11$

★ 20. $x^2 + x + \frac{1}{4} = \frac{9}{16}$ $\left\{-\frac{5}{4}, \frac{1}{4}\right\}$

★ 21. $x^2 + 1.4x + 0.49 = 0.81$ $\{-1.6, 0.2\}$

22. **MOVIE SCREENS** The area A in square feet of a projected picture on a movie screen is given by $A = 0.16d^2$, where d is the distance from the projector to the screen in feet. At what distance will the projected picture have an area of 100 square feet? **25 ft**

Answers

2. The value of c that makes $ax^2 + bx + c$ a perfect square trinomial is the square of $\frac{b}{2}$ and the square of a number can never be negative.
3. Before completing the square, you must first check to see that the coefficient of the quadratic term is 1. If it is not, you must first divide the equation by that coefficient.

23. **ENGINEERING** In an engineering test, a rocket sled is propelled into a target. The sled's distance d in meters from the target is given by the formula $d = -1.5t^2 + 120$, where t is the number of seconds after rocket ignition. How many seconds have passed since rocket ignition when the sled is 10 meters from the target? **about 8.56 s**

More About. . .

Engineering

Reverse ballistic testing—accelerating a target on a sled to impact a stationary test item at the end of the track—was pioneered at the Sandia National Laboratories' Rocket Sled Track Facility in Albuquerque, New Mexico. This facility provides a 10,000-foot track for testing items at very high speeds.

Source: www.sandia.gov

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

24. $x^2 + 16x + c$ $\mathbf{64; (x+8)^2}$

25. $x^2 - 18x + c$ $\mathbf{81; (x-9)^2}$

26. $x^2 - 15x + c$ $\mathbf{\frac{225}{4}; \left(x - \frac{15}{2}\right)^2}$

27. $x^2 + 7x + c$ $\mathbf{\frac{49}{4}; \left(x + \frac{7}{2}\right)^2}$

28. $x^2 + 0.6x + c$ $\mathbf{0.09; (x+0.3)^2}$

29. $x^2 - 2.4x + c$ $\mathbf{1.44; (x-1.2)^2}$

30. $x^2 - \frac{8}{3}x + c$ $\mathbf{\frac{16}{9}; \left(x - \frac{4}{3}\right)^2}$

31. $x^2 + \frac{5}{2}x + c$ $\mathbf{\frac{25}{16}; \left(x + \frac{5}{4}\right)^2}$

Solve each equation by completing the square.

32. $x^2 - 8x + 15 = 0$ $\mathbf{\{3, 5\}}$

33. $x^2 + 2x - 120 = 0$ $\mathbf{\{-12, 10\}}$

34. $x^2 + 2x - 6 = 0$ $\mathbf{\{-1 \pm \sqrt{7}\}}$

35. $x^2 - 4x + 1 = 0$ $\mathbf{\{2 \pm \sqrt{3}\}}$

36. $x^2 - 4x + 5 = 0$ $\mathbf{\{2 \pm i\}}$

37. $x^2 + 6x + 13 = 0$ $\mathbf{\{-3 \pm 2i\}}$

38. $2x^2 + 3x - 5 = 0$ $\mathbf{\left\{-\frac{5}{2}, 1\right\}}$

39. $2x^2 - 3x + 1 = 0$ $\mathbf{\left\{\frac{1}{2}, 1\right\}}$

40. $3x^2 - 5x + 1 = 0$

41. $3x^2 - 4x - 2 = 0$

42. $2x^2 - 7x + 12 = 0$

43. $3x^2 + 5x + 4 = 0$

44. $x^2 + 1.4x = 1.2$ $\mathbf{\{-2, 0.6\}}$

45. $x^2 - 4.7x = -2.8$ $\mathbf{\{0.7, 4\}}$

46. $x^2 - \frac{2}{3}x - \frac{26}{9} = 0$ $\mathbf{\left\{\frac{1}{3} \pm \sqrt{3}\right\}}$

47. $x^2 - \frac{3}{2}x - \frac{23}{16} = 0$ $\mathbf{\left\{\frac{3}{4} \pm \sqrt{2}\right\}}$

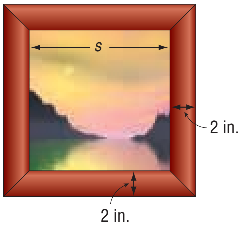
40. $\mathbf{\left\{\frac{5 \pm \sqrt{13}}{6}\right\}}$

41. $\mathbf{\left\{\frac{2 \pm \sqrt{10}}{3}\right\}}$

42. $\mathbf{\left\{\frac{7 \pm i\sqrt{47}}{4}\right\}}$

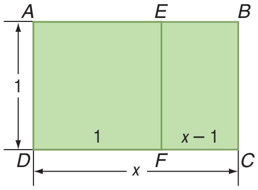
43. $\mathbf{\left\{\frac{-5 \pm i\sqrt{23}}{6}\right\}}$

48. **FRAMING** A picture has a square frame that is 2 inches wide. The area of the picture is one-third of the total area of the picture and frame. What are the dimensions of the picture to the nearest quarter of an inch? $\mathbf{5\frac{1}{2}}$ **in. by** $\mathbf{5\frac{1}{2}}$ **in.**



GOLDEN RECTANGLE **For Exercises 49–51, use the following information.**

A *golden rectangle* is one that can be divided into a square and a second rectangle that is geometrically similar to the original rectangle. The ratio of the length of the longer side to the shorter side of a golden rectangle is called the *golden ratio*.



49. Find the ratio of the length of the longer side to the length of the shorter side for rectangle *ABCD* and for rectangle *EBCF*.

50. Find the exact value of the golden ratio by setting the two ratios in Exercise 49 equal and solving for x. (*Hint*: The golden ratio is a positive value.)

51. **RESEARCH** Use the Internet or other reference to find examples of the golden rectangle in architecture. What applications does the reciprocal of the golden ratio have in music? **See margin.**

49. $\mathbf{\frac{x}{1}, \frac{1}{x-1}}$

50. $\mathbf{\frac{1+\sqrt{5}}{2}}$

52. **CRITICAL THINKING** Find all values of n such that $x^2 + bx + \left(\frac{b}{2}\right)^2 = n$ has

a. one real root. $\mathbf{n = 0}$

b. two real roots. $\mathbf{n > 0}$

c. two imaginary roots. $\mathbf{n < 0}$

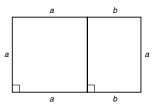
www.algebra2.com/self_check_quiz

51. Sample answers: The golden rectangle is found in much of ancient Greek architecture, such as the Parthenon, as well as in modern architecture, such as in the windows of the United Nations building. Many songs have their climax at a point occurring 61.8% of the way through the piece, with 0.618 being about the reciprocal of the golden ratio. The reciprocal of the golden ratio is also used in the design of some violins.

Enrichment, p. 336

The Golden Quadratic Equations

A **golden rectangle** has the property that its length can be written as $a + b$, where a is the width of the rectangle and $\frac{a+b}{a} = \frac{a}{b}$. Any golden rectangle can be divided into a square and a smaller golden rectangle, as shown.



The proportion used to define golden rectangles can be used to derive two quadratic equations. These are sometimes called *golden quadratic equations.*

Solve each problem.

1. In the proportion for the golden rectangle, let a equal 1. Write the resulting quadratic equation and solve for b.

$\mathbf{b^2 + b - 1 = 0}$

$\mathbf{b = \frac{-1 + \sqrt{5}}{2}}$

Study Guide and Intervention, p. 331 (shown) and p. 332

Square Root Property Use the following property to solve a quadratic equation that is in the form "perfect square trinomial = constant."

Square Root Property	For any real number x if $x^2 = n$, then $x = \pm n$.

Example **Solve each equation by using the Square Root Property.**

a. $\mathbf{x^2 - 8x + 16 = 25}$

$x^2 - 8x + 16 = 25$

$(x - 4)^2 = 25$

$x - 4 = \sqrt{25}$ or $x - 4 = -\sqrt{25}$

$x = 5 + 4 = 9$ or $x = -5 + 4 = -1$

The solution set is {9, −1}.

b. $\mathbf{4x^2 - 20x + 25 = 32}$

$4x^2 - 20x + 25 = 32$

$(2x - 5)^2 = 32$

$2x - 5 = \sqrt{32}$ or $2x - 5 = -\sqrt{32}$

$2x - 5 = 4\sqrt{2}$ or $2x - 5 = -4\sqrt{2}$

$x = \frac{5 \pm 4\sqrt{2}}{2}$

The solution set is $\left\{\frac{5 \pm 4\sqrt{2}}{2}\right\}$.

Exercises

Solve each equation by using the Square Root Property.

1. $x^2 - 18x + 81 = 49$ $\mathbf{\{2, 16\}}$

2. $x^2 + 20x + 100 = 64$ $\mathbf{\{-2, -18\}}$

3. $4x^2 + 4x + 1 = 16$ $\mathbf{\left\{\frac{3}{2}, -\frac{5}{2}\right\}}$

4. $36x^2 + 12x + 1 = 18$ $\mathbf{\left\{\frac{-1 \pm 3\sqrt{2}}{6}\right\}}$

5. $9x^2 - 12x + 4 = 4$ $\mathbf{\left\{0, \frac{4}{3}\right\}}$

6. $25x^2 + 40x + 16 = 28$ $\mathbf{\left\{\frac{-4 \pm 2\sqrt{7}}{5}\right\}}$

7. $4x^2 - 28x + 49 = 64$ $\mathbf{\left\{\frac{15}{2}, -\frac{1}{2}\right\}}$

8. $16x^2 + 24x + 9 = 81$ $\mathbf{\left\{\frac{3}{2}, -3\right\}}$

9. $100x^2 - 60x + 9 = 121$ $\mathbf{\{-0.8, 1.4\}}$

10. $25x^2 + 20x + 4 = 75$ $\mathbf{\left\{\frac{-2 \pm 5\sqrt{3}}{5}\right\}}$

11. $36x^2 + 48x + 16 = 12$ $\mathbf{\left\{\frac{-2 \pm \sqrt{3}}{3}\right\}}$

12. $25x^2 - 30x + 9 = 96$ $\mathbf{\left\{\frac{3 \pm 4\sqrt{6}}{5}\right\}}$

Skills Practice, p. 333 and Practice, p. 334 (shown)

Solve each equation by using the Square Root Property.

1. $x^2 + 8x + 16 = 1$ $\mathbf{-5, -3}$

2. $x^2 + 6x + 9 = 1$ $\mathbf{-4, -2}$

3. $x^2 + 10x + 25 = 16$ $\mathbf{-9, -1}$

4. $x^2 - 14x + 49 = 9$ $\mathbf{4, 10}$

5. $4x^2 + 12x + 9 = 4$ $\mathbf{-\frac{1}{2}, -\frac{5}{2}}$

6. $x^2 - 8x + 16 = 8$ $\mathbf{4 \pm 2\sqrt{2}}$

7. $x^2 - 6x + 9 = 5$ $\mathbf{3 \pm \sqrt{5}}$

8. $x^2 - 2x + 1 = 2$ $\mathbf{1 \pm \sqrt{2}}$

9. $9x^2 - 6x + 1 = 2$ $\mathbf{\frac{1 \pm \sqrt{2}}{3}}$

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

10. $x^2 + 12x + c$ $\mathbf{36; (x + 6)^2}$

11. $x^2 - 20x + c$ $\mathbf{100; (x - 10)^2}$

12. $x^2 + 11x + c$ $\mathbf{\frac{121}{4}; \left(x + \frac{11}{2}\right)^2}$

13. $x^2 + 0.8x + c$ $\mathbf{0.16; (x + 0.4)^2}$

14. $x^2 - 2.2x + c$ $\mathbf{1.21; (x - 1.1)^2}$

15. $x^2 - 0.36x + c$ $\mathbf{0.0324; (x - 0.18)^2}$

16. $x^2 + \frac{5}{6}x + c$ $\mathbf{\frac{25}{144}; \left(x + \frac{5}{12}\right)^2}$

17. $x^2 - \frac{1}{4}x + c$ $\mathbf{\frac{1}{64}; \left(x - \frac{1}{8}\right)^2}$

18. $x^2 - \frac{5}{3}x + c$ $\mathbf{\frac{25}{36}; \left(x - \frac{5}{6}\right)^2}$

Solve each equation by completing the square.

19. $x^2 + 6x + 8 = 0$ $\mathbf{-4, -2}$

20. $3x^2 + x - 2 = 0$ $\mathbf{\frac{2}{3}, -1}$

21. $3x^2 - 5x + 2 = 0$ $\mathbf{1, \frac{2}{3}}$

22. $x^2 + 18 = 9x$ $\mathbf{6, 3}$

23. $x^2 - 14x + 19 = 0$ $\mathbf{7 \pm \sqrt{30}}$

24. $x^2 + 16x - 7 = 0$ $\mathbf{-8 \pm \sqrt{71}}$

25. $2x^2 + 8x - 3 = 0$ $\mathbf{\frac{-4 \pm \sqrt{22}}{2}}$

26. $x^2 + x - 5 = 0$ $\mathbf{\frac{-1 \pm \sqrt{21}}{2}}$

27. $2x^2 - 10x + 5 = 0$ $\mathbf{\frac{5 \pm \sqrt{15}}{2}}$

28. $x^2 + 3x + 6 = 0$ $\mathbf{\frac{-3 \pm i\sqrt{15}}{2}}$

29. $2x^2 + 5x + 6 = 0$ $\mathbf{\frac{-5 \pm i\sqrt{23}}{4}}$

30. $7x^2 + 6x + 2 = 0$ $\mathbf{\frac{-3 \pm i\sqrt{5}}{7}}$

31. **GEOMETRY** When the dimensions of a cube are reduced by 4 inches on each side, the surface area of the new cube is 864 square inches. What were the dimensions of the original cube? **16 in. by 16 in. by 16 in.**

32. **INVESTMENTS** The amount of money A in an account in which P dollars is invested for 2 years is given by the formula $A = P(1 + r)^2$, where r is the interest rate compounded annually. If an investment of \$800 in the account grows to \$882 in two years, at what interest rate was it invested? **5%**

Reading to Learn Mathematics, p. 335

ELL

Pre-Activity **How can you find the time it takes an accelerating race car to reach the finish line?**

Read the introduction to Lesson 6-4 at the top of page 306 in your textbook.

Explain what it means to say that the driver accelerates at a constant rate of 8 feet per second square.

If the driver is traveling at a certain speed at a particular moment, then one second later, the driver is traveling 8 feet per second faster.

Reading the Lesson

1. Give the reason for each step in the following solution of an equation by using the Square Root Property.

$x^2 - 12x + 36 = 81$	Original equation
$(x - 6)^2 = 81$	**Factor the perfect square trinomial.**
$x - 6 = \pm\sqrt{81}$	**Square Root Property**
$x - 6 = \pm 9$	**81 = 9**
$x - 6 = 9$ or $x - 6 = -9$	**Rewrite as two equations.**
$x = 15$ $\quad$ $x = -3$	**Solve each equation.**

2. Explain how to find the constant that must be added to make a binomial into a perfect square trinomial.

Sample answer: Find half of the coefficient of the linear term and square it.

3. a. What is the first step in solving the equation $3x^2 + 6x = 5$ by completing the square? **Divide the equation by 3.**

b. What is the first step in solving the equation $x^2 + 5x - 12 = 0$ by completing the square? **Add 12 to each side.**

Helping You Remember

4. How can you use the rules for squaring a binomial to help you remember the procedure for changing a binomial into a perfect square trinomial?

One of the rules for squaring a binomial is $(x + y)^2 = x^2 + 2xy + y^2$. In completing the square, you are starting with $x^2 + bx$ and need to find y^2. This shows you that $b = 2y$, so $y = \frac{b}{2}$. That is why you must take half of the coefficient and square it to get the constant that must be added to complete the square.

4 Assess

Open-Ended Assessment

Writing Have students write a summary of the various techniques for solving quadratic equations using the Square Root Property. Students should provide written examples of each technique.

Tips for New Teachers

Intervention Suggest students write a summary of the various methods that can be used to solve quadratic equations. Ask them which method they prefer to use, and why they like that method the best.

Getting Ready for Lesson 6-5

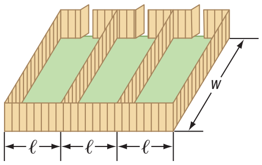
PREREQUISITE SKILL Lesson 6-5 presents the Quadratic Formula. The first step in evaluating the formula is to evaluate the expression under the radical sign. Use Exercises 69–72 to determine your students' familiarity with evaluating expressions.

Assessment Options

Quiz (Lessons 6-3 and 6-4) is available on p. 369 of the *Chapter 6 Resource Masters*.

Mid-Chapter Test (Lessons 6-1 through 6-4) is available on p. 371 of the *Chapter 6 Resource Masters*.

★ 53. **KENNEL** A kennel owner has 164 feet of fencing with which to enclose a rectangular region. He wants to subdivide this region into three smaller rectangles of equal length, as shown. If the total area to be enclosed is 576 square feet, find the dimensions of the entire enclosed region. (*Hint*: Write an expression for ℓ in terms of w.) **18 ft by 32 ft or 64 ft by 9 ft**



54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you find the time it takes an accelerating race car to reach the finish line?

Include the following in your answer:

- an explanation of why $t^2 + 22t + 121 = 246$ cannot be solved by factoring, and
- a description of the steps you would take to solve the equation $t^2 + 22t + 121 = 246$.

Standardized Test Practice

55. What is the absolute value of the product of the two solutions for x in $x^2 - 2x - 2 = 0$? **D**

Ⓐ −1 Ⓑ 0 Ⓒ 1 Ⓓ 2

56. For which value of c will the roots of $x^2 + 4x + c = 0$ be real and equal? **D**

Ⓐ 1 Ⓑ 2 Ⓒ 3 Ⓓ 4 Ⓔ 5

Maintain Your Skills

Mixed Review **Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers.** *(Lesson 6-3)*

57. 2, 1 **$x^2 - 3x + 2 = 0$**
58. −3, 9 **$x^2 - 6x - 27 = 0$**
59. $6, \frac{1}{3}$ **$3x^2 - 19x + 6 = 0$**
60. $-\frac{1}{3}, -\frac{3}{4}$ **$12x^2 + 13x + 3 = 0$**

Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located. *(Lesson 6-2)*

61. $3x^2 = 4 - 8x$ **61. between −4 and −3; between 0 and 1**
62. $x^2 + 48 = 14x$ **6, 8**
63. $2x^2 + 11x = -12$ **$-4, -1\frac{1}{2}$**

64. Write *the seventh root of 5 cubed* using exponents. *(Lesson 5-7)* **$5^{\frac{3}{7}}$**

Solve each system of equations by using inverse matrices. *(Lesson 4-8)*

65. $5x + 3y = -5$
$7x + 5y = -11$ **(2, −5)**

66. $6x + 5y = 8$
$3x - y = 7$ **$\left(\frac{43}{21}, -\frac{6}{7}\right)$**

CHEMISTRY **For Exercises 67 and 68, use the following information.**
For hydrogen to be a liquid, its temperature must be within 2°C of −257°C. *(Lesson 1-4)*

67. Write an equation to determine the greatest and least temperatures for this substance. **$|x - (-257)| = 2$**
68. Solve the equation. **greatest: −255°C; least: −259°C**

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Evaluate $b^2 - 4ac$ for the given values of a, b, and c.**
*(To review **evaluating expressions**, see Lesson 1-1.)*

69. $a = 1, b = 7, c = 3$ **37**
70. $a = 1, b = 2, c = 5$ **−16**
71. $a = 2, b = -9, c = -5$ **121**
72. $a = 4, b = -12, c = 9$ **0**

Answer

54. To find the distance traveled by the accelerating race car in the given situation, you must solve the equation $t^2 + 22t + 121 = 246$ or $t^2 + 22t - 125 = 0$. Answers should include the following.

- **Since the expression $t^2 + 22t - 125$ is prime, the solutions of $t^2 + 22t + 121 = 246$ cannot be obtained by factoring.**
- **Rewrite $t^2 + 22t + 121$ as $(t + 11)^2$. Solve $(t + 11)^2 = 246$ by applying the Square Root Property. Then, subtract 11 from each side. Using a calculator, the two solutions are about 4.7 or −26.7. Since time cannot be negative, the driver takes about 4.7 seconds to reach the finish line.**

6-5 The Quadratic Formula and the Discriminant

What You'll Learn

- Solve quadratic equations by using the Quadratic Formula.
- Use the discriminant to determine the number and type of roots of a quadratic equation.

Vocabulary

- Quadratic Formula
- discriminant

How is blood pressure related to age?

As people age, their arteries lose their elasticity, which causes blood pressure to increase. For healthy women, average systolic blood pressure is estimated by $P = 0.01A^2 + 0.05A + 107$, where P is the average blood pressure in millimeters of mercury (mm Hg) and A is the person's age. For healthy men, average systolic blood pressure is estimated by $P = 0.006A^2 - 0.02A + 120$.

QUADRATIC FORMULA You have seen that exact solutions to some quadratic equations can be found by graphing, by factoring, or by using the Square Root Property. While completing the square can be used to solve any quadratic equation, the process can be tedious if the equation contains fractions or decimals. Fortunately, a formula exists that can be used to solve any quadratic equation of the form $ax^2 + bx + c = 0$. This formula can be derived by solving the general form of a quadratic equation.

$$ax^2 + bx + c = 0 \qquad \text{General quadratic equation}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide each side by } a.$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Subtract } \frac{c}{a} \text{ from each side.}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{Complete the square.}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \qquad \text{Factor the left side. Simplify the right side.}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Square Root Property}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Subtract } \frac{b}{2a} \text{ from each side.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Simplify.}$$

This equation is known as the **Quadratic Formula**.

Study Tip

Reading Math

The Quadratic Formula is read *x equals the opposite of b, plus or minus the square root of b squared minus 4ac, all divided by 2a.*

Key Concept *Quadratic Formula*

The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

6-5 Lesson Notes

1 Focus

5-Minute Check Transparency 6-5 Use as a quiz or review of Lesson 6-4.

Mathematical Background notes are available for this lesson on p. 284D.

Building on Prior Knowledge

In Lesson 6-4, students solved quadratic equations by completing the square. In this lesson, students generalize this procedure as they complete the square for the general quadratic equation to derive the Quadratic Formula.

How is blood pressure related to age?

Ask students:

- As the value of A increases in these equations, what happens to the value of P? **It increases.**
- Which way do the parabolas open that are the graphs of these equations? **upward**

Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 337–338
- Skills Practice, p. 339
- Practice, p. 340
- Reading to Learn Mathematics, p. 341
- Enrichment, p. 342

Graphing Calculator and Spreadsheet Masters, p. 38

Transparencies

5-Minute Check Transparency 6-5
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lessons 11, 12
Interactive Chalkboard

2 Teach

QUADRATIC FORMULA

In-Class Examples

Power Point®

1 Solve $x^2 - 8x = 33$ by using the Quadratic Formula. **−3, 11**

Teaching Tip Encourage students to write down the values of a, b, and c from the standard form of the quadratic equation before they begin substituting into the formula.

2 Solve $x^2 - 34x + 289 = 0$ by using the Quadratic Formula. **17**

Study Tip

Quadratic Formula

Although factoring may be an easier method to solve the equations in Examples 1 and 2, the Quadratic Formula can be used to solve any quadratic equation.

Example 1 Two Rational Roots

Solve $x^2 - 12x = 28$ by using the Quadratic Formula.

First, write the equation in the form $ax^2 + bx + c = 0$ and identify a, b, and c.

$$x^2 - 12x = 28 \longrightarrow \begin{array}{c} ax^2 + bx + c = 0 \\ \downarrow \quad \downarrow \quad \downarrow \\ 1x^2 - 12x - 28 = 0 \end{array}$$

Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(-28)}}{2(1)}$$ Replace a with 1, b with −12, and c with −28.

$$x = \frac{12 \pm \sqrt{144 + 112}}{2}$$ Simplify.

$$x = \frac{12 \pm \sqrt{256}}{2}$$ Simplify.

$$x = \frac{12 \pm 16}{2}$$ $\sqrt{256} = 16$

$$x = \frac{12 + 16}{2} \quad \text{or} \quad x = \frac{12 - 16}{2}$$ Write as two equations.

$$= 14 \qquad\qquad = -2$$ Simplify.

The solutions are −2 and 14. Check by substituting each of these values into the original equation.

When the value of the radicand in the Quadratic Formula is 0, the quadratic equation has exactly one rational root.

Example 2 One Rational Root

Solve $x^2 + 22x + 121 = 0$ by using the Quadratic Formula.

Identify a, b, and c. Then, substitute these values into the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic Formula

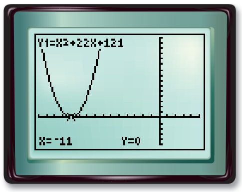
$$x = \frac{-(22) \pm \sqrt{(22)^2 - 4(1)(121)}}{2(1)}$$ Replace a with 1, b with 22, and c with 121.

$$x = \frac{-22 \pm \sqrt{0}}{2}$$ Simplify.

$$x = \frac{-22}{2} \text{ or } -11$$ $\sqrt{0} = 0$

The solution is −11.

CHECK A graph of the related function shows that there is one solution at $x = -11$.



[−15, 5] scl: 1 by [−5, 15] scl: 1

Teacher to Teacher

Lori Haldorson & Cathy Hokkanen Blaine H.S., Blaine, MN

"To help students memorize the Quadratic Formula, we sing it to the tune of 'Pop Goes the Weasel'."

You can express irrational roots exactly by writing them in radical form.

Example 3 Irrational Roots

Solve $2x^2 + 4x - 5 = 0$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

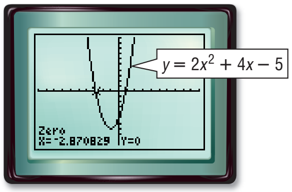
$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-5)}}{2(2)}$ Replace a with 2, b with 4, and c with -5.

$x = \frac{-4 \pm \sqrt{56}}{4}$ Simplify.

$x = \frac{-4 \pm 2\sqrt{14}}{4}$ or $\frac{-2 \pm \sqrt{14}}{2}$ $\sqrt{56} = \sqrt{4 \cdot 14}$ or $2\sqrt{14}$

The exact solutions are $\frac{-2 - \sqrt{14}}{2}$ and $\frac{-2 + \sqrt{14}}{2}$. The approximate solutions are -2.9 and 0.9.

CHECK Check these results by graphing the related quadratic function, $y = 2x^2 + 4x - 5$. Using the ZERO function of a graphing calculator, the approximate zeros of the related function are -2.9 and 0.9.



[−10, 10] scl: 1 by [−10, 10] scl: 1

When using the Quadratic Formula, if the radical contains a negative value, the solutions will be complex. Complex solutions always appear in conjugate pairs.

Example 4 Complex Roots

Solve $x^2 - 4x = -13$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$ Replace a with 1, b with -4, and c with 13.

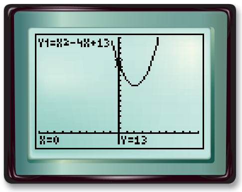
$x = \frac{4 \pm \sqrt{-36}}{2}$ Simplify.

$x = \frac{4 \pm 6i}{2}$ $\sqrt{-36} = \sqrt{36(-1)}$ or $6i$

$x = 2 \pm 3i$ Simplify.

The solutions are the complex numbers $2 + 3i$ and $2 - 3i$.

A graph of the related function shows that the solutions are complex, but it cannot help you find them.



[−15, 5] scl: 1 by [−2, 18] scl: 1

Study Tip

Using the Quadratic Formula

Remember that to correctly identify a, b, and c for use in the Quadratic Formula, the equation must be written in the form $ax^2 + bx + c = 0$.

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

Teaching Tip Some students may have commented that the equations in Examples 1 and 2 could have been solved by factoring. Before beginning Example 3, stress that many quadratic equations cannot easily be solved by factoring. State that the quadratic equation presented in Example 3 is such an equation. Emphasize that the Quadratic Formula provides a way to find the roots for *any* quadratic equation.

3 Solve $x^2 - 6x + 2 = 0$ by using the Quadratic Formula. **$3 \pm \sqrt{7}$, or approximately 0.4 and 5.6**

Teaching Tip Remind students that conjugate pairs are two complex numbers of the form $a + bi$ and $a - bi$.

4 Solve $x^2 + 13 = 6x$ by using the Quadratic Formula. **$3 \pm 2i$**

Concept Check

Real Roots or Imaginary Roots Ask students to look back at the first four examples in this lesson and see how they might predict whether the roots will be real or imaginary. **If the values of a and c have the same sign and 4 times their product is greater than the square of the value of b, then the roots will be imaginary.**

ROOTS AND THE DISCRIMINANT

In-Class Example Power Point®

5 Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $x^2 + 6x + 9 = 0$
0; one rational root

b. $x^2 + 3x + 5 = 0$
−11; two complex roots

c. $x^2 + 8x - 4 = 0$
80; two irrational roots

d. $x^2 - 11x + 10 = 0$
81; two rational roots

CHECK To check complex solutions, you must substitute them into the original equation. The check for 2 + 3*i* is shown below.

$$\begin{aligned} x^2 - 4x &= -13 && \text{Original equation} \\ (2+3i)^2 - 4(2+3i) &\stackrel{?}{=} -13 && x = 2 + 3i \\ 4 + 12i + 9i^2 - 8 - 12i &\stackrel{?}{=} -13 && \text{Sum of a square; Distributive Property} \\ -4 + 9i^2 &\stackrel{?}{=} -13 && \text{Simplify.} \\ -4 - 9 &= -13 \checkmark && i^2 = -1 \end{aligned}$$

Study Tip

Reading Math
Remember that the solutions of an equation are called *roots*.

ROOTS AND THE DISCRIMINANT

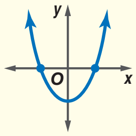
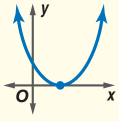
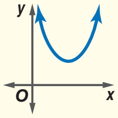
In Examples 1, 2, 3, and 4, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longleftarrow \text{discriminant}$$

The value of the discriminant can be used to determine the number and type of roots of a quadratic equation.

Key Concept *Discriminant*

Consider $ax^2 + bx + c = 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real, rational root	
$b^2 - 4ac < 0$	2 complex roots	

Study Tip

Using the Discriminant
The discriminant can help you check the solutions of a quadratic equation. Your solutions must match in number and in type to those determined by the discriminant.

Example 5 Describe Roots

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation.

a. $9x^2 - 12x + 4 = 0$

$a = 9, b = -12, c = 4$

$$\begin{aligned} b^2 - 4ac &= (-12)^2 - 4(9)(4) \\ &= 144 - 144 \\ &= 0 \end{aligned}$$

The discriminant is 0, so there is one rational root.

b. $2x^2 + 16x + 33 = 0$

$a = 2, b = 16, c = 33$

$$\begin{aligned} b^2 - 4ac &= (16)^2 - 4(2)(33) \\ &= 256 - 264 \\ &= -8 \end{aligned}$$

The discriminant is negative, so there are two complex roots.

DAILY INTERVENTION

Differentiated Instruction

Logical Have students use their classification skills to create a classroom poster listing the four different types of roots that can result when solving a quadratic equation. Each listing should include a sample equation that results in that type of roots and an explanation of how the value of the discriminant is indicative of the root type. Graphs like those shown on p. 316 can be added to the poster.

c. $-5x^2 + 8x - 1 = 0$

$a = -5, b = 8, c = -1$

$$\begin{aligned} b^2 - 4ac &= (8)^2 - 4(-5)(-1) \\ &= 64 - 20 \\ &= 44 \end{aligned}$$

The discriminant is 44, which is not a perfect square. Therefore, there are two irrational roots.

d. $-7x + 15x^2 - 4 = 0$

$a = 15, b = -7, c = -4$

$$\begin{aligned} b^2 - 4ac &= (-7)^2 - 4(15)(-4) \\ &= 49 + 240 \\ &= 289 \text{ or } 17^2 \end{aligned}$$

The discriminant is 289, which is a perfect square. Therefore, there are two rational roots.

You have studied a variety of methods for solving quadratic equations. The table below summarizes these methods.

Concept Summary — *Solving Quadratic Equations*

Method	Can be Used	When to Use
Graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
Factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 3x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x + 13)^2 = 9$
Completing the Square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 14x - 9 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $3.4x^2 - 2.5x + 7.9 = 0$

Check for Understanding

Concept Check

1. **OPEN ENDED** Sketch the graph of a quadratic equation whose discriminant is
   a. positive. **b.** negative. **c.** zero. **a–c. See margin.**
2. **Explain** why the roots of a quadratic equation are complex if the value of the discriminant is less than 0.

2. The square root of a negative number is a complex number.

3. **Describe** the relationship that must exist between a, b, and c in the equation $ax^2 + bx + c = 0$ in order for the equation to have exactly one solution.
   $b^2 - 4ac$ must equal 0.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1–5
8–11	1–4
12, 13	1–4

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant. **4–7. See margin.**

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

4. $8x^2 + 18x - 5 = 0$
5. $2x^2 - 4x + 1 = 0$
6. $4x^2 + 4x + 1 = 0$
7. $x^2 + 3x + 8 = 0$

3 Practice/Apply

Study Notebook

Have students—

- *add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.*
- *copy the information provided in the Concept Summary on p. 317 into their notebook.*
- *include any other item(s) that they find helpful in mastering the skills in this lesson.*

About the Exercises...

Organization by Objective

- **Quadratic Formula:** 14–39, 42–44
- **Roots and the Discriminant:** 14–27, 40, 41

Odd/Even Assignments

Exercises 14–39 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–25 odd, 29–39 odd, 40, 41, 45–66

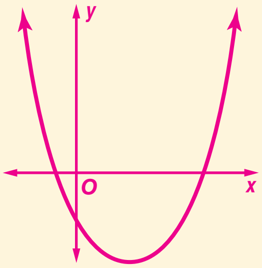
Average: 15–39 odd, 40–43, 45–66

Advanced: 14–38 even, 42–60 (optional: 61–66)

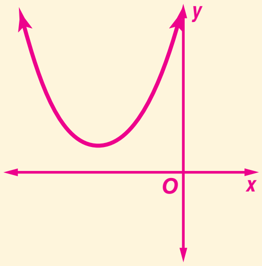
Answers

4a. 484	4b. 2 rational
4c. $\frac{1}{4}, -\frac{5}{2}$	
5a. 8	5b. 2 irrational
5c. $\frac{2 \pm \sqrt{2}}{2}$	
6a. 0	6b. one rational
6c. $-\frac{1}{2}$	
7a. −3	7b. 2 complex
7c. $\frac{-3 \pm i\sqrt{3}}{2}$	

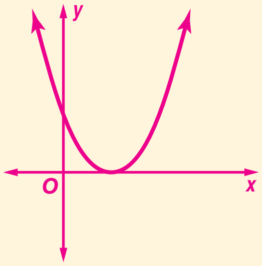
1a. Sample answer:



1b. Sample answer:



1c. Sample answer:



Study Guide and Intervention, p. 337 (shown) and p. 338

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example **Solve $x^2 - 5x = 14$ by using the Quadratic Formula.**

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ Replace *a* with 1, *b* with −5, and *c* with −14.

$= \frac{5 \pm \sqrt{81}}{2}$ Simplify.

$= \frac{5 \pm 9}{2}$

$= 7 \text{ or } -2$

The solutions are −2 and 7.

Exercises

Solve each equation by using the Quadratic Formula.

1. $x^2 + 2x - 35 = 0$ **5, −7**
2. $x^2 + 10x + 24 = 0$ **−4, −6**
3. $x^2 - 11x + 24 = 0$ **3, 8**
4. $4x^2 + 19x - 5 = 0$ $\frac{1}{4}, -5$
5. $14x^2 + 9x + 1 = 0$ $-\frac{1}{2}, -\frac{1}{7}$
6. $2x^2 - x - 15 = 0$ $3, -\frac{5}{2}$
7. $3x^2 + 5x = 2$ $-2, \frac{1}{3}$
8. $2y^2 + y - 15 = 0$ $\frac{5}{2}, -3$
9. $3x^2 - 16x + 16 = 0$ $4, \frac{4}{3}$
10. $8x^2 + 6x - 9 = 0$ $-\frac{3}{2}, \frac{3}{4}$
11. $r^2 - \frac{3r}{5} + \frac{2}{25} = 0$ $\frac{2}{5}, \frac{1}{5}$
12. $x^2 - 10x - 50 = 0$ $5 \pm 5\sqrt{3}$
13. $x^2 + 6x - 23 = 0$ $-3 \pm 4\sqrt{2}$
14. $4x^2 - 12x - 63 = 0$ $\frac{3 \pm 6\sqrt{2}}{2}$
15. $x^2 - 6x + 21 = 0$ $3 \pm 2i\sqrt{3}$

Skills Practice, p. 339 and Practice, p. 340 (shown)

Complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

1. $x^2 - 16x + 64 = 0$ **0; 1 rational; 8**
2. $x^2 = 3x$ **9; 2 rational; 0, 3**
3. $9x^2 - 24x + 16 = 0$ **0; 1 rational;** $\frac{4}{3}$
4. $x^2 - 3x = 40$ **169; 2 rational; −5, 8**
5. $3x^2 + 9x - 2 = 0$ **105; 2 irrational;** $\frac{-9 \pm \sqrt{105}}{6}$
6. $2x^2 + 7x = 0$ **49; 2 rational; 0,** $-\frac{7}{2}$
7. $5x^2 - 2x + 4 = 0$ **−76; 2 complex;** $\frac{1 \pm i\sqrt{19}}{5}$
8. $12x^2 - x - 6 = 0$ **289; 2 rational;** $\frac{3}{4}, -\frac{2}{3}$
9. $7x^2 + 6x + 2 = 0$ **−20; 2 complex;** $\frac{-3 \pm i\sqrt{5}}{7}$
10. $12x^2 + 2x - 4 = 0$ **196; 2 rational;** $\frac{1}{2}, -\frac{2}{3}$
11. $6x^2 - 2x - 1 = 0$ **28; 2 irrational;** $\frac{1 \pm \sqrt{7}}{6}$
12. $x^2 + 3x + 6 = 0$ **−15; 2 complex;** $\frac{-3 \pm i\sqrt{15}}{2}$
13. $4x^2 - 3x^2 - 6 = 0$ **105; 2 irrational;** $\frac{3 \pm \sqrt{105}}{8}$
14. $16x^2 - 8x + 1 = 0$ **0; 1 rational;** $\frac{1}{4}$
15. $2x^2 - 5x - 6 = 0$ **73; 2 irrational;** $\frac{5 \pm \sqrt{73}}{4}$

Solve each equation by using the method of your choice. Find exact solutions.

16. $7x^2 - 5x = 0$ $0, \frac{5}{7}$
17. $4x^2 - 9 = 0$ $\pm\frac{3}{2}$
18. $3x^2 + 8x = 3$ $\frac{1}{3}, -3$
19. $x^2 - 21 = 4x$ **−3, 7**
20. $3x^2 - 13x + 4 = 0$ $\frac{1}{3}, 4$
21. $15x^2 + 22x = -8$ $-\frac{2}{3}, -\frac{4}{5}$
22. $x^2 - 6x + 3 = 0$ $3 \pm \sqrt{6}$
23. $x^2 - 14x + 53 = 0$ $7 \pm 2i$
24. $3x^2 = -54$ $\pm 3i\sqrt{2}$
25. $25x^2 - 20x - 6 = 0$ $\frac{2 \pm \sqrt{10}}{5}$
26. $4x^2 - 4x + 17 = 0$ $\frac{1 \pm 4i}{2}$
27. $8x - 1 = 4x^2$ $\frac{2 \pm \sqrt{3}}{2}$
28. $x^2 = 4x - 15$ $2 \pm i\sqrt{11}$
29. $4x^2 - 12x + 7 = 0$ $\frac{3 \pm \sqrt{2}}{2}$

30. **GRAVITATION** The height $h(t)$ in feet of an object *t* seconds after it is propelled straight up from the ground with an initial velocity of 60 feet per second is modeled by the equation $h(t) = -16t^2 + 60t$. At what times will the object be at a height of 56 feet? **1.75 s, 2 s**

31. **STOPPING DISTANCE** The formula $d = 0.05s^2 + 1.1s$ estimates the minimum stopping distance *d* in feet for a car traveling *s* miles per hour. If a car stops in 200 feet, what is the

Reading to Learn Mathematics, p. 341

ELL

Pre-Activity **How is blood pressure related to age?**

Read the introduction to Lesson 6-5 at the top of page 313 in your textbook.

Describe how you would calculate your normal blood pressure using one of the formulas in your textbook.

Sample answer: Substitute your age for *A* in the appropriate formula (for females or males) and evaluate the expression.

Reading the Lesson

1. a. Write the Quadratic Formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   b. Identify the values of *a*, *b*, and *c* that you would use to solve $2x^2 - 5x = -7$, but do not actually solve the equation.

   $a =$ __2__ $b =$ __−5__ $c =$ __7__

2. Suppose that you are solving four quadratic equations with rational coefficients and have found the value of the discriminant for each equation. In each case, give the number of roots and describe the type of roots that the equation will have.

Value of Discriminant	Number of Roots	Type of Roots
64	2	real, rational
−8	2	complex
21	2	real, irrational
0	1	real, rational

Helping You Remember

3. How can looking at the Quadratic Formula help you remember the relationships between the value of the discriminant and the number of roots of a quadratic equation and whether the roots are real or complex?

Sample answer: The discriminant is the expression under the radical in the Quadratic Formula. Look at the Quadratic Formula and consider what happens when you take the principal square root of $b^2 - 4ac$ and apply ± in front of the result. If $b^2 - 4ac$ is positive, its principal square root will be a positive number and applying ± will give two different real solutions, which may be rational or irrational. If $b^2 - 4ac = 0$, its principal square root is 0, so applying ± in the Quadratic Formula will only lead to one solution, which will be rational (assuming *a*, *b*, and *c* are integers). If $b^2 - 4ac$ is negative, since the square roots of negative numbers are not real numbers, you will get two complex roots, corresponding to the + and − in the ± symbol.

Solve each equation using the method of your choice. Find exact solutions.

8. $x^2 + 8x = 0$ **0, −8**
9. $x^2 + 5x + 6 = 0$ **−3, −2**
10. $x^2 - 2x - 2 = 0$ $1 \pm \sqrt{3}$
11. $4x^2 + 20x + 25 = -2$ $\frac{-5 \pm i\sqrt{2}}{2}$

Application

PHYSICS **For Exercises 12 and 13, use the following information.**
The height $h(t)$ in feet of an object *t* seconds after it is propelled straight up from the ground with an initial velocity of 85 feet per second is modeled by $h(t) = -16t^2 + 85t$.

12. at about 0.7 s and again at about 4.6 s

12. When will the object be at a height of 50 feet?
13. Will the object ever reach a height of 120 feet? Explain your reasoning. **No; see margin for explanation.**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1–5
28–39, 42–44	1–4
40–41	5

Extra Practice
See page 841.

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant. **14–27. See margin.**

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

14. $x^2 + 3x - 3 = 0$
15. $x^2 - 16x + 4 = 0$
16. $x^2 - 2x + 5 = 0$
17. $x^2 - x + 6 = 0$
18. $-12x^2 + 5x + 2 = 0$
19. $-3x^2 - 5x + 2 = 0$
20. $x^2 + 4x + 3 = 4$
21. $2x - 5 = -x^2$
22. $9x^2 - 6x - 4 = -5$
23. $25 + 4x^2 = -20x$
24. $4x^2 + 7 = 9x$
25. $3x + 6 = -6x^2$
26. ★ $\frac{3}{4}x^2 - \frac{1}{3}x - 1 = 0$
27. ★ $0.4x^2 + x - 0.3 = 0$

28–39. See pp. 343A–343F.

Solve each equation by using the method of your choice. Find exact solutions.

28. $x^2 - 30x - 64 = 0$
29. $7x^2 + 3 = 0$
30. $x^2 - 4x + 7 = 0$
31. $2x^2 + 6x - 3 = 0$
32. $4x^2 - 8 = 0$
33. $4x^2 + 81 = 36x$
34. $-4(x + 3)^2 = 28$
35. $3x^2 - 10x = 7$
36. $x^2 + 9 = 8x$
37. $10x^2 + 3x = 0$
38. $2x^2 - 12x + 7 = 5$
39. $21 = (x - 2)^2 + 5$

More About. . .

Bridges

The Golden Gate, located in San Francisco, California, is the tallest bridge in the world, with its towers extending 746 feet above the water and the floor of the bridge extending 220 feet above water.

Source: www.goldengatebridge.org

BRIDGES **For Exercises 40 and 41, use the following information.**
The supporting cables of the Golden Gate Bridge approximate the shape of a parabola. The parabola can be modeled by the quadratic function $y = 0.00012x^2 + 6$, where *x* represents the distance from the axis of symmetry and *y* represents the height of the cables. The related quadratic equation is $0.00012x^2 + 6 = 0$.

40. Calculate the value of the discriminant. **≈ −0.00288**
41. What does the discriminant tell you about the supporting cables of the Golden Gate Bridge? **See pp. 343A–343F.**

FOOTBALL **For Exercises 42 and 43, use the following information.**
The average NFL salary $A(t)$ (in thousands of dollars) from 1975 to 2000 can be estimated using the function $A(t) = 2.3t^2 - 12.4t + 73.7$, where *t* is the number of years since 1975. **42. D: $0 \le t \le 25$, R: $73.7 \le A(t) \le 1201.2$**

42. Determine a domain and range for which this function makes sense.
43. According to this model, in what year did the average salary first exceed 1 million dollars? **1998**

Online Research **Data Update** What is the current average NFL salary? How does this average compare with the average given by the function used in Exercises 42 and 43? Visit www.algebra2.com/data_update to learn more.

Enrichment, p. 342

Sum and Product of Roots

Sometimes you may know the roots of a quadratic equation without knowing the equation itself. Using your knowledge of factoring to solve an equation, you can work backward to find the quadratic equation. The rule for finding the sum and product of roots is as follows:

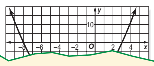
Sum and Product of Roots	If the roots of $ax^2 + bx + c = 0$, with $a \neq 0$, are s_1 and s_2, then $s_1 + s_2 = -\frac{b}{a}$ and $s_1 \cdot s_2 = \frac{c}{a}$.

Example **A road with an initial gradient, or slope, of 3% can be represented by the formula $y = ax^2 + 0.03x + c$, where *y* is the elevation and *x* is the distance along the curve. Suppose the elevation of the road is 1105 feet at points 200 feet and 1000 feet along the curve. You can find the equation of the transition curve. Equations of transition curves are used by civil engineers to design smooth and safe roads.**

The roots are $x = 3$ and $x = -8$.

$3 + (-8) = -5$ Add the roots.

$3(-8) = -24$ Multiply the roots.



Answer

13. The discriminant of $-16t^2 + 85t = 120$ is −455, indicating that the equation has no real solutions.

44. **HIGHWAY SAFETY** Highway safety engineers can use the formula $d = 0.05s^2 + 1.1s$ to estimate the minimum stopping distance d in feet for a vehicle traveling s miles per hour. If a car is able to stop after 125 feet, what is the fastest it could have been traveling when the driver first applied the brakes? **about 40.2 mph**

45. **CRITICAL THINKING** Find all values of k such that $x^2 - kx + 9 = 0$ has

a. one real root. **$k = \pm 6$**
b. two real roots. **$k < -6$ or $k > 6$**
c. no real roots. **$-6 < k < 6$**

46. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 343A–343F.**

How is blood pressure related to age?

Include the following in your answer:

- an expression giving the average systolic blood pressure for a person of your age, and
- an example showing how you could determine A in either formula given a specific value of P.

Standardized Test Practice

47. If $2x^2 - 5x - 9 = 0$, then x could equal which of the following? **D**

Ⓐ −1.12 Ⓑ 1.54 Ⓒ 2.63 Ⓓ 3.71

48. Which best describes the nature of the roots of the equation $x^2 - 3x + 4 = 0$? **C**

Ⓐ real and equal
Ⓑ real and unequal
Ⓒ complex
Ⓓ real and complex

Maintain Your Skills

Mixed Review

Solve each equation by using the Square Root Property. *(Lesson 6-4)*

49. $x^2 + 18x + 81 = 25$ **$-14, -4$**
50. $x^2 - 8x + 16 = 7$ **$4 \pm \sqrt{7}$**
51. $4x^2 - 4x + 1 = 8$ **$\frac{1 \pm 2\sqrt{2}}{2}$**

Solve each equation by factoring. *(Lesson 6-3)*

52. $4x^2 + 8x = 0$ **$-2, 0$**
53. $x^2 - 5x = 14$ **$-2, 7$**
54. $3x^2 + 10 = 17x$ **$\frac{2}{3}, 5$**

Simplify. *(Lesson 5-5)*

55. $\sqrt{a^8b^{20}}$ **a^4b^{10}**
56. $\sqrt{100p^{12}q^2}$ **$10p^6|q|$**
57. $\sqrt[3]{64b^6c^6}$ **$4b^2c^2$**

58. **ANIMALS** The fastest-recorded physical action of any living thing is the wing beat of the common midge. This tiny insect normally beats its wings at a rate of 133,000 times per minute. At this rate, how many times would the midge beat its wings in an hour? Write your answer in scientific notation. *(Lesson 5-1)* **7.98×10^6**

Solve each system of inequalities. *(Lesson 3-3)* **59–60. See pp. 343A–343F.**

59. $x + y \leq 9$
$x - y \leq 3$
$y - x \geq 4$

60. $x \geq 1$
$y \leq -1$
$y \leq x$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **State whether each trinomial is a perfect square. If it is, factor it.** *(To review **perfect square trinomials**, see Lesson 5-4.)*

61. $x^2 - 5x - 10$ **no**
62. $x^2 - 14x + 49$ **yes; $(x - 7)^2$**
63. $4x^2 + 12x + 9$ **yes; $(2x + 3)^2$**
64. $25x^2 + 20x + 4$ **yes; $(5x + 2)^2$**
65. $9x^2 - 12x + 16$ **no**
66. $36x^2 - 60x + 25$ **yes; $(6x - 5)^2$**

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Modeling Ask students to sketch graphs of parabolas that illustrate each of the four types of roots for quadratic equations. Have them label each sketch with the type of value the discriminant of the corresponding quadratic equation would have.

Getting Ready for Lesson 6-6

PREREQUISITE SKILL Lesson 6-6 presents the analysis of the graphs of quadratic functions. To graph a quadratic function, it is helpful if the function is written in vertex form, which often requires students to complete the square. Recognition of perfect square trinomials is an important part of completing the square. Exercises 61–66 should be used to determine your students' familiarity with perfect square trinomials.

Answers

14a. 21
14b. 2 irrational
14c. $\frac{-3 \pm \sqrt{21}}{2}$
15a. 240
15b. 2 irrational
15c. $8 \pm 2\sqrt{15}$
16a. −16
16b. 2 complex
16c. $1 \pm 2i$
17a. −23
17b. 2 complex
17c. $\frac{1 \pm i\sqrt{23}}{2}$
18a. 121
18b. 2 rational
18c. $-\frac{1}{4}, \frac{2}{3}$
19a. 49
19b. 2 rational
19c. $-2, \frac{1}{3}$
20a. 20
20b. 2 irrational
20c. $-2 \pm \sqrt{5}$
21a. 24
21b. 2 irrational
21c. $-1 \pm \sqrt{6}$

Answers

22a. 0
22b. one rational
22c. $\frac{1}{3}$
23a. 0
23b. one rational
23c. $-\frac{5}{2}$
24a. −31
24b. 2 complex
24c. $\frac{9 \pm i\sqrt{31}}{8}$
25a. −135
25b. 2 complex
25c. $\frac{-1 \pm i\sqrt{15}}{4}$
26a. $\frac{28}{9}$
26b. 2 irrational
26c. $\frac{2 \pm 4\sqrt{7}}{9}$
27a. 1.48
27b. 2 irrational
27c. $\frac{-1 \pm 2\sqrt{0.37}}{0.8}$

Graphing Calculator Investigation

A Preview of Lesson 6-6

Getting Started

Know Your Calculator Students can use the calculator to confirm the location of the vertex of each parabola. A good way to do this is to change the window settings for the x-axis to $[-9.4, 9.4]$. Then use the Trace feature and symmetry properties of parabolas to check that the graph is symmetric with respect to the vertical line through the point that appears to be the vertex.

Teach

- Ask students to describe the three constants (a, h, and k) in the general form of a quadratic equation $y = a(x - h)^2 + k$. **Sample answer: a: coefficient of the squared quantity involving the variable x; h: value subtracted from x in the quantity being squared and then multiplied by a; k: value added at the end**
- Before discussing the examples, have students make a conjecture about the effect of the value of each of the constants a, h, and k on the graph of the parabola.
- After completing the discussion of Example 3, have students compare the conjectures they made at the beginning of the investigation to the knowledge they gained during the discussions.
- Have students complete Exercises 1–15.

Graphing Calculator

A Preview of Lesson 6-6

Families of Parabolas

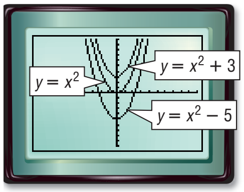
The general form of a quadratic equation is $y = a(x - h)^2 + k$. Changing the values of a, h, and k results in a different parabola in the family of quadratic functions. You can use a TI-83 Plus graphing calculator to analyze the effects that result from changing each of these parameters.

Example 1

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$$y = x^2,\ y = x^2 + 3,\ y = x^2 - 5$$

The graphs have the same shape, and all open up. The vertex of each graph is on the y-axis. However, the graphs have different vertical positions.



Example 1 shows how changing the value of k in the equation $y = a(x - h)^2 + k$ *translates* the parabola along the y-axis. If $k > 0$, the parabola is translated k units up, and if $k < 0$, it is translated k units down.

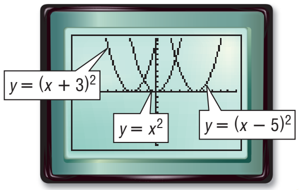
How do you think changing the value of h will affect the graph of $y = x^2$?

Example 2

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

$$y = x^2,\ y = (x + 3)^2,\ y = (x - 5)^2$$

These three graphs all open up and have the same shape. The vertex of each graph is on the x-axis. However, the graphs have different horizontal positions.



Example 2 shows how changing the value of h in the equation $y = a(x - h)^2 + k$ *translates* the graph horizontally. If $h > 0$, the graph translates to the right h units. If $h < 0$, the graph translates to the left h units.

www.algebra2.com/other_calculator_keystrokes

Investigation

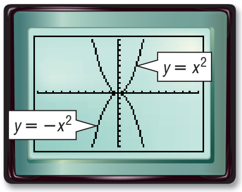
How does the value a affect the graph of $y = x^2$?

Example 3

Graph each set of equations on the same screen in the standard viewing window. Describe any similarities and differences among the graphs.

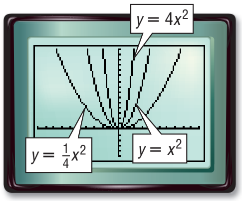
a. $y = x^2$, $y = -x^2$

The graphs have the same vertex and the same shape. However, the graph of $y = x^2$ opens up and the graph of $y = -x^2$ opens down.



b. $y = x^2$, $y = 4x^2$, $y = \frac{1}{4}x^2$

The graphs have the same vertex, (0, 0), but each has a different shape. The graph of $y = 4x^2$ is narrower than the graph of $y = x^2$. The graph of $y = \frac{1}{4}x^2$ is wider than the graph of $y = x^2$.



[−10, 10] scl: 1 by [−5, 15] scl: 1

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph. If $a > 0$, the graph opens up, and if $a < 0$, the graph opens down or is *reflected* over the x-axis. If $|a| > 1$, the graph is narrower than the graph of $y = x^2$. If $|a| < 1$, the graph is wider than the graph of $y = x^2$. Thus, a change in the absolute value of a results in a *dilation* of the graph of $y = x^2$.

Exercises

1–3. See margin.

Consider $y = a(x - h)^2 - k$.

1. How does changing the value of h affect the graph? Give an example.
2. How does changing the value of k affect the graph? Give an example.
3. How does using $-a$ instead of a affect the graph? Give an example.

Examine each pair of equations and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your predictions. Write a sentence or two comparing the two graphs. **4–15. See pp. 343A–343F.**

4. $y = x^2$, $y = x^2 + 2.5$
5. $y = -x^2$, $y = x^2 - 9$
6. $y = x^2$, $y = 3x^2$
7. $y = x^2$, $y = -6x^2$
8. $y = x^2$, $y = (x + 3)^2$
9. $y = -\frac{1}{3}x^2$, $y = -\frac{1}{3}x^2 + 2$
10. $y = x^2$, $y = (x - 7)^2$
11. $y = x^2$, $y = 3(x + 4)^2 - 7$
12. $y = x^2$, $y = -\frac{1}{4}x^2 + 1$
13. $y = (x + 3)^2 - 2$, $y = (x + 3)^2 + 5$
14. $y = 3(x + 2)^2 - 1$, $y = 6(x + 2)^2 - 1$
15. $y = 4(x - 2)^2 - 3$, $y = \frac{1}{4}(x - 2)^2 - 1$

Assess

Ask students:

- In the general form of a quadratic equation, which constant would you change to move the graph left or right? ***h***
- Which constant would you change to move the graph up or down? ***k***
- Which constant would you change to make the graph wider or narrower? ***a***

Answers

1. Changing the value of h moves the graph to the left and the right. If $h > 0$, the graph translates to the right, and if $h < 0$, it translates to the left. In $y = x^2$, the vertex is at (0, 0) and in $y = (x - 2)^2$, the vertex is at (2, 0). The graph has been translated to the right.

2. Changing the value of k moves the graph up and down. If $k > 0$, the graph translates upward, and if $k < 0$, it translates downward. In $y = x^2$, the vertex is at (0, 0) and in $y = x^2 - 3$, the vertex is at (0, −3). The graph has been translated downward.

3. Using $-a$ instead of a reflects the graph over the x-axis. The graph of $y = x^2$ opens upward, while the graph of $y = -x^2$ opens downward.

6-6 Lesson Notes

1 Focus

5-Minute Check Transparency 6-6 Use as a quiz or review of Lesson 6-5.

Mathematical Background notes are available for this lesson on p. 284D.

How can the graph of $y = x^2$ be used to graph any quadratic function?

Ask students:

- For the function $y = x^2$, what value of x makes y equal 0? **0** What value of x makes y equal 0 if the function is $y = (x - 3)^2$? **3**
- Compare the graph of $y = x^2 + 2$ with the graph of $y = (x + 2)^2$. What difference does adding the 2 within the parentheses make? **Sample answer: Adding the 2 inside the parentheses moves the graph 2 units to the left rather than 2 units up when compared to the graph of $y = x^2$.**

6-6 Analyzing Graphs of Quadratic Functions

What You'll Learn

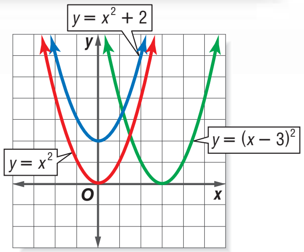
- Analyze quadratic functions of the form $y = a(x - h)^2 + k$.
- Write a quadratic function in the form $y = a(x - h)^2 + k$.

Vocabulary

- vertex form

How can the graph of $y = x^2$ be used to graph any quadratic function?

A *family of graphs* is a group of graphs that displays one or more similar characteristics. The graph of $y = x^2$ is called the *parent graph* of the family of quadratic functions. Study the graphs of $y = x^2$, $y = x^2 + 2$, and $y = (x - 3)^2$. Notice that adding a constant to x^2 moves the graph up. Subtracting a constant from x before squaring it moves the graph to the right.



ANALYZE QUADRATIC FUNCTIONS

Notice that each function above can be written in the form $y = (x - h)^2 + k$, where (h, k) is the vertex of the parabola and $x = h$ is its axis of symmetry. This is often referred to as the **vertex form** of a quadratic function.

Equation	Vertex	Axis of Symmetry
$y = x^2$ or $y = (x - 0)^2 + 0$	(0, 0)	$x = 0$
$y = x^2 + 2$ or $y = (x - 0)^2 + 2$	(0, 2)	$x = 0$
$y = (x - 3)^2$ or $y = (x - 3)^2 + 0$	(3, 0)	$x = 3$

In Chapter 4, you learned that a translation slides a figure on the coordinate plane without changing its shape or size. As the values of h and k change, the graph of $y = a(x - h)^2 + k$ is the graph of $y = x^2$ translated

- $|h|$ units *left* if h is negative or $|h|$ units *right* if h is positive, and
- $|k|$ units *up* if k is positive or $|k|$ units *down* if k is negative.

Example 1 Graph a Quadratic Function in Vertex Form

Analyze $y = (x + 2)^2 + 1$. Then draw its graph.

This function can be rewritten as $y = [x - (-2)]^2 + 1$. Then $h = -2$ and $k = 1$.

The vertex is at (h, k) or $(-2, 1)$, and the axis of symmetry is $x = -2$. The graph has the same shape as the graph of $y = x^2$, but is translated 2 units left and 1 unit up.

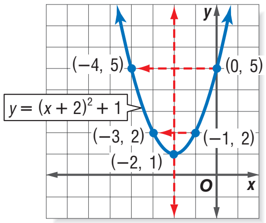
Now use this information to draw the graph.

Step 1 Plot the vertex, $(-2, 1)$.

Step 2 Draw the axis of symmetry, $x = -2$.

Step 3 Find and plot two points on one side of the axis of symmetry, such as $(-1, 2)$ and $(0, 5)$.

Step 4 Use symmetry to complete the graph.



Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 343–344
- Skills Practice, p. 345
- Practice, p. 346
- Reading to Learn Mathematics, p. 347
- Enrichment, p. 348
- Assessment, p. 370

Teaching Algebra With Manipulatives Masters, pp. 247–248

Transparencies

5-Minute Check Transparency 6-6
Answer Key Transparencies

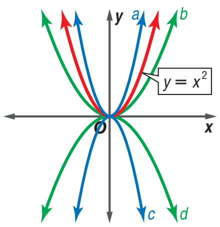
Technology

Interactive Chalkboard

How does the value of a in the general form $y = a(x - h)^2 + k$ affect a parabola? Compare the graphs of the following functions to the parent function, $y = x^2$.

a. $y = 2x^2$　　**b.** $y = \frac{1}{2}x^2$

c. $y = -2x^2$　　**d.** $y = -\frac{1}{2}x^2$



All of the graphs have the vertex (0, 0) and axis of symmetry $x = 0$.

Notice that the graphs of $y = 2x^2$ and $y = \frac{1}{2}x^2$ are *dilations* of the graph of $y = x^2$. The graph of $y = 2x^2$ is narrower than the graph of $y = x^2$, while the graph of $y = \frac{1}{2}x^2$ is wider. The graphs of $y = -2x^2$ and $y = 2x^2$ are *reflections* of each other over the x-axis, as are the graphs of $y = -\frac{1}{2}x^2$ and $y = \frac{1}{2}x^2$.

Changing the value of a in the equation $y = a(x - h)^2 + k$ can affect the direction of the opening and the shape of the graph.

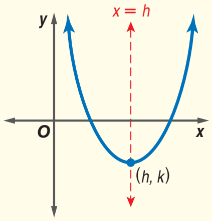
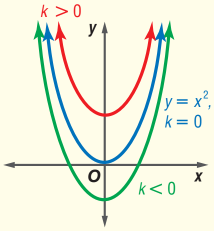
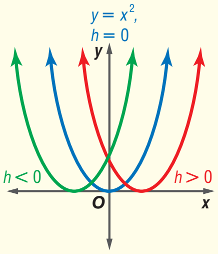
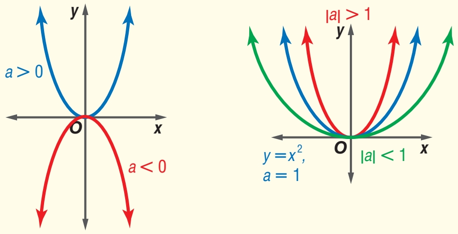
- If $a > 0$, the graph opens up.
- If $a < 0$, the graph opens down.
- If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
- If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Study Tip

Reading Math
$|a| < 1$ means that a is a rational number between 0 and 1, such as $\frac{2}{5}$, or a rational number between −1 and 0, such as −0.3.

Concept Summary　*Quadratic Functions in Vertex Form*

The vertex form of a quadratic function is $y = a(x - h)^2 + k$.

h* and *k	***k***
Vertex and Axis of Symmetry  	**Vertical Translation** 
h	***a***
Horizontal Translation  	**Direction of Opening and Shape of Parabola** 

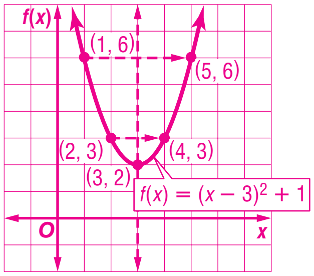
www.algebra2.com/extra_examples

2 Teach

ANALYZE QUADRATIC FUNCTIONS

In-Class Example　Power Point®

1 Analyze $y = (x - 3)^2 + 2$. Then draw its graph. **The vertex of the graph is at (3, 2) and the axis of symmetry is $x = 3$. The graph has the same shape as the graph of $y = x^2$, but is translated 3 units right and 2 units up.**



Teaching Tip To help students remember that as $|a|$ increases, the graph gets narrower and not wider, discuss the fact that a greater multiplier for the quantity $(x - h)^2$ will make the corresponding y value greater as well. Point out that greater values of y result in a steeper (and thus narrower) graph.

Tips for New Teachers

Intervention Encourage students to ask questions about any aspects they may find confusing that are covered in the Concept Summary chart on this page. Ask them to write and use their own summary on an index card. Explain to students that a thorough understanding of these concepts will save them time since this knowledge will enable them to sketch approximate graphs quickly.

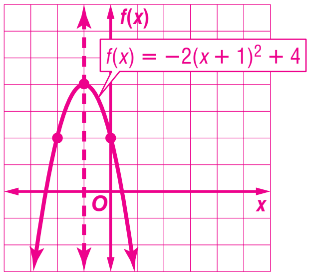
WRITE QUADRATIC FUNCTIONS IN VERTEX FORM

In-Class Examples

Power Point®

2 Write $y = x^2 + 2x + 4$ in vertex form. Then analyze the function. **$y = (x + 1)^2 + 3$; vertex: $(-1, 3)$; axis of symmetry: $x = -1$; opens up; The graph has the same shape as the graph of $y = x^2$, but it is translated 1 unit left and 3 units up.**

3 Write $y = -2x^2 - 4x + 2$ in vertex form. Then analyze and graph the function. **$y = -2(x + 1)^2 + 4$; vertex: $(-1, 4)$; axis of symmetry: $x = -1$; opens down; The graph is narrower than the graph of $y = x^2$, and it is translated 1 unit left and 4 units up.**



WRITE QUADRATIC FUNCTIONS IN VERTEX FORM Given a function of the form $y = ax^2 + bx + c$, you can complete the square to write the function in vertex form.

Example 2 Write $y = x^2 + bx + c$ in Vertex Form

Write $y = x^2 + 8x - 5$ in vertex form. Then analyze the function.

$y = x^2 + 8x - 5$	Notice that $x^2 + 8x - 5$ is not a perfect square.
$y = (x^2 + 8x + 16) - 5 - 16$	Complete the square by adding $\left(\frac{8}{2}\right)^2$ or 16. Balance this addition by subtracting 16.
$y = (x + 4)^2 - 21$	Write $x^2 + 8x + 16$ as a perfect square.

This function can be rewritten as $y = [x - (-4)]^2 + (-21)$. Written in this way, you can see that $h = -4$ and $k = -21$.

The vertex is at $(-4, -21)$, and the axis of symmetry is $x = -4$. Since $a = 1$, the graph opens up and has the same shape as the graph of $y = x^2$, but it is translated 4 units left and 21 units down.

CHECK You can check the vertex and axis of symmetry using the formula $x = -\frac{b}{2a}$. In the original equation, $a = 1$ and $b = 8$, so the axis of symmetry is $x = -\frac{8}{2(1)}$ or -4. Thus, the x-coordinate of the vertex is -4, and the y-coordinate of the vertex is $y = (-4)^2 + 8(-4) - 5$ or -21.

Study Tip

Check

As an additional check, graph the function in Example 2 to verify the location of its vertex and axis of symmetry.

When writing a quadratic function in which the coefficient of the quadratic term is not 1 in vertex form, the first step is to factor out that coefficient from the quadratic and linear terms. Then you can complete the square and write in vertex form.

Example 3 Write $y = ax^2 + bx + c$ in Vertex Form, $a \neq 1$

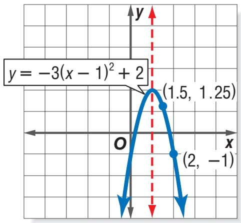
Write $y = -3x^2 + 6x - 1$ in vertex form. Then analyze and graph the function.

$y = -3x^2 + 6x - 1$	Original equation
$y = -3(x^2 - 2x) - 1$	Group $ax^2 + bx$ and factor, dividing by a.
$y = -3(x - 2x + 1) - 1 - (-3)(1)$	Complete the square by adding 1 inside the parentheses. Notice that this is an overall addition of $-3(1)$. Balance this addition by subtracting $-3(1)$.
$y = -3(x - 1)^2 + 2$	Write $x^2 - 2x + 1$ as a perfect square.

The vertex form of this function is $y = -3(x - 1)^2 + 2$. So, $h = 1$ and $k = 2$.

The vertex is at $(1, 2)$, and the axis of symmetry is $x = 1$. Since $a = -3$, the graph opens downward and is narrower than the graph of $y = x^2$. It is also translated 1 unit right and 2 units up.

Now graph the function. Two points on the graph to the right of $x = 1$ are $(1.5, 1.25)$ and $(2, -1)$. Use symmetry to complete the graph.



DAILY INTERVENTION

Differentiated Instruction

Naturalist Have students observe or research some natural events that can be modeled by parabolas, such as the fountain's water stream discussed in Exercise 14 on p. 326. Students should report their observations and findings to the class. If students are able to determine a quadratic function that models the event, they should present the function and explain how the characteristics of the equation can be used to analyze its graph.

If the vertex and one other point on the graph of a parabola are known, you can write the equation of the parabola in vertex form.

Example 4 Write an Equation Given Points

Write an equation for the parabola whose vertex is at (−1, 4) and passes through (2, 1).

The vertex of the parabola is at (−1, 4), so $h = -1$ and $k = 4$. Since (2, 1) is a point on the graph of the parabola, let $x = 2$ and $y = 1$. Substitute these values into the vertex form of the equation and solve for a.

$y = a(x - h)^2 + k$ Vertex form

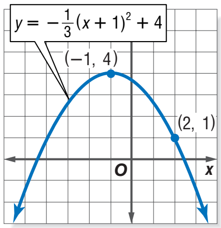
$1 = a[2 - (-1)]^2 + 4$ Substitute 1 for y, 2 for x, −1 for h, and 4 for k.

$1 = a(9) + 4$ Simplify.

$-3 = 9a$ Subtract 4 from each side.

$-\frac{1}{3} = a$ Divide each side by 9.

The equation of the parabola in vertex form is $y = -\frac{1}{3}(x + 1)^2 + 4$.



CHECK A graph of $y = -\frac{1}{3}(x + 1)^2 + 4$ verifies that the parabola passes through the point at (2, 1).

Check for Understanding

Concept Check

1d. $y = 2(x - 2)^2 + 3$
1e. Sample answer: $y = 4(x + 1)^2 + 3$
1f. Sample answer: $y = (x + 1)^2 + 3$
3. Sample answer: $y = 2(x - 2)^2 - 1$

4. Jenny; when completing the square is used to write a quadratic function in vertex form, the quantity added is then subtracted from the same side of the equation to maintain equality.

1. **Write** a quadratic equation that transforms the graph of $y = 2(x + 1)^2 + 3$ so that it is:
   - a. 2 units up. $y = 2(x + 1)^2 + 5$
   - b. 3 units down. $y = 2(x + 1)^2$
   - c. 2 units to the left. $y = 2(x + 3)^2 + 3$
   - d. 3 units to the right.
   - e. narrower.
   - f. wider.
   - g. opening in the opposite direction. $y = -2(x + 1)^2 + 3$
2. **Explain** how you can find an equation of a parabola using its vertex and one other point on its graph. See margin.
3. **OPEN ENDED** Write the equation of a parabola with a vertex of (2, −1).
4. **FIND THE ERROR** Jenny and Ruben are writing $y = x^2 - 2x + 5$ in vertex form.

Jenny

$y = x^2 - 2x + 5$
$y = (x^2 - 2x + 1) + 5 - 1$
$y = (x - 1)^2 + 4$

Ruben

$y = x^2 - 2x + 5$
$y = (x^2 - 2x + 1) + 5 + 1$
$y = (x - 1)^2 + 6$

Who is correct? Explain your reasoning.

Guided Practice

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. 5–7. See margin.

5. $y = 5(x + 3)^2 - 1$
6. $y = x^2 + 8x - 3$
7. $y = -3x^2 - 18x + 11$

In-Class Example

Power Point®

4 Write an equation for the parabola whose vertex is at (1, 2) and passes through (3, 4). $y = \frac{1}{2}(x - 1)^2 + 2$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 6.
- write a summary in their own words of everything you can tell about the graph of a parabola when the function is written in vertex form.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Students may find that it will help them avoid errors of this type if they specifically write the addition and subtraction in a separate step before placing parentheses around the perfect square trinomial, as in $y = x^2 - 2x + 1 - 1 + 5$.

Answers

2. Substitute the x-coordinate of the vertex for h and the y-coordinate of the vertex for k in the equation $y = a(x - h)^2 + k$. Then substitute the x-coordinate of the other point for x and the y-coordinate for y into this equation and solve for a. Replace a with this value in the equation you wrote with h and k.
5. (−3, −1); $x = -3$; up
6. $y = (x + 4)^2 - 19$, (−4, −19); $x = -4$; up
7. $y = -3(x + 3)^2 + 38$; (−3, 38); $x = -3$; down

About the Exercises…

Organization by Objective

- **Analyze Quadratic Functions:** 15, 16, 27–30, 37, 38, 47, 51, 52
- **Write Quadratic Functions in Vertex Form:** 19–26, 31–36, 39–46, 48–50

Odd/Even Assignments

Exercises 15–46 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–33 odd, 39–45 odd, 53–71

Average: 15–47 odd, 48–50, 53–71

Advanced: 16–46 even, 48–67 (optional: 68–71)

All: Practice Quiz 2 (1–10)

GUIDED PRACTICE KEY

Exercises	Examples
5–7	2
8–10	1, 3
11–14	4

Graph each function. 8–10. See margin.

8. $y = 3(x + 3)^2$

9. $y = \frac{1}{3}(x - 1)^2 + 3$

10. $y = -2x^2 + 16x - 31$

Write an equation for the parabola with the given vertex that passes through the given point.

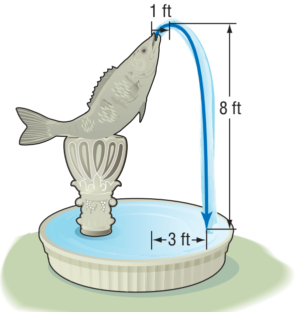
11. vertex: (2, 0)
point: (1, 4)
$y = 4(x - 2)^2$

12. vertex: (−3, 6)
point: (−5, 2)
$y = -(x + 3)^2 + 6$

13. vertex: (−2, −3)
point: (−4, −5)
$y = -\frac{1}{2}(x + 2)^2 - 3$

Application

14. FOUNTAINS The height of a fountain's water stream can be modeled by a quadratic function. Suppose the water from a jet reaches a maximum height of 8 feet at a distance 1 foot away from the jet. If the water lands 3 feet away from the jet, find a quadratic function that models the height $h(d)$ of the water at any given distance d feet from the jet. $h(d) = -2d^2 + 4d + 6$



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
15–26	2
27–38, 47, 48, 50–52	1, 3
39–46, 49	4

Extra Practice

See page 841.

38. Sample answer: the graphs have the same shape, but the graph of $y = 2(x - 4)^2 + 1$ is 1 unit to the left and 5 units below the graph of $y = 2(x - 5)^2 - 4$.

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. 19–26. See margin.

15. $y = -2(x + 3)^2$ (−3, 0); $x = -3$; down

16. $y = \frac{1}{3}(x - 1)^2 + 2$ (1, 2); $x = 1$; up

17. $y = 5x^2 - 6$ (0, −6); $x = 0$; up

18. $y = -8x^2 + 3$ (0, 3); $x = 0$; down

19. $y = -x^2 - 4x + 8$

20. $y = x^2 - 6x + 1$

21. $y = -3x^2 + 12x$

22. $y = 4x^2 + 24x$

23. $y = 4x^2 + 8x - 3$

24. $y = -2x^2 + 20x - 35$

25. $y = 3x^2 + 3x - 1$

26. $y = 4x^2 - 12x - 11$

Graph each function. 27–36. See pp. 343A–343F.

27. $y = 4(x + 3)^2 + 1$

28. $y = -(x - 5)^2 - 3$

29. $y = \frac{1}{4}(x - 2)^2 + 4$

30. $y = \frac{1}{2}(x - 3)^2 - 5$

31. $y = x^2 + 6x + 2$

32. $y = x^2 - 8x + 18$

33. $y = -4x^2 + 16x - 11$

34. $y = -5x^2 - 40x - 80$

★ **35.** $y = -\frac{1}{2}x^2 + 5x - \frac{27}{2}$

★ **36.** $y = \frac{1}{3}x^2 - 4x + 15$

★ **37.** Write one sentence that compares the graphs of $y = 0.2(x + 3)^2 + 1$ and $y = 0.4(x + 3)^2 + 1$. Sample answer: the graph of $y = 0.4(x + 3)^2 + 1$ is narrower than the graph of $y = 0.2(x + 3)^2 + 1$.

★ **38.** Compare the graphs of $y = 2(x - 5)^2 + 4$ and $y = 2(x - 4)^2 - 1$.

WebQuest

You can use a quadratic function to model the world population. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

Write an equation for the parabola with the given vertex that passes through the given point.

39. vertex: (6, 1) $y = 9(x - 6)^2 + 1$
point: (5, 10)

40. vertex: (−4, 3) $y = 3(x + 4)^2 + 3$
point: (−3, 6)

41. vertex: (3, 0) $y = -\frac{2}{3}(x - 3)^2$
point: (6, −6)

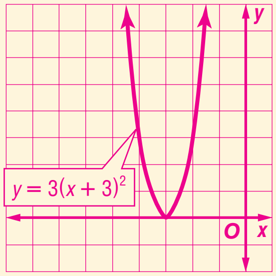
42. vertex: (5, 4) $y = -3(x - 5)^2 + 4$
point: (3, −8)

43. vertex: (0, 5) $y = \frac{1}{3}x^2 + 5$
point: (3, 8)

44. vertex: (−3, −2) $y = \frac{5}{2}(x + 3)^2 - 2$
point: (−1, 8)

Answers

8.



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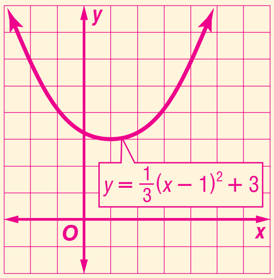

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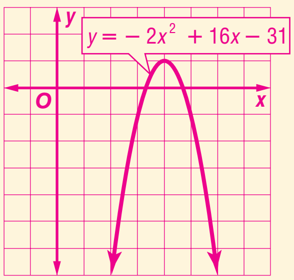


19. $y = -(x + 2)^2 + 12$; (−2, 12); $x = -2$; down

20. $y = (x - 3)^2 - 8$; (3, −8); $x = 3$; up

21. $y = -3(x - 2)^2 + 12$; (2, 12); $x = 2$; down

22. $y = 4(x + 3)^2 - 36$; (−3, −36); $x = -3$; up

23. $y = 4(x + 1)^2 - 7$; (−1, −7); $x = -1$; up

24. $y = -2(x - 5)^2 + 15$; (5, 15); $x = 5$; down

25. $y = 3\left(x + \frac{1}{2}\right)^2 - \frac{7}{4}$; $\left(-\frac{1}{2}, -\frac{7}{4}\right)$; $x = -\frac{1}{2}$; up

26. $y = 4\left(x - \frac{3}{2}\right)^2 - 20$; $\left(\frac{3}{2}, -20\right)$; $x = \frac{3}{2}$; up

45. Write an equation for a parabola whose vertex is at the origin and passes through (2, −8). $y = -2x^2$

46. Write an equation for a parabola with vertex at (−3, −4) and y-intercept 8. $y = \frac{4}{3}(x + 3)^2 - 4$

47. **AEROSPACE** NASA's KC135A aircraft flies in parabolic arcs to simulate the weightlessness experienced by astronauts in space. The height h of the aircraft (in feet) t seconds after it begins its parabolic flight can be modeled by the equation $h(t) = -9.09(t - 32.5)^2 + 34{,}000$. What is the maximum height of the aircraft during this maneuver and when does it occur? 34,000 feet; 32.5 s after the aircraft begins its parabolic flight

More About. . .

Aerospace

The KC135A has the nickname "Vomit Comet." It starts its ascent at 24,000 feet. As it approaches maximum height, the engines are stopped, and the aircraft is allowed to free-fall at a determined angle. Zero gravity is achieved for 25 seconds as the plane reaches the top of its flight and begins its descent.

Source: NASA

DIVING **For Exercises 48–50, use the following information.**
The distance of a diver above the water $d(t)$ (in feet) t seconds after diving off a platform is modeled by the equation $d(t) = -16t^2 + 8t + 30$.

48. Find the time it will take for the diver to hit the water. about 1.6 s

49. Write an equation that models the diver's distance above the water if the platform were 20 feet higher. $d(t) = -16t^2 + 8t + 50$

50. Find the time it would take for the diver to hit the water from this new height. about 2.0 s

LAWN CARE **For Exercises 51 and 52, use the following information.**
The path of water from a sprinkler can be modeled by a quadratic function. The three functions below model paths for three different angles of the water.

Angle A: $y = -0.28(x - 3.09)^2 + 3.27$

Angle B: $y = -0.14(x - 3.57)^2 + 2.39$

Angle C: $y = -0.09(x - 3.22)^2 + 1.53$

51. Which sprinkler angle will send water the highest? Explain your reasoning.

52. Which sprinkler angle will send water the farthest? Explain your reasoning.

51. Angle A; the graph of the equation for angle A is higher than the other two since 3.27 is greater than 2.39 or 1.53.

52. Angle B; the vertex of the equation for angle B is farther to the right than the other two since 3.57 is greater than 3.09 or 3.22.

53. **CRITICAL THINKING** Given $y = ax^2 + bx + c$ with $a \neq 0$, derive the equation for the axis of symmetry by completing the square and rewriting the equation in the form $y = a(x - h)^2 + k$. See pp. 343A–343F.

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 343A–343F.

How can the graph $y = x^2$ be used to graph any quadratic function?

Include the following in your answer:
- a description of the effects produced by changing a, h, and k in the equation $y = a(x - h)^2 + k$, and
- a comparison of the graph of $y = x^2$ and the graph of $y = a(x - h)^2 + k$ using values of your own choosing for a, h, and k.

Standardized Test Practice

55. If $f(x) = x^2 - 5x$ and $f(n) = -4$, then which of the following could be n? D

Ⓐ −5   Ⓑ −4   Ⓒ −1   Ⓓ 1

56. The vertex of the graph of $y = 2(x - 6)^2 + 3$ is located at which of the following points? B

Ⓐ (2, 3)   Ⓑ (6, 3)   Ⓒ (6, −3)   Ⓓ (−2, 3)

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 343 (shown) and p. 344

Analyze Quadratic Functions

Vertex Form of a Quadratic Function	The graph of $y = a(x - h)^2 + k$ has the following characteristics: • Vertex: (h, k) • Axis of symmetry: $x = h$ • Opens up if $a > 0$ • Opens down if $a < 0$ • Narrower than the graph of $y = x^2$ if $\lvert a \rvert > 1$ • Wider than the graph of $y = x^2$ if $\lvert a \rvert < 1$

Example **Identify the vertex, axis of symmetry, and direction of opening of each graph.**

a. $y = 2(x + 4)^2 - 11$

The vertex is at (h, k) or $(-4, -11)$, and the axis of symmetry is $x = -4$. The graph opens up, and is narrower than the graph of $y = x^2$.

a. $y = -\frac{1}{4}(x - 2)^2 + 10$

The vertex is at (h, k) or $(2, 10)$, and the axis of symmetry is $x = 2$. The graph opens down, and is wider than the graph of $y = x^2$.

Exercises

Each quadratic function is given in vertex form. Identify the vertex, axis of symmetry, and direction of opening of the graph.

1. $y = (x - 2)^2 + 16$ (2, 16); $x = 2$; up
2. $y = 4(x + 3)^2 - 7$ (−3, −7); $x = -3$; up
3. $y = \frac{1}{2}(x - 5)^2 + 3$ (5, 3); $x = 5$; up
4. $y = -7(x + 1)^2 - 9$ (−1, −9); $x = -1$; down
5. $y = \frac{1}{5}(x - 4)^2 - 12$ (4, −12); $x = 4$; up
6. $y = 6(x + 6)^2 + 6$ (−6, 6); $x = -6$; up
7. $y = \frac{2}{5}(x - 9)^2 + 12$ (9, 12); $x = 9$; up
8. $y = 8(x - 3)^2 - 2$ (3, −2); $x = 3$; up
9. $y = -3(x - 1)^2 - 2$ (1, −2); $x = 1$; down
10. $y = -\frac{5}{2}(x + 5)^2 + 12$ (−5, 12); $x = -5$; down
11. $y = \frac{4}{3}(x - 7)^2 + 22$ (7, 22); $x = 7$; up
12. $y = 16(x - 4)^2 + 1$ (4, 1); $x = 4$; up
13. $y = 3(x - 1.2)^2 + 2.7$ (1.2, 2.7); $x = 1.2$; up
14. $y = -0.4(x - 0.6)^2 - 0.2$ (0.6, −0.2); $x = 0.6$; down
15. $y = 1.2(x + 0.8)^2 + 6.5$ (−0.8, 6.5); $x = -0.8$; up

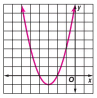
Skills Practice, p. 345 and Practice, p. 346 (shown)

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

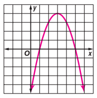
1. $y = -6(x + 2)^2 - 1$ $y = -6(x + 2)^2 - 1$; (−2, −1); $x = -2$; down
2. $y = 2x^2 + 2$ $y = 2(x + 0)^2 + 2$; (0, 2); $x = 0$; up
3. $y = -4x^2 + 8x$ $y = -4(x - 1)^2 + 4$; (1, 4); $x = 1$; down
4. $y = x^2 + 10x + 20$ $y = (x + 5)^2 - 5$; (−5, −5); $x = -5$; up
5. $y = 2x^2 + 12x + 18$ $y = 2(x + 3)^2$; (−3, 0); $x = -3$; up
6. $y = 3x^2 - 6x + 5$ $y = 3(x - 1)^2 + 2$; (1, 2); $x = 1$; up
7. $y = -2x^2 - 16x - 32$ $y = -2(x + 4)^2$; (−4, 0); $x = -4$; down
8. $y = -3x^2 + 18x - 21$ $y = -3(x - 3)^2 + 6$; (3, 6); $x = 3$; down
9. $y = 2x^2 + 16x + 29$ $y = 2(x + 4)^2 - 3$; (−4, −3); $x = -4$; up

Graph each function.

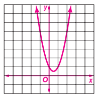
10. $y = (x + 3)^2 - 1$



11. $y = -x^2 + 6x - 5$



12. $y = 2x^2 - 2x + 1$



Write an equation for the parabola with the given vertex that passes through the given point.

13. vertex: (1, 3)
point: (−2, −15)
$y = -2(x - 1)^2 + 3$

14. vertex: (−3, 0)
point: (3, 18)
$y = \frac{1}{2}(x + 3)^2$

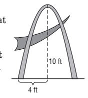
15. vertex: (10, −4)
point: (5, 6)
$y = \frac{2}{5}(x - 10)^2 - 4$

16. Write an equation for a parabola with vertex at (4, 4) and x-intercept 6. $y = -(x - 4)^2 + 4$

17. Write an equation for a parabola with vertex at (−3, −1) and y-intercept 2. $y = \frac{1}{3}(x + 3)^2 - 1$

18. **BASEBALL** The height h of a baseball t seconds after being hit is given by $h(t) = -16t^2 + 80t + 3$. What is the maximum height that the baseball reaches, and when does this occur? 103 ft; 2.5 s

19. **SCULPTURE** A modern sculpture in a park contains a parabolic arc that starts at the ground and reaches a maximum height of 10 feet after a horizontal distance of 4 feet. Write a quadratic function in vertex form that describes the shape of the outside of the arc, where y is the height of a point on the arc and x is its horizontal distance from the left-hand starting point of the arc. $y = -\frac{5}{8}(x - 4)^2 + 10$



Reading to Learn Mathematics, p. 347

ELL

Pre-Activity **How can the graph of $y = x^2$ be used to graph any quadratic function?**

Read the introduction to Lesson 6-6 at the top of page 322 in your textbook.

- What does adding a positive number to x^2 do to the graph of $y = x^2$? It moves the graph up.
- What does subtracting a positive number to x before squaring do to the graph of $y = x^2$? It moves the graph to the right.

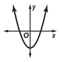
Reading the Lesson

1. Complete the following information about the graph of $y = a(x - h)^2 + k$.
   - **a.** What are the coordinates of the vertex? (h, k)
   - **b.** What is the equation of the axis of symmetry? $x = h$
   - **c.** In which direction does the graph open if $a > 0$? If $a < 0$? up; down
   - **d.** What do you know about the graph if $|a| < 1$? It is wider than the graph of $y = x^2$.
     If $|a| > 1$? It is narrower than the graph of $y = x^2$.

2. Match each graph with the description of the constants in the equation in vertex form.

a. $a > 0, h > 0, k < 0$ iii
b. $a < 0, h < 0, k < 0$ iv
c. $a < 0, h < 0, k > 0$ ii
d. $a > 0, h = 0, k < 0$ i

i.



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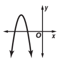

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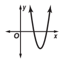

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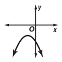


Helping You Remember

3. When graphing quadratic functions such as $y = (x + 4)^2$ and $y = (x - 5)^2$, many students have trouble remembering which represents a translation of the graph of $y = x^2$ to the left and which represents a translation to the right. What is an easy way to remember this? Sample answer: In functions like $y = (x + 4)^2$, the plus sign puts the graph "ahead" so that the vertex comes "sooner" than the origin and the translation is to the left. In functions like $y = (x - 5)^2$, the minus puts the graph "behind" so that the vertex comes "later" than the origin and the translation is to the right.

Enrichment, p. 348

Patterns with Differences and Sums of Squares

Some whole numbers can be written as the difference of two squares and some cannot. Formulas can be developed to describe the sets of numbers algebraically.

If possible, write each number as the difference of two squares. Look for patterns.

1. 0 $0^2 - 0^2$
2. 1 $1^2 - 0^2$
3. 2 cannot
4. 3 $2^2 - 1^2$
5. 4 $2^2 - 0^2$
6. 5 $3^2 - 2^2$
7. 6 cannot
8. 7 $4^2 - 3^2$
9. 8 $3^2 - 1^2$
10. 9 $3^2 - 0^2$
11. 10 cannot
12. 11 $6^2 - 5^2$
13. 12 $4^2 - 2^2$
14. 13 $7^2 - 6^2$
15. 14 cannot
16. 15 $4^2 - 1^2$

Even numbers can be written as $2n$, where n is one of the numbers 0, 1, 2, 3, and so on. Odd numbers can be written $2n + 1$. Use these

4 Assess

Open-Ended Assessment

Writing Give students the equation of a specific parabola and ask them to put it in vertex form, analyze it, and sketch the graph. Instruct them to show all the steps in their procedures, with notes and explanations for each step, similar to the notes in the right column of the Examples.

Getting Ready for Lesson 6-7

PREREQUISITE SKILL Lesson 6-7 presents quadratic inequalities. Use Exercises 68–71 to determine your students' familiarity with inequalities.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 6-4 through 6-6. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 6-5 and 6-6) is available on p. 370 of the *Chapter 6 Resource Masters*.

Answers

63. $2t^2 + 2t - \frac{3}{t-1}$

64. $t^2 - 2t + 1$

65. $n^3 - 3n^2 - 15n - 21$

66. $y^3 + 1 - \frac{4}{y+3}$

Answers (Practice Quiz 2)

8. $y = (x + 4)^2 + 2$; $(-4, 2)$, $x = -4$; up

9. $y = -(x - 6)^2$; $(6, 0)$, $x = 6$; down

10. $y = 2(x + 3)^2 - 5$; $(-3, -5)$, $x = -3$; up

Maintain Your Skills

Mixed Review

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation. *(Lesson 6-5)*

57. $3x^2 - 6x + 2 = 0$ **12; 2 irrational**

58. $4x^2 + 7x = 11$ **225; 2 rational**

59. $2x^2 - 5x + 6 = 0$ **−23; 2 complex**

Solve each equation by completing the square. *(Lesson 6-4)*

60. $x^2 + 10x + 17 = 0$ $\{-5 \pm 2\sqrt{2}\}$

61. $x^2 - 6x + 18 = 0$ $\{3 \pm 3i\}$

62. $4x^2 + 8x = 9$ $\left\{\frac{-2 \pm \sqrt{13}}{2}\right\}$

Find each quotient. *(Lesson 5-3)* **63–66. See margin.**

63. $(2t^3 - 2t - 3) \div (t - 1)$

64. $(t^3 - 3t + 2) \div (t + 2)$

65. $(n^4 - 8n^3 + 54n + 105) \div (n - 5)$

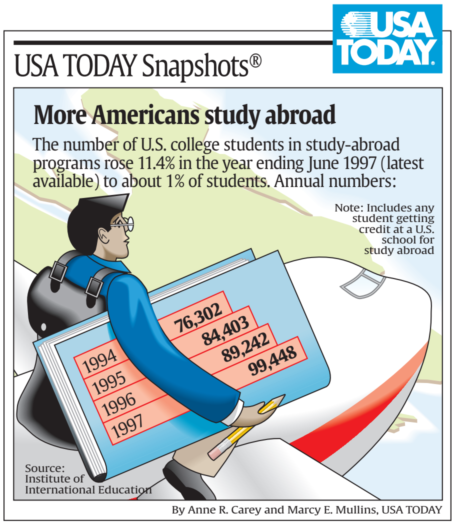
66. $(y^4 + 3y^3 + y - 1) \div (y + 3)$

67. **EDUCATION** The graph shows the number of U.S. students in study-abroad programs. *(Lesson 2-5)*

a. Write a prediction equation from the data given.

b. Use your equation to predict the number of students in these programs in 2005. **Sample answer: 161,167**

67a. Sample answer using (1994, 76,302) and (1997, 99,448): $y = 7715x - 15{,}307{,}408$



USA TODAY Snapshots®

More Americans study abroad

The number of U.S. college students in study-abroad programs rose 11.4% in the year ending June 1997 (latest available) to about 1% of students. Annual numbers:

Note: Includes any student getting credit at a U.S. school for study abroad

Source: Institute of International Education

By Anne R. Carey and Marcy E. Mullins, USA TODAY

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Determine whether the given value satisfies the inequality.**
(To review ***inequalities****, see Lesson 1-6.)*

68. $-2x^2 + 3 < 0$; $x = 5$ **yes**

69. $4x^2 + 2x - 3 \geq 0$; $x = -1$ **no**

70. $4x^2 - 4x + 1 \leq 10$; $x = 2$ **yes**

71. $6x^2 + 3x > 8$; $x = 0$ **no**

Practice Quiz 2

Lessons 6-4 through 6-6

Solve each equation by completing the square. *(Lesson 6-4)*

1. $x^2 + 14x + 37 = 0$ $\{-7 \pm 2\sqrt{3}\}$

2. $2x^2 - 2x + 5 = 0$ $\left\{\frac{1 \pm 3i}{2}\right\}$

Find the value of the discriminant for each quadratic equation. Then describe the number and type of roots for the equation. *(Lesson 6-5)*

3. $5x^2 - 3x + 1 = 0$ **−11; 2 complex**

4. $3x^2 + 4x - 7 = 0$ **100; 2 rational**

Solve each equation by using the Quadratic Formula. *(Lesson 6-5)*

5. $x^2 + 9x - 11 = 0$ $\left\{\frac{-9 \pm 5\sqrt{5}}{2}\right\}$

6. $-3x^2 + 4x = 4$ $\left\{\frac{2 \pm 2i\sqrt{2}}{3}\right\}$

7. Write an equation for a parabola with vertex at (2, −5) that passes through (−1, 1). *(Lesson 6-6)* $y = \frac{2}{3}(x - 2)^2 - 5$

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. *(Lesson 6-6)* **8–10. See margin.**

8. $y = x^2 + 8x + 18$

9. $y = -x^2 + 12x - 36$

10. $y = 2x^2 + 12x + 13$

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

6-7 Graphing and Solving Quadratic Inequalities

What You'll Learn

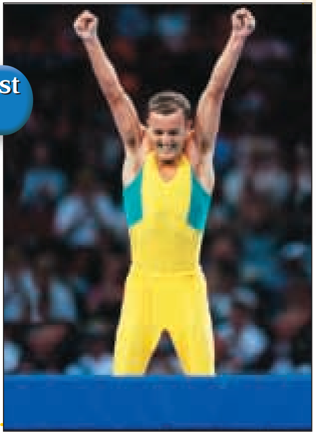
- Graph quadratic inequalities in two variables.
- Solve quadratic inequalities in one variable.

Vocabulary

- quadratic inequality

How can you find the time a trampolinist spends above a certain height?

Trampolining was first featured as an Olympic sport at the 2000 Olympics in Sydney, Australia. The competitors performed two routines consisting of 10 different skills. Suppose the height $h(t)$ in feet of a trampolinist above the ground during one bounce is modeled by the quadratic function $h(t) = -16t^2 + 42t + 3.75$. We can solve a quadratic inequality to determine how long this performer is more than a certain distance above the ground.



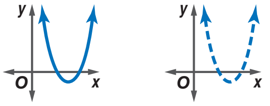
GRAPH QUADRATIC INEQUALITIES

You can graph **quadratic inequalities** in two variables using the same techniques you used to graph linear inequalities in two variables.

Study Tip

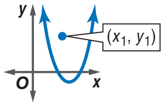
Look Back
For review of **graphing linear inequalities,** see Lesson 2-7.

Step 1 Graph the related quadratic equation, $y = ax^2 + bx + c$. Decide if the parabola should be solid or dashed.



≤ or ≥ < or >

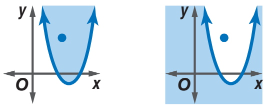
Step 2 Test a point (x_1, y_1) inside the parabola. Check to see if this point is a solution of the inequality.



$y_1 \overset{?}{\geq} a(x_1)^2 + b(x_1) + c$

TEACHING TIP
Remind students that (0, 0) is a good point to use as a test point.

Step 3 If (x_1, y_1) is a solution, shade the region *inside* the parabola. If (x_1, y_1) is *not* a solution, shade the region *outside* the parabola.



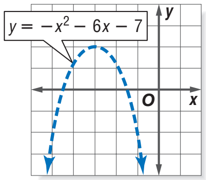
(x_1, y_1) is a solution. (x_1, y_1) is not a solution.

Example 1 Graph a Quadratic Inequality

Graph $y > -x^2 - 6x - 7$.

Step 1 Graph the related quadratic equation, $y = -x^2 - 6x - 7$.

Since the inequality symbol is >, the parabola should be dashed.



(continued on the next page)

6-7 Lesson Notes

1 Focus

5-Minute Check Transparency 6-7 Use as a quiz or review of Lesson 6-6.

Mathematical Background notes are available for this lesson on p. 284D.

Building on Prior Knowledge

In Lesson 6-6, students analyzed and graphed equations. In this lesson, students use the same techniques to graph and solve inequalities.

How can you find the time a trampolinist spends above a certain height?

Ask students:

- What is a trampoline and how does a trampolinist use it? **Ask a student who is familiar with this sport to explain it to those who may not have seen it.**
- Which way does the parabola for the given quadratic equation open? **downward**

Resource Manager

Workbook and Reproducible Masters

Chapter 6 Resource Masters

- Study Guide and Intervention, pp. 349–350
- Skills Practice, p. 351
- Practice, p. 352
- Reading to Learn Mathematics, p. 353
- Enrichment, p. 354
- Assessment, p. 370

Graphing Calculator and Spreadsheet Masters, p. 37
School-to-Career Masters, p. 12

Transparencies

5-Minute Check Transparency 6-7
Answer Key Transparencies

Technology

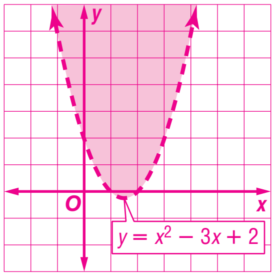
Interactive Chalkboard

2 Teach

GRAPH QUADRATIC INEQUALITIES

In-Class Example Power Point®

1 Graph $y > x^2 - 3x + 2$.

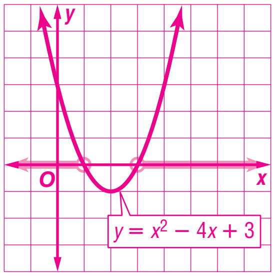


Teaching Tip For In-Class Example 1, encourage students to write the calculations as in Step 2 of Example 1 when testing a point inside the parabola. Emphasize that they must write a question mark over each inequality sign after substituting the coordinates of the point for the variables in the inequality.

SOLVE QUADRATIC INEQUALITIES

In-Class Example Power Point®

2 Solve $x^2 - 4x + 3 > 0$ by graphing.



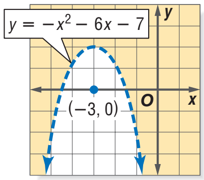
$\{x \mid x < 1 \text{ or } x > 3\}$

Teaching Tip Suggest that students try solving first by factoring, but if that does not quickly yield a solution, then use the Quadratic Formula.

Step 2 Test a point inside the parabola, such as $(-3, 0)$.

$$y > -x^2 - 6x - 7$$
$$0 \overset{?}{>} -(-3)^2 - 6(-3) - 7$$
$$0 \overset{?}{>} -9 + 18 - 7$$
$$0 \overset{?}{>} 2 \quad \times$$

So, $(-3, 0)$ is *not* a solution of the inequality.

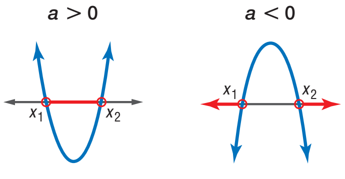


Step 3 Shade the region outside the parabola.

SOLVE QUADRATIC INEQUALITIES

To solve a quadratic inequality in one variable, you can use the graph of the related quadratic function.

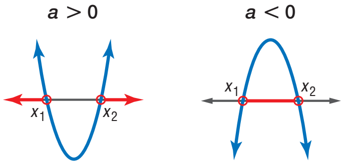
To solve $ax^2 + bx + c < 0$, graph $y = ax^2 + bx + c$. Identify the x values for which the graph lies *below* the x-axis.



$\{x \mid x_1 < x < x_2\}$

For $\leq$, include the x-intercepts in the solution.

To solve $ax^2 + bx + c > 0$, graph $y = ax^2 + bx + c$. Identify the x values for which the graph lies *above* the x-axis.



$\{x \mid x < x_1 \text{ or } x > x_2\}$

For $\geq$, include the x-intercepts in the solution.

Study Tip

Solving Quadratic Inequalities by Graphing

A precise graph of the related quadratic function is not necessary since the zeros of the function were found algebraically.

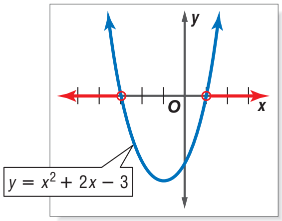
Example 2 Solve $ax^2 + bx + c > 0$

Solve $x^2 + 2x - 3 > 0$ by graphing.

The solution consists of the x values for which the graph of the related quadratic function lies *above* the x-axis. Begin by finding the roots of the related equation.

$x^2 + 2x - 3 = 0$	Related equation
$(x + 3)(x - 1) = 0$	Factor.
$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$	Zero Product Property
$x = -3 \qquad\qquad x = 1$	Solve each equation.

Sketch the graph of a parabola that has x-intercepts at -3 and 1. The graph should open up since $a > 0$.



The graph lies above the x-axis to the left of $x = -3$ and to the right of $x = 1$. Therefore, the solution set is $\{x \mid x < -3 \text{ or } x > 1\}$.

Example 3 Solve $ax^2 + bx + c \le 0$

Solve $0 \ge 3x^2 - 7x - 1$ by graphing.

This inequality can be rewritten as $3x^2 - 7x - 1 \le 0$. The solution consists of the x values for which the graph of the related quadratic function lies *on and below* the x-axis. Begin by finding the roots of the related equation.

$$3x^2 - 7x - 1 = 0 \quad \text{Related equation}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Use the Quadratic Formula.}$$

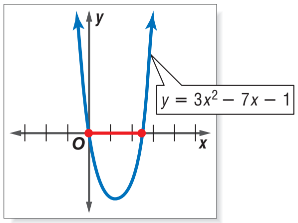
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-1)}}{2(3)} \quad \text{Replace } a \text{ with 3, } b \text{ with } -7\text{, and } c \text{ with } -1.$$

$$x = \frac{7 + \sqrt{61}}{6} \quad \text{or} \quad x = \frac{7 - \sqrt{61}}{6} \quad \text{Simplify and write as two equations.}$$

$$x \approx 2.47 \qquad x \approx -0.14 \quad \text{Simplify.}$$

Sketch the graph of a parabola that has x-intercepts of 2.47 and -0.14. The graph should open up since $a > 0$.

The graph lies on and below the x-axis at $x = -0.14$ and $x = 2.47$ and between these two values. Therefore, the solution set of the inequality is approximately $\{x \mid -0.14 \le x \le 2.47\}$.



CHECK Test one value of x less than -0.14, one between -0.14 and 2.47, and one greater than 2.47 in the original inequality.

Test $x = -1$.
$0 \ge 3x^2 - 7x - 1$
$0 \overset{?}{\ge} 3(-1)^2 - 7(-1) - 1$
$0 \ge 9$ X

Test $x = 0$.
$0 \ge 3x^2 - 7x - 1$
$0 \overset{?}{\ge} 3(0)^2 - 7(0) - 1$
$0 \ge -1$ ✓

Test $x = 3$.
$0 \ge 3x^2 - 7x - 1$
$0 \overset{?}{\ge} 3(3)^2 - 7(3) - 1$
$0 \ge 5$ X

More About. . .

Football

A long hang time allows the kicking team time to provide good coverage on a punt return. The suggested hang time for high school and college punters is 4.5–4.6 seconds.

Source: www.takeaknee.com

Real-world problems that involve vertical motion can often be solved by using a quadratic inequality.

Example 4 Write an Inequality

FOOTBALL The height of a punted football can be modeled by the function $H(x) = -4.9x^2 + 20x + 1$, where the height $H(x)$ is given in meters and the time x is in seconds. At what time in its flight is the ball within 5 meters of the ground?

The function $H(x)$ describes the height of the football. Therefore, you want to find the values of x for which $H(x) \le 5$.

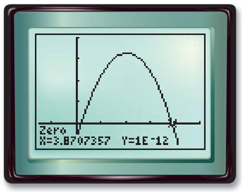
$$H(x) \le 5 \quad \text{Original inequality}$$

$$-4.9x^2 + 20x + 1 \le 5 \quad H(x) = -4.9x^2 + 20x + 1$$

$$-4.9x^2 + 20x - 4 \le 0 \quad \text{Subtract 5 from each side.}$$

Graph the related function $y = -4.9x^2 + 20x - 4$ using a graphing calculator. The zeros of the function are about 0.21 and 3.87, and the graph lies below the x-axis when $x < 0.21$ or $x > 3.87$.

Thus, the ball is within 5 meters of the ground for the first 0.21 second of its flight and again after 3.87 seconds until the ball hits the ground at 4.13 seconds.



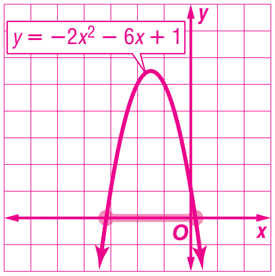
[−1.5, 5] scl: 1 by [−5, 20] scl: 5

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

3 Solve $0 \le -2x^2 - 6x + 1$ by graphing.



$\{x \mid -3.16 \le x \le 0.16\}$

Teaching Tip When discussing Example 4, be aware that some students may not be familiar with all of the aspects of the game of football. Ask students who are familiar with the terms in this example and the margin note to explain them to the class.

4 **SPORTS** The height of a ball above the ground after it is thrown upwards at 40 feet per second can be modeled by the function $h(x) = 40x - 16x^2$, where the height $h(x)$ is given in feet and the time x is in seconds. At what time in its flight is the ball within 15 feet of the ground? **The ball is within 15 feet of the ground for the first 0.46 second of its flight and again after 2.04 seconds until the ball hits the ground at 2.5 seconds.**

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students think about how the graph of a quadratic inequality helps them understand what the inequality means. Ask them to explore which is more meaningful to them (and therefore easier for them to grasp), the quadratic inequality itself or the graph of the inequality. Ask them to give an explanation of their choice.

In-Class Example

Power Point®

Teaching Tip An alternative way to solve the inequality in Example 5 is to first subtract 6 from both sides of the inequality, obtaining $x^2 + x - 6 > 0$. After factoring the left side as $(x + 3)(x - 2)$, point out that for the product $(x + 3)(x - 2)$ to be greater than 0, either both binomials must be positive or both must be negative. This fact can be used to test the three intervals of the number line.

5 Solve $x^2 + x \leq 2$ algebraically. $\{x \mid -2 \leq x \leq 1\}$

3 Practice/Apply

Study Notebook

Have students—
- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 6.
- write the basic steps shown in Example 1 for solving a quadratic inequality by graphing.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

You can also solve quadratic inequalities algebraically.

Study Tip

Solving Quadratic Inequalities Algebraically

As with linear inequalities, the solution set of a quadratic inequality can be all real numbers or the empty set, ∅. The solution is all real numbers when all three test points satisfy the inequality. It is the empty set when none of the tests points satisfy the inequality.

Example 5 Solve a Quadratic Inequality

Solve $x^2 + x > 6$ algebraically.

First solve the related quadratic equation $x^2 + x = 6$.

$x^2 + x = 6$ Related quadratic equation

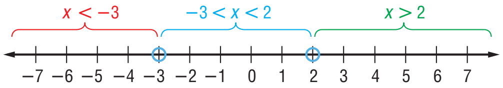
$x^2 + x - 6 = 0$ Subtract 6 from each side.

$(x + 3)(x - 2) = 0$ Factor.

$x + 3 = 0$ or $x - 2 = 0$ Zero Product Property

$x = -3$ $x = 2$ Solve each equation.

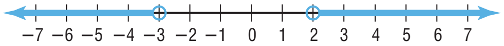
Plot –3 and 2 on a number line. Use circles since these values are not solutions of the original inequality. Notice that the number line is now separated into three intervals.



Test a value in each interval to see if it satisfies the original inequality.

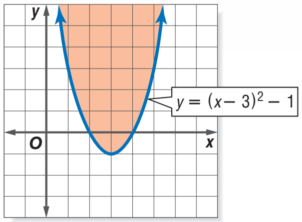
$x < -3$	$-3 < x < 2$	$x > 2$
Test $x = -4$.	Test $x = 0$.	Test $x = 4$.
$x^2 + x > 6$	$x^2 + x > 6$	$x^2 + x > 6$
$(-4)^2 + (-4) \overset{?}{>} 6$	$0^2 + 0 \overset{?}{>} 6$	$4^2 + 4 \overset{?}{>} 6$
$12 > 6$ ✓	$0 > 6$ ✗	$20 > 6$ ✓

The solution set is $\{x \mid x < -3 \text{ or } x > 2\}$. This is shown on the number line below.



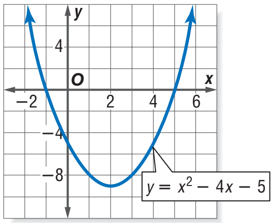
Check for Understanding

Concept Check



1. **Determine** which inequality, $y \geq (x - 3)^2 - 1$ or $y \leq (x - 3)^2 - 1$, describes the graph at the right. $y \geq (x - 3)^2 - 1$

2. **OPEN ENDED** List three points you might test to find the solution of $(x + 3)(x - 5) < 0$. Sample answer: one number less than −3, one number between −3 and 5, and one number greater than 5



3. **Examine** the graph of $y = x^2 - 4x - 5$ at the right. a. $x = -1, 5$ b. $x \leq -1$ or $x \geq 5$
   a. What are the solutions of $0 = x^2 - 4x - 5$?
   b. What are the solutions of $x^2 - 4x - 5 \geq 0$?
   c. What are the solutions of $x^2 - 4x - 5 \leq 0$? $-1 \leq x \leq 5$

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–7	1
8	2, 3
9–12	2, 3, 5
13	4

Graph each inequality. **4–7. See margin.**

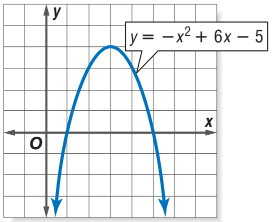
4. $y \ge x^2 - 10x + 25$

5. $y < x^2 - 16$

6. $y > -2x^2 - 4x + 3$

7. $y \le -x^2 + 5x + 6$

8. Use the graph of the related function of $-x^2 + 6x - 5 < 0$, which is shown at the right, to write the solutions of the inequality. **$x < 1$ or $x > 5$**



Solve each inequality algebraically.

9. $x^2 - 6x - 7 < 0$ **$\{x \mid -1 < x < 7\}$**

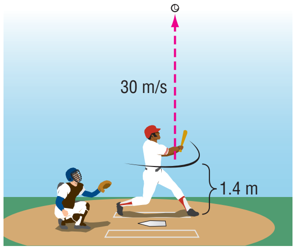
10. $x^2 - x - 12 > 0$ **$\{x \mid x < -3 \text{ or } x > 4\}$**

11. $x^2 < 10x - 25$ **$\varnothing$**

12. $x^2 \le 3$ **$\{x \mid -\sqrt{3} \le x \le \sqrt{3}\}$**

Application

13. **BASEBALL** A baseball player hits a high pop-up with an initial upward velocity of 30 meters per second, 1.4 meters above the ground. The height $h(t)$ of the ball in meters t seconds after being hit is modeled by $h(t) = -4.9t^2 + 30t + 1.4$. How long does a player on the opposing team have to catch the ball if he catches it 1.7 meters above the ground? **about 6.1 s**



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–25	1
26–29	2, 3
30–42	2, 3, 5
43–48	4

Extra Practice

See page 841.

Graph each inequality. **14–25. See pp. 343A–343F.**

14. $y \ge x^2 + 3x - 18$

15. $y < -x^2 + 7x + 8$

16. $y \le x^2 + 4x + 4$

17. $y \le x^2 + 4x$

18. $y > x^2 - 36$

19. $y > x^2 + 6x + 5$

20. $y \le -x^2 - 3x + 10$

21. $y \ge -x^2 - 7x + 10$

22. $y > -x^2 + 10x - 23$

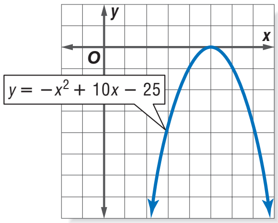
23. $y < -x^2 + 13x - 36$

24. $y < 2x^2 + 3x - 5$

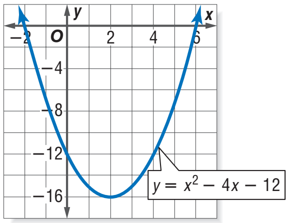
25. $y \ge 2x^2 + x - 3$

Use the graph of its related function to write the solutions of each inequality.

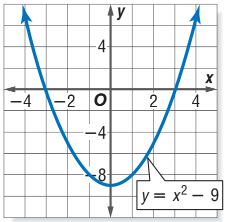
26. $-x^2 + 10x - 25 \ge 0$ **5**



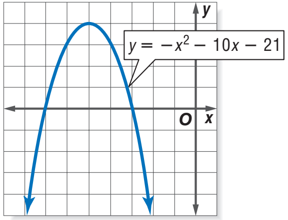
27. $x^2 - 4x - 12 \le 0$ **$-2 \le x \le 6$**



28. $x^2 - 9 > 0$ **$x < -3$ or $x > 3$**



29. $-x^2 - 10x - 21 < 0$ **$x < -7$ or $x > -3$**



www.algebra2.com/self_check_quiz

About the Exercises...

Organization by Objective

- **Graph Quadratic Inequalities:** 14–25
- **Solve Quadratic Inequalities:** 26–48

Odd/Even Assignments

Exercises 14–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 53–58 require a graphing calculator.

Assignment Guide

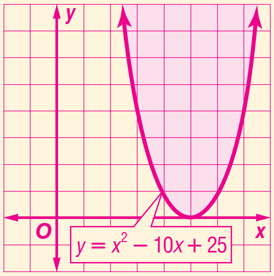
Basic: 15–41 odd, 45, 49–52, 59–71

Average: 15–45 odd, 49–52, 59–71 (optional: 53–58)

Advanced: 14–44 even, 46–71

Answers

4.



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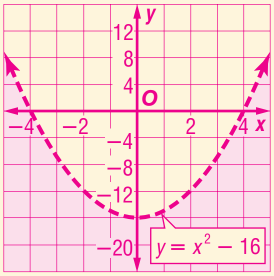

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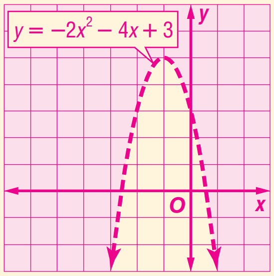

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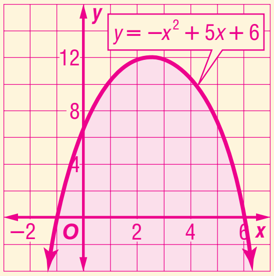


Solve each inequality algebraically.

30. $x^2 - 3x - 18 > 0$ $\{x \mid x < -3 \text{ or } x > 6\}$

31. $x^2 + 3x - 28 < 0$ $\{x \mid -7 < x < 4\}$

32. $x^2 - 4x \leq 5$ $\{x \mid -1 \leq x \leq 5\}$

33. $x^2 + 2x \geq 24$ $\{x \mid x \leq -6 \text{ or } x \geq 4\}$

34. $-x^2 - x + 12 \geq 0$ $\{x \mid -4 \leq x \leq 3\}$

35. $-x^2 - 6x + 7 \leq 0$ $\{x \mid x \leq -7 \text{ or } x \geq 1\}$

36. $9x^2 - 6x + 1 \leq 0$ $\left\{x \mid x = \frac{1}{3}\right\}$

37. $4x^2 + 20x + 25 \geq 0$ all reals

38. $x^2 + 12x < -36$ $\varnothing$

39. $-x^2 + 14x - 49 \geq 0$ $\{x \mid x = 7\}$

40. $18x - x^2 \leq 81$ all reals

41. $16x^2 + 9 < 24x$ $\varnothing$

★**42.** Solve $(x - 1)(x + 4)(x - 3) > 0$. $\{x \mid -4 < x < 1 \text{ or } x > 3\}$

43. LANDSCAPING Kinu wants to plant a garden and surround it with decorative stones. She has enough stones to enclose a rectangular garden with a perimeter of 68 feet, but she wants the garden to cover no more than 240 square feet. What could the width of her garden be? **0 to 10 ft or 24 to 34 ft**

44. BUSINESS A mall owner has determined that the relationship between monthly rent charged for store space r (in dollars per square foot) and monthly profit $P(r)$ (in thousands of dollars) can be approximated by the function $P(r) = -8.1r^2 + 46.9r - 38.2$. Solve each quadratic equation or inequality. Explain what each answer tells about the relationship between monthly rent and profit for this mall. **a–d. See margin.**

a. $-8.1r^2 + 46.9r - 38.2 = 0$

b. $-8.1r^2 + 46.9r - 38.2 > 0$

c. $-8.1r^2 + 46.9r - 38.2 > 10$

d. $-8.1r^2 + 46.9r - 38.2 < 10$

45. GEOMETRY A rectangle is 6 centimeters longer than it is wide. Find the possible dimensions if the area of the rectangle is more than 216 square centimeters. **The width should be greater than 12 cm and the length shoud be greater than 18 cm.**

FUND-RAISING **For Exercises 46–48, use the following information.**
The girls' softball team is sponsoring a fund-raising trip to see a professional baseball game. They charter a 60-passenger bus for \$525. In order to make a profit, they will charge \$15 per person if all seats on the bus are sold, but for each empty seat, they will increase the price by \$1.50 per person.

★**46.** Write a quadratic function giving the softball team's profit $P(n)$ from this fund-raiser as a function of the number of passengers n.

★**47.** What is the minimum number of passengers needed in order for the softball team not to lose money? **6**

★**48.** What is the maximum profit the team can make with this fund-raiser, and how many passengers will it take to achieve this maximum? **\$1312.50; 35 passengers**

49. CRITICAL THINKING Graph the intersection of the graphs of $y \leq -x^2 + 4$ and $y \geq x^2 - 4$. **See margin.**

50. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you find the time a trampolinist spends above a certain height?

Include the following in your answer:

- a quadratic inequality that describes the time the performer spends more than 10 feet above the ground, and
- two approaches to solving this quadratic inequality.

Career Choices

Landscape Architect

Landscape architects design outdoor spaces so that they are not only functional, but beautiful and compatible with the natural environment.

Online Research
For information about a career as a landscape architect, visit:
www.algebra2.com/careers

46. $P(n) = n[15 + 1.5(60 - n)] - 525$ or $-1.5n^2 + 105n - 525$

Study Guide and Intervention, p. 349 (shown) and p. 350

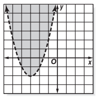
Graph Quadratic Inequalities To graph a quadratic inequality in two variables, use the following steps:

1. Graph the related quadratic equation, $y = ax^2 + bx + c$.
   Use a dashed line for $<$ or $>$; use a solid line for $\leq$ or $\geq$.
2. Test a point inside the parabola.
   If it satisfies the inequality, shade the region inside the parabola; otherwise, shade the region outside the parabola.

Example **Graph the inequality $y > x^2 + 6x + 7$.**

First graph the equation $y = x^2 + 6x + 7$. By completing the square, you get the vertex form of the equation $y = (x + 3)^2 - 2$, so the vertex is $(-3, -2)$. Make a table of values around $x = -3$, and graph. Since the inequality includes $>$, use a dashed line.

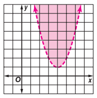
Test the point $(-3, 0)$, which is inside the parabola. Since $(-3)^2 + 6(-3) + 7 = -2$, and $0 > -2$, $(-3, 0)$ satisfies the inequality. Therefore, shade the region inside the parabola.



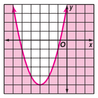
Exercises

Graph each inequality.

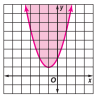
1. $y > x^2 - 8x + 17$



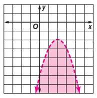
2. $y \leq x^2 + 6x + 4$



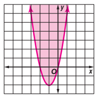
3. $y \geq x^2 + 2x + 2$



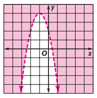
4. $y < -x^2 + 4x - 6$



5. $y \geq 2x^2 + 4x$



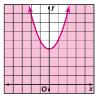
6. $y > -2x^2 - 4x + 2$



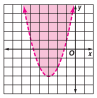
Skills Practice, p. 351 and Practice, p. 352 (shown)

Graph each inequality.

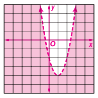
1. $y \leq x^2 + 4$



2. $y > x^2 + 6x + 6$

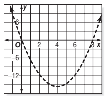


3. $y < 2x^2 - 4x - 2$



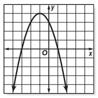
Use the graph of its related function to write the solutions of each inequality.

4. $x^2 - 8x > 0$



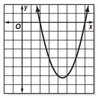
$x < 0$ or $x > 8$

5. $-x^2 - 2x + 3 \geq 0$



$-3 \leq x \leq 1$

6. $x^2 - 9x + 14 \leq 0$



$2 \leq x \leq 7$

Solve each inequality algebraically.

7. $x^2 - x - 20 > 0$ $\{x \mid x < -4 \text{ or } x > 5\}$

8. $x^2 - 10x + 16 < 0$ $\{x \mid 2 < x < 8\}$

9. $x^2 + 4x + 5 \leq 0$ $\varnothing$

10. $x^2 + 14x + 49 \geq 0$ all reals

11. $x^2 - 5x > 14$ $\{x \mid x < -2 \text{ or } x > 7\}$

12. $-x^2 - 15 \geq 8x$ $\{x \mid -5 \leq x \leq -3\}$

13. $-x^2 + 5x - 7 \leq 0$ all reals

14. $9x^2 + 36x + 36 \leq 0$ $\{x \mid x = -2\}$

15. $9x \leq 12x^2$ $\left\{x \mid x \leq 0 \text{ or } x \geq \frac{3}{4}\right\}$

16. $4x^2 + 4x + 1 > 0$ $\left\{x \mid x \neq -\frac{1}{2}\right\}$

17. $5x^2 + 10 \geq 27x$ $\left\{x \mid x \leq \frac{2}{5} \text{ or } x \geq 5\right\}$

18. $9x^2 + 31x + 12 \leq 0$ $\left\{x \mid -3 \leq x \leq -\frac{4}{9}\right\}$

19. FENCING Vanessa has 180 feet of fencing that she intends to use to build a rectangular play area for her dog. She wants the play area to enclose at least 1800 square feet. What are the possible widths of the play area? **30 ft to 60 ft**

20. BUSINESS A bicycle maker sold 300 bicycles last year at a profit of \$300 each. The maker wants to increase the profit margin this year, but predicts that each \$20 increase in profit will reduce the number of bicycles sold by 10. How many \$20 increases in profit can the maker add in and expect to make a total profit of at least \$100,000? **from 5 to 10**

Reading to Learn Mathematics, p. 353 ELL

Pre-Activity **How can you find the time a trampolinist spends above a certain height?**

Read the introduction to Lesson 6-7 at the top of page 329 in your textbook.

- How far above the ground is the trampoline surface? **3.75 feet**
- Using the quadratic function given in the introduction, write a quadratic inequality that describes the times at which the trampolinist is more than 20 feet above the ground. $-16t^2 + 42t + 3.75 > 20$

Reading the Lesson

1. Answer the following questions about how you would graph the inequality $y \geq x^2 + x - 6$.

a. What is the related quadratic equation? $y = x^2 + x - 6$

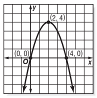
b. Should the parabola be solid or dashed? How do you know?
solid; The inequality symbol is ≥.

c. The point (0, 2) is inside the parabola. To use this as a test point, substitute ___0___ for x and ___2___ for y in the quadratic inequality.

d. Is the statement $2 \geq 0^2 + 0 - 6$ true or false? **true**

e. Should the region inside or outside the parabola be shaded? **inside**

2. The graph of $y = -x^2 + 4x$ is shown at the right. Match each of the following related inequalities with its solution set.



a. $-x^2 + 4x > 0$ **ii**

b. $-x^2 + 4x \leq 0$ **iii**

c. $-x^2 + 4x \geq 0$ **iv**

d. $-x^2 + 4x < 0$ **i**

i. $\{x \mid x < 0 \text{ or } x > 4\}$

ii. $\{x \mid 0 < x < 4\}$

iii. $\{x \mid x \leq 0 \text{ or } x \geq 4\}$

iv. $\{x \mid 0 \leq x \leq 4\}$

Helping You Remember

3. A quadratic inequality in two variables may have the form $y > ax^2 + bx + c$, $y < ax^2 + bx + c$, $y \geq ax^2 + bx + c$, or $y \leq ax^2 + bx + c$. Describe a way to remember which region to shade by looking at the inequality symbol and without using a test point.
Sample answer: If the symbol is > or ≥, shade the region above the parabola. If the symbol is < or ≤, shade the region below the parabola.

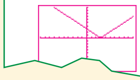
Enrichment, p. 354

Graphing Absolute Value Inequalities

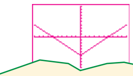
You can solve absolute value inequalities by graphing in much the same manner you graphed quadratic inequalities. Graph the related absolute function for each inequality by using a graphing calculator. For $>$ and $\geq$, identify the x-values, if any, for which the graph lies *below* the x-axis. For $<$ and $\leq$, identify the x values, if any, for which the graph lies *above* the x-axis.

For each inequality, make a sketch of the related graph and find the solutions rounded to the nearest hundredth.

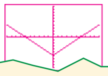
1. $|x - 3| > 0$
$x > 3$ or $x < 3$



2. $|x| - 6 < 0$
$-6 < x < 6$

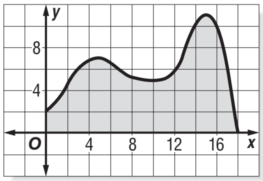


3. $-|x + 4| + 8 < 0$
$-12 < x < 4$



Standardized Test Practice

51. Which is a reasonable estimate of the area under the curve from $x = 0$ to $x = 18$? **C**



Ⓐ 29 square units

Ⓑ 58 square units

Ⓒ 116 square units

Ⓓ 232 square units

52. If $(x + 1)(x - 2)$ is positive, then **A**

Ⓐ $x < -1$ or $x > 2$.
Ⓑ $x > -1$ or $x < 2$.
Ⓒ $-1 < x < 2$.
Ⓓ $-2 < x < 1$.

Extending the Lesson

SOLVE ABSOLUTE VALUE INEQUALITIES BY GRAPHING Similar to quadratic inequalities, you can solve absolute value inequalities by graphing.

Graph the related absolute value function for each inequality using a graphing calculator. For > and ≥, identify the *x* values, if any, for which the graph lies *below* the *x*-axis. For < and ≤, identify the *x* values, if any, for which the graph lies *above* the *x*-axis.

53. $|x - 2| > 0$ **$\{x \mid \text{all reals}, x \neq 2\}$**

54. $|x| - 7 < 0$ **$\{x \mid -7 < x < 7\}$**

55. $-|x + 3| + 6 < 0$ **$\{x \mid x < -9 \text{ or } x > 3\}$**

56. $2|x + 3| - 1 \geq 0$ **$\{x \mid x \leq -3.5 \text{ or } x \geq -2.5\}$**

57. $|5x + 4| - 2 \leq 0$ **$\{x \mid -1.2 \leq x \leq -0.4\}$**

58. $|4x - 1| + 3 < 0$ **no real solutions**

Maintain Your Skills

Mixed Review

59. $y = (x - 1)^2 + 8$; $(1, 8)$, $x = 1$; up
60. $y = -2(x - 4)^2$; $(4, 0)$, $x = 4$; down
61. $y = \frac{1}{2}(x + 6)^2$; $(-6, 0)$, $x = -6$; up
63. $\frac{-5 \pm i\sqrt{3}}{2}$

Write each equation in vertex form. Then identify the vertex, axis of symmetry, and direction of opening. *(Lesson 6-6)*

59. $y = x^2 - 2x + 9$
60. $y = -2x^2 + 16x - 32$
61. $y = \frac{1}{2}x^2 + 6x + 18$

Solve each equation using the method of your choice. Find exact solutions. *(Lesson 6-5)*

62. $x^2 + 12x + 32 = 0$ **$-4, -8$**
63. $x^2 + 7 = -5x$
64. $3x^2 + 6x - 2 = 3$ **$\frac{-3 \pm 2\sqrt{6}}{3}$**

Simplify. *(Lesson 5-2)* **65. $4a^2b^2 + 2a^2b + 4ab^2 + 12a - 7b$**

65. $(2a^2b - 3ab^2 + 5a - 6b) + (4a^2b^2 + 7ab^2 - b + 7a)$

66. $(x^3 - 3x^2y + 4xy^2 + y^3) - (7x^3 + x^2y - 9xy^2 + y^3)$ **$-6x^3 - 4x^2y + 13xy^2$**

67. $x^{-3}y^2(x^4y + x^3y^{-1} + x^2y^{-2})$ **$xy^3 + y + \frac{1}{x}$**

68. $(5a - 3)(1 - 3a)$ **$-15a^2 + 14a - 3$**

Find each product, if possible. *(Lesson 4-3)*

69. $\begin{bmatrix} -6 & 3 \\ 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ -3 & 6 \end{bmatrix}$ **$\begin{bmatrix} -21 & 48 \\ -13 & 22 \end{bmatrix}$**

70. $\begin{bmatrix} 2 & -6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 9 & 0 \\ -2 & 4 \end{bmatrix}$ **$\begin{bmatrix} -54 & 6 \end{bmatrix}$**

71. LAW ENFORCEMENT Thirty-four states classify drivers having at least a 0.1 blood alcohol content (BAC) as intoxicated. An infrared device measures a person's BAC through an analysis of his or her breath. A certain detector measures BAC to within 0.002. If a person's actual blood alcohol content is 0.08, write and solve an absolute value equation to describe the range of BACs that might register on this device. *(Lesson 1-6)* **$|x - 0.08| \leq 0.002$; $0.078 \leq x \leq 0.082$**

4 Assess

Open-Ended Assessment

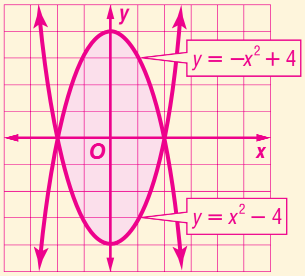
Speaking Have students explain how to test points in the coordinate plane in order to determine which region represents the solution to a quadratic inequality. Also ask them to explain how to analyze the graph of a quadratic equation in order to determine the solution set for a quadratic inequality.

Assessment Options

Quiz (Lesson 6-7) is available on p. 370 of the *Chapter 6 Resource Masters*.

Answers

49.



50. Answers should include the following.

- $-16t^2 + 42t + 3.75 > 10$
- One method of solving this inequality is to graph the related quadratic function $h(t) = -16t^2 + 42t + 3.75 - 10$. The interval(s) at which the graph is above the x-axis represents the times when the trampolinist is above 10 feet. A second method of solving this inequality would be find the roots of the related quadratic equation $-16t^2 + 42t + 3.75 - 10 = 0$ and then test points in the three intervals determined by these roots to see if they satisfy the inequality. The interval(s) at which the inequality is satisfied represent the times when the trampolinist is above 10 feet.

Answers

44a. 0.98, 4.81; The owner will break even if he charges \$0.98 or \$4.81 per square foot.

44b. $0.98 < r < 4.81$; The owner will make a profit if the rent is between \$0.98 and \$4.81.

44c. $1.34 < r < 4.45$; If rent is set between \$1.34 and \$4.45 per sq ft, the profit will be greater than \$10,000.

44d. $r < 1.34$ or $r > 4.45$; If rent is set between \$0 and \$1.34 or above \$4.45 per sq ft, the profit will be less than \$10,000.

Chapter 6 Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 6 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 6 is available on p. 368 of the *Chapter 6 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Chapter 6 Study Guide and Review

Vocabulary and Concept Check

axis of symmetry (p. 287)
completing the square (p. 307)
constant term (p. 286)
discriminant (p. 316)
linear term (p. 286)
maximum value (p. 288)
minimum value (p. 288)
parabola (p. 286)
quadratic equation (p. 294)
Quadratic Formula (p. 313)
quadratic function (p. 286)
quadratic inequality (p. 329)
quadratic term (p. 286)
roots (p. 294)
Square Root Property (p. 306)
vertex (p. 287)
vertex form (p. 322)
Zero Product Property (p. 301)
zeros (p. 294)

Choose the letter of the term that best matches each phrase.

1. the graph of any quadratic function f
2. process used to create a perfect square trinomial b
3. the line passing through the vertex of a parabola and dividing the parabola into two mirror images a
4. a function described by an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ h
5. the solutions of an equation i
6. $y = a(x - h)^2 + k$ j
7. in the Quadratic Formula, the expression under the radical sign, $b^2 - 4ac$ c
8. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ g

a. axis of symmetry
b. completing the square
c. discriminant
d. constant term
e. linear term
f. parabola
g. Quadratic Formula
h. quadratic function
i. roots
j. vertex form

Lesson-by-Lesson Review

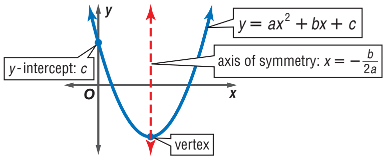
6-1 Graphing Quadratic Functions

See pages 286–293.

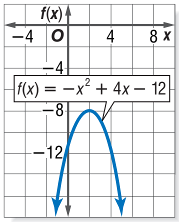
Concept Summary

The graph of $y = ax^2 + bx + c, a \neq 0$,

- opens up, and the function has a minimum value when $a > 0$, and
- opens down, and the function has a maximum value when $a < 0$.



Example **Find the maximum or minimum value of $f(x) = -x^2 + 4x - 12$.**



Since $a > 0$, the graph opens down and the function has a maximum value. The maximum value of the function is the y-coordinate of the vertex. The x-coordinate of the vertex is $x = -\frac{4}{2(-1)}$ or 2. Find the y-coordinate by evaluating the function for $x = 2$.

$f(x) = -x^2 + 4x - 12$ Original function
$f(2) = -(2)^2 + 4(2) - 12$ or -8 Replace x with 2.

Therefore, the maximum value of the function is -8.

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FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Discuss with students how they might recognize a key concept that needs to be included in the Foldable. Ask them to include a transition sentence or two in their notes that relates one topic to the next. Suggest that they use this discussion as they review their Foldable to add, delete, or reorganize material in order to make it more useful to them.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Exercises Complete parts a–c for each quadratic function.

a. Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.

b. Make a table of values that includes the vertex.

c. Use this information to graph the function. *(See Example 2 on pages 287 and 288.)*

9. $f(x) = x^2 + 6x + 20$ **10.** $f(x) = x^2 - 2x - 15$ **11.** $f(x) = x^2 - 8x + 7$

12. $f(x) = -2x^2 + 12x - 9$ **13.** $f(x) = -x^2 - 4x - 3$ **14.** $f(x) = 3x^2 + 9x + 6$

9–14. See margin.

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

(See Example 3 on pages 288 and 289.)

15. $f(x) = 4x^2 - 3x - 5$ **16.** $f(x) = -3x^2 + 2x - 2$ **17.** $f(x) = -2x^2 + 7$

min.; $-\frac{89}{16}$ **max.; $-\frac{5}{3}$** **max.; 7**

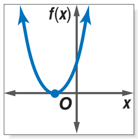
6-2 Solving Quadratic Equations by Graphing

See pages 294–299.

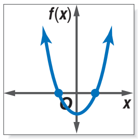
Concept Summary

- The solutions, or roots, of a quadratic equation are the zeros of the related quadratic function. You can find the zeros of a quadratic function by finding the x-intercepts of its graph.
- A quadratic equation can have one real solution, two real solutions, or no real solution.

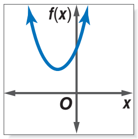
One Real Solution



Two Real Solutions



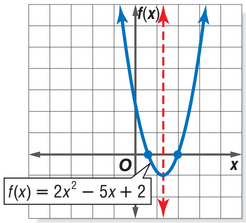
No Real Solution



Example Solve $2x^2 - 5x + 2 = 0$ by graphing.

The equation of the axis of symmetry is $x = -\frac{-5}{2(2)}$ or $x = \frac{5}{4}$.

x	0	$\frac{1}{2}$	$\frac{5}{4}$	2	$\frac{5}{2}$
f(x)	2	0	$-\frac{9}{8}$	0	2



The zeros of the related function are $\frac{1}{2}$ and 2. Therefore, the solutions of the equation are $\frac{1}{2}$ and 2.

Exercises Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

(See Examples 1–3 on pages 294 and 295.) **19. 2, −5**

18. $x^2 - 36 = 0$ **6, −6** **19.** $-x^2 - 3x + 10 = 0$ **20.** $2x^2 + x - 3 = 0$ **1, $-\frac{3}{2}$**

21. $-x^2 - 40x - 80 = 0$ **22.** $-3x^2 - 6x - 2 = 0$ **23.** $\frac{1}{5}(x + 3)^2 - 5 = 0$ **2, −8**

21. between −3 and −2; between −38 and −37

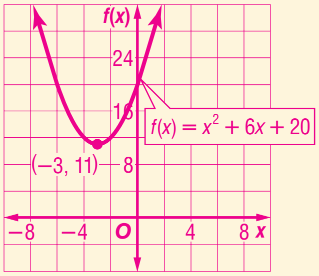
22. between −2 and −1; between −1 and 0

9a. 20; $x = -3$; −3

9b.

x	f(x)
−5	15
−4	12
−3	11
−2	12
−1	15

9c.



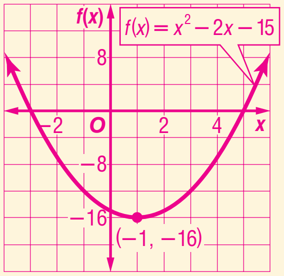
10a. −15; $x = 1$; 1

10b.

x	f(x)
−1	−12
0	−15
1	−16
2	−15
3	−12

Answers

10c.

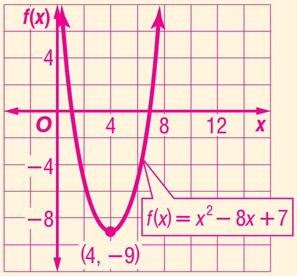


11a. 7; $x = 4$; 4

11b.

x	f(x)
2	−5
3	−8
4	−9
5	−8
6	−5

11c.

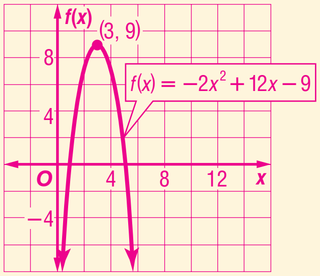


12a. −9; $x = 3$; 3

12b.

x	f(x)
1	1
2	7
3	9
4	7
5	1

12c.



(continued on the next page)

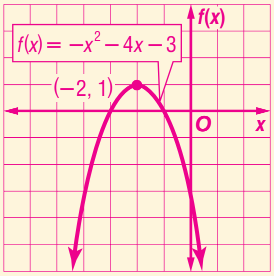
Answers

13a. -3; $x = -2$; -2

13b.

x	$f(x)$
-4	-3
-3	0
-2	1
-1	0
0	-3

13c.

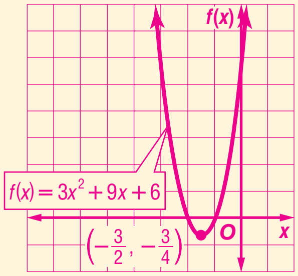


14a. 6; $x = -\frac{3}{2}$; $-\frac{3}{2}$

14b.

x	$f(x)$
-3	6
-2	0
$-\frac{3}{2}$	$-\frac{3}{4}$
-1	0
0	6

14c.



6-3 Solving Quadratic Equations by Factoring

See pages 301–305.

Concept Summary

- Zero Product Property: For any real numbers a and b, if $ab = 0$, then either $a = 0$, $b = 0$, or both a and $b = 0$.

Example **Solve $x^2 + 9x + 20 = 0$ by factoring.**

$x^2 + 9x + 20 = 0$ Original equation

$(x + 4)(x + 5) = 0$ Factor the trinomial.

$x + 4 = 0$ or $x + 5 = 0$ Zero Product Property

$x = -4$ $\quad x = -5$ The solution set is $\{-5, -4\}$.

Exercises **Solve each equation by factoring.** *(See Examples 1–3 on pages 301 and 302.)*

24. $x^2 - 4x - 32 = 0$ **25.** $3x^2 + 6x + 3 = 0$ $\{-1\}$ **26.** $5y^2 = 80$ $\{-4, 4\}$

27. $2c^2 + 18c - 44 = 0$ **28.** $25x^2 - 30x = -9$ $\left\{\frac{3}{5}\right\}$ **29.** $6x^2 + 7x = 3$ $\left\{\frac{1}{3}, -\frac{3}{2}\right\}$

24. $\{-4, 8\}$ **27.** $\{-11, 2\}$

Write a quadratic equation with the given root(s). Write the equation in the form $ax^2 + bx + c$, where a, b, and c are integers. *(See Example 4 on page 303.)*

30. $-4, -25$ **31.** $10, -7$ **32.** $\frac{1}{3}, 2$ $3x^2 - 7x + 2 = 0$

$x^2 + 29x + 100 = 0$ $\quad x^2 - 3x - 70 = 0$

6-4 Completing the Square

See pages 306–312.

Concept Summary

- To complete the square for any quadratic expression $x^2 + bx$:

Step 1 Find one half of b, the coefficient of x.

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$. $\quad x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Example **Solve $x^2 + 10x - 39 = 0$ by completing the square.**

$x^2 + 10x - 39 = 0$ Notice that $x^2 + 10x - 39 = 0$ is not a perfect square.

$x^2 + 10x = 39$ Rewrite so the left side is of the form $x^2 + bx$.

$x^2 + 10x + 25 = 39 + 25$ Since $\left(\frac{10}{2}\right)^2 = 25$, add 25 to each side.

$(x + 5)^2 = 64$ Write the left side as a perfect square by factoring.

$x + 5 = \pm 8$ Square Root Property

$x + 5 = 8$ or $x + 5 = -8$ Rewrite as two equations.

$x = 3$ $\quad x = -13$ The solution set is $\{-13, 3\}$.

Exercises **Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.** *(See Example 3 on page 307.)*

33. $x^2 + 34x + c$ $\quad 289; (x + 17)^2$ **34.** $x^2 - 11x + c$ **35.** $x^2 + \frac{7}{2}x + c$ $\frac{49}{16}; \left(x + \frac{7}{4}\right)^2$

Solve each equation by completing the square. *(See Examples 4–6 on pages 308 and 309.)*

36. $2x^2 - 7x - 15 = 0$ **37.** $2n^2 - 12n - 22 = 0$ **38.** $2x^2 - 5x + 7 = 3$

34. $\frac{121}{4}$; $\left(x - \frac{11}{2}\right)^2$

36. $\left\{-\frac{3}{2}, 5\right\}$

37. $\{3 \pm 2\sqrt{5}\}$

38. $\left\{\frac{5 \pm i\sqrt{7}}{4}\right\}$

6-5 The Quadratic Formula and the Discriminant

See pages 313–319.

Concept Summary

- Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$

Solve $x^2 - 5x - 66 = 0$ by using the Quadratic Formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-66)}}{2(1)}$ Replace a with 1, b with −5, and c with −66.

$= \frac{5 \pm 17}{2}$ Simplify.

$x = \frac{5 + 17}{2}$ or $x = \frac{5 - 17}{2}$ Write as two equations.

$= 11$ $= -6$ The solution set is {11, −6}.

Exercises **Complete parts a–c for each quadratic equation.**

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

(See Examples 1–4 on pages 314–316.) **39–41. See margin.**

39. $x^2 + 2x + 7 = 0$ **40.** $-2x^2 + 12x - 5 = 0$ **41.** $3x^2 + 7x - 2 = 0$

6-6 Analyzing Graphs of Quadratic Functions

See pages 322–328.

Concept Summary

- As the values of h and k change, the graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated
  - $|h|$ units left if h is negative or $|h|$ units right if h is positive.
  - $|k|$ units up if k is positive or $|k|$ units down if k is negative.
- Consider the equation $y = a(x - h)^2 + k$.
  - If $a > 0$, the graph opens up; if $a < 0$ the graph opens down.
  - If $|a| > 1$, the graph is narrower than the graph of $y = x^2$.
  - If $|a| < 1$, the graph is wider than the graph of $y = x^2$.

Example **Write the quadratic function $y = 3x^2 + 42x + 142$ in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.**

$y = 3x^2 + 42x + 142$ Original equation

$y = 3(x^2 + 14x) + 142$ Group $ax^2 + bx$ and factor, dividing by a.

$y = 3(x^2 + 14x + 49) + 142 - 3(49)$ Complete the square by adding $3\left(\frac{14}{2}\right)^2$. Balance this with a subtraction of 3(49).

$y = 3(x + 7)^2 - 5$ Write $x^2 + 14x + 7$ as a perfect square.

So, $a = 3$, $h = -7$, and $k = -5$. The vertex is at (−7, −5), and the axis of symmetry is $x = -7$. Since a is positive, the graph opens up.

Answers

39a. −24

39b. 2 complex

39c. $-1 \pm i\sqrt{6}$

40a. 104

40b. 2 irrational

40c. $3 \pm \frac{\sqrt{26}}{2}$

41a. 73

41b. 2 irrational

41c. $\frac{-7 \pm \sqrt{73}}{6}$

Chapter 6 For More ...
- Extra Practice, see pages 839–841.
- Mixed Problem Solving, see page 867.

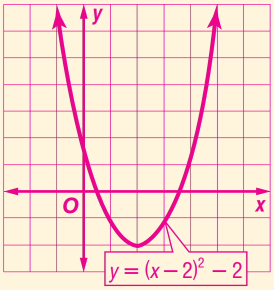
Answers

42. $(-2, 3)$; $x = -2$; down

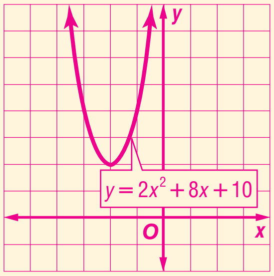
43. $y = 5\left(x + \frac{7}{2}\right)^2 - \frac{13}{4}$; $\left(-\frac{7}{2}, -\frac{13}{4}\right)$; $x = -\frac{7}{2}$; up

44. $y = -\frac{1}{3}(x - 12)^2 + 48$; $(12, 48)$; $x = 12$; down

45.



46.



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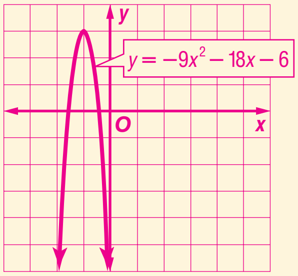

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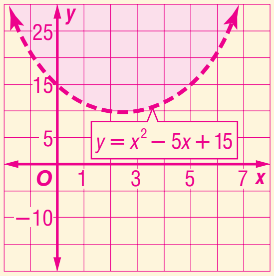


Exercises Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. *(See Examples 1 and 3 on pages 322 and 324.)* **42–44. See margin.**

42. $y = -6(x + 2)^2 + 3$ **43.** $y = 5x^2 + 35x + 58$ **44.** $y = -\frac{1}{3}x^2 + 8x$

Graph each function. *(See Examples 1–3 on pages 322 and 324.)* **45–47. See margin.**

45. $y = (x - 2)^2 - 2$ **46.** $y = 2x^2 + 8x + 10$ **47.** $y = -9x^2 - 18x - 6$

Write an equation for the parabola with the given vertex that passes through the given point. *(See Example 4 on page 325.)*

48. vertex: $(4, 1)$
point: $(2, 13)$
$y = 3(x - 4)^2 + 1$

49. vertex: $(-2, 3)$
point: $(-6, 11)$
$y = \frac{1}{2}(x + 2)^2 + 3$

50. vertex: $(-3, -5)$
point: $(0, -14)$
$y = -(x + 3)^2 - 5$

6-7 Graphing and Solving Quadratic Inequalities

See pages 329–335.

Concept Summary

- Graph quadratic inequalities in two variables as follows.

  Step 1 Graph the related quadratic equation, $y = ax^2 + bx + c$. Decide if the parabola should be solid or dashed.

  Step 2 Test a point (x_1, y_1) inside the parabola. Check to see if this point is a solution of the inequality.

  Step 3 If (x_1, y_1) *is* a solution, shade the region *inside* the parabola. If (x_1, y_1) is *not* a solution, shade the region *outside* the parabola.

- To solve a quadratic inequality in one variable, graph the related quadratic function. Identify the x values for which the graph lies *below* the x-axis for $<$ and $\leq$. Identify the x values for which the graph lies *above* the x-axis for $>$ and $\geq$.

Example **Solve $x^2 + 3x - 10 < 0$ by graphing.**

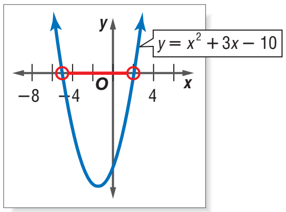
Find the roots of the related equation.

$0 = x^2 + 3x - 10$ Related equation

$0 = (x + 5)(x - 2)$ Factor.

$x + 5 = 0$ or $x - 2 = 0$ Zero Product Property

$x = -5$ $x = 2$ Solve each equation.



Sketch the graph of the parabola that has x-intercepts at -5 and 2. The graph should open up since $a > 0$. The graph lies below the x-axis between $x = -5$ and $x = 2$. Therefore, the solution set is $\{x \mid -5 < x < 2\}$.

Exercises Graph each inequality. *(See Example 1 on pages 329 and 330.)* **51–53. See margin.**

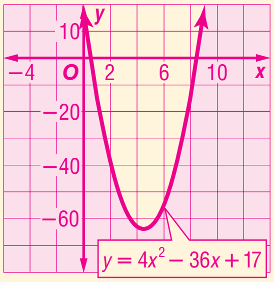
51. $y > x^2 - 5x + 15$ **52.** $y \leq 4x^2 - 36x + 17$ **53.** $y \geq -x^2 + 7x - 11$

Solve each inequality. *(See Examples 2, 3, and 5 on pages 330–332.)* **54–59. See pp. 343A–343F.**

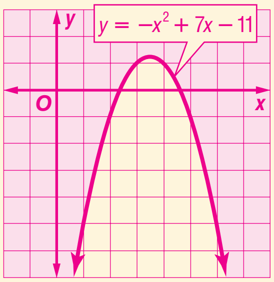
54. $6x^2 + 5x > 4$ **55.** $8x + x^2 \geq -16$ **56.** $2x^2 + 5x < 12$

57. $2x^2 - 5x > 3$ **58.** $4x^2 - 9 \leq -4x$ **59.** $3x^2 - 5 > 6x$

52.



53.



Chapter 6 Practice Test

Vocabulary and Concepts

Choose the word or term that best completes each statement.

1. The y-coordinate of the vertex of the graph of $y = ax^2 + bx + c$ is the (*maximum*, *minimum*) value obtained by the function when a is positive.
2. (*The Square Root Property*, *Completing the square*) can be used to solve any quadratic equation.

Skills and Applications

Complete parts a–c for each quadratic function.

a. **Find the y-intercept, the equation of the axis of symmetry, and the x-coordinate of the vertex.**
b. **Make a table of values that includes the vertex.**
c. **Use this information to graph the function.** **3–5. See pp. 343A–343F.**

3. $f(x) = x^2 - 2x + 5$
4. $f(x) = -3x^2 + 8x$
5. $f(x) = -2x^2 - 7x - 1$

Determine whether each function has a maximum or a minimum value. Then find the maximum or minimum value of each function.

6. $f(x) = x^2 + 6x + 9$ **min.; 0**
7. $f(x) = 3x^2 - 12x - 24$ **min.; −36**
8. $f(x) = -x^2 + 4x$ **max.; 4**

9. Write a quadratic equation with roots −4 and 5. Write the equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers. $x^2 - x - 20 = 0$

Solve each equation using the method of your choice. Find exact solutions. **10–18. See margin.**

10. $x^2 + x - 42 = 0$
11. $-1.6x^2 - 3.2x + 18 = 0$
12. $15x^2 + 16x - 7 = 0$
13. $x^2 + 8x - 48 = 0$
14. $x^2 + 12x + 11 = 0$
15. $x^2 - 9x - \frac{19}{4} = 0$
16. $3x^2 + 7x - 31 = 0$
17. $10x^2 + 3x = 1$
18. $-11x^2 - 174x + 221 = 0$

19. **BALLOONING** At a hot-air balloon festival, you throw a weighted marker straight down from an altitude of 250 feet toward a bull's eye below. The initial velocity of the marker when it leaves your hand is 28 feet per second. Find how long it will take the marker to hit the target by solving the equation $-16t^2 - 28t + 250 = 0$. **about 3.17 s**

Write each equation in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening. **20–22. See pp. 343A–343F.**

20. $y = (x + 2)^2 - 3$
21. $y = x^2 + 10x + 27$
22. $y = -9x^2 + 54x - 8$

Graph each inequality. **23–25. See pp. 343A–343F.**

23. $y \leq x^2 + 6x - 7$
24. $y > -2x^2 + 9$
25. $y \geq -\frac{1}{2}x^2 - 3x + 1$

Solve each inequality. **26.** $\{x \mid -7 < x < 5\}$ **27–28. See pp. 343A–343F.**

26. $(x - 5)(x + 7) < 0$
27. $3x^2 \geq 16$
28. $-5x^2 + x + 2 < 0$

29. **PETS** A rectangular turtle pen is 6 feet long by 4 feet wide. The pen is enlarged by increasing the length and width by an equal amount in order to double its area. What are the dimensions of the new pen? **8 ft by 6 ft**

30. **STANDARDIZED TEST PRACTICE** Which of the following is the sum of both solutions of the equation $x^2 + 8x - 48 = 0$? **B**

Ⓐ −16 Ⓑ −8 Ⓒ −4 Ⓓ 12

www.algebra2.com/chapter_test

Portfolio Suggestion

Introduction In this chapter quadratic equations have been graphed and solved using many different methods, often following a process that involved numerous steps.

Ask Students Select an item from this chapter that shows your best work, including a graph, and place it in your portfolio. Explain why you believe it to be your best work and how you came to choose this particular piece of work.

Chapter 6 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 6 can be found on p. 368 of the *Chapter 6 Resource Masters*.

Chapter Tests There are six Chapter 6 Tests and an Open-Ended Assessment task available in the *Chapter 6 Resource Masters*.

Chapter 6 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	355–356
2A	MC	average	357–358
2B	MC	average	359–360
2C	FR	average	361–362
2D	FR	average	363–364
3	FR	advanced	365–366

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment Performance tasks for Chapter 6 can be found on p. 367 of the *Chapter 6 Resource Masters*. A sample scoring rubric for these tasks appears on p. A28.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Answers

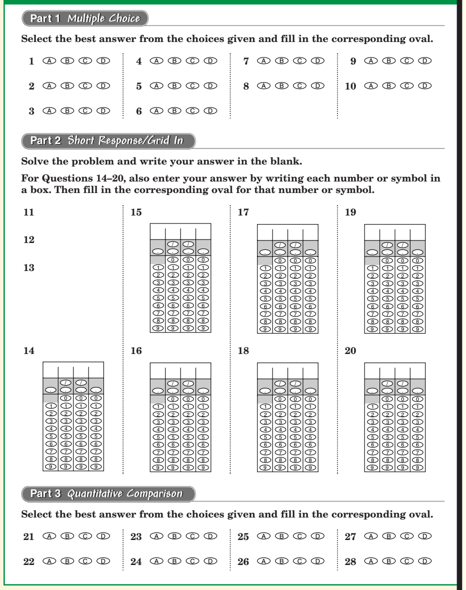
10. −7, 6
11. $-\frac{9}{2}, \frac{5}{2}$
12. $-\frac{7}{5}, \frac{1}{3}$
13. −12, 4
14. −11, −1
15. $-\frac{1}{2}, \frac{19}{2}$
16. $\frac{-7 \pm \sqrt{421}}{6}$
17. $-\frac{1}{2}, \frac{1}{5}$
18. $-17, \frac{13}{11}$

Chapter 6 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 6 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1



Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ

2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ 10 Ⓐ Ⓑ Ⓒ Ⓓ

3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 14–20, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 15 17 19

12

13

14 16 18 20

Part 3 *Quantitative Comparison*

Select the best answer from the choices given and fill in the corresponding oval.

21 Ⓐ Ⓑ Ⓒ Ⓓ 23 Ⓐ Ⓑ Ⓒ Ⓓ 25 Ⓐ Ⓑ Ⓒ Ⓓ 27 Ⓐ Ⓑ Ⓒ Ⓓ

22 Ⓐ Ⓑ Ⓒ Ⓓ 24 Ⓐ Ⓑ Ⓒ Ⓓ 26 Ⓐ Ⓑ Ⓒ Ⓓ 28 Ⓐ Ⓑ Ⓒ Ⓓ

Additional Practice

See pp. 373–374 in the *Chapter 6 Resource Masters* for additional standardized test practice.

Chapter 6 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. In a class of 30 students, half are girls and 24 ride the bus to school. If 4 of the girls do not ride the bus to school, how many boys in this class ride the bus to school? **C**

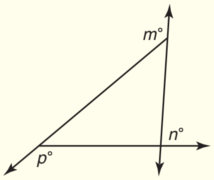
Ⓐ 2 Ⓑ 11

Ⓒ 13 Ⓓ 15

2. In the figure below, the measures of $\angle m + \angle n + \angle p = \underline{\ \ ?\ \ }$ **D**

Ⓐ 90 Ⓑ 180

Ⓒ 270 Ⓓ 360



3. Of the points $(-4, -2)$, $(1, -3)$, $(-1, 3)$, $(3, 1)$, and $(-2, 1)$, which three lie on the same side of the line $y - x = 0$? **C**

Ⓐ $(-4, -2), (1, -3), (-2, 1)$

Ⓑ $(-4, -2), (1, -3), (3, 1)$

Ⓒ $(-4, -2), (-1, 3), (-2, 1)$

Ⓓ $(1, -3), (-1, 3), (3, 1)$

4. If k is an integer, then which of the following must also be integers? **B**

I. $\frac{5k + 5}{5k}$ **II.** $\frac{5k + 5}{k + 1}$ **III.** $\frac{5k^2 + k}{5k}$

Ⓐ I only Ⓑ II only

Ⓒ I and II Ⓓ II and III

5. Which of the following is a factor of $x^2 - 7x - 8$? **D**

Ⓐ $x + 2$ Ⓑ $x - 1$

Ⓒ $x - 4$ Ⓓ $x - 8$

6. If $x > 0$, then $\frac{\sqrt{16x^2 + 64x + 64}}{x + 2} = \underline{\ \ ?\ \ }$ **B**

Ⓐ 2 Ⓑ 4

Ⓒ 8 Ⓓ 16

7. If x and p are both greater than zero and $4x^2p^2 + xp - 33 = 0$, then what is the value of p in terms of x? **D**

Ⓐ $-\frac{3}{x}$ Ⓑ $-\frac{11}{4x}$

Ⓒ $\frac{3}{4x}$ Ⓓ $\frac{11}{4x}$

8. For all positive integers n, $\langle n \rangle = 3\sqrt{n}$. Which of the following equals 12? **C**

Ⓐ $\langle 4 \rangle$ Ⓑ $\langle 8 \rangle$

Ⓒ $\langle 16 \rangle$ Ⓓ $\langle 32 \rangle$

9. Which number is the sum of both solutions of the equation $x^2 - 3x - 18 = 0$? **C**

Ⓐ -6 Ⓑ -3

Ⓒ 3 Ⓓ 6

10. One of the roots of the polynomial $6x^2 + kx + 20 = 0$ is $-\frac{5}{2}$. What is the value of k? **C**

Ⓐ -23 Ⓑ $-\frac{4}{3}$

Ⓒ 23 Ⓓ 7

The Princeton Review Test-Taking Tip

Questions 8, 11, 13, 16, 21, and 27 Be sure to use the information that describes the variables in any standardized test item. For example, if an item says that $x > 0$, check to be sure that your solution for x is not a negative number.

The Princeton Review *Log On* for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

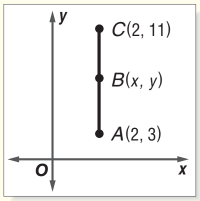
11. If n is a three-digit number that can be expressed as the product of three consecutive *even* integers, what is one possible value of n? **192, 480, or 960**

12. If x and y are *different* positive integers and $x + y = 6$, what is one possible value of $3x + 5y$? **20, 22, 26, or 28**

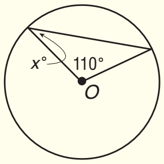
13. If a circle of radius 12 inches has its radius decreased by 6 inches, by what percent is its area decreased? **75%**

14. What is the least positive integer k for which $12k$ is the cube of an integer? **18**

15. If $AB = BC$ in the figure, what is the y-coordinate of point B? **7**



16. In the figure, if O is the center of the circle, what is the value of x? **35**



17. Let $a \blacklozenge b$ be defined as the sum of all integers greater than a and less than b. For example, $6 \blacklozenge 10 = 7 + 8 + 9$ or 25. What is the value of $(75 \blacklozenge 90) - (76 \blacklozenge 89)$? **165**

18. If $x^2 - y^2 = 42$ and $x + y = 6$, what is the value of $x - y$? **7**

19. By what amount does the sum of the roots exceed the product of the roots of the equation $(x - 7)(x + 3) = 0$? **25**

20. If $x^2 = 36$ and $y^2 = 9$, what is the greatest possible value of $(x - y)^2$? **81**

www.algebra2.com/standardized_test

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

21. $s > 0$

Column A	Column B	
s increased by 300% of s	$4s$	C

22. In $\triangle ABC$, side $\overline{AB}$ has length 8, and side $\overline{BC}$ has length 4.

the length of side $\overline{AC}$	10	D

23.

the perimeter of a rectangle with area 8 units	the perimeter of a rectangle with area 10 units	D

24.

$2^{350} - 2^{349}$	2^{349}	C

25. $t + 5 > 9$

$t + 3$	7	A

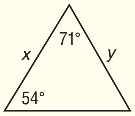
26. $x^2 + 12x + 36 = 0$

x	-5	B

27. $p > q$

$\lvert p \rvert$	$\lvert q \rvert$	D

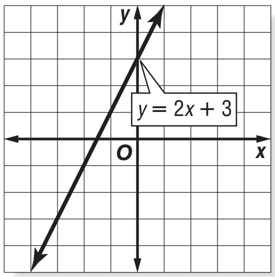
28.



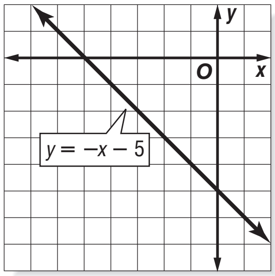
the measure of side x	the measure of side y	A

Page 284, Chapter 6 Getting Started

1.



2.



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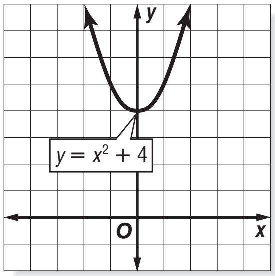

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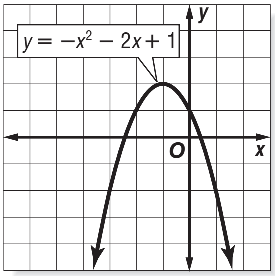


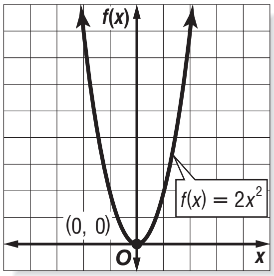
Page 291, Lesson 6-1

14a. 0; $x = 0$; 0

14b.

x	$f(x)$
-2	8
-1	2
0	0
1	2
2	8

14c.

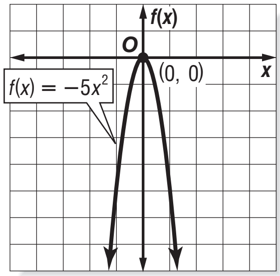


15a. 0; $x = 0$; 0

15b.

x	$f(x)$
-2	-20
-1	-5
0	0
1	-5
2	-20

15c.

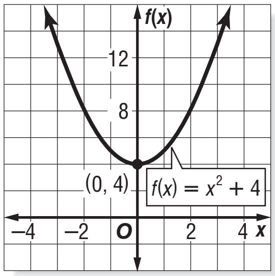


16a. 4; $x = 0$; 0

16b.

x	$f(x)$
-2	8
-1	5
0	4
1	5
2	8

16c.

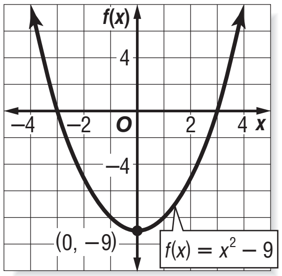


17a. -9; $x = 0$; 0

17b.

x	$f(x)$
-2	-5
-1	-8
0	-9
1	-8
2	-5

17c.

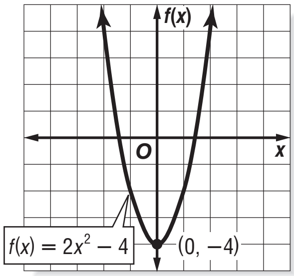


18a. -4; $x = 0$; 0

18b.

x	$f(x)$
-2	4
-1	-2
0	-4
1	-2
2	4

18c.

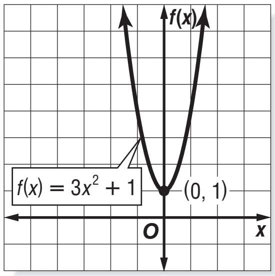


19a. 1; $x = 0$; 0

19b.

x	$f(x)$
-2	13
-1	4
0	1
1	4
2	13

19c.

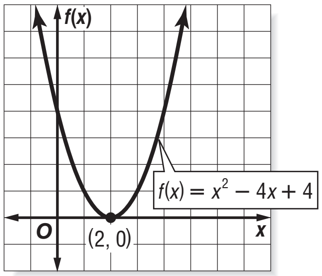


20a. 4; $x = 2$; 2

20b.

x	$f(x)$
0	4
1	1
2	0
3	1
4	4

20c.

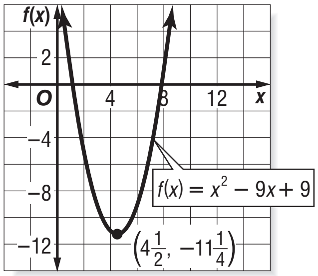


21a. 9; $x = 4.5$; 4.5

21b.

x	$f(x)$
3	-9
4	-11
4.5	-11.25
5	-11
6	-9

21c.

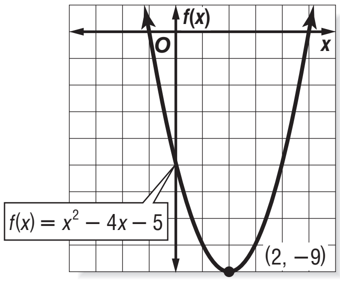


22a. -5; $x = 2$; 2

22b.

x	$f(x)$
0	-5
1	-8
2	-9
3	-8
4	-5

22c.

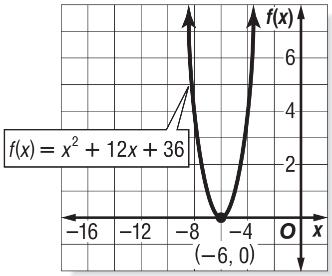


23a. 36; $x = -6$; -6

23b.

x	$f(x)$
-8	4
-7	1
-6	0
-5	1
-4	4

23c.

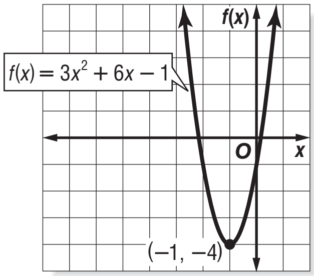


24a. -1; $x = -1$; -1

24b.

x	$f(x)$
-3	8
-2	-1
-1	-4
0	-1
1	8

24c.

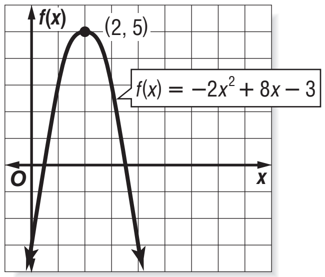


25a. -3; $x = 2$, 2

25b.

x	$f(x)$
0	-3
1	3
2	5
3	3
4	-3

25c.

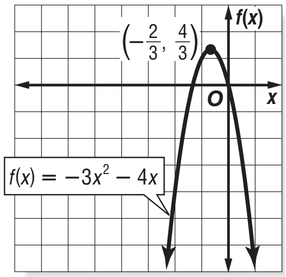


26a. 0; $x = -\frac{2}{3}$, $-\frac{2}{3}$

26b.

x	$f(x)$
-2	-4
-1	1
$-\frac{2}{3}$	$\frac{4}{3}$
0	0
1	-7

26c.

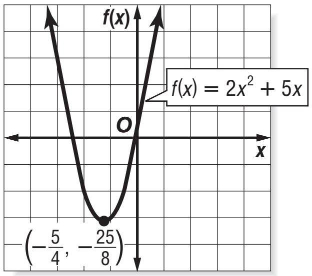


27a. 0; $x = -\frac{5}{4}$; $-\frac{5}{4}$

27b.

x	$f(x)$
-3	3
-2	-2
$-\frac{5}{4}$	$-\frac{25}{8}$
-1	-3
0	0

27c.

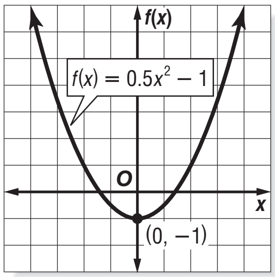


28a. -1; $x = 0$; 0

28b.

x	$f(x)$
-2	1
-1	$-\frac{1}{2}$
0	-1
1	$-\frac{1}{2}$
2	1

28c.

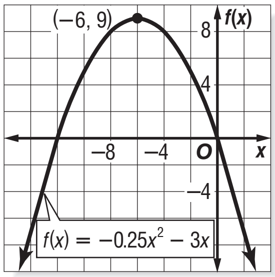


29a. 0; $x = -6$; -6

29b.

x	$f(x)$
-8	8
-7	8.75
-6	9
-5	8.75
-4	8

29c.

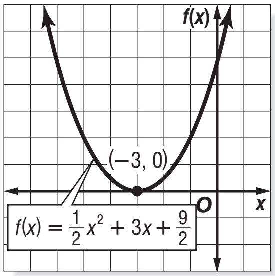


30a. $\frac{9}{2}$; $x = -3$, -3

30b.

x	$f(x)$
-5	2
-4	0.5
-3	0
-2	0.5
-1	2

30c.

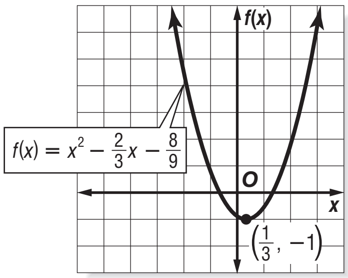


31a. $-\frac{8}{9}$; $x = \frac{1}{3}$; $\frac{1}{3}$

31b.

x	$f(x)$
-1	$\frac{7}{9}$
0	$-\frac{8}{9}$
$\frac{1}{3}$	-1
1	$-\frac{5}{9}$
2	$1\frac{7}{9}$

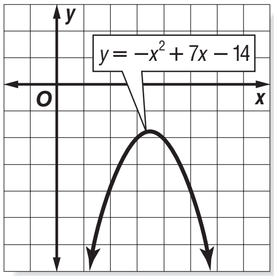
31c.



Pages 298–299, Lesson 6-2

39. Let x be the first number.
Then, $7 - x$ is the other number.

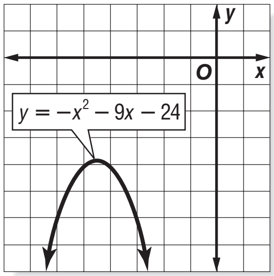
$$x(7 - x) = 14$$
$$-x^2 + 7x - 14 = 0$$



Since the graph of the related function does not intersect the x-axis, this equation has no real solutions. Therefore no such numbers exist.

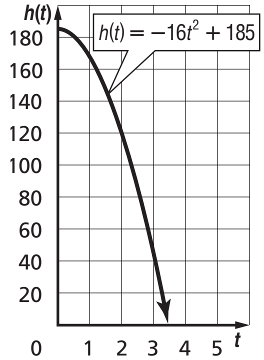
40. Let x be the first number.
Then, $-9 - x$ is the other number.

$$x(-9 - x) = 24$$
$$-x^2 - 9x - 24 = 0$$



Since the graph of the related function does not intersect the x-axis, this equation has no real solutions. Therefore no such numbers exist.

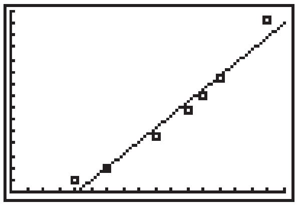
48. Answers should include the following.

- 

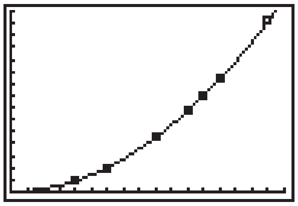
- Locate the positive x-intercept at about 3.4. This represents the time when the height of the ride is 0. Thus, if the ride were allowed to fall to the ground, it would take about 3.4 seconds.

Page 300, Follow-Up of Lesson 6-2
Graphing Calculator Investigation

1. linear: $y = 4.343x - 89.669$;
quadratic: $y = 0.044x^2 - 0.003x + 0.218$



[0, 85] scl: 5 by [0, 300] scl: 20



[0, 85] scl: 5 by [0, 300] scl: 20

The quadratic equation fits the data better.

Page 305, Lesson 6-3

50. Answers should include the following.

- Subtract 24 from each side of $x^2 + 5x = 24$ so that the equation becomes $x^2 + 5x - 24 = 0$. Factor the left side as $(x - 3)(x + 8)$. Set each factor equal to zero. Solve each equation for x. The solutions to the equation are 3 and -8. Since length cannot be negative, the width of the rectangle is 3 inches, and the length is $3 + 5$ or 8 inches.
- To use the Zero Product Property, one side of the equation must equal zero.

Pages 318–319, Lesson 6-5

28. $-2, 32$

29. $\pm i\frac{\sqrt{21}}{7}$

30. $2 \pm i\sqrt{3}$

31. $\frac{-3 \pm \sqrt{15}}{2}$

32. $\pm\sqrt{2}$

33. $\frac{9}{2}$

34. $-3 \pm i\sqrt{7}$

35. $\frac{5 \pm \sqrt{46}}{3}$

36. $4 \pm \sqrt{7}$

37. $0, -\frac{3}{10}$

38. $3 \pm 2\sqrt{2}$

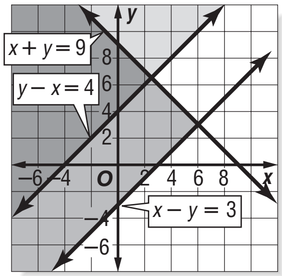
39. $-2, 6$

41. This means that the cables do not touch the floor of the bridge, since the graph does not intersect the x-axis and the roots are imaginary.

46. The person's age can be substituted for A in the appropriate formula, depending upon their gender, and their average blood pressure calculated. See student's work.

- If a woman's blood pressure is given to be 118, then solve the equation $118 = 0.01A^2 + 0.05A + 107$ to find the value of A. Use the Quadratic Formula, substituting 0.01 for a, 0.05 for b, and -11 for c. This gives solutions of about -35.8 or 30.8. Since age cannot be negative, the only valid solution for A is 30.8.

59.



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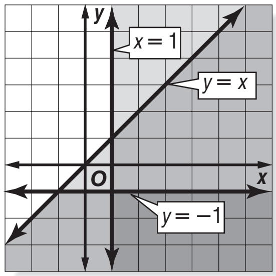
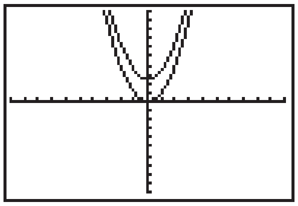
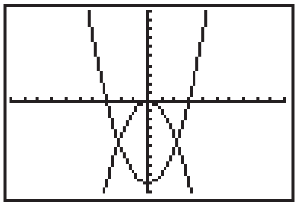


Page 321, Preview of Lesson 6-6
Graphing Calculator Investigation

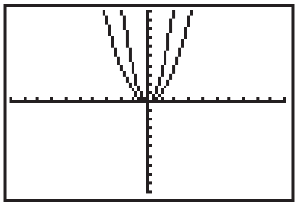
4. Both graphs have the same shape, but the graph of $y = x^2 + 2.5$ is 2.5 units above the graph of $y = x^2$.



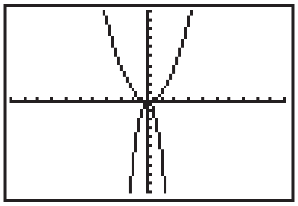
5. Both graphs have the same shape, but the graph of $y = -x^2$ opens downward while the graph of $y = x^2 - 9$ opens upward and is 9 units lower than the graph of $y = x^2$.



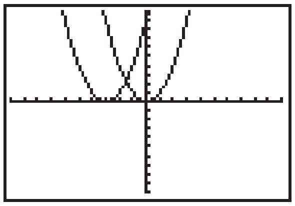
6. The graph of $y = 3x^2$ is narrower than the graph of $y = x^2$.



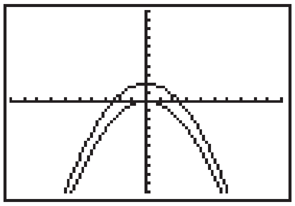
7. The graph of $y = -6x^2$ opens downward and is narrower than the graph of $y = x^2$.



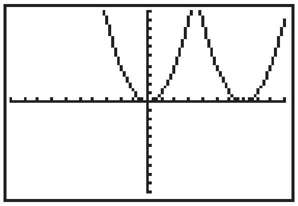
8. The graphs have the same shape, but the graph of $y = (x + 3)^2$ is 3 units to the left of the graph of $y = x^2$.



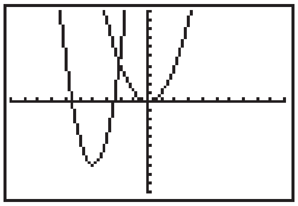
9. The graphs have the same shape and open downward, but the graph of $y = -\frac{1}{3}x^2 + 2$ is 2 units above the graph of $y = -\frac{1}{3}x^2$.



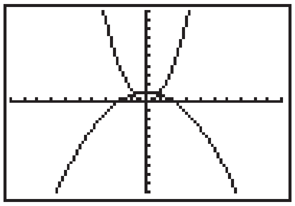
10. The graphs have the same shape, but the graph of $y = (x - 7)^2$ is 7 units to the right of the graph of $y = x^2$.



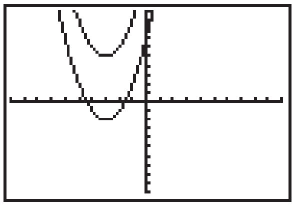
11. The graph of $y = 3(x + 4)^2 - 7$ is 4 units to the left, 7 units below, and narrower than the graph of $y = x^2$.



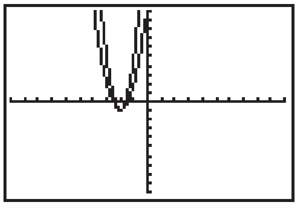
12. The graph of $y = -\frac{1}{4}x^2 + 1$ opens downward, is wider than and 1 unit above the graph of $y = -\frac{1}{4}x^2$.



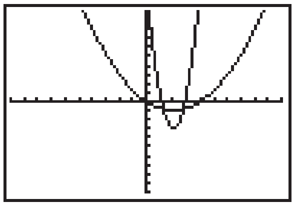
13. The graphs have the same shape, but the graph of $y = (x + 3)^2 + 5$ is 7 units above the graph of $y = (x + 3)^2 - 2$.



14. The graph of $y = 6(x + 2)^2 - 1$ is narrower than the graph of $y = 3(x + 2)^2 - 1$.

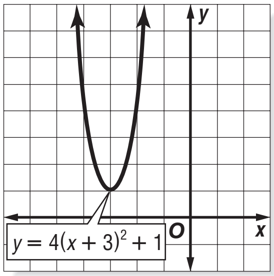


15. The graph of $y = \frac{1}{4}(x - 2)^2 - 1$ is wider than the graph of $y = 4(x - 2)^2 - 3$, and its vertex is 2 units above the vertex of $y = 4(x - 2)^2 - 3$.



Pages 326–327, Lesson 6-6

27.



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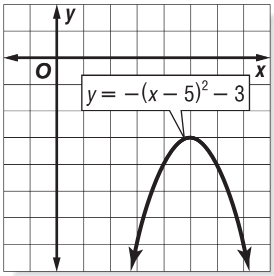

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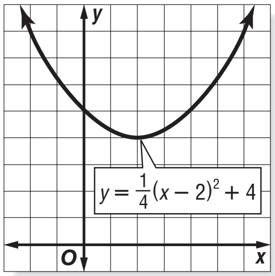

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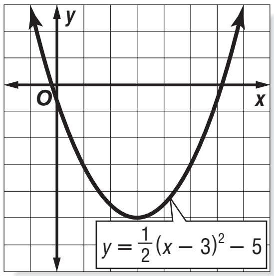

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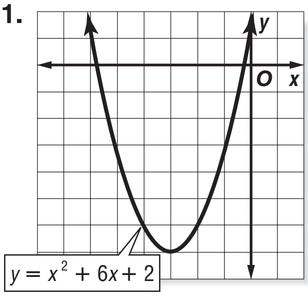

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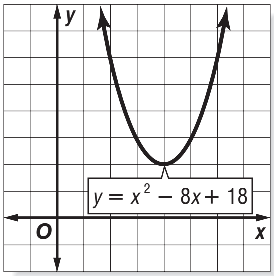

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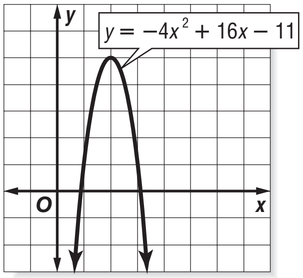

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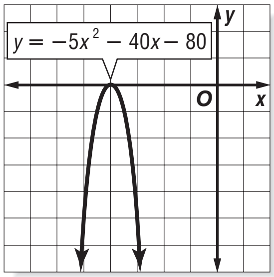

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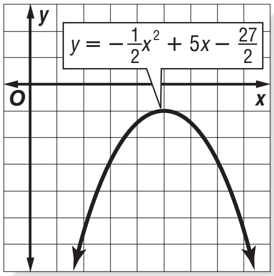

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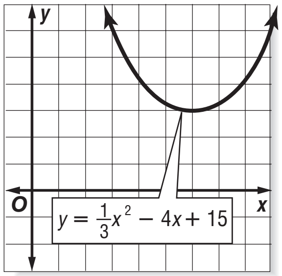

53. $y = ax^2 + bx + c$

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$

$$y = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] + c - a\left(\frac{b}{2a}\right)^2$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

The axis of symmetry is $x = h$ or $-\frac{b}{2a}$.

54. All quadratic equations are a transformation of the parent graph $y = x^2$. By identifying these transformations when a quadratic function is written in vertex form, you can redraw the graph of $y = x^2$. Answers should include the following.

- In the equation $y = a(x - h)^2 + k$, h translated the graph of $y = x^2$ h units to the right when h is positive and h units to the left when h is negative. The graph of $y = x^2$ is translated k units up when k is positive and k units down when k is negative. When a is positive, the graph opens upward and when a is negative, the graph opens downward. If the absolute value of a is less than 1, the graph will be narrower than the graph of $y = x^2$, and if the absolute value of a is greater than 1, the graph will be wider than the graph of $y = x^2$.
- Sample answer: $y = 2(x + 2)^2 - 3$ is the graph of $y = x^2$ translated 2 units left and 3 units down. The graph opens upward, but is narrower that the graph of $y = x^2$.

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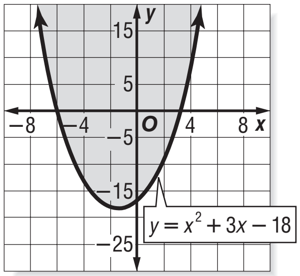
14.



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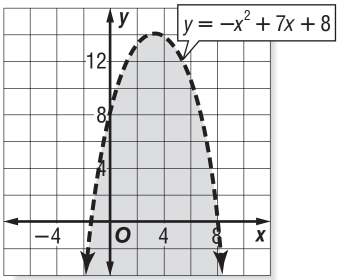

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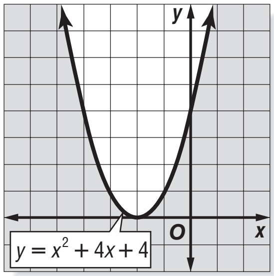

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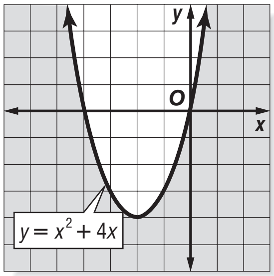

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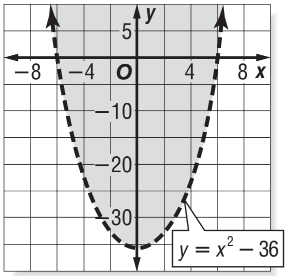

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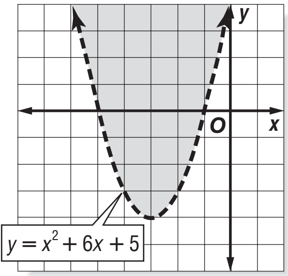

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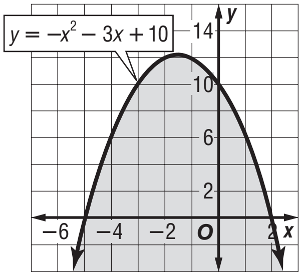

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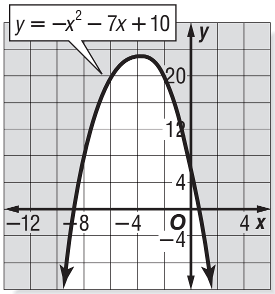

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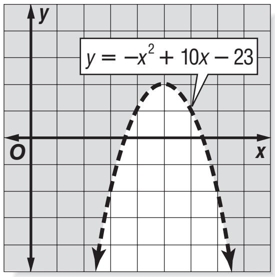

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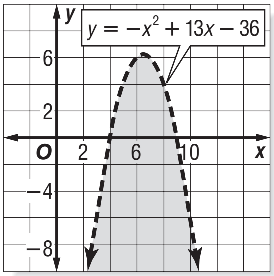

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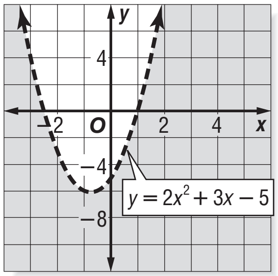

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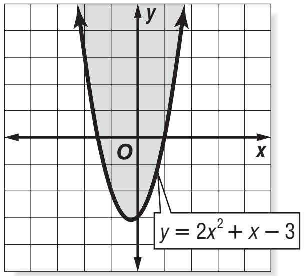


Page 340, Chapter 6 Study Guide and Review

54. $\left\{x \middle| x < -\frac{4}{3} \text{ or } x > \frac{1}{2}\right\}$

55. all reals

56. $\left\{x \middle| -4 < x < \frac{3}{2}\right\}$

57. $\left\{x \middle| x < -\frac{1}{2} \text{ or } x > 3\right\}$

58. $\left\{x \middle| \frac{-1 - \sqrt{10}}{2} \le x \le \frac{-1 + \sqrt{10}}{2}\right\}$

59. $\left\{x \middle| x < \frac{3 - 2\sqrt{6}}{3} \text{ or } x > \frac{3 + 2\sqrt{6}}{3}\right\}$

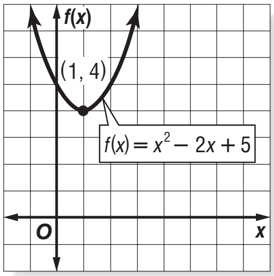
Page 341, Chapter 6 Practice Test

3a. $(0, 5)$; $x = 1$; 1

3b.

x	$f(x)$
-1	8
0	5
1	4
2	5
3	8

3c.

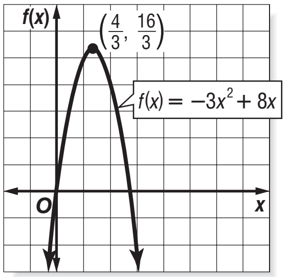


4a. $(0, 0)$; $x = \frac{4}{3}$; $\frac{4}{3}$

4b.

x	$f(x)$
0	0
1	5
$\frac{4}{3}$	$\frac{16}{3}$
2	4
3	-3

4c.

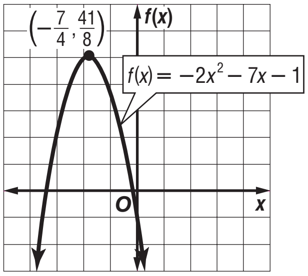


5a. $(0, -1)$; $x = -\frac{7}{4}$; $-\frac{7}{4}$

5b.

x	$f(x)$
-3	2
-2	5
$-\frac{7}{4}$	$\frac{41}{8}$
-1	4
0	-1

5c.

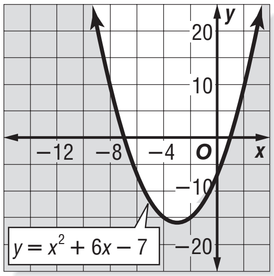


20. $(-2, -3)$; $x = -2$; up

21. $y = (x + 5)^2 + 2$; $(-5, 2)$; $x = -5$; up

22. $y = -9(x - 3)^2 + 73$; $(3, 73)$; $x = 3$; down

23.



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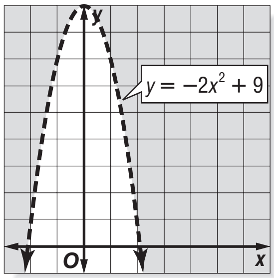

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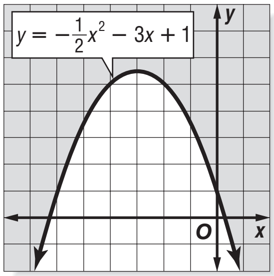

27. $\left\{x \middle| x \le -\frac{4\sqrt{3}}{3} \text{ or } x \ge \frac{4\sqrt{3}}{3}\right\}$

28. $\left\{x \middle| x < \frac{1 - \sqrt{41}}{10} \text{ or } x > \frac{1 + \sqrt{41}}{10}\right\}$