UNIT 3

Notes

Introduction

At the beginning of this unit, students are reacquainted with the Midpoint and Distance Formulas before exploring conic sections. They then learn to combine rational expressions, which leads to graphing rational functions where they examine asymptotes and holes. This knowledge of functions is applied to direct, joint, and inverse variations.

The unit concludes with an investigation of exponential and logarithmic functions. Finally, logarithms with base *e* and natural logarithms are investigated and applied to real-world situations involving investigating growth and decay.

Assessment Options

Unit 3 Test Pages 629–630 of the *Chapter 10 Resource Masters* may be used as a test or review for Unit 3. This assessment contains both multiple-choice and short answer items.

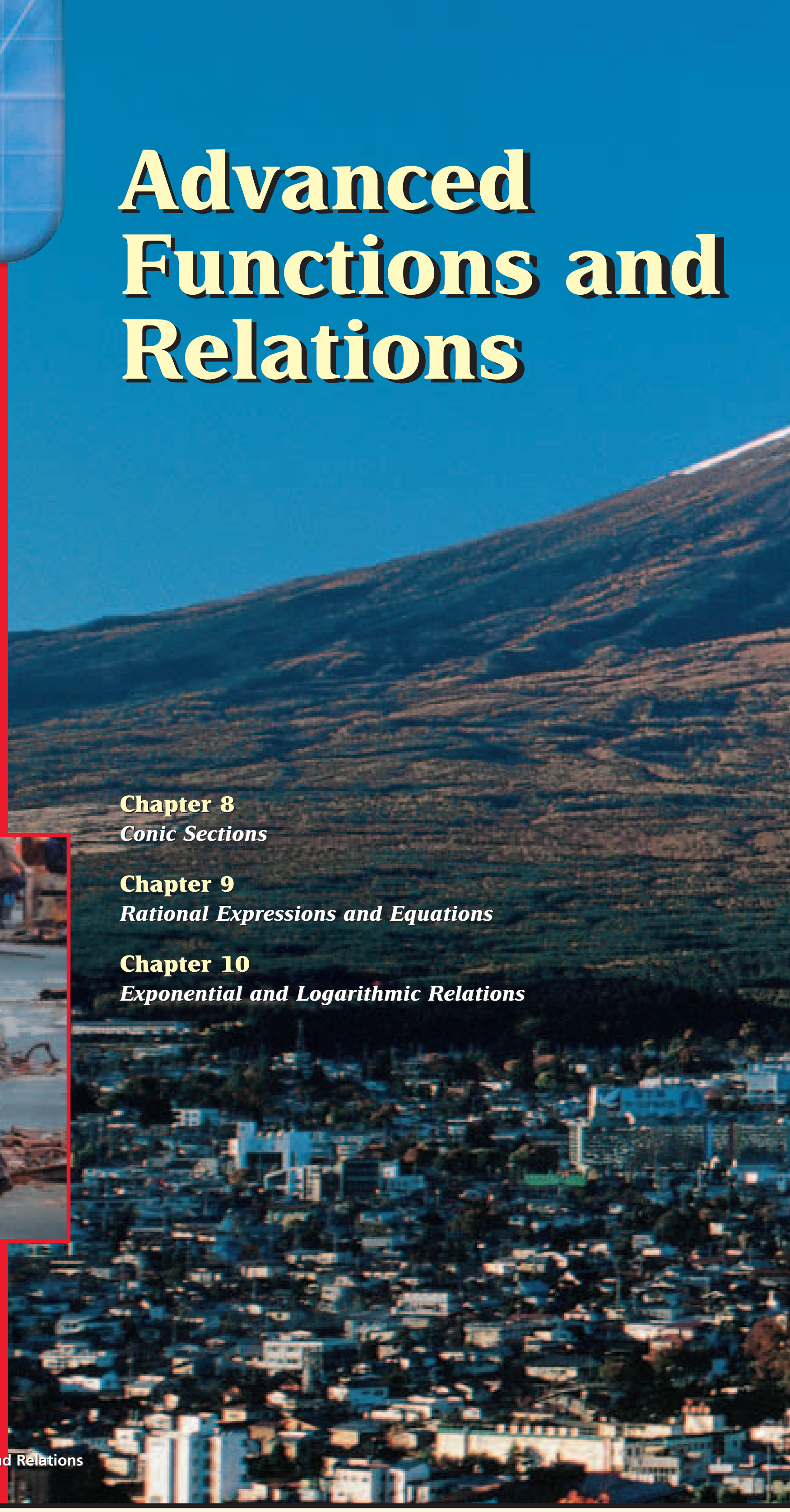
TestCheck and Worksheet Builder

This CD-ROM can be used to create additional unit tests and review worksheets.

UNIT 3

Advanced Functions and Relations

You can use functions and relations to investigate events like earthquakes. In this unit, you will learn about conic sections, rational expressions and equations, and exponential and logarithmic functions.



WebQuest Internet Project

On Quake Anniversary, Japan Still Worries

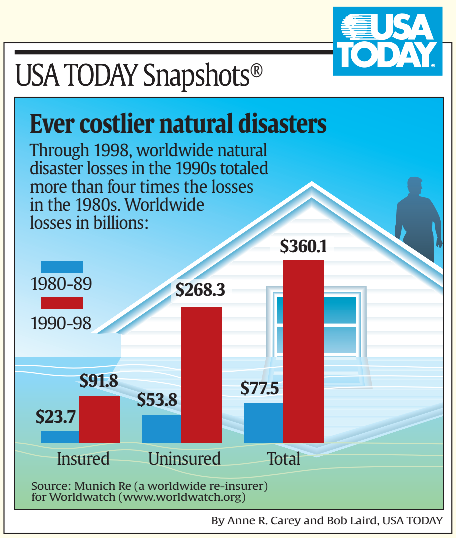
Source: *USA TODAY,* January 16, 2001

"As Japan marks the sixth anniversary of the devastating Kobe earthquake this week, a different seismic threat is worrying the country: Mount Fuji. Researchers have measured a sudden increase of small earthquakes on the volcano, indicating there is movement of magma underneath its snowcapped, nearly symmetrical cone about 65 miles from Tokyo." In this project, you will explore how functions and relations are related to locating, measuring, and classifying earthquakes.

Log on to www.algebra2.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 3.

Lesson	8-3	9-5	10-1
Page	429	502	529



USA TODAY Teaching Suggestions

Have students study the USA TODAY Snapshot®.

- Ask students to revisit the population data from page 219 of Unit 2 and make a conjecture of how it relates to the increase in natural disaster losses provided here.
- Is the ratio of the losses for 1980–89 to the losses for 1990–98 greater for insured losses or for uninsured losses? **insured**
- Ask students to suggest what types of disasters, other than earthquakes, qualify as natural disasters and why the uninsured costs have grown so much.

Additional USA TODAY Snapshots® appearing in Unit 3:

Chapter 8 Staying in touch (p. 448)

Chapter 9 College high-tech spending (p. 492)

Chapter 10 July 4th can be loud. Be careful. (p. 535)

Georgia led pecan production in 2000 (p. 565)

WebQuest Internet Project

A WebQuest is an online project in which students do research on the Internet, gather data, and make presentations using word processing, graphing, page-making, or presentation software. In each chapter, students advance to the next step in their WebQuest. At the end of Chapter 10, the project culminates with a presentation of their findings.

Teaching suggestions and sample answers are available in the *WebQuest and Project Resources.*

Chapter 8

Conic Sections

Chapter Overview and Pacing

LESSON OBJECTIVES	PACING (days) Regular Basic/Average	Regular Advanced	Block Basic/Average	Block Advanced
8-1 Midpoint and Distance Formulas *(pp. 412–418)* • Find the midpoint of a segment on the coordinate plane. • Find the distance between two points on the coordinate plane. ***Follow-Up:*** Midpoint and Distance Formulas in Three Dimensions	1	2 (with 8-1 Follow-Up)	0.5	1
8-2 Parabolas *(pp. 419–425)* • Write equations of parabolas in standard form. • Graph parabolas.	1	1	0.5	0.5
8-3 Circles *(pp. 426–431)* • Write equations of circles. • Graph circles.	1	1	0.5	0.5
8-4 Ellipses *(pp. 432–440)* ***Preview:*** Investigating Ellipses • Write equations of ellipses. • Graph ellipses.	2 (with 8-4 Preview)	2	1 (with 8-4 Preview)	1
8-5 Hyperpolas *(pp. 441–448)* • Write equations of hyperbolas. • Graph hyperbolas.	2	2	1	1
8-6 Conic Sections *(pp. 449–454)* • Write equations of conic sections in standard form. • Identify conic sections from their equations. ***Follow-Up:*** Conic Sections	1	2 (with 8-6 Follow-Up)	0.5	1
8-7 Solving Quadratic Systems *(pp. 455–460)* • Solve systems of quadratic equations algebraically and graphically. • Solve systems of quadratic inequalities graphically.	2	2	1	1
Study Guide and **Practice Test** *(pp. 461–467)* **Standardized Test Practice** *(pp. 468–469)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**12**	**14**	**6**	**7**

Pacing suggestions for the entire year can be found on pages T20–T21.

Chapter Resource Manager

Timesaving Tools
TeacherWorks™
All-In-One Planner and Resource Center
See pages T12–T13.

Chapter 8 Resource Masters

Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
455–456	457–458	459	460			8-1	8-1		(*Follow-Up:* shoe box or tissue box; wire, spaghetti, yarn, or thread)
461–462	463–464	465	466	511	GCS 42, SC 15, SM 119–122	8-2	8-2		wax paper, inch ruler, pen, posterboard, masking tape, yardsticks or meter sticks
467–468	469–470	471	472		GCS 41	8-3	8-3	15	
473–474	475–476	477	478	511, 513		8-4	8-4		(*Preview:* thumbtacks, cardboard, string, pencil, grid paper) grid paper, compass, index cards
479–480	481–482	483	484			8-5	8-5	16	posterboard
485–486	487–488	489	490	512		8-6	8-6		(*Follow-Up:* conic graph paper, concentric-circle graph paper)
491–492	493–494	495	496	512	SC 16	8-7	8-7		graphing calculator
				497–510, 514–516					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 8

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students have found midpoints of segments and distances between points on the coordinate plane. In prior lessons they worked with equations for parabolas, and in previous courses they may have worked with equations for circles. They have also used graphs and algebraic techniques to explore systems of linear equations and inequalities.

This Chapter

This chapter begins with a review of the midpoint and distance formulas. Students then explore the characteristics and equations of the conic sections. They will study the effect that each number in the standard form of the equations has on the graph of the equations. Students also graph and solve systems of quadratic equations and inequalities.

Future Connections

This study of conic sections lays the foundation for future mathematics study in coordinate geometry. In future courses, students will apply their knowledge when they study parametric equations and the polar coordinate system.

8-1 Midpoint and Distance Formulas

In this lesson, students explore the formulas relating to line segments on a coordinate plane. The coordinates of the midpoint of a line segment are the means of the corresponding coordinates of the endpoints. The distance formula is an application of the Pythagorean Theorem. These two formulas are used often in the remaining lessons in the chapter as students investigate the general forms of the equations of conic sections.

8-2 Parabolas

A parabola is the set of all points in a plane that are the same distance from a given point called the *focus* and a given line called the *directrix*. Students will use this definition with the distance formula to derive the formula $y = a(x - h)^2 + k$, which is the standard form of the equation of a parabola. Students explore how the values a, h, and k are related to the parabola's vertex, axis of symmetry, focus, and directrix, and to whether it opens up or down. The coefficient a also is associated with a segment, called the *latus rectum*, which has endpoints on the parabola, contains the focus, and is perpendicular to the axis of symmetry.

8-3 Circles

A circle is the set of points in a plane that are a given distance (the radius) from a given point (the center). Using (h, k) as the given point and r as the given distance, the equation of a circle is $\sqrt{(x - h)^2 + (y - k)^2} = r$. Squaring both sides of that equation gives the standard form of the equation, $(x - h)^2 + (y - k)^2 = r^2$.

8-4 Ellipses

In this lesson students explore an ellipse, the set of points in a plane such that the sum of the distances from each point to two fixed points is constant. The lesson uses the definition of an ellipse and the distance formula to derive the standard form of an equation for an ellipse centered at the origin, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Students find the ellipse's foci, and they examine how the values of a and b determine the length of the major and minor axes and whether the direction of the major ellipse is horizontal or vertical. Also, students examine equations that have the form $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$, exploring how such an equation represents an ellipse whose center is translated to the point (h, k).

8-5 Hyperbolas

This lesson begins by considering the distances between a general point and two fixed points, and defines a hyperbola as the set of all points in a plane for which the absolute value of the difference of those distances is constant. Then for a general point (x, y), two specific points, and a specific constant, the lesson finds the distances between (x, y) and each specific point, subtracts the distances, and equates that difference to the constant. The result is the standard form of the equation of a hyperbola centered at the origin, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Students identify the difference between the equations for ellipses and hyperbolas, and they examine the names of the parts of a hyperbola, its asymptotes $y = \pm\frac{b}{a}x$, its two branches, and whether the transverse axis is horizontal or vertical. They relate the variables a and b to the foci and vertices of the hyperbola, to the equations of the asymptotes, and to the lengths of the transverse and conjugate axes.

8-6 Conic Sections

This lesson presents the general quadratic equation for a conic section, $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, and explores how that equation is related to the standard forms of the equations of the four conic sections. In one activity, students are given specific values for the coefficients A through F and manipulate the resulting equation until it is in the standard form for one of the four conic sections. In another activity, students analyze how the different relationships between the coefficients A and C determine whether a particular equation represents a parabola, circle, ellipse, or hyperbola. The lesson also reviews why the four curves are called conic sections; that is, how to slice a double cone with a plane to illustate a parabola, circle, ellipse, or hyperbola.

8-7 Solving Quadratic Systems

This lesson shows how to use graphing techniques to find the number of solutions to a quadratic system, and then how to use algebraic techniques to find those solutions. For a linear-quadratic system, a graph indicates whether the conic section and the line intersect in 0, 1, or 2 points; then substitution can be used as the first step in writing a one-variable equation from the two-variable system. For a quadratic-quadratic system, a graph indicates the number of solutions (0, 1, 2, 3, or 4); elimination can be used to generate a one-variable equation from the two-variable system. The lesson also explores systems of quadratic inequalities. By graphing the related equations, shading the appropriate regions, deciding when boundary lines are part of a solution region, and solving related systems of equations to find specific points, the solution to a system of quadratic inequalities can be illustrated graphically and described algebraically.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Integration: Geometry/Midpoint of a Line Segment (Lesson 14)

Chapter 8

DAILY INTERVENTION and Assessment

INTERVENTION / ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 411, 416, 425, 431, 440, 448, 452 Practice Quiz 1, p. 431 Practice Quiz 2, p. 448	5-Minute Check Transparencies Quizzes, *CRM* pp. 511–512 Mid-Chapter Test, *CRM* p. 513 Study Guide and Intervention, *CRM* pp. 455–456, 461–462, 467–468, 473–474, 479–480, 485–486, 491–492	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
Mixed Review	pp. 416, 425, 431, 440, 447, 452, 460	Cumulative Review, *CRM* p. 514	
Error Analysis	Find the Error, pp. 423, 428	Find the Error, *TWE* pp. 423, 429 Unlocking Misconceptions, *TWE* pp. 420, 435, 442 Tips for New Teachers, *TWE* pp. 416, 440, 448	
Standardized Test Practice	pp. 413, 414, 416, 425, 431, 439, 440, 446, 447, 452, 459, 468–469	*TWE* p. 413 Standardized Test Practice, *CRM* pp. 515–516	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 416, 425, 430, 439, 447, 452, 459 Open Ended, pp. 414, 423, 437, 445, 450, 458	Modeling: *TWE* pp. 416, 452 Speaking: *TWE* pp. 431, 440, 460 Writing: *TWE* pp. 425, 448 Open-Ended Assessment, *CRM* p. 509	
Chapter Assessment	Study Guide, pp. 461–466 Practice Test, p. 467	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 497–502 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 503–508 Vocabulary Test/Review, *CRM* p. 510	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*
The Princeton Review's *Cracking the ACT*
ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
8-3	**15** *Graphing Parabolas and Circles*
8-5	**16** *Graphing Ellipses and Hyperbolas*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 411
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 414, 423, 428, 437, 445, 450, 458, 461)
- Writing in Math questions in every lesson, pp. 416, 425, 430, 439, 447, 452, 459
- Reading Study Tip, pp. 442, 449

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 411, 461
- Study Notebook suggestions, pp. 414, 418, 423, 429, 432, 437, 445, 450, 454, 458
- Modeling activities, pp. 416, 452
- Speaking activities, pp. 431, 440, 460
- Writing activities, pp. 425, 448
- ELL Resources, pp. 410, 415, 424, 430, 439, 447, 451, 459, 461

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 8 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 8 Resource Masters*, pp. 459, 465, 471, 477, 483, 489, 495)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 8 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
8-1	2, 3, 6, 8, 9, 10	
8-1 Follow-Up	3, 6, 8, 10	
8-2	2, 3, 6, 8, 9, 10	
8-3	2, 3, 6, 8, 9, 10	
8-4 Preview	3, 4, 7, 8	
8-4	2, 3, 6, 8, 9, 10	
8-5	2, 3, 6, 8, 9, 10	
8-6	2, 3, 6, 8, 9, 10	
8-6 Follow-Up	3, 7, 10	
8-7	2, 3, 6, 8, 9, 10	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 8 Conic Sections

What You'll Learn

- **Lesson 8-1** Use the Midpoint and Distance Formulas.
- **Lessons 8-2 through 8-5** Write and graph equations of parabolas, circles, ellipses, and hyperbolas.
- **Lesson 8-6** Identify conic sections.
- **Lesson 8-7** Solve systems of quadratic equations and inequalities.

Key Vocabulary

- **parabola** (p. 419)
- **conic section** (p. 419)
- **circle** (p. 426)
- **ellipse** (p. 433)
- **hyperbola** (p. 441)

Why It's Important

Many planets, comets, and satellites have orbits in curves called *conic sections*. These curves include parabolas, circles, ellipses, and hyperbolas. The Moon's orbit is almost a perfect circle. *You will learn more about the orbits in Lessons 8-2 through 8-7.*

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 8 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 8 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 8.

For Lessons 8-2 through 8-6 — Completing the Square

Solve each equation by completing the square. *(For review, see Lesson 6-4.)*

1. $x^2 + 10x + 24 = 0$ **−4, −6**
2. $x^2 - 2x + 2 = 0$ **1 + *i*, 1 − *i***
3. $2x^2 + 5x - 12 = 0$ **$\frac{3}{2}$, −4**

For Lessons 8-2 through 8-6 — Translation Matrices

A translation is given for each figure.

a. Write the vertex matrix for the given figure.

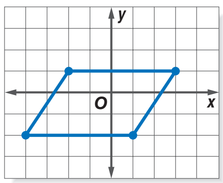
b. Write the translation matrix.

c. Find the coordinates in matrix form of the vertices of the translated figure.

(For review, see Lesson 4-4.)

4a. $\begin{bmatrix} -2 & 3 & 1 & -4 \\ 1 & 1 & -2 & -2 \end{bmatrix}$

4.

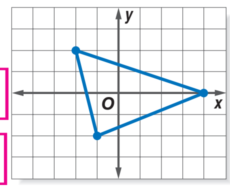


b. $\begin{bmatrix} -4 & -4 & -4 & -4 \\ 2 & 2 & 2 & 2 \end{bmatrix}$

c. $\begin{bmatrix} -6 & -1 & -3 & -8 \\ 3 & 3 & 0 & 0 \end{bmatrix}$

translated 4 units left and 2 units up

5.



a. $\begin{bmatrix} -2 & 4 & 1 \\ 2 & 0 & -2 \end{bmatrix}$

b. $\begin{bmatrix} 5 & 5 & 5 \\ -3 & -3 & -3 \end{bmatrix}$

c. $\begin{bmatrix} 3 & 9 & 4 \\ -1 & -3 & -5 \end{bmatrix}$

translated 5 units right and 3 units down

For Lesson 8-7 — Graph Linear Inequalities

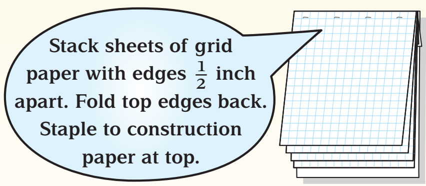
Graph each inequality. *(For review, see Lesson 2-7.)* **6–8. See margin.**

6. $y < x + 2$
7. $x + y \leq 3$
8. $2x - 3y > 6$

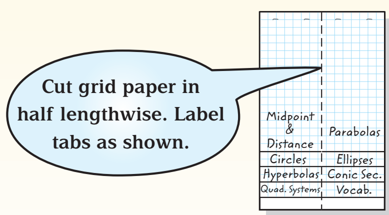
FOLDABLES™ Study Organizer

Make this Foldable to help you organize information about conic sections. Begin with four sheets of grid paper and one piece of construction paper.

Step 1 Fold and Staple



Step 2 Cut and Label



Reading and Writing As you read and study the chapter, use each tab to write notes, formulas, and examples for each conic section.

Getting Started

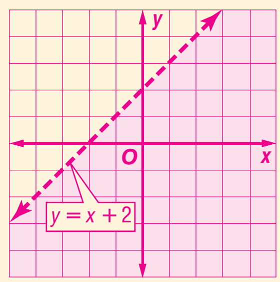
This section provides a review of the basic concepts needed before beginning Chapter 8. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

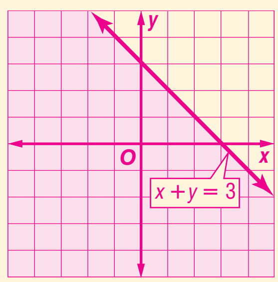
For Lesson	Prerequisite Skill
8-2	Completing the Square (p. 416)
8-3	Simplifying Radicals (p. 425)
8-4	Solving Quadratic Equations (p. 431)
8-5	Graphing Lines (p. 440)
8-6	Identifying Coefficients (p. 448)
8-7	Solving Systems of Linear Equations (p. 452)

Answers

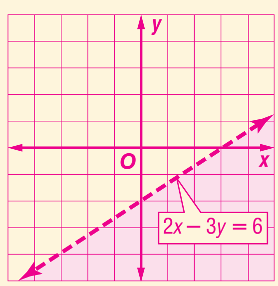
6.



7.



8.



FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Expository Writing and Organizing Data After students make their Foldables, have them label each tab to correspond to a lesson in Chapter 8. Use the extra tab for vocabulary. Students use their Foldables to take notes, define terms, record concepts, and write examples. Ask students to use their notes to write expositions on conic sections so that someone who did not know or understand conic sections before will understand them after reading what students have written. Explain that textbooks are examples of expository writing.

8-1 Lesson Notes

1 Focus

5-Minute Check Transparency 8-1 Use as a quiz or review of Chapter 7.

Mathematical Background notes are available for this lesson on p. 410C.

Building on Prior Knowledge

In Chapter 5, students simplified radical expressions. In this lesson, students will solve problems using the Pythagorean Theorem, which will require that they simplify radical expressions.

How are the Midpoint and Distance Formulas used in emergency medicine?

Ask students:

- Is an emergency in Fremont closer to Lincoln or to Omaha? **Omaha**
- Is an emergency in Wahoo closer to Lincoln or to Omaha? **They are about equally far.**
- What route would a helicopter follow to get from Fremont to Omaha? **Helicopters do not follow roads so they could fly directly from Fremont to Omaha.**

8-1 Midpoint and Distance Formulas

What You'll Learn

- Find the midpoint of a segment on the coordinate plane.
- Find the distance between two points on the coordinate plane.

How are the Midpoint and Distance Formulas used in emergency medicine?

A square grid is superimposed on a map of eastern Nebraska where emergency medical assistance by helicopter is available from both Lincoln and Omaha. Each side of a square represents 10 miles. You can use the formulas in this lesson to determine whether the site of an emergency is closer to Lincoln or to Omaha.

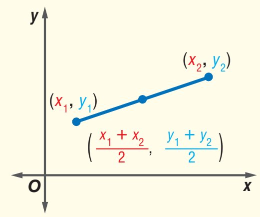
THE MIDPOINT FORMULA Recall that point M is the midpoint of segment PQ if M is between P and Q and $PM = MQ$. There is a formula for the coordinates of the midpoint of a segment in terms of the coordinates of the endpoints. *You will show that this formula is correct in Exercise 41.*

Study Tip

Midpoints
The coordinates of the midpoint are the means of the coordinates of the endpoints.

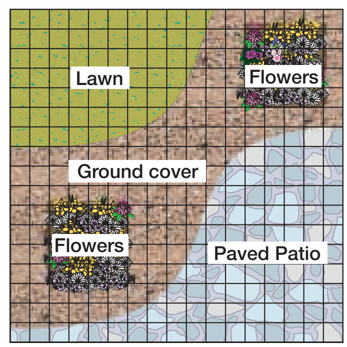
Key Concept — Midpoint Formula

- **Words** If a line segment has endpoints at (x_1, y_1) and (x_2, y_2), then the midpoint of the segment has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- **Symbols** midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- **Model**



Example 1 Find a Midpoint

LANDSCAPING A landscape design includes two square flower beds and a sprinkler halfway between them. Find the coordinates of the sprinkler if the origin is at the lower left corner of the grid.



The centers of the flower beds are at (4, 5) and (14, 13). The sprinkler will be at the midpoint of the segment joining these points.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 14}{2}, \frac{5 + 13}{2}\right)$$

$$= \left(\frac{18}{2}, \frac{18}{2}\right) \text{ or } (9, 9)$$

The sprinkler will have coordinates (9, 9).

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters
- Study Guide and Intervention, pp. 455–456
- Skills Practice, p. 457
- Practice, p. 458
- Reading to Learn Mathematics, p. 459
- Enrichment, p. 460

Transparencies

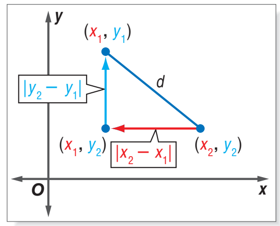
5-Minute Check Transparency 8-1
Answer Key Transparencies

Technology

Interactive Chalkboard

THE DISTANCE FORMULA Recall that the distance between two points on a number line whose coordinates are a and b is $|a - b|$ or $|b - a|$. You can use this fact and the Pythagorean Theorem to derive a formula for the distance between two points on a coordinate plane.

Suppose (x_1, y_1) and (x_2, y_2) name two points. Draw a right triangle with vertices at these points and the point (x_1, y_2). The lengths of the legs of the right triangle are $|x_2 - x_1|$ and $|y_2 - y_1|$. Let d represent the distance between (x_1, y_1) and (x_2, y_2). Now use the Pythagorean Theorem.



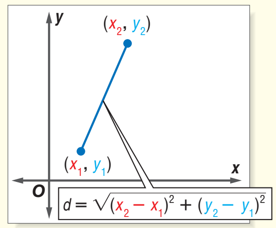
$c^2 = a^2 + b^2$	Pythagorean Theorem
$d^2 = \|x_2 - x_1\|^2 + \|y_2 - y_1\|^2$	Replace c with d, a with $\|x_2 - x_1\|$, and b with $\|y_2 - y_1\|$.
$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	$\|x_2 - x_1\|^2 = (x_2 - x_1)^2$; $\|y_2 - y_1\|^2 = (y_2 - y_1)^2$
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Find the nonnegative square root of each side.

Study Tip

Distance
In mathematics, distances are always positive.

Key Concept — Distance Formula

- **Words** The distance between two points with coordinates (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- **Model**



Example 2 Find the Distance Between Two Points

What is the distance between $A(-3, 6)$ and $B(4, -4)$?

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{[4 - (-3)]^2 + (-4 - 6)^2}$	Let $(x_1, y_1) = (-3, 6)$ and $(x_2, y_2) = (4, -4)$.
$= \sqrt{7^2 + (-10)^2}$	Subtract.
$= \sqrt{49 + 100}$ or $\sqrt{149}$	Simplify.

The distance between the points is $\sqrt{149}$ units.

Standardized Test Practice

Example 3 Find the Farthest Point

Multiple-Choice Test Item

Which point is farthest from $(-1, 3)$?

Ⓐ $(2, 4)$ Ⓑ $(-4, 1)$ Ⓒ $(0, 5)$ Ⓓ $(3, -2)$

Read the Test Item

The word *farthest* refers to the greatest distance.

(continued on the next page)

www.algebra2.com/extra_examples

2 Teach

THE MIDPOINT FORMULA

In-Class Example Power Point®

1 **COMPUTERS** A graphing program draws a line segment on a computer screen so that its ends are at (5, 2) and (7, 8). What are the coordinates of its midpoint? **(6, 5)**

Teaching Tip Have students draw a graph to check the coordinates.

THE DISTANCE FORMULA

In-Class Examples Power Point®

2 What is the distance between $P(-1, 4)$ and $Q(2, -3)$? $\sqrt{58}$

Teaching Tip Ask students if the Distance Formula can be used when the two points are both on the same vertical line. **yes**

3 Which point is farthest from $(2, -3)$? **C**

A $(0, 0)$ **B** $(3, 2)$
C $(-3, 0)$ **D** $(4, 1)$

Teaching Tip Tell students that a distance in radical form, such as $\sqrt{41}$, is an exact mathematical representation of the value. However, if they are using this value to measure the length of some real object, such as a piece of wood, they can find an approximate value with a calculator. For example, $\sqrt{41} \approx 6.4$.

Standardized Test Practice

Example 3 Point out that writing the steps in the calculations can be done very quickly, and helps to prevent careless errors in calculations with integers. This means that taking the time to write the steps is more efficient than trying to do all the calculations mentally.

Interactive Chalkboard
PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- add the Test-Taking Tip for Example 3 to their list of tips to review before a test.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **The Midpoint Formula:** 10–23
- **The Distance Formula:** 24–40

Odd/Even Assignments
Exercises 10–19 and 24–33 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercises 22–23 require the Internet.

Assignment Guide

Basic: 11, 13, 21–23, 25–31 odd, 35, 38–39, 41–44, 47–59

Average: 11–19 odd, 21–23, 25–37 odd, 38–39, 41–44, 47–59 (optional: 45, 46)

Advanced: 10–20 even, 24–36 even, 38–53 (optional: 54–59)

The Princeton Review
Test-Taking Tip
If you forget the Distance Formula, you can draw a right triangle and use the Pythagorean Theorem, as shown on the previous page.

Solve the Test Item
Use the Distance Formula to find the distance from (−1, 3) to each point.

Distance to (2, 4)
$d = \sqrt{[2-(-1)]^2 + (4-3)^2}$
$= \sqrt{3^2 + 1^2}$ or $\sqrt{10}$

Distance to (−4, 1)
$d = \sqrt{[-4-(-1)]^2 + (1-3)^2}$
$= \sqrt{(-3)^2 + (-2)^2}$ or $\sqrt{13}$

Distance to (0, 5)
$d = \sqrt{[0-(-1)]^2 + (5-3)^2}$
$= \sqrt{1^2 + 2^2}$ or $\sqrt{5}$

Distance to (3, −2)
$d = \sqrt{[3-(-1)]^2 + (-2-3)^2}$
$= \sqrt{4^2 + (-5)^2}$ or $\sqrt{41}$

The greatest distance is $\sqrt{41}$ units. So, the farthest point from (−1, 3) is (3, −2). The answer is D.

Check for Understanding

Concept Check

1. **Explain** how you can determine in which quadrant the midpoint of the segment with endpoints at (−6, 8) and (4, 3) lies without actually calculating the coordinates. **See margin.**
2. **Identify** all of the points that are equidistant from the endpoints of a given segment. **all of the points on the perpendicular bisector of the segment**
3. **OPEN ENDED** Find two points that are $\sqrt{29}$ units apart. **Sample answer: (0, 0) and (5, 2)**

GUIDED PRACTICE KEY

Exercises	Examples
4, 5	1
6–8	2, 3
9	3

Guided Practice

Find the midpoint of each line segment with endpoints at the given coordinates.

4. (−5, 6), (1, 7) $\left(-2, \frac{13}{2}\right)$
5. (8, 9), (−3, −4.5) **(2.5, 2.25)**

Find the distance between each pair of points with the given coordinates.

6. (2, −4), (10, −10) **10 units**
7. (7, 8), (−4, 9) $\sqrt{122}$ **units**
8. (0.5, 1.4), (1.1, 2.9) $\sqrt{2.61}$ **units**

Standardized Test Practice

9. Which of the following points is closest to (2, −4)? **D**
(A) (3, 1) (B) (−2, 0) (C) (1, 5) (D) (4, −2)

★ **indicates increased difficulty**

Practice and Apply

Homework Help

For Exercises	See Examples
10–23	1
24–40	2, 3

Extra Practice
See page 845.

Find the midpoint of each line segment with endpoints at the given coordinates.

10. (8, 3), (16, 7) **(12, 5)**
11. (−5, 3), (−3, −7) **(−4, −2)**
12. (6, −5), (−2, −7) **(2, −6)**
13. (5, 9), (12, 18) $\left(\frac{17}{2}, \frac{27}{2}\right)$
★ 14. (0.45, 7), (−0.3, −0.6) **(0.075, 3.2)**
★ 15. (4.3, −2.1), (1.9, 7.5) **(3.1, 2.7)**
★ 16. $\left(\frac{1}{2}, -\frac{2}{3}\right), \left(\frac{1}{3}, \frac{1}{4}\right)$ $\left(\frac{5}{12}, -\frac{5}{24}\right)$
★ 17. $\left(\frac{1}{3}, \frac{3}{4}\right), \left(-\frac{1}{4}, \frac{1}{2}\right)$ $\left(\frac{1}{24}, \frac{5}{8}\right)$

18. **GEOMETRY** Triangle *MNP* has vertices *M*(3, 5), *N*(−2, 8), and *P*(7, −4). Find the coordinates of the midpoint of each side. $\left(\frac{1}{2}, \frac{13}{2}\right), \left(\frac{5}{2}, 2\right), \left(5, \frac{1}{2}\right)$

★ 19. **GEOMETRY** Circle *Q* has a diameter $\overline{AB}$. If *A* is at (−3, −5) and the center is at (2, 3), find the coordinates of *B*. **(7, 11)**

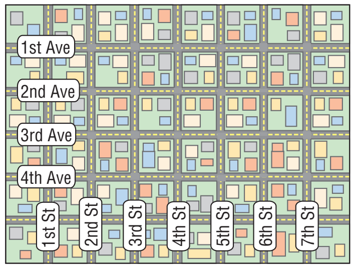
Answer

1. Since the sum of the *x*-coordinates of the given points is negative, the *x*-coordinate of the midpoint is negative. Since the sum of the *y*-coordinates of the given points is positive, the *y*-coordinate of the midpoint is positive. Therefore, the midpoint is in Quadrant II.

DAILY INTERVENTION — Differentiated Instruction

Visual/Spatial Learners Encourage students to relate coordinates to models and drawings of the lines and figures. Suggest that they construct and demonstrate models of some of the exercises to help other students who might have problems visualizing.

20. REAL ESTATE In John's town, the numbered streets and avenues form a grid. He belongs to a gym at the corner of 12th Street and 15th Avenue, and the deli where he works is at the corner of 4th Street and 5th Avenue. He wants to rent an apartment halfway between the two. In what area should he look? **around 8th Street and 10th Avenue**



GEOGRAPHY **For Exercises 21–23, use the following information.**
The U.S. Geological Survey (USGS) has determined the official center of the continental United States.

21. Describe a method that might be used to approximate the geographical center of the continental United States. **See pp. 469A–469J.**

22. RESEARCH Use the Internet or other reference to look up the USGS geographical center of the continental United States. **near Lebanon, KS**

23. How does the location given by USGS compare to the result of your method? **See students' work.**

Find the distance between each pair of points with the given coordinates.

24. $(-4, 9), (1, -3)$ **13 units**

25. $(1, -14), (-6, 10)$ **25 units**

26. $(-4, -10), (-3, -11)$ **$\sqrt{2}$ units**

27. $(9, -2), (12, -14)$ **$3\sqrt{17}$ units**

28. $(0.23, 0.4), (0.68, -0.2)$ **0.75 unit**

29. $(2.3, -1.2), (-4.5, 3.7)$ **$\sqrt{70.25}$ units**

30. $\left(-3, -\frac{2}{11}\right), \left(5, \frac{9}{11}\right)$ **$\sqrt{65}$ units**

31. $\left(0, \frac{1}{5}\right), \left(\frac{3}{5}, -\frac{3}{5}\right)$ **1 unit**

★ **32.** $(2\sqrt{3}, -5), (-3\sqrt{3}, 9)$ **$\sqrt{271}$ units**

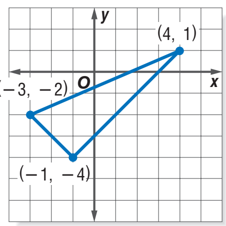
★ **33.** $\left(\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{4}\right), \left(-\frac{2\sqrt{3}}{3}, \frac{\sqrt{5}}{2}\right)$

33. $\frac{\sqrt{813}}{12}$ units

34. GEOMETRY A circle has a radius with endpoints at (2, 5) and (−1, −4). Find the circumference and area of the circle. **$6\sqrt{10}\pi$ units, 90π units^2**

35. GEOMETRY Find the perimeter and area of the triangle shown at the right.

35. $7\sqrt{2} + \sqrt{58}$ units, 10 units^2



★ **36. GEOMETRY** Quadrilateral *RSTV* has vertices *R*(−4, 6), *S*(4, 5), *T*(6, 3), and *V*(5, −8). Find the perimeter of the quadrilateral.

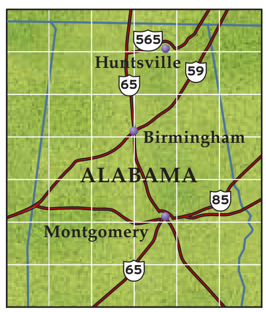
36. $\sqrt{65} + 2\sqrt{2} + \sqrt{122} + \sqrt{277}$ units

★ **37. GEOMETRY** Triangle *CAT* has vertices *C*(4, 9), *A*(8, −9), and *T*(−6, 5). *M* is the midpoint of $\overline{TA}$. Find the length of median $\overline{CM}$. (*Hint*: A median connects a vertex of a triangle to the midpoint of the opposite side.) **$\sqrt{130}$ units**

TRAVEL **For Exercises 38 and 39, use the figure at the right, where a grid is superimposed on a map of a portion of the state of Alabama.**

38. About how far is it from Birmingham to Montgomery if each unit on the grid represents 40 miles? **about 85 mi**

39. How long would it take a plane to fly from Huntsville to Montgomery if its average speed is 180 miles per hour? **about 0.9 h**



www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 455 (shown) and p. 456

The Midpoint Formula

Midpoint Formula	The midpoint *M* of a segment with endpoints (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1 **Find the midpoint of the line segment with endpoints at (4, −7) and (−2, 3).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-2)}{2}, \frac{-7 + 3}{2}\right) = \left(\frac{2}{2}, \frac{-4}{2}\right) \text{ or } (1, -2)$$

The midpoint of the segment is (1, −2).

Example 2 **A diameter $\overline{AB}$ of a circle has endpoints *A*(5, −11) and *B*(−7, 6). What are the coordinates of the center of the circle?**

The center of the circle is the midpoint of all of its diameters.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + (-7)}{2}, \frac{-11 + 6}{2}\right) = \left(\frac{-2}{2}, \frac{-5}{2}\right) \text{ or } \left(-1, -2\frac{1}{2}\right)$$

The circle has center $\left(-1, -2\frac{1}{2}\right)$.

Exercises

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (12, 7) and (−2, 11) **(5, 9)**

2. (−8, −3) and (10, 9) **(1, 3)**

3. (4, 15) and (10, 1) **(7, 8)**

4. (−3, −3) and (3, 3) **(0, 0)**

5. (15, 6) and (12, 14) **(13.5, 10)**

6. (22, −8) and (−10, 6) **(6, −1)**

7. (3, 5) and (−6, 11) **$\left(-\frac{3}{2}, 8\right)$**

8. (8, −15) and (−7, 13) **$\left(\frac{1}{2}, -1\right)$**

9. (2.5, −6.1) and (7.9, 13.7) **(5.2, 3.8)**

10. (−7, −6) and (−1, 24) **(−4, 9)**

11. (3, −10) and (30, −20) **$\left(\frac{33}{2}, -15\right)$**

12. (−9, 1.7) and (−11, 1.3) **(−10, 1.5)**

13. Segment $\overline{MN}$ has midpoint *P*. If *M* has coordinates (14, −3) and *P* has coordinates (−8, 6), what are the coordinates of *N*? **(−30, 15)**

14. Circle *R* has a diameter $\overline{ST}$. If *R* has coordinates (−4, −8) and *S* has coordinates (1, 4), what are the coordinates of *T*? **(−9, −20)**

15. Segment $\overline{AD}$ has midpoint *B*, and $\overline{BD}$ has midpoint *C*. If *A* has coordinates (−5, 4) and *C* has coordinates (10, 11), what are the coordinates of *B* and *D*?
***B* is $\left(5, 8\frac{2}{3}\right)$, *D* is $\left(15, 13\frac{1}{3}\right)$.**

Skills Practice, p. 457 and Practice, p. 458 (shown)

Find the midpoint of each line segment with endpoints at the given coordinates.

1. (8, −3), (−6, −11) **(1, −7)**

2. (−14, 5), (10, 6) **$\left(-2, \frac{11}{2}\right)$**

3. (−7, −6), (1, −2) **(−3, −4)**

4. (8, −2), (8, −8) **(8, −5)**

5. (9, −4), (1, −1) **$\left(5, -\frac{5}{2}\right)$**

6. (3, 3), (4, 9) **$\left(\frac{7}{2}, 6\right)$**

7. (4, −2), (3, −7) **$\left(\frac{7}{2}, -\frac{9}{2}\right)$**

8. (6, 7), (4, 4) **$\left(5, \frac{11}{2}\right)$**

9. (−4, −2), (−8, 2) **(−6, 0)**

10. (5, −2), (3, 7) **$\left(4, \frac{5}{2}\right)$**

11. (−6, 3), (−5, −7) **$\left(-\frac{11}{2}, -2\right)$**

12. (−9, −8), (8, 3) **$\left(-\frac{1}{2}, -\frac{5}{2}\right)$**

13. (2.6, −4.7), (8.4, 2.5) **(5.5, −1.1)**

14. $\left(-\frac{1}{3}, 6\right), \left(\frac{2}{3}, 4\right)$ **$\left(\frac{1}{6}, 5\right)$**

15. (−2.5, −4.2), (8.1, 4.2) **(2.8, 0)**

16. $\left(\frac{1}{8}, \frac{1}{2}\right), \left(-\frac{5}{8}, -\frac{1}{2}\right)$ **$\left(-\frac{1}{4}, 0\right)$**

Find the distance between each pair of points with the given coordinates.

17. (5, 2), (2, −2) **5 units**

18. (−2, −4), (4, 4) **10 units**

19. (−3, 8), (−1, −5) **$\sqrt{173}$ units**

20. (0, 1), (9, −6) **$\sqrt{130}$ units**

21. (−5, 6), (−6, 6) **1 unit**

22. (−3, 5), (12, −3) **17 units**

23. (−2, −3), (9, 3) **$\sqrt{157}$ units**

24. (−9, −8), (−7, 8) **$2\sqrt{65}$ units**

25. (9, 3), (9, −2) **5 units**

26. (−1, −7), (0, 6) **$\sqrt{170}$ units**

27. (10, −3), (−2, −8) **13 units**

28. (−0.5, −6), (1.5, 0) **$2\sqrt{10}$ units**

29. $\left(\frac{2}{5}, \frac{3}{5}\right), \left(1, \frac{7}{5}\right)$ **1 unit**

30. $(-4\sqrt{2}, -\sqrt{5}), (-5\sqrt{2}, 4\sqrt{5})$ **$\sqrt{127}$ units**

31. GEOMETRY Circle *O* has a diameter $\overline{AB}$. If *A* is at (−6, −2) and *B* is at (−3, 4), find the center of the circle and the length of its diameter. **$\left(-\frac{9}{2}, 1\right)$; $3\sqrt{5}$ units**

32. GEOMETRY Find the perimeter of a triangle with vertices at (1, −3), (−4, 9), and (−2, 1). **$18 + 2\sqrt{17}$ units**

Reading to Learn Mathematics, p. 459

ELL

Pre-Activity **How are the Midpoint and Distance Formulas used in emergency medicine?**

Read the introduction to Lesson 8-1 at the top of page 412 in your textbook.

How do you find distances on a road map?

Sample answer: Use the scale of miles on the map. You might also use a ruler.

Reading the Lesson

1. a. Write the coordinates of the midpoint of a segment with endpoints (x_1, y_1) and (x_2, y_2). **$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$**

b. Explain how to find the midpoint of a segment if you know the coordinates of the endpoints. Do not use subscripts in your explanation.

Sample answer: To find the *x*-coordinate of the midpoint, add the *x*-coordinates of the endpoints and divide by two. To find the *y*-coordinate of the midpoint, do the same with the *y*-coordinates of the endpoints.

2. a. Write an expression for the distance between two points with coordinates (x_1, y_1) and (x_2, y_2). **$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$**

b. Explain how to find the distance between two points. Do not use subscripts in your explanation.

Sample answer: Find the difference between the *x*-coordinates and square it. Find the difference between the *y*-coordinates and square it. Add the squares. Then find the square root of the sum.

3. Consider the segment connecting the points (−3, 5) and (9, 11).

a. Find the midpoint of this segment. **(3, 8)**

b. Find the length of the segment. Write your answer in simplified radical form. **$6\sqrt{5}$**

Helping You Remember

4. How can the "mid" in *midpoint* help you remember the midpoint formula?

Sample answer: The *midpoint* is the point in the *middle* of a segment. It is halfway between the endpoints. The coordinates of the midpoint are found by finding the average of the two *x*-coordinates (add them and divide by 2) and the average of the two *y*-coordinates.

Enrichment, p. 460

Quadratic Form

Consider two methods for solving the following equation.

$$(y - 2)^2 - 5(y - 2) + 6 = 0$$

One way to solve the equation is to simplify first, then use factoring.

$$y^2 - 4y + 4 - 5y + 10 + 6 = 0$$
$$y^2 - 9y + 20 = 0$$
$$(y - 4)(y - 5) = 0$$

Thus, the solution set is {4, 5}.

Another way to solve the equation is first to replace $y - 2$ by a single variable. This will produce an equation that is easier to solve than the original equation. Let $t = y - 2$ and then solve the new equation.

$$(y - 2)^2 - 5(y - 2) + 6 = 0$$
$$t^2 - 5t + 6 = 0$$
$$(\ldots t - 3) = 0$$

4 Assess

Open-Ended Assessment

Modeling Have students plot two points on a coordinate grid and explain how to find the distance between them and the coordinates of the midpoint of the segment joining them.

Tips for New Teachers

Intervention Some students may have difficulty understanding and manipulating radicals. Take time to clear up any problems in this area before going on.

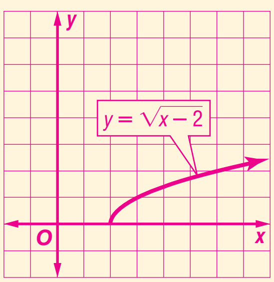
Getting Ready for Lesson 8-2

PREREQUISITE SKILL Students will analyze equations of parabolas in Lesson 8-2. When writing equations for parabolas into standard form, students frequently need to complete the square. Use Exercises 54–59 to determine your students' familiarity with completing squares.

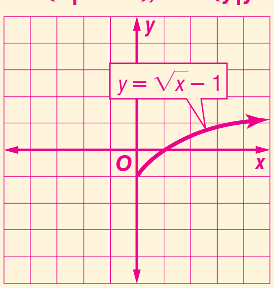
Answers

46. -1; $\overleftrightarrow{AA'}$ is perpendicular to the line with equation $y = x$, which has slope 1.

47. $D = \{x \mid x \geq 2\}$, $R = \{y \mid y \geq 0\}$



48. $D = \{x \mid x \geq 0\}$, $R = \{y \mid y \geq -1\}$



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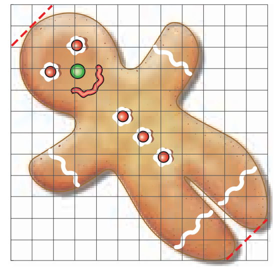
40. **WOODWORKING** A stage crew is making the set for a children's play. They want to make some gingerbread shapes out of some leftover squares of wood with sides measuring 1 foot. They can make taller shapes by cutting them out of the wood diagonally. To the nearest inch, how tall is the gingerbread shape in the drawing at the right? **14 in.**



41. **CRITICAL THINKING** Verify the Midpoint Formula. (*Hint*: You must show that the formula gives the coordinates of a point on the line through the given endpoints and that the point is equidistant from the endpoints.) **See pp. 469A–469J.**

42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 469A–469J.**

How are the Midpoint and Distance Formulas used in emergency medicine?

Include the following in your answer:
- a few sentences explaining how to use the Distance Formula to approximate the distance between two cities on a map, and
- which city, Lincoln or Omaha, an emergency medical helicopter should be dispatched from to pick up a patient in Fremont.

Standardized Test Practice

43. What is the distance between the points $A(4, -2)$ and $B(-4, -8)$? **C**

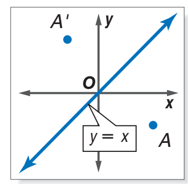
Ⓐ 6 Ⓑ 8 Ⓒ 10 Ⓓ 14

44. Point $D(5, -1)$ is the midpoint of segment $\overline{CE}$. If point C has coordinates $(3, 2)$, then what are the coordinates of point E?

Ⓐ $(8, 1)$ Ⓑ $(7, -4)$ Ⓒ $(2, -3)$ Ⓓ $\left(4, \frac{1}{2}\right)$ **B**

Extending the Lesson

For Exercises 45 and 46, use the following information.
You can use midpoints and slope to describe some transformations. Suppose point A' is the image when point A is reflected over the line with equation $y = x$.



45. Where is the midpoint of $\overline{AA'}$? **45. on the line with equation $y = x$**

46. What is the slope of $\overleftrightarrow{AA'}$? Explain. **See margin.**

Maintain Your Skills

Mixed Review

Graph each function. State the domain and range. *(Lesson 7-9)* **47–49. See margin.**

47. $y = \sqrt{x-2}$ 48. $y = \sqrt{x} - 1$ 49. $y = 2\sqrt{x} + 1$

50. Determine whether the functions $f(x) = x - 2$ and $g(x) = 2x$ are inverse functions. *(Lesson 7-8)* **no**

Simplify. *(Lesson 5-9)*

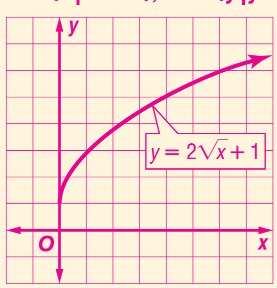
51. $(2 + 4i) + (-3 + 9i)$ **$-1 + 13i$**
52. $(4 - i) - (-2 + i)$ **$6 - 2i$**
53. $(1 - 2i)(2 + i)$ **$4 - 3i$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write each equation in the form $y = a(x - h)^2 + k$.**
(To review ***completing the square****, see Lesson 6-4.)* **54–59. See margin.**

54. $y = x^2 + 6x + 9$ 55. $y = x^2 - 4x + 1$ 56. $y = 2x^2 + 20x + 50$
57. $y = 3x^2 - 6x + 5$ 58. $y = -x^2 - 4x + 6$ 59. $y = -3x^2 - 18x - 10$

49. $D = \{x \mid x \geq 0\}$, $R = \{y \mid y \geq 1\}$



54. $y = (x + 3)^2$
55. $y = (x - 2)^2 - 3$
56. $y = 2(x + 5)^2$
57. $y = 3(x - 1)^2 + 2$
58. $y = -(x + 2)^2 + 10$
59. $y = -3(x + 3)^2 + 17$

Algebra Activity

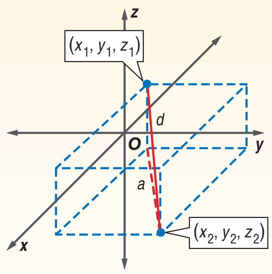
A Follow-Up of Lesson 8-1

Midpoint and Distance Formulas in Three Dimensions

You can derive a formula for distance in three-dimensional space. It may seem that the formula would involve a cube root, but it actually involves a square root, similar to the formula in two dimensions.

Suppose (x_1, y_1, z_1) and (x_2, y_2, z_2) name two points in space. Draw the rectangular box that has opposite vertices at these points. The dimensions of the box are $|x_2 - x_1|$, $|y_2 - y_1|$, and $|z_2 - z_1|$. Let a be the length of a diagonal of the bottom of the box. By the Pythagorean Theorem, $a_2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$.

To find the distance d between (x_1, y_1, z_1) and (x_2, y_2, z_2), apply the Pythagorean Theorem to the right triangle whose legs are a diagonal of the bottom of the box and a vertical edge of the box.



$d^2 = a^2 + |z_2 - z_1|^2$ Pythagorean Theorem

$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 + |z_2 - z_1|^2$ $a_2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$ $|x_2 - x_1|^2 = (x_2 - x_1)^2$, and so on

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ Take the square root of each side.

The distance d between the points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Example 1

Find the distance between (2, 0, −3) and (4, 2, 9).

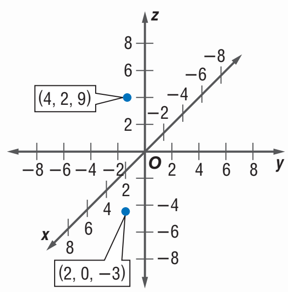
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ Distance Formula

$= \sqrt{(4 - 2)^2 + (2 - 0)^2 + [9 - (-3)]^2}$ $(x_1, y_1, z_1) = (2, 0, -3)$, $(x_2, y_2, z_2) = (4, 2, 9)$

$= \sqrt{2^2 + 2^2 + 12^2}$

$= \sqrt{152}$ or $2\sqrt{38}$

The distance is $2\sqrt{38}$ or about 12.33 units.



In three dimensions, the midpoint of the segment with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$. Notice how similar this is to the Midpoint Formula in two dimensions.

(continued on the next page)

Algebra Activity

A Follow-Up of Lesson 8-1

Getting Started

Objective To derive a formula for finding the distance between two points in three-dimensional space.

Materials
none

Teach

- To help students visualize this activity, use a box, such as a shoe box or tissue box, and mark points using the corners. Model the line between points with a piece of wire, dry spaghetti, yarn, or thread.
- Make sure that all students understand and pay attention to the difference between subscripts and superscripts.
- Ask students to explain how $\sqrt{152}$ simplifies to $2\sqrt{38}$.

Resource Manager

Teaching Algebra with Manipulatives

- p. 263 (student recording sheet)

Assess

In **Exercises 1–17**, students should

- find the distance between two points in a three-dimensional coordinate plane.
- find the midpoint of the segment between two points in a three-dimensional coordinate plane.
- use the midpoint and distance formulas to solve problems in a three-dimensional coordinate plane.

In **Exercises 12–15**, if students solve these problems correctly, they will have demonstrated their ability to use points in a three-dimensional coordinate plane.

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

Answer

11. The distance between the points with coordinates (2, −4, 2) and (3, 1, 5) is $\sqrt{35}$ units. The distance between the points with coordinates (2, −4, 2) and (6, −3, −1) is $\sqrt{26}$ units. The distance between the points with coordinates (3, 1, 5) and (6, −3, −1) is $\sqrt{61}$ units. Since $(\sqrt{61})^2 = (\sqrt{35})^2 + (\sqrt{26})^2$, the triangle is a right triangle.

Algebra Activity

Example 2

Find the coordinates of the midpoint of the segment with endpoints (6, −5, 1) and (−2, 4, 0).

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right) = \left(\frac{6 + (-2)}{2}, \frac{-5 + 4}{2}, \frac{1 + 0}{2}\right) \quad (x_1, y_1, z_1) = (6, -5, 1),\ (x_2, y_2, z_2) = (-2, 4, 0)$$

$$= \left(\frac{4}{2}, \frac{-1}{2}, \frac{1}{2}\right) \quad \text{Add.}$$

$$= \left(2, -\frac{1}{2}, \frac{1}{2}\right) \quad \text{Simplify.}$$

The midpoint has coordinates $\left(2, -\frac{1}{2}, \frac{1}{2}\right)$.

Exercises

Find the distance between each pair of points with the given coordinates.

1. (2, 4, 5), (1, 2, 3) **3 units**
2. (−1, 6, 2), (4, −3, 0) **$\sqrt{110}$ units**
3. (−2, 1, 7), (−2, 6, −3) **$5\sqrt{5}$ units**
4. (0, 7, −1), (−4, 1, 3) **$2\sqrt{17}$ units**

Find the midpoint of each line segment with endpoints at the given coordinates.

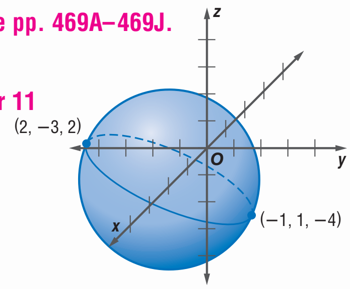
5. (2, 6, −1), (−4, 8, 5) **(−1, 7, 2)**
6. (4, −3, 2), (−2, 7, 6) **(1, 2, 4)**
7. (1, 3, 7), (−4, 2, −1) **$\left(-\frac{3}{2}, \frac{5}{2}, 3\right)$**
8. (2.3, −1.7, 0.6), (−2.7, 3.1, 1.8) **(−0.2, 0.7, 1.2)**
9. The coordinates of one endpoint of a segment are (4, −2, 3), and the coordinates of the midpoint are (3, 2, 5). Find the coordinates of the other endpoint. **(2, 6, 7)**
10. Two of the opposite vertices of a rectangular solid are at (4, 1, −1) and (2, 3, 5). Find the coordinates of the other six vertices. **(2, 3, −1), (4, 3, −1), (2, 1, −1), (4, 3, 5), (4, 1, 5), (2, 1, 5)**
11. Determine whether a triangle with vertices at (2, −4, 2), (3, 1, 5), and (6, −3, −1) is a right triangle. Explain. **Yes; see margin for explanation.**

The vertices of a rectangular solid are at (−2, 3, 2), (3, 3, 2), (3, 1, 2), (−2, 1, 2), (−2, 3, 6), (3, 3, 6), (3, 1, 6), and (−2, 1, 6).

12. Find the volume of the solid. **40 units^3**
13. Find the length of a diagonal of the solid. **$3\sqrt{5}$ units**
14. Show that the point with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ is equidistant from the points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2). **See pp. 469A–469J.**
15. Find the value of c so that the point with coordinates (2, 3, c) is $3\sqrt{6}$ units from the point with coordinates (−1, 0, 5). **−1 or 11**

The endpoints of a diameter of a sphere are at (2, −3, 2) and (−1, 1, −4).

16. Find the length of a radius of the sphere. **$\frac{\sqrt{61}}{2}$**
17. Find the coordinates of the center of the sphere. **$\left(\frac{1}{2}, -1, -1\right)$**



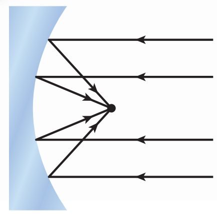
8-2 Parabolas

What You'll Learn

- Write equations of parabolas in standard form.
- Graph parabolas.

How are parabolas used in manufacturing?

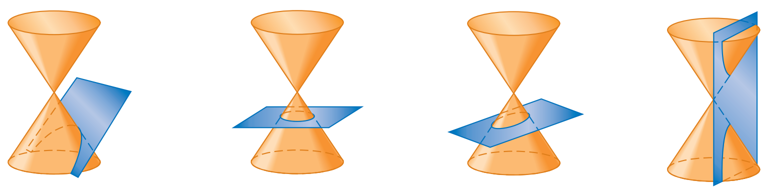
A mirror or other reflective object in the shape of a parabola has the property that parallel incoming rays are all reflected to the same point. Or, if that point is the source of rays, the rays become parallel when they are reflected.



Vocabulary

- parabola
- conic section
- focus
- directrix
- latus rectum

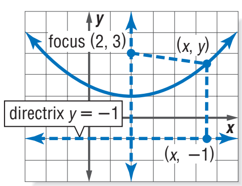
EQUATIONS OF PARABOLAS In Chapter 6, you learned that the graph of an equation of the form $y = ax^2 + bx + c$ is a **parabola**. A parabola can also be obtained by slicing a double cone on a slant as shown below on the left. Any figure that can be obtained by slicing a double cone is called a **conic section**. Other conic sections are also shown below.



Study Tip

Focus of a Parabola
The focus is the special point referred to at the beginning of the lesson.

A parabola can also be defined as the set of all points in a plane that are the same distance from a given point called the **focus** and a given line called the **directrix**. The parabola at the right has its focus at (2, 3), and the equation of its directrix is $y = -1$. You can use the Distance Formula to find an equation of this parabola.



Let (x, y) be any point on this parabola. The distance from this point to the focus must be the same as the distance from this point to the directrix. The distance from a point to a line is measured along the perpendicular from the point to the line.

distance from (x, y) to $(2, 3)$ = distance from (x, y) to $(x, -1)$

$$\sqrt{(x-2)^2 + (y-3)^2} = \sqrt{(x-x)^2 + [y-(-1)]^2}$$

$$(x-2)^2 + (y-3)^2 = 0^2 + (y+1)^2 \quad \text{Square each side.}$$

$$(x-2)^2 + y^2 - 6y + 9 = y^2 + 2y + 1 \quad \text{Square } y - 3 \text{ and } y + 1.$$

$$(x-2)^2 + 8 = 8y \quad \text{Isolate the } y\text{-terms.}$$

$$\frac{1}{8}(x-2)^2 + 1 = y \quad \text{Divide each side by 8.}$$

8-2 Lesson Notes

1 Focus

5-Minute Check Transparency 8-2 Use as a quiz or review of Lesson 8-1.

Mathematical Background notes are available for this lesson on p. 410C.

Building on Prior Knowledge

In Chapter 6, students graphed quadratic functions. In this lesson, students will discover that parabolas are conic sections.

How are parabolas used in manufacturing?

Ask students:

- What are some applications in which reflecting light rays are important? **Sample answers: projecting slides and film; astronomy; cameras, and lenses**
- The rays from a lightbulb radiate in all directions, but the beam of a flashlight comes out in a straight line. Why? **Lead students to recognize that a parabolic mirror inside a flashlight reflects the rays so they are parallel.**

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters
- Study Guide and Intervention, pp. 461–462
- Skills Practice, p. 463
- Practice, p. 464
- Reading to Learn Mathematics, p. 465
- Enrichment, p. 466
- Assessment, p. 511

Graphing Calculator and Spreadsheet Masters, p. 42
School-to-Career Masters, p. 15
Science and Mathematics Lab Manual, pp. 119–122
Teaching Algebra With Manipulatives Masters, pp. 264, 265

Transparencies

5-Minute Check Transparency 8-2
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

EQUATIONS OF PARABOLAS

Teaching Tip Discuss with students what they already know and remember about parabolas, their shapes, and their equations.

Teaching Tip Remind students that the distance from a point to a line is measured by the perpendicular from the point to the line.

In-Class Example

Power Point®

1 Write $y = -x^2 - 2x + 3$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola. **$y = -(x + 1)^2 + 4$; vertex: $(-1, 4)$; axis of symmetry: $x = -1$; opens downward**

Teaching Tip Point out that, while the standard form for a linear equation is $Ax + By = C$ (see Lesson 2-2), the standard form for the equation of a parabola is $y = a(x - h)^2 + k$.

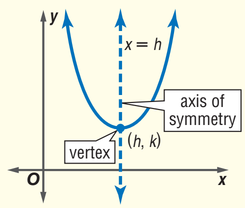
An equation of the parabola with focus at (2, 3) and directrix with equation $y = -1$ is $y = \frac{1}{8}(x - 2)^2 + 1$. The equation of the *axis of symmetry* for this parabola is $x = 2$. The axis of symmetry intersects the parabola at a point called the *vertex*. The vertex is the point where the graph turns. The vertex of this parabola is at (2, 1). Since $\frac{1}{8}$ is positive, the parabola opens upward.

Any equation of the form $y = ax^2 + bx + c$ can be written in standard form.

Key Concept — Equation of a Parabola

The standard form of the equation of a parabola with vertex (h, k) and axis of symmetry $x = h$ is $y = a(x - h)^2 + k$.

- If $a > 0$, k is the minimum value of the related function and the parabola opens upward.
- If $a < 0$, k is the maximum value of the related function and the parabola opens downward.



Example 1 Analyze the Equation of a Parabola

Write $y = 3x^2 + 24x + 50$ in standard form. Identify the vertex, axis of symmetry, and direction of opening of the parabola.

$y = 3x^2 + 24x + 50$	Original equation
$y = 3(x^2 + 8x) + 50$	Factor 3 from the x-terms.
$y = 3(x^2 + 8x + ■) + 50 - 3(■)$	Complete the square on the right side.
$y = 3(x^2 + 8x + 16) + 50 - 3(16)$	The 16 added when you complete the square is multiplied by 3.
$y = 3(x + 4)^2 + 2$	
$y = 3[x - (-4)]^2 + 2$	$(h, k) = (-4, 2)$

The vertex of this parabola is located at $(-4, 2)$, and the equation of the axis of symmetry is $x = -4$. The parabola opens upward.

Study Tip

Look Back
To review **completing the square**, see Lesson 6-4.

GRAPH PARABOLAS You can use symmetry and translations to graph parabolas. The equation $y = a(x - h)^2 + k$ can be obtained from $y = ax^2$ by replacing x with $x - h$ and y with $y - k$. Therefore, the graph of $y = a(x - h)^2 + k$ is the graph of $y = ax^2$ translated h units to the right and k units up.

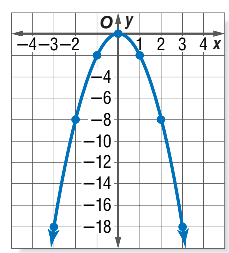
Example 2 Graph Parabolas

Graph each equation.

a. $y = -2x^2$

For this equation, $h = 0$ and $k = 0$. The vertex is at the origin. Since the equation of the axis of symmetry is $x = 0$, substitute some small positive integers for x and find the corresponding y-values.

x	y
1	−2
2	−8
3	−18



Since the graph is symmetric about the y-axis, the points at $(-1, -2)$, $(-2, -8)$, and $(-3, -18)$ are also on the parabola. Use all of these points to draw the graph.

Notice that each side of the graph is the reflection of the other side about the y-axis.

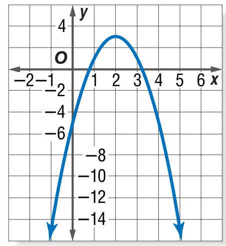
DAILY INTERVENTION — Unlocking Misconceptions

Some students may think that any curve can be called a parabola. Explain that only curves with a certain well-defined shape meet the definition of a parabola.

420 Chapter 8 Conic Sections

b. $y = -2(x - 2)^2 + 3$

The equation is of the form $y = a(x - h)^2 + k$, where $h = 2$ and $k = 3$. The graph of this equation is the graph of $y = -2x^2$ in part **a** translated 2 units to the right and up 3 units. The vertex is now at (2, 3).



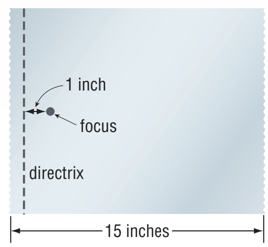
You can use paper folding to investigate the characteristics of a parabola.

Algebra Activity

Parabolas

Model

Step 1 Start with a sheet of wax paper that is about 15 inches long and 12 inches wide. Make a line that is perpendicular to the sides of the sheet by folding the sheet near one end. Open up the paper again. This line is the directrix. Mark a point about midway between the sides of the sheet so that the distance from the directrix is about 1 inch. This point is the focus.



Put the focus on top of any point on the directrix and crease the paper. Make about 20 more creases by placing the focus on top of other points on the directrix. The lines form the outline of a parabola.

Step 2 Start with a new sheet of wax paper. Form another outline of a parabola with a focus that is about 3 inches from the directrix.

Step 3 On a new sheet of wax paper, form a third outline of a parabola with a focus that is about 5 inches from the directrix.

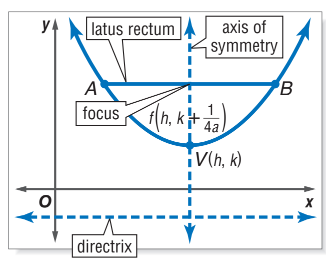
Analyze

Compare the shapes of the three parabolas. How does the distance between the focus and the directrix affect the shape of a parabola?

TEACHING TIP
The line does not have to be perfectly perpendicular to the sides.

As the distance between the directrix and the focus increases, the parabola becomes wider.

The shape of a parabola and the distance between the focus and directrix depend on the value of a in the equation. The line segment through the focus of a parabola and perpendicular to the axis of symmetry is called the **latus rectum**. The endpoints of the latus rectum lie on the parabola. In the figure at the right, the latus rectum is $\overline{AB}$. The length of the latus rectum of the parabola with equation $y = a(x - h)^2 + k$ is $\left|\frac{1}{a}\right|$ units. The endpoints of the latus rectum are $\left|\frac{1}{2a}\right|$ units from the focus.



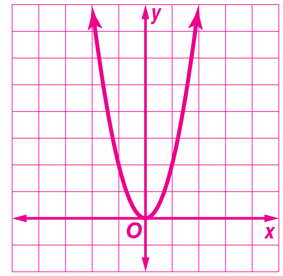
www.algebra2.com/extra_examples

GRAPH PARABOLAS

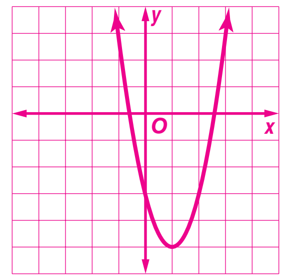
In-Class Example Power Point®

2 Graph each equation.

a. $y = 2x^2$



b. $y = 2(x - 1)^2 - 5$



Teaching Tip Remind students that in $y = a(x - h)^2 + k$, the vertex is (h, k) and the axis of symmetry is $x = h$.

Algebra Activity

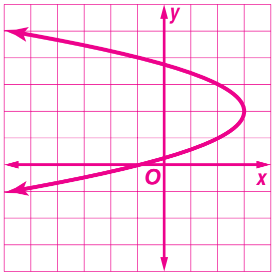
Materials: 3 sheets of wax paper about 15 inches by 12 inches, inch ruler

- You may wish to have students work in pairs. Caution students to avoid making unintentional creases as they fold the wax paper to find the focal point so that it is on the directrix.
- Suggest that students draw the directrix with a pen to make it easier to see.
- The creases that form the parabola may be easier to see against a dark background, such as a dark piece of posterboard.

In-Class Examples

Power Point®

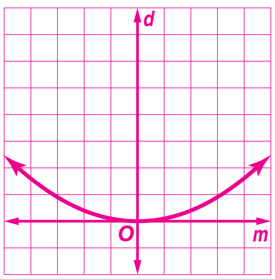
3 Graph $x + y^2 = 4y - 1$.



4 **TSUNAMIS** An undersea earthquake can cause a tsunami wave that travels at m miles per second when the ocean depth is d meters. The parabola that models the relationship between m and d opens upward and has the origin as its vertex. When the ocean depth is 1000 meters, the speed is 100 miles per second.

a. Write an equation for the parabola. $d = 0.1m^2$

b. Graph the equation.



Teaching Tip Suggest that students make a rough sketch of the graph before they begin to find the details of the graph. They can do this by finding the vertex and then deciding which axis the graph wraps around and in which direction it opens.

Equations of parabolas with vertical axes of symmetry are of the form $y = a(x - h)^2 + k$ and are functions. Equations of parabolas with horizontal axes of symmetry are of the form $x = a(y - k)^2 + h$ and are not functions.

Concept Summary — *Information About Parabolas*

Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\|\frac{1}{a}\right\|$ units	$\left\|\frac{1}{a}\right\|$ units

Example 3 Graph an Equation Not in Standard Form

Graph $4x - y^2 = 2y + 13$.

First, write the equation in the form $x = a(y - k)^2 + h$.

$4x - y^2 = 2y + 13$ — There is a y^2 term, so isolate the y and y^2 terms.

$4x = y^2 + 2y + 13$

$4x = (y^2 + 2y + \blacksquare) + 13 - \blacksquare$ — Complete the square.

$4x = (y^2 + 2y + 1) + 13 - 1$ — Add and subtract 1, since $\left(\frac{2}{2}\right)^2 = 1$.

$4x = (y + 1)^2 + 12$ — Write $y^2 + 2y + 1$ as a square.

$x = \frac{1}{4}(y + 1)^2 + 3$ — $(h, k) = (3, -1)$

Then use the following information to draw the graph.

vertex: $(3, -1)$

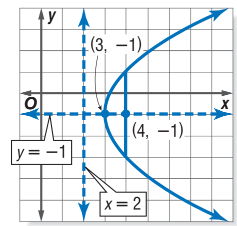
axis of symmetry: $y = -1$

focus: $\left(3 + \frac{1}{4\left(\frac{1}{4}\right)}, -1\right)$ or $(4, -1)$

directrix: $x = 3 - \frac{1}{4\left(\frac{1}{4}\right)}$ or 2

direction of opening: right, since $a > 0$

length of latus rectum: $\left|\frac{1}{\frac{1}{4}}\right|$ or 4 units



Remember that you can plot as many points as necessary to help you draw an accurate graph.

Example 4 Write and Graph an Equation for a Parabola

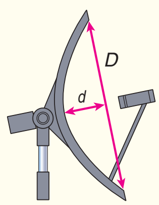
SATELLITE TV **Satellite dishes have parabolic cross sections.**

a. Use the information at the left to write an equation that models a cross section of a satellite dish. Assume that the focus is at the origin and the parabola opens to the right.

First, solve for f. Since $\frac{f}{D} = 0.6$, and $D = 60$, $f = 0.6(60)$ or 36.

The focus is at (0, 0), and the parabola opens to the right. So the vertex must be at $(-36, 0)$. Thus, $h = -36$ and $k = 0$. Use the x-coordinate of the focus to find a.

More About. . .



Satellite TV

The important characteristics of a satellite dish are the diameter D, depth d, and the ratio $\frac{f}{D}$, where f is the distance between the focus and the vertex. A typical dish has the values $D = 60$ cm, $d = 6.25$ cm, and $\frac{f}{D} = 0.6$.

Source: www.2000networks.com

DAILY INTERVENTION

Differentiated Instruction

Kinesthetic Have students make a giant parabola. They can use masking tape to mark a focus point and a directrix line on the classroom floor. Then have them use yardsticks or meter sticks to find and mark, with colored masking tape, a set of points that are equidistant from the point and the line. These points trace a parabola.

$$-36 + \frac{1}{4a} = 0 \quad h = -36\text{; The } x\text{-coordinate of the focus is } 0.$$

$$\frac{1}{4a} = 36 \quad \text{Add 36 to each side.}$$

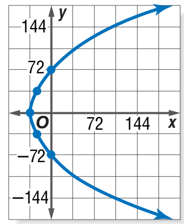
$$1 = 144a \quad \text{Multiply each side by } 4a.$$

$$\frac{1}{144} = a \quad \text{Divide each side by 144.}$$

An equation of the parabola is $x = \frac{1}{144}y^2 - 36$.

b. Graph the equation.

The length of the latus rectum is $\left|\frac{1}{\frac{1}{144}}\right|$ or 144 units, so the graph must pass through (0, 72) and (0, −72). According to the diameter and depth of the dish, the graph must pass through (−29.75, 30) and (−29.75, −30). Use these points and the information from part **a** to draw the graph.



Check for Understanding

Concept Check

1. **Identify** the vertex, focus, axis of symmetry, and directrix of the graph of $y = 4(x - 3)^2 - 7$.
   1. (3, −7), $\left(3, -6\frac{15}{16}\right)$, $x = 3$, $y = -7\frac{1}{16}$
2. **OPEN ENDED** Write an equation for a parabola that opens to the left.
   2. Sample answer: $x = -y^2$
3. **FIND THE ERROR** Katie is finding the standard form of the equation $y = x^2 + 6x + 4$. What mistake did she make in her work?

   $y = x^2 + 6x + 4$

   $y = x^2 + 6x + 9 + 4$

   $y = (x + 3)^2 + 4$

   When she added 9 to complete the square, she forgot to also subtract 9. The standard form is $y = (x + 3)^2 - 9 + 4$ or $y = (x + 3)^2 - 5$.

Guided Practice

4. Write $y = 2x^2 - 12x + 6$ in standard form. **$y = 2(x - 3)^2 - 12$**

GUIDED PRACTICE KEY	
Exercises	Examples
4	1
5–8	1–3
9, 10	2–4
11	4

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. **5–8. See pp. 469A–469J.**

5. $y = (x - 3)^2 - 4$
6. $y = 2(x + 7)^2 + 3$
7. $y = -3x^2 - 8x - 6$
8. $x = \frac{2}{3}y^2 - 6y + 12$

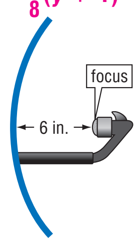
Write an equation for each parabola described below. Then draw the graph.

9–10. See margin for graphs.

9. focus (3, 8), directrix $y = 4$ **$y = \frac{1}{8}(x - 3)^2 + 6$**
10. vertex (5, −1), focus (3, −1) **$x = -\frac{1}{8}(y + 1)^2 + 5$**

Application

11. **COMMUNICATION** A microphone is placed at the focus of a parabolic reflector to collect sound for the television broadcast of a World Cup soccer game. Write an equation for the cross section, assuming that the focus is at the origin and the parabola opens to the right. **$x = \frac{1}{24}y^2 - 6$**



3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- copy the Concept Summary chart just before Example 3 into their notebooks and add some labeled sketches to illustrate.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Remind students that any change they make to the form of an equation must not change the value, so when they add a quantity to complete the square they must also subtract that quantity or add it to the other side of the equation.

About the Exercises...

Organization by Objective
- **Equations of Parabolas:** 12–15, 35
- **Graph Parabolas:** 16–34, 36–45

Odd/Even Assignments
Exercises 12–29 and 36–41 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

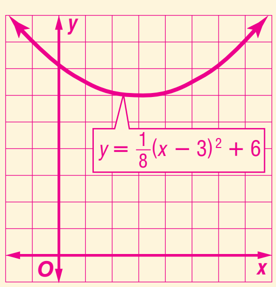
Basic: 13, 17–27 odd, 31–45 odd, 46–62

Average: 13–45 odd, 46–62

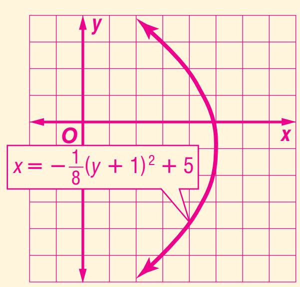
Advanced: 12–46 even, 47–54 (optional: 55–62)

Answers

9.



10.



Lesson 8-2 Parabolas 423

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
12–15, 35	1
16–34	1–3
36–41	2–4
42–45	4a

Extra Practice

See page 845.

Write each equation in standard form.

12. $y = x^2 - 6x + 11$ $y = (x - 3)^2 + 2$

13. $x = y^2 + 14y + 20$ $x = (y + 7)^2 - 29$

★ 14. $y = \frac{1}{2}x^2 + 12x - 8$ $y = \frac{1}{2}(x + 12)^2 - 80$

★ 15. $x = 3y^2 + 5y - 9$ $x = 3\left(y + \frac{5}{6}\right)^2 - 11\frac{1}{12}$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. 16–29. See pp. 469A–469J.

16. $-6y = x^2$

17. $y^2 = 2x$

18. $3(y - 3) = (x + 6)^2$

19. $-2(y - 4) = (x - 1)^2$

20. $4(x - 2) = (y + 3)^2$

21. $(y - 8)^2 = -4(x - 4)$

22. $y = x^2 - 12x + 20$

23. $x = y^2 - 14y + 25$

24. $x = 5y^2 + 25y + 60$

25. $y = 3x^2 - 24x + 50$

26. $y = -2x^2 + 5x - 10$

27. $x = -4y^2 + 6y + 2$

★ 28. $y = \frac{1}{2}x^2 - 3x + \frac{19}{2}$

★ 29. $x = -\frac{1}{3}y^2 - 12y + 15$

For Exercises 30–34, use the equation $x = 3y^2 + 4y + 1$.

30. Draw the graph. See margin.

31. Find the x-intercept(s). 1

32. Find the y-intercept(s). -1 and $-\frac{1}{3}$

33. What is the equation of the axis of symmetry? $y = -\frac{2}{3}$

34. What are the coordinates of the vertex? $\left(-\frac{1}{3}, -\frac{2}{3}\right)$

35. **MANUFACTURING** The reflective surface in a flashlight has a parabolic cross section that can be modeled by $y = \frac{1}{3}x^2$, where x and y are in centimeters. How far from the vertex should the filament of the light bulb be located? 0.75 cm

36–41. See pp. 469A–469J for graphs.

Write an equation for each parabola described below. Then draw the graph.

36. vertex (0, 1), focus (0, 5) $y = \frac{1}{16}x^2 + 1$

37. vertex (8, 6), focus (2, 6)

38. focus (−4, −2), directrix $x = -8$

39. vertex (1, 7), directrix $y = 3$

40. vertex (−7, 4), axis of symmetry $x = -7$, measure of latus rectum 6, $a < 0$

41. vertex (4, 3), axis of symmetry $y = 3$, measure of latus rectum 4, $a > 0$ $x = \frac{1}{4}(y - 3)^2 + 4$

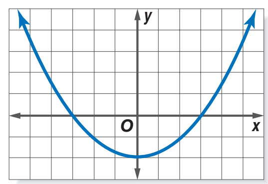
37. $x = -\frac{1}{24}(y - 6)^2 + 8$

38. $x = \frac{1}{8}(y + 2)^2 - 6$

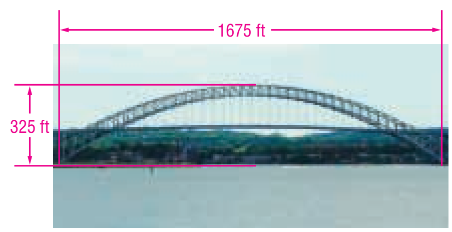
39. $y = \frac{1}{16}(x - 1)^2 + 7$

40. $y = -\frac{1}{6}(x + 7)^2 + 4$

42. Write an equation for the graph at the right. $y = \frac{2}{9}x^2 - 2$



43. **BRIDGES** The Bayonne Bridge connects Staten Island, New York, to New Jersey. It has an arch in the shape of a parabola that opens downward. Write an equation of a parabola to model the arch, assuming that the origin is at the surface of the water, beneath the vertex of the arch. about $y = -0.00046x^2 + 325$



Study Guide and Intervention, p. 461 (shown) and p. 462

Equations of Parabolas A parabola is a curve consisting of all points in the coordinate plane that are the same distance from a given point (the **focus**) and a given line (the **directrix**). The following chart summarizes important information about parabolas.

Standard Form of Equation	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Axis of Symmetry	$x = h$	$y = k$
Vertex	(h, k)	(h, k)
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$
Direction of Opening	upward if $a > 0$, downward if $a < 0$	right if $a > 0$, left if $a < 0$
Length of Latus Rectum	$\left\lvert\frac{1}{a}\right\rvert$ units	$\left\lvert\frac{1}{a}\right\rvert$ units

Example **Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with equation $y = 2x^2 - 12x - 25$.**

$y = 2x^2 - 12x - 25$ Original equation
$y = 2(x^2 - 6x) - 25$ Factor 2 from the x-terms.
$y = 2(x^2 - 6x + ■) - 25 - 2(■)$ Complete the square on the right side.
$y = 2(x^2 - 6x + 9) - 25 - 2(9)$ The 9 added to complete the square is multiplied by 2.
$y = 2(x - 3)^2 - 43$ Write in standard form.

The vertex of this parabola is located at (3, −43), the focus is located at $\left(3, -42\frac{7}{8}\right)$, the equation of the axis of symmetry is $x = 3$, and the equation of the directrix is $y = -43\frac{1}{8}$. The parabola opens upward.

Exercises

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation.

1. $y = x^2 + 6x - 4$ (−3, −13), $\left(-3, -12\frac{3}{4}\right)$, $x = -3$, $y = -13\frac{1}{4}$, up

2. $y = 8x - 2x^2 + 10$ (2, 18), $\left(2, 17\frac{1}{8}\right)$, $x = 2$, $y = 18\frac{1}{8}$, down

3. $x = y^2 - 8y + 6$ (−10, 4), $\left(-9\frac{3}{4}, 4\right)$, $y = 4$, $x = -10\frac{1}{4}$, right

Write an equation of each parabola described below.

4. focus (−2, 3), directrix $x = -2\frac{1}{12}$ $x = 6(y - 3)^2 - 2\frac{1}{24}$

5. vertex (5, 1), focus $\left(4\frac{11}{12}, 1\right)$ $x = -3(y - 1)^2 + 5$

Skills Practice, p. 463 and Practice, p. 464 (shown)

Write each equation in standard form.

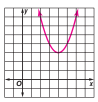
1. $y = 2x^2 - 12x + 19$ $y = 2(x - 3)^2 + 1$

2. $y = \frac{1}{2}x^2 + 3x + \frac{1}{2}$ $y = \frac{1}{2}[x - (-3)]^2 + (-4)$

3. $y = -3x^2 - 12x - 7$ $y = -3[x - (-2)]^2 + 5$

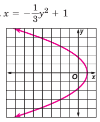
Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.

4. $y = (x - 4)^2 + 3$



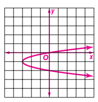
vertex: (4, 3); focus: $\left(4, 3\frac{1}{4}\right)$; axis: $x = 4$; directrix: $y = 2\frac{3}{4}$; opens up; latus rectum: 1 unit

5. $x = -\frac{1}{3}y^2 + 1$



vertex: (1, 0); focus: $\left(\frac{1}{4}, 0\right)$; axis: $y = 0$; directrix: $x = 1\frac{3}{4}$; opens left; latus rectum: 3 units

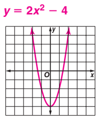
6. $x = 3(y + 1)^2 - 3$



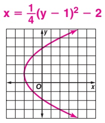
vertex: (−3, −1); focus: $\left(-2\frac{11}{12}, -1\right)$; axis: $y = -1$; directrix: $x = -3\frac{1}{12}$; opens right; latus rectum: $\frac{1}{3}$ unit

Write an equation for each parabola described below. Then draw the graph.

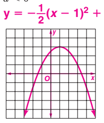
7. vertex (0, −4), focus $\left(0, -3\frac{7}{8}\right)$ $y = 2x^2 - 4$



8. vertex (−2, 1), directrix $x = -3$ $x = \frac{1}{4}(y - 1)^2 - 2$



9. vertex (1, 3), axis of symmetry $x = 1$, latus rectum: 2 units, $a < 0$ $y = -\frac{1}{2}(x - 1)^2 + 3$



10. **TELEVISION** Write the equation in the form $y = ax^2$ for a satellite dish. Assume that the bottom of the upward-facing dish passes through (0, 0) and that the distance from the bottom to the focus point is 8 inches. $y = \frac{1}{32}x^2$

Reading to Learn Mathematics, p. 465 ELL

Pre-Activity **How are parabolas used in manufacturing?**

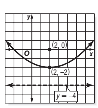
Read the introduction to Lesson 8-2 at the top of page 419 in your textbook.

Name at least two reflective objects that might have the shape of a parabola.

Sample answer: telescope mirror, satellite dish

Reading the Lesson

1. In the parabola shown in the graph, the point (2, −2) is called the __vertex__ and the point (2, 0) is called the __focus__. The line $y = -4$ is called the __directrix__, and the line $x = 2$ is called the __axis of symmetry__.



2. a. Write the standard form of the equation of a parabola that opens upward or downward. $y = a(x - h)^2 + k$

b. The parabola opens downward if __$a < 0$__ and opens upward if __$a > 0$__. The equation of the axis of symmetry is __$x = h$__, and the coordinates of the vertex are __(h, k)__.

3. A parabola has equation $x = -\frac{1}{8}(y - 2)^2 + 4$. This parabola opens to the __left__. It has vertex __(4, 2)__ and focus __(2, 2)__. The directrix is __$x = 6$__. The length of the latus rectum is __8__ units.

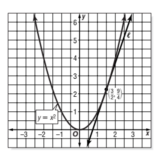
Helping You Remember

4. How can the way in which you plot points in a rectangular coordinate system help you to remember what the sign of a tells you about the direction in which a parabola opens? Sample answer: In plotting points, a positive x-coordinate tells you to move to the *right* and a negative x-coordinate tells you to move to the *left*. This is like a parabola whose equation is of the form "$x = …$"; it opens to the *right* if $a > 0$ and to the *left* if $a < 0$. Likewise, a positive y-coordinate tells you to move *up* and a negative y-coordinate tells you to move *down*. This is like a parabola whose equation is of the form "$y = …$"; it opens *upward* if $a > 0$ and *downward* if $a < 0$.

Enrichment, p. 466

Tangents to Parabolas

A line that intersects a parabola in exactly one point without crossing the curve is a **tangent** to the parabola. The point where a tangent line touches a parabola is the **point of tangency.** The line perpendicular to a tangent to a parabola at the point of tangency is called the **normal** to the parabola at that point. In the diagram, line ℓ is tangent to the parabola that is the graph of $y = x^2$ at $\left(\frac{3}{2}, \frac{9}{4}\right)$. The x-axis is tangent to the parabola at O, and the y-axis is the normal to the parabola at O.

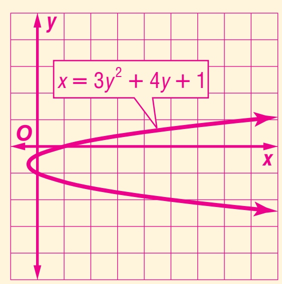


Solve each problem.

1. Find an equation for line ℓ in the diagram. *Hint:* A nonvertical line with an equation of the form $y = mx + b$ will be tangent to the graph of $y = x^2$ at

Answer

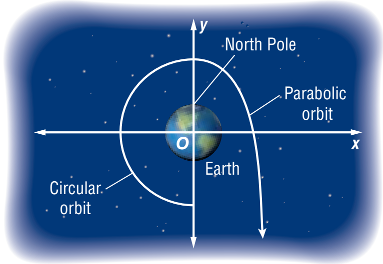
30.



424 Chapter 8 Conic Sections

44. **SPORTS** When a ball is thrown or kicked, the path it travels is shaped like a parabola. Suppose a football is kicked from ground level, reaches a maximum height of 25 feet, and hits the ground 100 feet from where it was kicked. Assuming that the ball was kicked at the origin, write an equation of the parabola that models the flight of the ball. $y = -\frac{1}{100}(x - 50)^2 + 25$

45. **AEROSPACE** A spacecraft is in a circular orbit 150 kilometers above Earth. Once it attains the velocity needed to escape Earth's gravity, the spacecraft will follow a parabolic path with the center of Earth as the focus. Suppose the spacecraft reaches escape velocity above the North Pole. Write an equation to model the parabolic path of the spacecraft, assuming that the center of Earth is at the origin and the radius of Earth is 6400 kilometers. $y = -\frac{1}{26{,}200}x^2 + 6550$



46. **CRITICAL THINKING** The parabola with equation $y = (x - 4)^2 + 3$ has its vertex at (4, 3) and passes through (5, 4). Find an equation of a different parabola with its vertex at (4, 3) that passes through (5, 4). $x = (y - 3)^2 + 4$

47. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How are parabolas used in manufacturing?

Include the following in your answer:
- how you think the focus of a parabola got its name, and
- why a car headlight with a parabolic reflector is better than one with an unreflected light bulb.

Standardized Test Practice

48. Which equation has a graph that opens downward? **B**
   (A) $y = 3x^2 - 2$ (B) $y = 2 - 3x^2$ (C) $x = 3y^2 - 2$ (D) $x = 2 - 3y^2$

49. Find the vertex of the parabola with equation $y = x^2 - 10x + 8$. **A**
   (A) (5, −17) (B) (10, 8) (C) (0, 8) (D) (5, 8)

Maintain Your Skills

Mixed Review

Find the distance between each pair of points with the given coordinates. *(Lesson 8-1)*

50. (7, 3), (−5, 8) **13 units** 51. (4, −1), (−2, 7) **10 units** 52. (−3, 1), (0, 6) $\sqrt{34}$ **units**

53. Graph $y \le \sqrt{x + 1}$. *(Lesson 7-9)* **See margin.**

54. **HEALTH** Ty's heart rate is usually 120 beats per minute when he runs. If he runs for 2 hours every day, about how many times will his heart beat during the amount of time he exercises in two weeks? Express the answer in scientific notation. *(Lesson 5-1)* 2.016×10^5

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each radical expression.**
*(To review **simplifying radicals**, see Lessons 5-5 and 5-6.)*

55. $\sqrt{16}$ **4** 56. $\sqrt{25}$ **5** 57. $\sqrt{81}$ **9** 58. $\sqrt{144}$ **12**
59. $\sqrt{12}$ $2\sqrt{3}$ 60. $\sqrt{18}$ $3\sqrt{2}$ 61. $\sqrt{48}$ $4\sqrt{3}$ 62. $\sqrt{72}$ $6\sqrt{2}$

www.algebra2.com/self_check_quiz

4 Assess

Open-Ended Assessment

Writing Have students draw and label four types of parabolas opening upward, downward, left, and right. Have them give the equation, vertex, axis of symmetry, focus, and directrix for each in terms of x, y, a, h, and k.

Getting Ready for Lesson 8-3

PREREQUISITE SKILL Students will write and analyze equations of circles in Lesson 8-3. Using the Distance Formula, students will simplify radicals to find the radii of circles. Exercises 55–62 should be used to determine your students' familiarity with simplifying radicals.

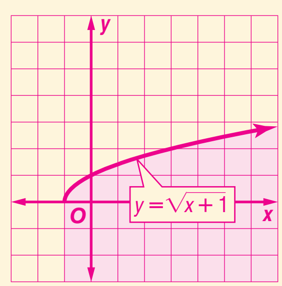
Assessment Options

Quiz (Lessons 8-1 and 8-2) is available on p. 511 of the *Chapter 8 Resource Masters*.

Answers

47. A parabolic reflector can be used to make a car headlight more effective. Answers should include the following.
- **Reflected rays are *focused* at that point.**
- **The light from an unreflected bulb would shine in all directions. With a parabolic reflector, most of the light can be directed forward toward the road.**

53.



8-3 Lesson Notes

1 Focus

5-Minute Check Transparency 8-3 Use as a quiz or review of Lesson 8-2.

Mathematical Background notes are available for this lesson on p. 410C.

Why are circles important in air traffic control?

Ask students:

- Describe any radar antenna you may have seen on a boat or at an airport. How did the antenna move? **constantly turning in a circle**
- What part of a circle is the range of 45 to 70 miles describing? **radius**

8-3 Circles

What You'll Learn

- Write equations of circles.
- Graph circles.

Vocabulary

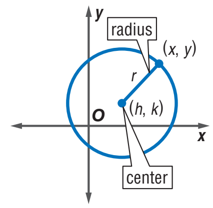
- circle
- center
- tangent

Why are circles important in air traffic control?

Radar equipment can be used to detect and locate objects that are too far away to be seen by the human eye. The radar systems at major airports can typically detect and track aircraft up to 45 to 70 miles in any direction from the airport. The boundary of the region that a radar system can monitor can be modeled by a circle.

EQUATIONS OF CIRCLES A **circle** is the set of all points in a plane that are equidistant from a given point in the plane, called the **center**. Any segment whose endpoints are the center and a point on the circle is a *radius* of the circle.

Assume that (x, y) are the coordinates of a point on the circle at the right. The center is at (h, k), and the radius is r. You can find an equation of the circle by using the Distance Formula.



$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \quad \text{Distance Formula}$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r \quad (x_1, y_1) = (h, k),\ (x_2, y_2) = (x, y),\ d = r$$

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Square each side.}$$

Key Concept *Equation of a Circle*

The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example 1 *Write an Equation Given the Center and Radius*

NUCLEAR POWER **In 1986, a nuclear reactor exploded at a power plant about 110 kilometers north and 15 kilometers west of Kiev. At first, officials evacuated people within 30 kilometers of the power plant. Write an equation to represent the boundary of the evacuated region if the origin of the coordinate system is at Kiev.**

Since Kiev is at (0, 0), the power plant is at (−15, 110). The boundary of the evacuated region is the circle centered at (−15, 110) with radius 30 kilometers.

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Equation of a circle}$$

$$[x - (-15)]^2 + (y - 110)^2 = 30^2 \quad (h, k) = (-15, 110),\ r = 30$$

$$(x + 15)^2 + (y - 110)^2 = 900 \quad \text{Simplify.}$$

The equation is $(x + 15)^2 + (y - 110)^2 = 900$.

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters

- Study Guide and Intervention, pp. 467–468
- Skills Practice, p. 469
- Practice, p. 470
- Reading to Learn Mathematics, p. 471
- Enrichment, p. 472

Graphing Calculator and Spreadsheet Masters, p. 41

Transparencies

5-Minute Check Transparency 8-3
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 15
Interactive Chalkboard

Example 2 Write an Equation Given a Diameter

Write an equation for a circle if the endpoints of a diameter are at (5, 4) and (−2, −6).

Explore To write an equation of a circle, you must know the center and the radius.

Plan You can find the center of the circle by finding the midpoint of the diameter. Then you can find the radius of the circle by finding the distance from the center to one of the given points.

Solve First find the center of the circle.

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$= \left(\frac{5 + (-2)}{2}, \frac{4 + (-6)}{2}\right) \quad (x_1, y_1) = (5, 4), (x_2, y_2) = (-2, -6)$$

$$= \left(\frac{3}{2}, \frac{-2}{2}\right) \quad \text{Add.}$$

$$= \left(\frac{3}{2}, -1\right) \quad \text{Simplify.}$$

Now find the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{\left(\frac{3}{2} - 5\right)^2 + (-1 - 4)^2} \quad (x_1, y_1) = (5, 4), (x_2, y_2) = \left(\frac{3}{2}, -1\right)$$

$$= \sqrt{\left(-\frac{7}{2}\right)^2 + (-5)^2} \quad \text{Subtract.}$$

$$= \sqrt{\frac{149}{4}} \quad \text{Simplify.}$$

The radius of the circle is $\sqrt{\frac{149}{4}}$ units, so $r^2 = \frac{149}{4}$.

Substitute h, k, and r^2 into the standard form of the equation of a circle.

An equation of the circle is $\left(x - \frac{3}{2}\right)^2 + [y - (-1)]^2 = \frac{149}{4}$ or $\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{149}{4}$.

Examine Each of the given points satisfies the equation, so the equation is reasonable.

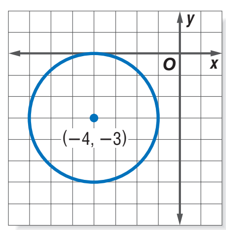
A line in the plane of a circle can intersect the circle in zero, one, or two points. A line that intersects the circle in exactly one point is said to be **tangent** to the circle. The line and the circle are tangent to each other at this point.

Example 3 Write an Equation Given the Center and a Tangent

Write an equation for a circle with center at (−4, −3) that is tangent to the *x*-axis.

Sketch the circle. Since the circle is tangent to the x-axis, its radius is 3.

An equation of the circle is $(x + 4)^2 + (y + 3)^2 = 9$.



www.algebra2.com/extra_examples

2 Teach

EQUATIONS OF CIRCLES

In-Class Examples

Power Point®

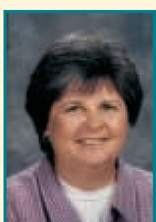
1 **LANDSCAPING** The plan for a park puts the center of a circular pond, of radius 0.6 miles, 2.5 miles east and 3.8 miles south of the park headquarters. Write an equation to represent the border of the pond, using the headquarters as the origin. $(x - 2.5)^2 + (y + 3.8)^2 = 0.36$

Teaching Tip Explain that h and k are the variables traditionally used, but other variables could be used just as well.

2 Write an equation for a circle if the endpoints of a diameter are at (2, 8) and (2, −2). $(x - 2)^2 + (y - 3)^2 = 25$

Teaching Tip Suggest that students draw a sketch showing the circle and the endpoints of the diameter to check their work.

3 Write an equation for a circle with center at (3, 5) that is tangent to the y-axis. $(x - 3)^2 + (y - 5)^2 = 9$



Teacher to Teacher

Nancy McKinney & Karen Rowe Camdenton H.S., Camdenton, MO

"While studying conic sections we have our students make a collage of the various shapes. They look in magazines, on the Internet, within computer print programs, etc."

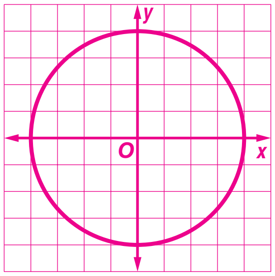
Lesson 8-3 Circles 427

GRAPH CIRCLES

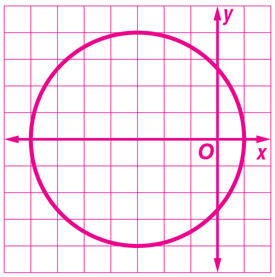
In-Class Examples

Power Point®

4 Find the center and radius of the circle with equation $x^2 + y^2 = 16$. Then graph the circle. **(0, 0); 4**



5 Find the center and radius of the circle with equation $x^2 + y^2 + 6x - 7 = 0$. Then graph the circle. **(−3, 0); 4**



Answer

3. Lucy; 36 is the *square* of the radius, so the radius is 6 units.

GRAPH CIRCLES You can use completing the square, symmetry, and transformations to help you graph circles. The equation $(x - h)^2 + (y - k)^2 = r^2$ is obtained from the equation $x^2 + y^2 = r^2$ by replacing x with $x - h$ and y with $y - k$. So, the graph of $(x - h)^2 + (y - k)^2 = r^2$ is the graph of $x^2 + y^2 = r^2$ translated h units to the right and k units up.

Example 4 Graph an Equation in Standard Form

Find the center and radius of the circle with equation $x^2 + y^2 = 25$. Then graph the circle.

The center of the circle is at (0, 0), and the radius is 5.

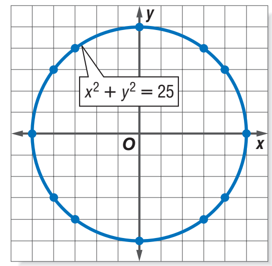
The table lists some integer values for x and y that satisfy the equation.

x	y
0	5
3	4
4	3
5	0

Since the circle is centered at the origin, it is symmetric about the y-axis. Therefore, the points at (−3, 4), (−4, 3) and (−5, 0) lie on the graph.

The circle is also symmetric about the x-axis, so the points at (−4, −3), (−3, −4), (0, −5), (3, −4), and (4, −3) lie on the graph.

Graph all of these points and draw the circle that passes through them.



Example 5 Graph an Equation Not in Standard Form

Find the center and radius of the circle with equation $x^2 + y^2 - 4x + 8y - 5 = 0$. Then graph the circle.

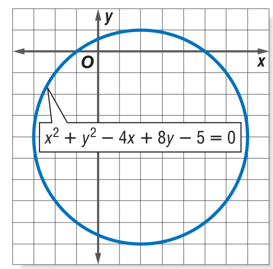
Complete the squares.

$$x^2 + y^2 - 4x + 8y - 5 = 0$$

$$x^2 - 4x + \blacksquare + y^2 + 8y + \blacksquare = 5 + \blacksquare + \blacksquare$$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 5 + 4 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 25$$



The center of the circle is at (2, −4), and the radius is 5. In the equation from Example 4, x has been replaced by $x - 2$, and y has been replaced by $y + 4$. The graph is the graph from Example 4 translated 2 units to the right and down 4 units.

Check for Understanding

Concept Check

1. Sample answer: $(x - 6)^2 + (y + 2)^2 = 16$

2. $(x + 3)^2 + (y - 1)^2 = 64$; left 3 units, up 1 unit

1. **OPEN ENDED** Write an equation for a circle with center at (6, −2).
2. Write $x^2 + y^2 + 6x - 2y - 54 = 0$ in standard form by completing the square. Describe the transformation that can be applied to the graph of $x^2 + y^2 = 64$ to obtain the graph of the given equation.
3. **FIND THE ERROR** Juwan says that the circle with equation $(x - 4)^2 + y^2 = 36$ has radius 36 units. Lucy says that the radius is 6 units. Who is correct? Explain your reasoning. **See margin.**

DAILY INTERVENTION

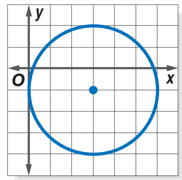
Differentiated Instruction

Naturalist Encourage students to find and describe objects in nature that are related to circles. Although circles in nature may not be mathematically perfect, they are often seen, as in the circle of leaves on a plant, or designs on an insect, or the shape of a flower, or ripples on a pond after a stone is tossed into the water.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7, 14, 15	1–3
8–13	4, 5

4. Write an equation for the graph at the right. $(x-3)^2+(y+1)^2=9$



Write an equation for the circle that satisfies each set of conditions.

5. center $(-1, -5)$, radius 2 units $(x+1)^2+(y+5)^2=4$
6. endpoints of a diameter at $(-4, 1)$ and $(4, -5)$ $x^2+(y+2)^2=25$
7. center $(3, -7)$, tangent to the y-axis $(x-3)^2+(y+7)^2=9$

Find the center and radius of the circle with the given equation. Then graph the circle. 8–13. See pp. 469A–469J for graphs.

8. $(x-4)^2+(y-1)^2=9$ (4, 1), 3 units
9. $x^2+(y-14)^2=34$
10. $(x-4)^2+y^2=\frac{16}{25}$ $(4, 0), \frac{4}{5}$ unit
11. $\left(x+\frac{2}{3}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{8}{9}$
12. $x^2+y^2+8x-6y=0$ $(-4, 3)$, 5 units
13. $x^2+y^2+4x=8=0$ $(-2, 0), 2\sqrt{3}$ units

9. $(0, 14), \sqrt{34}$ units

11. $\left(-\frac{2}{3}, \frac{1}{2}\right), \frac{2\sqrt{2}}{3}$ unit

Application

AEROSPACE **For Exercises 14 and 15, use the following information.**
In order for a satellite to remain in a circular orbit above the same spot on Earth, the satellite must be 35,800 kilometers above the equator.

14. Write an equation for the orbit of the satellite. Use the center of Earth as the origin and 6400 kilometers for the radius of Earth. $x^2+y^2=42{,}200^2$
15. Draw a labeled sketch of Earth and the orbit to scale. See margin.

★ indicates increased difficulty

Practice and Apply

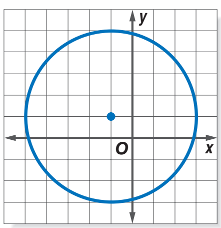
Homework Help

For Exercises	See Examples
16–29	1–3
30–48	4, 5

Extra Practice
See page 845.

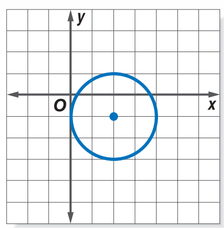
Write an equation for each graph.

16.



$(x+1)^2+(y-1)^2=16$

17.



$(x-2)^2+(y+1)^2=4$

26–27. See margin.

Write an equation for the circle that satisfies each set of conditions.

18. center $(0, 3)$, radius 7 units $x^2+(y-3)^2=49$
19. center $(-8, 7)$, radius $\frac{1}{2}$ unit $(x+8)^2+(y-7)^2=\frac{1}{4}$
20. endpoints of a diameter at $(-5, 2)$ and $(3, 6)$ $(x+1)^2+(y-4)^2=20$
21. endpoints of a diameter at $(11, 18)$ and $(-13, -19)$ $(x+1)^2+\left(y+\frac{1}{2}\right)^2=\frac{1945}{4}$
22. center $(8, -9)$, passes through $(21, 22)$ $(x-8)^2+(y+9)^2=1130$
23. center $\left(-\sqrt{13}, 42\right)$, passes through the origin $\left(x+\sqrt{13}\right)^2+(y-42)^2=1777$
24. center at $(-8, -7)$, tangent to y-axis $(x+8)^2+(y+7)^2=64$
25. center at $(4, 2)$, tangent to x-axis $(x-4)^2+(y-2)^2=4$

★ 26. center in the first quadrant; tangent to $x=-3$, $x=5$, and the x-axis

★ 27. center in the second quadrant; tangent to $y=-1$, $y=9$, and the y-axis

WebQuest

The epicenter of an earthquake can be located by using the equation of a circle. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

www.algebra2.com/self_check_quiz

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Have students review the equation of a circle in standard form and notice that the right side of the equation is the square of the radius.

About the Exercises…

Organization by Objective

- **Equations of Circles:** 16–29
- **Graph Circles:** 30–48

Odd/Even Assignments

Exercises 16–27 and 30–47 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–25 odd, 29–45 odd, 49–52, 57–71

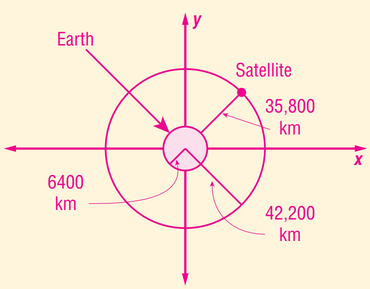
Average: 17–47 odd, 49–52, 57–71 (optional: 53–56)

Advanced: 16–48 even, 49–65 (optional: 66–71)

All: Practice Quiz 1 (1–5)

Answers

15.



26. $(x-1)^2+(y-4)^2=16$

27. $(x+5)^2+(y-4)^2=25$

Lesson 8-3 Circles 429

Study Guide and Intervention, p. 467 (shown) and p. 468

Equations of Circles The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

Example **Write an equation for a circle if the endpoints of a diameter are at (−4, 5) and (6, −3).**

Use the midpoint formula to find the center of the circle.

$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint formula

$= \left(\frac{-4 + 6}{2}, \frac{5 + (-3)}{2}\right)$ $(x_1, y_1) = (-4, 5), (x_2, y_2) = (6, -3)$

$= \left(\frac{2}{2}, \frac{2}{2}\right)$ or $(1, 1)$ Simplify.

Use the coordinates of the center and one endpoint of the diameter to find the radius.

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance formula

$r = \sqrt{(-4 - 1)^2 + (5 - 1)^2}$ $(x_1, y_1) = (1, 1), (x_2, y_2) = (-4, 5)$

$= \sqrt{(-5)^2 + 4^2} = \sqrt{41}$ Simplify.

The radius of the circle is $\sqrt{41}$, so $r^2 = 41$.

An equation of the circle is $(x - 1)^2 + (y - 1)^2 = 41$.

Exercises

Write an equation for the circle that satisfies each set of conditions.

1. center (8, −3), radius 6 $(x - 8)^2 + (y + 3)^2 = 36$
2. center (5, −6), radius 4 $(x - 5)^2 + (y + 6)^2 = 16$
3. center (−5, 2), passes through (−9, 6) $(x + 5)^2 + (y - 2)^2 = 32$
4. endpoints of a diameter at (6, 6) and (10, 12) $(x - 8)^2 + (y - 9)^2 = 13$
5. center (3, 6), tangent to the x-axis $(x - 3)^2 + (y - 6)^2 = 36$
6. center (−4, −7), tangent to $x = 2$ $(x + 4)^2 + (y + 7)^2 = 36$
7. center at (−2, 8), tangent to $y = -4$ $(x + 2)^2 + (y - 8)^2 = 144$
8. center (7, 7), passes through (12, 9) $(x - 7)^2 + (y - 7)^2 = 29$
9. endpoints of a diameter are (−4, −2) and (8, 4) $(x - 2)^2 + (y - 1)^2 = 45$
10. endpoints of a diameter are (−4, 3) and (6, −8) $(x - 1)^2 + (y + 2.5)^2 = 55.25$

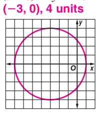
Skills Practice, p. 469 and Practice, p. 470 (shown)

Write an equation for the circle that satisfies each set of conditions.

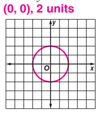
1. center (−4, 2), radius 8 units $(x + 4)^2 + (y - 2)^2 = 64$
2. center (0, 0), radius 4 units $x^2 + y^2 = 16$
3. center $\left(-\frac{1}{4}, -\sqrt{3}\right)$, radius $5\sqrt{2}$ units $\left(x + \frac{1}{4}\right)^2 + (y + \sqrt{3})^2 = 50$
4. center (2.5, 4.2), radius 0.9 unit $(x - 2.5)^2 + (y - 4.2)^2 = 0.81$
5. endpoints of a diameter at (−2, −9) and (0, −5) $(x + 1)^2 + (y + 7)^2 = 5$
6. center at (−9, −12), passes through (−4, −5) $(x + 9)^2 + (y + 12)^2 = 74$
7. center at (−6, 5), tangent to x-axis $(x + 6)^2 + (y - 5)^2 = 25$

Find the center and radius of the circle with the given equation. Then graph the circle.

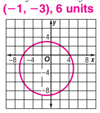
8. $(x + 3)^2 + y^2 = 16$ (−3, 0), 4 units



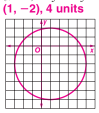
9. $3x^2 + 3y^2 = 12$ (0, 0), 2 units



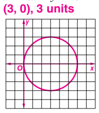
10. $x^2 + y^2 + 2x + 6y = 26$ (−1, −3), 6 units



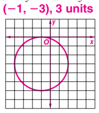
11. $(x - 1)^2 + y^2 + 4y = 12$ (1, −2), 4 units



12. $x^2 - 6x + y^2 = 0$ (3, 0), 3 units



13. $x^2 + y^2 + 2x + 6y = -1$ (−1, −3), 3 units



WEATHER **For Exercises 14 and 15, use the following information.**

On average, the circular eye of a hurricane is about 15 miles in diameter. Gale winds can affect an area up to 300 miles from the storm's center. In 1992, Hurricane Andrew devastated southern Florida. A satellite photo of Andrew's landfall showed the center of its eye on one coordinate system could be approximated by the point (80, 26).

14. Write an equation to represent a possible boundary of Andrew's eye. $(x - 80)^2 + (y - 26)^2 = 56.25$
15. Write an equation to represent a possible boundary of the area affected by gale winds. $(x - 80)^2 + (y - 26)^2 = 90{,}000$

Reading to Learn Mathematics, p. 471 ELL

Pre-Activity **Why are circles important in air traffic control?**

Read the introduction to Lesson 8-3 at the top of page 426 in your textbook.

A large home improvement chain is planning to enter a new metropolitan area and needs to select locations for its stores. Market research has shown that potential customers are willing to travel up to 12 miles to shop at one of their stores. How can circles help the managers decide where to place their store?

Sample answer: A store will draw customers who live inside a circle with center at the store and a radius of 12 miles. The management should select locations for which as many people as possible live within a circle of radius 12 miles around one of the stores.

Reading the Lesson

1. a. Write the equation of the circle with center (h, k) and radius r. $(x - h)^2 + (y - k)^2 = r^2$

   b. Write the equation of the circle with center (4, −3) and radius 5. $(x - 4)^2 + (y + 3)^2 = 25$

   c. The circle with equation $(x + 8)^2 + y^2 = 121$ has center (−8, 0) and radius 11.

   d. The circle with equation $(x - 10)^2 + (y + 10)^2 = 1$ has center (10, −10) and radius 1.

2. a. In order to find center and radius of the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$, it is necessary to complete the square. Fill in the missing parts of this process.

$$x^2 + y^2 + 4x - 6y - 3 = 0$$
$$x^2 + y^2 + 4x - 6y = 3$$
$$x^2 + 4x + 4 + y^2 - 6y + 9 = 3 + 4 + 9$$
$$(x + 2)^2 + (y - 3)^2 = 16$$

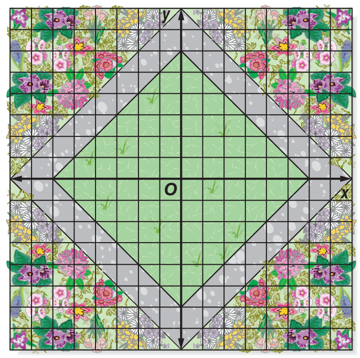
   b. This circle has radius 4 and center at (−2, 3).

Helping You Remember

3. How can the distance formula help you to remember the equation of a circle? **Sample answer: Write the distance formula. Replace (x_1, y_1) with (h, k) and (x_2, y_2) with (x, y). Replace d with r. Square both sides. Now you have the equation of a circle.**

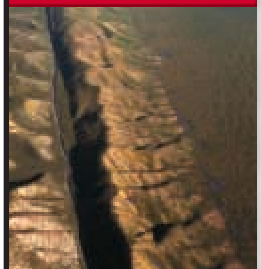
---

28. **LANDSCAPING** The design of a garden is shown at the right. A pond is to be built in the center region. What is the equation of the largest circular pond centered at the origin that would fit within the walkways? $x^2 + y^2 = 18$



29. **EARTHQUAKES** The University of Southern California is located about 2.5 miles west and about 2.8 miles south of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 40 miles from the university. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake. $(x + 2.5)^2 + (y + 2.8)^2 = 1600$

More About. . .



Earthquakes

Southern California has about 10,000 earthquakes per year many of which occur at or near the San Andreas fault. Most are too small to be felt.

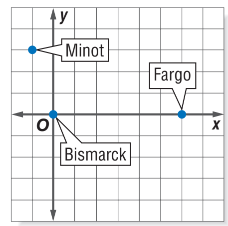
Source: www.earthquake.usgs.gov

Find the center and radius of the circle with the given equation. Then graph the circle. 30–47. See pp. 469A–469J for graphs.

30. $x^2 + (y + 2)^2 = 4$ (0, −2), 2 units
31. $x^2 + y^2 = 144$ (0, 0), 12 units
32. $(x - 3)^2 + (y - 1)^2 = 25$ (3, 1), 5 units
33. $(x + 3)^2 + (y + 7)^2 = 81$
34. $(x - 3)^2 + y^2 = 16$ (3, 0), 4 units
35. $(x - 3)^2 + (y + 7)^2 = 50$
36. $(x + \sqrt{5})^2 + y^2 - 8y = 9$
37. $x^2 + (y - \sqrt{3})^2 + 4x = 25$
38. $x^2 + y^2 + 6y = -50 - 14x$
39. $x^2 + y^2 - 6y - 16 = 0$ (0, 3), 5 units
40. $x^2 + y^2 + 2x - 10 = 0$
41. $x^2 + y^2 - 18x - 18y + 53 = 0$
42. $x^2 + y^2 + 9x - 8y + 4 = 0$
43. $x^2 + y^2 - 3x + 8y = 20$
44. $x^2 - 12x + 84 = -y^2 + 16y$
45. $x^2 + y^2 + 2x + 4y = 9$
46. ★ $3x^2 + 3y^2 + 12x - 6y + 9 = 0$ $(-2, 1)$, $\sqrt{2}$ units
47. ★ $4x^2 + 4y^2 + 36y + 5 = 0$

33. (−3, −7), 9 units
35. (3, −7), $5\sqrt{2}$ units
36. $(-\sqrt{5}, 4)$, 5 units
37. $(-2, \sqrt{3})$, $\sqrt{29}$ units
38. (−7, −3), $2\sqrt{2}$ units
40. (−1, 0), $\sqrt{11}$ units
41. (9, 9), $\sqrt{109}$ units
42. $\left(-\frac{9}{2}, 4\right)$, $\frac{\sqrt{129}}{2}$ units
43. $\left(\frac{3}{2}, -4\right)$, $\frac{3\sqrt{17}}{2}$ units
44. (6, 8), 4 units
45. (−1, −2), $\sqrt{14}$ units
47. $\left(0, -\frac{9}{2}\right)$, $\sqrt{19}$ units

48. **RADIO** The diagram at the right shows the relative locations of some cities in North Dakota. The x-axis represents Interstate 94. The scale is 1 unit = 30 miles. While driving west on the highway, Doralina is listening to a radio station in Minot. She estimates the range of the signal to be 120 miles. How far west of Bismarck will she be able to pick up the signal? about 109 mi



49. **CRITICAL THINKING** A circle has its center on the line with equation $y = 2x$. The circle passes through $(1, -3)$ and has a radius of $\sqrt{5}$ units. Write an equation of the circle. $(x + 1)^2 + (y + 2)^2 = 5$

50. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson. See margin.

Why are circles important in air traffic control?

Include the following in your answer:

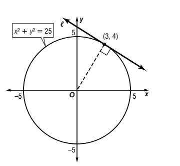
- an equation of the circle that determines the boundary of the region where planes can be detected if the range of the radar is 50 miles and the radar is at the origin, and
- how an air traffic controller's job would be different for a region whose boundary is modeled by $x^2 + y^2 = 4900$ instead of $x^2 + y^2 = 1600$.

Enrichment, p. 472

Tangents to Circles

A line that intersects a circle in exactly one point is a **tangent** to the circle. In the diagram, line ℓ is tangent to the circle with equation $x^2 + y^2 = 25$ at the point whose coordinates are (3, 4).

A line is tangent to a circle at a point P on the circle if and only if the line is perpendicular to the radius from the center of the circle to point P. This fact enables you to find an equation of the tangent to a circle at a point P if you know an equation for the circle and the coordinates of P.



Use the diagram above to solve each problem.

1. What is the slope of the radius to the point with coordinates (3, 4)? What is

Standardized Test Practice

51. Find the radius of the circle with equation $x^2 + y^2 + 8x + 8y + 28 = 0$. **A**

(A) 2 (B) 4 (C) 8 (D) 28

52. Find the center of the circle with equation $x^2 + y^2 - 10x + 6y + 27 = 0$. **D**

(A) $(-10, 6)$ (B) $(1, 1)$ (C) $(10, -6)$ (D) $(5, -3)$

Graphing Calculator

CIRCLES **For Exercises 53–56, use the following information.**
Since a circle is not the graph of a function, you cannot enter its equation directly into a graphing calculator. Instead, you must solve the equation for y. The result will contain a $\pm$ symbol, so you will have two functions.

53. Solve $(x + 3)^2 + y^2 = 16$ for y. **$y = \pm\sqrt{16 - (x+3)^2}$**
54. What two functions should you enter to graph the given equation?
55. Graph $(x + 3)^2 + y^2 = 16$ on a graphing calculator. **See margin.**
56. Solve $(x + 3)^2 + y^2 = 16$ for x. What parts of the circle do the two expressions for x represent? **See margin.**

54. $y = \sqrt{16 - (x+3)^2}$, $y = -\sqrt{16 - (x+3)^2}$

Maintain Your Skills

Mixed Review

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. *(Lesson 8-2)* **57–59. See pp. 469A–469J.**

57. $x = -3y^2 + 1$
58. $y + 2 = -(x - 3)^2$
59. $y = x^2 + 4x$

Find the midpoint of the line segment with endpoints at the given coordinates. *(Lesson 8-1)*

60. $(5, -7), (3, -1)$ **$(4, -4)$**
61. $(2, -9), (-4, 5)$ **$(-1, -2)$**
62. $(8, 0), (-5, 12)$ **$\left(\frac{3}{2}, 6\right)$**

Find all of the rational zeros for each function. *(Lesson 7-5)*

63. $f(x) = x^3 + 5x^2 + 2x - 8$ **$-4, -2, 1$**
64. $g(x) = 2x^3 - 9x^2 + 7x + 6$ **$-\frac{1}{2}, 2, 3$**

65. **PHOTOGRAPHY** The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture? *(Lesson 3-2)* **28 in. by 15 in.**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Assume that all variables are positive.**
(To review ***solving quadratic equations****, see Lesson 6-4.)*

66. $c^2 = 13^2 - 5^2$ **12**
67. $c^2 = 10^2 - 8^2$ **6**
68. $(\sqrt{7})^2 = a^2 - 3^2$ **4**
69. $24^2 = a^2 - 7^2$ **25**
70. $4^2 = 6^2 - b^2$ **$2\sqrt{5}$**
71. $(2\sqrt{14})^2 = 8^2 - b^2$ **$2\sqrt{2}$**

Practice Quiz 1

Lessons 8-1 through 8-3

Find the distance between each pair of points with the given coordinates. *(Lesson 8-1)*

1. $(9, 5), (4, -7)$ **13 units**
2. $(0, -5), (10, -3)$ **$2\sqrt{26}$ units**

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. *(Lesson 8-2)* **3–4. See pp. 469A–469J.**

3. $y^2 = 6x$
4. $y = x^2 + 8x + 20$

5. Find the center and radius of the circle with equation $x^2 + (y - 4)^2 = 49$. Then graph the circle. *(Lesson 8-3)* **(0, 4), 7 units; see pp. 469A–469J for graph.**

4 Assess

Open-Ended Assessment

Speaking Have students explain how to tell from a given equation of a circle how the equation of a circle with its center at the origin and the same radius can be translated to give the graph of the given equation.

Getting Ready for Lesson 8-4

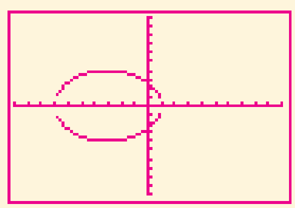
PREREQUISITE SKILL In the process of analyzing and simplifying equations of ellipses in Lesson 8-4, students will solve quadratic equations. Exercises 66–71 should be used to determine your students' familiarity with solving quadratic equations.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 8-1 through 8-3. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Answers

55.



$[-10, 10]$ scl:1 by $[-10, 10]$ scl:1

56. $x = -3 \pm \sqrt{16 - y^2}$; The equations with the $+$ symbol and $-$ symbol represent the right and left halves of the circle, respectively.

Answer

50. A circle can be used to represent the limit at which planes can be detected by radar. Answers should include the following.

- $x^2 + y^2 = 2500$
- The region whose boundary is modeled by $x^2 + y^2 = 4900$ is larger, so there would be more planes to track.

Algebra Activity

A Preview of Lesson 8-4

Getting Started

Objective To derive an understanding of an ellipse as the set of points for which the sum of the distances from two fixed points is constant.

Materials

two thumbtacks	cardboard
string	pencil
grid paper	ruler

Teach

- It may be easier to manipulate the string and pencil if students work in pairs. Make sure that each member of the pair has the chance to draw an ellipse.
- Ask students to stop at one point in the curve they are drawing and ask what the total distance is from that point to one thumbtack plus the distance from the same point to the other thumbtack. **the length of the string minus the distance between the thumbtacks**
- Ask students what the total distance to the tacks is for any point on the ellipse. **the length of the string minus the distance between the thumbtacks**

Assess

In **Exercises 1–12**, students should

- be able to see what changes result from varying the positions of the thumbtacks and the length of the string.
- be able to predict what changes will result from varying the positions of the foci.

Algebra Activity

A Preview of Lesson 8-4

Investigating Ellipses

Follow the steps below to construct another type of conic section.

Step 1 Place two thumbtacks in a piece of cardboard, about 1 foot apart.

Step 2 Tie a knot in a piece of string and loop it around the thumbtacks.

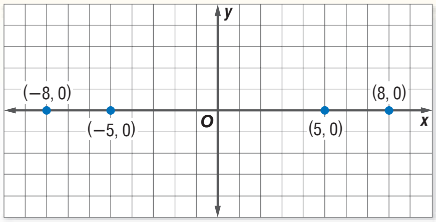
Step 3 Place your pencil in the string. Keep the string tight and draw a curve.

Step 4 Continue drawing until you return to your starting point.

The curve you have drawn is called an **ellipse**. The points where the thumbtacks are located are called the **foci** of the ellipse. *Foci* is the plural of *focus*.

Model and Analyze

Place a large piece of grid paper on a piece of cardboard. **1. See students' work.**



1. Place the thumbtacks at (8, 0) and (−8, 0). Choose a string long enough to loop around both thumbtacks. Draw an ellipse.

2. Repeat Exercise 1, but place the thumbtacks at (5, 0) and (−5, 0). Use the same loop of string and draw an ellipse. How does this ellipse compare to the one in Exercise 1? **See students' work; the ellipse is more circular.**

Place the thumbtacks at each set of points and draw an ellipse.

You may change the length of the loop of string if you like. **3–5. See students' work.**

3. (12, 0), (−12, 0) **4.** (2, 0), (−2, 0) **5.** (14, 4), (−10, 4)

Make a Conjecture

In Exercises 6–10, describe what happens to the shape of an ellipse when each change is made.

6. The thumbtacks are moved closer together. **The ellipse becomes more circular.**

7. The thumbtacks are moved farther apart. **The ellipse becomes more elongated.**

8. The length of the loop of string is increased. **The ellipse becomes larger.**

9. The thumbtacks are arranged vertically. **See pp. 469A–469J.**

10. One thumbtack is removed, and the string is looped around the remaining thumbtack. **The ellipse is a circle.**

11. Pick a point on one of the ellipses you have drawn. Use a ruler to measure the distances from that point to the points where the thumbtacks were located. Add the distances. Repeat for other points on the same ellipse. What relationship do you notice? **The sum of the distances is constant.**

12. Could this activity be done with a rubber band instead of a piece of string? Explain. **See pp. 469A–469J.**

Resource Manager

Teaching Algebra with Manipulatives

- p. 1 (master for grid paper)
- p. 24 (master for rulers)
- p. 266 (student recording sheet)

Glencoe Mathematics Classroom Manipulative Kit

- rulers

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

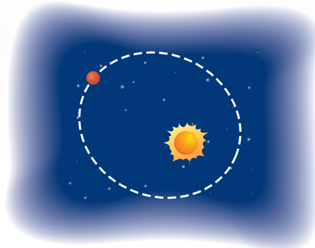
8-4 Ellipses

What You'll Learn

- Write equations of ellipses.
- Graph ellipses.

Why are ellipses important in the study of the solar system?

Fascination with the sky has caused people to wonder, observe, and make conjectures about the planets since the beginning of history. Since the early 1600s, the orbits of the planets have been known to be ellipses with the Sun at a focus.

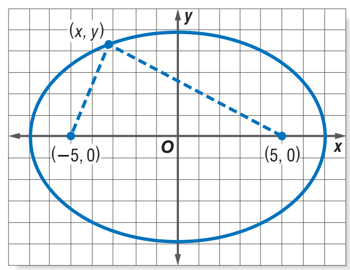


Vocabulary

- ellipse
- foci
- major axis
- minor axis
- center

EQUATIONS OF ELLIPSES As you discovered in the Algebra Activity on page 432, an **ellipse** is the set of all points in a plane such that the sum of the distances from two fixed points is constant. The two fixed points are called the **foci** of the ellipse.

The ellipse at the right has foci at (5, 0) and (−5, 0). The distances from either of the x-intercepts to the foci are 2 units and 12 units, so the sum of the distances from any point with coordinates (x, y) on the ellipse to the foci is 14 units.



You can use the Distance Formula and the definition of an ellipse to find an equation of this ellipse.

$$\underbrace{\text{The distance between } (x, y) \text{ and } (-5, 0)} + \underbrace{\text{the distance between } (x, y) \text{ and } (5, 0)} = 14.$$

$$\sqrt{(x+5)^2+y^2} + \sqrt{(x-5)^2+y^2} = 14$$

$$\sqrt{(x+5)^2+y^2} = 14 - \sqrt{(x-5)^2+y^2} \quad \text{Isolate the radicals.}$$

$$(x+5)^2+y^2 = 196 - 28\sqrt{(x-5)^2+y^2} + (x-5)^2+y^2 \quad \text{Square each side.}$$

$$x^2+10x+25+y^2 = 196 - 28\sqrt{(x-5)^2+y^2} + x^2-10x+25+y^2$$

$$20x - 196 = -28\sqrt{(x-5)^2+y^2} \quad \text{Simplify.}$$

$$5x - 49 = -7\sqrt{(x-5)^2+y^2} \quad \text{Divide each side by 4.}$$

$$25x^2 - 490x + 2401 = 49[(x-5)^2+y^2] \quad \text{Square each side.}$$

$$25x^2 - 490x + 2401 = 49x^2 - 490x + 1225 + 49y^2 \quad \text{Distributive Property}$$

$$-24x^2 - 49y^2 = -1176 \quad \text{Simplify.}$$

$$\frac{x^2}{49} + \frac{y^2}{24} = 1 \quad \text{Divide each side by } -1176.$$

An equation for this ellipse is $\frac{x^2}{49} + \frac{y^2}{24} = 1$.

8-4 Lesson Notes

1 Focus

5-Minute Check Transparency 8-4 Use as a quiz or review of Lesson 8-3.

Mathematical Background notes are available for this lesson on p. 410C.

Why are ellipses important in the study of the solar system?

Ask students:

- What is the solar system? **The Sun and the group of objects orbiting around the Sun.**
- Does the Sun travel around Earth or does Earth travel around the Sun? **Earth travels around the Sun.**
- Describe some of the earlier conjectures about the Sun and Earth. **Sample answers: The Earth was the center of the solar system; the Earth was flat.**

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters

- Study Guide and Intervention, pp. 473–474
- Skills Practice, p. 475
- Practice, p. 476
- Reading to Learn Mathematics, p. 477
- Enrichment, p. 478
- Assessment, pp. 511, 513

Teaching Algebra With Manipulatives Masters, p. 267

Transparencies

5-Minute Check Transparency 8-4
Answer Key Transparencies

Technology

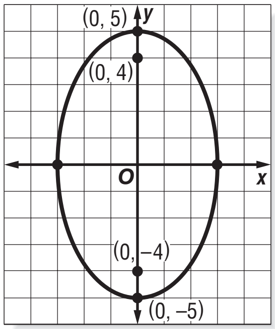
Interactive Chalkboard

2 Teach

EQUATIONS OF ELLIPSES

In-Class Example Power Point®

1 Write an equation for the ellipse shown.



$\frac{y^2}{25} + \frac{x^2}{9} = 1$

Teaching Tip Lead a discussion to clarify how ellipses are different from circles and parabolas in their equations, foci, and other characteristics.

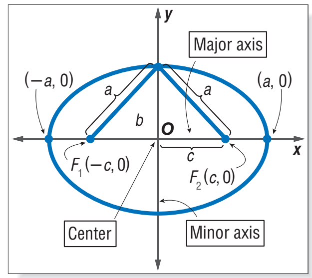
TEACHING TIP
Point out to students that a, not c, is the length of the hypotenuse.

Study Tip

Vertices of Ellipses
The endpoints of each axis are called the *vertices* of the ellipse.

Every ellipse has two axes of symmetry. The points at which the ellipse intersects its axes of symmetry determine two segments with endpoints on the ellipse called the **major axis** and the **minor axis**. The axes intersect at the **center** of the ellipse. The foci of an ellipse always lie on the major axis.

Study the ellipse at the right. The sum of the distances from the foci to any point on the ellipse is the same as the length of the major axis, or $2a$ units. The distance from the center to either focus is c units. By the Pythagorean Theorem, a, b, and c are related by the equation $c^2 = a^2 - b^2$. Notice that the x- and y-intercepts, $(\pm a, 0)$ and $(0, \pm b)$, satisfy the quadratic equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. This is the standard form of the equation of an ellipse with its center at the origin and a horizontal major axis.



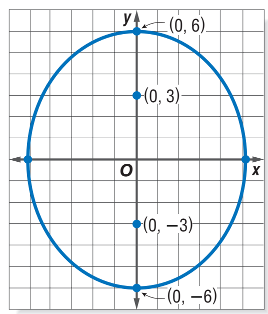
Key Concept *Equations of Ellipses with Centers at the Origin*

Standard Form of Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

In either case, $a^2 \geq b^2$ and $c^2 = a^2 - b^2$. You can determine if the foci are on the x-axis or the y-axis by looking at the equation. If the x^2 term has the greater denominator, the foci are on the x-axis. If the y^2 term has the greater denominator, the foci are on the y-axis.

Example 1 Write an Equation for a Graph

Write an equation for the ellipse shown at the right.



In order to write the equation for the ellipse, we need to find the values of a and b for the ellipse. We know that the length of the major axis of any ellipse is $2a$ units. In this ellipse, the length of the major axis is the distance between the points at (0, 6) and (0, −6). This distance is 12 units.

$2a = 12$ Length of major axis = 12

$a = 6$ Divide each side by 2.

The foci are located at (0, 3) and (0, −3), so $c = 3$. We can use the relationship between a, b, and c to determine the value of b.

$c^2 = a^2 - b^2$ Equation relating a, b, and c

$9 = 36 - b^2$ $c = 3$ and $a = 6$

$b^2 = 27$ Solve for b^2.

Since the major axis is vertical, substitute 36 for a^2 and 27 for b^2 in the form $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. An equation of the ellipse is $\frac{y^2}{36} + \frac{x^2}{27} = 1$.

DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Have students describe some symbols in musical notation that use circles or ellipses.

434 Chapter 8 Conic Sections

Example 2 Write an Equation Given the Lengths of the Axes

MUSEUMS **In an ellipse, sound or light coming from one focus is reflected to the other focus. In a whispering gallery, a person can hear another person whisper from across the room if the two people are standing at the foci. The whispering gallery at the Museum of Science and Industry in Chicago has an elliptical cross section that is 13 feet 6 inches by 47 feet 4 inches.**

a. Write an equation to model this ellipse. Assume that the center is at the origin and the major axis is horizontal.

The length of the major axis is $47\frac{1}{3}$ or $\frac{142}{3}$ feet.

$2a = \frac{142}{3}$ Length of major axis $= \frac{142}{3}$

$a = \frac{71}{3}$ Divide each side by 2.

The length of the minor axis is $13\frac{1}{2}$ or $\frac{27}{2}$ feet.

$2b = \frac{27}{2}$ Length of minor axis $= \frac{27}{2}$

$b = \frac{27}{4}$ Divide each side by 2.

Substitute $a = \frac{71}{3}$ and $b = \frac{27}{4}$ into the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. An equation of the ellipse is $\frac{x^2}{\left(\frac{71}{3}\right)^2} + \frac{y^2}{\left(\frac{27}{4}\right)^2} = 1$.

b. How far apart are the points at which two people should stand to hear each other whisper?

People should stand at the two foci of the ellipse. The distance between the foci is $2c$ units.

$c^2 = a^2 - b^2$ Equation relating a, b, and c

$c = \sqrt{a^2 - b^2}$ Take the square root of each side.

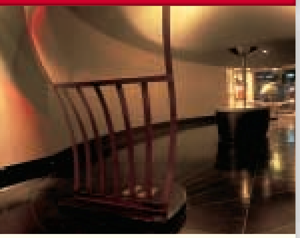
$2c = 2\sqrt{a^2 - b^2}$ Multiply each side by 2.

$2c = 2\sqrt{\left(\frac{71}{3}\right)^2 - \left(\frac{27}{4}\right)^2}$ Substitute $a = \frac{71}{3}$ and $b = \frac{27}{4}$.

$2c \approx 45.37$ Use a calculator.

The points where two people should stand to hear each other whisper are about 45.37 feet or 45 feet 4 inches apart.

More About. . .



Museums

The whispering gallery at Chicago's Museum of Science and Industry has a parabolic dish at each focus to help collect sound.

Source: www.msichicago.org

GRAPH ELLIPSES As with circles, you can use completing the square, symmetry, and transformations to help graph ellipses. An ellipse with its center at the origin is represented by an equation of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$. The ellipse could be translated h units to the right and k units up. This would move the center to the point (h, k). Such a move would be equivalent to replacing x with $x - h$ and replacing y with $y - k$.

Key Concept: Equations of Ellipses with Centers at (h, k)

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical
Foci	$(h \pm c, k)$	$(h, k \pm c)$

www.algebra2.com/extra_examples

In-Class Example

Power Point®

2 **SOUND** A listener is standing in an elliptical room 150 feet wide and 320 feet long. When a speaker stands at one focus and whispers, the best place for the listener to stand is at the other focus.

a. Write an equation to model this ellipse, assuming the major axis is horizontal and the center is at the origin. $\frac{x^2}{160^2} + \frac{y^2}{75^2} = 1$

b. How far apart should the speaker and the listener be in this room? **approximately 282.7 feet**

Teaching Tip Suggest that students make a rough sketch of the situation in these problems.

DAILY INTERVENTION

Unlocking Misconceptions

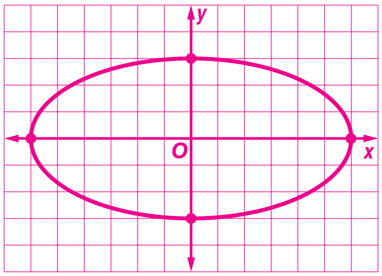
- **Symmetry in Ellipses** Make sure that students understand that an ellipse has two axes of symmetry, the major axis and the minor axis.
- **Identifying Axes** Ask students how they can tell which is the major and which is the minor axis. **The major axis is longer.**

GRAPH ELLIPSES

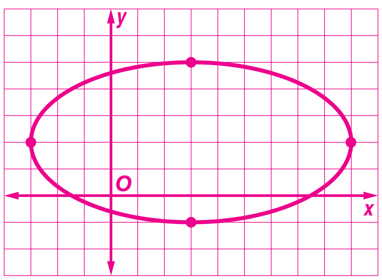
In-Class Examples

Power Point®

3 Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{9} = 1$. Then graph the ellipse. **center: (0, 0); foci: $(3\sqrt{3}, 0)$, $(-3\sqrt{3}, 0)$; major axis: 12; minor axis: 6**



4 Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 - 6x - 16y - 11 = 0$. Then graph the ellipse.
center: (3, 2); foci: $(3\sqrt{3} + 3, 2)$, $(-3\sqrt{3} + 3, 2)$; major axis: 12; minor axis: 6



Study Tip

Graphing Calculator

You can graph an ellipse on a graphing calculator by first solving for y. Then graph the two equations that result on the same screen.

Example 3 Graph an Equation in Standard Form

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $\frac{x^2}{16} + \frac{y^2}{4} = 1$. Then graph the ellipse.

The center of this ellipse is at (0, 0).

Since $a^2 = 16$, $a = 4$. Since $b^2 = 4$, $b = 2$.

The length of the major axis is 2(4) or 8 units, and the length of the minor axis is 2(2) or 4 units. Since the x^2 term has the greater denominator, the major axis is horizontal.

$c^2 = a^2 - b^2$	Equation relating a, b, and c
$c^2 = 4^2 - 2^2$ or 12	$a = 4$, $b = 2$
$c = \sqrt{12}$ or $2\sqrt{3}$	Take the square root of each side.

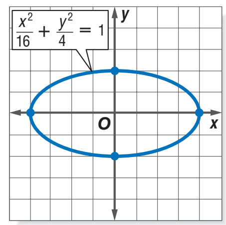
The foci are at $(2\sqrt{3}, 0)$ and $(-2\sqrt{3}, 0)$.

You can use a calculator to find some approximate nonnegative values for x and y that satisfy the equation. Since the ellipse is centered at the origin, it is symmetric about the y-axis. Therefore, the points at (−4, 0), (−3, 1.3), (−2, 1.7), and (−1, 1.9) lie on the graph.

x	y
0	2.0
1	1.9
2	1.7
3	1.3
4	0.0

The ellipse is also symmetric about the x-axis, so the points at (−3, −1.3), (−2, −1.7), (−1, −1.9), (0, −2), (1, −1.9), (2, −1.7), and (3, −1.3) lie on the graph.

Graph the intercepts, (−4, 0), (4, 0), (0, 2), and (0, −2), and draw the ellipse that passes through them and the other points.



If you are given an equation of an ellipse that is not in standard form, write it in standard form first. This will make graphing the ellipse easier.

Example 4 Graph an Equation Not in Standard Form

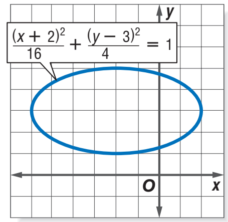
Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $x^2 + 4y^2 + 4x - 24y + 24 = 0$. Then graph the ellipse.

Complete the square for each variable to write this equation in standard form.

$x^2 + 4y^2 + 4x - 24y + 24 = 0$	Original equation
$(x^2 + 4x + ■) + 4(y^2 - 6y + ■) = -24 + ■ + 4(■)$	Complete the squares.
$(x^2 + 4x + 4) + 4(y^2 - 6y + 9) = -24 + 4 + 4(9)$	$\left(\frac{4}{2}\right)^2 = 4$, $\left(\frac{-6}{2}\right)^2 = 9$
$(x + 2)^2 + 4(y - 3)^2 = 16$	Write the trinomials as perfect squares.
$\frac{(x + 2)^2}{16} + \frac{(y - 3)^2}{4} = 1$	Divide each side by 16.

436 Chapter 8 Conic Sections

The graph of this ellipse is the graph from Example 3 translated 2 units to the left and up 3 units. The center is at $(-2, 3)$ and the foci are at $(-2 + 2\sqrt{3}, 0)$ and $(-2 - 2\sqrt{3}, 0)$. The length of the major axis is still 8 units, and the length of the minor axis is still 4 units.



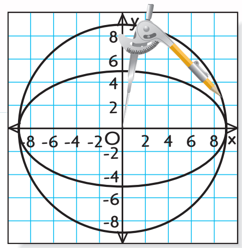
You can use a circle to locate the foci on the graph of a given ellipse.

Algebra Activity

Locating Foci

You can locate the foci of an ellipse by using the following method.

Step 1 Graph an ellipse so that its center is at the origin. Let the endpoints of the major axis be at $(-9, 0)$ and $(9, 0)$, and let the endpoints of the minor axis be at $(0, -5)$ and $(0, 5)$.



Step 2 Use a compass to draw a circle with center at $(0, 0)$ and radius 9 units.

Step 3 Draw the line with equation $y = 5$ and mark the points at which the line intersects the circle.

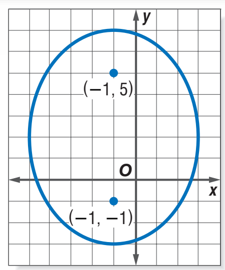
Step 4 Draw perpendicular lines from the points of intersection to the x-axis. The foci of the ellipse are located at the points where the perpendicular lines intersect the x-axis.

Make a Conjecture **See students' work; see margin for explanation.**
Draw another ellipse and locate its foci. Why does this method work?

Check for Understanding

Concept Check

1. **Identify** the axes of symmetry of the ellipse at the right. $x = -1,\ y = 2$



2. **Explain** why a circle is a special case of an ellipse.

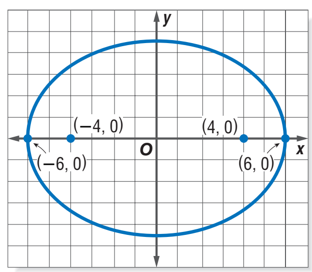
2. See margin.

3. **OPEN ENDED** Write an equation for an ellipse with its center at $(2, -5)$ and a horizontal major axis.
Sample answer: $\frac{(x-2)^2}{4} + \frac{(y+5)^2}{1} = 1$

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6, 11	1, 2
7–10	3, 4

4. Write an equation for the ellipse shown at the right. $\frac{x^2}{36} + \frac{y^2}{20} = 1$



Write an equation for the ellipse that satisfies each set of conditions.

5. endpoints of major axis at $(2, 2)$ and $(2, -10)$, endpoints of minor axis at $(0, -4)$ and $(4, -4)$

5. $\frac{(y+4)^2}{36} + \frac{(x-2)^2}{4} = 1$

6. endpoints of major axis at $(0, 10)$ and $(0, -10)$, foci at $(0, 8)$ and $(0, -8)$ $\frac{y^2}{100} + \frac{x^2}{36} = 1$

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- copy both of the Key Concept summaries of ellipses into their notebooks, with labeled illustrations.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answer

Algebra Activity

Let $(d, 0)$ be the coordinates of the point located on the positive x-axis. This point, the origin, and the point of intersection in the first quadrant of the circle and the ellipse form a right triangle. The length of the hypotenuse is the radius of the circle, which is half the length of the major axis of the ellipse, or a. One leg of the triangle has length d and the other has half the length of the minor axis of the ellipse, or b. By the Pythagorean Theorem, $a^2 = d^2 + b^2$ or $d^2 = a^2 - b^2$. Therefore, d satisfies the equation relating a, b, and c for an ellipse. Thus, one focus of the ellipse is at $(d, 0)$. By symmetry, the other focus is at $(-d, 0)$, which is the other point located by this method.

Answer

2. Let the equation of a circle be $(x - h)^2 + (y - k)^2 = r^2$. Divide each side by r^2 to get $\frac{(x-h)^2}{r^2} + \frac{(y-k)^2}{r^2} = 1$. This is the equation of an ellipse with a and b both equal to r. In other words, a circle is an ellipse whose major and minor axes are both diameters.

Algebra Activity

Materials: grid paper, compass, straightedge

- Suggest that students look at the point where the line intersects the circle (in Step 3) and think about how far that point is from the foci.
- Lead students to recall the relationship that they discovered in the Algebra Activity that was a preview of Lesson 8-4.

About the Exercises…

Organization by Objective

- **Equations of Ellipses:** 12–24
- **Graph Ellipses:** 25–38

Odd/Even Assignments

Exercises 12–21 and 27–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–31 odd, 37, 39–42, 44–57

Average: 13–39 odd, 40–42, 44–57 (optional: 43)

Advanced: 12–38 even, 39–51 (optional: 52–57)

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

7. $\frac{y^2}{18} + \frac{x^2}{9} = 1$

8. $\frac{(x-1)^2}{20} + \frac{(y+2)^2}{4} = 1$

9. $4x^2 + 8y^2 = 32$

10. $x^2 + 25y^2 - 8x + 100y + 91 = 0$

7–10. See margin.

Application

11. **ASTRONOMY** At its closest point, Mercury is 29.0 million miles from the center of the Sun. At its farthest point, Mercury is 43.8 million miles from the center of the Sun. Write an equation for the orbit of Mercury, assuming that the center of the orbit is the origin and the Sun lies on the x-axis. **about $\frac{x^2}{1.33 \times 10^{15}} + \frac{y^2}{1.27 \times 10^{15}} = 1$**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
12–24	1, 2
25–38	3, 4

Extra Practice

See page 846.

16. $\frac{(x+2)^2}{81} + \frac{(y-5)^2}{16} = 1$

17. $\frac{(y-4)^2}{64} + \frac{(x-2)^2}{4} = 1$

18. $\frac{(y-2)^2}{100} + \frac{(x-4)^2}{9} = 1$

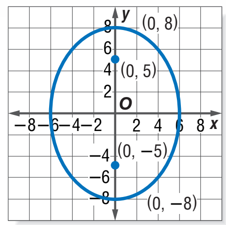
19. $\frac{(x-5)^2}{64} + \frac{(y-4)^2}{\frac{81}{4}} = 1$

20. $\frac{(x-1)^2}{81} + \frac{(y-2)^2}{56} = 1$

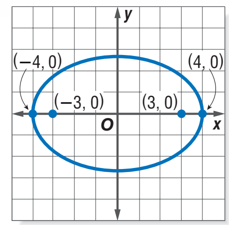
21. $\frac{x^2}{169} + \frac{y^2}{25} = 1$

Write an equation for each ellipse. 12–15. See margin.

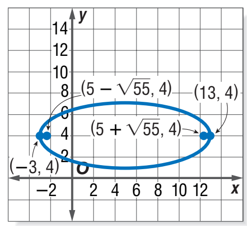
12.



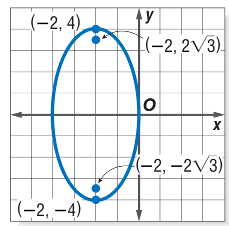
13.



14.



15.



Write an equation for the ellipse that satisfies each set of conditions.

16. endpoints of major axis at (−11, 5) and (7, 5), endpoints of minor axis at (−2, 9) and (−2, 1)

17. endpoints of major axis at (2, 12) and (2, −4), endpoints of minor axis at (4, 4) and (0, 4)

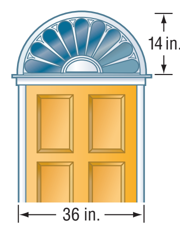
18. major axis 20 units long and parallel to y-axis, minor axis 6 units long, center at (4, 2)

19. major axis 16 units long and parallel to x-axis, minor axis 9 units long, center at (5, 4)

20. endpoints of major axis at (10, 2) and (−8, 2), foci at (6, 2) and (−4, 2)

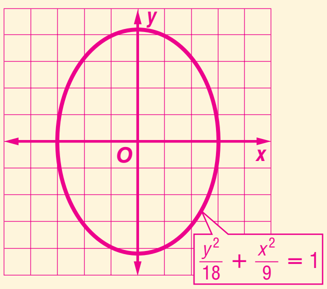
21. endpoints of minor axis at (0, 5) and (0, −5), foci at (12, 0) and (−12, 0)

22. **INTERIOR DESIGN** The rounded top of the window is the top half of an ellipse. Write an equation for the ellipse if the origin is at the midpoint of the bottom edge of the window. **$\frac{x^2}{324} + \frac{y^2}{196} = 1$**

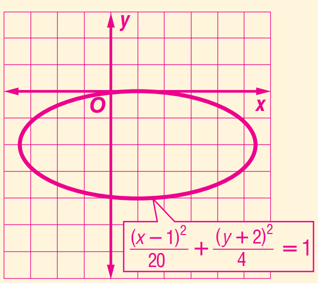


Answers

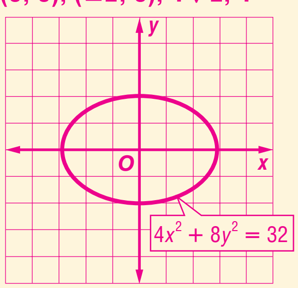
7. (0, 0); (0, ±3); $6\sqrt{2}$; 6



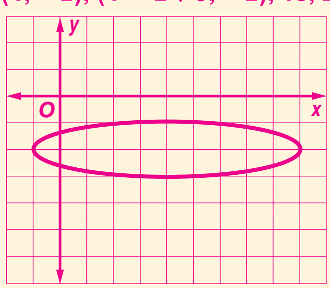
8. (1, −2); (5, −2), (−3, −2); $4\sqrt{5}$; 4



9. (0, 0); (±2, 0); $4\sqrt{2}$; 4



10. (4, −2); $(4 \pm 2\sqrt{6}, -2)$; 10; 2



12. $\frac{y^2}{64} + \frac{x^2}{39} = 1$

13. $\frac{x^2}{16} + \frac{y^2}{7} = 1$

14. $\frac{(x-5)^2}{64} + \frac{(y-4)^2}{9} = 1$

15. $\frac{y^2}{16} + \frac{(x+2)^2}{4} = 1$

23. about $\frac{x^2}{2.02 \times 10^{16}} + \frac{y^2}{2.00 \times 10^{16}} = 1$

23. ASTRONOMY At its closest point, Mars is 128.5 million miles from the Sun. At its farthest point, Mars is 155.0 million miles from the Sun. Write an equation for the orbit of Mars. Assume that the center of the orbit is the origin, the Sun lies on the x-axis, and the radius of the Sun is 400,000 miles.

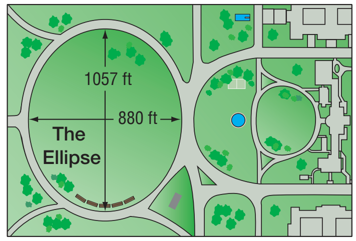
More About. . .

White House

The Ellipse, also known as President's Park South, has an area of about 16 acres.

Source: www.nps.gov

24. WHITE HOUSE There is an open area south of the White House known as the Ellipse. Write an equation to model the Ellipse. Assume that the origin is at the center of the Ellipse.



24. $\frac{x^2}{193{,}600} + \frac{y^2}{279{,}312.25} = 1$

25. Write the equation $10x^2 + 2y^2 = 40$ in standard form. $\frac{y^2}{20} + \frac{x^2}{4} = 1$

26. What is the standard form of the equation $x^2 + 6y^2 - 2x + 12y - 23 = 0$? $\frac{(x-1)^2}{30} + \frac{(y+1)^2}{5} = 1$

27–38. See pp. 469A–469J for graphs.

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

27. $\frac{y^2}{10} + \frac{x^2}{5} = 1$ $(0, 0); (0, \pm\sqrt{5}); 2\sqrt{10}; 2\sqrt{5}$

28. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ $(0, 0); (\pm 4, 0); 10; 6$

29. $\frac{(x+8)^2}{144} + \frac{(y-2)^2}{81} = 1$

30. $\frac{(y+11)^2}{144} + \frac{(x-5)^2}{121} = 1$

31. $3x^2 + 9y^2 = 27$ $(0, 0); (\pm\sqrt{6}, 0); 6; 2\sqrt{3}$

32. $27x^2 + 9y^2 = 81$ $(0, 0); (0, \pm\sqrt{6}); 6; 2\sqrt{3}$

★ 33. $16x^2 + 9y^2 = 144$

34. $36x^2 + 81y^2 = 2916$

★ 35. $3x^2 + y^2 + 18x - 2y + 4 = 0$

★ 36. $x^2 + 5y^2 + 4x - 70y + 209 = 0$

37. $7x^2 + 3y^2 - 28x - 12y = -19$ $(2, 2); (2, 4), (2, 0); 2\sqrt{7}; 2\sqrt{3}$

★ 38. $16x^2 + 25y^2 + 32x - 150y = 159$ $(-1, 3); (2, 3), (-4, 3); 10; 8$

29. $(-8, 2); (-8 \pm 3\sqrt{7}, 2); 24; 18$

30. $(5, -11); (5, -11 \pm \sqrt{23}); 24; 22$

33. $(0, 0); (0, \pm\sqrt{7}); 8; 6$

34. $(0, 0); (\pm 3\sqrt{5}, 0); 18; 12$

35. $(-3, 1); (-3, 5), (-3, -3); 4\sqrt{6}; 4\sqrt{2}$

36. $(-2, 7); (-2 \pm 4\sqrt{2}, 7); 4\sqrt{10}; 4\sqrt{2}$

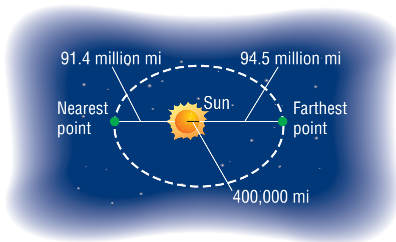
39. CRITICAL THINKING Find an equation for the ellipse with foci at $(\sqrt{3}, 0)$ and $(-\sqrt{3}, 0)$ that passes through $(0, 3)$. $\frac{x^2}{12} + \frac{y^2}{9} = 1$

40. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 469A–469J.**

Why are ellipses important in the study of the solar system?

Include the following in your answer:

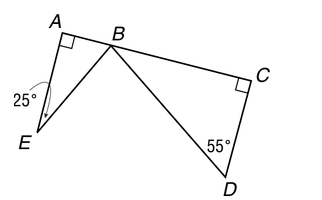
- why an equation that is an accurate model of the path of a planet might be useful, and
- the distance from the center of Earth's orbit to the center of the Sun given that the Sun is at a focus of the orbit of Earth. Use the information in the figure at the right.



Standardized Test Practice

41. In the figure, A, B, and C are collinear. What is the measure of $\angle DBE$? **C**

(A) 40° (B) 65° (C) 80° (D) 100°



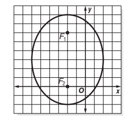
www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 473 (shown) and p. 474

Equations of Ellipses An **ellipse** is the set of all points in a plane such that the *sum* of the distances from two given points in the plane, called the foci, is constant. An ellipse has two axes of symmetry which contain the **major** and **minor axes**. In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 - b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Direction of Major Axis	Horizontal	Vertical
Foci	$(h + c, k), (h - c, k)$	$(h, k - c), (h, k + c)$
Length of Major Axis	$2a$ units	$2a$ units
Length of Minor Axis	$2b$ units	$2b$ units

Example **Write an equation for the ellipse shown.**



The length of the major axis is the distance between $(-2, -2)$ and $(-2, 8)$. This distance is 10 units.

$2a = 10$, so $a = 5$

The foci are located at $(-2, 6)$ and $(-2, 0)$, so $c = 3$.

$b^2 = a^2 - c^2$
$= 25 - 9$
$= 16$

The center of the ellipse is at $(-2, 3)$, so $h = -2$, $k = 3$, $a^2 = 25$, and $b^2 = 16$. The major axis is vertical.

An equation of the ellipse is $\frac{(y-3)^2}{25} + \frac{(x+2)^2}{16} = 1$.

Exercises

Write an equation for the ellipse that satisfies each set of conditions.

1. endpoints of major axis at $(-7, 2)$ and $(5, 2)$, endpoints of minor axis at $(-1, 0)$ and $(-1, 4)$ $\frac{(x+1)^2}{36} + \frac{(y-2)^2}{4} = 1$

2. major axis 8 units long and parallel to the x-axis, minor axis 2 units long, center at $(-2, -5)$ $\frac{(x+2)^2}{16} + (y+5)^2 = 1$

3. endpoints of major axis at $(-8, 4)$ and $(4, 4)$, foci at $(-3, 4)$ and $(-1, 4)$ $\frac{(x+2)^2}{36} + \frac{(y-4)^2}{35} = 1$

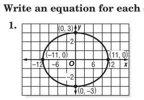
4. endpoints of major axis at $(3, 2)$ and $(3, -14)$, endpoints of minor axis at $(-1, -6)$ and $(7, -6)$ $\frac{(y+6)^2}{64} + \frac{(x-3)^2}{16} = 1$

5. minor axis 6 units long and parallel to the x-axis, major axis 12 units long, center at $(6, 1)$ $\frac{(y-1)^2}{36} + \frac{(x-6)^2}{9} = 1$

Skills Practice, p. 475 and Practice, p. 476 (shown)

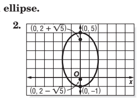
Write an equation for each ellipse.

1.



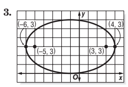
$\frac{x^2}{121} + \frac{y^2}{9} = 1$

2.



$\frac{(y-2)^2}{9} + \frac{x^2}{4} = 1$

3.



$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$

Write an equation for the ellipse that satisfies each set of conditions.

4. endpoints of major axis at $(-9, 0)$ and $(9, 0)$, endpoints of minor axis at $(0, 3)$ and $(0, -3)$ $\frac{x^2}{81} + \frac{y^2}{9} = 1$

5. endpoints of major axis at $(4, 2)$ and $(4, -8)$, endpoints of minor axis at $(1, -3)$ and $(7, -3)$ $\frac{(y+3)^2}{25} + \frac{(x-4)^2}{9} = 1$

6. major axis 20 units long and parallel to x-axis, minor axis 10 units long, center at $(2, 1)$ $\frac{(x-2)^2}{100} + \frac{(y-1)^2}{25} = 1$

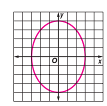
7. major axis 10 units long, minor axis 6 units long and parallel to x-axis, center at $(2, -4)$ $\frac{(y+4)^2}{25} + \frac{(x-2)^2}{9} = 1$

8. major axis 16 units long, center at $(0, 0)$, foci at $(0, 2\sqrt{15})$ and $(0, -2\sqrt{15})$ $\frac{y^2}{64} + \frac{x^2}{4} = 1$

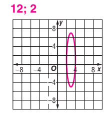
9. endpoints of minor axis at $(0, 2)$ and $(0, -2)$, foci at $(-4, 0)$ and $(4, 0)$ $\frac{x^2}{20} + \frac{y^2}{4} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

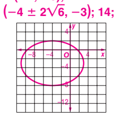
10. $\frac{y^2}{16} + \frac{x^2}{9} = 1$ $(0, 0); (0, \pm\sqrt{7}); 8; 6$



11. $\frac{(y-1)^2}{36} + \frac{(x-3)^2}{1} = 1$ $(3, 1); (3, 1 \pm \sqrt{35}); 12; 2$



12. $\frac{(x+4)^2}{49} + \frac{(y+3)^2}{25} = 1$ $(-4, -3); (-4 \pm 2\sqrt{6}, -3); 14; 10$



13. **SPORTS** An ice skater traces two congruent ellipses to form a figure eight. Assume that the center of the first loop is at the origin, with the second loop to its right. Write an equation to model the first loop if its major axis (along the x-axis) is 12 feet long and its minor axis is 6 feet long. Write another equation to model the second loop. $\frac{x^2}{36} + \frac{y^2}{9} = 1$; $\frac{(x-12)^2}{36} + \frac{y^2}{9} = 1$

Reading to Learn Mathematics, p. 477 ELL

Pre-Activity **Why are ellipses important in the study of the solar system?**

Read the introduction to Lesson 8-4 at the top of page 433 in your textbook.

Is the Earth always the same distance from the Sun? Explain your answer using the words *circle* and *ellipse*. **No; if the Earth's orbit were a circle, it would always be the same distance from the Sun because every point on a circle is the same distance from the center. However, the Earth's orbit is an ellipse, and the points on an ellipse are not all the same distance from the center.**

Reading the Lesson

1. An ellipse is the set of all points in a plane such that the **sum** of the distances from two fixed points is **constant**. The two fixed points are called the **foci** of the ellipse.

2. Consider the ellipse with equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$.
   a. For this equation, a = **3** and b = **2**.
   b. Write an equation that relates the values of a, b, and c. $c^2 = a^2 - b^2$
   c. Find the value of c for this ellipse. $\sqrt{5}$

3. Consider the ellipses with equations $\frac{y^2}{25} + \frac{x^2}{16} = 1$ and $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Complete the following table to describe characteristics of their graphs.

Standard Form of Equation	$\frac{y^2}{25} + \frac{x^2}{16} = 1$	$\frac{x^2}{9} + \frac{y^2}{4} = 1$
Direction of Major Axis	vertical	horizontal
Direction of Minor Axis	horizontal	vertical
Foci	$(0, 3), (0, -3)$	$(\sqrt{5}, 0), (-\sqrt{5}, 0)$
Length of Major Axis	10 units	6 units
Length of Minor Axis	8 units	4 units

Helping You Remember

4. Some students have trouble remembering the two standard forms for the equation of an ellipse. How can you remember which term comes first and where to place a and b in these equations? **The x-axis is horizontal. If the major axis is horizontal, the first term is $\frac{x^2}{a^2}$. The y-axis is vertical. If the major axis is vertical, the first term is $\frac{y^2}{a^2}$. a is always the larger of the numbers a and b.**

Enrichment, p. 478

Eccentricity

In an ellipse, the ratio $\frac{c}{d}$ is called the **eccentricity** and is denoted by the letter e. Eccentricity measures the elongation of an ellipse. The closer e is to 0, the more an ellipse looks like a circle. The closer e is to 1, the more elongated it is. Recall that the equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ where a is the length of the major axis, and that $c = \sqrt{a^2 - b^2}$.

Find the eccentricity of each ellipse rounded to the nearest hundredth.

1. $\frac{x^2}{9} + \frac{y^2}{36} = 1$ **0.87**

2. $\frac{x^2}{81} + \frac{y^2}{9} = 1$ **0.94**

3. $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **0.75**

4 Assess

Open-Ended Assessment

Speaking Have students draw and label the various parts of an ellipse whose major axis is on the x-axis and one whose major axis is on the y-axis, and then explain the differences.

Tips for New Teachers

Intervention Suggest that students make a summary of the various quadratic graphs on a large index card so that they can easily refer to it as they solve problems. Clear up any questions they may have about the meaning of the various variables and vocabulary words.

Getting Ready for Lesson 8-5

PREREQUISITE SKILL In the process of graphing hyperbolas in Lesson 8-5, students will graph lines that represent asymptotes of the hyperbolas. Exercises 52–57 should be used to determine your students' familiarity with graphing lines.

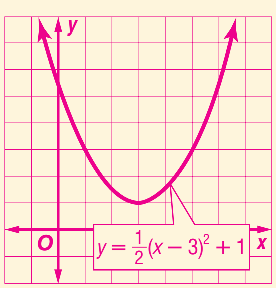
Assessment Options

Quiz (Lessons 8-3 and 8-4) is available on p. 511 of the *Chapter 8 Resource Masters*.

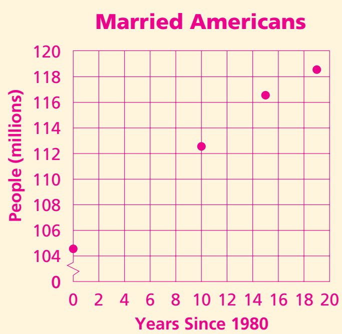
Mid-Chapter Test (Lessons 8-1 through 8-4) is available on p. 513 of the *Chapter 8 Resource Masters*.

Answers

48.



49.



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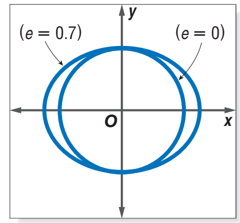
42. $\sqrt{25 + 144} =$ **B**

Ⓐ 7 Ⓑ 13 Ⓒ 17 Ⓓ 169

Extending the Lesson

43. **ASTRONOMY** In an ellipse, the ratio $\frac{c}{a}$ is called the **eccentricity** and is denoted by the letter e. Eccentricity measures the elongation of an ellipse. As shown in the graph at the right, the closer e is to 0, the more an ellipse looks like a circle. Pluto has the most eccentric orbit in our solar system with $e \approx 0.25$. Find an equation to model the orbit of Pluto, given that the length of the major axis is about 7.34 billion miles. Assume that the major axis is horizontal and that the center of the orbit is the origin.



about $\frac{x^2}{1.35 \times 10^{19}} + \frac{y^2}{1.26 \times 10^{19}} = 1$

Maintain Your Skills

Mixed Review

Write an equation for the circle that satisfies each set of conditions. *(Lesson 8-3)*

44. center $(3, -2)$, radius 5 units **$(x - 3)^2 + (y + 2)^2 = 25$**

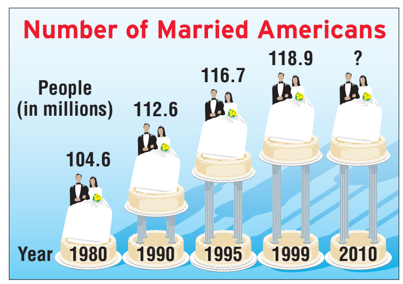
45. endpoints of a diameter at $(5, -9)$ and $(3, 11)$ **$(x - 4)^2 + (y - 1)^2 = 101$**

46. center $(-1, 0)$, passes through $(2, -6)$ **$(x + 1)^2 + y^2 = 45$**

47. center $(4, -1)$, tangent to y-axis **$(x - 4)^2 + (y + 1)^2 = 16$**

48. Write an equation of a parabola with vertex $(3, 1)$ and focus $\left(3, 1\frac{1}{2}\right)$. Then draw the graph. *(Lesson 8-2)* **$y = \frac{1}{2}(x - 3)^2 + 1$; See margin for graph.**

MARRIAGE **For Exercises 49–51, use the table at the right that shows the number of married Americans over the last few decades.** *(Lesson 2-5)*



Source: U.S. Census Bureau

49. See margin.

49. Draw a scatter plot in which x is the number of years since 1980.

50. Sample answer using (0, 104.6) and (10, 112.6): $y = 0.8x + 104.6$

50. Find a prediction equation.

51. Predict the number of married Americans in 2010. **Sample answer: 128,600,000**

Online Research **Data Update** For the latest statistics on marriage and other characteristics of the population, visit www.algebra2.com/data_update to learn more.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Graph the line with the given equation.**
*(To review **graphing lines**, see Lessons 2-1, 2-2, and 2-3.)* **52–57. See pp. 469A–469J.**

52. $y = 2x$
53. $y = -2x$
54. $y = -\frac{1}{2}x$
55. $y = \frac{1}{2}x$
56. $y + 2 = 2(x - 1)$
57. $y + 2 = -2(x - 1)$

8-5 Hyperbolas

What You'll Learn

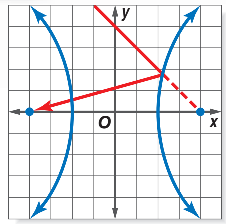
- Write equations of hyperbolas.
- Graph hyperbolas.

Vocabulary

- hyperbola
- foci
- center
- vertex
- asymptote
- transverse axis
- conjugate axis

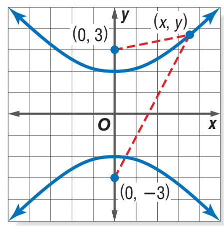
How are hyperbolas different from parabolas?

A hyperbola is a conic section with the property that rays directed toward one focus are reflected toward the other focus. Notice that, unlike the other conic sections, a hyperbola has two branches.



EQUATIONS OF HYPERBOLAS A **hyperbola** is the set of all points in a plane such that the absolute value of the difference of the distances from two fixed points, called the **foci**, is constant.

The hyperbola at the right has foci at (0, 3) and (0, −3). The distances from either of the y-intercepts to the foci are 1 unit and 5 units, so the difference of the distances from any point with coordinates (x, y) on the hyperbola to the foci is 4 or −4 units, depending on the order in which you subtract.



You can use the Distance Formula and the definition of a hyperbola to find an equation of this hyperbola.

The distance between (x, y) and $(0, 3)$ − the distance between (x, y) and $(0, -3)$ = ±4.

$$\sqrt{x^2 + (y-3)^2} - \sqrt{x^2 + (y+3)^2} = \pm 4$$

$$\sqrt{x^2 + (y-3)^2} = \pm 4 + \sqrt{x^2 + (y+3)^2} \quad \text{Isolate the radicals.}$$

$$x^2 + (y-3)^2 = 16 \pm 8\sqrt{x^2 + (y+3)^2} + x^2 + (y+3)^2 \quad \text{Square each side.}$$

$$x^2 + y^2 - 6y + 9 = 16 \pm 8\sqrt{x^2 + (y+3)^2} + x^2 + y^2 + 6y + 9$$

$$-12y - 16 = \pm 8\sqrt{x^2 + (y+3)^2} \quad \text{Simplify.}$$

$$3y + 4 = \pm 2\sqrt{x^2 + (y+3)^2} \quad \text{Divide each side by } -4.$$

$$9y^2 + 24y + 16 = 4[x^2 + (y+3)^2] \quad \text{Square each side.}$$

$$9y^2 + 24y + 16 = 4x^2 + 4y^2 + 24y + 36 \quad \text{Distributive Property}$$

$$5y^2 - 4x^2 = 20 \quad \text{Simplify.}$$

$$\frac{y^2}{4} - \frac{x^2}{5} = 1 \quad \text{Divide each side by 20.}$$

An equation of this hyperbola is $\frac{y^2}{4} - \frac{x^2}{5} = 1$.

8-5 Lesson Notes

1 Focus

5-Minute Check Transparency 8-5 Use as a quiz or review of Lesson 8-4.

Mathematical Background notes are available for this lesson on p. 410D.

How are hyperbolas different from parabolas?

Ask students:

- Why are parabolas, circles, ellipses, and hyperbolas called conic sections? **They are cross sections that result when a plane intersects a double right circular cone.**
- Where is a ray directed toward one focus and reflected toward the other? **inside or between the two branches of the hyperbola**

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters

- Study Guide and Intervention, pp. 479–480
- Skills Practice, p. 481
- Practice, p. 482
- Reading to Learn Mathematics, p. 483
- Enrichment, p. 484

Transparencies

5-Minute Check Transparency 8-5
Real-World Transparency 8
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 16
Interactive Chalkboard

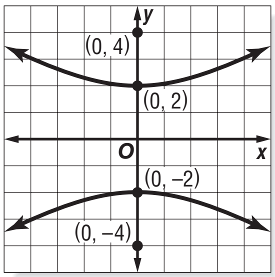
2 Teach

EQUATIONS OF HYPERBOLAS

In-Class Examples

Power Point®

1 Write an equation for the hyperbola shown.



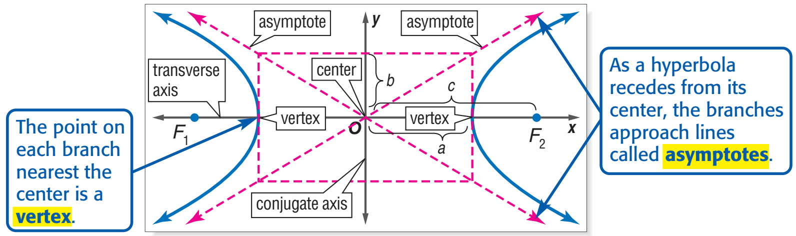
$\frac{y^2}{4} - \frac{x^2}{12} = 1$

2 **NAVIGATION** A ship notes that the difference of its distance from two LORAN stations that are located at (−70, 0) and (70, 0) is 70 nautical miles. Write an equation for the hyperbola on which the ship lies.

$\frac{x^2}{1225} - \frac{y^2}{3675} = 1$

Teaching Tip Tell students to make a rough sketch of the situation in problems such as these.

The diagram below shows the parts of a hyperbola.



A hyperbola has some similarities to an ellipse. The distance from the **center** to a vertex is a units. The distance from the center to a focus is c units. There are two axes of symmetry. The **transverse axis** is a segment of length $2a$ whose endpoints are the vertices of the hyperbola. The **conjugate axis** is a segment of length $2b$ units that is perpendicular to the transverse axis at the center. The values of a, b, and c are related differently for a hyperbola than for an ellipse. For a hyperbola, $c^2 = a^2 + b^2$. The table below summarizes many of the properties of hyperbolas with centers at the origin.

Study Tip

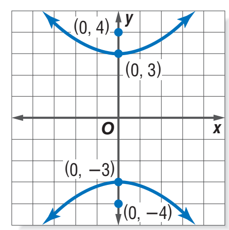
Reading Math
In the standard form of a hyperbola, the squared terms are subtracted (−). For an ellipse, they are added (+).

Key Concept *Equations of Hyperbolas with Centers at the Origin*

Standard Form of Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Foci	$(c, 0), (-c, 0)$	$(0, c), (0, -c)$
Vertices	$(a, 0), (-a, 0)$	$(0, a), (0, -a)$
Length of Transverse Axis	$2a$ units	$2a$ units
Length of Conjugate Axis	$2b$ units	$2b$ units
Equations of Asymptotes	$y = \pm\frac{b}{a}x$	$y = \pm\frac{a}{b}x$

Example 1 *Write an Equation for a Graph*

Write an equation for the hyperbola shown at the right.



The center is the midpoint of the segment connecting the vertices, or (0, 0).

The value of a is the distance from the center to a vertex, or 3 units. The value of c is the distance from the center to a focus, or 4 units.

$c^2 = a^2 + b^2$ Equation relating a, b, and c for a hyperbola

$4^2 = 3^2 + b^2$ $c = 4$, $a = 3$

$16 = 9 + b^2$ Evaluate the squares.

$7 = b^2$ Solve for b^2.

Since the transverse axis is vertical, the equation is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{7} = 1$.

DAILY INTERVENTION

Unlocking Misconceptions

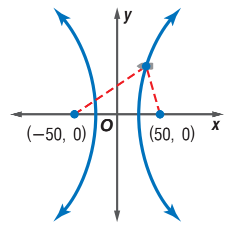
Some students may think that a hyperbola has the shape of two parabolas. Explain that this is not true, and encourage students to draw a parabola on thin paper and place it over a hyperbola to see that the shapes of these curves are different.

442 Chapter 8 Conic Sections

Example 2 Write an Equation Given the Foci and Transverse Axis

NAVIGATION **The LORAN navigational system is based on hyperbolas. Two stations send out signals at the same time. A ship notes the difference in the times at which it receives the signals. The ship is on a hyperbola with the stations at the foci. Suppose a ship determines that the difference of its distances from two stations is 50 nautical miles. The stations are 100 nautical miles apart. Write an equation for a hyperbola on which the ship lies if the stations are at (−50, 0) and (50, 0).**

First, draw a figure. By studying either of the x-intercepts, you can see that the difference of the distances from any point on the hyperbola to the stations at the foci is the same as the length of the transverse axis, or $2a$. Therefore, $2a = 50$, or $a = 25$. According to the coordinates of the foci, $c = 50$.



Use the values of a and c to determine the value of b for this hyperbola.

$c^2 = a^2 + b^2$	Equation relating a, b, and c for a hyperbola
$50^2 = 25^2 + b^2$	$c = 50$, $a = 25$
$2500 = 625 + b^2$	Evaluate the squares.
$1875 = b^2$	Solve for b^2.

Since the transverse axis is horizontal, the equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Substitute the values for a^2 and b^2. An equation of the hyperbola is $\frac{x^2}{625} - \frac{y^2}{1875} = 1$.

More About. . .

Navigation

LORAN stands for *Long Range Navigation*. The LORAN system is generally accurate to within 0.25 nautical mile.

Source: U.S. Coast Guard

GRAPH HYPERBOLAS

So far, you have studied hyperbolas that are centered at the origin. A hyperbola may be translated so that its center is at (h, k). This corresponds to replacing x by $x - h$ and y by $y - k$ in both the equation of the hyperbola and the equations of the asymptotes.

Key Concept *Equations of Hyperbolas with Centers at (h, k)*

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Direction of Transverse Axis	horizontal	vertical
Equations of Asymptotes	$y - k = \pm\frac{b}{a}(x - h)$	$y - k = \pm\frac{a}{b}(x - h)$

It is easier to graph a hyperbola if the asymptotes are drawn first. To graph the asymptotes, use the values of a and b to draw a rectangle with dimensions $2a$ and $2b$. The diagonals of the rectangle should intersect at the center of the hyperbola. The asymptotes will contain the diagonals of the rectangle.

Example 3 Graph an Equation in Standard Form

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$. Then graph the hyperbola.

The center of this hyperbola is at the origin. According to the equation, $a^2 = 9$ and $b^2 = 4$, so $a = 3$ and $b = 2$. The coordinates of the vertices are (3, 0) and (−3, 0).

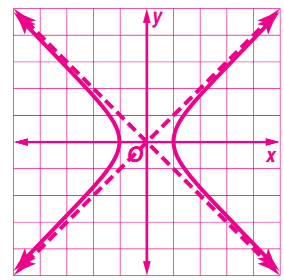
(continued on the next page)

www.algebra2.com/extra_examples

GRAPH HYPERBOLAS

In-Class Example Power Point®

3 Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 = 1$. Then graph the hyperbola.
vertices: (−1, 0), (1, 0);
foci: $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$;
asymptotes: $y = x$, $y = -x$



DAILY INTERVENTION

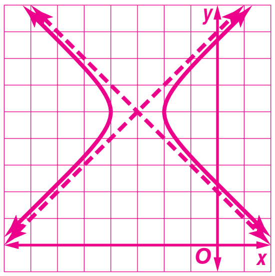
Differentiated Instruction

Logical Have students create a chart for a classroom poster that summarizes all the facts and equations for each type of conic section, with illustrations.

In-Class Example

Power Point®

4 Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $x^2 - y^2 + 6x + 10y - 17 = 0$. Then graph the hyperbola. **vertices: $(-4, 5)$, $(-2, 5)$; foci: $(\sqrt{2} - 3, 5)$, $(-\sqrt{2} - 3, 5)$; asymptotes: $y = x + 8$, $y = -x + 2$**



Teaching Tip Suggest that students make a list of the values of a, b, c, h, and k. This will help avoid confusions about the signs of h and k.

$c^2 = a^2 + b^2$ Equation relating a, b, and c for a hyperbola

$c^2 = 3^2 + 2^2$ $a = 3$, $b = 2$

$c^2 = 13$ Simplify.

$c = \sqrt{13}$ Take the square root of each side.

The foci are at $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$.

The equations of the asymptotes are $y = \pm\frac{b}{a}x$ or $y = \pm\frac{2}{3}x$.

You can use a calculator to find some approximate nonnegative values for x and y that satisfy the equation. Since the hyperbola is centered at the origin, it is symmetric about the y-axis. Therefore, the points at $(-8, 4.9)$, $(-7, 4.2)$, $(-6, 3.5)$, $(-5, 2.7)$, $(-4, 1.8)$, and $(-3, 0)$ lie on the graph.

x	y
3	0
4	1.8
5	2.7
6	3.5
7	4.2
8	4.9

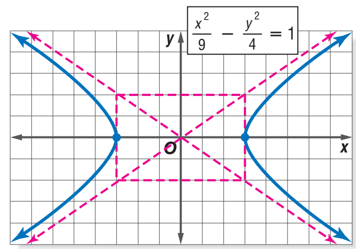
The hyperbola is also symmetric about the x-axis, so the points at $(-8, -4.9)$, $(-7, -4.2)$, $(-6, -3.5)$, $(-5, -2.7)$, $(-4, -1.8)$, $(4, -1.8)$, $(5, -2.7)$, $(6, -3.5)$, $(7, -4.2)$, and $(8, -4.9)$ also lie on the graph.

Study Tip

Graphing Calculator

You can graph a hyperbola on a graphing calculator. Similar to an ellipse, first solve the equation for y. Then graph the two equations that result on the same screen.

Draw a 6-unit by 4-unit rectangle. The asymptotes contain the diagonals of the rectangle. Graph the vertices, which, in this case, are the x-intercepts. Use the asymptotes as a guide to draw the hyperbola that passes through the vertices and the other points. The graph does not intersect the asymptotes.



When graphing a hyperbola given an equation that is not in standard form, begin by rewriting the equation in standard form.

Example 4 Graph an Equation Not in Standard Form

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with equation $4x^2 - 9y^2 - 32x - 18y + 19 = 0$. Then graph the hyperbola.

Complete the square for each variable to write this equation in standard form.

$4x^2 - 9y^2 - 32x - 18y + 19 = 0$ Original equation

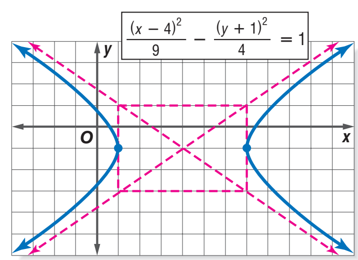
$4(x^2 - 8x + \blacksquare) - 9(y^2 + 2y + \blacksquare) = -19 + 4(\blacksquare) - 9(\blacksquare)$ Complete the squares.

$4(x^2 - 8x + 16) - 9(y^2 + 2y + 1) = -19 + 4(16) - 9(1)$

$4(x - 4)^2 - 9(y + 1)^2 = 36$ Write the trinomials as perfect squares.

$\frac{(x-4)^2}{9} - \frac{(y+1)^2}{4} = 1$ Divide each side by 36.

The graph of this hyperbola is the graph from Example 3 translated 4 units to the right and down 1 unit. The vertices are at $(7, -1)$ and $(1, -1)$, and the foci are at $(4 + \sqrt{13}, -1)$ and $(4 - \sqrt{13}, -1)$. The equations of the asymptotes are $y + 1 = \pm\frac{2}{3}(x - 4)$.



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Check for Understanding

Concept Check

2. As k increases, the branches of the hyperbola become wider.

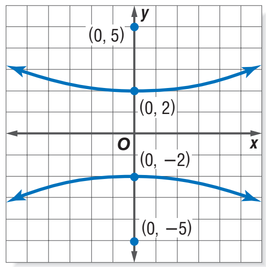
1. **Determine** whether the statement is *sometimes*, *always*, or *never* true. *The graph of a hyperbola is symmetric about the x-axis.* **sometimes**
2. **Describe** how the graph of $y^2 - \frac{x^2}{k^2} = 1$ changes as k increases.
3. **OPEN ENDED** Find a counterexample to the following statement. *If the equation of a hyperbola is* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, *then* $a^2 \geq b^2$. **Sample answer:** $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1, 2
6–10	3, 4

4. $\frac{y^2}{4} - \frac{x^2}{21} = 1$

4. Write an equation for the hyperbola shown at the right.
5. A hyperbola has foci at (4, 0) and (−4, 0). The value of a is 1. Write an equation for the hyperbola. $\frac{x^2}{1} - \frac{y^2}{15} = 1$



6–9. See margin.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

6. $\frac{y^2}{18} - \frac{x^2}{20} = 1$
7. $\frac{(y+6)^2}{20} - \frac{(x-1)^2}{25} = 1$
8. $x^2 - 36y^2 = 36$
9. $5x^2 - 4y^2 - 40x - 16y - 36 = 0$

Application

10. $(0, \pm 15)$; $(0, \pm 25)$; $y = \pm\frac{3}{4}x$; See margin for graph.

10. **ASTRONOMY** Comets that pass by Earth only once may follow hyperbolic paths. Suppose a comet's path is modeled by a branch of the hyperbola with equation $\frac{y^2}{225} - \frac{x^2}{400} = 1$. Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola. Then graph the hyperbola.

★ indicates increased difficulty

Practice and Apply

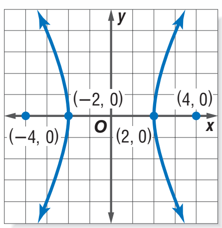
Homework Help

For Exercises	See Examples
11–20, 35	1, 2
21–34, 36–38	3, 4

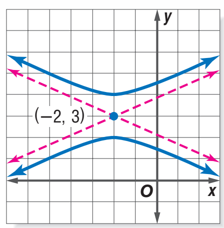
Extra Practice

See page 846.

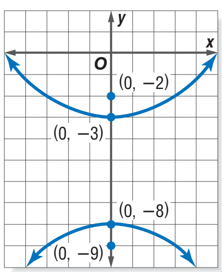
Write an equation for each hyperbola.

11. 

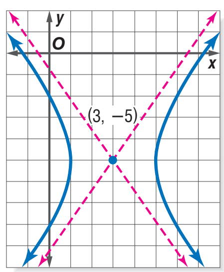
$\frac{x^2}{4} - \frac{y^2}{12} = 1$

12. 

$\frac{(y-3)^2}{1} - \frac{(x+2)^2}{4} = 1$

13. 

$\frac{\left(y + \frac{11}{2}\right)^2}{\frac{25}{4}} - \frac{x^2}{6} = 1$

14. 

$\frac{(x-3)^2}{4} - \frac{(y+5)^2}{9} = 1$

www.algebra2.com/self_check_quiz

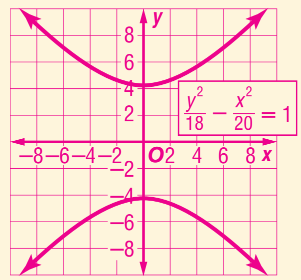
3 Practice/Apply

Study Notebook

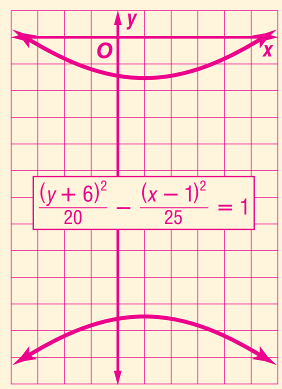
Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- copy both of the Key Concept summaries of hyperbolas into their notebooks, with labeled illustrations.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answers

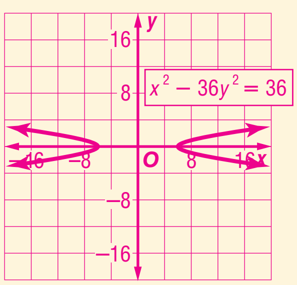
6. $(0, \pm 3\sqrt{2})$; $(0, \pm\sqrt{38})$; $y = \pm\frac{3\sqrt{10}}{10}x$



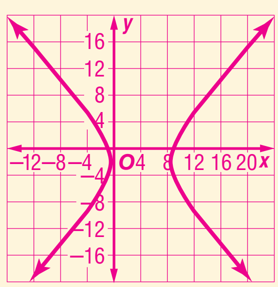
7. $(1, -6 \pm 2\sqrt{5})$; $(1, -6 \pm 3\sqrt{5})$; $y + 6 = \pm\frac{2\sqrt{5}}{5}(x - 1)$



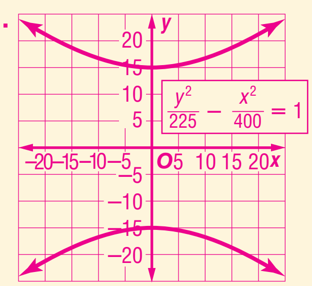
8. $(\pm 6, 0)$; $(\pm\sqrt{37}, 0)$; $y = \pm\frac{1}{6}x$



9. $(4 \pm 2\sqrt{5}, -2)$; $(4 \pm 3\sqrt{5}, -2)$; $y + 2 = \pm\frac{\sqrt{5}}{2}(x - 4)$



10.



About the Exercises...

Organization by Objective

- **Equations of Hyperbolas:** 11–20, 35
- **Graph Hyperbolas:** 21–34, 36–38

Odd/Even Assignments

Exercises 11–34 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 11–31 odd, 35–38, 40–42, 47–49

Average: 11–35 odd, 36–42, 47–49 (optional: 43–46)

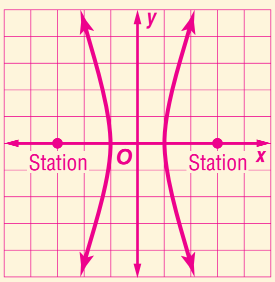
Advanced: 12–34 even, 35, 36, 38–46 (optional: 47–49)

All: Practice Quiz 2 (1–5)

Answers

35. $\frac{x^2}{1.1025} - \frac{y^2}{7.8975} = 1$

36.



40. Hyperbolas and parabolas have different graphs and different reflective properties. Answers should include the following.

- Hyperbolas have two branches, two foci, and two vertices. Parabolas have only one branch, one focus, and one vertex. Hyperbolas have asymptotes, but parabolas do not.
- Hyperbolas reflect rays directed at one focus toward the other focus. Parabolas reflect parallel incoming rays toward the only focus.

16. $\frac{y^2}{16} - \frac{x^2}{49} = 1$

19. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

23. $(0, \pm 4)$; $(0, \pm\sqrt{41})$; $y = \pm\frac{4}{5}x$

24. $(\pm 3, 0)$; $(\pm\sqrt{34}, 0)$; $y = \pm\frac{5}{3}x$

25. $(\pm\sqrt{2}, 0)$; $(\pm\sqrt{3}, 0)$; $y = \pm\frac{\sqrt{2}}{2}x$

26. $(\pm 2, 0)$; $(\pm 2\sqrt{2}, 0)$; $y = \pm x$

27. $(0, \pm 6)$; $(0, \pm 3\sqrt{5})$; $y = \pm 2x$

28. $(0, \pm\sqrt{2})$; $(0, \pm 2\sqrt{2})$; $y = \pm\frac{\sqrt{3}}{3}x$

Write an equation for the hyperbola that satisfies each set of conditions.

15. vertices $(-5, 0)$ and $(5, 0)$, conjugate axis of length 12 units **$\frac{x^2}{25} - \frac{y^2}{36} = 1$**

16. vertices $(0, -4)$ and $(0, 4)$, conjugate axis of length 14 units

17. vertices $(9, -3)$ and $(-5, -3)$, foci $(2 \pm \sqrt{53}, -3)$ **$\frac{(x-2)^2}{49} - \frac{(y+3)^2}{4} = 1$**

18. vertices $(-4, 1)$ and $(-4, 9)$, foci $(-4, 5 \pm \sqrt{97})$ **$\frac{(y-5)^2}{16} - \frac{(x+4)^2}{81} = 1$**

19. Find an equation for a hyperbola centered at the origin with a horizontal transverse axis of length 8 units and a conjugate axis of length 6 units.

20. What is an equation for the hyperbola centered at the origin with a vertical transverse axis of length 12 units and a conjugate axis of length 4 units? **$\frac{y^2}{36} - \frac{x^2}{4} = 1$**

21–28. See pp. 469A–469J for graphs.

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

21. $\frac{x^2}{81} - \frac{y^2}{49} = 1$ **$(\pm 9, 0)$; $(\pm\sqrt{130}, 0)$; $y = \pm\frac{7}{9}x$**

22. $\frac{y^2}{36} - \frac{x^2}{4} = 1$ **$(0, \pm 6)$; $(0, \pm 2\sqrt{10})$; $y = \pm 3x$**

23. $\frac{y^2}{16} - \frac{x^2}{25} = 1$

24. $\frac{x^2}{9} - \frac{y^2}{25} = 1$

25. $x^2 - 2y^2 = 2$

26. $x^2 - y^2 = 4$

27. $y^2 = 36 + 4x^2$

28. $6y^2 = 2x^2 + 12$

29. $\frac{(y-4)^2}{16} - \frac{(x+2)^2}{9} = 1$

30. $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

31. $\frac{(x+1)^2}{4} - \frac{(y+3)^2}{9} = 1$

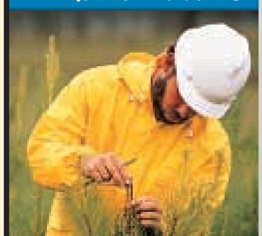
32. $\frac{(x+6)^2}{36} - \frac{(y+3)^2}{9} = 1$

★ **33.** $y^2 - 3x^2 + 6y + 6x - 18 = 0$

★ **34.** $4x^2 - 25y^2 - 8x - 96 = 0$

29–34. pp. 469A–469J.

Career Choices



Forester

Foresters work for private companies or governments to protect and manage forest land. They also supervise the planting of trees and use controlled burning to clear weeds, brush, and logging debris.

Online Research
For information about a career as a forester, visit: www.algebra2.com/careers

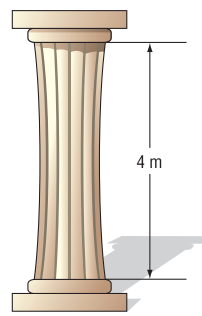
FORESTRY **For Exercises 35 and 36, use the following information.**
A forester at an outpost and another forester at the primary station both heard an explosion. The outpost and the primary station are 6 kilometers apart.

35. If one forester heard the explosion 6 seconds before the other, write an equation that describes all the possible locations of the explosion. Place the two forester stations on the x-axis with the midpoint between the stations at the origin. The transverse axis is horizontal. (*Hint*: The speed of sound is about 0.35 kilometer per second.)

36. Draw a sketch of the possible locations of the explosion. Include the ranger stations in the drawing.

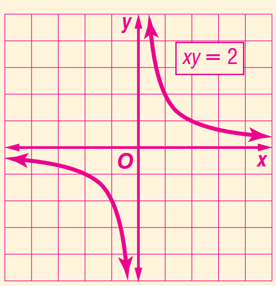
35–36. See margin.

37. STRUCTURAL DESIGN An architect's design for a building includes some large pillars with cross sections in the shape of hyperbolas. The curves can be modeled by the equation $\frac{x^2}{0.25} - \frac{y^2}{9} = 1$, where the units are in meters. If the pillars are 4 meters tall, find the width of the top of each pillar and the width of each pillar at the narrowest point in the middle. Round to the nearest centimeter. **120 cm, 100 cm**

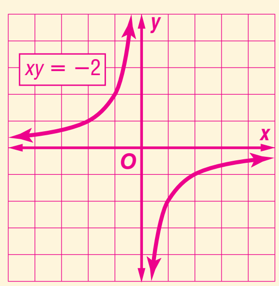


38. CRITICAL THINKING A hyperbola with a horizontal transverse axis contains the point at $(4, 3)$. The equations of the asymptotes are $y - x = 1$ and $y + x = 5$. Write the equation for the hyperbola. **$\frac{(x-2)^2}{4} - \frac{(y-3)^2}{4} = 1$**

43.



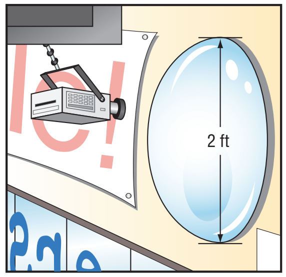
45.



46. The graph of $xy = -2$ can be obtained by reflecting the graph of $xy = 2$ over the x-axis or over the y-axis. The graph of $xy = -2$ can also be obtained by rotating the graph of $xy = 2$ by 90°.

39. about 47.32 ft

★ **39. PHOTOGRAPHY** A curved mirror is placed in a store for a wide-angle view of the room. The right-hand branch of $\frac{x^2}{1} - \frac{y^2}{3} = 1$ models the curvature of the mirror. A small security camera is placed so that all of the 2-foot diameter of the mirror is visible. If the back of the room lies on $x = -18$, what width of the back of the room is visible to the camera? (*Hint*: Find the equations of the lines through the focus and each edge of the mirror.)



40. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How are hyperbolas different from parabolas?

Include the following in your answer:
- differences in the graphs of hyperbolas and parabolas, and
- differences in the reflective properties of hyperbolas and parabolas.

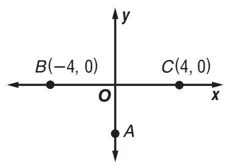
Standardized Test Practice

41. A leg of an isosceles right triangle has a length of 5 units. What is the length of the hypotenuse? **C**

(A) $\frac{5\sqrt{2}}{2}$ units (B) 5 units (C) $5\sqrt{2}$ units (D) 10 units

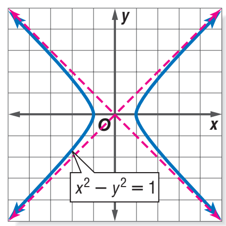
42. In the figure, what is the sum of the slopes of $\overline{AB}$ and $\overline{AC}$? **B**

(A) −1 (B) 0
(C) 1 (D) 8



Extending the Lesson

A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. Most of the hyperbolas you have studied so far are nonrectangular. A **rectangular hyperbola** has perpendicular asymptotes. For example, the graph of $x^2 - y^2 = 1$ is a rectangular hyperbola. The graphs of equations of the form $xy = c$, where c is a constant, are rectangular hyperbolas with the coordinate axes as their asymptotes.



For Exercises 43 and 44, consider the equation $xy = 2$.

43. Plot some points and use them to graph the equation. Be sure to consider negative values for the variables. **See margin.**

44. Find the coordinates of the vertices of the graph of the equation.

44. $(\sqrt{2}, \sqrt{2})$, $(-\sqrt{2}, -\sqrt{2})$

45. Graph $xy = -2$. **See margin.**

46. Describe the transformations that can be applied to the graph of $xy = 2$ to obtain the graph of $xy = -2$. **See margin.**

Maintain Your Skills

Mixed Review

47–49. See margin.

Write an equation for the ellipse that satisfies each set of conditions. *(Lesson 8-4)*

47. endpoints of major axis at (1, 2) and (9, 2), endpoints of minor axis at (5, 1) and (5, 3)

48. major axis 8 units long and parallel to y-axis, minor axis 6 units long, center at (−3, 1)

49. foci at (5, 4) and (−3, 4), major axis 10 units long

Answers

47. $\frac{(x-5)^2}{16} + \frac{(y-2)^2}{1} = 1$

48. $\frac{(y-1)^2}{16} + \frac{(x+3)^2}{9} = 1$

49. $\frac{(x-1)^2}{25} + \frac{(y-4)^2}{9} = 1$

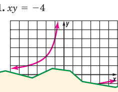
Enrichment, p. 484

Rectangular Hyperbolas

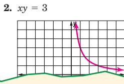
A **rectangular hyperbola** is a hyperbola with perpendicular asymptotes. For example, the graph of $x^2 - y^2 = 1$ is a rectangular hyperbola. A hyperbola with asymptotes that are not perpendicular is called a **nonrectangular hyperbola**. The graphs of equations of the form $xy = c$, where c is a constant, are rectangular hyperbolas.

Make a table of values and plot points to graph each rectangular hyperbola below. Be sure to consider negative values for the variables. See students' tables.

1. $xy = -4$



2. $xy = 3$



Study Guide and Intervention, p. 479 (shown) and p. 480

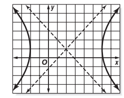
Equations of Hyperbolas A **hyperbola** is the set of all points in a plane such that the absolute value of the *difference* of the distances from any point on the hyperbola to any two given points in the plane, called the **foci**, is constant.

In the table, the lengths a, b, and c are related by the formula $c^2 = a^2 + b^2$.

Standard Form of Equation	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Equations of the Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$
Transverse Axis	Horizontal	Vertical
Foci	$(h-c, k), (h+c, k)$	$(h, k-c), (h, k+c)$
Vertices	$(h-a, k), (h+a, k)$	$(h, k-a), (h, k+a)$

Example **Write an equation for the hyperbola with vertices (−2, 1) and (6, 1) and foci (−4, 1) and (8, 1).**

Use a sketch to orient the hyperbola correctly. The center of the hyperbola is the midpoint of the segment joining the two vertices. The center is $(\frac{-2+6}{2}, 1)$, or (2, 1). The value of a is the distance from the center to a vertex, so $a = 4$. The value of c is the distance from the center to a focus, so $c = 6$.



$c^2 = a^2 + b^2$
$6^2 = 4^2 + b^2$
$b^2 = 36 - 16 = 20$

Use h, k, a^2, and b^2 to write an equation of the hyperbola.

$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{20} = 1$

Exercises

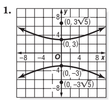
Write an equation for the hyperbola that satisfies each set of conditions.

1. vertices (−7, 0) and (7, 0), conjugate axis of length 10 $\frac{x^2}{49} - \frac{y^2}{25} = 1$
2. vertices (−2, −3) and (4, −3), foci (−5, −3) and (7, −3) $\frac{(x-1)^2}{9} - \frac{(y+3)^2}{27} = 1$
3. vertices (4, 3) and (4, −5), conjugate axis of length 4 $\frac{(y+1)^2}{16} - \frac{(x-4)^2}{4} = 1$
4. vertices (−8, 0) and (8, 0), equation of asymptotes $y = \pm\frac{1}{6}x$ $\frac{x^2}{64} - \frac{9y^2}{16} = 1$
5. vertices (−4, 6) and (−4, −2), foci (−4, 10) and (−4, −6) $\frac{(y-2)^2}{16} - \frac{(x+4)^2}{48} = 1$

Skills Practice, p. 481 and Practice, p. 482 (shown)

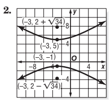
Write an equation for each hyperbola.

1.



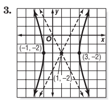
$\frac{y^2}{9} - \frac{x^2}{36} = 1$

2.



$\frac{(y-2)^2}{9} - \frac{(x+3)^2}{25} = 1$

3.



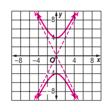
$\frac{(x-1)^2}{4} - \frac{(y+2)^2}{16} = 1$

Write an equation for the hyperbola that satisfies each set of conditions.

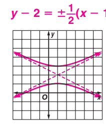
4. vertices (0, 7) and (0, −7), conjugate axis of length 18 units $\frac{y^2}{49} - \frac{x^2}{81} = 1$
5. vertices (1, −1) and (1, −9), conjugate axis of length 6 units $\frac{(y+5)^2}{16} - \frac{(x-1)^2}{9} = 1$
6. vertices (−5, 0) and (5, 0), foci $(\pm\sqrt{26}, 0)$ $\frac{x^2}{25} - \frac{y^2}{1} = 1$
7. vertices (1, 1) and (1, −3), foci $(1, -1 \pm \sqrt{5})$ $\frac{(y+1)^2}{4} - \frac{(x-1)^2}{1} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

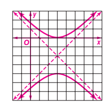
8. $\frac{y^2}{16} - \frac{x^2}{4} = 1$
$(0, \pm 4)$; $(0, \pm 2\sqrt{5})$;
$y = \pm 2x$



9. $\frac{(y-2)^2}{1} - \frac{(x-1)^2}{4} = 1$
(1, 3), (1, 1);
$(1, 2 \pm \sqrt{5})$;
$y - 2 = \pm\frac{1}{2}(x - 1)$



10. $\frac{(y+2)^2}{4} - \frac{(x-3)^2}{4} = 1$
(3, 0), (3, −4);
$(3, -2 \pm 2\sqrt{2})$;
$y + 2 = \pm(x - 3)$



11. **ASTRONOMY** Astronomers use special X-ray telescopes to observe the sources of celestial X rays. Some X-ray telescopes are fitted with a metal mirror in the shape of a hyperbola, which reflects the X rays to a focus. Suppose the vertices of such a mirror are located at (−3, 0) and (3, 0), and one focus is located at (5, 0). Write an equation that models the hyperbola formed by the mirror. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Reading to Learn Mathematics, p. 483

ELL

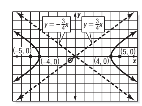
Pre-Activity **How are hyperbolas different from parabolas?**

Read the introduction to Lesson 8-5 at the top of page 441 in your textbook.

Look at the sketch of a hyperbola in the introduction to this lesson. List three ways in which hyperbolas are different from parabolas.
Sample answer: A hyperbola has two branches, while a parabola is one continuous curve. A hyperbola has two foci, while a parabola has one focus. A hyperbola has two vertices, while a parabola has one vertex.

Reading the Lesson

1. The graph at the right shows the hyperbola whose equation in standard form is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.



The point (0, 0) is the **center** of the hyperbola.

The points (4, 0) and (−4, 0) are the **vertices** of the hyperbola.

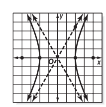
The points (5, 0) and (−5, 0) are the **foci** of the hyperbola.

The segment connecting (4, 0) and (−4, 0) is called the **transverse** axis.

The segment connecting (0, 3) and (0, −3) is called the **conjugate** axis.

The lines $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$ are called the **asymptotes**.

2. Study the hyperbola graphed at the right.



The center is **(0, 0)**.

The value of a is **2**.

The value of c is **4**.

To find b^2, solve the equation **c^2** = **a^2** + **b^2**.

The equation in standard form for this hyperbola is **$\frac{x^2}{4} - \frac{y^2}{12} = 1$**.

Helping You Remember

3. What is an easy way to remember the equation relating the values of a, b, and c for a hyperbola? **This equation looks just like the Pythagorean Theorem, although the variables represent different lengths in a hyperbola than in a right triangle.**

50. Find the center and radius of the circle with equation $x^2 + y^2 - 10x + 2y + 22 = 0$. Then graph the circle. *(Lesson 8-3)* **(5, −1), 2 units; See margin for graph.**

Solve each equation by factoring. *(Lesson 6-2)*

51. $x^2 + 6x + 8 = 0$ **−4, −2**

52. $2q^2 + 11q = 21$ **$-7, \frac{3}{2}$**

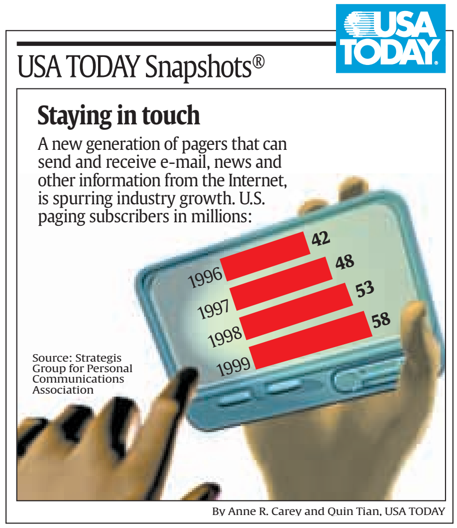
Perform the indicated operations, if possible. *(Lesson 4-5)*

53. $\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}$ $\begin{bmatrix} -7 & 0 \\ 5 & 20 \end{bmatrix}$

54. $[1 \quad -3] \cdot \begin{bmatrix} 4 & -2 & 1 \\ -3 & 2 & 0 \end{bmatrix}$ **[13 −8 1]**

55. about 5,330,000 subscribers per year

55. **PAGERS** Refer to the graph at the right. What was the average rate of change of the number of pager subscribers from 1996 to 1999? *(Lesson 2-3)*



56. Solve $|2x + 1| = 9$. *(Lesson 1-4)* **−5, 4**

57. Simplify $7x + 8y + 9y - 5x$. *(Lesson 1-2)* **$2x + 17y$**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Each equation is of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$. Identify the values of A, B, and C.**
*(To review **coefficients**, see Lesson 5-1.)*

58. $2x^2 + 3xy - 5y^2 = 0$ **2, 3, −5**

59. $x^2 - 2xy + 9y^2 = 0$ **1, −2, 9**

60. $-3x^2 + xy + 2y^2 + 4x - 7y = 0$ **60. −3, 1, 2**

61. $5x^2 - 2y^2 + 5x - y = 0$ **5, 0, −2**

62. $x^2 - 4x + 5y + 2 = 0$ **1, 0, 0**

63. $xy - 2x - 3y + 6 = 0$ **0, 1, 0**

Practice Quiz 2 — *Lessons 8-4 and 8-5*

1. Write an equation of the ellipse with foci at (3, 8) and (3, −6) and endpoints of the major axis at (3, −8) and (3, 10). *(Lesson 8-4)* $\frac{(y-1)^2}{81} + \frac{(x-3)^2}{32} = 1$

2–3. See pp. 469A–469J for graphs.

Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with the given equation. Then graph the ellipse. *(Lesson 8-4)*

2. $\frac{(x-4)^2}{9} + \frac{(y+2)^2}{1} = 1$ **$(4, -2)$; $(4 \pm 2\sqrt{2}, -2)$; 6; 2**

3. $16x^2 + 5y^2 + 32x - 10y - 59 = 0$ **$(-1, 1)$; $(-1, 1 \pm \sqrt{11})$; 8; $2\sqrt{5}$**

Write an equation for the hyperbola that satisfies each set of conditions. *(Lesson 8-5)*

4. vertices (−3, 0) and (3, 0), conjugate axis of length 8 units $\frac{x^2}{9} - \frac{y^2}{16} = 1$

5. vertices (−2, 2) and (6, 2), foci $(2 \pm \sqrt{21}, 2)$ **5.** $\frac{(x-2)^2}{16} - \frac{(y-2)^2}{5} = 1$

4 Assess

Open-Ended Assessment

Writing Have students write a paragraph that explains how hyperbolas are related to other conic sections, and how they are alike and different from the others.

Tips for New Teachers

Intervention
To diagnose confusion, ask students to draw a quick sketch, without labels, of each of the conic sections and to name the shape of each of the graphs they drew.

Getting Ready for Lesson 8-6

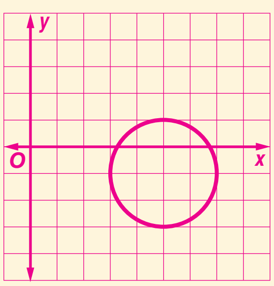
PREREQUISITE SKILL In Lesson 8-6, students will learn how to identify conic sections from their equations, including comparing specific coefficients within equations. Exercises 58–63 should be used to determine your students' familiarity with identifying coefficients.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 8-4 and 8-5. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Answer

50.



Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

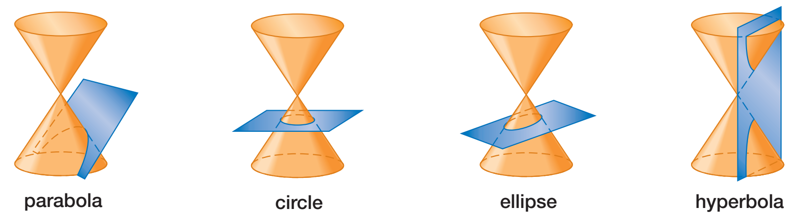
8-6 Conic Sections

What You'll Learn

- Write equations of conic sections in standard form.
- Identify conic sections from their equations.

How can you use a flashlight to make conic sections?

Recall that parabolas, circles, ellipses, and hyperbolas are called conic sections because they are the cross sections formed when a double cone is sliced by a plane. You can use a flashlight and a flat surface to make patterns in the shapes of conic sections.



STANDARD FORM The equation of any conic section can be written in the form of the general quadratic equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C are not all zero. If you are given an equation in this general form, you can complete the square to write the equation in one of the standard forms you have learned.

Study Tip

Reading Math In this lesson, the word *ellipse* means an ellipse that is not a circle.

Concept Summary — *Standard Form of Conic Sections*

Conic Section	Standard Form of Equation
Parabola	$y = a(x - h)^2 + k$ or $x = a(y - k)^2 + h$
Circle	$(x - h)^2 + (y - k)^2 = r^2$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$, $a \neq b$
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ or $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Example 1 *Rewrite an Equation of a Conic Section*

Write the equation $x^2 + 4y^2 - 6x - 7 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

Write the equation in standard form.

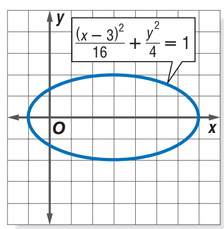
$x^2 + 4y^2 - 6x - 7 = 0$ — Original equation

$x^2 - 6x + \blacksquare + 4y^2 = 7 + \blacksquare$ — Isolate terms.

$x^2 - 6x + 9 + 4y^2 = 7 + 9$ — Complete the square.

$(x - 3)^2 + 4y^2 = 16$ — $x^2 - 6x + 9 = (x - 3)^2$

$\frac{(x-3)^2}{16} + \frac{y^2}{4} = 1$ — Divide each side by 16.



The graph of the equation is an ellipse with its center at (3, 0).

www.algebra2.com/extra_examples

8-6 Lesson Notes

1 Focus

5-Minute Check Transparency 8-6 Use as a quiz or review of Lesson 8-5.

Mathematical Background notes are available for this lesson on p. 410D.

Building on Prior Knowledge

In Chapter 6, students learned how to analyze graphs of quadratic equations and rewrite the equations in different forms. In this lesson, students will use similar techniques to analyze graphs of conic sections.

How can you use a flashlight to make conic sections?

Ask students:

- Describe the plane that forms a hyperbola. **perpendicular to the base of the cone**
- Describe the plane that forms a parabola. **parallel to the slant height of the cone**

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters

- Study Guide and Intervention, pp. 485–486
- Skills Practice, p. 487
- Practice, p. 488
- Reading to Learn Mathematics, p. 489
- Enrichment, p. 490
- Assessment, p. 512

Teaching Algebra With Manipulatives Masters, pp. 268–269

Transparencies

5-Minute Check Transparency 8-6
Answer Key Transparencies

Technology

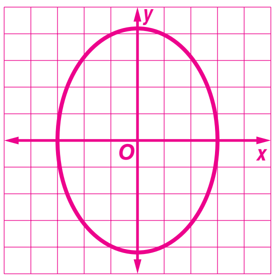
Interactive Chalkboard

2 Teach

STANDARD FORM

In-Class Example (Power Point®)

1 Write the equation $y^2 = 18 - 2x^2$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. $\frac{x^2}{9} + \frac{y^2}{18} = 1$; **ellipse**



IDENTIFY CONIC SECTIONS

In-Class Example (Power Point®)

2 Without writing the equation in standard form, state whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a. $3y^2 - x^2 - 9 = 0$ **hyperbola**

b. $2x^2 + 2y^2 + 16x - 20y = -32$ **circle**

c. $y^2 - 2x - 4y + 10 = 0$ **parabola**

3 Practice/Apply

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 8.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

IDENTIFY CONIC SECTIONS Instead of writing the equation in standard form, you can determine what type of conic section an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $B = 0$, represents by looking at A and C.

Concept Summary — *Identifying Conic Sections*

Conic Section	Relationship of *A* and *C*
Parabola	$A = 0$ or $C = 0$, but not both.
Circle	$A = C$
Ellipse	A and C have the same sign and $A \neq C$.
Hyperbola	A and C have opposite signs.

Example 2 Analyze an Equation of a Conic Section

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

a. $y^2 - 2x^2 - 4x - 4y - 4 = 0$

$A = -2$ and $C = 1$. Since A and C have opposite signs, the graph is a hyperbola.

b. $4x^2 + 4y^2 + 20x - 12y + 30 = 0$

$A = 4$ and $C = 4$. Since $A = C$, the graph is a circle.

c. $y^2 - 3x + 6y + 12 = 0$

$C = 1$. Since there is no x^2 term, $A = 0$. The graph is a parabola.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an equation of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where $A = 2$, that represents a circle. **Sample answer: $2x^2 + 2y^2 - 1 = 0$**
2. **Write** the general quadratic equation for which $A = 2$, $B = 0$, $C = 0$, $D = -4$, $E = 7$, and $F = 1$. **$2x^2 - 4x + 7y + 1 = 0$**
3. **Explain** why the graph of $x^2 + y^2 - 4x + 2y + 5 = 0$ is a single point. **See margin.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7, 10, 11	1
8, 9	2

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

4. $y = x^2 + 3x + 1$ **parabola**
5. $y^2 - 2x^2 - 16 = 0$ **hyperbola**
6. $x^2 + y^2 = x + 2$ **circle**
7. $x^2 + 4y^2 + 2x - 24y + 33 = 0$ **ellipse**

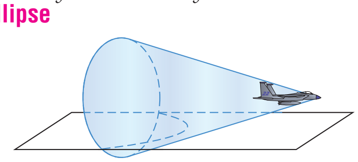
4–7. See pp. 469A–469J for equations and graphs.

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

8. $y^2 - x - 10y + 34 = 0$ **parabola**
9. $3x^2 + 2y^2 + 12x - 28y + 104 = 0$ **ellipse**

Application

AVIATION **For Exercises 10 and 11, use the following information.**
When an airplane flies faster than the speed of sound, it produces a shock wave in the shape of a cone. Suppose the shock wave intersects the ground in a curve that can be modeled by $x^2 - 14x + 4 = 9y^2 - 36y$.



10. Identify the shape of the curve. **hyperbola**
11. Graph the equation. **See pp. 469A–469J.**

DAILY INTERVENTION — **Differentiated Instruction**

Intrapersonal Encourage students to make a list of the techniques and hints that they use as they answer questions like the ones in this lesson. Invite students to share their techniques with the class.

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
12–32	1
33–43	2

Extra Practice

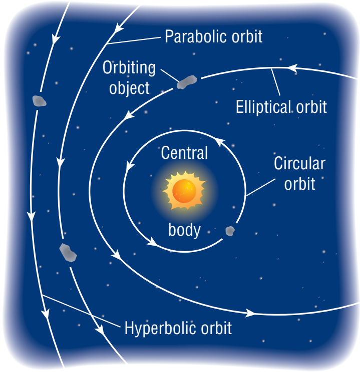
See page 846.

12–29. See pp. 469A–469J for equations and graphs.

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

12. $6x^2 + 6y^2 = 162$ circle
13. $4x^2 + 2y^2 = 8$ ellipse
14. $x^2 = 8y$ parabola
15. $4y^2 - x^2 + 4 = 0$ hyperbola
16. $(x - 1)^2 - 9(y - 4)^2 = 36$ hyperbola
17. $y + 4 = (x - 2)^2$ parabola
18. $(y - 4)^2 = 9(x - 4)$ parabola
19. $x^2 + y^2 + 4x - 6y = -4$ circle
20. $x^2 + y^2 + 6y + 13 = 40$ circle
21. $x^2 - y^2 + 8x = 16$ hyperbola
22. $x^2 + 2y^2 = 2x + 8$ ellipse
23. $x^2 - 8y + y^2 + 11 = 0$ circle
24. $9y^2 + 18y = 25x^2 + 216$ hyperbola
25. $3x^2 + 4y^2 + 8y = 8$ ellipse
26. $x^2 + 4y^2 - 11 = 2(4y - x)$ ellipse
27. $y + x^2 = -(8x + 23)$ parabola
28. ★ $6x^2 - 24x - 5y^2 - 10y - 11 = 0$ hyperbola
29. ★ $25y^2 + 9x^2 - 50y - 54x = 119$ ellipse

30. **ASTRONOMY** The orbits of comets follow paths in the shapes of conic sections. For example, Halley's Comet follows an elliptical orbit with the Sun located at one focus. What type(s) of orbit(s) pass by the Sun only once? parabolas and hyperbolas



WATER For Exercises 31 and 32, use the following information.
If two stones are thrown into a lake at different points, the points of intersection of the resulting ripples will follow a conic section. Suppose the conic section has the equation $x^2 - 2y^2 - 2x - 5 = 0$.

31. Identify the shape of the curve. hyperbola
32. Graph the equation. See pp. 469A–469J.

More About. . .

Astronomy

Halley's Comet orbits the Sun about once every 76 years. It will return next in 2061.

Source: www.solarviews.com

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

33. $x^2 + y^2 - 8x - 6y + 5 = 0$ circle
34. $3x^2 - 2y^2 + 32y - 134 = 0$ hyperbola
35. $y^2 + 18y - 2x = -84$ parabola
36. $7x^2 - 28x + 4y^2 + 8y = -4$ ellipse
37. ★ $5x^2 + 6x - 4y = x^2 - y^2 - 2x$ ellipse
38. ★ $2x^2 + 12x + 18 - y^2 = 3(2 - y^2) + 4y$ circle
39. Identify the shape of the graph of the equation $2x^2 + 3x - 4y + 2 = 0$. parabola
40. What type of conic section is represented by the equation $y^2 - 6y = x^2 - 8$? hyperbola

For Exercises 41–43, match each equation below with the situation that it could represent.

a. $9x^2 + 4y^2 - 36 = 0$
b. $0.004x^2 - x + y - 3 = 0$
c. $x^2 + y^2 - 20x + 30y - 75 = 0$

41. **SPORTS** the flight of a baseball b
42. **PHOTOGRAPHY** the oval opening in a picture frame a
43. **GEOGRAPHY** the set of all points that are 20 miles from a landmark c

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 485 (shown) and p. 486

Standard Form Any conic section in the coordinate plane can be described by an equation of the form

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, where A, B, and C are not all zero.

One way to tell what kind of conic section an equation represents is to rearrange terms and complete the square, if necessary, to get one of the standard forms from an earlier lesson. This method is especially useful if you are going to graph the equation.

Example **Write the equation $3x^2 - 4y^2 - 30x - 8y + 59 = 0$ in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

$3x^2 - 4y^2 - 30x - 8y + 59 = 0$	Original equation
$3x^2 - 30x - 4y^2 - 8y = -59$	Isolate terms.
$3(x^2 - 10x + ■) - 4(y^2 + 2y + ■) = -59 + ■ + ■$	Factor out common multiples.
$3(x^2 - 10x + 25) - 4(y^2 + 2y + 1) = -59 + 3(25) + (-4)(1)$	Complete the squares.
$3(x - 5)^2 - 4(y + 1)^2 = 12$	Simplify.
$\frac{(x-5)^2}{4} - \frac{(y+1)^2}{3} = 1$	Divide each side by 12.

The graph of the equation is a hyperbola with its center at (5, −1). The length of the transverse axis is 4 units and the length of the conjugate axis is $2\sqrt{3}$ units.

Exercises

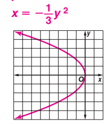
Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

1. $x^2 + y^2 - 6x + 4y + 3 = 0$ $(x - 3)^2 + (y + 2)^2 = 10$; circle
2. $x^2 + 2y^2 + 6x - 20y + 53 = 0$ $\frac{(x+3)^2}{6} + \frac{(y-5)^2}{3} = 1$; ellipse
3. $6x^2 - 60x - y + 161 = 0$ $y = 6(x - 5)^2 + 11$; parabola
4. $x^2 + y^2 - 4x - 14y + 29 = 0$ $(x - 2)^2 + (y - 7)^2 = 24$; circle
5. $6x^2 - 5y^2 + 24x + 20y - 56 = 0$ $\frac{(x+2)^2}{10} - \frac{(y-2)^2}{12} = 1$; hyperbola
6. $3y^2 + x - 24y + 46 = 0$ $x = -3(y - 4)^2 + 2$; parabola
7. $x^2 - 4y^2 - 16x + 24y - 36 = 0$ $\frac{(x-8)^2}{64} - \frac{(y-3)^2}{16} = 1$; hyperbola
8. $x^2 + 2y^2 + 8x + 4y + 2 = 0$ $\frac{(x+4)^2}{16} + \frac{(y+1)^2}{8} = 1$; ellipse
9. $4x^2 + 48x + y + 158 = 0$ $y = -4(x + 6)^2 - 14$; parabola
10. $3x^2 + y^2 - 48x - 4y + 184 = 0$ $\frac{(x-8)^2}{4} + \frac{(y-2)^2}{12} = 1$; ellipse
11. $-3x^2 + 2y^2 - 18x + 20y + 5 = 0$ $\frac{(y+5)^2}{9} - \frac{(x+3)^2}{6} = 1$; hyperbola
12. $x^2 + y^2 + 8x + 2y + 8 = 0$ $(x + 4)^2 + (y + 1)^2 = 9$; circle

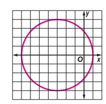
Skills Practice, p. 487 and Practice, p. 488 (shown)

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

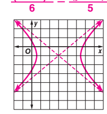
1. $y^2 = -3x$ parabola $x = -\frac{1}{3}y^2$



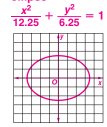
2. $x^2 + y^2 + 6x = 7$ circle $(x + 3)^2 + y^2 = 16$



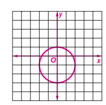
3. $5x^2 - 6y^2 - 30x - 12y = -9$ hyperbola $\frac{(x-3)^2}{6} - \frac{(y+1)^2}{5} = 1$



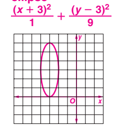
4. $196y^2 = 1225 - 100x^2$ ellipse $\frac{x^2}{12.25} + \frac{y^2}{6.25} = 1$



5. $3x^2 = 9 - 3y^2 - 6y$ circle $x^2 + (y + 1)^2 = 4$



6. $9x^2 + y^2 + 54x - 6y = -81$ ellipse $\frac{(x+3)^2}{1} + \frac{(y-3)^2}{9} = 1$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7. $6x^2 + 6y^2 = 36$ circle
8. $4x^2 - y^2 = 16$ hyperbola
9. $9x^2 + 16y^2 - 64y - 80 = 0$ ellipse
10. $5x^2 + 5y^2 - 45 = 0$ circle
11. $x^2 + 2x = y$ parabola
12. $4y^2 - 36x^2 + 4x - 144 = 0$ hyperbola
13. **ASTRONOMY** A satellite travels in an hyperbolic orbit. It reaches the vertex of its orbit at (5, 0) and then travels along a path that gets closer and closer to the line $y = \frac{2}{5}x$. Write an equation that describes the path of the satellite if the center of its hyperbolic orbit is at (0, 0). $\frac{x^2}{25} - \frac{y^2}{4} = 1$

Reading to Learn Mathematics, p. 489 ELL

Pre-Activity **How can you use a flashlight to make conic sections?**

Read the introduction to Lesson 8-6 at the top of page 449 in your textbook.

The figures in the introduction show how a plane can slice a double cone to form the conic sections. Name the conic section that is formed if the plane slices the double cone in each of the following ways:

- The plane is parallel to the base of the double cone and slices through one of the cones that form the double cone. circle
- The plane is perpendicular to the base of the double cone and slices through both of the cones that form the double cone. hyperbola

Reading the Lesson

1. Name the conic section that is the graph of each of the following equations. Give the coordinates of the vertex if the conic section is a parabola and of the center if it is a circle, an ellipse, or a hyperbola.
   a. $\frac{(x-3)^2}{36} + \frac{(y+5)^2}{15} = 1$ ellipse; (3, −5)
   b. $x = -2(y + 1)^2 + 7$ parabola; (7, −1)
   c. $(x - 5)^2 - (y + 5)^2 = 1$ hyperbola; (5, −5)
   d. $(x + 6)^2 + (y - 2)^2 = 1$ circle; (−6, 2)
2. Each of the following is the equation of a conic section. For each equation, identify the values of A and C. Then, without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle, ellipse*, or *hyperbola*.
   a. $2x^2 + y^2 - 6x + 8y + 12 = 0$ A = 2; C = 1; type of graph: ellipse
   b. $2x^2 + 3x - 2y - 5 = 0$ A = 2; C = 0; type of graph: parabola
   c. $5x^2 + 10x + 5y^2 - 20y + 1 = 0$ A = 5; C = 5; type of graph: circle
   d. $x^2 - y^2 + 4x + 2y - 5 = 0$ A = 1; C = −1; type of graph: hyperbola

Helping You Remember

3. What is an easy way to recognize that an equation represents a parabola rather than one of the other conic sections?
   If the equation has an x^2 term and y term but no y^2 term, then the graph is a parabola. Likewise, if the equation has a y^2 term and x term but no x^2 term, then the graph is a parabola.

Answer

3. The standard form of the equation is $(x - 2)^2 + (y + 1)^2 = 0$. This is an equation of a circle centered at (2, −1) with radius 0. In other words, (2, −1) is the only point that satisfies the equation.

Enrichment, p. 490

Loci

A *locus* (plural, *loci*) is the set of all points, and only those points, that satisfy a given set of conditions. In geometry, figures often are defined as loci. For example, a circle is the locus of points of a plane that are a given distance from a given point. The definition leads naturally to an equation whose graph is the curve described.

Example **Write an equation of the locus of points that are the same distance from (3, 4) and $y = -4$.**

Recognizing that the locus is a parabola with focus (3, 4) and directrix $y = -4$, you can find that $h = 3$, $k = 0$, and $a = 4$ where (h, k) is the vertex and 4 units is the distance from the vertex to both the focus and directrix.

Thus, an equation for the parabola is $y = \frac{1}{16}(x - 3)^2$.

The problem also may be approached analytically as follows:

Let (x, y) be a point of the locus.

The distance from (3, 4) to (x, y) = the distance from $y = -4$ to (x, y)

About the Exercises...

Organization by Objective

- **Standard Form:** 12–32
- **Identify Conic Sections:** 33–43

Odd/Even Assignments

Exercises 12–29 and 33–38 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 13–27 odd, 31–35 odd, 39–43 odd, 44–48, 50–59

Average: 13–43 odd, 44–48, 50–59 (optional: 49)

Advanced: 12–30 even, 31, 32–44 even, 45–56 (optional: 57–59)

4 Assess

Open-Ended Assessment

Modeling Have students use paper folding to construct double right circular cones. Use them to demonstrate the various conic sections.

Getting Ready for Lesson 8-7

PREREQUISITE SKILL In Lesson 8-7, students will solve systems of quadratic equations. Students should be sure they can solve systems of simpler linear equations before continuing. Exercises 57–59 should be used to determine your students' familiarity with solving systems of linear equations.

Assessment Options

Quiz (Lessons 8-5 and 8-6) is available on p. 512 of the *Chapter 8 Resource Masters*.

CRITICAL THINKING **For Exercises 44 and 45, use the following information.**

The graph of an equation of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ is a special case of a hyperbola.

44. Identify the graph of such an equation. **2 intersecting lines**

45. Explain how to obtain such a set of points by slicing a double cone with a plane.

45. The plane should be vertical and contain the axis of the double cone.

46. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. **See margin.**

How can you use a flashlight to make conic sections?

Include the following in your answer:

- an explanation of how you could point the flashlight at a ceiling or wall to make a circle, and
- an explanation of how you could point the flashlight to make a branch of a hyperbola.

Standardized Test Practice

47. Which conic section is not symmetric about the y-axis? **D**

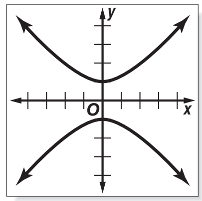
Ⓐ $x^2 - y + 3 = 0$ Ⓑ $y^2 - x^2 - 1 = 0$

Ⓒ $6x^2 + y^2 - 6 = 0$ Ⓓ $x^2 + y^2 - 2x - 3 = 0$

48. What is the equation of the graph at the right? **C**

Ⓐ $y = x^2 + 1$ Ⓑ $y - x = 1$

Ⓒ $y^2 - x^2 = 1$ Ⓓ $x^2 + y^2 = 1$



Extending the Lesson

49. Refer to Exercise 43 on page 440. Eccentricity can be studied for conic sections other than ellipses. The expression for the eccentricity of a hyperbola is $\frac{c}{a}$, just as for an ellipse. The eccentricity of a parabola is 1. Find inequalities for the eccentricities of noncircular ellipses and hyperbolas, respectively. $0 < e < 1, e > 1$

Maintain Your Skills

Mixed Review **Write an equation of the hyperbola that satisfies each set of conditions.** *(Lesson 8-5)*

50. vertices (5, 10) and (5, −2), conjugate axis of length 8 units

50. $\frac{(y-4)^2}{36} - \frac{(x-5)^2}{16} = 1$

51. vertices (6, −6) and (0, −6), foci $(3 \pm \sqrt{13}, -6)$ $\frac{(x-3)^2}{9} - \frac{(y+6)^2}{4} = 1$

52. Find the coordinates of the center and foci and the lengths of the major and minor axes of the ellipse with equation $4x^2 + 9y^2 - 24x + 72y + 144 = 0$. Then graph the ellipse. *(Lesson 8-4)* $(3, -4)$ $(3 \pm \sqrt{5}, -4)$; 6; 4; **See pp. 469A-469J for graph.**

Simplify. Assume that no variable equals 0. *(Lesson 5-1)*

53. $(x^3)^4$ x^{12}

54. $(m^5n^{-3})^2m^2n^7$ $m^{12}n$

55. $\frac{x^2y^{-3}}{x^{-5}y}$ $\frac{x^7}{y^4}$

56. HEALTH The prediction equation $y = 205 - 0.5x$ relates a person's maximum heart rate for exercise y and age x. Use the equation to find the maximum heart rate for an 18-year-old. *(Lesson 2-5)* **196 beats per min**

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each system of equations.**

*(To review **solving systems of linear equations**, see Lesson 3-2.)*

57. $y = x + 4$
$2x + y = 10$ **(2, 6)**

58. $4x + y = 14$
$4x - y = 10$ **(3, 2)**

59. $x + 5y = 10$
$3x - 2y = -4$ **(0, 2)**

Teacher to Teacher

Judie Campbell — Derry Area H.S., Derry, PA

"As a final project, I have students make a poster with 6 different types of graphs, explaining how each was derived, and then have them label each part of the graph. An explanation of each graph is also required."

Algebra Activity

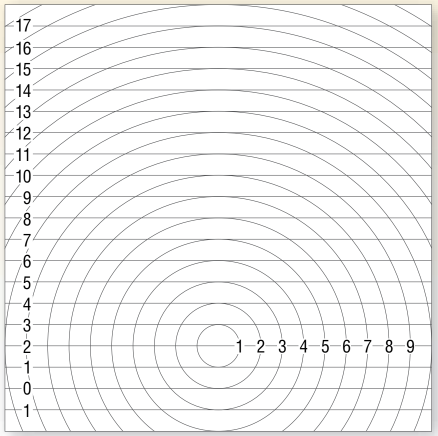
A Follow-Up of Lesson 8-6

Conic Sections

Recall that a parabola is the set of all points that are equidistant from the focus and the directrix.

You can draw a parabola based on this definition by using special conic graph paper. This graph paper contains a series of concentric circles equally spaced from each other and a series of parallel lines tangent to the circles.

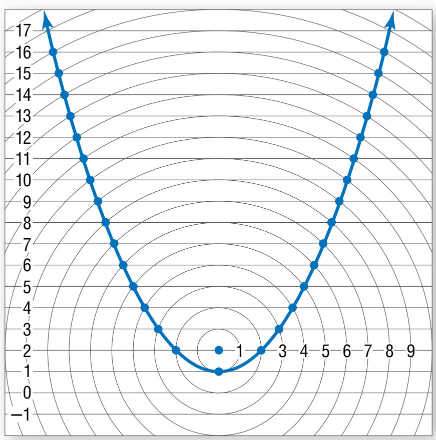
Number the circles consecutively beginning with the smallest circle. Number the lines with consecutive integers as shown in the sample at the right. Be sure that line 1 is tangent to circle 1.



Activity 1

Mark the point at the intersection of circle 1 and line 1. Mark both points that are on line 2 and circle 2. Continue this process, marking both points on line 3 and circle 3, and so on. Then connect the points with a smooth curve.

Look at the diagram at the right. What shape is the graph? Note that every point on the graph is equidistant from the center of the small circle and the line labeled 0. The center of the small circle is the focus of the parabola, and line 0 is the directrix.



Algebra Activity

A Follow-Up of Lesson 8-6

Getting Started

Objective To illustrate definitions of the conic sections by graphing them on conic graph paper.

Materials
conic graph paper

Teach

- Explain to students that there are many different kinds of graph paper (logarithmic, polar, and isometric, for example) in addition to the familiar rectangular grid. Each type helps visualize various mathematical relationships.
- Some students may have some visual difficulties with this kind of graph. Suggest that students move a pointer (finger or pencil) to keep track of where they are.
- To correct numbering errors, have students work in pairs to check the numbering before graphing.

Resource Manager

Teaching Algebra with Manipulatives

- pp. 8–9 (masters for conic paper)
- p. 270 (student recording sheet)

Assess

In **Exercises 1–3**, students should

- use conic graphing paper.
- see how the distance definitions of the conic sections define their shapes.

Study Notebook

You may wish to have students summarize this activity and what they learned from it.

Answers

1. The points on each graph are equidistant from the focus and the directrix.

2a. There are no intersecting circles whose sum is less than 10.

2b. The ellipses become more circular; the ellipses become more oblong.

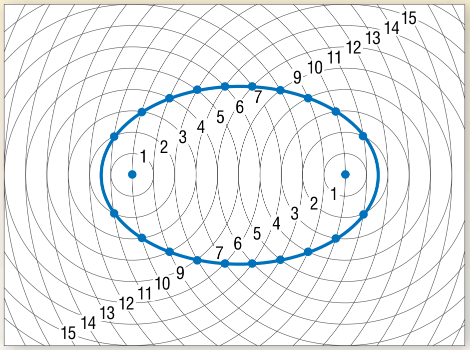
3. Each branch of the hyperbola becomes more narrow and the vertices become farther apart; each branch of the hyperbola becomes wider and the vertices become closer.

Algebra Activity

Activity 2

An ellipse is the set of points such that the sum of the distances from two fixed points is constant. The two fixed points are called the foci.

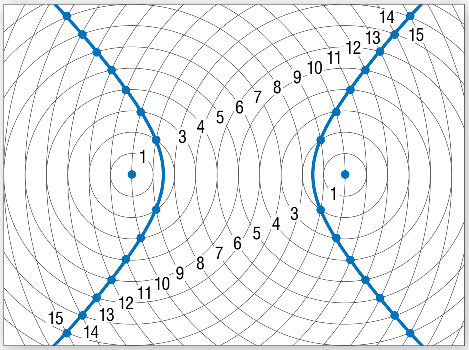
- Use graph paper like that shown. It contains two small circles and a series of concentric circles from each. The concentric circles are tangent to each other as shown.
- Choose the constant 13. Mark the points at the intersections of circle 9 and circle 4, because 9 + 4 = 13. Continue this process until you have marked the intersection of all circles whose sum is 13.
- Connect the points to form a smooth curve. The curve is an ellipse whose foci are the centers of the two small circles on the graph paper.



Activity 3

A hyperbola is the set of points such that the difference of the distances from two fixed points is constant. The two fixed points are called the foci.

- Use the same type of graph paper that you used for the ellipse in Activity 2. Choose the constant 7. Mark the points at the intersections of circle 9 and circle 2, because 9 − 2 = 7. Continue this process until you have marked the intersections of all circles whose difference in radius is 7.
- Connect the points to form a hyperbola.



Model and Analyze 1–3. See margin.

1. Use the type of graph paper you used in Activity 1. Mark the intersection of line 0 and circle 2. Then mark the two points on line 1 and circle 3, the two points on line 2 and circle 4, and so on. Draw the new parabola. Continue this process and make as many parabolas as you can on one sheet of the graph paper. The focus is always the center of the small circle. Why are the resulting graphs parabolas?

2. In Activity 2, you drew an ellipse such that the sum of the distances from two fixed points was 13. Choose 10, 11, 12, 14, and so on for that sum, and draw as many ellipses as you can on one piece of the graph paper.

a. Why can you not start with 9 as the sum?

b. What happens as the sum increases? decreases?

3. In Activity 3, you drew a hyperbola such that the difference of the distances from two fixed points was 7. Choose other numbers and draw as many hyperbolas as you can on one piece of graph paper. What happens as the difference increases? decreases?

454 Chapter 8 Conic Sections

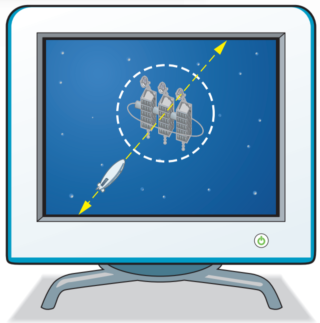
8-7 Solving Quadratic Systems

What You'll Learn

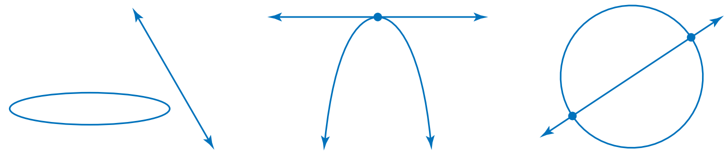
- Solve systems of quadratic equations algebraically and graphically.
- Solve systems of quadratic inequalities graphically.

How do systems of equations apply to video games?

Computer software often uses a coordinate system to keep track of the locations of objects on the screen. Suppose an enemy space station is located at the center of the screen, which is the origin in a coordinate system. The space station is surrounded by a circular force field of radius 50 units. If the spaceship you control is flying toward the center along the line with equation $y = 3x$, the point where the ship hits the force field is a solution of a system of equations.



SYSTEMS OF QUADRATIC EQUATIONS If the graphs of a system of equations are a conic section and a line, the system may have zero, one, or two solutions. Some of the possible situations are shown below.



no solutions   one solution   two solutions

You have solved systems of linear equations graphically and algebraically. You can use similar methods to solve systems involving quadratic equations.

Example 1 *Linear-Quadratic System*

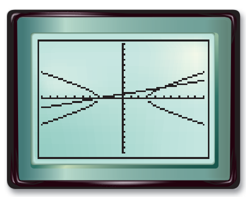
Solve the system of equations.

$x^2 - 4y^2 = 9$

$4y - x = 3$

You can use a graphing calculator to help visualize the relationships of the graphs of the equations and predict the number of solutions.

Solve each equation for y to obtain $y = \pm\frac{\sqrt{x^2 - 9}}{2}$ and $y = \frac{1}{4}x + \frac{3}{4}$. Enter the functions $y = \frac{\sqrt{x^2 - 9}}{2}$, $y = -\frac{\sqrt{x^2 - 9}}{2}$, and $y = \frac{1}{4}x + \frac{3}{4}$ on the Y= screen. The graph indicates that the hyperbola and line intersect in two points. So the system has two solutions.



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(continued on the next page)

8-7 Lesson Notes

1 Focus

5-Minute Check Transparency 8-7 Use as a quiz or review of Lesson 8-6.

Mathematical Background notes are available for this lesson on p. 410D.

How do systems of equations apply to video games?

Ask students:

- Describe the graph of $y = 3x$. **a line through the origin and steeper than the 45 degree line in quadrants I and III**
- Describe the graph of the force field. **a circle with a radius of 50 and a center at the center of the screen**

Resource Manager

Workbook and Reproducible Masters

Chapter 8 Resource Masters

- Study Guide and Intervention, pp. 491–492
- Skills Practice, p. 493
- Practice, p. 494
- Reading to Learn Mathematics, p. 495
- Enrichment, p. 496
- Assessment, p. 512

School-to-Career Masters, p. 16

Transparencies

5-Minute Check Transparency 8-7
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

SYSTEMS OF QUADRATIC EQUATIONS

In-Class Examples Power Point®

1 Solve the system of equations.
$4x^2 - 16y^2 = 25$
$2y + x = 2$
$\left(\frac{41}{16}, -\frac{9}{32}\right)$

Teaching Tip Point out that there are combinations of graphs other than those shown just before Example 2 which are possible for each number of solutions. For example, the ellipse in the third figure could be shifted to the right to intersect the circle in four points.

2 Solve the system of equations.
$x^2 + y^2 = 16$
$4x^2 + y^2 = 23$
$\left(\frac{\sqrt{21}}{3}, \frac{\sqrt{123}}{3}\right), \left(-\frac{\sqrt{21}}{3}, \frac{\sqrt{123}}{3}\right),$
$\left(\frac{\sqrt{21}}{3}, -\frac{\sqrt{123}}{3}\right),$
$\left(-\frac{\sqrt{21}}{3}, -\frac{\sqrt{123}}{3}\right)$

Use substitution to solve the system. First rewrite $4y - x = 3$ as $x = 4y - 3$.

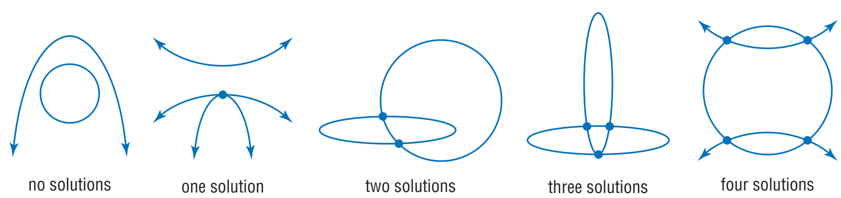
$x^2 - 4y^2 = 9$	First equation in the system
$(4y - 3)^2 - 4y^2 = 9$	Substitute $4y - 3$ for x.
$12y^2 - 24y = 0$	Simplify.
$y^2 - 2y = 0$	Divide each side by 12.
$y(y - 2) = 0$	Factor.
$y = 0$ or $y - 2 = 0$	Zero Product Property
$y = 2$	Solve for y.

Now solve for x.

$x = 4y - 3$	$x = 4y - 3$	Equation for x in terms of y
$= 4(0) - 3$	$= 4(2) - 3$	Substitute the y values.
$= -3$	$= 5$	Simplify.

The solutions of the system are $(-3, 0)$ and $(5, 2)$. Based on the graph, these solutions are reasonable.

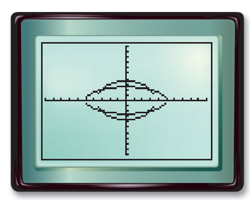
If the graphs of a system of equations are two conic sections, the system may have zero, one, two, three, or four solutions. Some of the possible situations are shown below.



Example 2 Quadratic-Quadratic System

Solve the system of equations.
$y^2 = 13 - x^2$
$x^2 + 4y^2 = 25$

A graphing calculator indicates that the circle and ellipse intersect in four points. So, this system has four solutions.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Use the elimination method to solve the system.

$y^2 = 13 - x^2$
$x^2 + 4y^2 = 25$

$-x^2 - y^2 = -13$	Rewrite the first original equation.
$(+)\ x^2 + 4y^2 = 25$	Second original equation
$3y^2 = 12$	Add.
$y^2 = 4$	Divide each side by 3.
$y = \pm 2$	Take the square root of each side.

Study Tip

Graphing Calculators
If you use ZSquare on the ZOOM menu, the graph of the first equation will look like a circle.

DAILY INTERVENTION

Differentiated Instruction

Logical Challenge students to sketch as many different possibilities as they can think of to add to the figures just before Example 2.

Substitute 2 and -2 for y in either of the original equations and solve for x.

$x^2 + 4y^2 = 25$	$x^2 + 4y^2 = 25$	Second original equation
$x^2 + 4(2)^2 = 25$	$x^2 + 4(-2)^2 = 25$	Substitute for y.
$x^2 = 9$	$x^2 = 9$	Subtract 16 from each side.
$x = \pm 3$	$x = \pm 3$	Take the square root of each side.

The solutions are (3, 2), $(-3, 2)$, $(-3, -2)$, and $(3, -2)$.

A graphing calculator can be used to approximate the solutions of a system of equations.

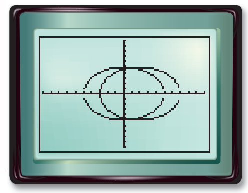
Graphing Calculator Investigation

Quadratic Systems

The calculator screen shows the graphs of two circles.

Think and Discuss

1. Write the system of equations represented by the graph. $x^2 + y^2 = 25$; $(x - 2)^2 + y^2 = 25$
2. Enter the equations into a TI-83 Plus and use the intersect feature on the CALC menu to solve the system. Round to the nearest hundredth.
3. Solve the system algebraically. $(1, \pm 2\sqrt{6})$
4. Can you always find the exact solution of a system using a graphing calculator? Explain.



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Use a graphing calculator to solve each system of equations. Round to the nearest hundredth.

5. $y = x + 2$, $x^2 + y^2 = 9$ (0.87, 2.87), (−2.87, −0.87)
6. $3x^2 + y^2 = 11$, $y = x^2 + x + 1$ (−1.57, 1.90), (0.96, 2.87)

2. $(1, \pm 4.90)$

4. No; a calculator only gives decimal approximations. If the solution involves irrational numbers or unfamiliar fractions, you may not be able to recognize them.

SYSTEMS OF QUADRATIC INEQUALITIES You have learned how to solve systems of linear inequalities by graphing. Systems of quadratic inequalities are also solved by graphing.

The graph of an inequality involving a parabola, circle, or ellipse is either the interior or the exterior of the conic section. The graph of an inequality involving a hyperbola is either the region between the branches or the two regions inside the branches. As with linear inequalities, examine the inequality symbol to determine whether to include the boundary.

Example 3 System of Quadratic Inequalities

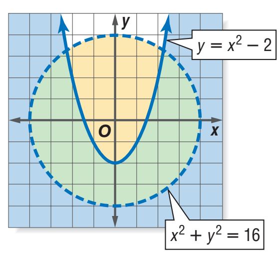
Solve the system of inequalities by graphing.

$y \le x^2 - 2$
$x^2 + y^2 < 16$

The graph of $y \le x^2 - 2$ is the parabola $y = x^2 - 2$ and the region outside or below it. This region is shaded blue.

The graph of $x^2 + y^2 < 16$ is the interior of the circle $x^2 + y^2 = 16$. This region is shaded yellow.

The intersection of these regions, shaded green, represents the solution of the system of inequalities.



Study Tip

Graphing Quadratic Inequalities

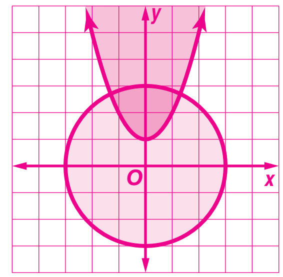
If you are unsure about which region to shade, you can test one or more points, as you did with linear inequalities.

www.algebra2.com/extra_examples

SYSTEMS OF QUADRATIC INEQUALITIES

In-Class Example Power Point®

3 Solve the system of inequalities by graphing.
$y > x^2 + 1$
$x^2 + y^2 \le 9$



Teaching Tip Remind students that it is always a good idea to check a point inside the solution area as well as one outside that area.

Graphing Calculator Investigation

- **Rounding** Discuss the fact that a coordinate such as $\sqrt{3}$ is an exact symbol for this irrational number, while the decimal value used for a graph, 1.73, is an approximation rounded to the nearest hundredth.
- **Viewing Window** Remind students that the circles will not appear circular on the calculator screen unless the viewing window ranges have been set for a square grid.

3 Practice/Apply

Study Notebook

Have students—
- complete the definitions/examples of the remaining terms on their Vocabulary Builder worksheets for Chapter 8.
- summarize what they learned about graphing systems of quadratic equations and quadratic inequalities.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Systems of Quadratic Equations:** 11–31
- **Systems of Quadratic Inequalities:** 32–37

Odd/Even Assignments

Exercises 11–26 and 32–37 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 11–21 odd, 25–27, 29–31, 33–37 odd, 38–45, 52–72

Average: 11–27 odd, 29–31, 33, 35, 37–45, 52–72 (optional: 46–51)

Advanced: 12–30 even, 31, 32–38 even, 39–72

Check for Understanding

Concept Check

1. See margin for graphs.

1. **Graph** each system of equations. Use the graph to solve the system.

a. $4x - 3y = 0$ $\quad x^2 + y^2 = 25$ **(−3, −4), (3, 4)**

b. $y = 5 - x^2$ $\quad y = 2x^2 + 2$ **(±1, 4)**

2. **Sketch** a parabola and an ellipse that intersect at exactly three points. **See margin.**

3. **OPEN ENDED** Write a system of quadratic equations for which (2, 6) is a solution. **Sample answer: $x^2 + y^2 = 40$, $y = x^2 + x$**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4, 5	1
6, 7, 10	2
8, 9	3

Find the exact solution(s) of each system of equations.

4. $y = 5$, $y^2 = x^2 + 9$ **(±4, 5)**

5. $y - x = 1$, $x^2 + y^2 = 25$ **(−4, −3), (3, 4)**

6. $3x = 8y^2$, $8y^2 - 2x^2 = 16$ **no solution**

7. $5x^2 + y^2 = 30$, $9x^2 - y^2 = -16$ **(1, ±5), (−1, ±5)**

Solve each system of inequalities by graphing. **8–9. See pp. 469A–469J.**

8. $x + y < 4$, $9x^2 - 4y^2 \geq 36$

9. $x^2 + y^2 < 25$, $4x^2 - 9y^2 < 36$

Application

10. **EARTHQUAKES** In a coordinate system where a unit represents one mile, the epicenter of an earthquake was determined to be 50 miles from a station at the origin. It was also 40 miles from a station at (0, 30) and 13 miles from a station at (35, 18). Where was the epicenter located? **(40, 30)**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
11–16, 25, 29	1
17–24, 26–28, 30, 31	2
32–37	3

Extra Practice

See page 847.

Find the exact solution(s) of each system of equations.

11. $y = x + 2$, $y = x^2$ **(2, 4), (−1, 1)**

12. $y = x + 3$, $y = 2x^2$ **$\left(\frac{3}{2}, \frac{9}{2}\right)$, (−1, 2)**

13. $x^2 + y^2 = 36$, $y = x + 2$ **$(-1 + \sqrt{17}, 1 + \sqrt{17})$, $(-1 - \sqrt{17}, 1 - \sqrt{17})$**

14. $y^2 + x^2 = 9$, $y = 7 - x$ **no solution**

15. $\frac{x^2}{30} + \frac{y^2}{6} = 1$, $x = y$ **$(\sqrt{5}, \sqrt{5})$, $(-\sqrt{5}, -\sqrt{5})$**

16. $\frac{x^2}{36} - \frac{y^2}{4} = 1$, $x = y$ **no solution**

17. $4x + y^2 = 20$, $4x^2 + y^2 = 100$ **(5, 0), (−4, ±6)**

18. $y + x^2 = 3$, $x^2 + 4y^2 = 36$ **(0, 3), $\left(\pm\frac{\sqrt{23}}{2}, -\frac{11}{4}\right)$**

19. $x^2 + y^2 = 64$, $x^2 + 64y^2 = 64$ **(±8, 0)**

20. $y^2 + x^2 = 25$, $y^2 + 9x^2 = 25$ **(0, ±5)**

21. $y^2 = x^2 - 25$, $x^2 - y^2 = 7$ **no solution**

22. $y^2 = x^2 - 7$, $x^2 + y^2 = 25$ **(4, ±3), (−4, ±3)**

★ **23.** $2x^2 + 8y^2 + 8x - 48y + 30 = 0$, $2x^2 - 8y^2 = -48y + 90$ **(−5, 5), (−5, 1), (3, 3)**

★ **24.** $3x^2 - 20y^2 - 12x + 80y - 96 = 0$, $3x^2 + 20y^2 = 80y + 48$ **(6, 3), (6, 1), (−4, 4), (−4, 0)**

25. $\left(-\frac{5}{3}, -\frac{7}{3}\right)$, (1, 3)

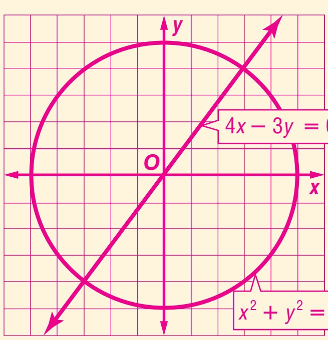
25. Where do the graphs of the equations $y = 2x + 1$ and $2x^2 + y^2 = 11$ intersect?

26. What are the coordinates of the points that lie on the graphs of both $x^2 + y^2 = 25$ and $2x^2 + 3y^2 = 66$? **(3, ±4), (−3, ±4)**

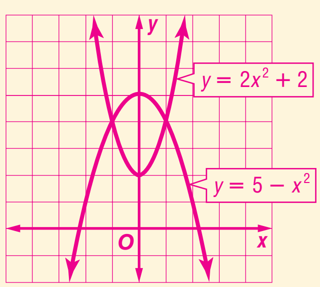
27. **ROCKETS** Two rockets are launched at the same time, but from different heights. The height y in feet of one rocket after t seconds is given by $y = -16t^2 + 150t + 5$. The height of the other rocket is given by $y = -16t^2 + 160t$. After how many seconds are the rockets at the same height? **0.5 s**

Answers

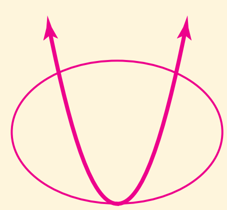
1a.



1b.



2. The vertex of the parabola is on the ellipse. The parabola opens toward the interior of the ellipse and is narrow enough to intersect the ellipse in two other points. Thus, there are exactly three points of intersection.



458 Chapter 8 Conic Sections

★ **28. ADVERTISING** The corporate logo for an automobile manufacturer is shown at the right. Write a system of three equations to model this logo. **Sample answer:**

$$\frac{x^2}{36} + \frac{y^2}{16} = 1,\ \frac{x^2}{16} + \frac{(y-2)^2}{4} = 1,\ \frac{x^2}{2} + \frac{y^2}{16} = 1$$

29. MIRRORS A hyperbolic mirror is a mirror in the shape of one branch of a hyperbola. Such a mirror reflects light rays directed at one focus toward the other focus. Suppose a hyperbolic mirror is modeled by the upper branch of the hyperbola with equation $\frac{y^2}{9} - \frac{x^2}{16} = 1$. A light source is located at $(-10, 0)$. Where should the light from the source hit the mirror so that the light will be reflected to $(0, -5)$?

29. $\left(\frac{40 - 24\sqrt{5}}{5}, \frac{45 - 12\sqrt{5}}{5}\right)$

More About. . .

Astronomy

The astronomical unit (AU) is the mean distance between Earth and the Sun. One AU is about 93 million miles or 150 million kilometers.

Source: www.infoplease.com

ASTRONOMY **For Exercises 30 and 31, use the following information.**

The orbit of Pluto can be modeled by the equation $\frac{x^2}{39.5^2} + \frac{y^2}{38.3^2} = 1$, where the units are astronomical units. Suppose a comet is following a path modeled by the equation $x = y^2 + 20$.

30. Find the point(s) of intersection of the orbits of Pluto and the comet. Round to the nearest tenth. **$(39.2, \pm 4.4)$**

31. Will the comet necessarily hit Pluto? Explain. **No; the comet and Pluto may not be at either point of intersection at the same time.**

Solve each system of inequalities by graphing. **32–37. See pp. 469A–469J.**

32. $x + 2y > 1$, $x^2 + y^2 \le 25$ **33.** $x + y \le 2$, $4x^2 - y^2 \ge 4$ **34.** $x^2 + y^2 \ge 4$, $4y^2 + 9x^2 \le 36$

35. $x^2 + y^2 < 36$, $4x^2 + 9y^2 > 36$ **36.** $y^2 < x$, $x^2 - 4y^2 < 16$ **37.** $x^2 \le y$, $y^2 - x^2 \ge 4$

CRITICAL THINKING **For Exercises 38–42, find all values of k for which the system of equations has the given number of solutions. If no values of k meet the condition, write *none*.** **38. $k < -3$, $-2 < k < 2$, or $k > 3$**

$$x^2 + y^2 = k^2 \qquad \frac{x^2}{9} + \frac{y^2}{4} = 1$$

40. $k = \pm 2$ or $k = \pm 3$

38. no solutions **39.** one solution **none** **40.** two solutions

41. three solutions **none** **42.** four solutions **$-3 < k < -2$ or $2 < k < 3$**

43. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson. **See pp. 469A–469J.**

How do systems of equations apply to video games?

Include the following in your answer:

- a linear-quadratic system of equations that applies to this situation,
- an explanation of how you know that the spaceship is headed directly toward the center of the screen, and
- the coordinates of the point at which the spaceship will hit the force field, assuming that the spaceship moves from the bottom of the screen toward the center.

Standardized Test Practice

44. If $\boxed{x}$ is defined to be $x^2 - 4x$ for all numbers x, which of the following is the greatest? **A**

Ⓐ $\boxed{0}$ Ⓑ $\boxed{1}$ Ⓒ $\boxed{2}$ Ⓓ $\boxed{3}$

45. How many three-digit numbers are divisible by 3? **B**

Ⓐ 299 Ⓑ 300 Ⓒ 301 Ⓓ 302

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 491 (shown) and p. 492

Systems of Quadratic Equations Like systems of linear equations, systems of quadratic equations can be solved by substitution and elimination. If the graphs are a conic section and a line, the system will have 0, 1, or 2 solutions. If the graphs are two conic sections, the system will have 0, 1, 2, 3, or 4 solutions.

Example **Solve the system of equations. $y = x^2 - 2x - 15$, $x + y = -3$**

Rewrite the second equation as $y = -x - 3$ and substitute into the first equation.

$-x - 3 = x^2 - 2x - 15$

$0 = x^2 - x - 12$ Add $x + 3$ to each side.

$0 = (x - 4)(x + 3)$ Factor.

Use the Zero Product property to get $x = 4$ or $x = -3$.

Substitute these values for x in $x + y = -3$:

$4 + y = -3$ or $-3 + y = -3$

$y = -7$ $y = 0$

The solutions are $(4, -7)$ and $(-3, 0)$.

Exercises

Find the exact solution(s) of each system of equations.

1. $y = x^2 - 5$, $y = x - 3$ **$(2, -1), (-1, -4)$**

2. $x^2 + (y - 5)^2 = 25$, $y = -x^2$ **$(0, 0)$**

3. $x^2 + (y - 5)^2 = 25$, $y = x^2$ **$(0, 0), (3, 9), (-3, 9)$**

4. $x^2 + y^2 = 9$, $x^2 + y = 3$ **$(0, 3), (\sqrt{5}, -2), (-\sqrt{5}, -2)$**

5. $x^2 - y^2 = 1$, $x^2 + y^2 = 16$ **$\left(\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, \frac{\sqrt{30}}{2}\right), \left(-\frac{\sqrt{34}}{2}, -\frac{\sqrt{30}}{2}\right)$**

6. $y = x - 3$, $x = y^2 - 4$ **$\left(\frac{7 + \sqrt{29}}{2}, \frac{1 + \sqrt{29}}{2}\right), \left(\frac{7 - \sqrt{29}}{2}, \frac{1 - \sqrt{29}}{2}\right)$**

Skills Practice, p. 493 and Practice, p. 494 (shown)

Find the exact solution(s) of each system of equations.

1. $(x - 2)^2 + y^2 = 5$, $x - y = 1$ **$(0, -1), (3, 2)$**

2. $x = 2(y + 1)^2 - 6$, $x + y = 3$ **$(2, 1), (6.5, -3.5)$**

3. $y^2 - 3x^2 = 6$, $y = 2x - 1$ **$(-1, -3), (5, 9)$**

4. $x^2 + 2y^2 = 1$, $y = -x + 1$ **$(1, 0), \left(\frac{1}{3}, \frac{2}{3}\right)$**

5. $4y^2 - 9x^2 = 36$, $4x^2 - 9y^2 = 36$ **no solution**

6. $y = x^2 - 3$, $x^2 + y^2 = 9$ **$(0, -3), (\pm\sqrt{5}, 2)$**

7. $x^2 + y^2 = 25$, $4y = 3x$ **$(4, 3), (-4, -3)$**

8. $y^2 = 10 - 6x^2$, $4y^2 = 40 - 2x^2$ **$(0, \pm\sqrt{10})$**

9. $x^2 + y^2 = 25$, $x = 3y - 5$ **$(-5, 0), (4, 3)$**

10. $4x^2 + 9y^2 = 36$, $2x^2 - 9y^2 = 18$ **$(\pm 3, 0)$**

11. $x = -(y - 3)^2 + 2$, $x = (y - 3)^2 + 3$ **no solution**

12. $\frac{x^2}{9} - \frac{y^2}{16} = 1$, $x^2 + y^2 = 9$ **$(\pm 3, 0)$**

13. $25x^2 + 4y^2 = 100$, $x = -\frac{5}{2}$ **no solution**

14. $x^2 + y^2 = 4$, $\frac{x^2}{4} + \frac{y^2}{8} = 1$ **$(\pm 2, 0)$**

15. $x^2 - y^2 = 3$, $y^2 - x^2 = 3$ **no solution**

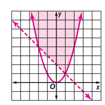
16. $\frac{x^2}{7} + \frac{y^2}{7} = 1$, $3x^2 - y^2 = 9$ **$(\pm 2, \pm\sqrt{3})$**

17. $x + 2y = 3$, $x^2 + y^2 = 9$ **$(3, 0), \left(-\frac{9}{5}, \frac{12}{5}\right)$**

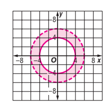
18. $x^2 + y^2 = 64$, $x^2 - y^2 = 8$ **$(\pm 6, \pm 2\sqrt{7})$**

Solve each system of inequalities by graphing.

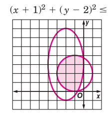
19. $y \ge x^2$, $y > -x + 2$



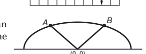
20. $x^2 + y^2 < 36$, $x^2 + y^2 \ge 16$



21. $\frac{(y - 3)^2}{16} + \frac{(x + 2)^2}{4} \le 1$, $(x + 1)^2 + (y - 2)^2 \le 4$



22. GEOMETRY The top of an iron gate is shaped like half an ellipse with two congruent segments from the center of the ellipse to the ellipse as shown. Assume that the center of the ellipse is at $(0, 0)$. If the ellipse can be modeled by the equation $x^2 + 4y^2 = 4$ for $y \ge 0$ and the two congruent segments can be modeled by $y = \frac{\sqrt{3}}{2}x$ and $y = -\frac{\sqrt{3}}{2}x$, what are the coordinates of points A and B?



$\left(-1, \frac{\sqrt{3}}{2}\right)$ and $\left(1, \frac{\sqrt{3}}{2}\right)$

Reading to Learn Mathematics, p. 495 ELL

Pre-Activity **How do systems of equations apply to video games?**

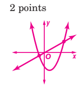
Read the introduction to Lesson 8-7 at the top of page 455 in your textbook.

The figure in your textbook shows that the spaceship hits the circular force field in two points. Is it possible for the spaceship to hit the force field in either fewer or more than two points? State all possibilities and explain how these could happen. **Sample answer: The spaceship could hit the force field in zero points if the spaceship missed the force field all together. The spaceship could also hit the force field in one point if the spaceship just touched the edge of the force field.**

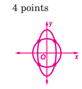
Reading the Lesson

1. Draw a sketch to illustrate each of the following possibilities.

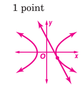
a. a parabola and a line that intersect in 2 points



b. an ellipse and a circle that intersect in 4 points



c. a hyperbola and a line that intersect in 1 point



2. Consider the following system of equations.

$x^2 = 3 + y^2$

$2x^2 + 3y^2 = 11$

a. What kind of conic section is the graph of the first equation? **hyperbola**

b. What kind of conic section is the graph of the second equation? **ellipse**

c. Based on your answers to parts a and b, what are the possible numbers of solutions that this system could have? **0, 1, 2, 3, or 4**

Helping You Remember

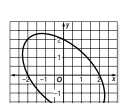
3. Suppose that the graph of a quadratic inequality is a region whose boundary is a circle. How can you remember whether to shade the interior or the exterior of the circle? **Sample answer: The solutions of an inequality of the form $x^2 + y^2 < r^2$ are all points that are *less than* r units from the origin, so the graph is the *interior* of the circle. The solutions of an inequality of the form $x^2 + y^2 > r^2$ are the points that are *more than* r units from the origin, so the graph is the *exterior* of the circle.**

Enrichment, p. 496

Graphing Quadratic Equations with xy-Terms

You can use a graphing calculator to examine graphs of quadratic equations that contain xy-terms.

Example **Use a graphing calculator to display the graph of $x^2 + xy + y^2 = 4$.**



Solve the equation for y in terms of x by using the quadratic formula.

$y^2 + xy + (x^2 - 4) = 0$

To use the formula, let $a = 1$, $b = x$, and $c = (x^2 - 4)$.

$$y = \frac{-x \pm \sqrt{x^2 - 4(1)(x^2 - 4)}}{2}$$

$$y = \frac{-x \pm \sqrt{16 - 3x^2}}{2}$$

4 Assess

Open-Ended Assessment

Speaking Have students use a sketch to explain how they know which regions to shade for systems of quadratic inequalities.

Assessment Options

Quiz (Lesson 8-7) is available on p. 512 of the *Chapter 8 Resource Masters*.

Answers

46. Sample answer: $y = x^2$, $x = (y - 2)^2$

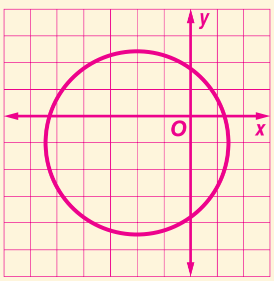
47. Sample answer: $x^2 + y^2 = 36$, $\frac{(x+2)^2}{16} - \frac{y^2}{4} = 1$

48. Sample answer: $x^2 + y^2 = 100$, $\frac{x^2}{16} + \frac{y^2}{4} = 1$

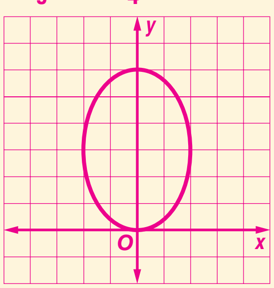
49. Sample answer: $x^2 + y^2 = 81$, $\frac{x^2}{4} + \frac{y^2}{100} = 1$

50. Sample answer: $\frac{x^2}{64} + \frac{y^2}{36} = 1$, $\frac{x^2}{64} - \frac{y^2}{36} = 1$

52. $(x + 2)^2 + (y + 1)^2 = 11$, circle



53. $\frac{(y-3)^2}{9} + \frac{x^2}{4} = 1$, ellipse



Graphing Calculator

46–50. See margin for sample answers.

SYSTEMS OF EQUATIONS **Write a system of equations that satisfies each condition. Use a graphing calculator to verify that you are correct.**

46. two parabolas that intersect in two points
47. a hyperbola and a circle that intersect in three points
48. a circle and an ellipse that do not intersect
49. a circle and an ellipse that intersect in four points
50. a hyperbola and an ellipse that intersect in two points
51. two circles that intersect in three points **impossible**

Maintain Your Skills

Mixed Review

Write each equation in standard form. State whether the graph of the equation is a *parabola, circle, ellipse,* or *hyperbola*. Then graph the equation. *(Lesson 8-6)*

52. $x^2 + y^2 + 4x + 2y - 6 = 0$
53. $9x^2 + 4y^2 - 24y = 0$

52–53. See margin.

54. Find the coordinates of the vertices and foci and the equations of the asymptotes of the hyperbola with the equation $6y^2 - 2x^2 = 24$. Then graph the hyperbola. *(Lesson 8-5)* **See margin.**

Solve each equation by factoring. *(Lesson 6-5)*

55. $x^2 + 7x = 0$ **$-7, 0$**
56. $x^2 - 3x = 0$ **$0, 3$**
57. $22 = 9x^2 + 4x$ **$-7, 3$**
58. $35 = -2x + x^2$ **$7, -5$**
59. $9x^2 + 24 = -16$ **$-\frac{4}{3}$**
60. $8x^2 + 2x = 3$ **$-\frac{3}{4}, \frac{1}{2}$**

For Exercises 61 and 62, complete parts a–c for each quadratic equation. *(Lesson 6-5)*

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

61. $5x^2 = 2$
62. $-3x^2 + 6x - 7 = 0$

61a. 40
61b. two real, irrational
61c. $\pm\frac{\sqrt{10}}{5}$
62a. -48
62b. two imaginary
62c. $1 \pm \frac{2i\sqrt{3}}{3}$

Simplify. *(Lesson 5-9)*

63. $(3 + 2i) - (1 - 7i)$ **$2 + 9i$**
64. $(8 - i)(4 - 3i)$ **$29 - 28i$**
65. $\frac{2 + 3i}{1 + 2i}$ **$\frac{8}{5} - \frac{1}{5}i$**

66. **CHEMISTRY** The mass of a proton is about 1.67×10^{-27} kilogram. The mass of an electron is about 9.11×10^{-31} kilogram. About how many times as massive as an electron is a proton? *(Lesson 5-1)* **about 1830 times**

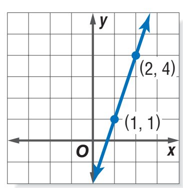
Evaluate each determinant. *(Lesson 4-3)*

67. $\begin{vmatrix} 2 & -3 \\ 2 & 0 \end{vmatrix}$ **6**
68. $\begin{vmatrix} -4 & -2 \\ 5 & 3 \end{vmatrix}$ **-2**
69. $\begin{vmatrix} 2 & 1 & -2 \\ 4 & 0 & 3 \\ -3 & 1 & 7 \end{vmatrix}$ **-51**

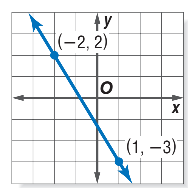
70. Solve the system of equations. *(Lesson 3-5)* **(5, 3, 7)**

$r + s + t = 15$
$r + t = 12$
$s + t = 10$

Write an equation in slope-intercept form for each graph. *(Lesson 2-4)*

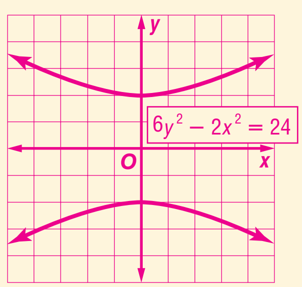
71. 

$y = 3x - 2$

72. 

$y = -\frac{5}{3}x - \frac{4}{3}$

54. $(0, \pm 2)$; $(0, \pm 4)$; $y = \pm\frac{\sqrt{3}}{3}x$



Chapter 8

Study Guide and Review

Vocabulary and Concept Check

asymptote (p. 442)	Distance Formula (p. 413)	major axis (p. 434)
center of a circle (p. 426)	ellipse (p. 433)	Midpoint Formula (p. 412)
center of a hyperbola (p. 442)	foci of a hyperbola (p. 441)	minor axis (p. 434)
center of an ellipse (p. 434)	foci of an ellipse (p. 433)	parabola (p. 419)
circle (p. 426)	focus of a parabola (p. 419)	tangent (p. 427)
conic section (p. 419)	hyperbola (p. 441)	transverse axis (p. 442)
conjugate axis (p. 442)	latus rectum (p. 421)	vertex of a hyperbola (p. 442)
directrix (p. 419)		

Tell whether each statement is *true* or *false*. If the statement is false, correct it to make it true. **4, 6, 8, 9. See pp. 469A–469J for correct statements.**

1. An ellipse is the set of all points in a plane such that the sum of the distances from two given points in the plane, called the foci, is constant. **true**
2. The major axis is the longer of the two axes of symmetry of an ellipse. **true**
3. The formula used to find the distance between two points in a coordinate plane is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. **true**
4. A parabola is the set of all points that are the same distance from a given point called the directrix and a given line called the focus. **false**
5. The radius is the distance from the center of a circle to any point on the circle. **true**
6. The conjugate axis of a hyperbola is a line segment parallel to the transverse axis. **false**
7. A conic section is formed by slicing a double cone by a plane. **true**
8. A hyperbola is the set of all points in a plane such that the absolute value of the sum of the distances from any point on the hyperbola to two given points is constant. **false**
9. The midpoint formula is given by $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$. **false**
10. The set of all points in a plane that are equidistant from a given point in a plane, called the center, forms a circle. **true**

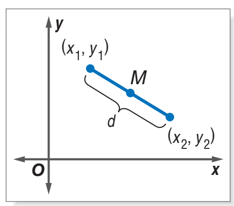
Lesson-by-Lesson Review

8-1 Midpoint and Distance Formulas

See pages 412–416.

Concept Summary

- Midpoint Formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Examples

1 **Find the midpoint of a segment whose endpoints are at (−5, 9) and (11, −1).**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 11}{2}, \frac{9 + (-1)}{2}\right) \quad \text{Let } (x_1, y_1) = (-5, 9) \text{ and } (x_2, y_2) = (11, -1).$$

$$= \left(\frac{6}{2}, \frac{8}{2}\right) \text{ or } (3, 4) \quad \text{Simplify.}$$

www.algebra2.com/vocabulary_review

FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

As students review their Foldable for this chapter, ask them if they have used titles and headings to make it clear what the topic is for each section they write. Ask volunteers for some possible headings within a lesson, and for titles that might tie the whole chapter together. Encourage students to use both their own informal phrasing and the correct mathematical terminology of the textbook as they write.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Chapter 8

Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 8 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 8 is available on p. 510 of the *Chapter 8 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

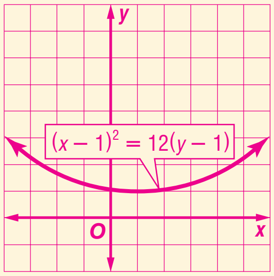
MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

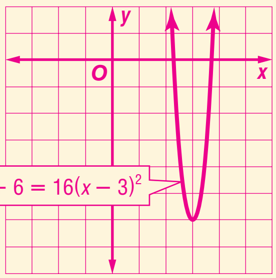
Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

Answers

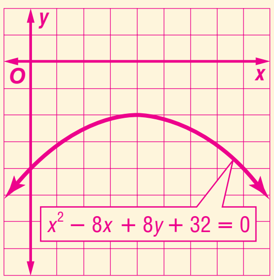
17. (1, 1); (1, 4); $x = 1$; $y = -2$; upward; 12 units



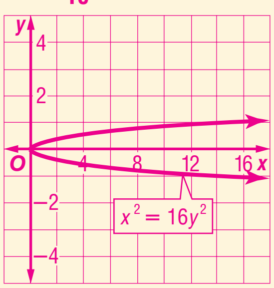
18. (3, −6); $\left(3, -5\frac{63}{64}\right)$; $x = 3$; $y = -6\frac{1}{64}$; upward; $\frac{1}{16}$ unit



19. (4, −2); (4, −4); $x = 4$; $y = 0$; downward; 8 units



20. (0, 0); $\left(\frac{1}{64}, 0\right)$; $y = 0$; $x = -\frac{1}{64}$; right; $\frac{1}{16}$ unit



Chapter 8 Study Guide and Review

2 Find the distance between $P(6, -4)$ and $Q(-3, 8)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Distance Formula

$= \sqrt{(-3 - 6)^2 + [8 - (-4)]^2}$ Let $(x_1, y_1) = (6, -4)$ and $(x_2, y_2) = (-3, 8)$.

$= \sqrt{81 + 144}$ Subtract.

$= \sqrt{225}$ or 15 units Simplify.

Exercises **Find the midpoint of the line segment with endpoints at the given coordinates.** *See Example 1 on page 412.*

11. (1, 2), (4, 6) $\left(\frac{5}{2}, 4\right)$ **12.** (−8, 0), (−2, 3) $\left(-5, \frac{3}{2}\right)$ **13.** $\left(\frac{3}{5}, -\frac{7}{4}\right), \left(\frac{1}{4}, -\frac{2}{5}\right)$ $\left(\frac{17}{40}, -\frac{43}{40}\right)$

Find the distance between each pair of points with the given coordinates. *See Examples 2 and 3 on pages 413 and 414.*

14. (−2, 10), (−2, 13) 3 units **15.** (8, 5), (−9, 4) $\sqrt{290}$ units **16.** (7, −3), (1, 2) $\sqrt{61}$ units

8-2 Parabolas

See pages 419–425.

Concept Summary

Parabolas		
Standard Form	$y = a(x - h)^2 + k$	$x = a(y - k)^2 + h$
Vertex	(h, k)	(h, k)
Axis of Symmetry	$x = h$	$y = k$
Focus	$\left(h, k + \frac{1}{4a}\right)$	$\left(h + \frac{1}{4a}, k\right)$
Directrix	$y = k - \frac{1}{4a}$	$x = h - \frac{1}{4a}$

Example Graph $4y - x^2 = 14x - 27$.

First write the equation in the form $y = a(x - h)^2 + k$.

$4y - x^2 = 14x - 27$ Original equation

$4y = x^2 + 14x - 27$ Isolate the terms with x.

$4y = (x^2 + 14x + ■) - 27 - ■$ Complete the square.

$4y = (x^2 + 14x + 49) - 27 - 49$ Add and subtract 49, since $\left(\frac{14}{2}\right)^2 = 49$.

$4y = (x + 7)^2 - 76$ $x^2 + 14x + 49 = (x + 7)^2$

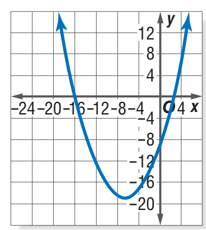
$y = \frac{1}{4}(x + 7)^2 - 19$ Divide each side by 4.

vertex: (−7, −19) axis of symmetry: $x = -7$

focus: $\left(-7, -19 + \frac{1}{4\left(\frac{1}{4}\right)}\right)$ or (−7, −18)

directrix: $y = -19 - \frac{1}{4\left(\frac{1}{4}\right)}$ or $y = -20$

direction of opening: upward since $a > 0$



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Exercises **Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.** *See Examples 2–4 on pages 420–423.* **17–21. See margin.**

17. $(x-1)^2 = 12(y-1)$ **18.** $y + 6 = 16(x-3)^2$

19. $x^2 - 8x + 8y + 32 = 0$ **20.** $x = 16y^2$

21. Write an equation for a parabola with vertex (0, 1) and focus (0, −1). Then graph the parabola. *See Example 4 on pages 422 and 423.*

8-3 Circles

See pages 426–431.

Concept Summary

- The equation of a circle with center (h, k) and radius r can be written in the form $(x-h)^2 + (y-k)^2 = r^2$.

Example **Graph $x^2 + y^2 + 8x - 24y + 16 = 0$.**

First write the equation in the form $(x-h)^2 + (y-k)^2 = r^2$.

$$x^2 + y^2 + 8x - 24y + 16 = 0 \quad \text{Original equation}$$

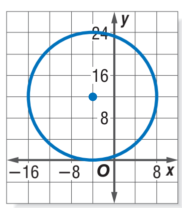
$$x^2 + 8x + \blacksquare + y^2 - 24y + \blacksquare = -16 + \blacksquare + \blacksquare \quad \text{Complete the squares.}$$

$$x^2 + 8x + 16 + y^2 - 24y + 144 = -16 + 16 + 144 \quad \left(\frac{8}{2}\right)^2 = 16, \left(\frac{-24}{2}\right)^2 = 144$$

$$(x+4)^2 + (y-12)^2 = 144 \quad \text{Write the trinomials as squares.}$$

The center of the circle is at (−4, 12) and the radius is 12.

Now draw the graph.



Exercises **Write an equation for the circle that satisfies each set of conditions.** *See Example 1 on page 426.*

22. center (2, −3), radius 5 units $(x-2)^2 + (y+3)^2 = 25$

23. center (−4, 0), radius $\frac{3}{4}$ unit $(x+4)^2 + y^2 = \frac{9}{16}$

24. endpoints of a diameter at (9, 4) and (−3, −2) $(x-3)^2 + (y-1)^2 = 45$

25. center at (−1, 2), tangent to x-axis $(x+1)^2 + (y-2)^2 = 4$

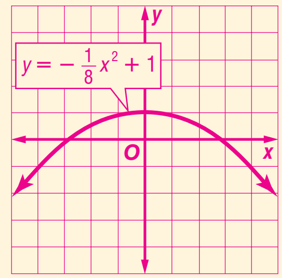
Find the center and radius of the circle with the given equation. Then graph the circle. *See Examples 4 and 5 on page 428.* **26–29. See margin for graphs.**

26. $x^2 + y^2 = 169$ **(0, 0), 13 units** **27.** $(x+5)^2 + (y-11)^2 = 49$ **(−5, 11), 7 units**

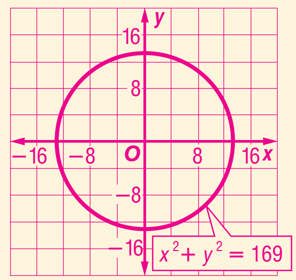
28. $x^2 + y^2 - 6x + 16y - 152 = 0$ **(3, −8), 15 units** **29.** $x^2 + y^2 + 6x - 2y - 15 = 0$ **(−3, 1), 5 units**

Answers

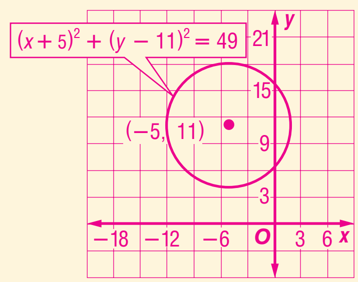
21. $y = -\frac{1}{8}x^2 + 1$



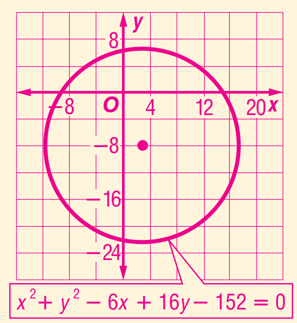
26.



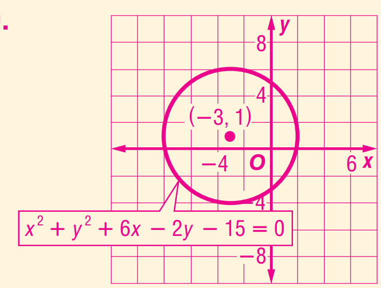
27.



28.

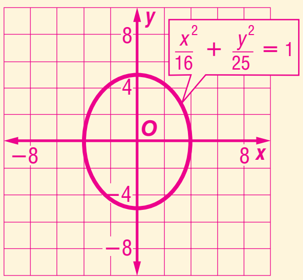


29.

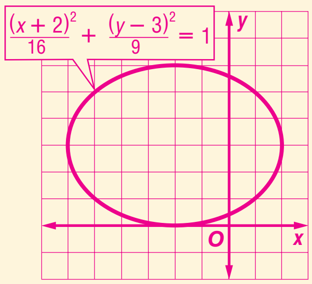


Answers

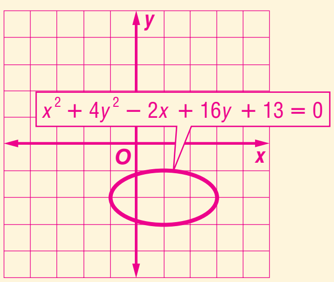
31. (0, 0); (0, ±3); 10; 8



32. (−2, 3); $(-2 \pm \sqrt{7}, 3)$; 8; 6



33. (1, −2); $(1 \pm \sqrt{3}, -2)$; 4; 2



8-4 Ellipses

See pages 433–440.

Concept Summary

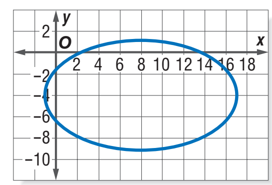
Ellipses		
Standard Form of Equation	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Direction of Major Axis	horizontal	vertical

Example **Graph $x^2 + 3y^2 - 16x + 24y + 31 = 0$.**

First write the equation in standard form by completing the squares.

$x^2 + 3y^2 - 16x + 24y + 31 = 0$	Original equation
$x^2 - 16x + ■ + 3(y^2 + 8y + ■) = -31 + ■ + 3(■)$	Complete the squares.
$x^2 - 16x + 64 + 3(y^2 + 8y + 16) = -31 + 64 + 3(16)$	$\left(\frac{-16}{2}\right)^2 = 64$, $\left(\frac{8}{2}\right)^2 = 16$
$(x-8)^2 + 3(y+4)^2 = 81$	Write the trinomials as squares.
$\frac{(x-8)^2}{81} + \frac{(y+4)^2}{27} = 1$	Divide each side by 81.

The center of the ellipse is at (8, −4). The length of the major axis is 18, and the length of the minor axis is $6\sqrt{3}$.



Exercises

30. Write an equation for the ellipse with endpoints of the major axis at (4, 1) and (−6, 1) and endpoints of the minor axis at (−1, 3) and (−1, −1).
See Examples 1 and 2 on pages 434 and 435. $\frac{(x+1)^2}{25} + \frac{(y-1)^2}{4} = 1$

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.
See Examples 3 and 4 on pages 436 and 437. **31–33. See margin.**

31. $\frac{x^2}{16} + \frac{y^2}{25} = 1$ **32.** $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{9} = 1$ **33.** $x^2 + 4y^2 - 2x + 16y + 13 = 0$

8-5 Hyperbolas

See pages 441–448.

Concept Summary

Hyperbolas		
Standard Form	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transverse Axis	horizontal	vertical
Asymptotes	$y - k = \pm\frac{b}{a}(x-h)$	$y - k = \pm\frac{a}{b}(x-h)$

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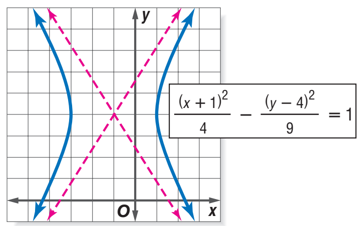
Example **Graph $9x^2 - 4y^2 + 18x + 32y - 91 = 0$.**

Complete the square for each variable to write this equation in standard form.

$9x^2 - 4y^2 + 18x + 32y - 91 = 0$	Original equation
$9(x^2 + 2x + \blacksquare) - 4(y^2 - 8y + \blacksquare) = 91 + 9(\blacksquare) - 4(\blacksquare)$	Complete the squares.
$9(x^2 + 2x + 1) - 4(y^2 - 8y + 16) = 91 + 9(1) - 4(16)$	$\left(\frac{2}{2}\right)^2 = 1$, $\left(\frac{-8}{2}\right)^2 = 16$
$9(x + 1)^2 - 4(y - 4)^2 = 36$	Write the trinomials as squares.
$\frac{(x + 1)^2}{4} - \frac{(y - 4)^2}{9} = 1$	Divide each side by 36.

The center is at $(-1, 4)$. The vertices are at $(-3, 4)$ and $(1, 4)$ and the foci are at $\left(-1 \pm \sqrt{13}, 4\right)$. The equations of the asymptotes are $y - 4 = \pm\frac{3}{2}(x + 1)$.



Exercises

34. Write an equation for a hyperbola that has vertices at (2, 5) and (2, 1) and a conjugate axis of length 6 units. *See Example 1 on page 442.* $\frac{(y-3)^2}{4} - \frac{(x-2)^2}{9} = 1$

Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.
See Examples 3 and 4 on pages 443 and 444. **35–38. See margin.**

35. $\frac{y^2}{4} - \frac{x^2}{9} = 1$

36. $\frac{(x - 2)^2}{1} - \frac{(y + 1)^2}{9} = 1$

37. $9y^2 - 16x^2 = 144$

38. $16x^2 - 25y^2 - 64x - 336 = 0$

8-6 Conic Sections

See pages 449–452.

Concept Summary

- Conic sections can be identified directly from their equations of the form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, assuming $B = 0$.

Conic Section	Relationship of *A* and *C*
Parabola	$A = 0$ or $C = 0$, but not both.
Circle	$A = C$
Ellipse	A and C have the same sign and $A \neq C$.
Hyperbola	A and C have opposite signs.

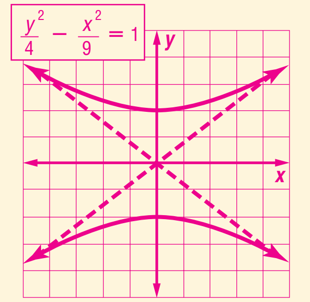
Example **Without writing the equation in standard form, state whether the graph of $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ is a *parabola*, *circle*, *ellipse*, or *hyperbola*.**

In this equation, $A = 4$ and $C = 9$. Since A and C are both positive and $A \neq C$, the graph is an ellipse.

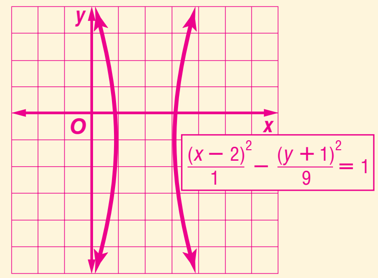
Study Guide and Review

Answers

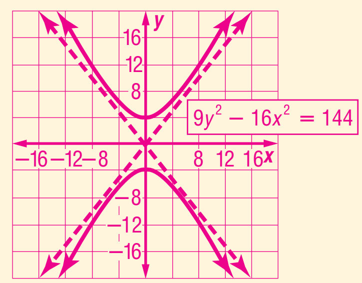
35. $(0, \pm2)$; $\left(0, \pm\sqrt{13}\right)$; $y = \pm\frac{2}{3}x$



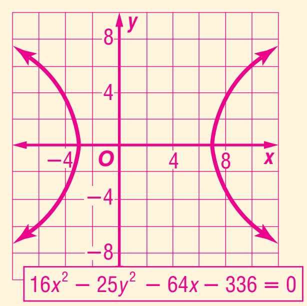
36. $(1, -1)$, $(3, -1)$; $\left(2 \pm \sqrt{10}, -1\right)$; $y + 1 = \pm3(x - 2)$



37. $(0, \pm4)$; $(0, \pm5)$; $y \pm\frac{4}{3}x$



38. $(-3, 0)$, $(7, 0)$; $\left(2 \pm \sqrt{41}, 0\right)$; $y = \pm\frac{4}{5}(x - 2)$



Chapter 8 **For More ...**

- Extra Practice, see pages 845–847.
- Mixed Problem Solving, see page 869.

Exercises **Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.** *See Example 1 on page 449.* **39–42. See margin for equations and graphs.**

39. $x^2 + 4x - y = 0$ **parabola**
40. $9x^2 + 4y^2 = 36$ **ellipse**
41. $-4x^2 + y^2 + 8x - 8 = 0$ **hyperbola**
42. $x^2 + y^2 - 4x - 6y + 4 = 0$ **circle**

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. *See Example 2 on page 450.*

43. $7x^2 + 9y^2 = 63$ **ellipse**
44. $x^2 - 8x + 16 = 6y$ **parabola**
45. $x^2 + 4x + y^2 - 285 = 0$ **circle**
46. $5y^2 + 2y + 4x - 13x^2 = 81$ **hyperbola**

8-7 Solving Quadratic Systems

See pages 455–460.

Concept Summary

- Systems of quadratic equations can be solved using substitution and elimination.
- A system of quadratic equations can have zero, one, two, three, or four solutions.

Example **Solve the system of equations.**
$x^2 + y^2 + 2x - 12y + 12 = 0$
$y + x = 0$

Use substitution to solve the system.

First, rewrite $y + x = 0$ as $y = -x$.

$x^2 + y^2 + 2x - 12y + 12 = 0$	First original equation
$x^2 + (-x)^2 + 2x - 12(-x) + 12 = 0$	Substitute $-x$ for y.
$2x^2 + 14x + 12 = 0$	Simplify.
$x^2 + 7x + 6 = 0$	Divide each side by 2.
$(x + 6)(x + 1) = 0$	Factor.
$x + 6 = 0$ or $x + 1 = 0$	Zero Product Property.
$x = -6$ $\quad$ $x = -1$	Solve for x.

Now solve for y.

$y = -x$	$y = -x$	Equation for y in terms of x
$= -(-6)$ or 6	$= -(-1)$ or 1	Substitute the x values.

The solutions of the system are $(-6, 6)$ and $(-1, 1)$.

Exercises **Find the exact solution(s) of each system of equations.**
See Examples 1 and 2 on pages 455–457.

47. $x^2 + y^2 - 18x + 24y + 200 = 0$
$4x + 3y = 0$ **(6, −8), (12, −16)**
48. $4x^2 + y^2 = 16$
$x^2 + 2y^2 = 4$ **(±2, 0)**

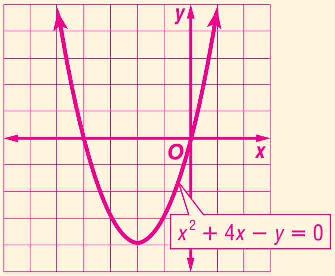
49–50. See margin.

Solve each system of inequalities by graphing. *See Example 3 on page 457.*

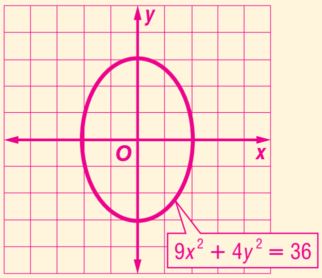
49. $y < x$
$y > x^2 - 4$
50. $x^2 + y^2 \le 9$
$x^2 + 4y^2 \le 16$

Answers

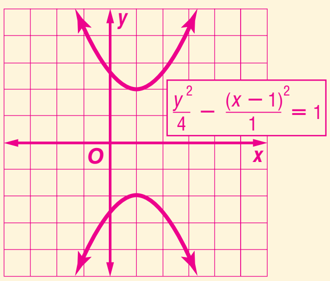
39. $y = (x + 2)^2 - 4$



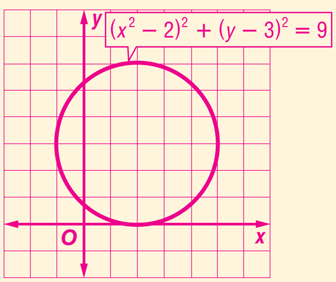
40. $\frac{y^2}{9} + \frac{x^2}{4} = 1$



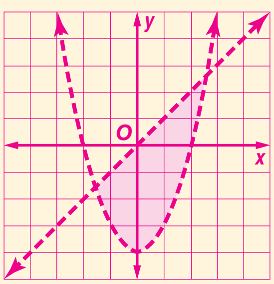
41. $\frac{y^2}{4} - \frac{(x-1)^2}{1} = 1$



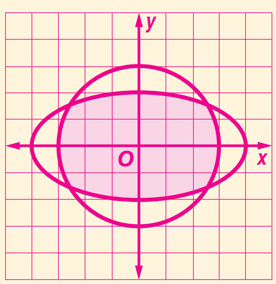
42. $(x - 2)^2 + (y - 3)^2 = 9$



49.



50.



466 Chapter 8 Conic Sections

Chapter 8 Practice Test

Vocabulary and Concepts

Choose the letter that best matches each description.

a. ellipse	
b. parabola	
c. hyperbola	

1. the set of all points in a plane that are the same distance from a given point, the focus, and a given line, the directrix **b**
2. the set of all points in a plane such that the absolute value of the difference of the distances from two fixed points, the foci, is constant **c**
3. the set of all points in a plane such that the sum of the distances from two fixed points, the foci, is constant **a**

Skills and Applications

Find the midpoint of the line segment with endpoints at the given coordinates.

4. $(7, 1), (-5, 9)$ **$(1, 5)$**
5. $\left(\frac{3}{8}, -1\right), \left(-\frac{8}{5}, 2\right)$ **$\left(-\frac{49}{80}, \frac{1}{2}\right)$**
6. $(-13, 0), (-1, -8)$ **$(-7, -4)$**

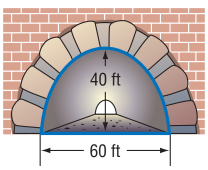
Find the distance between each pair of points with the given coordinates.

7. $(-6, 7), (3, 2)$ **$\sqrt{106}$ units**
8. $\left(\frac{1}{2}, \frac{5}{2}\right), \left(-\frac{3}{4}, -\frac{11}{4}\right)$ **$\frac{\sqrt{466}}{4}$ units**
9. $(8, -1), (8, -9)$ **8 units**

State whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. **10–19. See pp. 469A–469J for graphs.**

10. $x^2 + 4y^2 = 25$ **ellipse**
11. $y = 4x^2 + 1$ **parabola**
12. $x^2 = 36 - y^2$ **circle**
13. $(x + 4)^2 = 7(y + 5)$ **parabola**
14. $4x^2 - 26y^2 + 10 = 0$ **hyperbola**
15. $25x^2 + 49y^2 = 1225$ **ellipse**
16. $-(y^2 - 24) = x^2 + 10x$ **circle**
17. $5x^2 - y^2 = 49$ **hyperbola**
18. $\frac{1}{3}x^2 - 4 = y$ **parabola**
19. $\frac{y^2}{9} - \frac{x^2}{25} = 1$ **hyperbola**

20. **TUNNELS** The opening of a tunnel is in the shape of a semielliptical arch. The arch is 60 feet wide and 40 feet high. Find the height of the arch 12 feet from the edge of the tunnel. **32 ft**



Find the exact solution(s) of each system of equations.

21. $x^2 + y^2 = 100$
    $y = 2 - x$ **$(-6, 8), (8, -6)$**
22. $x^2 + 2y^2 = 6$
    $x + y = 1$ **$\left(-\frac{2}{3}, \frac{5}{3}\right), (2, -1)$**
23. $x^2 - y^2 - 12x + 12y = 36$
    $x^2 + y^2 - 12x - 12y + 36 = 0$ **$(0, 6), (12, 6)$**
24. Solve the system of inequalities by graphing. **See pp. 469A–469J.**
    $x^2 - y^2 \geq 1$
    $x^2 + y^2 \leq 16$

25. **STANDARDIZED TEST PRACTICE** Which is *not* the equation of a parabola? **C**
    (A) $y = 2x^2 + 4x - 9$
    (B) $3x + 2y^2 + y + 1 = 0$
    (C) $x^2 + 2y^2 + 8y = 8$
    (D) $x = \frac{1}{2}(y - 1)^2 + 5$

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Chapter 8 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 8 can be found on p. 510 of the *Chapter 8 Resource Masters*.

Chapter Tests There are six Chapter 8 Tests and an Open-Ended Assessment task available in the *Chapter 8 Resource Masters*.

Chapter 8 Tests

Form	Type	Level	Pages
1	MC	basic	497–498
2A	MC	average	499–500
2B	MC	average	501–502
2C	FR	average	503–504
2D	FR	average	505–506
3	FR	advanced	507–508

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 8 can be found on p. 509 of the *Chapter 8 Resource Masters*. A sample scoring rubric for these tasks appears on p. A28.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction In this chapter you have worked with four different conic sections.

Ask Students Describe what you have learned about conic sections and how they are related. List and compare their standard equations. Then draw each conic section on a separate coordinate grid, labeling important characteristics, such as center, foci, vertex, axes of symmetry, and so on. Place these drawings in your portfolio.

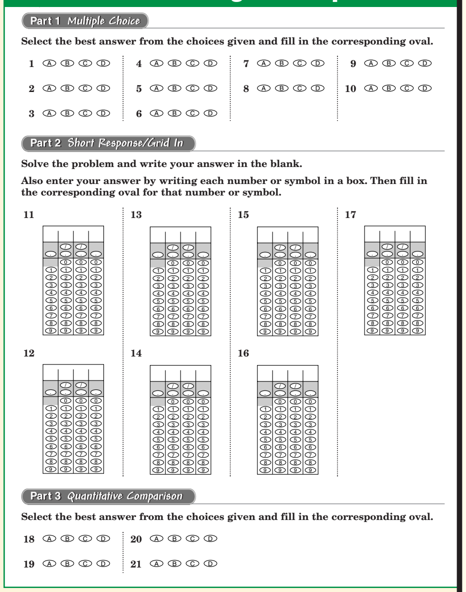
Chapter 8 Practice Test 467

Chapter 8 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 8 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 2 3 4 5 6 7 8 9 10

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

Also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

11 12 13 14 15 16 17

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

18 19 20 21

Additional Practice

See pp. 515–516 in the *Chapter 8 Resource Masters* for additional standardized test practice.

Chapter 8 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The product of a prime number and a composite number must be **B**

(A) prime. (B) composite.
(C) even. (D) negative.

2. In 1990, the population of Clayton was 54,200, and the population of Montrose was 47,500. By 2000, the population of each city had decreased by exactly 5%. How many more people lived in Clayton than in Montrose in 2000? **C**

(A) 335 (B) 5085
(C) 6365 (D) 6700

3. If 4% of n is equal to 40% of p, then n is what percent of $10p$? **C**

(A) $\frac{1}{1000}\%$ (B) 10%
(C) 100% (D) 1,000%

4. Leroy bought m magazines at d dollars per magazine and p paperback books at $2d + 1$ dollars per book. Which of the following represents the total amount Leroy spent? **A**

(A) $d(m + 2p) + p$ (B) $(m + p)(3d + 1)$
(C) $md + 2pd + 1$ (D) $pd(m + 2)$

The Princeton Review Test-Taking Tip

Questions 3, 4 In problems with variables, you can substitute values to try to eliminate some of the answer choices. For example, in Question 3, choose a value for n and compute the corresponding value of p. Then find $\frac{n}{10p}$ to answer the question.

5. What is the midpoint of the line segment whose endpoints are at $(-5, -3)$ and $(-1, 4)$? **B**

(A) $\left(-3, -\frac{1}{2}\right)$ (B) $\left(-3, \frac{1}{2}\right)$
(C) $\left(-2, \frac{7}{2}\right)$ (D) $\left(-2, \frac{1}{2}\right)$

6. Point $M(-2, 3)$ is the midpoint of line segment NP. If point N has coordinates $(-7, 1)$, then what are the coordinates of point P? **D**

(A) $(-5, 2)$ (B) $(-4, 6)$
(C) $\left(-\frac{9}{2}, 2\right)$ (D) $(3, 5)$

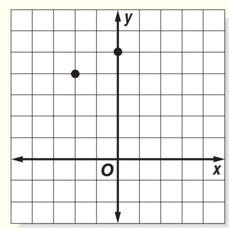
7. Which equation's graph is a parabola? **D**

(A) $3x^2 - 2y^2 = 10$
(B) $4x^2 + 3y^2 = 20$
(C) $2x^2 + 2y^2 = 15$
(D) $3x^2 + 4y = 8$

8. What is the center of the circle with equation $x^2 + y^2 - 4x + 6y - 9 = 0$? **C**

(A) $(-4, 6)$ (B) $(-2, 3)$
(C) $(2, -3)$ (D) $(3, 3)$

9. What is the distance between the points shown in the graph? **B**



(A) $\sqrt{3}$ units
(B) $\sqrt{5}$ units
(C) 3 units
(D) $\sqrt{17}$ units

10. The median of seven test scores is 52, the mode is 64, the lowest score is 40, and the average is 53. If the scores are integers, what is the greatest possible test score? **A**

(A) 68 (B) 72 (C) 76 (D) 84

The Princeton Review *Log On* for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

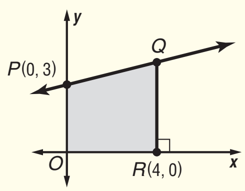
11. What is the least positive integer p for which $2^{2p} + 3$ is not a prime number? **4**

12. The ratio of cars to SUVs in a parking lot is 4 to 5. After 6 cars leave the parking lot, the ratio of cars to SUVs becomes 1 to 2. How many SUVs are in the parking lot? **20**

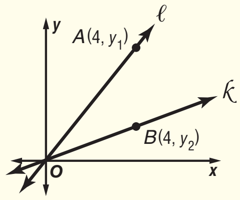
13. Each dimension of a rectangular box is an integer greater than 1. If the area of one side of the box is 27 square units and the area of another side is 12 square units, what is the volume of the box in cubic units? **108**

14. Let the operation * be defined as $a * b = 2ab - (a + b)$. If $4 * x = 10$, then what is the value of x? **2**

15. If the slope of line PQ in the figure is $\frac{1}{4}$, what is the area of quadrilateral $OPQR$? **14**



16. In the figure, the slope of line ℓ is $\frac{5}{4}$, and the slope of line k is $\frac{3}{8}$. What is the distance from point A to point B? **7/2 or 3.5**



17. If $(2x - 3)(4x + n) = ax^2 + bx - 15$ for all values of x, what is the value of $a + b$? **6**

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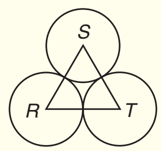
Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

Column A	Column B

18. Tangent circles R, S, and T each have an area of 16π square units. **B**



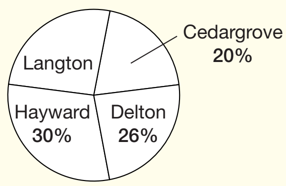
perimeter of △RST	circumference of ⊙R

19.

the ratio of girls to boys in Class A that contains 4 more boys than girls	the ratio of girls to boys in Class B that contains 4 more girls than boys

B

20. Percent of Lakewood School Students Living in Each Town



the number of students who do *not* live in Langton	the number of students who do *not* live in Delton

A

21. $2 < nk < 10$

n and k are positive integers.

nk	$n + k$

D

Chapter 8 Standardized Test Practice 469

Pages 415–416, Lesson 8-1

21. Sample answer: Draw several line segments across the U.S. One should go from the northeast corner to the southwest corner; another should go from the southeast corner to the northwest corner; another should go across the middle of the U.S. from east to west; and so on. Find the midpoints of these segments. Locate a point to represent all of these midpoints.

41. The slope of the line through (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$ and the point-slope form of the equation of the line is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. Substitute $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ into this equation. The left side is $\frac{y_1 + y_2}{2} - y_1$ or $\frac{y_2 - y_1}{2}$. The right side is $\frac{y_2 - y_1}{x_2 - x_1}\left(\frac{x_1 + x_2}{2} - x_1\right) = \frac{y_2 - y_1}{x_2 - x_1}\left(\frac{x_2 - x_1}{2}\right)$ or $\frac{y_2 - y_1}{2}$.

Therefore, the point with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ lies on the line through (x_1, y_1) and (x_2, y_2). The distance from $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to (x_1, y_1) is $\sqrt{\left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_1 - \frac{y_1 + y_2}{2}\right)^2}$ or $\sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$. The distance from $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ to (x_2, y_2) is $\sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2} = \sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2}$ or $\sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2}$. Therefore the point with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ is equidistant from (x_1, y_1) and (x_2, y_2).

42. The formulas can be used to decide from which location an emergency squad should be dispatched. Answers should include the following.

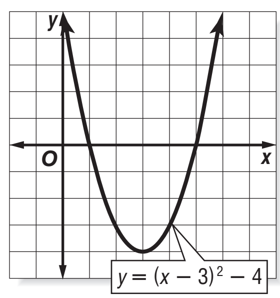
- Most maps have a superimposed grid. Think of the grid as a coordinate system and assign approximate coordinates to the two cities. Then use the Distance Formula to find the distance between the points with those coordinates.
- Suppose the bottom left of the grid is the origin. Then the coordinates of Lincoln are about (0.7, 0.2); the coordinates of Omaha are about (4.4, 3.9); and the coordinates of Fremont are about (1.7, 4.6). The distance from Omaha to Fremont is about $10\sqrt{(1.7 - 4.4)^2 + (4.6 - 3.9)^2}$ or about 28 miles. The distance from Lincoln to Fremont is about $10\sqrt{(1.7 - 0.7)^2 + (4.6 - 0.2)^2}$ or about 45 miles. Since Omaha is closer than Lincoln, the helicopter should be dispatched from Omaha.

Page 418, Follow-Up of Lesson 8-1
Algebra Activity

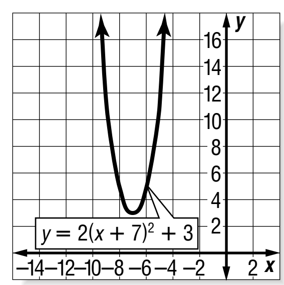
14. The distance between the points with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ and (x_1, y_1, z_1) is $\sqrt{\left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_1 - \frac{y_1 + y_2}{2}\right)^2 + \left(z_1 - \frac{z_1 + z_2}{2}\right)^2}$ or $\sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2 + \left(\frac{z_1 - z_2}{2}\right)^2}$ units. The distance between the points with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$ and (x_2, y_2, z_2) is $\sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2 + \left(z_2 - \frac{z_1 + z_2}{2}\right)^2}$ or $\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2 + \left(\frac{z_2 - z_1}{2}\right)^2}$ units. Since $\sqrt{\left(\frac{x_2 - x_1}{2}\right)^2 + \left(\frac{y_2 - y_1}{2}\right)^2 + \left(\frac{z_2 - z_1}{2}\right)^2} = \sqrt{\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2 + \left(\frac{z_1 - z_2}{2}\right)^2}$, the distances are equal.

Pages 423–424, Lesson 8-2

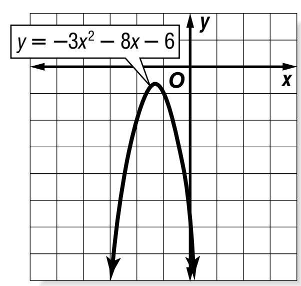
5. $(3, -4)$, $\left(3, -3\frac{3}{4}\right)$, $x = 3$, $y = -4\frac{1}{4}$, upward, 1 unit



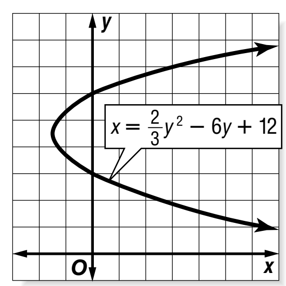
6. $(-7, 3)$, $\left(-7, 3\frac{1}{8}\right)$, $x = -7$, $y = 2\frac{7}{8}$, upward, $\frac{1}{2}$ unit



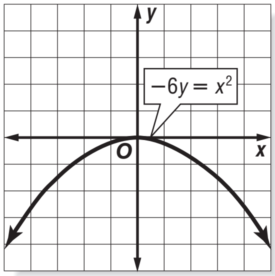
7. $\left(-\frac{4}{3}, -\frac{2}{3}\right)$, $\left(-\frac{4}{3}, -\frac{3}{4}\right)$, $x = -\frac{4}{3}$, $y = -\frac{7}{12}$, downward, $\frac{1}{3}$ unit



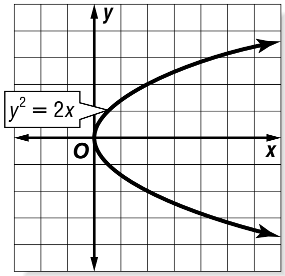
8. $\left(-\frac{3}{2}, \frac{9}{2}\right)$, $\left(-\frac{9}{8}, \frac{9}{2}\right)$, $y = \frac{9}{2}$, $x = -\frac{15}{8}$, right, $\frac{3}{2}$ units



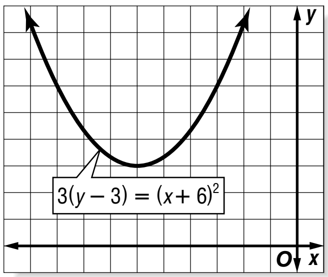
16. $(0, 0), \left(0, -\frac{3}{2}\right), x = 0, y = \frac{3}{2}$, downward, 6 units



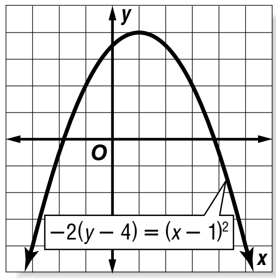
17. $(0, 0), \left(\frac{1}{2}, 0\right), y = 0, x = -\frac{1}{2}$, right, 2 units



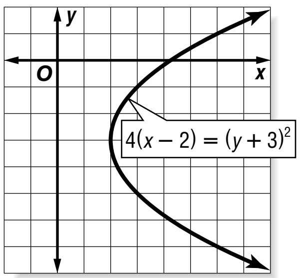
18. $(-6, 3), \left(-6, 3\frac{3}{4}\right), x = -6, y = 2\frac{1}{4}$, upward, 3 units



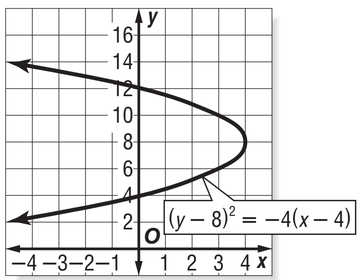
19. $(1, 4), \left(1, 3\frac{1}{2}\right), x = 1, y = 4\frac{1}{2}$, downward, 2 units



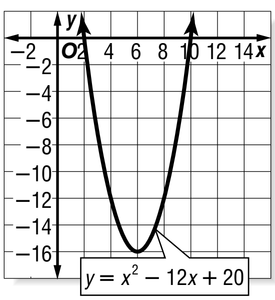
20. $(2, -3), (3, -3), y = -3, x = 1$, right, 4 units



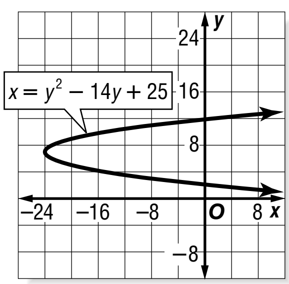
21. $(4, 8), (3, 8), y = 8, x = 5$, left, 4 units



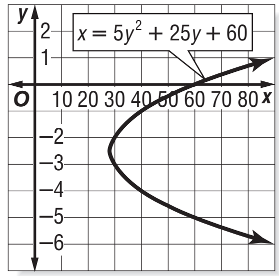
22. $(6, -16), \left(6, -15\frac{3}{4}\right), x = 6, y = -16\frac{1}{4}$, upward, 1 unit



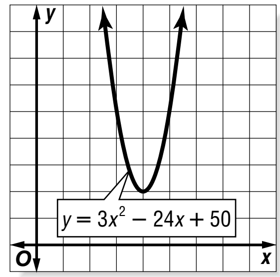
23. $(-24, 7), \left(-23\frac{3}{4}, 7\right), y = 7, x = -24\frac{1}{4}$, right, 1 unit



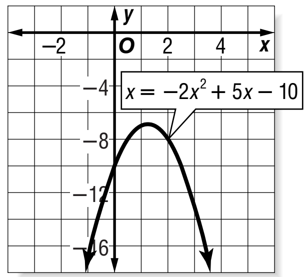
24. $\left(\frac{115}{4}, -\frac{5}{2}\right), \left(\frac{144}{5}, -\frac{5}{2}\right), y = -\frac{5}{2}, x = \frac{287}{10}$, right, $\frac{1}{5}$ unit



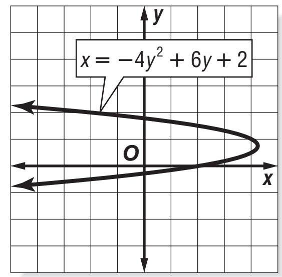
25. $(4, 2), \left(4, 2\frac{1}{12}\right), x = 4, y = 1\frac{11}{12}$, upward, $\frac{1}{3}$ unit



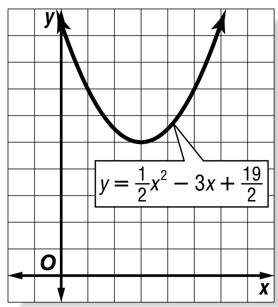
26. $\left(\frac{5}{4}, -\frac{55}{8}\right), \left(\frac{5}{4}, -7\right), x = \frac{5}{4}, y = -\frac{27}{4}$, downward, $\frac{1}{2}$ unit



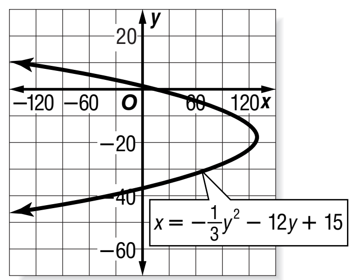
27. $\left(\frac{17}{4}, \frac{3}{4}\right), \left(\frac{67}{16}, \frac{3}{4}\right), y = \frac{3}{4}, x = \frac{69}{16}$, left, $\frac{1}{4}$ unit



28. $(3, 5), \left(3, 5\frac{1}{2}\right), x = 3, y = 4\frac{1}{2}$, upward, 2 units



29. $(123, -18), \left(122\frac{1}{4}, -18\right), y = -18, x = 123\frac{3}{4}$, left, 3 units



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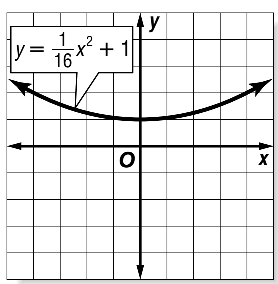

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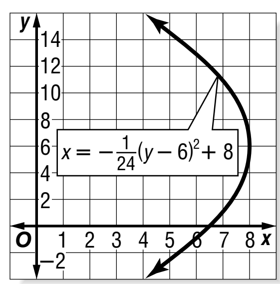

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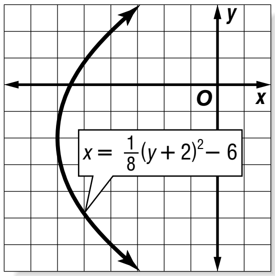

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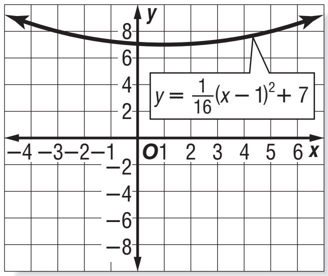

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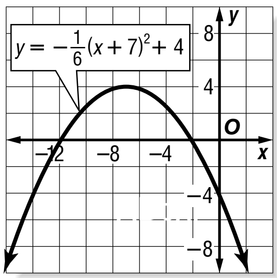

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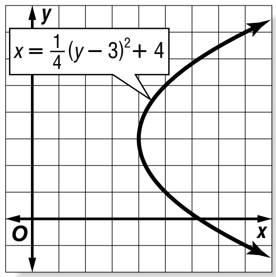

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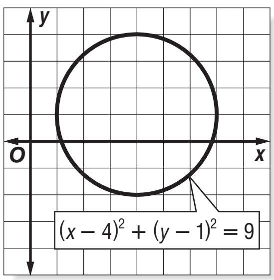
8.



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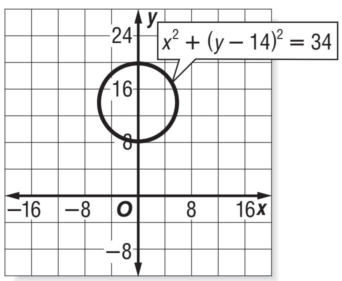

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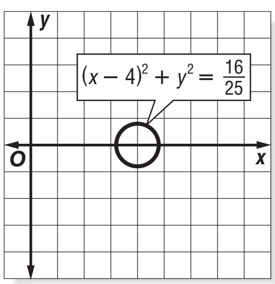

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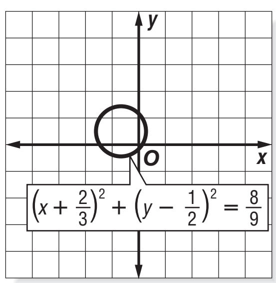

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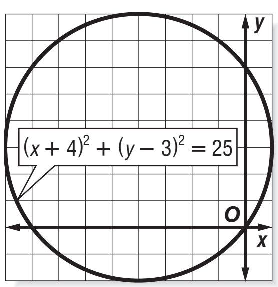

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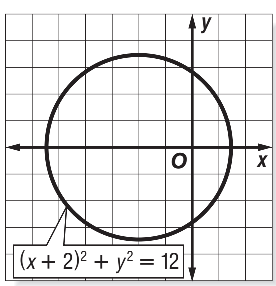

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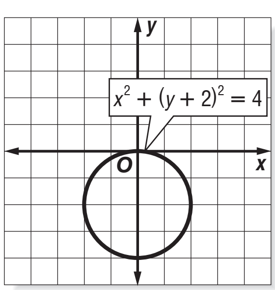

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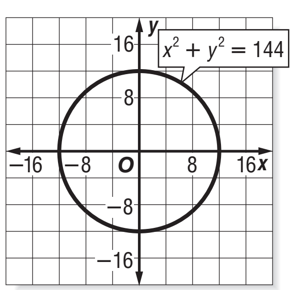

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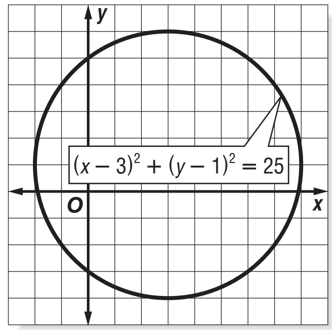

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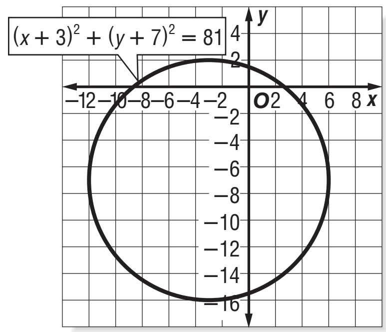

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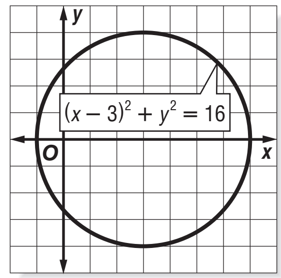

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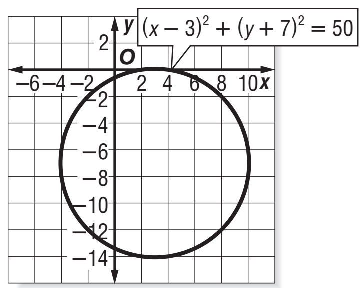

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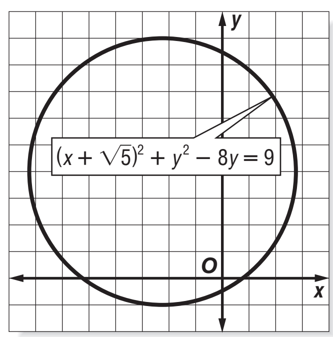

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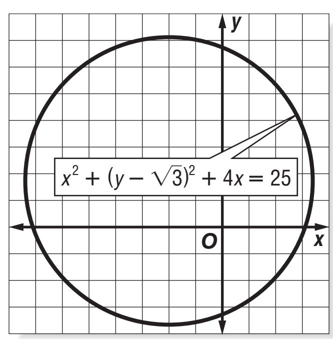

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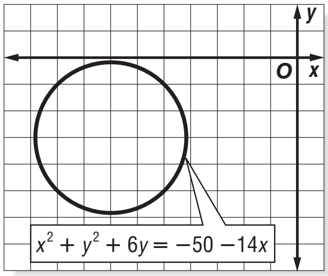

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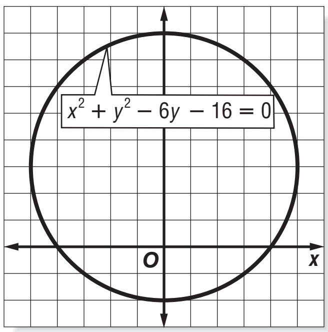

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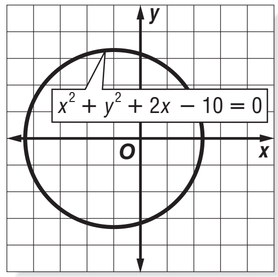

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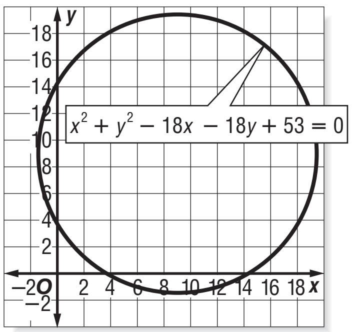

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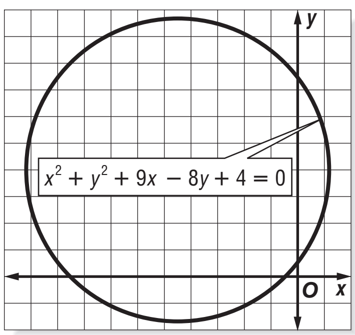

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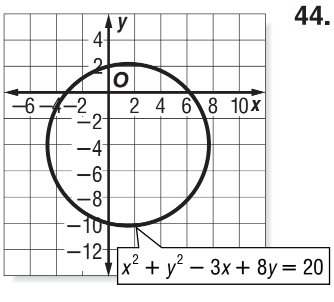

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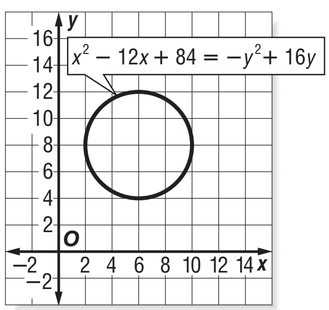

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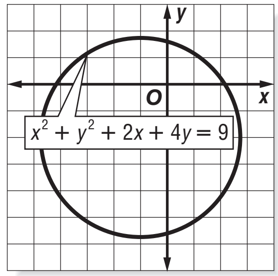

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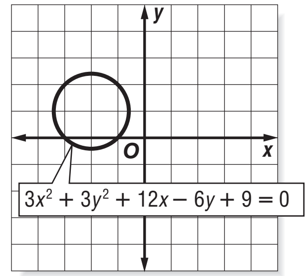

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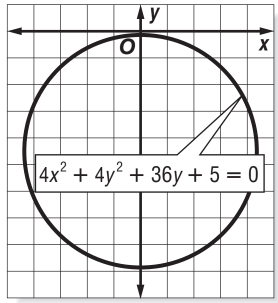
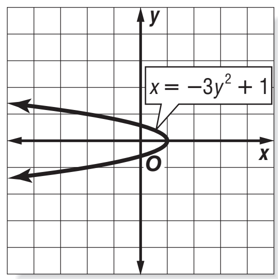
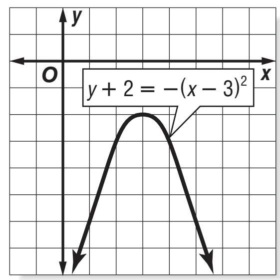


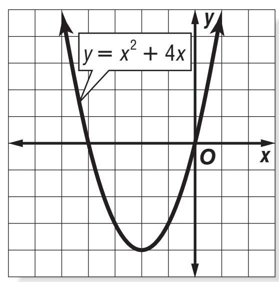
57. (1, 0), $\left(\frac{11}{12}, 0\right)$, $y = 0$, $x = 1\frac{1}{12}$, left, $\frac{1}{3}$ unit



58. (3, −2), $\left(3, -2\frac{1}{4}\right)$, $x = 3$, $y = -1\frac{3}{4}$, downward, 1 unit



59. (−2, −4), $\left(-2, -3\frac{3}{4}\right)$, $x = -2$, $y = -4\frac{1}{4}$, upward, 1 unit



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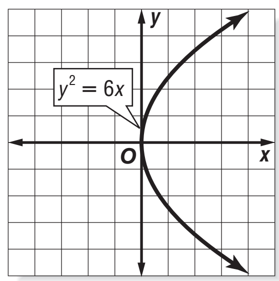
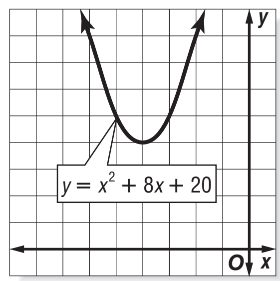
3. (0, 0), $(1\frac{1}{2}, 0)$, $y = 0$, $x = -1\frac{1}{2}$, right, 6 units



4. (−4, 4), $\left(-4, 4\frac{1}{4}\right)$, $x = -4$, $y = 3\frac{3}{4}$, upward, 1 unit



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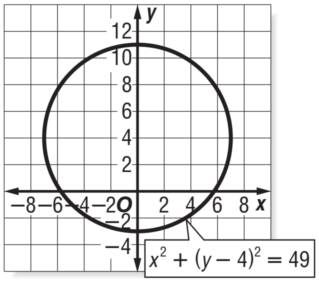

Page 432, Preview of Lesson 8-4
Algebra Activity

9. The ellipse is longer in the vertical direction than in the horizontal direction.

12. No; a rubber band might stretch so that the sum of the distances to the thumbtacks would not be constant.

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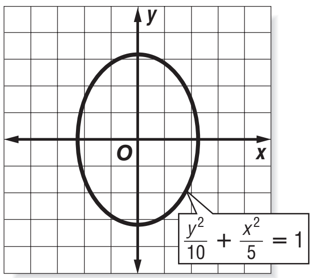
27.



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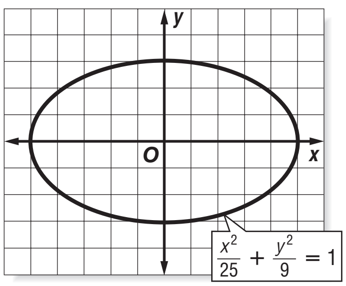

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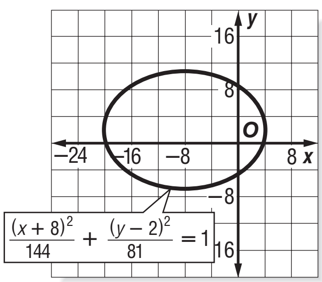

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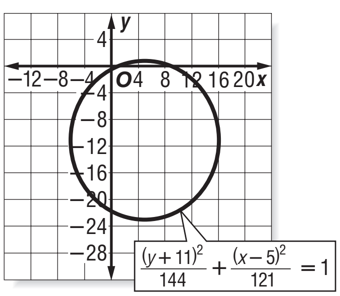

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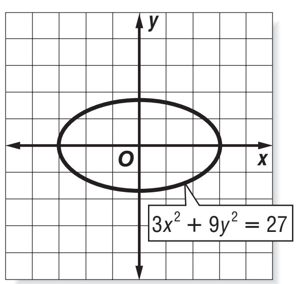

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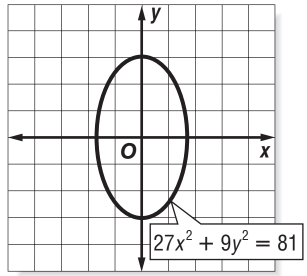

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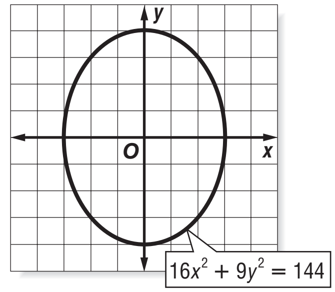

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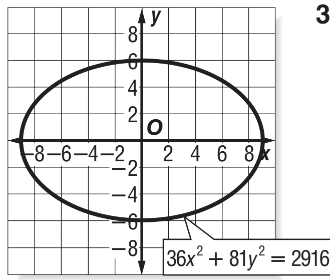

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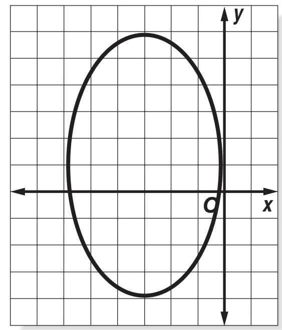

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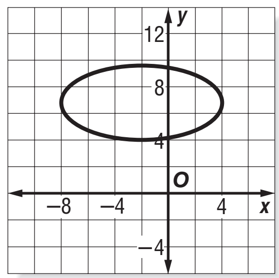

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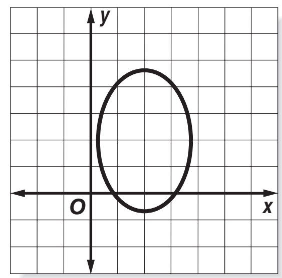

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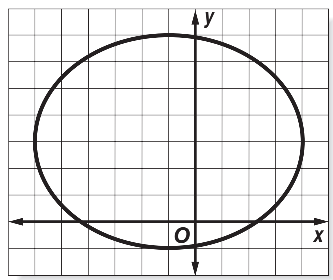
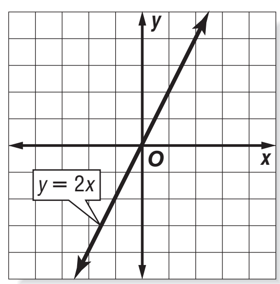


40. Knowledge of the orbit of Earth can be used in predicting the seasons and in space exploration. Answers should include the following.

- Knowledge of the path of another planet would be needed if we wanted to send a spacecraft to that planet.
- 1.55 million miles

52.



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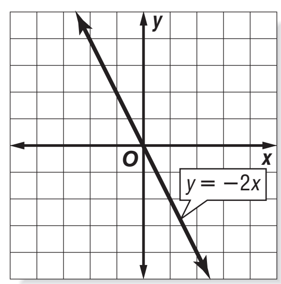

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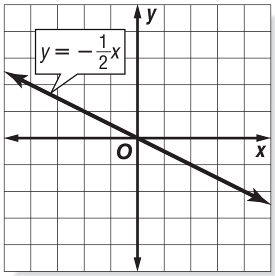

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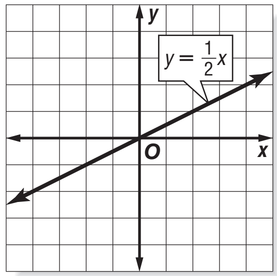

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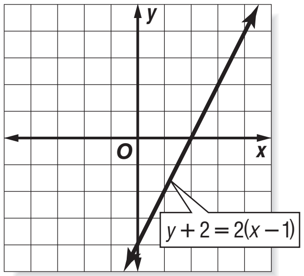

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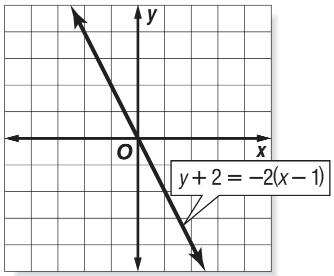

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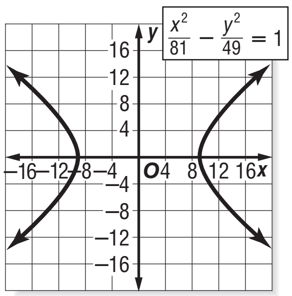
21.



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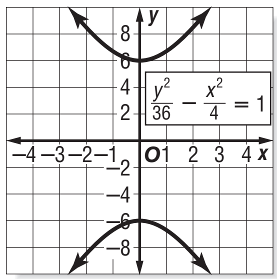

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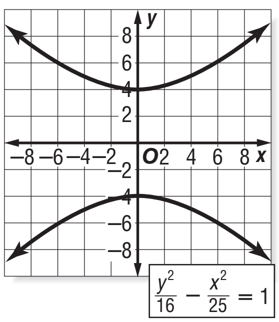
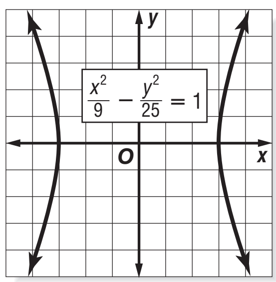


24.



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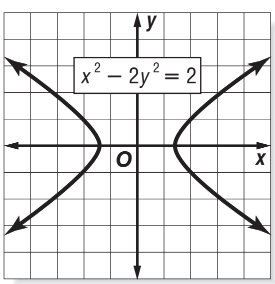
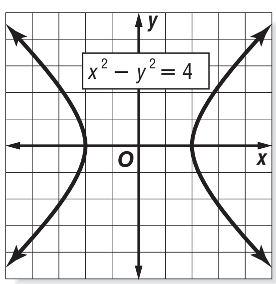


26.



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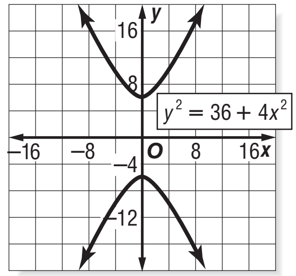

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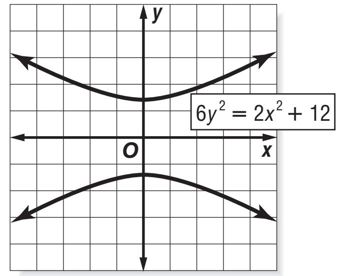
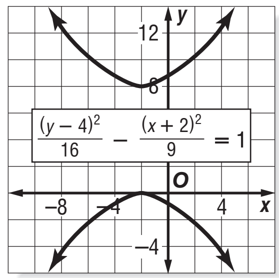
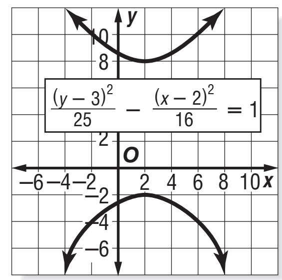


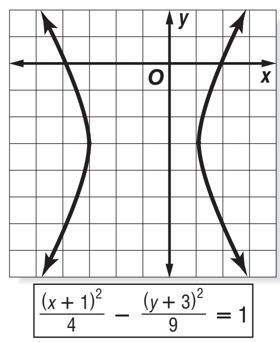
29. $(-2, 0)$, $(-2, 8)$; $(-2, -1)$, $(-2, 9)$; $y - 4 = \pm\frac{4}{3}(x + 2)$



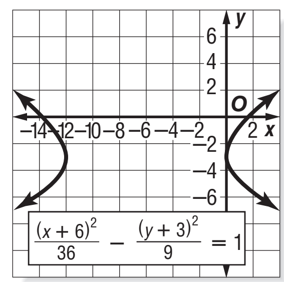
30. $(2, -2)$, $(2, 8)$; $\left(2, 3 \pm \sqrt{41}\right)$; $y - 3 = \pm\frac{5}{4}(x - 2)$



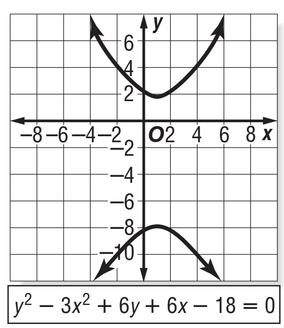
31. $(-3, -3)$, $(1, -3)$; $\left(-1 \pm \sqrt{13}, -3\right)$; $y + 3 = \pm\frac{3}{2}(x + 1)$



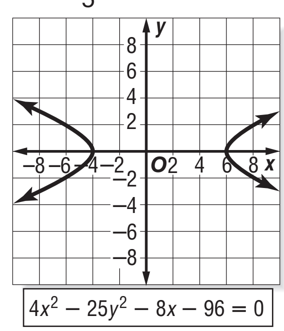
32. $(-12, -3)$, $(0, -3)$; $\left(-6 \pm 3\sqrt{5}, -3\right)$; $y + 3 = \pm\frac{1}{2}(x + 6)$



33. $\left(1, -3 \pm 2\sqrt{6}\right)$; $\left(1, -3 \pm 4\sqrt{2}\right)$; $y + 3 = \pm\sqrt{3}(x - 1)$



34. $(-4, 0)$, $(6, 0)$; $\left(1 \pm \sqrt{29}, 0\right)$; $y = \pm\frac{2}{5}(x - 1)$



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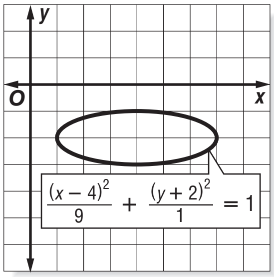
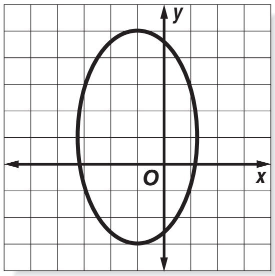
2.



3.



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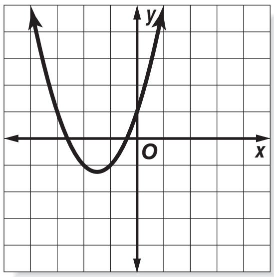
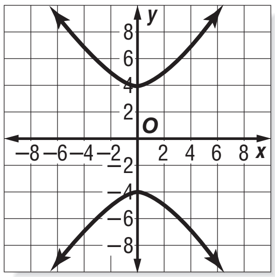
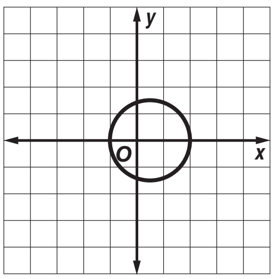
4. $y = \left(x + \frac{3}{2}\right)^2 - \frac{5}{4}$



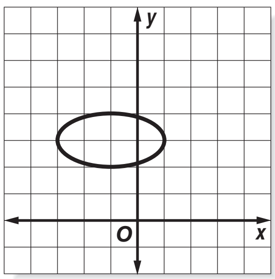
5. $\frac{y^2}{16} - \frac{x^2}{8} = 1$



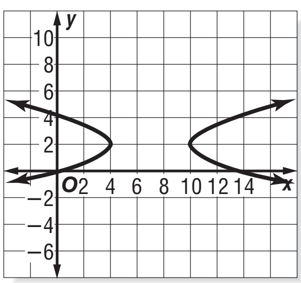
6. $\left(x - \frac{1}{2}\right)^2 + y^2 = \frac{9}{4}$



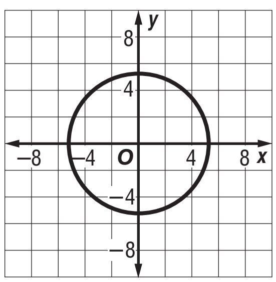
7. $\frac{(x+1)^2}{4} + \frac{(y-3)^2}{1} = 1$



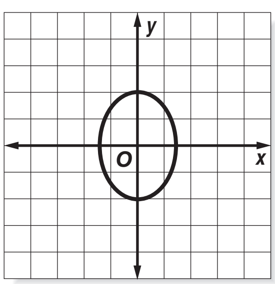
11.



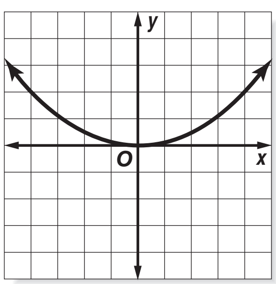
12. $x^2 + y^2 = 27$



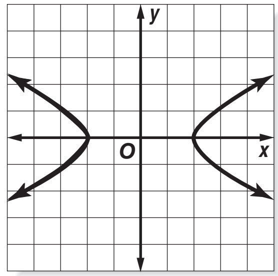
13. $\frac{y^2}{4} + \frac{x^2}{2} = 1$



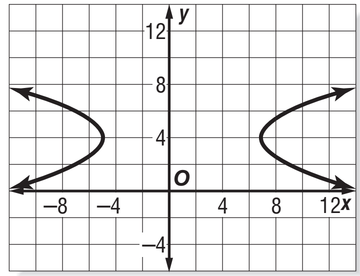
14. $y = \frac{1}{8}x^2$



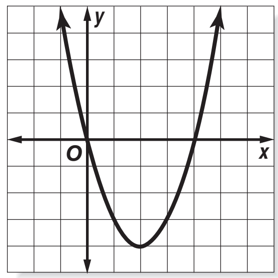
15. $\frac{x^2}{4} - \frac{y^2}{1} = 1$



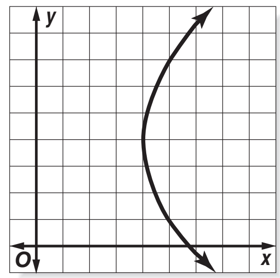
16. $\frac{(x-1)^2}{36} - \frac{(y-4)^2}{4} = 1$



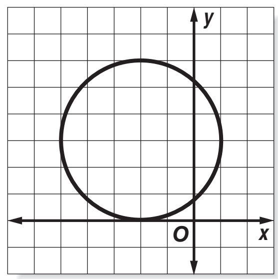
17. $y = (x-2)^2 - 4$



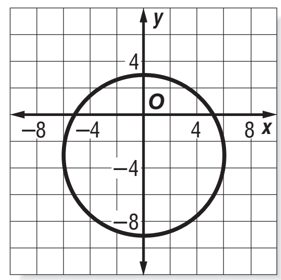
18. $x = \frac{1}{9}(y-4)^2 + 4$



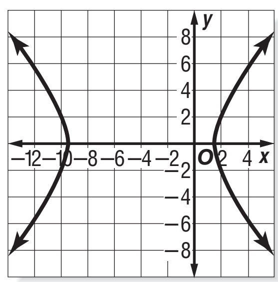
19. $(x+2)^2 + (y-3)^2 = 9$



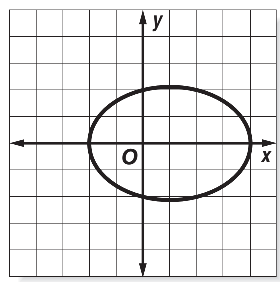
20. $x^2 + (y+3)^2 = 36$



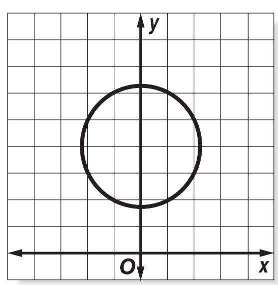
21. $\frac{(x+4)^2}{32} - \frac{y^2}{32} = 1$



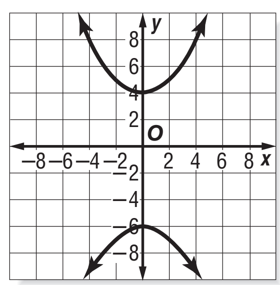
22. $\frac{(x-1)^2}{9} + \frac{y^2}{\frac{9}{2}} = 1$



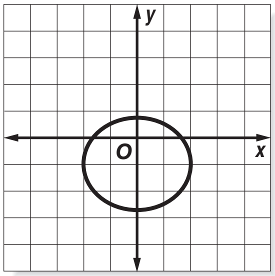
23. $x^2 + (y-4)^2 = 5$



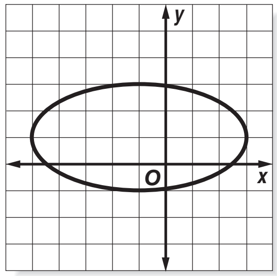
24. $\frac{(y+1)^2}{25} - \frac{x^2}{9} = 1$



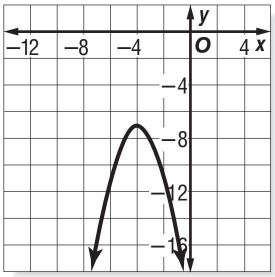
25. $\frac{x^2}{4} + \frac{(y+1)^2}{3} = 1$



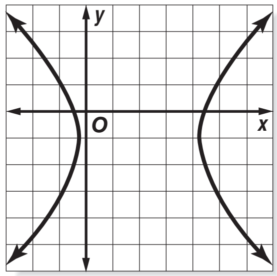
26. $\frac{(x+1)^2}{16} + \frac{(y-1)^2}{4} = 1$



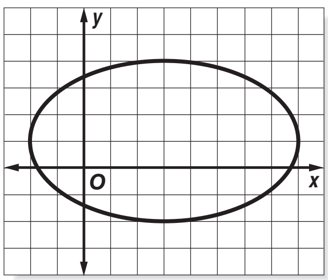
27. $y = -(x+4)^2 - 7$



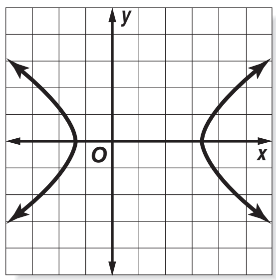
28. $\frac{(x-2)^2}{5} - \frac{(y+1)^2}{6} = 1$



29. $\frac{(x-3)^2}{25} + \frac{(y-1)^2}{9} = 1$



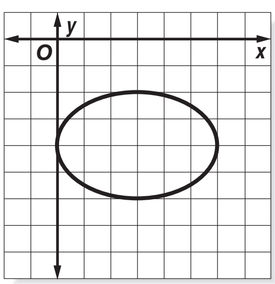
32.



46. If you point a flashlight at a flat surface, you can make different conic sections by varying the angle at which you point the flashlight. Answers should include the following.

- Point the flashlight directly at a ceiling or wall. The light from the flashlight is in the shape of a cone and the ceiling or wall acts as a plane perpendicular to the axis of the cone.
- Hold the flashlight close to a wall and point it directly vertically toward the ceiling. A branch of a hyperbola will appear on the wall. In this case, the wall acts as a plane parallel to the axis of the cone.

52.



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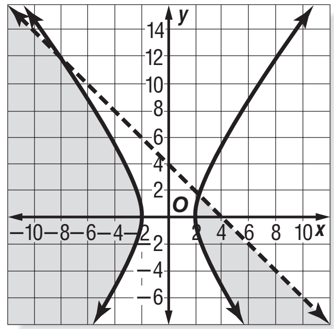
8.



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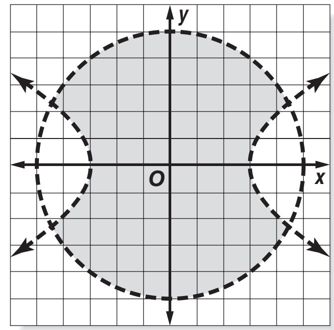

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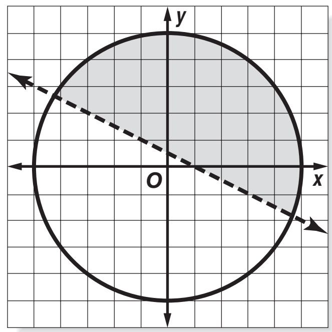

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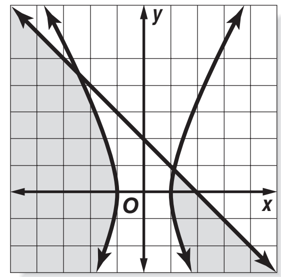
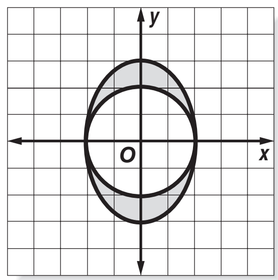
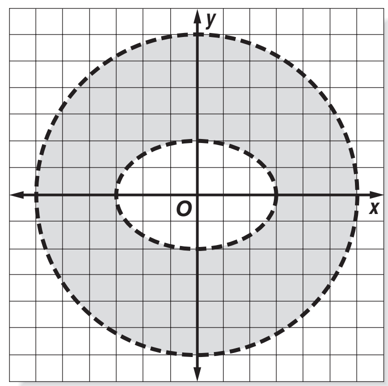


34.



35.



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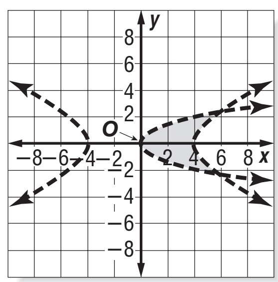
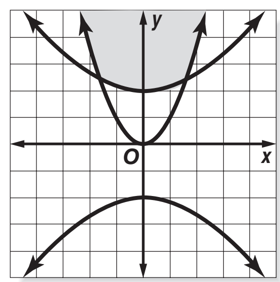


37.



43. Systems of equations can be used to represent the locations and/or paths of objects on the screen. Answers should include the following.

- $y = 3x$, $x^2 + y^2 = 2500$
- The y-intercept of the graph of the equation $y = 3x$ is 0, so the path of the spaceship contains the origin.
- $\left(-5\sqrt{10}, -15\sqrt{10}\right)$ or about $(-15.81, -47.43)$

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4. A parabola is the set of all points that are the same distance from a given point called the focus and a given line called the directrix.
6. The conjugate axis of a hyperbola is a line segment perpendicular to the transverse axis.
8. A hyperbola is the set of all points in a plane such that the absolute value of the difference of the distances from any point on the hyperbola to two given points is constant.
9. The midpoint formula is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

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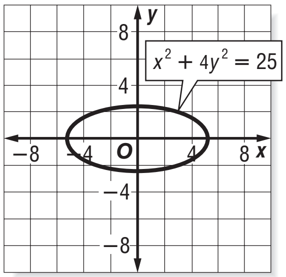
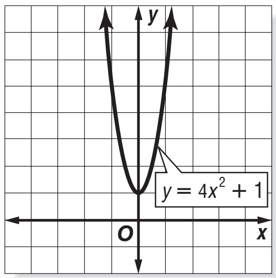
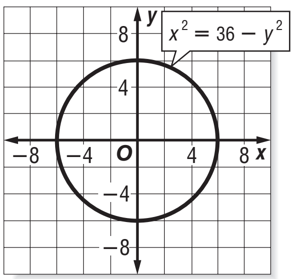
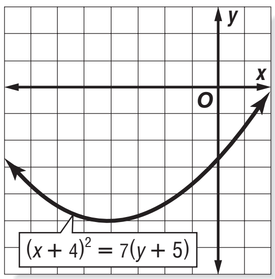
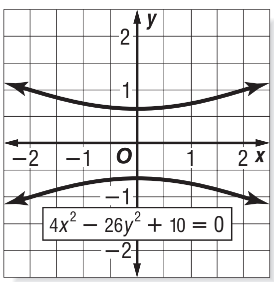
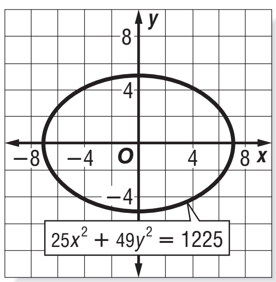
10. 

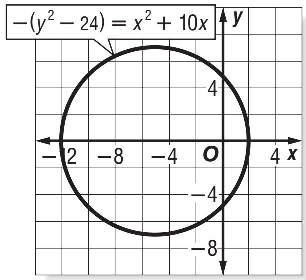
11. 

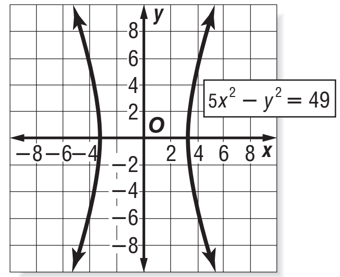
12. 

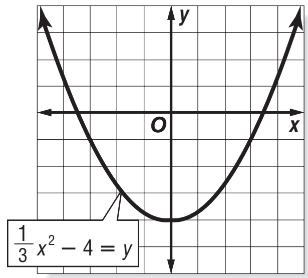
13. 

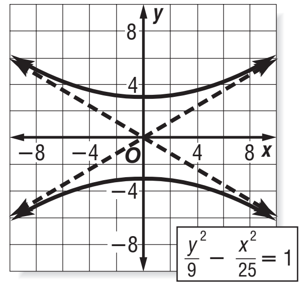
14. 

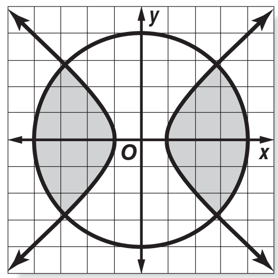
15. 

16. 

17. 

18. 

19. 

24. 

Notes