Chapter 9

Rational Expressions and Equations

Chapter Overview and Pacing

LESSON OBJECTIVES

	PACING (days)			
	Regular		Block	
	Basic/ Average	Advanced	Basic/ Average	Advanced
9-1 Multiplying and Dividing Rational Expressions *(pp. 472–478)* • Simplify rational expressions. • Simplify complex fractions.	1	1	0.5	0.5
9-2 Adding and Subtracting Rational Expressions *(pp. 479–484)* • Determine the LCM of polynomials. • Add and subtract rational expressions.	2	2	1	1
9-3 Graphing Rational Functions *(pp. 485–491)* • Determine the vertical asymptotes and the point discontinuity for the graphs of rational functions. • Graph rational functions. ***Follow-Up:*** Graphing Rational Functions	1	2 (with 9-3 Follow-Up)	0.5	1
9-4 Direct, Joint, and Inverse Variation *(pp. 492–498)* • Recognize and solve direct and joint variation problems. • Recognize and solve inverse variation problems.	2	2	1	1
9-5 Classes of Functions *(pp. 499–504)* • Identify graphs as different types of functions. • Identify equations as different types of functions.	1	1	0.5	0.5
9-6 Solving Rational Equations and Inequalities *(pp. 505–512)* • Solve rational equations. • Solve rational inequalities. ***Follow-Up:*** Solving Rational Equations by Graphing	2	3 (with 9-6 Follow-Up)	1.5	1
Study Guide and Practice Test *(pp. 513–517)* **Standardized Test Practice** *(pp. 518–519)*	1	1	0.5	0.5
Chapter Assessment	1	1	0.5	0.5
TOTAL	**11**	**13**	**6**	**6**

Pacing suggestions for the entire year can be found on pages T20–T21.

Timesaving Tools

TeacherWorks™

All-In-One Planner and Resource Center

See pages T12–T13.

Chapter Resource Manager

Study Guide and Intervention	Practice (Skills and Average)	Reading to Learn Mathematics	Enrichment	Assessment	Applications*	5-Minute Check Transparencies	Interactive Chalkboard	Alge2PASS: Tutorial Plus (lessons)	Materials
Chapter 9 Resource Masters									
517–518	519–520	521	522		SC 17	9-1	9-1	17	
523–524	525–526	527	528	567		9-2	9-2	18	
529–530	531–532	533	534	567, 569	GCS 43	9-3	9-3		balance, metric measuring cup, graph paper (*Follow-Up:* graphing calculator)
535–536	537–538	539	540		GCS 44, SC 18, SM 123–126	9-4	9-4		
541–542	543–544	545	546	568		9-5	9-5		string, grid paper
547–548	549–550	551	552	568		9-6	9-6		(*Follow-Up:* graphing calculator)
				553–566, 570–572					

**Key to Abbreviations:* GCS = Graphing Calculator and Speadsheet Masters,
SC = School-to-Career Masters,
SM = Science and Mathematics Lab Manual

Chapter 9

Mathematical Connections and Background

Continuity of Instruction

Prior Knowledge

Students have simplified rational numbers, written equivalent rational numbers, and found least common denominators. They have graphed functions from tables of values and they have explored functions whose graphs are lines or other shapes. Also, they have solved linear and polynomial equations.

This Chapter

Students extend basic arithmetic operations to rational expressions and extend solving equations to rational equations and inequalities. They graph rational functions and identify discontinuities in the graphs. They investigate equations that represent direct, inverse, and joint variation. They look at graphs of various shapes, including continuous curves, discontinuous curves, and lines, and associate each graph with a specific kind of function.

Future Connections

Students will extend operations on rational expressions to finding powers and roots of rational expressions. They will continue to study situations involving direct or inverse variation, and they will extend the study of joint variation to variables with exponents. Also, they will continue to see how the shape of a graph is used to classify the function represented by the graph.

9-1 Multiplying and Dividing Rational Expressions

In this lesson students look at some familiar ideas from fractions and apply them to rational expressions. One idea is that a fraction is undefined if the denominator is zero. To extend that idea, students examine polynomials that are the denominators of rational expressions and use their skill in factoring to identify values of variables for which the denominator would be zero. Students extend another idea to rational expressions, simplifying fractions, by finding common factors in the numerator and denominator, and replacing the quotient of those factors with 1. As another extension of simplifying, students identify factors of the form $a - b$ and $b - a$ in the numerator and denominator of a rational expression, and replace the quotient of those factors with -1. The ideas of multiplying or dividing fractions and simplifying complex fractions have direct extensions. To multiply two rational expressions, students divide the product of the numerators by the product of the denominators; to divide by a rational expression, they multiply by the reciprocal of that expression; and to simplify a complex fraction involving rational expressions, they rewrite it and treat it as a division expression.

9-2 Adding and Subtracting Rational Expressions

Students continue to look at familiar ideas from fractions and extend those ideas to rational expressions. For the idea of a least common multiple, students factor two or more polynomials. They write each factor of either of the given polynomials, with each factor having an exponent that indicates the maximum number of times that factor appears in any one of the given polynomials. Another familiar idea is writing two fractions as equivalent fractions with a common denominator. To extend this idea to rational expressions, students find the LCM of the given denominators and rewrite each rational expression as an equivalent expression whose denominator is that LCM. The ideas of adding and subtracting fractions are extended to rational expressions by writing the rational expressions with a common denominator (again, the common denominator is the LCM of the given denominators) and then adding or subtracting the numerators.

9-3 Graphing Rational Functions

In this lesson (and the next two) students use graphs to examine properties of functions. This lesson introduces graphs of rational functions, which are functions whose numerator and denominator are both polynomials. To look at values of x for which the denominator and thus the function is undefined, students study two kinds of rational functions. In one kind, the denominator is a factor of the numerator; for example, $f(x) = \frac{(x + 1)^5}{x + 1}$ or $g(x) = \frac{(2x + 1)(3x - 2)(5x + 3)}{(2x + 1)(5x + 3)}$. These functions can be simplified, and the graph of the simplified function is a continuous curve. However, there are one or more values of the variable for which the denominator of the original polynomial is zero. These values, called point discontinuities, represent places when the function is undefined. Students explore these types of functions by reducing the function, graphing the reduced function, and identifying the "holes" in the graph. Another kind of rational expression is one in which the entire denominator is not a factor of the numerator. For these functions, each value of the variable for which the denominator is zero is associated with a vertical asymptote on the graph. Students explore these functions by identifying all the vertical asymptotes, and then using tables of values and the asymptotes to graph the function.

9-4 Direct, Joint, and Inverse Variation

In this lesson students examine graphs for two-variable equations that represent two types of relationships. For variables x and y and constant k, the relationship $y = kx$ (also written as $\frac{y}{x} = k$) is called direct variation. The graph of a direct variation is a line; the line goes through the origin (0, 0) and has slope k. The relationship $y = \frac{k}{x}$ (or $xy = k$) is called inverse variation; its graph is a hyperbola. Students explore direct and inverse variation by finding the constant or a missing value of a variable for a given type of variation or by stating the type of variation for a given graph or set of values. Students also explore the relationship $y = kxz$ among the variables x, y, and z and constant k. Students explore this type of variation, called joint variation, by using given values to find the value of the constant or a missing value of a variable.

9-5 Classes of Functions

In this lesson students organize information learned previously about graphs and functions. Given the graph of a line, they describe that line as the graph of a constant function, a direct variation function, or the identity function; if a line has a hole, they identify it as the graph of one type of rational function. Given a graph that is a continuous curve, they describe that curve as the graph of a quadratic function or a square root function. Also, they relate V-shaped graphs to absolute value functions and relate discontinuous graphs to greatest integer functions, inverse variation functions, and rational functions.

9-6 Solving Rational Equations and Inequalities

In this lesson students return to the topics of the chapter's first two lessons and solve equations involving rational expressions. In general, the first step is to multiply both sides of the equation by the least common denominator of all the denominators. The result is to rewrite the original equation as an equation with no denominators; the new equation can be solved using familiar methods for solving linear or polynomial equations. Students apply these methods to several kinds of word problems. One kind is "work problems," in which one complete job is the sum of partial jobs, each partial job being the quotient of some number of time units divided by a per-unit rate. Another kind is "rate problems," in which a total duration is the sum of two smaller durations, each one being the quotient of distance divided by rate. Also in this lesson students explore rational inequalities by finding values that make the denominator equal to 0, solving a related equation, and identifying intervals on the number line.

www.algebra2.com/key_concepts

Additional mathematical information and teaching notes are available in Glencoe's **Algebra 2 Key Concepts: Mathematical Background and Teaching Notes**, which is available at www.algebra2.com/key_concepts. The lessons appropriate for this chapter are as follows.

- Simplifying Rational Expressions (Lesson 35)
- Multiplying Rational Expressions (Lesson 36)
- Dividing Rational Expressions (Lesson 37)
- Rational Expressions with Unlike Denominators (Lesson 38)

Chapter 9

DAILY INTERVENTION and Assessment

INTERVENTION / ASSESSMENT

Type	Student Edition	Teacher Resources	Technology/Internet
Ongoing	Prerequisite Skills, pp. 471, 478, 484, 490, 498, 504 Practice Quiz 1, p. 484 Practice Quiz 2, p. 498	5-Minute Check Transparencies Quizzes, *CRM* pp. 567–568 Mid-Chapter Test, *CRM* p. 569 Study Guide and Intervention, *CRM* pp. 517–518, 523–524, 529–530, 535–536, 541–542, 547–548	Alge2PASS: Tutorial Plus www.algebra2.com/self_check_quiz www.algebra2.com/extra_examples
Mixed Review	pp. 478, 484, 490, 498, 504, 511	Cumulative Review, *CRM* p. 570	
Error Analysis	Find the Error, pp. 481, 509	Find the Error, *TWE* pp. 481, 509 Unlocking Misconceptions, *TWE* pp. 474, 486, 494 Tips for New Teachers, *TWE* pp. 478, 484, 487, 498, 504, 511	
Standardized Test Practice	pp. 473, 476, 478, 484, 490, 498, 503, 504, 511, 517, 518–519	*TWE* p. 473 Standardized Test Practice, *CRM* pp. 571–572	Standardized Test Practice CD-ROM www.algebra2.com/standardized_test
Open-Ended Assessment	Writing in Math, pp. 477, 484, 490, 498, 503, 511 Open Ended, pp. 476, 478, 482, 488, 495, 501, 509	Modeling: *TWE* pp. 498, 504 Speaking: *TWE* p. 478 Writing: *TWE* pp. 484, 490, 511 Open-Ended Assessment, *CRM* p. 565	
Chapter Assessment	Study Guide, pp. 513–516 Practice Test, p. 517	Multiple-Choice Tests (Forms 1, 2A, 2B), *CRM* pp. 553–558 Free-Response Tests (Forms 2C, 2D, 3), *CRM* pp. 559–564 Vocabulary Test/Review, *CRM* p. 566	TestCheck and Worksheet Builder (see below) MindJogger Videoquizzes www.algebra2.com/vocabulary_review www.algebra2.com/chapter_test

Key to Abbreviations: TWE = Teacher Wraparound Edition; CRM = Chapter Resource Masters

Additional Intervention Resources

The Princeton Review's *Cracking the SAT & PSAT*

The Princeton Review's *Cracking the ACT*

ALEKS

TestCheck and Worksheet Builder

This **networkable** software has three modules for intervention and assessment flexibility:

- **Worksheet Builder** to make worksheet and tests
- **Student Module** to take tests on screen (optional)
- **Management System** to keep student records (optional)

Special banks are included for SAT, ACT, TIMSS, NAEP, and End-of-Course tests.

Intervention Technology

Alge2PASS: Tutorial Plus CD-ROM offers a complete, self-paced algebra curriculum.

Algebra 2 Lesson	Alge2PASS Lesson
9-1	**17** *Simplifying Rational Expressions*
9-2	**18** *Operations with Rational Functions*

ALEKS is an online mathematics learning system that adapts assessment and tutoring to the student's needs. Subscribe at www.k12aleks.com.

Intervention at Home

Log on for student study help.

- For each lesson in the Student Edition, there are Extra Examples and Self-Check Quizzes.
  www.algebra2.com/extra_examples
  www.algebra2.com/self_check_quiz
- For chapter review, there is vocabulary review, test practice, and standardized test practice.
  www.algebra2.com/vocabulary_review
  www.algebra2.com/chapter_test
  www.algebra2.com/standardized_test

For more information on Intervention and Assessment, see pp. T8–T11.

Reading and Writing in Mathematics

Glencoe Algebra 2 provides numerous opportunities to incorporate reading and writing into the mathematics classroom.

Student Edition

- Foldables Study Organizer, p. 471
- Concept Check questions require students to verbalize and write about what they have learned in the lesson. (pp. 476, 481, 488, 495, 501, 509, 513)
- Writing in Math questions in every lesson, pp. 477, 484, 490, 498, 503, 511
- WebQuest, p. 502

Teacher Wraparound Edition

- Foldables Study Organizer, pp. 471, 513
- Study Notebook suggestions, pp. 476, 481, 488, 495, 501, 509
- Modeling activities, pp. 498, 504
- Speaking activities, p. 478
- Writing activities, pp. 484, 490, 511
- ELL Resources, pp. 470, 477, 483, 489, 496, 503, 510, 513

Additional Resources

- Vocabulary Builder worksheets require students to define and give examples for key vocabulary terms as they progress through the chapter. (*Chapter 9 Resource Masters*, pp. vii-viii)
- Reading to Learn Mathematics master for each lesson (*Chapter 9 Resource Masters*, pp. 521, 527, 533, 539, 545, 551)
- *Vocabulary PuzzleMaker* software creates crossword, jumble, and word search puzzles using vocabulary lists that you can customize.
- *Teaching Mathematics with Foldables* provides suggestions for promoting cognition and language.
- *Reading and Writing in the Mathematics Classroom*
- *WebQuest and Project Resources*

For more information on Reading and Writing in Mathematics, see pp. T6–T7.

Chapter 9 Notes

What You'll Learn

Have students read over the list of objectives and make a list of any words with which they are not familiar.

Why It's Important

Point out to students that this is only one of many reasons why each objective is important. Others are provided in the introduction to each lesson.

Lesson	NCTM Standards	Local Objectives
9-1	2, 6, 8, 9, 10	
9-2	2, 6, 8, 9, 10	
9-3	2, 6, 8, 9, 10	
9-3 Follow-Up	2, 6, 8	
9-4	1, 2, 6, 8, 9, 10	
9-5	2, 6, 8, 9, 10	
9-6	2, 6, 8, 9, 10	
9-6 Follow-Up	2, 6	

Key to NCTM Standards:

1=Number & Operations, 2=Algebra, 3=Geometry, 4=Measurement, 5=Data Analysis & Probability, 6=Problem Solving, 7=Reasoning & Proof, 8=Communication, 9=Connections, 10=Representation

Chapter 9 Rational Expressions and Equations

What You'll Learn

- **Lessons 9-1 and 9-2** Simplify rational expressions.
- **Lesson 9-3** Graph rational functions.
- **Lesson 9-4** Solve direct, joint, and inverse variation problems.
- **Lesson 9-5** Identify graphs and equations as different types of functions.
- **Lesson 9-6** Solve rational equations and inequalities.

Key Vocabulary

- **rational expression** (p. 472)
- **asymptote** (p. 485)
- **point discontinuity** (p. 485)
- **direct variation** (p. 492)
- **inverse variation** (p. 493)

Why It's Important

Rational expressions, functions, and equations can be used to solve problems involving mixtures, photography, electricity, medicine, and travel, to name a few. Direct, joint, and inverse variation are important applications of rational expressions. For example, scuba divers can use direct variation to determine the amount of pressure at various depths. *You will learn how to determine the amount of pressure exerted on the ears of a diver in Lesson 9-4.*

Vocabulary Builder

ELL

The Key Vocabulary list introduces students to some of the main vocabulary terms included in this chapter. For a more thorough vocabulary list with pronunciations of new words, give students the Vocabulary Builder worksheets found on pages vii and viii of the *Chapter 9 Resource Masters*. Encourage them to complete the definition of each term as they progress through the chapter. You may suggest that they add these sheets to their study notebooks for future reference when studying for the Chapter 9 test.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 9.

For Lesson 9-1 — Solve Equations with Rational Numbers

Solve each equation. Write your answer in simplest form. *(For review, see Lesson 1-3.)*

1. $\frac{8}{5}x = \frac{4}{15}$ **$\frac{1}{6}$**
2. $\frac{27}{14}t = \frac{6}{7}$ **$\frac{4}{9}$**
3. $\frac{3}{10} = \frac{12}{25}a$ **$\frac{5}{8}$**
4. $\frac{6}{7} = 9m$ **$\frac{2}{21}$**
5. $\frac{9}{8}b = 18$ **16**
6. $\frac{6}{7}s = \frac{3}{4}$ **$\frac{7}{8}$**
7. $\frac{1}{3}r = \frac{5}{6}$ **$2\frac{1}{2}$**
8. $\frac{2}{3}n = 7$ **$10\frac{1}{2}$**
9. $\frac{4}{5}r = \frac{5}{6}$ **$1\frac{1}{24}$**

For Lesson 9-3 — Determine Asymptotes and Graph Equations

Draw the asymptotes and graph each hyperbola. *(For review, see Lesson 8-5.)*

10. $\frac{(x-3)^2}{4} - \frac{(y+5)^2}{9} = 1$
11. $\frac{y^2}{4} - \frac{(x+4)^2}{1} = 1$
12. $\frac{(x+2)}{4} - \frac{(y-3)^2}{25} = 1$

10–12. See margin.

For Lesson 9-4 — Solve Proportions

Solve each proportion.

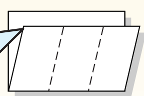
13. $\frac{3}{4} = \frac{r}{16}$ **12**
14. $\frac{8}{16} = \frac{5}{y}$ **10**
15. $\frac{6}{8} = \frac{m}{20}$ **15**
16. $\frac{t}{3} = \frac{5}{24}$ **$\frac{5}{8}$**
17. $\frac{5}{a} = \frac{6}{18}$ **15**
18. $\frac{3}{4} = \frac{b}{6}$ **$4\frac{1}{2}$**
19. $\frac{v}{9} = \frac{12}{18}$ **6**
20. $\frac{7}{p} = \frac{1}{4}$ **28**
21. $\frac{2}{5} = \frac{3}{z}$ **$7\frac{1}{2}$**

FOLDABLES™ Study Organizer

Make this Foldable to help you organize what you learn about rational expressions and equations. Begin with a sheet of plain $8\frac{1}{2}$" × 11" paper.

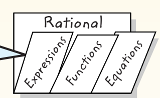
Step 1 Fold

Fold in half lengthwise leaving a $1\frac{1}{2}$" margin at the top. Fold again in thirds.



Step 2 Cut and Label

Open. Cut along the second folds to make three tabs. Label as shown.



Reading and Writing As you read and study the chapter, write notes and examples for each concept under the tabs.

Getting Started

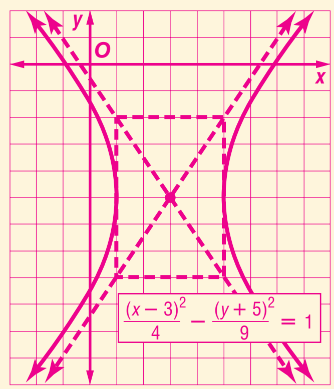
This section provides a review of the basic concepts needed before beginning Chapter 9. Page references are included for additional student help.

Prerequisite Skills in the Getting Ready for the Next Lesson section at the end of each exercise set review a skill needed in the next lesson.

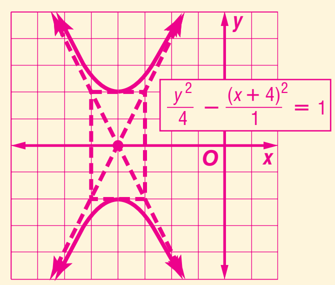
For Lesson	Prerequisite Skill
9-2	Solving Equations (p. 478)
9-3	Graphing Hyperbolas (p. 484)
9-4	Solving Proportions (p. 490)
9-5	Special Functions (p. 498)
9-6	Least Common Multiples of Polynomials (p. 504)

Answers

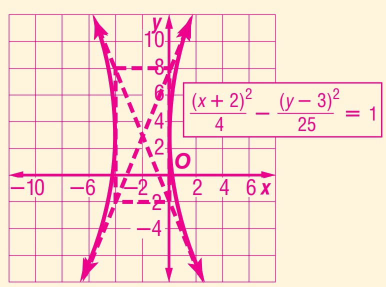
10.



11.



12.



FOLDABLES™ Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Organization of Data with a Concept Map Concept maps are visual study guides that allow students to view main ideas or key words and use them to recall and organize what they know and what they have learned. Begin by writing *Rational* on the base of the Foldable and the words *Expressions*, *Functions*, and *Equations* on the tabs of the concept map. Under the tabs of their Foldable, have students take notes, define terms, record concepts, and write examples. Students can check their responses and memory by reviewing their notes under the tabs.

9-1 Lesson Notes

1 Focus

5-Minute Check Transparency 9-1 Use as a quiz or review of Chapter 8.

Mathematical Background notes are available for this lesson on p. 470C.

How are rational expressions used in mixtures?

Ask students:

- How can the term *rational expression* help you recall what it means? **The word "rational" contains the word "ratio."**
- What does it mean to say that 6 is the GCF of 12 and 30? **It is the greatest integer that divides into both 12 and 30 without a remainder.**

9-1 Multiplying and Dividing Rational Expressions

What You'll Learn

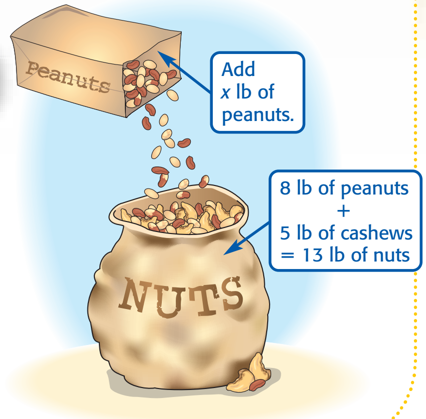
- Simplify rational expressions.
- Simplify complex fractions.

Vocabulary

- rational expression
- complex fraction

How are rational expressions used in mixtures?

The Goodie Shoppe sells candy and nuts by the pound. One of their items is a mixture of peanuts and cashews. This mixture is made with 8 pounds of peanuts and 5 pounds of cashews. Therefore, $\frac{8}{8+5}$ or $\frac{8}{13}$ of the mixture is peanuts. If the store manager adds an additional x pounds of peanuts to the mixture, then $\frac{8+x}{13+x}$ of the mixture will be peanuts.



SIMPLIFY RATIONAL EXPRESSIONS A ratio of two polynomial expressions such as $\frac{8+x}{13+x}$ is called a **rational expression**. Because variables in algebra represent real numbers, operations with rational numbers and rational expressions are similar.

To write a fraction in simplest form, you divide both the numerator and denominator by their greatest common factor (GCF). To simplify a rational expression, you use similar properties.

Example 1 Simplify a Rational Expression

a. Simplify $\frac{2x(x-5)}{(x-5)(x^2-1)}$.

Look for common factors.

$$\frac{2x(x-5)}{(x-5)(x^2-1)} = \frac{2x}{x^2-1} \cdot \frac{\overset{1}{\cancel{x-5}}}{\underset{1}{\cancel{x-5}}}$$ How is this similar to simplifying $\frac{10}{15}$?

$$= \frac{2x}{x^2-1}$$ Simplify.

b. Under what conditions is this expression undefined?

Just as with a fraction, a rational expression is undefined if the denominator is equal to 0. To find when this expression is undefined, completely factor the original denominator.

$$\frac{2x(x-5)}{(x-5)(x^2-1)} = \frac{2x(x-5)}{(x-5)(x-1)(x+1)}$$ $x^2 - 1 = (x-1)(x+1)$

The values that would make the denominator equal to 0 are 5, 1, or −1. So the expression is undefined when $x = 5$, $x = 1$, or $x = -1$. These numbers are called *excluded values*.

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters
- Study Guide and Intervention, pp. 517–518
- Skills Practice, p. 519
- Practice, p. 520
- Reading to Learn Mathematics, p. 521
- Enrichment, p. 522

School-to-Career Masters, p. 17
Teaching Algebra With Manipulatives Masters, p. 272

Transparencies

5-Minute Check Transparency 9-1
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 17
Interactive Chalkboard

Standardized Test Practice

Example 2 Use the Process of Elimination

Multiple-Choice Test Item

For what value(s) of x is $\frac{x^2 + x - 12}{x^2 + 7x + 12}$ undefined?

(A) $-4, -3$ (B) -4 (C) 0 (D) $-4, 3$

Read the Test Item

You want to determine which values of x make the denominator equal to 0.

Solve the Test Item

Look at the possible answers. Notice that if x equals 0 or a positive number, $x^2 + 7x + 12$ must be greater than 0. Therefore, you can eliminate choices C and D. Since both choices A and B contain -4, determine whether the denominator equals 0 when $x = -3$.

$x^2 + 7x + 12 = (-3)^2 + 7(-3) + 12$ $x = -3$

$= 9 - 21 + 12$ Multiply.

$= 0$ Simplify.

Since the denominator equals 0 when $x = -3$, the answer is A.

The Princeton Review

Test-Taking Tip

Sometimes you can save time by looking at the possible answers and eliminating choices, rather than actually evaluating an expression or solving an equation.

Sometimes you can factor out -1 in the numerator or denominator to help simplify rational expressions.

Example 3 Simplify by Factoring Out −1

Simplify $\frac{z^2w - z^2}{z^3 - z^3w}$.

$\frac{z^2w - z^2}{z^3 - z^3w} = \frac{z^2(w - 1)}{z^3(1 - w)}$ Factor the numerator and the denominator.

$= \frac{\overset{1}{\cancel{z^2}}(-1)\overset{1}{\cancel{(1 - w)}}}{\underset{z}{\cancel{z^3}}\underset{1}{\cancel{(1 - w)}}}$ $w - 1 = -(-w + 1)$ or $-1(1 - w)$

$= \frac{-1}{z}$ or $-\frac{1}{z}$ Simplify.

Remember that to multiply two fractions, you first multiply the numerators and then multiply the denominators. To divide two fractions, you multiply by the multiplicative inverse, or reciprocal, of the divisor.

Multiplication

$\frac{5}{6} \cdot \frac{4}{15} = \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3 \cdot 3 \cdot \underset{1}{\cancel{5}}}$

$= \frac{2}{3 \cdot 3}$ or $\frac{2}{9}$

Division

$\frac{3}{7} \div \frac{9}{14} = \frac{3}{7} \cdot \frac{14}{9}$

$= \frac{\overset{1}{\cancel{3}} \cdot 2 \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{3}} \cdot 3}$

$= \frac{2}{3}$

The same procedures are used for multiplying and dividing rational expressions.

www.algebra2.com/extra_examples

2 Teach

SIMPLIFY RATIONAL EXPRESSIONS

In-Class Examples

PowerPoint®

1

a. Simplify $\frac{3y(y + 7)}{(y + 7)(y^2 - 9)}$. $\frac{3y}{y^2 - 9}$

b. Under what conditions is this expression undefined? when $y = -7$, $y = -3$, or $y = 3$

Teaching Tip Ask students to explain why division by zero is undefined.

2 For what value(s) of p is $\frac{p^2 + 2p - 3}{p^2 - 2p - 15}$ undefined? B

A 5 **B** $-3, 5$

C $3, -5$ **D** $5, 1, -3$

3 Simplify $\frac{a^4b - 2a^4}{2a^3 - a^3b}$. $-a$

Teaching Tip Point out that rational expressions are usually used without specifically excluding those values that make the expression undefined. It is understood that only those values for which the expression has meaning are included.

Interactive Chalkboard

PowerPoint® Presentations

This CD-ROM is a customizable Microsoft® PowerPoint® presentation that includes:

- Step-by-step, dynamic solutions of each In-Class Example from the Teacher Wraparound Edition
- Additional, Your Turn exercises for each example
- The 5-Minute Check Transparencies
- Hot links to Glencoe Online Study Tools

Standardized Test Practice

Example 2 Make sure students know to study only the denominator to determine the values that make the expression undefined. In this example, the numerator is irrelevant.

In-Class Examples

Power Point®

4 Simplify each expression.

a. $\frac{8x}{21y^3} \cdot \frac{7y^2}{16x^3}$ $\frac{1}{6x^2y}$

b. $\frac{5a^4c}{12b} \cdot \frac{24bc^2}{15a^3b^2}$ $\frac{2ac^3}{3b^2}$

5 Simplify $\frac{10ps^2}{3c^2d} \div \frac{5ps}{6c^2d^2}$. $4ds$

Teaching Tip To help students understand why division is equivalent to multiplying by the reciprocal, discuss simple examples such as this: dividing 18 marbles between 2 people means that each person gets one-half, or 9, of the marbles.

Key Concept — *Rational Expressions*

Multiplying Rational Expressions

- **Words** To multiply two rational expressions, multiply the numerators and the denominators.
- **Symbols** For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.

Dividing Rational Expressions

- **Words** To divide two rational expressions, multiply by the reciprocal of the divisor.
- **Symbols** For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

The following examples show how these rules are used with rational expressions.

Study Tip

Alternative Method

When multiplying rational expressions, you can multiply first and then divide by the common factors. For instance, in Example 4,

$\frac{4a}{5b} \cdot \frac{15b^2}{16a^3} = \frac{60ab^2}{80a^3b}$.

Now divide the numerator and denominator by the common factors.

$\frac{\overset{3}{\cancel{60}}\,\overset{1}{\cancel{a}}\,\overset{b}{\cancel{b^2}}}{\underset{4}{\cancel{80}}\,\underset{a^2}{\cancel{a^3}}\,\underset{1}{\cancel{b}}} = \frac{3b}{4a^2}$

Example 4 *Multiply Rational Expressions*

Simplify each expression.

a. $\frac{4a}{5b} \cdot \frac{15b^2}{16a^3}$

$\frac{4a}{5b} \cdot \frac{15b^2}{16a^3} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{a}} \cdot 3 \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{b}} \cdot b}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{a}} \cdot a \cdot a}$ Factor.

$= \frac{3 \cdot b}{2 \cdot 2 \cdot a \cdot a}$ Simplify.

$= \frac{3b}{4a^2}$ Simplify.

b. $\frac{8t^2s}{5r^2} \cdot \frac{15sr}{12t^3s^2}$

$\frac{8t^2s}{5r^2} \cdot \frac{15sr}{12t^3s^2} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{t}} \cdot \overset{1}{\cancel{s}} \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{s}} \cdot \overset{1}{\cancel{r}}}{\underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{r}} \cdot r \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{t}} \cdot \underset{1}{\cancel{t}} \cdot t \cdot \underset{1}{\cancel{s}} \cdot \underset{1}{\cancel{s}}}$ Factor.

$= \frac{2}{rt}$ Simplify.

Example 5 *Divide Rational Expressions*

Simplify $\frac{4x^2y}{15a^3b^3} \div \frac{2xy^2}{5ab^3}$.

$\frac{4x^2y}{15a^3b^3} \div \frac{2xy^2}{5ab^3} = \frac{4x^2y}{15a^3b^3} \cdot \frac{5ab^3}{2xy^2}$ Multiply by the reciprocal of the divisor.

$= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{x}} \cdot x \cdot \overset{1}{\cancel{y}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}} \cdot \overset{1}{\cancel{b}}}{3 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{a}} \cdot a \cdot a \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{b}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{y}} \cdot y}$ Factor.

$= \frac{2 \cdot x}{3 \cdot a \cdot a \cdot y}$ Simplify.

$= \frac{2x}{3a^2y}$ Simplify.

DAILY INTERVENTION — Unlocking Misconceptions

- **Simplifying the Quotient of Opposites** Help students understand why the quotient of $(x - y)$ and $(y - x)$ is -1 by pointing out that these two expressions are opposites (or additive inverses) just as are 2 and -2.
- **Division by Zero** By definition, $\frac{a}{b} = c$ if $a = bc$. If students think $\frac{6}{0} = 0$, use the definition to show $\frac{6}{0} = 0$ if $6 = 0 \cdot 0$, which is false.

474 Chapter 9 Rational Expressions and Equations

These same steps are followed when the rational expressions contain numerators and denominators that are polynomials.

Example 6 Polynomials in the Numerator and Denominator

Simplify each expression.

a. $\frac{x^2 + 2x - 8}{x^2 + 4x + 3} \cdot \frac{3x + 3}{x - 2}$

$\frac{x^2 + 2x - 8}{x^2 + 4x + 3} \cdot \frac{3x + 3}{x - 2} = \frac{(x + 4)\overset{1}{\cancel{(x - 2)}}}{(x + 3)\underset{1}{\cancel{(x + 1)}}} \cdot \frac{3\overset{1}{\cancel{(x + 1)}}}{\underset{1}{\cancel{(x - 2)}}}$ Factor.

$= \frac{3(x + 4)}{(x + 3)}$ Simplify.

$= \frac{3x + 12}{x + 3}$ Simplify.

b. $\frac{a + 2}{a + 3} \div \frac{a^2 + a - 12}{a^2 - 9}$

$\frac{a + 2}{a + 3} \div \frac{a^2 + a - 12}{a^2 - 9} = \frac{a + 2}{a + 3} \cdot \frac{a^2 - 9}{a^2 + a - 12}$ Multiply by the reciprocal of the divisor.

$= \frac{(a + 2)\overset{1}{\cancel{(a + 3)}}\overset{1}{\cancel{(a - 3)}}}{\underset{1}{\cancel{(a + 3)}}(a + 4)\underset{1}{\cancel{(a - 3)}}}$ Factor.

$= \frac{a + 2}{a + 4}$ Simplify.

Study Tip

Factor First

As in Example 6, sometimes you must factor the numerator and/or the denominator first before you can simplify a quotient of rational expressions.

TEACHING TIP

Have students replace a with a value such as 2. Then simplify both $\frac{a + 2}{a + 3} \div \frac{a^2 + a - 12}{a^2 - 9}$ and $\frac{a + 2}{a + 4}$. The values are the same.

SIMPLIFY COMPLEX FRACTIONS

A **complex fraction** is a rational expression whose numerator and/or denominator contains a rational expression. The expressions below are complex fractions.

$$\frac{\frac{a}{5}}{3b} \qquad \frac{\frac{3}{t}}{t + 5} \qquad \frac{\frac{m^2 - 9}{8}}{\frac{3 - m}{12}} \qquad \frac{\frac{1}{p} + 2}{\frac{3}{p} - 4}$$

Remember that a fraction is nothing more than a way to express a division problem. For example, $\frac{2}{5}$ can be expressed as $2 \div 5$. So to simplify any complex fraction, rewrite it as a division expression and use the rules for division.

Example 7 Simplify a Complex Fraction

Simplify $\frac{\frac{r^2}{r^2 - 25s^2}}{\frac{r}{5s - r}}$.

$\frac{\frac{r^2}{r^2 - 25s^2}}{\frac{r}{5s - r}} = \frac{r^2}{r^2 - 25s^2} \div \frac{r}{5s - r}$ Express as a division expression.

$= \frac{r^2}{r^2 - 25s^2} \cdot \frac{5s - r}{r}$ Multiply by the reciprocal of the divisor.

$= \frac{\overset{1}{\cancel{r}} \cdot r(-1)\overset{1}{\cancel{(r - 5s)}}}{(r + 5s)\underset{1}{\cancel{(r - 5s)}}\underset{1}{\cancel{r}}}$ Factor.

$= \frac{-r}{r + 5s}$ or $-\frac{r}{r + 5s}$ Simplify.

In-Class Example — Power Point®

6 Simplify each expression.

a. $\frac{k - 3}{k + 1} \div \frac{k^2 - 4k + 3}{1 - k^2}$. -1

b. $\frac{2d + 6}{d^2 + d - 2} \div \frac{d + 3}{d^2 + 3d + 2}$ $\frac{2(d + 1)}{d - 1}$

SIMPLIFY COMPLEX FRACTIONS

In-Class Example — Power Point®

7 Simplify $\frac{\frac{x^2}{9x^2 - 4y^2}}{\frac{x^3}{2y - 3x}}$ $-\frac{1}{x(3x + 2y)}$ or $\frac{-1}{x(3x + 2y)}$

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 9.
- add the Key Concepts in this lesson to their notebook, adding their own examples for each one.
- add the Test-Taking Tip to their list of test-taking tips for review as they prepare for standardized tests.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises…

Organization by Objective

- **Simplify Rational Expressions:** 14–35
- **Simplify Complex Fractions:** 36–41

Odd/Even Assignments

Exercises 14–43 and 46–47 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–37 odd, 43, 47–70

Average: 15–43 odd, 44, 45, 47–70

Advanced: 14–42 even, 44–46, 48–64 (optional: 65–70)

Answer

2. To multiply rational numbers or rational expressions, you multiply the numerators and multiply the denominators. To divide rational numbers or rational expressions, you multiply by the reciprocal of the divisor. In either case, you can reduce your answer by dividing the numerator and the denominator of the results by any common factors.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write two rational expressions that are equivalent.

1. Sample answer: $\frac{4}{6}$, $\frac{4(x+2)}{6(x+2)}$

2. **Explain** how multiplication and division of rational expressions are similar to multiplication and division of rational numbers. **See margin.**

3. **Determine** whether $\frac{2d+5}{3d+5} = \frac{2}{3}$ is *sometimes*, *always*, or *never* true. Explain. **Never; solving the equation using cross products leads to 15 = 10, which is never true.**

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–6	1, 3
7–10	4–6
11, 12	7
13	2

Simplify each expression.

4. $\frac{45mn^3}{20n^7}$ $\frac{9m}{4n^4}$

5. $\frac{a+b}{a^2-b^2}$ $\frac{1}{a-b}$

6. $\frac{6y^3-9y^2}{2y^2+5y-12}$ $\frac{3y^2}{y+4}$

7. $\frac{2a^2}{5b^2c} \cdot \frac{3bc^2}{8a^2}$ $\frac{3c}{20b}$

8. $\frac{35}{16x^2} \div \frac{21}{4x}$ $\frac{5}{12x}$

9. $\frac{3t+6}{7t-7} \cdot \frac{14t-14}{5t+10}$ $\frac{6}{5}$

10. $\frac{12p^2+6p-6}{4(p+1)^2} \div \frac{6p-3}{2p+10}$ $\frac{p+5}{p+1}$

11. $\frac{\frac{c^3d^3}{a}}{\frac{xc^2d}{ax^2}}$ cd^2x

12. $\frac{\frac{2y}{y^2-4}}{\frac{3}{y^2-4y+4}}$ $\frac{2y(y-2)}{3(y+2)}$

Standardized Test Practice

13. Identify all of the values of y for which the expression $\frac{y-4}{y^2-4y-12}$ is undefined. **D**

(A) −2, 4, 6 (B) −6, −4, 2 (C) −2, 0, 6 (D) −2, 6

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 3
22–35	4–6
36–41	7
42, 43, 50	2

Extra Practice

See page 847.

Simplify each expression.

14. $\frac{30bc}{12b^2}$ $\frac{5c}{2b}$

15. $\frac{-3mn^4}{21m^2n^2}$ $\frac{-n^2}{7m}$

16. $\frac{(-3x^2y)^3}{9x^2y^2}$ $-3x^4y$

17. $\frac{(-2rs^2)^2}{12r^2s^3}$ $\frac{s}{3}$

18. $\frac{5t-5}{t^2-1}$ $\frac{5}{t+1}$

19. $\frac{c+5}{2c+10}$ $\frac{1}{2}$

20. $\frac{y^2+4y+4}{3y^2+5y-2}$ $\frac{y+2}{3y-1}$

21. $\frac{a^2+2a+1}{2a^2+3a+1}$ $\frac{a+1}{2a+1}$

22. $\frac{3xyz}{4xz} \cdot \frac{6x^2}{3y^2}$ $\frac{3x^2}{2y}$

23. $\frac{-4ab}{21c} \cdot \frac{14c^2}{18a^2}$ $-\frac{4bc}{27a}$

24. $\frac{3}{5d} \div \left(\frac{-9}{15df}\right)$ $-f$

25. $\frac{p^3}{2q} \div \frac{-p}{4q}$ $-2p^2$

26. $\frac{2x^3y}{z^5} \div \left(\frac{4xy}{z^3}\right)^2$ $\frac{xz}{8y}$

27. $\frac{xy}{a^3} \div \left(\frac{xy}{ab}\right)^3$ $\frac{b^3}{x^2y^2}$

28. $\frac{3t^2}{t+2} \cdot \frac{t+2}{t^2}$ 3

29. $\frac{4w+4}{3} \cdot \frac{1}{w+1}$ $\frac{4}{3}$

30. $\frac{4t^2-4}{9(t+1)^2} \cdot \frac{3t+3}{2t-2}$ $\frac{2}{3}$

31. $\frac{3p-21}{p^2-49} \cdot \frac{p^2+7p}{3p}$ 1

32. $\frac{5x^2+10x-75}{4x^2-24x-28} \cdot \frac{2x^2-10x-28}{x^2+7x+10}$

32. $\frac{5(x-3)}{2(x+1)}$

33. $\frac{w^2-11w+24}{w^2-18w+80} \cdot \frac{w^2-15w+50}{w^2-9w+20}$ $\frac{w-3}{w-4}$

34. $\frac{r^2+2r-8}{r^2+4r+3} \div \frac{r-2}{3r+3}$ $\frac{3(r+4)}{r+3}$

35. $\frac{a^2+2a-15}{a-3} \div \frac{a^2-4}{2}$ $\frac{2(a+5)}{(a-2)(a+2)}$

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students think about what aspects of multiplying and dividing rational expressions they find most challenging. Have them write a paragraph explaining why, and what steps they can take to help their challenges or confusions.

★ 36. $\dfrac{\dfrac{m^3}{3n}}{-\dfrac{m^4}{9n^2}}$ $-\dfrac{3n}{m}$

37. $\dfrac{\dfrac{p^3}{2q}}{\dfrac{-p^2}{4q}}$ $-2p$

38. $\dfrac{\dfrac{m+n}{5}}{\dfrac{m^2+n^2}{5}}$ $\dfrac{m+n}{m^2+n^2}$

★ 39. $\dfrac{\dfrac{x+y}{2x-y}}{\dfrac{x+y}{2x+y}}$ $\dfrac{2x+y}{2x-y}$

40. $\dfrac{\dfrac{6y^2-6}{8y^2+8y}}{\dfrac{3y-3}{4y^2+4y}}$ $y+1$

41. $\dfrac{\dfrac{5x^2-5x-30}{45-15x}}{\dfrac{6+x-x^2}{4x-12}}$ $\dfrac{4}{3}$

42. Under what conditions is $\dfrac{2d(d+1)}{(d+1)(d^2-4)}$ undefined? $d = -2, -1, \text{or } 2$

43. Under what conditions is $\dfrac{a^2+ab+b^2}{a^2-b^2}$ undefined? $a = -b$ or b

More About. . .

Basketball

After graduating from the U.S. Naval Academy, David Robinson became the NBA Rookie of the Year in 1990. He has played basketball in 3 different Olympic Games.

Source: NBA

BASKETBALL **For Exercises 44 and 45, use the following information.**

At the end of the 2000–2001 season, David Robinson had made 6827 field goals out of 13,129 attempts during his NBA career.

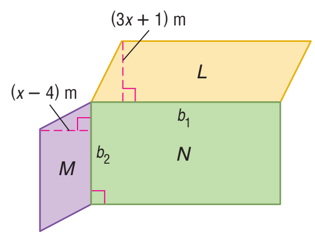
44. Write a fraction to represent the ratio of the number of career field goals made to career field goals attempted by David Robinson at the end of the 2000–2001 season. $\dfrac{6827}{13{,}129}$

45. Suppose David Robinson attempted a field goals and made m field goals during the 2001–2002 season. Write a rational expression to represent the number of career field goals made to the number of career field goals attempted at the end of the 2001–2002 season. $\dfrac{6827+m}{13{,}129+a}$

Online Research **Data Update** What are the current scoring statistics of your favorite NBA player? Visit www.algebra2.com/data_update to learn more.

46. **GEOMETRY** A parallelogram with an area of $6x^2 - 7x - 5$ square units has a base of $3x - 5$ units. Determine the height of the parallelogram. $2x + 1$ units

47. **GEOMETRY** Parallelogram L has an area of $3x^2 + 10x + 3$ square meters and a height of $3x + 1$ meters. Parallelogram M has an area of $2x^2 - 13x + 20$ square meters and a height of $x - 4$ meters. Find the area of rectangle N. $(2x^2 + x - 15)$ m^2



48. **CRITICAL THINKING** Simplify $\dfrac{(a^2-5a+6)^{-1}}{(a-2)^{-2}} \div \dfrac{(a-3)^{-1}}{(a-2)^{-2}}$. $\dfrac{1}{a-2}$

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. See pp. 519A–519D.

How are rational expressions used in mixtures?

Include the following in your answer:

- an explanation of how to determine whether the rational expression representing the nut mixture is in simplest form, and
- an example of a mixture problem that could be represented by $\dfrac{8+x}{13+x+y}$.

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 517 (shown) and p. 518

Simplify Rational Expressions A ratio of two polynomial expressions is a **rational expression**. To simplify a rational expression, divide both the numerator and the denominator by their greatest common factor (GCF).

Multiplying Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, if $b \neq 0$ and $d \neq 0$.
Dividing Rational Expressions	For all rational expressions $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$, if $b \neq 0$, $c \neq 0$, and $d \neq 0$.

Example Simplify each expression.

a. $\dfrac{24a^5b^2}{(2ab)^4}$

$\dfrac{24a^5b^2}{(2ab)^4} = \dfrac{2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b}{2 \cdot 2 \cdot 2 \cdot 2 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b} = \dfrac{3a}{2b^2}$

b. $\dfrac{3r^2s^3}{5t^4} \cdot \dfrac{20t^2}{9r^3s}$

$\dfrac{3r^2s^3}{5t^4} \cdot \dfrac{20t^2}{9r^3s} = \dfrac{3 \cdot r \cdot r \cdot s \cdot s \cdot s \cdot 2 \cdot 2 \cdot 5 \cdot t \cdot t}{5 \cdot t \cdot t \cdot t \cdot t \cdot 3 \cdot 3 \cdot r \cdot r \cdot r \cdot s} = \dfrac{2 \cdot 2 \cdot s \cdot s}{3 \cdot r \cdot t \cdot t} = \dfrac{4s^2}{3rt^2}$

c. $\dfrac{x^2+8x+16}{2x-2} \div \dfrac{x^2+2x-8}{x-1}$

$\dfrac{x^2+8x+16}{2x-2} \div \dfrac{x^2+2x-8}{x-1} = \dfrac{x^2+8x+16}{2x-2} \cdot \dfrac{x-1}{x^2+2x-8}$

$= \dfrac{(x+4)(x+4)(x-1)}{2(x-1)(x-2)(x+4)} = \dfrac{x+4}{2(x-2)}$

Exercises

Simplify each expression.

1. $\dfrac{(-2ab^2)^3}{20ab^4}$ $-\dfrac{2a^2b^2}{5}$
2. $\dfrac{4x^2-12x+9}{9-6x}$ $\dfrac{3-2x}{3}$
3. $\dfrac{x^2+x-6}{x^2-6x-27}$ $\dfrac{x-2}{x-9}$
4. $\dfrac{3m^3-3m}{6m^4} \cdot \dfrac{4m^5}{m+1}$ $2m^2(m-1)$
5. $\dfrac{c^2-3c}{c^2-25} \cdot \dfrac{c^2+4c-5}{c^2-4c+3}$ $\dfrac{c}{c-5}$
6. $\dfrac{(m-3)^2}{m^2-6m+9} \cdot \dfrac{m^3-9m}{m^2-9}$ m
7. $\dfrac{6xy^4}{25z^3} \div \dfrac{18xz^2}{5y}$ $\dfrac{y^5}{15z^5}$
8. $\dfrac{16p^2-8p+1}{14p^4} \div \dfrac{4p^2+7p-2}{7p^5}$ $\dfrac{p(4p-1)}{2(p+2)}$
9. $\dfrac{2m-1}{m^2-3m-10} \div \dfrac{4m^2-1}{4m+8}$ $\dfrac{4}{(2m+1)(m-5)}$

Skills Practice, p. 519 and Practice, p. 520 (shown)

Simplify each expression.

1. $\dfrac{9a^2b^3}{27a^4b^4c}$ $\dfrac{1}{3a^2bc}$
2. $\dfrac{(2m^3n^2)^3}{-18m^5n^4}$ $-\dfrac{4m^4n^2}{9}$
3. $\dfrac{10y^2+15y}{35y^2-5y}$ $\dfrac{2y+3}{7y-1}$
4. $\dfrac{2k^2-k-15}{k^2-9}$ $\dfrac{2k+5}{k+3}$
5. $\dfrac{25-v^2}{3v^2-13v-10}$ $-\dfrac{v+5}{3v+2}$
6. $\dfrac{x^4+x^3-2x^2}{x^4-x^3}$ $\dfrac{x+2}{x}$
7. $\dfrac{-2u^3y}{15xz^5} \cdot \dfrac{25x^3}{14u^2y^2}$ $-\dfrac{5ux^2}{21yz^5}$
8. $\dfrac{a+y}{6} \cdot \dfrac{4}{y+a}$ $\dfrac{2}{3}$
9. $\dfrac{n^5}{n-6} \cdot \dfrac{n^2-6n}{n^8}$ $\dfrac{1}{n^2}$
10. $\dfrac{a-y}{w+n} \cdot \dfrac{w^2-n^2}{y-a}$ $n-w$
11. $\dfrac{x^2-5x-24}{6x+2x^2} \cdot \dfrac{5x^2}{8-x}$ $-\dfrac{5x}{2}$
12. $\dfrac{x-5}{10x-2} \cdot \dfrac{25x^2-1}{x^2-10x+25}$ $\dfrac{5x+1}{2(x-5)}$
13. $\dfrac{a^5y^3}{wy^7} \div \dfrac{a^3w^2}{w^5y^2}$ $\dfrac{a^2w^2}{y^2}$
14. $\left(\dfrac{2xy}{w^2}\right)^3 \div \dfrac{24x^2}{w^5}$ $\dfrac{xy^3}{3w}$
15. $\dfrac{x+y}{6} \div \dfrac{x^2-y^2}{3}$ $\dfrac{1}{2(x-y)}$
16. $\dfrac{3x+6}{x^2-9} \div \dfrac{6x^2+12x}{4x+12}$ $\dfrac{2}{x(x-3)}$
17. $\dfrac{2s^2-7s-15}{(s+4)^2} \div \dfrac{s^2-10s+25}{s+4}$ $\dfrac{2s+3}{(s+4)(s-5)}$
18. $\dfrac{9-a^2}{a^2+5a+6} \div \dfrac{2a-6}{5a+10}$ $-\dfrac{5}{2}$
19. $\dfrac{\frac{2x+1}{x}}{\frac{4-x}{x}}$ $\dfrac{2x+1}{4-x}$
20. $\dfrac{\frac{x^2-9}{4}}{\frac{3-x}{8}}$ $-2(x+3)$
21. $\dfrac{\frac{x^3+2^3}{x^2-2x}}{\frac{(x+2)^3}{x^2+4x+4}}$ $\dfrac{x^2-2x+4}{x(x-2)}$

22. **GEOMETRY** A right triangle with an area of $x^2 - 4$ square units has a leg that measures $2x + 4$ units. Determine the length of the other leg of the triangle. $x - 2$ units

23. **GEOMETRY** A rectangular pyramid has a base area of $\dfrac{x^2+3x-10}{2x}$ square centimeters and a height of $\dfrac{x^2-3x}{x^2-5x+6}$ centimeters. Write a rational expression to describe the volume of the rectangular pyramid. $\dfrac{x+5}{6}$ cm^3

Reading to Learn Mathematics, p. 521

ELL

Pre-Activity **How are rational expressions used in mixtures?**

Read the introduction to Lesson 9-1 at the top of page 472 in your textbook.

- Suppose that the Goodie Shoppe also sells a candy mixture of chocolate mints and caramels. If this mixture is made with 4 pounds of chocolate mints and 3 pounds of caramels, then $\frac{4}{7}$ of the mixture is mints and $\frac{3}{7}$ of the mixture is caramels.
- If the store manager adds another y pounds of mints to the mixture, what fraction of the mixture will be mints? $\dfrac{4+y}{7+y}$

Reading the Lesson

1. a. In order to simplify a rational number or rational expression, factor the numerator and denominator and divide both of them by their greatest common factor.
   b. A rational expression is undefined when its denominator is equal to 0. To find the values that make the expression undefined, completely factor the original denominator and set each factor equal to 0.
2. a. To multiply two rational expressions, multiply the numerators and multiply the denominators.
   b. To divide two rational expressions, multiply by the reciprocal of the divisor.
3. a. Which of the following expressions are complex fractions? ii, iv, v

   i. $\dfrac{7}{12}$ ii. $\dfrac{\frac{3}{8}}{\frac{5}{16}}$ iii. $\dfrac{r+5}{r-5}$ iv. $\dfrac{\frac{z+1}{z}}{z}$ v. $\dfrac{\frac{r^2-25}{9}}{\frac{r+5}{3}}$

   b. Does a complex fraction express a multiplication or division problem? division How is multiplication used in simplifying a complex fraction? Sample answer: To divide the numerator of the complex fraction by the denominator, multiply the numerator by the reciprocal of the denominator.

Helping You Remember

4. One way to remember something new is to see how it is similar to something you already know. How can your knowledge of division of fractions in arithmetic help you to understand how to divide rational expressions? Sample answer: To divide rational expressions, multiply the first expression by the reciprocal of the second. This is the same “invert and multiply” process that is used when dividing arithmetic fractions.

Enrichment, p. 522

Reading Algebra

In mathematics, the term *group* has a special meaning. The following numbered sentences discuss the idea of group and one interesting example of a group.

01 To be a group, a set of elements and a binary operation must satisfy four conditions: the set must be closed under the operation, the operation must be associative, there must be an identity element, and every element must have an inverse.

02 The following six functions form a group under the operation of composition of functions: $f_1(x) = x$, $f_2(x) = \frac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \frac{(x-1)}{x}$, $f_5(x) = \frac{x}{(x-1)}$, and $f_6(x) = \frac{1}{(1-x)}$.

03 This group is an example of a noncommutative group. For example, $f_3 \circ f_2 = f_4$, but $f_2 \circ f_3 = f_6$.

04 Some experimentation with this group will show that the identity element is f_1.

4 Assess

Open-Ended Assessment

Speaking Have students explain the procedures and cautions for multiplying and dividing rational expressions, demonstrating with examples.

Tips for New Teachers

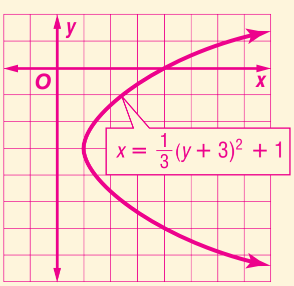
Intervention Encourage students who are having difficulty with these problems to use several steps, writing each one below the previous one, and keeping each line equivalent to the one above. Caution them to make only one change per step.

Getting Ready for Lesson 9-2

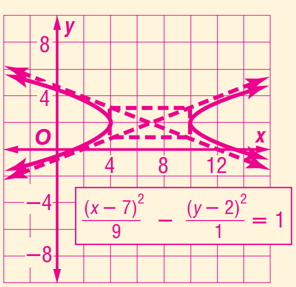
PREREQUISITE SKILL Students will add and subtract rational expressions in Lesson 9-2. As with equations containing fractions, students will find common denominators, combine like terms, and simplify equations. Use Exercises 65–70 to determine your students' familiarity with solving equations containing fractions.

Answers

54.



55.



Standardized Test Practice

50. For what value(s) of x is the expression $\frac{4x}{x^2 - x}$ undefined? **C**

(A) −1, 1 (B) −1, 0, 1 (C) 0, 1 (D) 0 (E) 1, 2

51. **Compare the quantity in Column A and the quantity in Column B. Then determine whether: A**

(A) the quantity in Column A is greater,
(B) the quantity in Column B is greater,
(C) the two quantities are equal, or
(D) the relationship cannot be determined from the information given.

Column A	Column B
$\frac{a^2 + 3a - 10}{a - 2}$	$\frac{a^2 + a - 6}{a + 3}$

Maintain Your Skills

Mixed Review **Find the exact solution(s) of each system of equations.** *(Lesson 8–7)*

52. $x^2 + 2y^2 = 33$
$x^2 + y^2 - 19 = 2x$
(−1, ±4), (5, ±2)

53. $x^2 + 2y^2 = 33$
$x^2 - y^2 = 9$
$(\pm\sqrt{17}, \pm 2\sqrt{2})$

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation. *(Lesson 8–6)*

54. $y^2 - 3x + 6y + 12 = 0$ 55. $x^2 - 14x + 4 = 9y^2 - 36y$

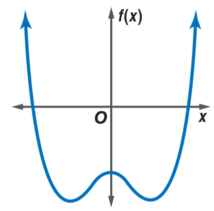
54–55. See margin for graphs.

54. $x = \frac{1}{3}(y + 3)^2 + 1$; parabola

55. $\frac{(x-7)^2}{9} - \frac{(y-2)^2}{1} = 1$; hyperbola

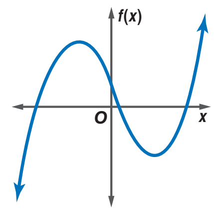
Determine whether each graph represents an odd-degree function or an even-degree function. Then state how many real zeros each function has. *(Lesson 7–1)*

56.



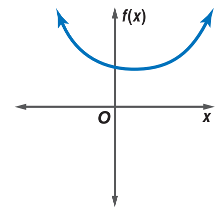
even; 2

57.



odd; 3

58.



even; 0

Solve each equation by factoring. *(Lesson 6–3)*

59. $r^2 - 3r = 4$ **−1, 4** 60. $18u^2 - 3u = 1$ **$-\frac{1}{6}, \frac{1}{3}$** 61. $d^2 - 5d = 0$ **0, 5**

62. **ASTRONOMY** Earth is an average 1.496×10^8 kilometers from the Sun. If light travels 3×10^5 kilometers per second, how long does it take sunlight to reach Earth? *(Lesson 5–1)* **4.99×10^2 s or about 8 min 19 s**

Solve each equation. *(Lesson 1–4)*

63. $|2x + 7| + 5 = 0$ **∅** 64. $5|3x - 4| = x + 1$ **$\frac{3}{2}, \frac{19}{16}$**

Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Solve each equation.** *(To review **solving equations**, see Lesson 1–3.)*

65. $\frac{2}{3} + x = -\frac{4}{9}$ **$-1\frac{1}{9}$** 66. $x + \frac{5}{8} = -\frac{5}{6}$ **$-1\frac{11}{24}$** 67. $x - \frac{3}{5} = \frac{2}{3}$ **$1\frac{4}{15}$**

68. $x + \frac{3}{16} = -\frac{1}{2}$ **$-\frac{11}{16}$** 69. $x - \frac{1}{6} = -\frac{7}{9}$ **$-\frac{11}{18}$** 70. $x - \frac{3}{8} = -\frac{5}{24}$ **$\frac{1}{6}$**

478 Chapter 9 Rational Expressions and Equations

9-2 Adding and Subtracting Rational Expressions

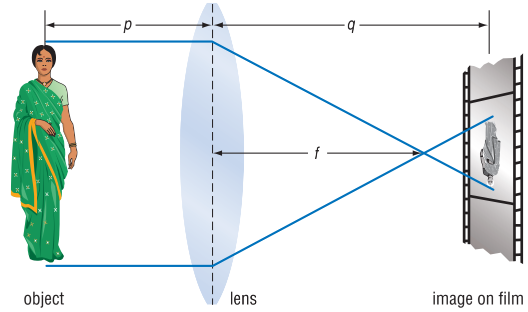
What You'll Learn

- Determine the LCM of polynomials.
- Add and subtract rational expressions.

How is subtraction of rational expressions used in photography?

To take sharp, clear pictures, a photographer must focus the camera precisely. The distance from the object to the lens p and the distance from the lens to the film q must be accurately calculated to ensure a sharp image. The focal length of the lens is f.

The formula $\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$ can be used to determine how far the film should be placed from the lens to create a perfect photograph.



LCM OF POLYNOMIALS To find $\frac{5}{6} - \frac{1}{4}$ or $\frac{1}{f} - \frac{1}{p}$, you must first find the least common denominator (LCD). The LCD is the least common multiple (LCM) of the denominators.

To find the LCM of two or more numbers or polynomials, factor each number or polynomial. The LCM contains *each* factor the greatest number of times it appears as a factor.

LCM of 6 and 4	LCM of $a^2 - 6a + 9$ and $a^2 + a - 12$
$6 = 2 \cdot 3$	$a^2 - 6a + 9 = (a - 3)^2$
$4 = 2^2$	$a^2 + a - 12 = (a - 3)(a + 4)$
LCM $= 2^2 \cdot 3$ or 12	LCM $= (a - 3)^2(a + 4)$

Example 1 LCM of Monomials

Find the LCM of $18r^2s^5$, $24r^3st^2$, and $15s^3t$.

$18r^2s^5 = 2 \cdot 3^2 \cdot r^2 \cdot s^5$ — Factor the first monomial.

$24r^3st^2 = 2^3 \cdot 3 \cdot r^3 \cdot s \cdot t^2$ — Factor the second monomial.

$15s^3t = 3 \cdot 5 \cdot s^3 \cdot t$ — Factor the third monomial.

LCM $= 2^3 \cdot 3^2 \cdot 5 \cdot r^3 \cdot s^5 \cdot t^2$ — Use each factor the greatest number of times it appears as a factor and simplify.

$= 360r^3s^5t^2$

9-2 Lesson Notes

1 Focus

5-Minute Check Transparency 9-2 Use as a quiz or review of Lesson 9-1.

Mathematical Background notes are available for this lesson on p. 470C.

How is subtraction of rational expressions used in photography?

Ask students:

- If you drew a box to represent the camera, which of the variables shown would describe dimensions within the camera? **f and q**
- In a camera, what is a typical value for q? **Sample answer: 0.5 in.**

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters

- Study Guide and Intervention, pp. 523–524
- Skills Practice, p. 525
- Practice, p. 526
- Reading to Learn Mathematics, p. 527
- Enrichment, p. 528
- Assessment, p. 567

Transparencies

5-Minute Check Transparency 9-2
Answer Key Transparencies

Technology

Alge2PASS: Tutorial Plus, Lesson 18
Interactive Chalkboard

2 Teach

LCM OF POLYNOMIALS

In-Class Examples Power Point®

1 Find the LCM of $15a^2bc^3$, $16b^5c^2$, and $20a^3c^6$. $240a^3b^5c^6$

2 Find the LCM of $x^3 - x^2 - 2x$ and $x^2 - 4x + 4$.
$x(x + 1)(x - 2)^2$

ADD AND SUBTRACT RATIONAL EXPRESSIONS

In-Class Examples Power Point®

3 Simplify $\frac{5a^2}{6b} + \frac{9}{14a^2b^2}$.

$\frac{35a^4b + 27}{42a^2b^2}$

Teaching Tip Have students discuss the differences between procedures for adding and multiplying fractions.

4 Simplify $\frac{x + 10}{3x - 15} - \frac{3x + 15}{6x - 30}$.
$-\frac{1}{6}$

Example 2 LCM of Polynomials

Find the LCM of $p^3 + 5p^2 + 6p$ and $p^2 + 6p + 9$.

$p^3 + 5p^2 + 6p = p(p + 2)(p + 3)$ Factor the first polynomial.

$p^2 + 6p + 9 = (p + 3)^2$ Factor the second polynomial.

$\text{LCM} = p(p + 2)(p + 3)^2$ Use each factor the greatest number of times it appears as a factor.

ADD AND SUBTRACT RATIONAL EXPRESSIONS As with fractions, to add or subtract rational expressions, you must have common denominators.

Specific Case		General Case
$\frac{2}{3} + \frac{3}{5} = \frac{2 \cdot 5}{3 \cdot 5} + \frac{3 \cdot 3}{5 \cdot 3}$	Find equivalent fractions that have a common denominator.	$\frac{a}{c} + \frac{b}{d} = \frac{a \cdot d}{c \cdot d} + \frac{b \cdot c}{d \cdot c}$
$= \frac{10}{15} + \frac{9}{15}$	Simplify each numerator and denominator.	$= \frac{ad}{cd} + \frac{bc}{cd}$
$= \frac{19}{15}$	Add the numerators.	$= \frac{ad + bc}{cd}$

Example 3 Monomial Denominators

Simplify $\frac{7x}{15y^2} + \frac{y}{18xy}$.

$\frac{7x}{15y^2} + \frac{y}{18xy} = \frac{7x \cdot 6x}{15y^2 \cdot 6x} + \frac{y \cdot 5y}{18xy \cdot 5y}$ The LCD is $90xy^2$. Find equivalent fractions that have this denominator.

$= \frac{42x^2}{90xy^2} + \frac{5y^2}{90xy^2}$ Simplify each numerator and denominator.

$= \frac{42x^2 + 5y^2}{90xy^2}$ Add the numerators.

Example 4 Polynomial Denominators

Simplify $\frac{w + 12}{4w - 16} - \frac{w + 4}{2w - 8}$.

$\frac{w + 12}{4w - 16} - \frac{w + 4}{2w - 8} = \frac{w + 12}{4(w - 4)} - \frac{w + 4}{2(w - 4)}$ Factor the denominators.

$= \frac{w + 12}{4(w - 4)} - \frac{(w + 4)(2)}{2(w - 4)(2)}$ The LCD is $4(w-4)$.

$= \frac{(w + 12) - (2)(w + 4)}{4(w - 4)}$ Subtract the numerators.

$= \frac{w + 12 - 2w - 8}{4(w - 4)}$ Distributive Property

$= \frac{-w + 4}{4(w - 4)}$ Combine like terms.

$= \frac{-1\cancel{(w - 4)}^{1}}{4\cancel{(w - 4)}_{1}}$ or $-\frac{1}{4}$ Simplify.

Study Tip

Common Factors
Sometimes when you simplify the numerator, the polynomial contains a factor common to the denominator. Thus, the rational expression can be further simplified.

Sometimes simplifying complex fractions involves adding or subtracting rational expressions. One way to simplify a complex fraction is to simplify the numerator and the denominator separately, and then simplify the resulting expressions.

480 Chapter 9 Rational Expressions and Equations

Example 5 Simplify Complex Fractions

Simplify $\dfrac{\frac{1}{x} - \frac{1}{y}}{1 + \frac{1}{x}}$.

$$\frac{\frac{1}{x} - \frac{1}{y}}{1 + \frac{1}{x}} = \frac{\frac{y}{xy} - \frac{x}{xy}}{\frac{x}{x} + \frac{1}{x}}$$ The LCD of the numerator is xy. The LCD of the denominator is x.

$$= \frac{\frac{y-x}{xy}}{\frac{x+1}{x}}$$ Simplify the numerator and denominator.

$$= \frac{y-x}{xy} \div \frac{x+1}{x}$$ Write as a division expression.

$$= \frac{y-x}{\cancel{x}y} \cdot \frac{\cancel{x}}{x+1}$$ Multiply by the reciprocal of the divisor.

$$= \frac{y-x}{y(x+1)} \text{ or } \frac{y-x}{xy+y}$$ Simplify.

TEACHING TIP

Point out that $\dfrac{\frac{1}{x} + \frac{1}{y}}{1 + \frac{1}{x}}$ can also be simplified by multiplying the numerator and denominator by xy.

Example 6 Use a Complex Fraction to Solve a Problem

COORDINATE GEOMETRY Find the slope of the line that passes through $A\left(\frac{2}{p}, \frac{1}{2}\right)$ and $B\left(\frac{1}{3}, \frac{3}{p}\right)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Definition of slope

$$= \frac{\frac{3}{p} - \frac{1}{2}}{\frac{1}{3} - \frac{2}{p}}$$ $y_2 = \frac{3}{p}, y_1 = \frac{1}{2}, x_2 = \frac{1}{3}$, and $x_1 = \frac{2}{p}$

$$= \frac{\frac{6-p}{2p}}{\frac{p-6}{3p}}$$ The LCD of the numerator is $2p$. The LCD of the denominator is $3p$.

$$= \frac{6-p}{2p} \div \frac{p-6}{3p}$$ Write as a division expression.

$$= \frac{\overset{-1}{\cancel{6-p}}}{\underset{1}{2\cancel{p}}} \cdot \frac{\overset{1}{3\cancel{p}}}{\underset{1}{\cancel{p-6}}} \text{ or } -\frac{3}{2}$$ The slope is $-\frac{3}{2}$.

Study Tip

Check Your Solution

You can check your answer be letting p equal any nonzero number, say 1. Use the definition of slope to find the slope of the line through the points.

Check for Understanding

Concept Check

1. **FIND THE ERROR** Catalina and Yong-Chan are simplifying $\frac{x}{a} - \frac{x}{b}$.

Catalina	Yong-Chan
$\frac{x}{a} - \frac{x}{b} = \frac{bx}{ab} - \frac{ax}{ab} = \frac{bx - ax}{ab}$	$\frac{x}{a} - \frac{x}{b} = \frac{x}{a - b}$

Who is correct? Explain your reasoning.

1. Catalina; you need a common denominator, not a common numerator, to subtract two rational expressions.

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

5 Simplify $\dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{b} - 1}$. $\dfrac{a+b}{a(1-b)}$ or $\dfrac{a+b}{a-ab}$

6 **COORDINATE GEOMETRY** Find the slope of the line that passes through $P\left(\frac{3}{k}, \frac{1}{3}\right)$ and $Q\left(\frac{1}{2}, \frac{2}{k}\right)$. $-\frac{2}{3}$

Teaching Tip Remind students that the slope of a line is the change in y divided by the change in x, or the rise over the run.

3 Practice/Apply

Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 9.
- include any item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR One way to find the error is to substitute values for the variables. With $x = 4$, $a = 5$, and $b = 3$, $\frac{x}{a} - \frac{x}{b}$ becomes $\frac{4}{5} - \frac{4}{3}$. Since the first fraction is less than 1 and the second is greater than 1, the result must be negative, which means that the answer $\frac{x}{a-b} = \frac{4}{2}$ or 2 cannot be correct.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students work with a partner, one in the role of coach and the other in the role of athlete. The athlete works the problem, using steps and explaining the thinking, while the coach listens and watches for errors, correcting as necessary. Then the partners exchange roles.

Lesson 9-2 Adding and Subtracting Rational Expressions 481

About the Exercises…

Organization by Objective

- **LCM of Polynomials:** 14–21
- **Add and Subtract Rational Expressions:** 22–49

Odd/Even Assignments

Exercises 14–43 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 15–37 odd, 50–61

Average: 15–43 odd, 49–61

Advanced: 14–42 even, 44–48, 50–58 (optional: 59–61)

All: Practice Quiz 1 (1–10)

Answers

3a. Since a, b, and c are factors of abc, abc is always a common denominator of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

3b. If a, b, and c have no common factors, then abc is the LCD of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

3c. If a and b have no common factors and c is a factor of ab, then ab is the LCD of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

3d. If a and c are factors of b, then b is the LCD of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

3e. Since $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{bc}{abc} + \frac{ac}{abc} + \frac{ab}{abc}$, the sum is always $\frac{bc + ac + ab}{abc}$.

---

2. Sample answer: $d^2 - d$, $d + 1$

2. **OPEN ENDED** Write two polynomials that have a LCM of $d^3 - d$.

★ 3. Consider $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ if a, b, and c are real numbers. Determine whether each statement is *sometimes*, *always*, or *never* true. Explain your answer.
   a. abc is a common denominator. **always** a–e. See margin for explanations.
   b. abc is the LCD. **sometimes**
   c. ab is the LCD. **sometimes**
   d. b is the LCD. **sometimes**
   e. The sum is $\frac{bc + ac + ab}{abc}$. **always**

Guided Practice

GUIDED PRACTICE KEY

Exercises	Examples
4–6	1, 2
7–11	3, 4
12	5
13	6

Find the LCM of each set of polynomials.

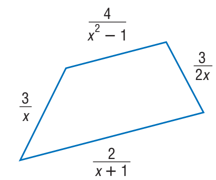
4. $12y^2, 6x^2$ $\mathbf{12x^2y^2}$
5. $16ab^3, 5b^2a^2, 20ac$ $\mathbf{80ab^3c^2}$
6. $x^2 - 2x, x^2 - 4$ $\mathbf{x(x - 2)(x + 2)}$

Simplify each expression.

7. $\frac{2}{x^2y} - \frac{x}{y}$ $\frac{2 - x^3}{x^2y}$
8. $\frac{7a}{15b^2} + \frac{b}{18ab}$ $\frac{42a^2 + 5b^2}{90ab^2}$
9. $\frac{5}{3m} - \frac{2}{7m} - \frac{1}{2m}$ $\frac{37}{42m}$
10. $\frac{6}{d^2 + 4d + 4} + \frac{5}{d + 2}$ $\frac{5d + 16}{(d + 2)^2}$
11. $\frac{a}{a^2 - a - 20} + \frac{2}{a + 4}$ $\frac{3a - 10}{(a - 5)(a + 4)}$
12. $\frac{x + \frac{x}{3}}{x - \frac{x}{6}}$ $\frac{8}{5}$

Application

13. **GEOMETRY** Find the perimeter of the quadrilateral. Express in simplest form. $\frac{13x^2 + 4x - 9}{2x(x - 1)(x + 1)}$ units



★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1, 2
22–39	3, 4
40–43	5
44–49	6

Extra Practice

See page 847.

Find the LCM of each set of polynomials.

14. $10s^2, 35s^2t^2$ $\mathbf{70s^2t^2}$
15. $36x^2y, 20xyz$ $\mathbf{180x^2yz}$
16. $14a^3, 15bc^3, 12b^3$ $\mathbf{420a^3b^3c^3}$
17. $9p^2q^3, 6pq^4, 4p^3$ $\mathbf{36p^3q^4}$
18. $4w - 12, 2w - 6$ $\mathbf{4(w - 3)}$
19. $x^2 - y^2, x^3 + x^2y$ $\mathbf{x^2(x - y)(x + y)}$
20. $2t^2 + t - 3, 2t^2 + 5t + 3$ $\mathbf{(2t + 3)(t - 1)(t + 1)}$
21. $n^2 - 7n + 12, n^2 - 2n - 8$ $\mathbf{(n - 4)(n - 3)(n + 2)}$

Simplify each expression.

22. $\frac{6}{ab} + \frac{8}{a}$ $\frac{6 + 8b}{ab}$
23. $\frac{5}{6v} + \frac{7}{4v}$ $\frac{31}{12v}$
24. $\frac{5}{r} + 7$ $\frac{5 + 7r}{r}$
25. $\frac{2x}{3y} + 5$ $\frac{2x + 15y}{3y}$
26. $\frac{3x}{4y^2} - \frac{y}{6x}$ $\frac{9x^2 - 2y^2}{12xy^2}$
27. $\frac{5}{a^2b} - \frac{7a}{5b^2}$ $\frac{25b - 7a^3}{5a^2b^2}$
28. $\frac{3}{4q} - \frac{2}{5q} - \frac{1}{2q}$ $-\frac{3}{20q}$
29. $\frac{11}{9} - \frac{7}{2w} - \frac{6}{5w}$ $\frac{110w - 423}{90w}$
30. $\frac{7}{y - 8} - \frac{6}{8 - y}$ $\frac{13}{y - 8}$
31. $\frac{a}{a - 4} - \frac{3}{4 - a}$ $\frac{a + 3}{a - 4}$
32. $\frac{m}{m^2 - 4} + \frac{2}{3m + 6}$ $\frac{5m - 4}{3(m + 2)(m - 2)}$
33. $\frac{y}{y + 3} - \frac{6y}{y^2 - 9}$ $\frac{y(y - 9)}{(y + 3)(y - 3)}$

482 Chapter 9 Rational Expressions and Equations

35. $\dfrac{-8d+20}{(d-4)(d+4)(d-2)}$

36. $\dfrac{-4h+15}{(h-4)(h-5)^2}$

37. $\dfrac{x^2-6}{(x+2)^2(x+3)}$

39. $\dfrac{2y^2+y-4}{(y-1)(y-2)}$

34. $\dfrac{5}{x^2-3x-28}+\dfrac{7}{2x-14}$ $\dfrac{7x+38}{2(x-7)(x+4)}$

35. $\dfrac{d-4}{d^2+2d-8}-\dfrac{d+2}{d^2-16}$

36. $\dfrac{1}{h^2-9h+20}-\dfrac{5}{h^2-10h+25}$

37. $\dfrac{x}{x^2+5x+6}-\dfrac{2}{x^2+4x+4}$

★ 38. $\dfrac{m^2+n^2}{m^2-n^2}+\dfrac{m}{n-m}+\dfrac{n}{m+n}$ 0

39. $\dfrac{y+1}{y-1}+\dfrac{y+2}{y-2}+\dfrac{y}{y^2-3y+2}$

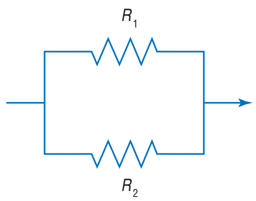
★ 40. $\dfrac{\frac{1}{b+2}+\frac{1}{b-5}}{\frac{2b^2-b-3}{b^2-3b-10}}$ $\dfrac{1}{b+1}$

41. $\dfrac{(x+y)\left(\frac{1}{x}-\frac{1}{y}\right)}{(x-y)\left(\frac{1}{x}+\frac{1}{y}\right)}$ -1

★ 42. Write $\left(\dfrac{2s}{2s+1}-1\right)\div\left(1+\dfrac{2s}{1-2s}\right)$ in simplest form. $\dfrac{2s-1}{2s+1}$

★ 43. What is the simplest form of $\left(3+\dfrac{5}{a+2}\right)\div\left(3-\dfrac{10}{a+7}\right)$? $\dfrac{a+7}{a+2}$

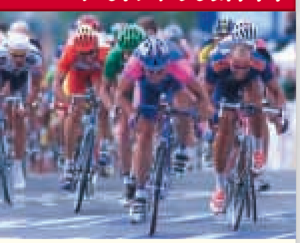
ELECTRICITY **For Exercises 44 and 45, use the the following information.**
In an electrical circuit, if two resistors with resistance R_1 and R_2 are connected in parallel as shown, the relationship between these resistances and the resulting combination resistance R is $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$.



44. If R_1 is x ohms and R_2 is 4 ohms less than twice x ohms, write an expression for $\dfrac{1}{R}$. $\dfrac{3x-4}{2x(x-2)}$

45. Find the effective resistance of a 30-ohm resistor and a 20-ohm resistor that are connected in parallel. 12 ohms

More About. . .



Bicycling

The Tour de France is the most popular bicycle road race. It lasts 24 days and covers 2500 miles.

Source: *World Book Encyclopedia*

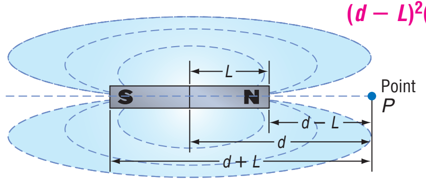
BICYCLING **For Exercises 46–48, use the following information.**
Jalisa is competing in a 48-mile bicycle race. She travels half the distance at one rate. The rest of the distance, she travels 4 miles per hour slower.

46. If x represents the faster pace in miles per hour, write an expression that represents the time spent at that pace. $\dfrac{24}{x}$ h

47. Write an expression for the amount of time spent at the slower pace. $\dfrac{24}{x-4}$ h

48. Write an expression for the amount of time Jalisa needed to complete the race. $\dfrac{48(x-2)}{x(x-4)}$ h

49. **MAGNETS** For a bar magnet, the magnetic field strength H at a point P along the axis of the magnet is $H=\dfrac{m}{2L(d-L)^2}-\dfrac{m}{2L(d+L)^2}$. Write a simpler expression for H. $\dfrac{2md}{(d-L)^2(d+L)^2}$ or $\dfrac{2md}{(d^2-L^2)^2}$



50. **CRITICAL THINKING** Find two rational expressions whose sum is $\dfrac{2x-1}{(x+1)(x-2)}$. Sample answer: $\dfrac{1}{x+1}$, $\dfrac{1}{x-2}$

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 523 (shown) and p. 524

LCM of Polynomials To find the least common multiple of two or more polynomials, factor each expression. The LCM contains each factor the greatest number of times it appears as a factor.

Example 1 **Find the LCM of $16p^2q^3r$, $40pq^4r^2$, and $15p^3r^4$.**

$16p^2q^3r = 2^4 \cdot p^2 \cdot q^3 \cdot r$
$40pq^4r^2 = 2^3 \cdot 5 \cdot p \cdot q^4 \cdot r^2$
$15p^3r^4 = 3 \cdot 5 \cdot p^3 \cdot r^4$
$\text{LCM} = 2^4 \cdot 3 \cdot 5 \cdot p^3 \cdot q^4 \cdot r^4$
$= 240p^3q^4r^4$

Example 2 **Find the LCM of $3m^2 - 3m - 6$ and $4m^2 + 12m - 40$.**

$3m^2 - 3m - 6 = 3(m+1)(m-2)$
$4m^2 + 12m - 40 = 4(m-2)(m+5)$
$\text{LCM} = 12(m+1)(m-2)(m+5)$

Exercises

Find the LCM of each set of polynomials.

1. $14ab^2, 42bc^3, 18a^2c$ $126a^2b^2c^3$
2. $8cdf^3, 28c^2f, 35d^4f^2$ $280c^2d^4f^3$
3. $65x^4y, 10x^2y^2, 26y^4$ $130x^4y^4$
4. $11mn^5, 18m^2n^3, 20mn^4$ $1980m^2n^5$
5. $15a^4b, 50a^2b^2, 40b^8$ $600a^4b^8$
6. $24p^7q, 30p^2q^2, 45pq^3$ $360p^7q^3$
7. $39b^2c^2, 52b^4c, 12c^3$ $156b^4c^3$
8. $12xy^4, 42x^2y, 30x^2y^3$ $420x^2y^4$
9. $56stv^2, 24s^2v^2, 70t^3v^3$ $840s^2t^3v^3$
10. $x^2+3x, 10x^2+25x-15$ $5x(x+3)(2x-1)$
11. $9x^2-12x+4, 3x^2+10x-8$ $(3x-2)^2(x+4)$
12. $22x^2+66x-220, 4x^2-16$ $44(x-2)(x+2)(x+5)$
13. $8x^2-36x-20, 2x^2+2x-60$ $4(x-5)(x+6)(2x+1)$
14. $5x^2-125, 5x^2+24x-5$ $5(x-5)(x+5)(5x-1)$
15. $3x^2-18x+27, 2x^3-4x^2-6x$ $6x(x-3)^2(x+1)$
16. $45x^2-6x-3, 45x^2-5$ $15(5x+1)(3x-1)(3x+1)$
17. x^3+4x^2-x-4, x^2+2x-3 $(x-1)(x+1)(x+3)(x+4)$
18. $54x^3-24x, 12x^2-26x+12$ $6x(3x+2)(3x-2)(2x-3)$

Skills Practice, p. 525 and Practice, p. 526 (shown)

Find the LCM of each set of polynomials.

1. x^2y, xy^3 x^2y^3
2. a^2b^3c, abc^4 $a^2b^3c^4$
3. $x+1, x+3$ $(x+1)(x+3)$
4. $g-1, g^2+3g-4$ $(g-1)(g+4)$
5. $2r+2, r^2+r, r+1$ $2r(r+1)$
6. $3, 4w+2, 4w^2-1$ $6(2w+1)(2w-1)$
7. $x^2+2x-8, x+4$ $(x+4)(x-2)$
8. x^2-x-6, x^2+6x+8 $(x+2)(x+4)(x-3)$
9. $d^2+6d+9, 2(d^2-9)$ $2(d-3)(d+3)^2$

Simplify each expression.

10. $\dfrac{5}{6ab}-\dfrac{7}{8a}$ $\dfrac{20-21b}{24ab}$
11. $\dfrac{5}{12x^4y}-\dfrac{1}{5x^2y^3}$ $\dfrac{25y^2-12x^2}{60x^4y^3}$
12. $\dfrac{1}{6c^2d}+\dfrac{3}{4cd^3}$ $\dfrac{2d^2+9c}{12c^2d^3}$
13. $\dfrac{4m}{3mn}+2$ $\dfrac{2(2+3n)}{3n}$
14. $2x-5-\dfrac{x-8}{x+4}$ $\dfrac{2(x+3)(x-2)}{x+4}$
15. $\dfrac{4}{a-3}+\dfrac{9}{a-5}$ $\dfrac{13a-47}{(a-3)(a-5)}$
16. $\dfrac{16}{x^2-16}+\dfrac{2}{x+4}$ $\dfrac{2}{x-4}$
17. $\dfrac{2-5m}{m-9}+\dfrac{4m-5}{9-m}$ $\dfrac{7-9m}{m-9}$
18. $\dfrac{y-5}{y^2-3y-10}+\dfrac{y}{y^2+y-2}$ $\dfrac{2y-1}{(y+2)(y-1)}$
19. $\dfrac{5}{2x-12}-\dfrac{20}{x^2-4x-12}$ $\dfrac{5}{2(x+2)}$
20. $\dfrac{2p-3}{p^2-5p+6}-\dfrac{5}{p^2-9}$ $\dfrac{2p^2-2p+1}{(p-2)(p+3)(p-3)}$
21. $\dfrac{1}{5n}-\dfrac{3}{4}+\dfrac{7}{10n}$ $\dfrac{3(6-5n)}{20n}$
22. $\dfrac{2a}{a-3}-\dfrac{2a}{a+3}+\dfrac{36}{a^2-9}$ $\dfrac{12}{a-3}$
23. $\dfrac{\frac{2}{x-y}+\frac{1}{x+y}}{\frac{1}{x-y}}$ $\dfrac{3x+y}{x+y}$
24. $\dfrac{\frac{r+6}{r}-\frac{1}{r+2}}{\frac{r^2+4r+3}{r^2+2r}}$ $\dfrac{r+4}{r+1}$

25. **GEOMETRY** The expressions $\dfrac{5x}{2}$, $\dfrac{20}{x+4}$, and $\dfrac{10}{x-4}$ represent the lengths of the sides of a triangle. Write a simplified expression for the perimeter of the triangle. $\dfrac{5(x^3-4x-16)}{2(x-4)(x+4)}$

26. **KAYAKING** Mai is kayaking on a river that has a current of 2 miles per hour. If r represents her rate in calm water, then $r+2$ represents her rate with the current, and $r-2$ represents her rate against the current. Mai kayaks 2 miles downstream and then back to her starting point. Use the formula for time, $t=\dfrac{d}{r}$, where d is the distance, to write a simplified expression for the total time it takes Mai to complete the trip. $\dfrac{4r}{(r+2)(r-2)}$ h

Reading to Learn Mathematics, p. 527 ELL

Pre-Activity **How is subtraction of rational expressions used in photography?**

Read the introduction to Lesson 9-2 at the top of page 479 in your textbook.

A person is standing 5 feet from a camera that has a lens with a focal length of 3 feet. Write an equation that you could solve to find how far the film should be from the lens to get a perfectly focused photograph.

$\dfrac{1}{q}=\dfrac{1}{3}-\dfrac{1}{5}$

Reading the Lesson

1. a. In work with rational expressions, LCD stands for least common denominator and LCM stands for least common multiple. The LCD is the LCM of the denominators.

b. To find the LCM of two or more numbers or polynomials, factor each number or polynomial. The LCM contains each factor the greatest number of times it appears as a factor.

2. To add $\dfrac{x^2-3}{x^2-5x+6}$ and $\dfrac{x-4}{x^3-4x^2+4x}$, you should first factor the denominator of each fraction. Then use the factorizations to find the LCM of x^2-5x+6 and x^3-4x^2+4x. This is the LCD for the two fractions.

3. When you add or subtract fractions, you often need to rewrite the fractions as equivalent fractions. You do this so that the resulting equivalent fractions will each have a denominator equal to the LCD of the original fractions.

4. To add or subtract two fractions that have the same denominator, you add or subtract their numerators and keep the same denominator.

5. The sum or difference of two rational expressions should be written as a polynomial or as a fraction in simplest form.

Helping You Remember

6. Some students have trouble remembering whether a common denominator is needed to add and subtract rational expressions or to multiply and divide them. How can your knowledge of working with fractions in arithmetic help you remember this?

Sample answer: In arithmetic, a common denominator is needed to add and subtract fractions, but not to multiply and divide them. The situation is the same for rational expressions.

Enrichment, p. 528

Superellipses

The circle and the ellipse are members of an interesting family of curves that were first studied by the French physicist and mathematician Gabriel Lamé (1795–1870). The general equation for the family is

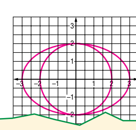
$\left|\dfrac{x}{a}\right|^n+\left|\dfrac{y}{b}\right|^n=1$, with $a \neq 0$, $b \neq 0$, and $n > 0$.

For even values of n greater than 2, the curves are called **superellipses.**

1. Consider two curves that are *not* superellipses. Graph each equation on the grid at the right. State the type of curve produced each time.

a. $\left|\dfrac{x}{2}\right|^2+\left|\dfrac{y}{2}\right|^2=1$ circle

b. $\left|\dfrac{x}{3}\right|^2+\left|\dfrac{y}{2}\right|^2=1$ ellipse



4 Assess

Open-Ended Assessment

Writing Have students write their own problems of the types in this lesson by beginning with an answer and working backward to create a problem.

Tips for New Teachers

Intervention The skills for combining and simplifying done in this lesson are used extensively in algebra. Take time to clear up student errors and misconceptions before proceeding.

Getting Ready for Lesson 9-3

PREREQUISITE SKILL Students will graph rational functions using asymptotes in Lesson 9-3. In previous course material, students graphed hyperbolas by using asymptotes and will apply these skills to graphing rational functions. Use Exercises 59–61 to determine your students' familiarity with graphing hyperbolas.

Assessment Options

Practice Quiz 1 The quiz provides students with a brief review of the concepts and skills in Lessons 9-1 and 9-2. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

Quiz (Lessons 9-1 and 9-2) is available on p. 567 of the *Chapter 9 Resource Masters*.

---

51. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See pp. 519A–519D.**

How is subtraction of rational expressions used in photography?
Include the following in your answer:
- an explanation of how to subtract rational expressions, and
- an equation that could be used to find the distance between the lens and the film if the focal length of the lens is 10 centimeters and the distance between the lens and the object is 60 centimeters.

Standardized Test Practice

52. For all $t \neq 5$, $\frac{t^2 - 25}{3t - 15} =$ **B**

(A) $\frac{t-5}{3}$. (B) $\frac{t+5}{3}$. (C) $t - 5$. (D) $t + 5$. (E) $\frac{t-5}{t-3}$.

53. What is the sum of $\frac{x-y}{5}$ and $\frac{x+y}{4}$? **C**

(A) $\frac{9x+9y}{20}$ (B) $\frac{x+9y}{20}$ (C) $\frac{9x+y}{20}$ (D) $\frac{9x-y}{20}$ (E) $\frac{x-9y}{20}$

Maintain Your Skills

Mixed Review **Simplify each expression.** *(Lesson 9-1)*

54. $\frac{9x^2y^3}{(5xyz)^2} \div \frac{(3xy)^3}{20x^2y}$ **$\frac{4}{15xyz^2}$**

55. $\frac{5a^2 - 20}{2a + 2} \cdot \frac{4a}{10a - 20}$ **$\frac{a(a+2)}{a+1}$**

Solve each system of inequalities by graphing. *(Lesson 8-7)* **56–57. See pp. 519A–519D.**

56. $9x^2 + y^2 < 81$
$x^2 + y^2 \geq 16$

57. $(y - 3)^2 \geq x + 2$
$x^2 \leq y + 4$

58. **GARDENS** Helene Jonson has a rectangular garden 25 feet by 50 feet. She wants to increase the garden on all sides by an equal amount. If the area of the garden is to be increased by 400 square feet, by how much should each dimension be increased? *(Lesson 6-4)* **2.5 ft**

Getting Ready for the Next Lesson **PREREQUISITE SKILL Draw the asymptotes and graph each hyperbola.**
*(To review **graphing hyperbolas**, see Lesson 8-5.)* **59–61. See pp. 519A–519D.**

59. $\frac{x^2}{16} - \frac{y^2}{20} = 1$

60. $\frac{y^2}{49} - \frac{x^2}{25} = 1$

61. $\frac{(x+2)^2}{16} - \frac{(y-5)^2}{25} = 1$

Practice Quiz 1 — *Lessons 9-1 and 9-2*

Simplify each expression. *(Lesson 9-1)*

1. $\frac{t^2 - t - 6}{t^2 - 6t + 9}$ **$\frac{t+2}{t-3}$**

2. $\frac{3ab^3}{8a^2b} \cdot \frac{4ac}{9b^4}$ **$\frac{c}{6b^2}$**

3. $-\frac{4}{8x} \div \frac{16}{xy^2}$ **$-\frac{y^2}{32}$**

4. $\frac{48}{6a + 42} \cdot \frac{7a + 49}{16}$ **$\frac{7}{2}$**

5. $\frac{w^2 + 5w + 4}{6} \div \frac{w + 1}{18w + 24}$ **$(w + 4)(3w + 4)$**

6. $\frac{\frac{x^2 + x}{x + 1}}{\frac{x}{x - 1}}$ **$x - 1$**

Simplify each expression. *(Lesson 9-2)*

7. $\frac{4a + 2}{a + b} + \frac{1}{-b - a}$ **$\frac{4a+1}{a+b}$**

8. $\frac{2x}{5ab^3} + \frac{4y}{3a^2b^2}$ **$\frac{6ax + 20by}{15a^2b^3}$**

9. $\frac{5}{n + 6} - \frac{4}{n - 1}$ **$\frac{n - 29}{(n + 6)(n - 1)}$**

10. $\frac{x - 5}{2x - 6} - \frac{x - 7}{4x - 12}$ **$\frac{1}{4}$**

484 Chapter 9 Rational Expressions and Equations

9-3 Graphing Rational Functions

What You'll Learn

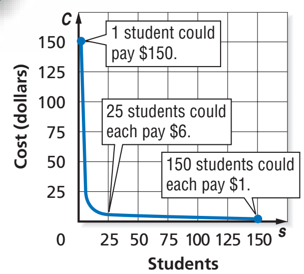
- Determine the vertical asymptotes and the point discontinuity for the graphs of rational functions.
- Graph rational functions.

Vocabulary

- rational function
- continuity
- asymptote
- point discontinuity

How can rational functions be used when buying a group gift?

A group of students want to get their favorite teacher, Mr. Salgado, a retirement gift. They plan to get him a gift certificate for a weekend package at a lodge in a state park. The certificate costs \$150. If c represents the cost for each student and s represents the number of students, then $c = \frac{150}{s}$.



VERTICAL ASYMPTOTES AND POINT DISCONTINUITY The function $c = \frac{150}{s}$ is an example of a rational function. A **rational function** is an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions and $q(x) \neq 0$. Here are other examples of rational functions.

$$f(x) = \frac{x}{x + 3} \qquad g(x) = \frac{5}{x - 6} \qquad h(x) = \frac{x + 4}{(x - 1)(x + 4)}$$

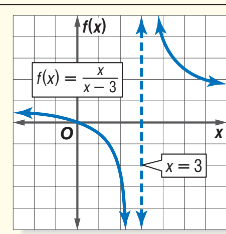
No denominator in a rational function can be zero because division by zero is not defined. In the examples above, the functions are not defined at $x = -3$, $x = 6$, and $x = 1$ and $x = -4$, respectively.

The graphs of rational functions may have breaks in **continuity**. This means that, unlike polynomial functions, which can be traced with a pencil never leaving the paper, not all rational functions are traceable. Breaks in continuity can appear as a vertical **asymptote** or as a **point discontinuity**. Recall that an asymptote is a line that the graph of the function approaches, but never crosses. Point discontinuity is like a hole in a graph.

Study Tip

Look Back
To review **asymptotes**, see Lesson 8-5.

Key Concept — *Vertical Asymptotes*

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.	For $f(x) = \frac{x}{x - 3}$, $x = 3$ is a vertical asymptote.	

9-3 Lesson Notes

1 Focus

5-Minute Check Transparency 9-3 Use as a quiz or review of Lesson 9-2.

Mathematical Background notes are available for this lesson on p. 470D.

Building on Prior Knowledge

In Chapter 7, students learned to graph polynomial equations. In this lesson, they will apply the same skills to graphing rational functions.

How can rational functions be used when buying a group gift?

Ask students:

- What does the cost for one student depend on? **the number of students who participate**
- What happens to the value of c as the value of s increases? **It decreases.**

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters

- Study Guide and Intervention, pp. 529–530
- Skills Practice, p. 531
- Practice, p. 532
- Reading to Learn Mathematics, p. 533
- Enrichment, p. 534
- Assessment, pp. 567, 569

Graphing Calculator and Spreadsheet Masters, p. 43

Teaching Algebra With Manipulatives Masters, p. 273

Transparencies

5-Minute Check Transparency 9-3
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

VERTICAL ASYMPTOTES AND POINT DISCONTINUITY

In-Class Example PowerPoint®

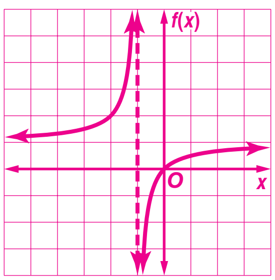
1 Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \dfrac{x^2 - 4}{x^2 + 5x + 6}$.

$x = -3$ is a vertical asymptote and $x = -2$ represents a hole in the graph.

GRAPH RATIONAL FUNCTIONS

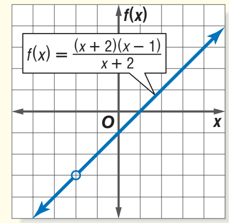
In-Class Example PowerPoint®

2 Graph $f(x) = \dfrac{x}{x + 1}$.



Teaching Tip Suggest that students choose a large unit on their grid paper and estimate points to the nearest tenth. Point out that they will probably not be able to see the shape of the graph as a whole unless they use a graphing calculator or computer program.

Key Concept *Point Discontinuity*

Property	Words	Example	Model
Point Discontinuity	If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.	$f(x) = \dfrac{(x + 2)(x - 1)}{x + 2}$ can be simplified to $f(x) = x - 1$. So, $x = -2$ represents a hole in the graph.	

Example 1 *Vertical Asymptotes and Point Discontinuity*

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \dfrac{x^2 - 1}{x^2 - 6x + 5}$.

First factor the numerator and denominator of the rational expression.

$$\frac{x^2 - 1}{x^2 - 6x + 5} = \frac{(x - 1)(x + 1)}{(x - 1)(x - 5)}$$

The function is undefined for $x = 1$ and $x = 5$. Since $\dfrac{\overset{1}{\cancel{(x - 1)}}(x + 1)}{\underset{1}{\cancel{(x - 1)}}(x - 5)} = \dfrac{x + 1}{x - 5}$,

$x = 5$ is a vertical asymptote, and $x = 1$ represents a hole in the graph.

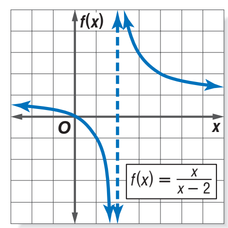
GRAPH RATIONAL FUNCTIONS You can use what you know about vertical asymptotes and point discontinuity to graph rational functions.

Example 2 *Graph with a Vertical Asymptote*

Graph $f(x) = \dfrac{x}{x - 2}$.

The function is undefined for $x = 2$. Since $\dfrac{x}{x - 2}$ is in simplest form, $x = 2$ is a vertical asymptote. Draw the vertical asymptote. Make a table of values. Plot the points and draw the graph.

x	f(x)
−50	0.96154
−30	0.9375
−20	0.90909
−10	0.83333
−2	0.5
−1	0.33333
0	0
1	−1
3	3
4	2
5	1.6667
10	1.25
20	1.1111
30	1.0714
50	1.0417



As $|x|$ increases, it appears that the y values of the function get closer and closer to 1. The line with the equation $f(x) = 1$ is a horizontal asymptote of the function.

Study Tip

Graphing Rational Functions
Finding the x- and y-intercepts is often useful when graphing rational functions.

DAILY INTERVENTION **Unlocking Misconceptions**

Asymptotes Students should understand that a graph continues to approach an asymptote and gets closer and closer to that value, but never reaches it. This is an abstract mathematical idea that cannot be represented accurately with any form of visual illustration.

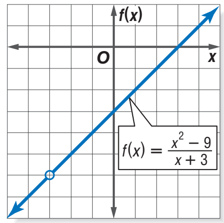
As you have learned, graphs of rational functions may have point discontinuity rather than vertical asymptotes. The graphs of these functions appear to have holes. These holes are usually shown as circles on graphs.

Example 3 Graph with Point Discontinuity

Graph $f(x) = \frac{x^2 - 9}{x + 3}$.

Notice that $\frac{x^2 - 9}{x + 3} = \frac{(x + 3)(x - 3)}{x + 3}$ or $x - 3$.

Therefore, the graph of $f(x) = \frac{x^2 - 9}{x + 3}$ is the graph of $f(x) = x - 3$ with a hole at $x = -3$.



Many real-life situations can be described by using rational functions.

Algebra Activity

Rational Functions

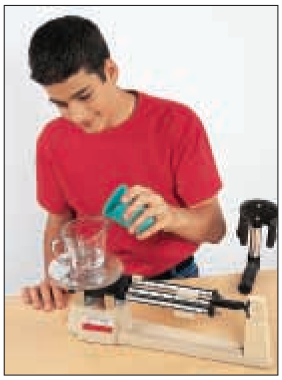
The density of a material can be expressed as $D = \frac{m}{V}$, where m is the mass of the material in grams and V is the volume in cubic centimeters. By finding the volume and density of 200 grams of each liquid, you can draw a graph of the function $D = \frac{200}{V}$.

Collect the Data

- Use a balance and metric measuring cups to find the volumes of 200 grams of different liquids such as water, cooking oil, isopropyl alcohol, sugar water, and salt water.
- Use $D = \frac{m}{V}$ to find the density of each liquid.

Analyze the Data

1. Graph the data by plotting the points (volume, density) on a graph. Connect the points. See pp. 519A–519D.
2. From the graph, find the asymptotes. $x = 0$, $y = 0$

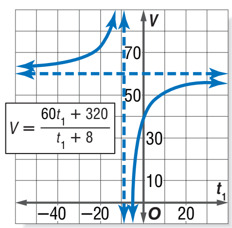


In the real world, sometimes values on the graph of a rational function are not meaningful.

Example 4 Use Graphs of Rational Functions

TRANSPORTATION A train travels at one velocity V_1 for a given amount of time t_1 and then another velocity V_2 for a different amount of time t_2. The average velocity is given by $V = \frac{V_1 t_1 + V_2 t_2}{t_1 + t_2}$.

a. Let t_1 be the independent variable and let V be the dependent variable. Draw the graph if $V_1 = 60$ miles per hour, $V_2 = 40$ miles per hour, and $t_2 = 8$ hours.



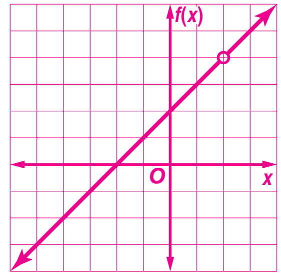
The function is $V = \frac{60t_1 + 40(8)}{t_1 + 8}$ or $V = \frac{60t_1 + 320}{t_1 + 8}$. The vertical asymptote is $t_1 = -8$. Graph the vertical asymptote and the function. Notice that the horizontal asymptote is $V = 60$.

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

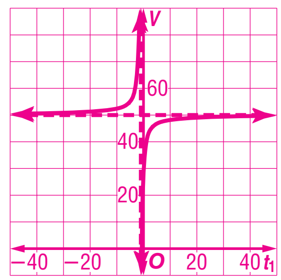
3 Graph $f(x) = \frac{x^2 - 4}{x - 2}$.



Teaching Tip Since the discontinuity is only one point (and a mathematical point has no dimensions), suggest that students draw a circle on their graphs to indicate the discontinuity.

4 **TRANSPORTATION** Use the situation and formula given in Example 4.

a. Draw the graph if $V_1 = 50$ miles per hour, $V_2 = 30$ miles per hour, and $t_2 = 1$ hour.



b. What is the V-intercept of the graph? **30**

c. What values of t_1 and V are meaningful in the context of the problem? **Positive values of t_1 and values of V between 30 and 50 are meaningful.**

Tips for New Teachers

Intervention To make sure the situation in Example 4 is meaningful, ask a student to explain the situation as a story without using letter names for variables. For example, the story might begin "A train travels 40 miles per hour as it goes through towns. Eight hours of its total trip are spent going through towns."

Algebra Activity

Materials: balance, metric measuring cups, different liquids, graph paper

- Choose liquids that are quite different in density. If you make sugar or salt water, dissolve as much of the substance as you can in it.
- Differences in volume for each 200 grams will be easier to read if you use the smallest measuring cup that holds the amount.

3 Practice/Apply

b. **What is the V-intercept of the graph?**
The V-intercept is 40.

c. **What values of t_1 and V are meaningful in the context of the problem?**
In the problem context, time and velocity are positive values. Therefore, only values of t_1 greater than 0 and values of V between 40 and 60 are meaningful.

Check for Understanding

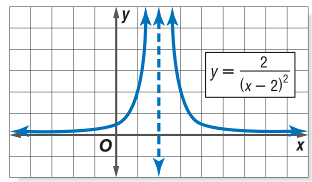
Concept Check

1. **OPEN ENDED** Write a function whose graph has two vertical asymptotes located at $x = -5$ and $x = 2$.

1. Sample answer: $f(x) = \frac{1}{(x+5)(x-2)}$

2. **Compare and contrast** the graphs of $f(x) = \frac{(x-1)(x+5)}{x-1}$ and $g(x) = x + 5$. **See margin.**

3. **Describe** the graph at the right. Include the equations of any asymptotes, the x values of any holes, and the x- and y-intercepts.
$x = 2$ and $y = 0$ are asymptotes of the graph. The y-intercept is 0.5 and there is no x-intercept because $y = 0$ is an asymptote.



Guided Practice

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

4. $f(x) = \frac{3}{x^2 - 4x + 4}$ **asymptote: $x = 2$**

5. $f(x) = \frac{x-1}{x^2 + 4x - 5}$ **asymptote: $x = -5$; hole: $x = 1$**

Graph each rational function. **6–11. See pp. 519A–519D.**

6. $f(x) = \frac{x}{x+1}$

7. $f(x) = \frac{6}{(x-2)(x+3)}$

8. $f(x) = \frac{x^2 - 25}{x - 5}$

9. $f(x) = \frac{x-5}{x+1}$

10. $f(x) = \frac{4}{(x-1)^2}$

11. $f(x) = \frac{x+2}{x^2 - x - 6}$

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6–11	2, 3
12–15	4

Application

MEDICINE **For Exercises 12–15, use the following information.**
For certain medicines, health care professionals may use Young's Rule, $C = \frac{y}{y+12} \cdot D$, to estimate the proper dosage for a child when the adult dosage is known. In this equation, C represents the child's dose, D represents the adult dose, and y represents the child's age in years.

12. Use Young's Rule to estimate the dosage of amoxicillin for an eight-year-old child if the adult dosage is 250 milligrams. **100 mg**

13. Graph $C = \frac{y}{y+12}$. **See pp. 519A–519D.**

14. Give the equations of any asymptotes and y- and C-intercepts of the graph.
$y = -12$, $C = 1$; 0; 0

15. What values of y and C are meaningful in the context of the problem?
$y > 0$ and $0 < C < 1$

Study Notebook

Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 9.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
- **Vertical Asymptotes and Point Discontinuity:** 16–21
- **Graph Rational Functions:** 22–45, 47–50

Odd/Even Assignments
Exercises 16–39 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide
Basic: 17–39 odd, 46, 51–66

Average: 17–39 odd, 40–42, 46–66

Advanced: 16–38 even, 40–62 (optional: 63–66)

Answer

2. **Each of the graphs is a straight line passing through $(-5, 0)$ and $(0, 5)$. However, the graph of $f(x) = \frac{(x-1)(x+5)}{x-1}$ has a hole at $(1, 6)$, and the graph of $g(x) = x + 5$ does not have a hole.**

DAILY INTERVENTION

Differentiated Instruction

Visual/Spatial Have students graph one of the examples from the lesson with colors on a large sheet of posterboard, to clearly show how a graph approaches but never reaches an asymptote or how a graph may have a hole in it for a certain value of the variable. Display the results in the classroom.

488 Chapter 9 Rational Expressions and Equations

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–39	2, 3
40–50	4

Extra Practice

See page 849.

19. asymptote: $x = -1$; hole: $x = 5$

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

16. $f(x) = \frac{2}{x^2 - 5x + 6}$ **asymptotes: $x = 2$, $x = 3$**

17. $f(x) = \frac{4}{x^2 + 2x - 8}$ **asymptotes: $x = -4$, $x = 2$**

18. $f(x) = \frac{x + 3}{x^2 + 7x + 12}$ **asymptote: $x = -4$; hole: $x = -3$**

19. $f(x) = \frac{x - 5}{x^2 - 4x - 5}$

20. $f(x) = \frac{x^2 - 8x + 16}{x - 4}$ **hole: $x = 4$**

21. $f(x) = \frac{x^2 - 3x + 2}{x - 1}$ **hole: $x = 1$**

Graph each rational function. **22–39. See pp. 519A–519D.**

22. $f(x) = \frac{1}{x}$
23. $f(x) = \frac{3}{x}$
24. $f(x) = \frac{1}{x + 2}$
25. $f(x) = \frac{-5}{x + 1}$
26. $f(x) = \frac{x}{x - 3}$
27. $f(x) = \frac{5x}{x + 1}$
28. $f(x) = \frac{-3}{(x - 2)^2}$
29. $f(x) = \frac{1}{(x + 3)^2}$
30. $f(x) = \frac{x + 4}{x - 1}$
31. $f(x) = \frac{x - 1}{x - 3}$
32. $f(x) = \frac{x^2 - 36}{x + 6}$
33. $f(x) = \frac{x^2 - 1}{x - 1}$
34. $f(x) = \frac{3}{(x - 1)(x + 5)}$
35. $f(x) = \frac{-1}{(x + 2)(x - 3)}$
36. $f(x) = \frac{x}{x^2 - 1}$
37. $f(x) = \frac{x - 1}{x^2 - 4}$
38. $f(x) = \frac{6}{(x - 6)^2}$
39. $f(x) = \frac{1}{(x + 2)^2}$

HISTORY For Exercises 40–42, use the following information.
In Maria Gaetana Agnesi's book *Analytical Institutions*, Agnesi discussed the characteristics of the equation $x^2y = a^2(a - y)$, whose graph is called the "curve of Agnesi." This equation can be expressed as $y = \frac{a^3}{x^2 + a^2}$.

40. Graph $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = 4$. **See pp. 519A–519D.**

41. Describe the graph.

42. **Make a conjecture** about the shape of the graph of $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = -4$. Explain your reasoning. **See pp. 519A–519D.**

41. The graph is bell-shaped with a horizontal asymptote at $f(x) = 0$.

More About. . .

History

Mathematician Maria Gaetana Agnesi was one of the greatest scholars of all time. Born in Milan, Italy, in 1718, she mastered Greek, Hebrew, and several modern languages by the age of 11.

Source: *A History of Mathematics*

AUTO SAFETY For Exercises 43–45, use the following information.
When a car has a front-end collision, the objects in the car (including passengers) keep moving forward until the impact occurs. After impact, objects are repelled. Seat belts and airbags limit how far you are jolted forward. The formula for the velocity at which you are thrown backward is $V_f = \frac{(m_1 - m_2)v_i}{m_1 + m_2}$, where m_1 and m_2 are masses of the two objects meeting and v_i is the initial velocity. **43. See pp. 519A–519D.**

43. Let m_1 be the independent variable, and let V_f be the dependent variable. Graph the function if $m_2 = 7$ kilograms and $v_i = 5$ meters per second.

44. Give the equation of the vertical asymptote and the m_1- and V_f-intercepts of the graph. **$m_1 = -7$; 7; −5**

45. Find the value of V_f when the value of m_1 is 5 kilograms. **about −0.83 m/s**

46. Sample answers: $f(x) = \frac{x + 2}{(x + 2)(x - 3)}$, $f(x) = \frac{2(x + 2)}{(x + 2)(x - 3)}$, $f(x) = \frac{5(x + 2)}{(x + 2)(x - 3)}$

46. **CRITICAL THINKING** Write three rational functions that have a vertical asymptote at $x = 3$ and a hole at $x = -2$.

www.algebra2.com/self_check_quiz

Study Guide and Intervention, p. 529 (shown) and p. 530

Vertical Asymptotes and Point Discontinuity

Rational Function	an equation of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial expressions and $q(x) \neq 0$
Vertical Asymptote of the Graph of a Rational Function	An asymptote is a line that the graph of a function approaches, but never crosses. If the simplified form of the related rational expression is undefined for $x = a$, then $x = a$ is a vertical asymptote.
Point Discontinuity of the Graph of a Rational Function	Point discontinuity is like a hole in a graph. If the original related expression is undefined for $x = a$ but the simplified expression is defined for $x = a$, then there is a hole in the graph at $x = a$.

Example **Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of $f(x) = \frac{4x^2 + x - 3}{x^2 - 1}$.**

First factor the numerator and the denominator of the rational expression.

$f(x) = \frac{4x^2 + x - 3}{x^2 - 1} = \frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)}$

The function is undefined for $x = 1$ and $x = -1$.

Since $\frac{(4x - 3)(x + 1)}{(x + 1)(x - 1)} = \frac{4x - 3}{x - 1}$, $x = 1$ is a vertical asymptote. The simplified expression is defined for $x = -1$, so this value represents a hole in the graph.

Exercises

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

1. $f(x) = \frac{4}{x^2 + 3x - 10}$ **asymptotes: $x = 2$, $x = -5$**
2. $f(x) = \frac{2x^2 - x - 10}{2x - 5}$ **hole: $x = \frac{5}{2}$**
3. $f(x) = \frac{x^2 - x - 12}{x^2 - 4x}$ **asymptote: $x = 0$; hole $x = 4$**
4. $f(x) = \frac{3x - 1}{3x^2 + 5x - 2}$ **asymptote: $x = -2$; hole: $x = \frac{1}{3}$**
5. $f(x) = \frac{x^2 - 6x - 7}{x^2 + 6x - 7}$ **asymptotes: $x = 1$, $x = -7$**
6. $f(x) = \frac{3x^2 - 5x - 2}{x + 3}$ **asymptote: $x = -3$**
7. $f(x) = \frac{x + 1}{x^2 - 6x + 5}$ **asymptotes: $x = 1$, $x = 5$**
8. $f(x) = \frac{2x^2 - x - 3}{2x^2 + 3x - 9}$ **asymptote: $x = -3$; hole: $x = \frac{3}{2}$**
9. $f(x) = \frac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3}$ **holes: $x = 1$, $x = 3$**

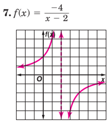
Skills Practice, p. 531 and Practice, p. 532 (shown)

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

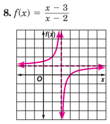
1. $f(x) = \frac{6}{x^2 + 3x - 10}$ **asymptotes: $x = 2$, $x = -5$**
2. $f(x) = \frac{x - 7}{x^2 - 10x + 21}$ **asymptote: $x = 3$; hole: $x = 7$**
3. $f(x) = \frac{x - 2}{x^2 + 4x + 4}$ **asymptote: $x = -2$**
4. $f(x) = \frac{x^2 - 100}{x + 10}$ **hole: $x = -10$**
5. $f(x) = \frac{x^2 - 2x - 24}{x - 6}$ **hole: $x = 6$**
6. $f(x) = \frac{x^2 + 9x + 20}{x + 5}$ **hole: $x = -5$**

Graph each rational function.

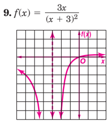
7. $f(x) = \frac{-4}{x - 2}$



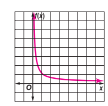
8. $f(x) = \frac{x - 3}{x - 2}$



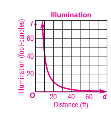
9. $f(x) = \frac{3x}{(x + 3)^2}$



10. **PAINTING** Working alone, Tawa can give the shed a coat of paint in 6 hours. It takes her father x hours working alone to give the shed a coat of paint. The equation $f(x) = \frac{6 + x}{6x}$ describes the portion of the job Tawa and her father working together can complete in 1 hour. Graph $f(x) = \frac{6 + x}{6x}$ for $x \geq 0$, $y \geq 0$. If Tawa's father can complete the job in 4 hours alone, what portion of the job can they complete together in 1 hour? **$\frac{5}{12}$**



11. **LIGHT** The relationship between the illumination an object receives from a light source of I foot-candles and the square of the distance d in feet of the object from the source can be modeled by $I(d) = \frac{4500}{d^2}$. Graph the function $I(d) = \frac{4500}{d^2}$ for $0 \leq I \leq 80$ and $0 \leq d \leq 80$. What is the illumination in foot-candles that the object receives at a distance of 20 feet from the light source? **11.25 foot-candles**



Reading to Learn Mathematics, p. 533 ELL

Pre-Activity **How can rational functions be used when buying a group gift?**

Read the introduction to Lesson 9-3 at the top of page 485 in your textbook.

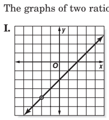
- If 15 students contribute to the gift, how much would each of them pay? **$10**
- If each student pays $5, how many students contributed? **30 students**

Reading the Lesson

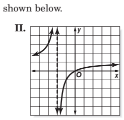
1. Which of the following are rational functions? **A and C**
   A. $f(x) = \frac{1}{x - 5}$ B. $g(x) = \sqrt{x}$ C. $h(x) = \frac{x^2 - 25}{x^2 + 6x + 9}$

2. a. Graphs of rational functions may have breaks in **continuity**. These may occur as vertical **asymptotes** or as point **discontinuities**.
   b. The graphs of two rational functions are shown below.

I.



II.



Graph I has a **point discontinuity** at $x =$ **−2**.
Graph II has a **vertical asymptote** at $x =$ **−2**.
Match each function with its graph above.
$f(x) = \frac{x}{x + 2}$ **II** $g(x) = \frac{x^2 - 4}{x + 2}$ **I**

Helping You Remember

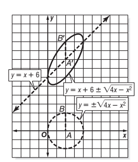
3. One way to remember something new is to see how it is related to something you already know. How can knowing that division by zero is undefined help you to remember how to find the places where a rational function has a point discontinuity or an asymptote?
Sample answer: A point discontinuity or vertical asymptote occurs where the function is undefined, that is, where the denominator of the related rational expression is equal to 0. Therefore, set the denominator equal to zero and solve for the variable.

Enrichment, p. 534

Graphing with Addition of y-Coordinates

Equations of parabolas, ellipses, and hyperbolas that are "tipped" with respect to the x- and y-axes are more difficult to graph than the equations you have been studying.

Often, however, you can use the graphs of two simpler equations to graph a more complicated equation. For example, the graph of the ellipse in the diagram at the right is obtained by adding the y-coordinate of each point on the circle and the y-coordinate of the corresponding point of the line.



Graph each equation. State the type of curve for each graph.

… 6 … $\sqrt{4 - x^2}$ **ellipse** 2. … $\sqrt{x}$ **parabola**

4 Assess

Open-Ended Assessment

Writing Have students write their own examples of rational functions and graph them, showing discontinuities.

Getting Ready for Lesson 9-4

BASIC SKILL Students will write and solve direct, joint, and inverse variation problems in Lesson 9-4. This will include students writing and solving proportions that relate the values in the variation. Use Exercises 63–66 to determine your students' familiarity with solving proportions.

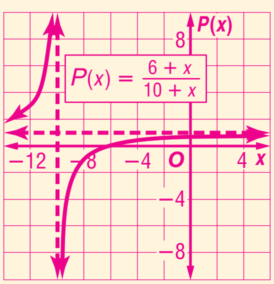
Assessment Options

Quiz (Lesson 9-3) is available on p. 567 of the *Chapter 9 Resource Masters*.

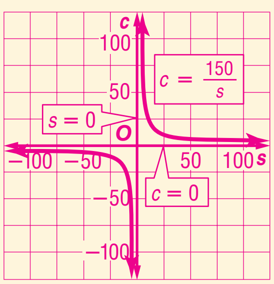
Mid-Chapter Test (Lessons 9-1 through 9-3) is available on p. 569 of the *Chapter 9 Resource Masters*.

Answers

47.



51. A rational function can be used to determine how much each person owes if the cost of the gift is known and the number of people sharing the cost is s. Answers should include the following.

- 

BASKETBALL **For Exercises 47–50, use the following information.**
Zonta plays basketball for Centerville High School. So far this season, she has made 6 out of 10 free throws. She is determined to improve her free-throw percentage. If she can make x consecutive free throws, her free-throw percentage can be determined using $P(x) = \frac{6 + x}{10 + x}$.

47. Graph the function. **See margin.**

48. What part of the graph is meaningful in the context of the problem? **48. the part in the first quadrant**

★ 49. Describe the meaning of the y-intercept. **49. It represents her original free-throw percentage of 60%.**

★ 50. What is the equation of the horizontal asymptote? Explain its meaning with respect to Zonta's shooting percentage. **50. $y = 1$; this represents 100% which she cannot achieve because she has already missed 4 free throws.**

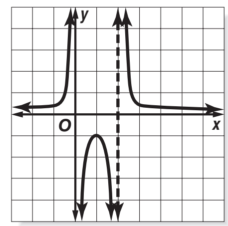
51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How can rational functions be used when buying a group gift?

Include the following in your answer:
- a complete graph of the function $c = \frac{150}{s}$ with asymptotes, and
- an explanation of why only part of the graph is meaningful in the context of the problem.

Standardized Test Practice

52. Which set is the domain of the function graphed at the right? **A**



(A) $\{x \mid x \neq 0, 2\}$
(B) $\{x \mid x \neq -2, 0\}$
(C) $\{x \mid x < 4\}$
(D) $\{x \mid x > -4\}$

53. Which set is the range of the function $y = \frac{x^2 + 8}{2}$? **B**

(A) $\{y \mid y \neq \pm 2\sqrt{2}\}$ (B) $\{y \mid y \geq 4\}$
(C) $\{y \mid y \geq 0\}$ (D) $\{y \mid y \leq 0\}$

Maintain Your Skills

Mixed Review

Simplify each expression. *(Lessons 9-2 and 9-1)*

54. $\frac{3m + 2}{m + n} + \frac{4}{2m + 2n}$ **54. $\frac{3m + 4}{m + n}$**

55. $\frac{5}{x + 3} - \frac{2}{x - 2}$ **55. $\frac{3x - 16}{(x + 3)(x - 2)}$**

56. $\frac{2w - 4}{w + 3} \div \frac{2w + 6}{5}$ **56. $\frac{5(w - 2)}{(w + 3)^2}$**

Find the coordinates of the center and the radius of the circle with the given equation. Then graph the circle. *(Lesson 8-3)* **57–58. See margin for graphs.**

57. $(x - 6)^2 + (y - 2)^2 = 25$ **(6, 2); 5**

58. $x^2 + y^2 + 4x = 9$ **(−2, 0); $\sqrt{13}$**

59. **ART** Joyce Jackson purchases works of art for an art gallery. Two years ago, she bought a painting for \$20,000, and last year, she bought one for \$35,000. If paintings appreciate 14% per year, how much are the two paintings worth now? *(Lesson 7-1)* **\$65,892**

Solve each equation by completing the square. *(Lesson 6-4)*

60. $x^2 + 8x + 20 = 0$ **$-4 \pm 2i$**

61. $x^2 + 2x - 120 = 0$ **−12, 10**

62. $x^2 + 7x - 17 = 0$ **$\frac{-7 \pm 3\sqrt{13}}{2}$**

Getting Ready for the Next Lesson

BASIC SKILL **Solve each proportion.**

63. $\frac{16}{v} = \frac{32}{9}$ **4.5**

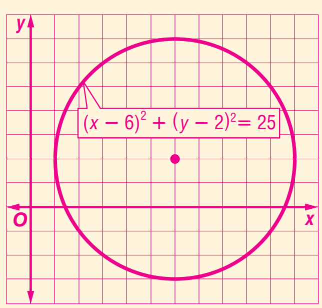
64. $\frac{7}{25} = \frac{a}{5}$ **1.4**

65. $\frac{6}{15} = \frac{8}{s}$ **20**

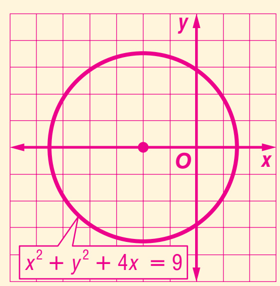
66. $\frac{b}{9} = \frac{40}{30}$ **12**

- Only the portion in the first quadrant is significant in the real world because there cannot be a negative number of people nor a negative amount of money owed for the gift.

57.



58.



490 Chapter 9 Rational Expressions and Equations

Graphing Calculator Investigation

A Follow-Up of Lesson 9-3

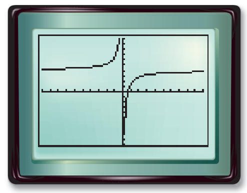
Graphing Rational Functions

A TI-83 Plus graphing calculator can be used to explore the graphs of rational functions. These graphs have some features that never appear in the graphs of polynomial functions.

Example 1 **Graph $y = \frac{8x - 5}{2x}$ in the standard viewing window. Find the equations of any asymptotes.**

- Enter the equation in the Y= list.

KEYSTROKES: [Y=] [(] 8 [X,T,θ,n] [–] 5 [)] [÷] [(] 2 [X,T,θ,n] [)] [ZOOM] 6



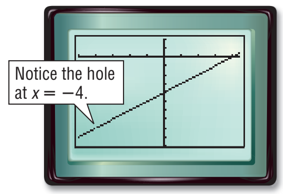
[−10, 10] scl: 1 by [−10, 10] scl: 1

By looking at the equation, we can determine that if $x = 0$, the function is undefined. The equation of the vertical asymptote is $x = 0$. Notice what happens to the y values as x grows larger and as x gets smaller. The y values approach 4. So, the equation for the horizontal asymptote is $y = 4$.

Example 2 **Graph $y = \frac{x^2 - 16}{x + 4}$ in the window [−5, 4.4] by [−10, 2] with scale factors of 1.**

- Because the function is not continuous, put the calculator in dot mode.

KEYSTROKES: [MODE] [▼] [▼] [▼] [▼] [▶] [ENTER]



[−5, 4.4] scl: 1 by [−10, 2] scl: 1

This graph looks like a line with a break in continuity at $x = -4$. This happens because the denominator is 0 when $x = -4$. Therefore, the function is undefined when $x = -4$.

If you TRACE along the graph, when you come to $x = -4$, you will see that there is no corresponding y value.

Exercises 1–6. See pp. 519A–519D for graphs.

Use a graphing calculator to graph each function. Be sure to show a complete graph. Draw the graph on a sheet of paper. Write the *x*-coordinates of any points of discontinuity and/or the equations of any asymptotes. 2. $x = -2$, $y = 1$ 4. $x = 2$, $y = \frac{2}{3}$ 5. $x = 1$, $y = 4$

1. $f(x) = \frac{1}{x}$ $x = 0$, $y = 0$
2. $f(x) = \frac{x}{x + 2}$
3. $f(x) = \frac{2}{x - 4}$ $x = 4$, $y = 0$
4. $f(x) = \frac{2x}{3x - 6}$
5. $f(x) = \frac{4x + 2}{x - 1}$
6. $f(x) = \frac{x^2 - 9}{x + 3}$ point discontinuity at $x = -3$
7. Which graph(s) has point discontinuity? 6
8. Describe functions that have point discontinuity. See margin.

www.algebra2.com/other_calculator_keystrokes

Graphing Calculator Investigation

A Follow-Up of Lesson 9-3

Getting Started

Graphing Window For the examples, students should use the settings shown below the diagrams. For all the exercises, a good window is [−10, 10] scl: 1 by [−10, 10] scl: 1.

Graph Style Students may find it instructive to experiment with the graph style. They can begin by using the usual line style. This is the style when the icon to the left of the equation on the Y= list is a backslash. The best alternate style to use is path style. The icon for this style is a small numeral 0 with a short minus sign attached to the left side of the 0.

Teach

Suggest that students try graphing Example 2 in Connected mode as well as Dot mode. Ask them which way makes it easier to see the discontinuity.

Assess

Ask: How can you use the graphing calculator to check the exact value where a discontinuity occurs? Use [TRACE] to find where there is no *y* value.

Answer

8. rational functions where a value of the function is not defined, but the rational expression in simplest form is defined for that value

9-4 Lesson Notes

1 Focus

5-Minute Check Transparency 9-4 Use as a quiz or review of Lesson 9-3.

Mathematical Background notes are available for this lesson on p. 470D.

How is variation used to find the total cost given the unit cost?

Ask students:

- If the number of students increases, what happens to the value of the total spending? **It increases.**
- If the number of students decreases, what happens to the value of the total spending? **It decreases.**

9-4 Direct, Joint, and Inverse Variation

What You'll Learn

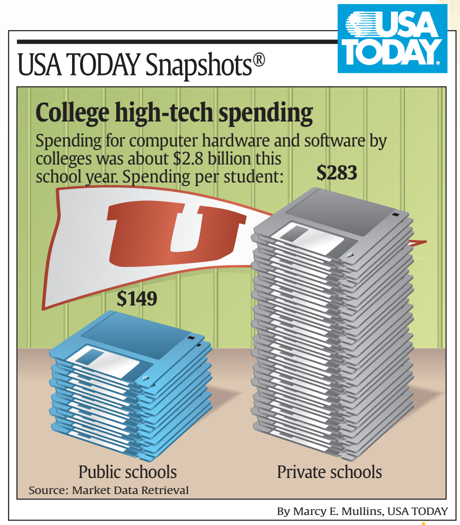
- Recognize and solve direct and joint variation problems.
- Recognize and solve inverse variation problems.

Vocabulary

- direct variation
- constant of variation
- joint variation
- inverse variation

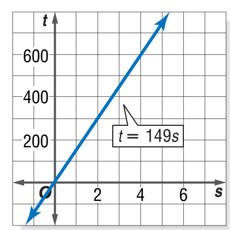
How is variation used to find the total cost given the unit cost?

The total high-tech spending t of an average public college can be found by using the equation $t = 149s$, where s is the number of students.



DIRECT VARIATION AND JOINT VARIATION The relationship given by $t = 149s$ is an example of direct variation. A **direct variation** can be expressed in the form $y = kx$. The k in this equation is a constant and is called the **constant of variation**.

Notice that the graph of $t = 149s$ is a straight line through the origin. An equation of a direct variation is a special case of an equation written in slope-intercept form, $y = mx + b$. When $m = k$ and $b = 0$, $y = mx + b$ becomes $y = kx$. So the slope of a direct variation equation is its constant of variation.



To express a direct variation, we say that y varies directly as x. In other words, as x increases, y increases or decreases at a constant rate.

Key Concept — Direct Variation

y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.

If you know that y varies directly as x and one set of values, you can use a proportion to find the other set of corresponding values.

$$y_1 = kx_1 \quad \text{and} \quad y_2 = kx_2$$

$$\frac{y_1}{x_1} = k \qquad \frac{y_2}{x_2} = k$$

Therefore, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$.

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters

- Study Guide and Intervention, pp. 535–536
- Skills Practice, p. 537
- Practice, p. 538
- Reading to Learn Mathematics, p. 539
- Enrichment, p. 540

Graphing Calculator and Spreadsheet Masters, p. 44

School-to-Career Masters, p. 18

Science and Mathematics Lab Manual, pp. 123–126

Transparencies

5-Minute Check Transparency 9-4
Real-World Transparency 9
Answer Key Transparencies

Technology

Interactive Chalkboard

Using the properties of equality, you can find many other proportions that relate these same x and y values.

Example 1 Direct Variation

If y varies directly as x and $y = 12$ when $x = -3$, find y when $x = 16$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1} = \frac{y_2}{x_2}$$ Direct proportion

$$\frac{12}{-3} = \frac{y_2}{16}$$ $y_1 = 12$, $x_1 = -3$, and $x_2 = 16$

$$16(12) = -3(y_2)$$ Cross multiply.

$$192 = -3y_2$$ Simplify.

$$-64 = y_2$$ Divide each side by -3.

When $x = 16$, the value of y is -64.

Another type of variation is joint variation. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities.

Key Concept — *Joint Variation*

y varies jointly as x and z if there is some number k such that $y = kxz$, where $k \neq 0$, $x \neq 0$, and $z \neq 0$.

If you know y varies jointly as x and z and one set of values, you can use a proportion to find the other set of corresponding values.

$$y_1 = kx_1z_1 \quad \text{and} \quad y_2 = kx_2z_2$$

$$\frac{y_1}{x_1z_1} = k \qquad \frac{y_2}{x_2z_2} = k$$

Therefore, $\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$.

Example 2 Joint Variation

Suppose y varies jointly as x and z. Find y when $x = 8$ and $z = 3$, if $y = 16$ when $z = 2$ and $x = 5$.

Use a proportion that relates the values.

$$\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$$ Joint variation

$$\frac{16}{5(2)} = \frac{y_2}{8(3)}$$ $y_1 = 16$, $x_1 = 5$, $z_1 = 2$, $x_2 = 8$, and $z_2 = 3$

$$8(3)(16) = 5(2)(y_2)$$ Cross multiply.

$$384 = 10y_2$$ Simplify.

$$38.4 = y_2$$ Divide each side by 10.

When $x = 8$ and $z = 3$, the value of y is 38.4.

INVERSE VARIATION Another type of variation is inverse variation. For two quantities with **inverse variation**, as one quantity increases, the other quantity decreases. For example, speed and time for a fixed distance vary inversely with each other. When you travel to a particular location, as your speed increases, the time it takes to arrive at that location decreases.

www.algebra2.com/extra_examples

2 Teach

DIRECT VARIATION AND JOINT VARIATION

In-Class Examples Power Point®

1. If y varies directly as x and $y = -15$ when $x = 5$, find y when $x = 3$. **-9**

2. Suppose y varies jointly as x and z. Find y when $x = 10$ and $z = 5$, if $y = 12$ when $z = 8$ and $x = 3$. **25**

Teaching Tip Discuss with students what happens to the constant of variation in the proportions used to solve these examples.

Online Lesson Plans

USA TODAY Education's Online site offers resources and interactive features connected to each day's newspaper. *Experience TODAY*, USA TODAY's daily lesson plan, is available on the site and delivered daily to subscribers. This plan provides instruction for integrating USA TODAY graphics and key editorial features into your mathematics classroom. Log on to www.education.usatoday.com.

INVERSE VARIATION

In-Class Examples

Power Point®

3 If a varies inversely as b and $a = -6$ when $b = 2$, find a when $b = -7$. $\frac{12}{7}$

4 **SPACE** The next closest planet to the Sun after Mercury is Venus, which is about 67 million miles away. How much larger would the diameter of the Sun appear on Venus than on Earth? **about 1.39 times as large as it appears from Earth**

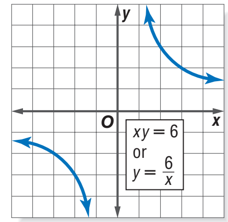
Teaching Tip To understand the situation in the problem, some students may find it useful to make a sketch showing the relative distances from the Sun to Earth, Mercury, and Venus.

Key Concept — *Inverse Variation*

y varies inversely as x if there is some nonzero constant k such that

$$xy = k \text{ or } y = \frac{k}{x}.$$

Suppose y varies inversely as x such that $xy = 6$ or $y = \frac{6}{x}$. The graph of this equation is shown at the right. Note that in this case, k is a positive value 6, so as the values of x increase, the values of y decrease.



Just as with direct variation and joint variation, a proportion can be used with inverse variation to solve problems where some quantities are known. The following proportion is only one of several that can be formed.

$x_1y_1 = k$ and $x_2y_2 = k$

$x_1y_1 = x_2y_2$ — Substitution Property of Equality

$\frac{x_1}{y_2} = \frac{x_2}{y_1}$ — Divide each side by y_1y_2.

TEACHING TIP
In Example 3, students may wish to solve the problem by using the equation $r_1t_1 = r_2t_2$.

Example 3 Inverse Variation

If r varies inversely as t and $r = 18$ when $t = -3$, find r when $t = -11$.

Use a proportion that relates the values.

$\frac{r_1}{t_2} = \frac{r_2}{t_1}$ — Inverse variation

$\frac{18}{-11} = \frac{r_2}{-3}$ — $r_1 = 18$, $t_1 = -3$, and $t_2 = -11$

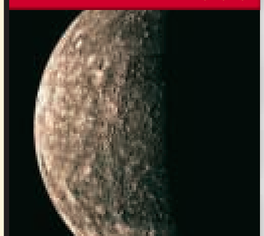
$18(-3) = -11(r_2)$ — Cross multiply.

$-54 = -11r_2$ — Simplify.

$4\frac{10}{11} = r_2$ — Divide each side by -11.

When $t = -11$, the value of r is $4\frac{10}{11}$.

More About. . .



Space
Mercury is about 36 million miles from the Sun, making it the closest planet to the Sun. Its proximity to the Sun causes its temperature to be as high as 800°F.
Source: *World Book Encyclopedia*

Example 4 Use Inverse Variation

SPACE The apparent length of an object is inversely proportional to one's distance from the object. Earth is about 93 million miles from the Sun. Use the information at the left to find how much larger the diameter of the Sun would appear on Mercury than on Earth.

Explore You know that the apparent diameter of the Sun varies inversely with the distance from the Sun. You also know Mercury's distance from the Sun and Earth's distance from the Sun. You want to determine how much larger the diameter of the Sun appears on Mercury than on Earth.

Plan Let the apparent diameter of the Sun from Earth equal 1 unit and the apparent diameter of the Sun from Mercury equal m. Then use a proportion that relates the values.

DAILY INTERVENTION

Unlocking Misconceptions

Direct and Inverse Variation Help students understand the difference between the two types of variation by using the example of gas in the tank of a car, distance, and driving time. The amount of distance increases as the driving time increases (direct). The amount of gas decreases as the driving time increases (inverse).

494 Chapter 9 Rational Expressions and Equations

Solve

$$\frac{\text{distance from Mercury}}{\text{apparent diameter from Earth}} = \frac{\text{distance from Earth}}{\text{apparent diameter from Mercury}}$$ Inverse variation

$$\frac{36 \text{ million miles}}{1 \text{ unit}} = \frac{93 \text{ million miles}}{m \text{ units}}$$ Substitution

$$(36 \text{ million miles})(m \text{ units}) = (93 \text{ million miles})(1 \text{ unit})$$ Cross multiply.

$$m = \frac{(93 \text{ million miles})(1 \text{ unit})}{36 \text{ million miles}}$$ Divide each side by 36 million miles.

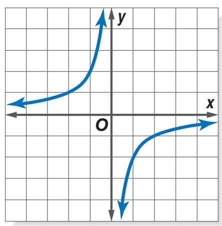
$$m \approx 2.58 \text{ units}$$ Simplify.

Examine Since distance between the Sun and Earth is between 2 and 3 times the distance between the Sun and Mercury, the answer seems reasonable. From Mercury, the diameter of the Sun will appear about 2.58 times as large as it appears from Earth.

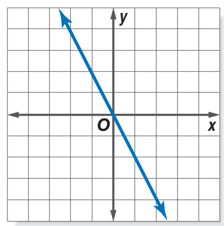
Check for Understanding

Concept Check

1. Determine whether each graph represents a *direct* or an *inverse* variation.

a. 

inverse

b. 

direct

2. Both are examples of direct variation. For $y = 5x$, y increases as x increases. For $y = -5x$, y decreases as x increases.

2. Compare and contrast $y = 5x$ and $y = -5x$.

3. OPEN ENDED Describe two quantities in real life that vary directly with each other and two quantities that vary inversely with each other. See margin.

Guided Practice

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1–3
10–13	4

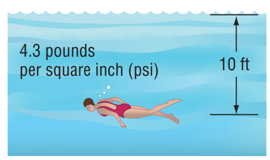
4. $ab = 20$ inverse; 20
5. $\frac{y}{x} = -0.5$ direct; -0.5
6. $A = \frac{1}{2}bh$ joint; $\frac{1}{2}$

Find each value.

7. If y varies directly as x and $y = 18$ when $x = 15$, find y when $x = 20$. 24
8. Suppose y varies jointly as x and z. Find y when $x = 9$ and $z = -5$, if $y = -90$ when $z = 15$ and $x = -6$. -45
9. If y varies inversely as x and $y = -14$ when $x = 12$, find x when $y = 21$. -8

Application

SWIMMING **For Exercises 10–13, use the following information.**
When a person swims underwater, the pressure in his or her ears varies directly with the depth at which he or she is swimming.



10. Write an equation of direct variation that represents this situation. $P = 0.43d$
11. Find the pressure at 60 feet. 25.8 psi
12. It is unsafe for amateur divers to swim where the water pressure is more than 65 pounds per square inch. How deep can an amateur diver safely swim?
13. Make a table showing the number of pounds of pressure at various depths of water. Use the data to draw a graph of pressure versus depth. See margin.

12. about 150 ft

3 Practice/Apply

Study Notebook

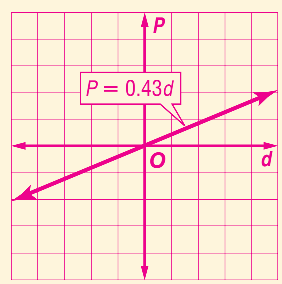
Have students—
- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 9.
- write the names and some examples from their lives for direct, joint, and inverse variation.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answers

3. Sample answers: wages and hours worked, total cost and number of pounds of apples purchased; distances traveled and amount of gas remaining in the tank, distance of an object and the size it appears

13.

Depth (ft)	Pressure (psi)
0	0
1	0.43
2	0.86
3	1.29
4	1.72



DAILY INTERVENTION

Differentiated Instruction

Auditory/Musical Have students find various kinds of variation in the sounds made by musical instruments. Suggest that they investigate the length and size of guitar strings relative to their vibrations, and the length and diameter of the columns of air used in wind and brass instruments for various notes.

Lesson 9-4 Direct, Joint, and Inverse Variation 495

★ indicates increased difficulty

Practice and Apply

Homework Help

Exercises	Examples
14–37	1–3
38–53	4

Extra Practice

See page 848.

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation. 14. direct; 1.5 16. inverse; −18

14. $\frac{n}{m} = 1.5$ **15.** $a = 5bc$ joint; 5 **16.** $vw = -18$ **17.** $3 = \frac{a}{b}$ direct; 3

18. $p = \frac{12}{q}$ inverse; 12 **19.** $y = -7x$ direct; −7 **20.** $V = \frac{1}{3}Bh$ joint; $\frac{1}{3}$ **21.** $\frac{2.5}{t} = s$ inverse; 2.5

22. CHEMISTRY Boyle's Law states that when a sample of gas is kept at a constant temperature, the volume varies inversely with the pressure exerted on it. Write an equation for Boyle's Law that expresses the variation in volume V as a function of pressure P. $V = \frac{k}{P}$

23. CHEMISTRY Charles' Law states that when a sample of gas is kept at a constant pressure, its volume V will increase as the temperature t increases. Write an equation for Charles' Law that expresses volume as a function. $V = kt$

24. GEOMETRY How does the circumference of a circle vary with respect to its radius? What is the constant of variation? directly; 2π

25. TRAVEL A map is scaled so that 3 centimeters represents 45 kilometers. How far apart are two towns if they are 7.9 centimeters apart on the map? 118.5 km

Find each value.

26. If y varies directly as x and $y = 15$ when $x = 3$, find y when $x = 12$. 60

27. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$. 20

28. Suppose y varies jointly as x and z. Find y when $x = 2$ and $z = 27$, if $y = 192$ when $x = 8$ and $z = 6$. 216

29. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$. 64

30. If y varies inversely as x and $y = 5$ when $x = 10$, find y when $x = 2$. 25

31. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$. 4

32. If y varies inversely as x and $y = 2$ when $x = 25$, find x when $y = 40$. 1.25

33. If y varies inversely as x and $y = 4$ when $x = 12$, find y when $x = 5$. 9.6

34. If y varies directly as x and $y = 9$ when x is -15, find y when $x = 21$. −12.6

35. If y varies directly as x and $x = 6$ when $y = 0.5$, find y when $x = 10$. 0.83

★ **36.** Suppose y varies jointly as x and z. Find y when $x = \frac{1}{2}$ and $z = 6$, if $y = 45$ when $x = 6$ and $z = 10$. $2\frac{1}{4}$

★ **37.** If y varies jointly as x and z and $y = \frac{1}{8}$ when $x = \frac{1}{2}$ and $z = 3$, find y when $x = 6$ and $z = \frac{1}{3}$. $\frac{1}{6}$

38. WORK Paul drove from his house to work at an average speed of 40 miles per hour. The drive took him 15 minutes. If the drive home took him 20 minutes and he used the same route in reverse, what was his average speed going home? 30 mph

39. WATER SUPPLY Many areas of Northern California depend on the snowpack of the Sierra Nevada Mountains for their water supply. If 250 cubic centimeters of snow will melt to 28 cubic centimeters of water, how much water does 900 cubic centimeters of snow produce? 100.8 cm^3

Career Choices

Travel Agent

Travel agents give advice and make arrangements for transportation, accommodations, and recreation. For international travel, they also provide information on customs and currency exchange.

Online Research

For information about a career as a travel agent, visit: www.algebra2.com/careers

Study Guide and Intervention, p. 535 (shown) and p. 536

Direct Variation and Joint Variation

Direct Variation	y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the constant of variation.
Joint Variation	y varies jointly as x and z if there is some number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.

Example **Find each value.**

a. If y varies directly as x and $y = 16$ when $x = 4$, find x when $y = 20$.

$\frac{y_1}{x_1} = \frac{y_2}{x_2}$ Direct proportion

$\frac{16}{4} = \frac{20}{x_2}$ $y_1 = 16$, $x_1 = 4$, and $y_2 = 20$

$16x_2 = (20)(4)$ Cross multiply.

$x_2 = 5$ Simplify.

The value of x is 5 when y is 20.

b. If y varies jointly as x and z and $y = 10$ when $x = 2$ and $z = 4$, find y when $x = 4$ and $z = 3$.

$\frac{y_1}{x_1z_1} = \frac{y_2}{x_2z_2}$ Joint variation

$\frac{10}{2 \cdot 4} = \frac{y_2}{4 \cdot 3}$ $y_1 = 10$, $x_1 = 2$, $z_1 = 4$, $x_2 = 4$, and $z_2 = 3$

$120 = 8y_2$ Simplify.

$y_2 = 15$ Divide each side by 8.

The value of y is 15 when $x = 4$ and $z = 3$.

Exercises

Find each value.

1. If y varies directly as x and $y = 9$ when $x = 6$, find y when $x = 8$. 12
2. If y varies directly as x and $y = 16$ when $x = 36$, find y when $x = 54$. 24
3. If y varies directly as x and $x = 15$ when $y = 5$, find x when $y = 9$. 27
4. If y varies directly as x and $x = 33$ when $y = 22$, find x when $y = 32$. 48
5. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 3$, if $y = 18$ when $x = 3$ and $z = 2$. 45
6. Suppose y varies jointly as x and z. Find y when $x = 6$ and $z = 8$, if $y = 6$ when $x = 4$ and $z = 2$. 36
7. Suppose y varies jointly as x and z. Find y when $x = 4$ and $z = 11$, if $y = 60$ when $x = 3$ and $z = 5$. 176
8. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 2$, if $y = 84$ when $x = 4$ and $z = 7$. 30
9. If y varies directly as x and $y = 14$ when $x = 35$, find y when $x = 12$. 4.8
10. If y varies directly as x and $x = 200$ when $y = 50$, find x when $y = 1000$. 4000
11. If y varies directly as x and $y = 39$ when $x = 52$, find y when $x = 22$. 16.5
12. If y varies directly as x and $x = 60$ when $y = 75$, find x when $y = 42$. 33.6
13. Suppose y varies jointly as x and z. Find y when $x = 6$ and $z = 11$, if $y = 120$ when $x = 5$ and $z = 12$. 132
14. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 10$, if $y = 12$ when $x = 8$ and $z = 6$. 12.5
15. Suppose y varies jointly as x and z. Find y when $x = 7$ and $z = 18$, if $y = 351$ when $x = 6$ and $z = 13$. 567
16. Suppose y varies jointly as x and z. Find y when $x = 5$ and $z = 27$, if $y = 480$ when $x = 9$ and $z = 20$. 360

Skills Practice, p. 537 and Practice, p. 538 (shown)

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1. $u = 8wz$ joint; 8 **2.** $p = 4s$ direct; 4 **3.** $L = \frac{5}{k}$ inverse; 5 **4.** $xy = 4.5$ inverse; 4.5

5. $\frac{C}{d} = \pi$ direct; π **6.** $2d = mn$ joint; $\frac{1}{2}$ **7.** $\frac{1.25}{g} = h$ inverse; 1.25 **8.** $y = \frac{3}{4x}$ inverse; $\frac{3}{4}$

Find each value.

9. If y varies directly as x and $y = 8$ when $x = 2$, find y when $x = 6$. 24
10. If y varies directly as x and $y = -16$ when $x = 6$, find x when $y = -4$. 1.5
11. If y varies directly as x and $y = 132$ when $x = 11$, find y when $x = 33$. 396
12. If y varies directly as x and $y = 7$ when $x = 1.5$, find y when $x = 4$. $\frac{56}{3}$
13. If y varies jointly as x and z and $y = 24$ when $x = 2$ and $z = 1$, find y when $x = 12$ and $z = 2$. 288
14. If y varies jointly as x and z and $y = 60$ when $x = 3$ and $z = 4$, find y when $x = 6$ and $z = 8$. 240
15. If y varies jointly as x and z and $y = 12$ when $x = -2$ and $z = 3$, find y when $x = 4$ and $z = -1$. 8
16. If y varies inversely as x and $y = 16$ when $x = 4$, find y when $x = 3$. $\frac{64}{3}$
17. If y varies inversely as x and $y = 3$ when $x = 5$, find x when $y = 2.5$. 6
18. If y varies inversely as x and $y = -18$ when $x = 6$, find y when $x = 5$. −21.6
19. If y varies directly as x and $y = 5$ when $x = 0.4$, find x when $y = 37.5$. 3
20. **GASES** The volume V of a gas varies inversely as its pressure P. If $V = 80$ cubic centimeters when $P = 2000$ millimeters of mercury, find V when $P = 320$ millimeters of mercury. 500 cm^3
21. **SPRINGS** The length S that a spring will stretch varies directly with the weight F that is attached to the spring. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached? 12 in.
22. **GEOMETRY** The area A of a trapezoid varies jointly as its height and the sum of its bases. If the area is 480 square meters when the height is 20 meters and the bases are 28 meters and 20 meters, what is the area of a trapezoid when its height is 8 meters and its bases are 10 meters and 15 meters? 100 m^2

Reading to Learn Mathematics, p. 539 ELL

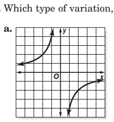
Pre-Activity **How is variation used to find the total cost given the unit cost?**

Read the introduction to Lesson 9-4 at the top of page 492 in your textbook.

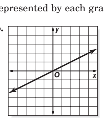
- For each additional student who enrolls in a public college, the total high-tech spending will ___increase___ (increase/decrease) by ___$149___.
- For each decrease in enrollment of 100 students in a public college, the total high-tech spending will ___decrease___ (increase/decrease) by ___$14,900___.

Reading the Lesson

1. Write an equation to represent each of the following variation statements. Use k as the constant of variation.
   a. m varies inversely as n. $m = \frac{k}{n}$
   b. s varies directly as r. $s = kr$
   c. t varies jointly as p and q. $t = kpq$
2. Which type of variation, direct or inverse, is represented by each graph?

a. 

inverse

b. 

direct

Helping You Remember

3. How can your knowledge of the equation of the slope-intercept form of the equation of a line help you remember the equation for direct variation?

Sample answer: The graph of an equation expressing direct variation is a line. The slope-intercept form of the equation of a line is $y = mx + b$. In direct variation, if one of the quantities is 0, the other quantity is also 0, so $b = 0$ and the line goes through the origin. The equation of a line through the origin is $y = mx$, where m is the slope. This is the same as the equation for direct variation with $k = m$.

Enrichment, p. 540

Expansions of Rational Expressions

Many rational expressions can be transformed into **power series.** A power series is an infinite series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$. The rational expression and the power series normally can be said to have the same values only for certain values of x. For example, the following equation holds only for values of x such that $-1 < x < 1$.

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \ldots \text{ for } -1 < x < 1$$

Example **Expand $\frac{2 + 3x}{1 + x + x^2}$ in ascending powers of x.**

Assume that the expression equals a series of the form $A + Bx + Cx^2 + Dx^3 + \ldots$. Then multiply both sides of the equation by the denominator $1 + x + x^2$.

$$\frac{2 + 3x}{1 + x + x^2} = A + Bx + Cx^2 + Dx^3 + \ldots$$

$$2 + 3x = (1 + x + x^2)(A + Bx + Cx^2 + Dx^3 + \ldots)$$

$2 + 3x = A + Bx + Cx^2 + Dx^3 +$

40. **RESEARCH** According to Johannes Kepler's third law of planetary motion, the ratio of the square of a planet's period of revolution around the Sun to the cube of its mean distance from the Sun is constant for all planets. Verify that this is true for at least three planets. **See students' work.**

BIOLOGY **For Exercises 41–43, use the information at the left.**

41. Write an equation to represent the amount of meat needed to sustain s Siberian tigers for d days. $m = 20sd$

42. Is your equation in Exercise 41 a *direct*, *joint*, or *inverse* variation? **joint**

43. How much meat do three Siberian tigers need for the month of January? **1860 lb**

LAUGHTER **For Exercises 44–46, use the following information.**
According to *The Columbus Dispatch*, the average American laughs 15 times per day.

44. Write an equation to represent the average number of laughs produced by m household members during a period of d days. $\ell = 15md$

45. Is your equation in Exercise 44 a *direct*, *joint*, or *inverse* variation? **joint**

46. Assume that members of your household laugh the same number of times each day as the average American. How many times would the members of your household laugh in a week? **See students' work.**

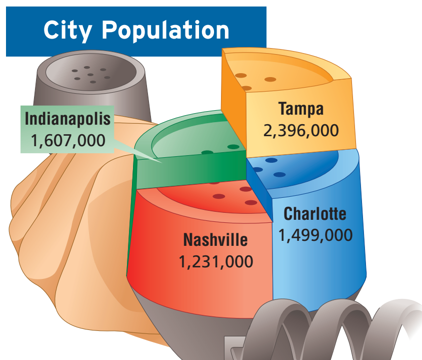
ARCHITECTURE **For Exercises 47–49, use the following information.**
When designing buildings such as theaters, auditoriums, or museums architects have to consider how sound travels. Sound intensity I is inversely proportional to the square of the distance from the sound source d.

★**47.** Write an equation that represents this situation. $I = \frac{k}{d^2}$

★**48.** If d is the independent variable and I is the dependent variable, graph the equation from Exercise 47 when $k = 16$. **See margin.**

★**49.** If a person in a theater moves to a seat twice as far from the speakers, compare the new sound intensity to that of the original.

TELECOMMUNICATIONS **For Exercises 50–53, use the following information.**
It has been found that the average number of daily phone calls C between two cities is directly proportional to the product of the populations P_1 and P_2 of two cities and inversely proportional to the square of the distance d between the cities. That is, $C = \frac{kP_1P_2}{d^2}$.



Source: U.S. Census Bureau

★**50.** The distance between Nashville and Charlotte is about 425 miles. If the average number of daily phone calls between the cities is 204,000, find the value of k and write the equation of variation. Round to the nearest hundredth.

★**51.** Nashville is about 680 miles from Tampa. Find the average number of daily phone calls between them. **about 127,572 calls**

★**52.** The average daily phone calls between Indianapolis and Charlotte is 133,380. Find the distance between Indianapolis and Charlotte. **about 601 mi**

★**53.** Could you use this formula to find the populations or the average number of phone calls between two adjoining cities? Explain. **no;** $d \neq 0$

54. **CRITICAL THINKING** Write a real-world problem that involves a joint variation. Solve the problem.

www.algebra2.com/self_check_quiz

More About. . .

Biology
In order to sustain itself in its cold habitat, a Siberian tiger requires 20 pounds of meat per day.
Source: *Wildlife Fact File*

49. The sound will be heard $\frac{1}{4}$ as intensely.

50. 0.02; $C = \frac{0.02P_1P_2}{d^2}$

54. Sample answer: If the average student spends $2.50 for lunch in the school cafeteria, write an equation to represent the amount *s* students will spend for lunch in *d* days. How much will 30 students spend in a week?
$a = 2.50sd$; **$375**

About the Exercises...

Organization by Objective
- **Direct Variation and Joint Variation:** 23–29, 34–37, 39–46
- **Inverse Variation:** 22, 30–33, 38, 47–49

Odd/Even Assignments
Exercises 14–39 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Alert! Exercise 40 requires reference materials for planetary data.

Assignment Guide

Basic: 15–35 odd, 39, 41–43, 54–73

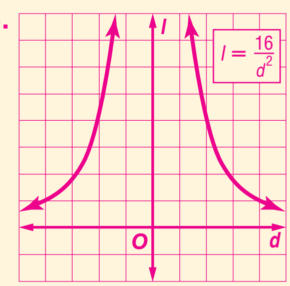
Average: 15–39 odd, 44–49, 54–73

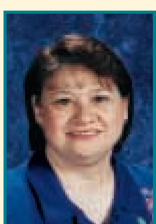
Advanced: 14–40 even, 44–67 (optional: 68–73)

All: Practice Quiz 2 (1–5)

Answer

48.





Teacher to Teacher

Susan Nelson — Spring H.S., Spring, TX

"I have my students do a data gathering activity called Rotations where we have the student do a regression for the diameter of a lid versus the number of rotations it takes to move across a fixed length of masking tape."

Lesson 9-4 Direct, Joint, and Inverse Variation 497

4 Assess

Open-Ended Assessment

Modeling Have students write equations for various variations in their life (time spent studying, hours of sleep, and so on). Ask them to write examples and explain them.

Tips for New Teachers

Intervention Make sure that students understand the essential differences between direct and inverse variation. For example, ask them if the number of candles on a birthday cake varies directly or indirectly with the age of the birthday person. Then ask how the length of the candle remaining varies with the time the candle has burned.

Getting Ready for Lesson 9-5

PREREQUISITE SKILL In Lesson 9-5, students will identify equations and graphs as different types of functions. Use Exercises 68–73 to determine your students' familiarity with identifying equations as step, constant, absolute value, or piecewise functions.

Assessment Options

Practice Quiz 2 The quiz provides students with a brief review of the concepts and skills in Lessons 9-3 and 9-4. Lesson numbers are given to the right of exercises or instruction lines so students can review concepts not yet mastered.

---

55. WRITING IN MATH Answer the question that was posed at the beginning of the lesson. **See margin.**

How is variation used to find the total cost given the unit cost?

Include the following in your answer:
- an explanation of why the equation for the total cost is a direct variation, and
- a problem involving unit cost and total cost of an item and its solution.

Standardized Test Practice

56. If the ratio of $2a$ to $3b$ is 4 to 5, what is the ratio of $5a$ to $4b$? **D**

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{9}{8}$ (D) $\frac{3}{2}$

57. Suppose b varies inversely as the square of a. If a is multiplied by 9, which of the following is true for the value of b? **C**

(A) It is multiplied by $\frac{1}{3}$. (B) It is multiplied by $\frac{1}{9}$.

(C) It is multiplied by $\frac{1}{81}$. (D) It is multiplied by 3.

Maintain Your Skills

Mixed Review

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function. *(Lesson 9-3)*

58. $f(x) = \frac{x+1}{x^2-1}$ **asymp.: $x = 1$; hole: $x = -1$**

59. $f(x) = \frac{x+3}{x^2+x-12}$ **asymp.: $x = -4$, $x = 3$**

60. $f(x) = \frac{x^2+4x+3}{x+3}$ **hole: $x = -3$**

Simplify each expression. *(Lesson 9-2)*

61. $\frac{3x}{x-y} + \frac{4x}{y-x}$ $\frac{x}{y-x}$

62. $\frac{t}{t+2} - \frac{2}{t^2-4}$ 62. $\frac{t^2-2t-2}{(t+2)(t-2)}$

63. $\frac{m - \frac{1}{m}}{1 + \frac{4}{m} - \frac{5}{m^2}}$ $\frac{m(m+1)}{m+5}$

64. **ASTRONOMY** The distance from Earth to the Sun is approximately 93,000,000 miles. Write this number in scientific notation. *(Lesson 5-1)* **9.3×10^7**

State the slope and the y-intercept of the graph of each equation. *(Lesson 2-4)*

65. $y = 0.4x + 1.2$ **0.4; 1.2**

66. $2y = 6x + 14$ **3; 7**

67. $3x + 5y = 15$ **$-\frac{3}{5}$; 3**

Getting Ready for the Next Lesson

PREREQUISITE SKILL Identify each function as S for step, C for constant, A for absolute value, or P for piecewise. *(To review* ***special functions,*** *see Lesson 2-6.)*

68. $h(x) = \frac{2}{3}$ **C**

69. $g(x) = 3|x|$ **A**

70. $f(x) = [\![2x]\!]$ **S**

71. $f(x) = \begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x \le 0 \end{cases}$ **P**

72. $h(x) = |x - 2|$ **A**

73. $g(x) = -3$ **C**

Practice Quiz 2 — *Lessons 9-3 and 9-4*

Graph each rational function. *(Lesson 9-3)* **1–2. See pp. 519A–519D.**

1. $f(x) = \frac{x-1}{x-4}$

2. $f(x) = \frac{-2}{x^2-6x+9}$

Find each value. *(Lesson 9-4)*

3. If y varies inversely as x and $x = 14$ when $y = 7$, find x when $y = 2$. **49**
4. If y varies directly as x and $y = 1$ when $x = 5$, find y when $x = 22$. **4.4**
5. If y varies jointly as x and z and $y = 80$ when $x = 25$ and $z = 4$, find y when $x = 20$ and $z = 7$. **112**

Answer

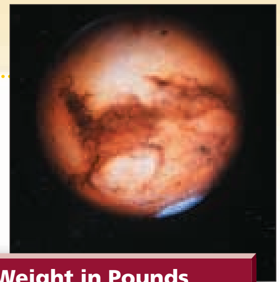
55. A direct variation can be used to determine the total cost when the cost per unit is known. Answers should include the following.
- Since the total cost T is the cost per unit u times the number of units n or $T = un$, the relationship is a direct variation. In this equation u is the constant of variation.
- Sample answer: The school store sells pencils for 20¢ each. John wants to buy 5 pencils. What is the total cost of the pencils? ($1.00)

9-5 Classes of Functions

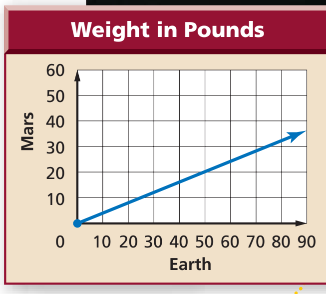
What You'll Learn

- Identify graphs as different types of functions.
- Identify equations as different types of functions.

How can graphs of functions be used to determine a person's weight on a different planet?



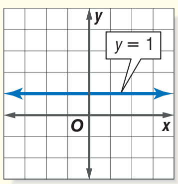
The purpose of the 2001 Mars Odyssey Mission is to study conditions on Mars. The findings will help NASA prepare for a possible mission with human explorers. The graph at the right compares a person's weight on Earth with his or her weight on Mars. This graph represents a direct variation, which you studied in the previous lesson.



IDENTIFY GRAPHS In this book, you have studied several types of graphs representing special functions. The following is a summary of these graphs.

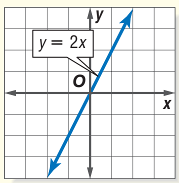
Concept Summary — *Special Functions*

Constant Function



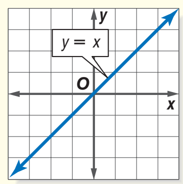
The general equation of a constant function is $y = a$, where a is any number. Its graph is a horizontal line that crosses the y-axis at a.

Direct Variation Function



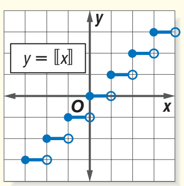
The general equation of a direct variation function is $y = ax$, where a is a nonzero constant. Its graph is a line that passes through the origin and is neither horizontal nor vertical.

Identity Function



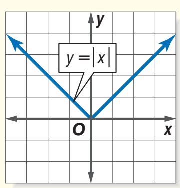
The identity function $y = x$ is a special case of the direct variation function in which the constant is 1. Its graph passes through all points with coordinates (a, a).

Greatest Integer Function



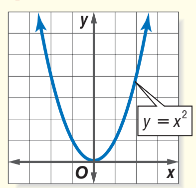
If an equation includes an expression inside the greatest integer symbol, the function is a greatest integer function. Its graph looks like steps.

Absolute Value Function



An equation with a direct variation expression inside absolute value symbols is an absolute value function. Its graph is in the shape of a V.

Quadratic Function



The general equation of a quadratic function is $y = ax^2 + bx + c$, where $a \neq 0$. Its graph is a parabola.

(continued on the next page)

9-5 Lesson Notes

1 Focus

5-Minute Check Transparency 9-5 Use as a quiz or review of Lesson 9-4.

Mathematical Background notes are available for this lesson on p. 470D.

Building on Prior Knowledge

In previous course material, students have learned about different kinds of functions. In this lesson, students will revisit different functions and group them into logical categories based on their characteristics.

How can graphs of functions be used to determine a person's weight on a different planet?

Ask students:

- According to the graph, what is the approximate weight on Mars of a person who weighs 50 pounds on Earth? **about 20 lb**
- According to the graph, what is the approximate weight on Earth of a person who would weigh 30 pounds on Mars? **about 75 lb**

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters

- Study Guide and Intervention, pp. 541–542
- Skills Practice, p. 543
- Practice, p. 544
- Reading to Learn Mathematics, p. 545
- Enrichment, p. 546
- Assessment, p. 568

Transparencies

5-Minute Check Transparency 9-5
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

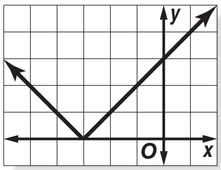
IDENTIFY GRAPHS

In-Class Example

Power Point®

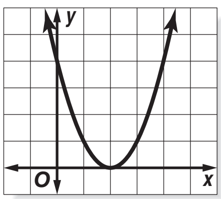
1 Identify the type of function represented by each graph.

a.



absolute value function

b.



quadratic function

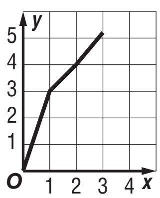
IDENTIFY EQUATIONS

In-Class Example

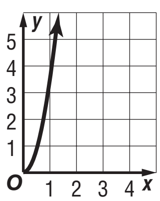
Power Point®

2 **SHIPPING CHARGES** A chart gives the shipping rates for an Internet company. They charge \$3.50 to ship less than 1 pound, \$3.95 for 1 pound and over up to 2 pounds, and \$5.20 for 2 pounds and over up to 3 pounds. Which graph depicts these rates? c, the step or greatest integer function

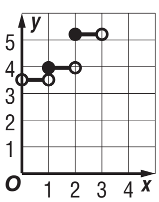
a.



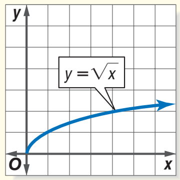
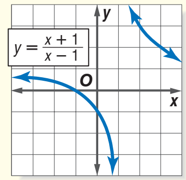
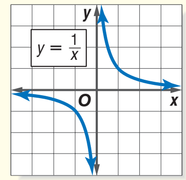
b.



c.



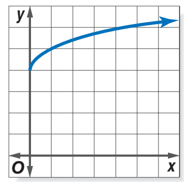
Concept Summary — *Special Functions*

Square Root Function	Rational Function	Inverse Variation Function
		
If an equation includes an expression inside the radical sign, the function is a square root function. Its graph is a curve that starts at a point and continues in only one direction.	The general equation for a rational function is $y = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions. Its graph has one or more asymptotes and/or holes.	The inverse variation function $y = \frac{a}{x}$ is a special case of the rational function where $p(x)$ is a constant and $q(x) = x$. Its graph has two asymptotes, $x = 0$ and $y = 0$.

Example 1 *Identify a Function Given the Graph*

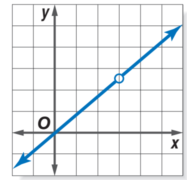
Identify the type of function represented by each graph.

a.



The graph has a starting point and curves in one direction. The graph represents a square root function.

b.



The graph appears to be a direct variation since it is a straight line passing through the origin. However, the hole indicates that it represents a rational function.

More About. . .

Rocketry

A rocket-powered airplane called the *X-15* set an altitude record for airplanes by flying 67 miles above Earth.

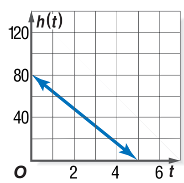
Source: *World Book Encyclopedia*

IDENTIFY EQUATIONS If you can identify an equation as a type of function, you can determine the shape of the graph.

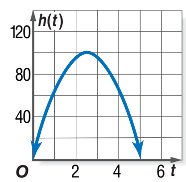
Example 2 *Match Equation with Graph*

ROCKETRY **Emily launched a toy rocket from ground level. The height above the ground level *h*, in feet, after *t* seconds is given by the formula $h(t) = -16t^2 + 80t$. Which graph depicts the height of the rocket during its flight?**

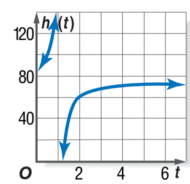
a.



b.



c.



The function includes a second-degree polynomial. Therefore, it is a quadratic function, and its graph is a parabola. Graph b is the only parabola. Therefore, the answer is graph **b**.

Teacher to Teacher

Deedee S. Adams — Oxford H.S., Oxford, AL

"I have my students play Simon Says by having them all stand and graph different types of functions with their arms. Students sit down if they don't illustrate the correct graph."

7-2 Graphing Polynomial Functions

See pages 353–358.

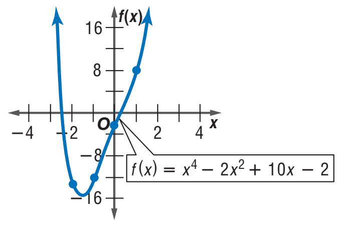
Concept Summary

- The Location Principle: Since zeros of a function are located at x-intercepts, there is also a zero between each pair of these zeros.
- Turning points of a function are called relative maxima and relative minima.

Example **Graph $f(x) = x^4 - 2x^2 + 10x - 2$ by making a table of values.**

Make a table of values for several values of x and plot the points. Connect the points with a smooth curve.

x	$f(x)$
−3	31
−2	−14
−1	−13
0	−2
1	7
2	26



Exercises **For Exercises 13–18, complete each of the following.**

a. Graph each function by making a table of values.

b. Determine consecutive values of x between which each real zero is located.

c. Estimate the x-coordinates at which the relative maxima and relative minima occur. *See Example 1 on page 353.* **13–18. See margin.**

13. $h(x) = x^3 - 6x - 9$
14. $f(x) = x^4 + 7x + 1$
15. $p(x) = x^5 + x^4 - 2x^3 + 1$
16. $g(x) = x^3 - x^2 + 1$
17. $r(x) = 4x^3 + x^2 - 11x + 3$
18. $f(x) = x^3 + 4x^2 + x - 2$

7-3 Solving Equations Using Quadratic Techniques

See pages 360–364.

Concept Summary

- Solve polynomial equations by using quadratic techniques.

Example **Solve $x^3 - 3x^2 - 54x = 0$.**

$x^3 - 3x^2 - 54x = 0$	Original equation
$x(x^2 - 3x - 54) = 0$	Factor out the GCF.
$x(x - 9)(x + 6) = 0$	Factor the trinomial.
$x = 0$ or $x - 9 = 0$ or $x + 6 = 0$	Zero Product Property
$x = 0$ $\quad x = 9$ $\quad x = -6$	

19. $\frac{5}{3}, -3, 0$
20. $-8, 0, 5$
21. $4, -2 \pm 2i\sqrt{3}$
23. $2, -2$

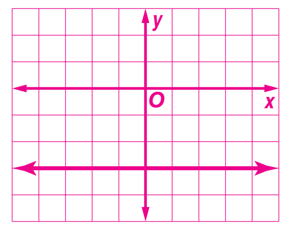
Exercises **Solve each equation.** *See Example 2 on page 361.*

19. $3x^3 + 4x^2 - 15x = 0$
20. $m^4 + 3m^3 = 40m^2$
21. $a^3 - 64 = 0$
22. $r + 9\sqrt{r} = -8$ $\varnothing$
23. $x^4 - 8x^2 + 16 = 0$
24. $x^{\frac{2}{3}} - 9x^{\frac{1}{3}} + 20 = 0$ **64, 125**

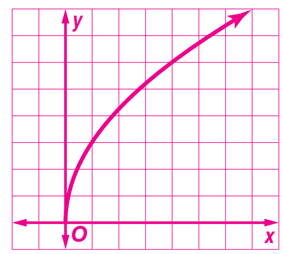
In-Class Example Power Point®

3 Identify the type of function represented by each equation. Then graph the equation.

a. $y = -3$ **constant function**



b. $y = \sqrt{9x}$ **square root function**



3 Practice/Apply

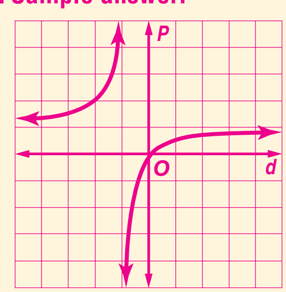
Study Notebook

Have students—

- add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 9.
- add sketches to illustrate each special function graph.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

Answer

1. Sample answer:



This graph is a rational function. It has an asymptote at $x = -1$.

DAILY INTERVENTION

Differentiated Instruction

Interpersonal Have students work with a partner or in small groups to do quick sketches of graphs and identify the type of function the graph could represent. Have each group make a list of the identifying characteristics of the graph; then ask groups to exchange and compare their lists.

About the Exercises...

Organization by Objective

- **Identify Graphs:** 13–22
- **Identify Equations:** 23–34

Odd/Even Assignments

Exercises 13–30 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

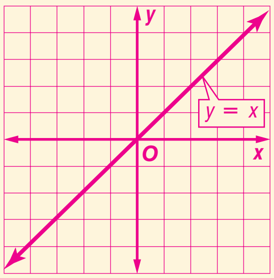
Basic: 13–29 odd, 31–33, 37–61

Average: 13–29 odd, 31–33, 37–61

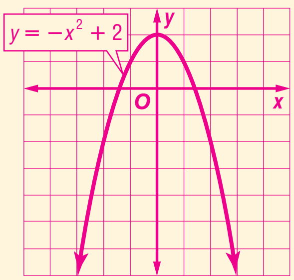
Advanced: 14–30 even, 31–55 (optional: 56–61)

Answers

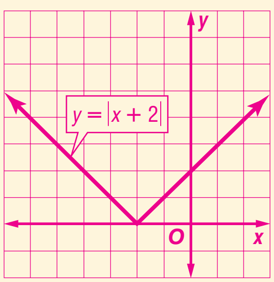
9.



10.



11.



9–11. See margin for graphs.

Identify the type of function represented by each equation. Then graph the equation.

9. $y = x$ identity or direct variation
10. $y = -x^2 + 2$ quadratic
11. $y = |x + 2|$ absolute value

Application

12. **GEOMETRY** Write the equation for the area of a circle. Identify the equation as a type of function. Describe the graph of the function.
$A = \pi r^2$; quadratic; the graph is a parabola.

★ indicates increased difficulty

Practice and Apply

Homework Help

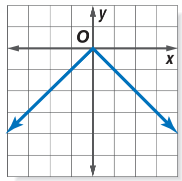
For Exercises	See Examples
13–18	1
19–22, 31–36	2
23–30	3

Extra Practice

See page 848.

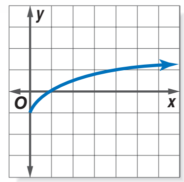
Identify the type of function represented by each graph.

13.



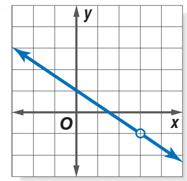
absolute value

14.



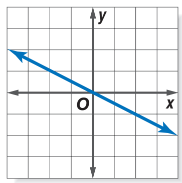
square root

15.



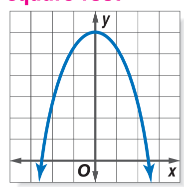
rational

16.



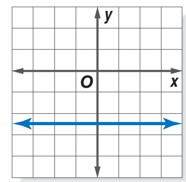
direct variation

17.



quadratic

18.



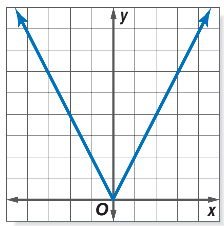
constant

WebQuest

You can use functions to determine the relationship between primary and secondary earthquake waves. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

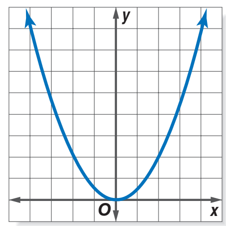
Match each graph with an equation at the right.

19.



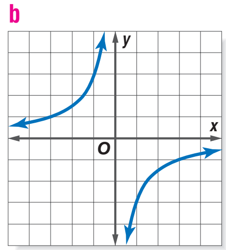
b

20.



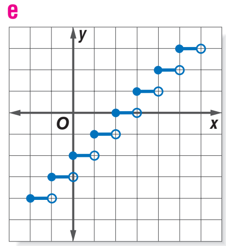
e

21.



g

22.



a

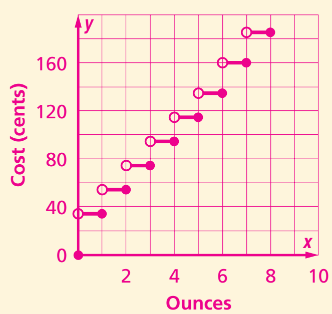
a. $y = [\![x]\!] - 2$
b. $y = 2|x|$
c. $y = 2\sqrt{x}$
d. $y = -3x$
e. $y = 0.5x^2$
f. $y = -\frac{3}{x+1}$
g. $y = -\frac{3}{x}$

23–30. See pp. 519A–519D for graphs.
23. constant
24. direct variation
25. square root
26. inverse variation or rational
27. rational
28. greatest integer
29. absolute value
30. quadratic

Identify the type of function represented by each equation. Then graph the equation.

23. $y = -1.5$
24. $y = 2.5x$
25. $y = \sqrt{9x}$
26. $y = \frac{4}{x}$
27. $y = \frac{x^2 - 1}{x - 1}$
28. $y = 3[\![x]\!]$
29. $y = |2x|$
30. $y = 2x^2$

35.



36. The graph is similar to the graph of the greatest integer function because both graphs look like a series of steps. In the graph of the postage rates, the solid dots are on the right and the circles are on the left. However, in the greatest integer function, the circles are on the right and the solid dots are on the left.

502 Chapter 9 Rational Expressions and Equations

HEALTH **For Exercises 31–33, use the following information.**
A woman painting a room will burn an average of 4.5 Calories per minute.

31. Write an equation for the number of Calories burned in m minutes. **$C = 4.5m$**

32. Identify the equation in Exercise 31 as a type of function. **direct variation**

33. Describe the graph of the function. **a line slanting to the right and passing through the origin**

More About. . .

Architecture

The Gateway Arch is 630 feet high and is the tallest monument in the United States.

Source: *World Book Encyclopedia*

34. **ARCHITECTURE** The shape of the Gateway Arch of the Jefferson National Expansion Memorial in St. Louis, Missouri, resembles the graph of the function $f(x) = -0.00635x^2 + 4.0005x - 0.07875$, where x is in feet. Describe the shape of the Gateway Arch. **similar to a parabola**

MAIL **For Exercises 35 and 36, use the following information.**
In 2001, the cost to mail a first-class letter was 34¢ for any weight up to and including 1 ounce. Each additional ounce or part of an ounce added 21¢ to the cost.

★35. Make a graph showing the postal rates to mail any letter from 0 to 8 ounces. **See pp. 519A–519D.**

★36. Compare your graph in Exercise 35 to the graph of the greatest integer function. **See pp. 519A–519D.**

37. **CRITICAL THINKING** Identify each table of values as a type of function.

a.

x	$f(x)$
−5	7
−3	5
−1	3
0	2
1	3
3	5
5	7
7	9

absolute value

b.

x	$f(x)$
−5	24
−3	8
−1	0
0	−1
1	0
3	8
5	24
7	48

quadratic

c.

x	$f(x)$
−1.3	−1
−1.7	−1
0	1
0.8	1
0.9	1
1	2
1.5	2
2.3	3

greatest integer

d.

x	$f(x)$
−5	undefined
−3	undefined
−1	undefined
0	0
1	1
4	2
9	3
16	4

square root

38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See pp. 519A–519D.**

How can graphs of functions be used to determine a person's weight on a different planet?

Include the following in your answer:
- an explanation of why the graph comparing weight on Earth and Mars represents a direct variation function, and
- an equation and a graph comparing a person's weight on Earth and Venus if a person's weight on Venus is 0.9 of his or her weight on Earth.

Standardized Test Practice

A B C D

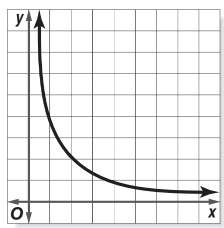
39. The curve at the right could be part of the graph of which function? **C**

Ⓐ $y = \sqrt{x}$

Ⓑ $y = x^2 - 5x + 4$

Ⓒ $xy = 4$

Ⓓ $y = -x + 20$



www.algebra2.com/self_check_quiz

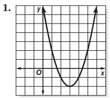
Study Guide and Intervention, p. 541 (shown) and p. 542

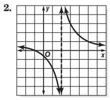
Identify Graphs You should be familiar with the graphs of the following functions.

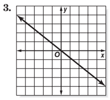
Function	Description of Graph
Constant	a horizontal line that crosses the y-axis at a
Direct Variation	a line that passes through the origin and is neither horizontal nor vertical
Identity	a line that passes through the point (a, a), where a is any real number
Greatest Integer	a step function
Absolute Value	V-shaped graph
Quadratic	a parabola
Square Root	a curve that starts at a point and curves in only one direction
Rational	a graph with one or more asymptotes and/or holes
Inverse Variation	a graph with 2 curved branches and 2 asymptotes, $x = 0$ and $y = 0$ (special case of rational function)

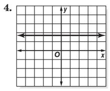
Exercises

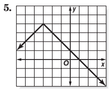
Identify the function represented by each graph.

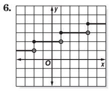
1. 
quadratic

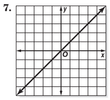
2. 
rational

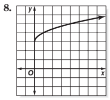
3. 
direct variation

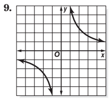
4. 
constant

5. 
absolute value

6. 
greatest integer

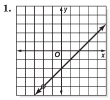
7. 
identity

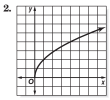
8. 
square root

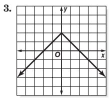
9. 
inverse variation

Skills Practice, p. 543 and Practice, p. 544 (shown)

Identify the type of function represented by each graph.

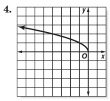
1. 
rational

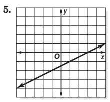
2. 
square root

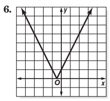
3. 
absolute value

Match each graph with an equation below.

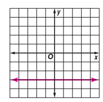
A. $y = |2x + 1|$ **B.** $y = [\![2x + 1]\!]$ **C.** $y = \frac{x - 3}{2}$ **D.** $y = \sqrt{-x}$

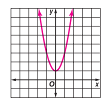
4. 
D

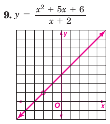
5. 
C

6. 
A

Identify the type of function represented by each equation. Then graph the equation.

7. $y = -3$

constant

8. $y = 2x^2 + 1$

quadratic

9. $y = \frac{x^2 + 5x + 6}{x + 2}$

rational

10. **BUSINESS** A startup company uses the function $P = 1.3x^2 + 3x - 7$ to predict its profit or loss during its first 7 years of operation. Describe the shape of the graph of the function. **The graph is U-shaped; it is a parabola.**

11. **PARKING** A parking lot charges $10 to park for the first day or part of a day. After that, it charges an additional $8 per day or part of a day. Describe the graph and find the cost of parking for $6\frac{1}{2}$ days. **The graph looks like a series of steps, similar to a greatest integer function, but with open circles on the left and closed circles on the right; $58.**

Reading to Learn Mathematics, p. 545 ELL

Pre-Activity **How can graphs of functions be used to determine a person's weight on a different planet?**

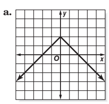
Read the introduction to Lesson 9-5 at the top of page 499 in your textbook.

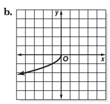
- Based on the graph, estimate the weight on Mars of a child who weighs 40 pounds on Earth.
about 15 pounds
- Although the graph does not extend far enough to the right to read it directly from the graph, use the weight you found above and your knowledge that this graph represents direct variation to estimate the weight on Mars of a woman who weighs 120 pounds on Earth.
about 45 pounds

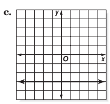
Reading the Lesson

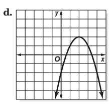
1. Match each graph below with the type of function it represents. Some types may be used more than once and others not at all.

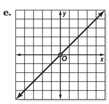
I. square root **II.** quadratic **III.** absolute value **IV.** rational
V. greatest integer **VI.** constant **VII.** identity

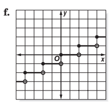
a.  **III**

b.  **I**

c.  **VI**

d.  **II**

e.  **IV**

f.  **V**

Helping You Remember

2. How can the symbolic definition of absolute value that you learned in Lesson 1-4 help you to remember the graph of the function $f(x) = |x|$? **Sample answer: Using the definition of absolute value, $f(x) = x$ if $x \geq 0$ and $f(x) = -x$ if $x < 0$. Therefore, the graph is made up of pieces of two lines, one with slope 1 and one with slope −1, meeting at the origin. This forms a V-shaped graph with "vertex" at the origin.**

Enrichment, p. 546

Partial Fractions

It is sometimes an advantage to rewrite a rational expression as the sum of two or more fractions. For example, you might do this in a calculus course while carrying out a procedure called integration.

You can resolve a rational expression into partial fractions if two conditions are met:

(1) The degree of the numerator must be less than the degree of the denominator; and
(2) The factors of the denominator must be known.

Example **Resolve $\frac{3}{x^3 + 1}$ into partial fractions.**

The denominator has two factors, a linear factor, $x + 1$, and a quadratic factor, $x^2 - x + 1$. Start by writing the following equation. Notice that the degree of the numerators of each partial fraction is less than its denominator.

4 Assess

Open-Ended Assessment

Modeling Have students use string on a coordinate grid to model some of the nine different types of functions in this lesson. Ask them to give an example of an equation that might have that sort of graph.

Tips for New Teachers

Intervention Help students associate the graphs and their functions by grouping the 9 types into 2 groups, those which involve straight lines and those which involve curves.

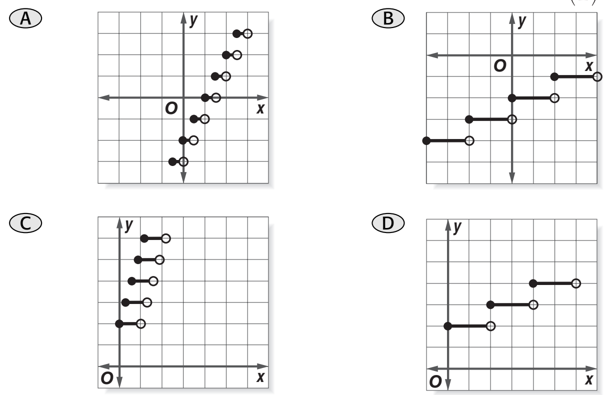
Getting Ready for Lesson 9-6

PREREQUISITE SKILL Students will solve rational equations in Lesson 9-6. These equations often contain fractions that are simplified by finding the LCD. Use Exercises 56–61 to determine your students' familiarity with finding LCMs of polynomials.

Assessment Options

Quiz (Lessons 9-4 and 9-5) is available on p. 568 of the *Chapter 9 Resource Masters*.

40. If $g(x) = [\![x]\!]$, which of the following is the graph of $g\left(\frac{x}{2}\right) + 2$? **D**



Maintain Your Skills

Mixed Review

41. If x varies directly as y and $y = \frac{1}{5}$ when $x = 11$, find x when $y = \frac{2}{5}$. *(Lesson 9–4)* **22**

Graph each rational function. *(Lesson 9-3)* **42–44. See pp. 519A–519D.**

42. $f(x) = \frac{3}{x + 2}$ 43. $f(x) = \frac{8}{(x - 1)(x + 3)}$ 44. $f(x) = \frac{x^2 - 5x + 4}{x - 4}$

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. *(Lesson 8-2)* **45–47. See margin.**

45. $\frac{1}{2}(y + 1) = (x - 8)^2$ 46. $x = \frac{1}{4}y^2 - \frac{1}{2}y - 3$ 47. $3x - y^2 = 8y + 31$

Find each product, if possible. *(Lesson 4-3)*

48. $\begin{bmatrix} 3 & -5 \\ 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} 5 & 1 & -3 \\ 8 & -4 & 9 \end{bmatrix}$ **48.** $\begin{bmatrix} -25 & 23 & -54 \\ 66 & -26 & 57 \end{bmatrix}$

49. $\begin{bmatrix} 4 & -1 & 6 \\ 1 & 5 & -8 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$ **impossible**

Solve each system of equations by using either substitution or elimination. *(Lesson 3-2)*

50. $3x + 5y = -4$; $2x - 3y = 29$ **(7, −5)**

51. $3a - 2b = -3$; $3a + b = 3$ $\left(\frac{1}{3}, 2\right)$

52. $3s - 2t = 10$; $4s + t = 6$ **(2, −2)**

Determine the value of r so that a line through the points with the given coordinates has the given slope. *(Lesson 2-3)*

53. $(r, 2), (4, -6)$; slope $= \frac{-8}{3}$ **1** 54. $(r, 6), (8, 4)$; slope $= \frac{1}{2}$ **12**

55. Evaluate $[(-7 + 4) \times 5 - 2] \div 6$. *(Lesson 1-1)* $-\frac{17}{6}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the LCM of each set of polynomials. *(To review **least common multiples of polynomials,** see Lesson 9-2.)*

56. $15ab^2c, 6a^3, 4bc^2$ **56.** $60a^3b^2c^2$

57. $9x^3, 5xy^2, 15x^2y^3$ **57.** $45x^3y^3$

58. $5d - 10, 3d - 6$ **58.** $15(d - 2)$

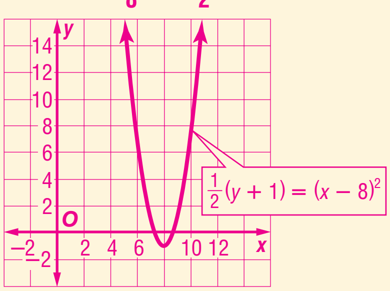
59. $x^2 - y^2, 3x + 3y$ $3(x - y)(x + y)$

60. $a^2 - 2a - 3, a^2 - a - 6$ $(a - 3)(a + 1)(a + 2)$

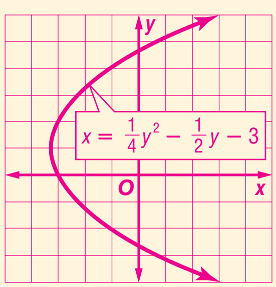
61. $2t^2 - 9t - 5, t^2 + t - 30$ $(t - 5)(t + 6)(2t + 1)$

Answers

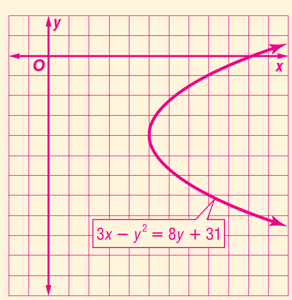
45. $(8, -1)$; $\left(8, -\frac{7}{8}\right)$; $x = 8$; $y = -1\frac{1}{8}$; up; $\frac{1}{2}$ unit



46. $\left(-3\frac{1}{4}, 1\right)$, $\left(-2\frac{1}{4}, 1\right)$; $y = 1$; $x = -4\frac{1}{4}$; right; 4 units



47. $(5, -4)$; $\left(5\frac{3}{4}, -4\right)$; $y = -4$; $x = 4\frac{1}{4}$; right; 3 units



9-6 Solving Rational Equations and Inequalities

What You'll Learn

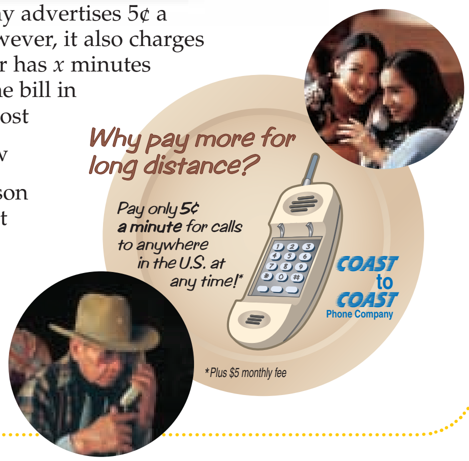
- Solve rational equations.
- Solve rational inequalities.

Vocabulary

- rational equation
- rational inequality

How are rational equations used to solve problems involving unit price?

The Coast to Coast Phone Company advertises 5¢ a minute for long-distance calls. However, it also charges a monthly fee of $5. If the customer has x minutes in long distance calls last month, the bill in cents will be $500 + 5x$. The actual cost per minute is $\frac{500 + 5x}{x}$. To find how many long-distance minutes a person would need to make the actual cost per minute 6¢, you would need to solve the equation $\frac{500 + 5x}{x} = 6$.



SOLVE RATIONAL EQUATIONS The equation $\frac{500 + 5x}{x} = 6$ is an example of a rational equation. In general, any equation that contains one or more rational expressions is called a **rational equation**.

Rational equations are easier to solve if the fractions are eliminated. You can eliminate the fractions by multiplying each side of the equation by the least common denominator (LCD). Remember that when you multiply each side by the LCD, each term on each side must be multiplied by the LCD.

Example 1 Solve a Rational Equation

Solve $\frac{9}{28} + \frac{3}{z + 2} = \frac{3}{4}$. Check your solution.

The LCD for the three denominators is $28(z + 2)$.

$\frac{9}{28} + \frac{3}{z+2} = \frac{3}{4}$	Original equation
$28(z+2)\left(\frac{9}{28} + \frac{3}{z+2}\right) = 28(z+2)\left(\frac{3}{4}\right)$	Multiply each side by $28(z + 2)$.
$\overset{1}{\cancel{28}}(z+2)\left(\frac{9}{\underset{1}{\cancel{28}}}\right) + 28\overset{1}{\cancel{(z+2)}}\left(\frac{3}{\underset{1}{\cancel{z+2}}}\right) = \overset{7}{\cancel{28}}(z+2)\left(\frac{3}{\underset{1}{\cancel{4}}}\right)$	Distributive Property
$(9z + 18) + 84 = 21z + 42$	Simplify.
$9z + 102 = 21z + 42$	Simplify.
$60 = 12z$	Subtract $9z$ and 42 from each side.
$5 = z$	Divide each side by 12.

9-6 Lesson Notes

1 Focus

5-Minute Check Transparency 9-6 Use as a quiz or review of Lesson 9-5.

Mathematical Background notes are available for this lesson on p. 470D.

Building on Prior Knowledge

In Chapter 1, students reviewed techniques for solving linear equations and inequalities. In this lesson, students will apply those same techniques to solving rational equations and inequalities.

How are rational equations used to solve problems involving unit price?

Ask students:

- Why does the equation use 500 instead of 5 for the monthly fee? **The fee and the per minute cost are both expressed in cents.**
- If a person makes 100 minutes of calls for a given month, how much did the monthly fee add to the per minute cost for these calls? **The fee adds 5 cents per minute.**

Resource Manager

Workbook and Reproducible Masters

Chapter 9 Resource Masters

- Study Guide and Intervention, pp. 547–548
- Skills Practice, p. 549
- Practice, p. 550
- Reading to Learn Mathematics, p. 551
- Enrichment, p. 552
- Assessment, p. 568

Transparencies

5-Minute Check Transparency 9-6
Answer Key Transparencies

Technology

Interactive Chalkboard

2 Teach

SOLVE RATIONAL EQUATIONS

In-Class Examples

Power Point®

1 Solve $\frac{5}{24} + \frac{2}{3-x} = \frac{1}{4}$. Check your solution. $x = -45$

Teaching Tip Discuss with students that, when checking a solution, it is not possible to simply substitute the solution for the variable and then multiply both sides of the equation by the LCD because the truth of the equation is being tested; it cannot be assumed.

2 Solve $\frac{p^2 - p + 1}{p+1} = \frac{p^2 - 7}{p^2 - 1} + p$. Check your solution. $p = 2$

Teaching Tip Remind students that solutions must always be checked in the original equation, rather than in any of the steps of the solution.

CHECK

$\frac{9}{28} + \frac{3}{z+2} = \frac{3}{4}$ Original equation

$\frac{9}{28} + \frac{3}{5+2} \stackrel{?}{=} \frac{3}{4}$ $z = 5$

$\frac{9}{28} + \frac{3}{7} \stackrel{?}{=} \frac{3}{4}$ Simplify.

$\frac{9}{28} + \frac{12}{28} \stackrel{?}{=} \frac{3}{4}$ Simplify.

$\frac{3}{4} = \frac{3}{4}$ ✓ The solution is correct.

The solution is 5.

When solving a rational equation, any possible solution that results in a zero in the denominator must be excluded from your list of solutions.

Example 2 Elimination of a Possible Solution

Solve $r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$. Check your solution.

The LCD is $(r^2 - 1)$.

$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$ Original equation

$(r^2-1)\left(r + \frac{r^2-5}{r^2-1}\right) = (r^2-1)\left(\frac{r^2+r+2}{r+1}\right)$ Multiply each side by the LCD, (r^2-1).

$(r^2-1)r + \overset{1}{\cancel{(r^2-1)}}\left(\frac{r^2-5}{\underset{1}{\cancel{r^2-1}}}\right) = \overset{(r-1)}{\cancel{(r^2-1)}}\left(\frac{r^2+r+2}{\underset{1}{\cancel{r+1}}}\right)$ Distributive Property

$(r^3 - r) + (r^2 - 5) = (r-1)(r^2+r+2)$ Simplify.

$r^3 + r^2 - r - 5 = r^3 + r - 2$ Simplify.

$r^2 - 2r - 3 = 0$ Subtract $(r^3 + r - 2)$ from each side.

$(r-3)(r+1) = 0$ Factor.

$r - 3 = 0$ or $r + 1 = 0$ Zero Product Property

$r = 3$ $\quad$ $r = -1$

CHECK

$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$ Original equation

$3 + \frac{3^2-5}{3^2-1} \stackrel{?}{=} \frac{3^2+3+2}{3+1}$ $r = 3$

$3 + \frac{4}{8} \stackrel{?}{=} \frac{14}{4}$ Simplify.

$\frac{7}{2} = \frac{7}{2}$ ✓

$r + \frac{r^2-5}{r^2-1} = \frac{r^2+r+2}{r+1}$ Original equation

$-1 + \frac{(-1)^2-5}{(-1)^2-1} \stackrel{?}{=} \frac{(-1)^2+(-1)+2}{-1+1}$ $r = -1$

$-1 + \frac{-4}{0} \stackrel{?}{=} \frac{2}{0}$ Simplify.

Since $r = -1$ results in a zero in the denominator, eliminate -1 from the list of solutions.

The solution is 3.

Study Tip

Extraneous Solutions
Multiplying each side of an equation by the LCD of rational expressions can yield results that are not solutions of the original equation. These solutions are called *extraneous solutions.*

506 Chapter 9 Rational Expressions and Equations

Some real-world problems can be solved with rational equations.

Example 3 Work Problem

TUNNELS **When building the Chunnel, the English and French each started drilling on opposite sides of the English Channel. The two sections became one in 1990. The French used more advanced drilling machinery than the English. Suppose the English could drill the Chunnel in 6.2 years and the French could drill it in 5.8 years. How long would it have taken the two countries to drill the tunnel?**

In 1 year, the English could complete $\frac{1}{6.2}$ of the tunnel.

In 2 years, the English could complete $\frac{1}{6.2} \cdot 2$ or $\frac{2}{6.2}$ of the tunnel.

In t years, the English could complete $\frac{1}{6.2} \cdot t$ or $\frac{t}{6.2}$ of the tunnel.

Likewise, in t years, the French could complete $\frac{1}{5.8} \cdot t$ or $\frac{t}{5.8}$ of the tunnel.

Together, they completed the whole tunnel.

Part completed by the English	plus	part completed by the French	equals	entire tunnel.
$\frac{t}{6.2}$	$+$	$\frac{t}{5.8}$	$=$	1

Solve the equation.

$\frac{t}{6.2} + \frac{t}{5.8} = 1$ — Original equation

$17.98\left(\frac{t}{6.2} + \frac{t}{5.8}\right) = 17.98(1)$ — Multiply each side by 17.98.

$17.98\left(\frac{t}{6.2}\right) + 17.98\left(\frac{t}{5.8}\right) = 17.98$ — Distributive Property

$2.9t + 3.1t = 17.98$ — Simplify.

$6t = 17.98$ — Simplify.

$t \approx 3.00$ — Divide each side by 6.

It would have taken about 3 years to build the Chunnel.

More About. . .

Tunnels

The Chunnel is a tunnel under the English Channel that connects England with France. It is 32 miles long with 23 miles of the tunnel under water.

Source: www.pbs.org

Rate problems frequently involve rational equations.

Example 4 Rate Problem

NAVIGATION **The speed of the current in the Puget sound is 5 miles per hour. A barge travels 26 miles with the current and returns in $10\frac{2}{3}$ hours. What is the speed of the barge in still water?**

WORDS The formula that relates distance, time, and rate is $d = rt$ or $\frac{d}{r} = t$.

VARIABLES Let r be the speed of the barge in still water. Then the speed of the barge with the current is $r + 5$, and the speed of the barge against the current is $r - 5$.

	Time going with the current	plus	time going against the current	equals	total time.
EQUATION	$\frac{26}{r+5}$	$+$	$\frac{26}{r-5}$	$=$	$10\frac{2}{3}$

(continued on the next page)

www.algebra2.com/extra_examples

In-Class Examples

Power Point®

3 **MOWING LAWNS** Tim and Ashley mow lawns together. Tim working alone could complete the job in 4.5 hours, and Ashley could complete it alone in 3.7 hours. How long does it take to complete the job when they work together? **about 2 h**

Teaching Tip Suggest to students that when the problem involves completing part of a job and working together, students think about what part of the work gets done in one day, or one year—whatever is one unit of the time.

4 **SWIMMING** Janine swims for 5 hours in a stream that has a current of 1 mile per hour. She leaves her dock and swims upstream for 2 miles and then back to her dock. What is her swimming speed in still water? **about 1.5 mi/h**

Teaching Tip Make sure that students understand the difference between the solutions to the quadratic equation (of which there are two) and the solution to the problem (of which there is only one).

Lesson 9-6 Solving Rational Equations and Inequalities 507

SOLVE RATIONAL INEQUALITIES

In-Class Example Power Point®

5 Solve $\frac{1}{3s} + \frac{2}{9s} < \frac{2}{3}$. $s < 0$ or $s > \frac{5}{6}$

Teaching Tip Suggest that students also verify whether the boundary indicated by the solution of the equation is or is not in the solution set of the inequality.

Study Tip

Look Back

To review the **Quadratic Formula,** see Lesson 6-5.

Solve the equation.

$\frac{26}{r+5} + \frac{26}{r-5} = 10\frac{2}{3}$ — Original equation

$3(r^2 - 25)\left(\frac{26}{r+5} + \frac{26}{r-5}\right) = 3(r^2 - 25)\left(10\frac{2}{3}\right)$ — Multiply each side by $3(r^2 - 25)$.

$3\overset{(r-5)}{\cancel{(r^2 - 25)}}\left(\frac{26}{\underset{1}{\cancel{r+5}}}\right) + 3\overset{(r+5)}{\cancel{(r^2 - 25)}}\left(\frac{26}{\underset{1}{\cancel{r-5}}}\right) = \overset{1}{\cancel{3}}(r^2 - 25)\left(\frac{32}{\underset{1}{\cancel{3}}}\right)$ — Distributive Property

$(78r - 390) + (78r + 390) = 32r^2 - 800$ — Simplify.

$156r = 32r^2 - 800$ — Simplify.

$0 = 32r^2 - 156r - 800$ — Subtract $156r$ from each side.

$0 = 8r^2 - 39r - 200$ — Divide each side by 4.

Use the Quadratic Formula to solve for r.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ — Quadratic Formula

$r = \frac{-(-39) \pm \sqrt{(-39)^2 - 4(8)(-200)}}{2(8)}$ — $x = r$, $a = 8$, $b = -39$, and $c = -200$

$r = \frac{39 \pm \sqrt{7921}}{16}$ — Simplify.

$r = \frac{39 \pm 89}{16}$ — Simplify.

$r = 8$ or -3.125 — Simplify.

Since the speed must be positive, the answer is 8 miles per hour.

SOLVE RATIONAL INEQUALITIES Inequalities that contain one or more rational expressions are called **rational inequalities**. To solve rational inequalities, complete the following steps.

Step 1 State the excluded values.

Step 2 Solve the related equation.

Step 3 Use the values determined in Steps 1 and 2 to divide a number line into regions. Test a value in each region to determine which regions satisfy the original inequality.

Example 5 *Solve a Rational Inequality*

Solve $\frac{1}{4a} + \frac{5}{8a} > \frac{1}{2}$.

Step 1 Values that make a denominator equal to 0 are excluded from the domain. For this inequality, the excluded value is 0.

Step 2 Solve the related equation.

$\frac{1}{4a} + \frac{5}{8a} = \frac{1}{2}$ — Related equation

$8a\left(\frac{1}{4a} + \frac{5}{8a}\right) = 8a\left(\frac{1}{2}\right)$ — Multiply each side by $8a$.

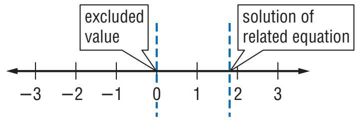
$2 + 5 = 4a$ — Simplify.

$7 = 4a$ — Add.

$1\frac{3}{4} = a$ — Divide each side by 4.

508 Chapter 9 Rational Expressions and Equations

Step 3 Draw vertical lines at the excluded value and at the solution to separate the number line into regions.



Now test a sample value in each region to determine if the values in the region satisfy the inequality.

Test $a = -1$.	Test $a = 1$.	Test $a = 2$.
$\frac{1}{4(-1)} + \frac{5}{8(-1)} \overset{?}{>} \frac{1}{2}$	$\frac{1}{4(1)} + \frac{5}{8(1)} \overset{?}{>} \frac{1}{2}$	$\frac{1}{4(2)} + \frac{5}{8(2)} \overset{?}{>} \frac{1}{2}$
$-\frac{1}{4} - \frac{5}{8} \overset{?}{>} \frac{1}{2}$	$\frac{1}{4} + \frac{5}{8} \overset{?}{>} \frac{1}{2}$	$\frac{1}{8} + \frac{5}{16} \overset{?}{>} \frac{1}{2}$
$-\frac{7}{8} \not> \frac{1}{2}$	$\frac{7}{8} > \frac{1}{2}$ ✓	$\frac{7}{16} \not> \frac{1}{2}$
$a < 0$ is *not* a solution.	$0 < a < 1\frac{3}{4}$ is a solution.	$a > 1\frac{3}{4}$ is *not* a solution.

The solution is $0 < a < 1\frac{3}{4}$.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a rational equation that can be solved by first multiplying each side by $5(a + 2)$.
2. State the number by which you would multiply each side of $\frac{x}{x+4} + \frac{1}{2} = 1$ in order to solve the equation. What value(s) of x cannot be a solution?
3. **FIND THE ERROR** Jeff and Dustin are solving $2 - \frac{3}{a} = \frac{2}{3}$.

Jeff	Dustin
$2 - \frac{3}{a} = \frac{2}{3}$	$2 - \frac{3}{a} = \frac{2}{3}$
$6a - 9 = 2a$	$2 - 9 = 2a$
$4a = 9$	$-7 = 2a$
$a = 2.25$	$-3.5 = a$

Who is correct? Explain your reasoning.

1. Sample answer: $\frac{1}{5} + \frac{2}{a+2} = 1$

2. $2(x + 4)$; -4

3. Jeff; when Dustin multiplied by $3a$, he forgot to multiply the 2 by $3a$.

Guided Practice

Solve each equation or inequality. Check your solutions.

4. $\frac{2}{d} + \frac{1}{4} = \frac{11}{12}$ **3**
5. $t + \frac{12}{t} - 8 = 0$ **2, 6**
6. $\frac{1}{x-1} + \frac{2}{x} = 0$ $\frac{2}{3}$
7. $\frac{12}{v^2 - 16} - \frac{24}{v-4} = 3$ **−6, −2**
8. $\frac{4}{c+2} > 1$ $-2 < c < 2$
9. $\frac{1}{3v} + \frac{1}{4v} < \frac{1}{2}$ $v < 0$ or $v > 1\frac{1}{7}$

GUIDED PRACTICE KEY	
Exercises	**Examples**
4–9	1, 2, 5
10	3, 4

Application

10. **WORK** A bricklayer can build a wall of a certain size in 5 hours. Another bricklayer can do the same job in 4 hours. If the bricklayers work together, how long would it take to do the job? $2\frac{2}{9}$ **h**

3 Practice/Apply

Study Notebook

Have students—

- complete the definitions/examples for the remaining terms on their Vocabulary Builder worksheets for Chapter 9.
- add the steps for solving rational inequalities given in this lesson to their notebooks, along with an example from their work.
- write a list of cautions, or checks, that must be done as part of working with problems such as those in the lesson.
- include any other item(s) that they find helpful in mastering the skills in this lesson.

DAILY INTERVENTION

FIND THE ERROR Ask students what value of a can be excluded at the beginning of the problem. **The value of a cannot be 0.**

About the Exercises…

Organization by Objective

- **Solve Rational Equations:** 11–14, 17, 18, 23–39
- **Solve Rational Inequalities:** 15, 16, 19–22

Odd/Even Assignments

Exercises 11–32 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 11–27 odd, 31, 33, 40–54

Average: 11–33 odd, 37–54

Advanced: 12–32 even, 34–36, 38–54

DAILY INTERVENTION

Differentiated Instruction

Logical Have students think about the difference between "pure" mathematics, such as solving an equation, and "applied" mathematics, such as solving a real-world problem. Ask them to list some ways in which these two are alike and some ways in which they are different.

Study Guide and Intervention, p. 547 (shown) and p. 548

Solve Rational Equations A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude any solution that would produce a denominator of zero.

Example **Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$.**

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$	Multiply each side by 10(x + 1).
$9(x+1) + 2(10) = 4(x+1)$	Multiply.
$9x + 9 + 20 = 4x + 4$	Distributive Property
$5x = -25$	Subtract 4x and 29 from each side.
$x = -5$	Divide each side by 5.

Check

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$	x = −5
$\frac{9}{10} + \frac{2}{-4} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{8}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{2}{5} = \frac{2}{5}$	

Exercises

Solve each equation.

1. $\frac{2y}{3} - \frac{y+3}{6} = 2$ **5**
2. $\frac{4t-3}{5} - \frac{4-2t}{3} = 1$ **2**
3. $\frac{2x+1}{3} - \frac{x-5}{4} = \frac{1}{2}$ **$-\frac{13}{5}$**
4. $\frac{3m+2}{5m} + \frac{2m-1}{2m} = 4$ **$-\frac{1}{24}$**
5. $\frac{4}{x-1} = \frac{x+1}{12}$ **±7**
6. $\frac{x}{x-2} + \frac{4}{x-2} = 10$ **$\frac{8}{3}$**
7. **NAVIGATION** The current in a river is 6 miles per hour. In her motorboat Marissa can travel 12 miles upstream or 16 miles downstream in the same amount of time. What is the speed of her motorboat in still water? **42 mph**
8. **WORK** Adam, Bethany, and Carlos own a painting company. To paint a particular house alone, Adam estimates that it would take him 4 days, Bethany estimates $5\frac{1}{2}$ days, and Carlos 6 days. If these estimates are accurate, how long should it take the three of them to paint the house if they work together? **about $1\frac{2}{3}$ days**

Skills Practice, p. 549 and Practice, p. 550 (shown)

Solve each equation or inequality. Check your solutions.

1. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$ **16**
2. $\frac{x}{x-1} - 1 = \frac{x}{2}$ **−1, 2**
3. $\frac{p+10}{p^2-2} = \frac{4}{p}$ **$-\frac{2}{3}$, 4**
4. $\frac{s}{s+2} + s = \frac{5s+8}{s+2}$ **4**
5. $\frac{5}{y-5} = \frac{y}{y-5} - 1$ **all reals except 5**
6. $\frac{1}{3x-2} + \frac{5}{x} = 0$ **$\frac{5}{8}$**
7. $\frac{5}{t} < \frac{9}{2t+1}$ **$t < -5$ or $-\frac{1}{2} < t < 0$**
8. $\frac{1}{2h} + \frac{5}{h} = \frac{3}{h-1}$ **$\frac{11}{5}$**
9. $\frac{4}{w-2} = \frac{-1}{w+3}$ **−2**
10. $5 - \frac{3}{a} < \frac{7}{a}$ **$0 < a < 2$**
11. $\frac{4}{5x} + \frac{1}{10} < \frac{3}{2x}$ **$0 < x < 7$**
12. $8 + \frac{3}{y} > \frac{19}{y}$ **$y < 0$ or $y > 2$**
13. $\frac{4}{p} + \frac{1}{3p} < \frac{1}{5}$ **$p < 0$ or $p > \frac{65}{3}$**
14. $\frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$ **∅**
15. $g + \frac{g}{g-2} = \frac{2}{g-2}$ **−1**
16. $b + \frac{2b}{b-1} = 1 - \frac{b-3}{b-1}$ **−2**
17. $2 = \frac{x+2}{x-3} + \frac{x-2}{x-6}$ **$\frac{14}{3}$**
18. $5 - \frac{3d+2}{d-1} = \frac{2d-4}{d+2}$ **6**
19. $\frac{1}{n+2} + \frac{1}{n-2} = \frac{3}{n^2-4}$ **$\frac{3}{2}$**
20. $\frac{c+1}{c-3} = 4 - \frac{12}{c^2-2c-3}$ **$-\frac{5}{3}$, 5**
21. $\frac{3}{k-3} + \frac{4}{k-4} = \frac{25}{k^2-7k+12}$ **7**
22. $\frac{4v}{v-1} - \frac{5v}{v-2} = \frac{2}{v^2-3v+2}$ **−1, −2**
23. $\frac{y}{y+2} + \frac{7}{y-5} = \frac{14}{y^2-3y-10}$ **0**
24. $\frac{x^2+4}{x^2-4} + \frac{x}{2-x} = \frac{2}{x+2}$ **∅**
25. $\frac{r}{r+4} + \frac{4}{r-4} = \frac{r^2+16}{r^2-16}$ **all reals except −4 and 4**
26. $3 = \frac{6a-1}{2a+7} + \frac{22}{a+5}$ **−2**
27. **BASKETBALL** Kiana has made 9 of 19 free throws so far this season. Her goal is to make 60% of her free throws. If Kiana makes her next x free throws in a row, the function $f(x) = \frac{9+x}{19+x}$ represents Kiana's new ratio of free throws made. How many successful free throws in a row will raise Kiana's percent made to 60%? **6**
28. **OPTICS** The lens equation $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ relates the distance p of an object from a lens, the distance q of the image of the object from the lens, and the focal length f of the lens. What is the distance of an object from a lens if the image of the object is 5 centimeters from the lens and the focal length of the lens is 4 centimeters? **20 cm**

Reading to Learn Mathematics, p. 551 — ELL

Pre-Activity **How are rational equations used to solve problems involving unit price?**

Read the introduction to Lesson 9-6 at the top of page 505 in your textbook.

- If you increase total number of minutes of long-distance calls from March to April, will your long-distance phone bill increase or decrease? **increase**
- Will your actual cost per minute increase or decrease? **decrease**

Reading the Lesson

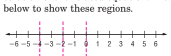
1. When solving a rational equation, any possible solution that results in 0 in the denominator must be excluded from the list of solutions.
2. Suppose that on a quiz you are asked to solve the rational inequality $\frac{3}{z+2} - \frac{6}{z} > 0$. Complete the steps of the solution.

Step 1 The excluded values are **−2** and **0**.

Step 2 The related equation is **$\frac{3}{z+2} - \frac{6}{z} = 0$**.

To solve this equation, multiply both sides by the LCD, which is **$z(z+2)$**. Solving this equation will show that the only solution is −4.

Step 3 Divide a number line into **4** regions using the excluded values and the solution of the related equation. Draw dashed vertical lines on the number line below to show these regions.



Consider the following values of $\frac{3}{z+2} - \frac{6}{z}$ for various test values of z.

If $z = -5$, $\frac{3}{z+2} - \frac{6}{z} = 0.2$.   If $z = -3$, $\frac{3}{z+2} - \frac{6}{z} = -1$.

If $z = -1$, $\frac{3}{z+2} - \frac{6}{z} = 9$.   If $z = 1$, $\frac{3}{z+2} - \frac{6}{z} = -5$.

Using this information and your number line, write the solution of the inequality.
$z < -4$ or $-2 < z < 0$

Helping You Remember

3. How are the processes of adding rational expressions with different denominators and of solving rational expressions alike, and how are they different? **Sample answer: They are alike because both use the LCD of all the rational expressions in the problem. They are different because in an addition problem, the LCD remains after the fractions are added, while in solving a rational equation, the LCD is eliminated.**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
11–30	1, 2, 5
31–39	3, 4

Extra Practice

See page 849.

19. $t < 0$ or $t > 3$
22. $p < 0$ or $p > 2\frac{1}{2}$

Solve each equation or inequality. Check your solutions.

11. $\frac{y}{y+1} = \frac{2}{3}$ **2**
12. $\frac{p}{p-2} = \frac{2}{5}$ **$-\frac{4}{3}$**
13. $s + 5 = \frac{6}{s}$ **−6, 1**
14. $a + 1 = \frac{6}{a}$ **−3, 2**
15. $\frac{7}{a+1} > 7$ **$-1 < a < 0$**
16. $\frac{10}{m+1} > 5$ **$-1 < m < 1$**
17. $\frac{9}{t-3} = \frac{t-4}{t-3} + \frac{1}{4}$ **11**
18. $\frac{w}{w-1} + w = \frac{4w-3}{w-1}$ **3**
19. $5 + \frac{1}{t} > \frac{16}{t}$
20. $7 - \frac{2}{b} < \frac{5}{b}$ **$0 < b < 1$**
21. $\frac{2}{3y} + \frac{5}{6y} > \frac{3}{4}$ **$0 < y < 2$**
22. $\frac{1}{2p} + \frac{3}{4p} < \frac{1}{2}$
23. $\frac{b-4}{b-2} = \frac{b-2}{b+2} + \frac{1}{b-2}$ **14**
24. $\frac{4n^2}{n^2-9} - \frac{2n}{n+3} = \frac{3}{n-3}$ **$\frac{3}{2}$**
25. $\frac{1}{d+4} = \frac{2}{d^2+3d-4} - \frac{1}{1-d}$ **∅**
26. $\frac{2}{y+2} - \frac{y}{2-y} = \frac{y^2+4}{y^2-4}$ **∅**
27. $\frac{3}{b^2+5b+6} + \frac{b-1}{b+2} = \frac{7}{b+3}$ **7**
28. $\frac{1}{n-2} = \frac{2n+1}{n^2+2n-8} + \frac{2}{n+4}$ **$\frac{7}{3}$**

★ 29. $\frac{2q}{2q+3} - \frac{2q}{2q-3} = 1$ **$\frac{-3 \pm 3\sqrt{2}}{2}$**

30. $\frac{4}{z-2} - \frac{z+6}{z+1} = 1$ **$\frac{1 \pm \sqrt{145}}{4}$**

31. **NUMBER THEORY** The ratio of 8 less than a number to 28 more than that number is 2 to 5. What is the number? **32**

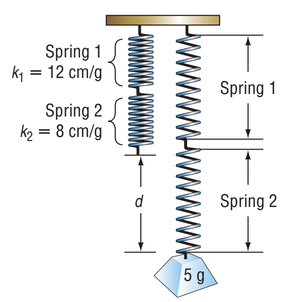
32. **NUMBER THEORY** The sum of a number and 8 times its reciprocal is 6. Find the number(s). **2 or 4**

33. **ACTIVITIES** The band has 30 more members than the school chorale. If each group had 10 more members, the ratio of their membership would be 3:2. How many members are in each group? **band, 80 members; chorale, 50 members**

PHYSICS **For Exercises 34 and 35, use the following information.**
The distance a spring stretches is related to the mass attached to the spring. This is represented by $d = km$, where d is the distance, m is the mass, and k is the spring constant. When two springs with spring constants k_1 and k_2 are attached in a series, the resulting spring constant k is found by the equation $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$.

34. If one spring with constant of 12 centimeters per gram is attached in a series with another spring with constant of 8 centimeters per gram, find the resultant spring constant. **4.8 cm/g**

35. If a 5-gram object is hung from the series of springs, how far will the springs stretch? **24 cm**



36. **CYCLING** On a particular day, the wind added 3 kilometers per hour to Alfonso's rate when he was cycling with the wind and subtracted 3 kilometers per hour from his rate on his return trip. Alfonso found that in the same amount of time he could cycle 36 kilometers with the wind, he could go only 24 kilometers against the wind. What is his normal bicycling speed with no wind? **15 km/h**

Enrichment, p. 552

Limits

Sequences of numbers with a rational expression for the general term often approach some number as a finite limit. For example, the reciprocals of the positive integers approach 0 as n gets larger and larger. This is written using the notation shown below. The symbol ∞ stands for infinity and $n \to \infty$ means that n is getting larger and larger, or "n goes to infinity."

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$ $\lim_{n \to \infty} \frac{1}{n} = 0$

Example **Find $\lim_{n \to \infty} \frac{n^2}{(n+1)^2}$**

It is not immediately apparent whether the sequence approaches a limit or not. But notice what happens if we divide the numerator and denominator of the general term by n^2.

510 Chapter 9 Rational Expressions and Equations

Career Choices

Chemist
Many chemists work for manufacturers developing products or doing quality control to ensure the products meet industry and government standards.

Online Research
For information about a career as a chemist, visit: www.algebra2.com/careers

37. CHEMISTRY Kiara adds an 80% acid solution to 5 milliliters of solution that is 20% acid. The function that represents the percent of acid in the resulting solution is $f(x) = \dfrac{5(0.20) + x(0.80)}{5 + x}$, where x is the amount of 80% solution added. How much 80% solution should be added to create a solution that is 50% acid? **5 mL**

STATISTICS **For Exercises 38 and 39, use the following information.**
A number x is the *harmonic mean* of y and z if $\frac{1}{x}$ is the average of $\frac{1}{y}$ and $\frac{1}{z}$.

★38. Find y if $x = 8$ and $z = 20$. **5**

★39. Find x if $y = 5$ and $z = 8$. **6.15**

40. **CRITICAL THINKING** Solve for a if $\frac{1}{a} - \frac{1}{b} = c$. $\boldsymbol{\frac{b}{bc + 1}}$

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

How are rational equations used to solve problems involving unit price?

Include the following in your answer:
- an explanation of how to solve $\frac{500 + 5x}{x} = 6$, and
- the reason why the actual price per minute could never be 5¢.

Standardized Test Practice

42. If $T = \frac{4st}{s - t}$, what is the value of s when $t = 5$ and $T = 40$? **B**

Ⓐ 20 Ⓑ 10 Ⓒ 5 Ⓓ 2

43. Amanda wanted to determine the average of her 6 test scores. She added the scores correctly to get T, but divided by 7 instead of 6. Her average was 12 less than the actual average. Which equation could be used to determine the value of T? **C**

Ⓐ $6T + 12 = 7T$ Ⓑ $\frac{T}{7} = \frac{T - 12}{6}$

Ⓒ $\frac{T}{7} + 12 = \frac{T}{6}$ Ⓓ $\frac{T}{6} = \frac{T - 12}{7}$

Maintain Your Skills

Mixed Review

Identify the type of function represented by each equation. Then graph the equation. *(Lesson 9-5)* **44–46. See margin for graphs.**

44. $y = 2x^2 + 1$ **quad.** 45. $y = 2\sqrt{x}$ **sq. root** 46. $y = 0.8x$ **direct var.**

47. If y varies inversely as x and $y = 24$ when $x = 9$, find y when $x = 6$. *(Lesson 9-4)* **36**

48. If y varies directly as x and $y = 9$ when $x = 4$, find y when $x = 15$. *(Lesson 9-4)* **33.75**

Find the distance between each pair of points with the given coordinates. *(Lesson 8-1)*

49. $(-5, 7), (9, -11)$ $\boldsymbol{2\sqrt{130}}$ 50. $(3, 5), (7, 3)$ $\boldsymbol{2\sqrt{5}}$ 51. $(-1, 3), (-5, -8)$ $\boldsymbol{\sqrt{137}}$

Solve each inequality. *(Lesson 6-7)*

52. $(x + 11)(x - 3) > 0$ 53. $x^2 - 4x \le 0$ 54. $2b^2 - b < 6$

52. $\{x \mid x < -11$ or $x > 3\}$
53. $\{x \mid 0 \le x \le 4\}$
54. $\left\{b \mid -1\frac{1}{2} < b < 2\right\}$

www.algebra2.com/self_check_quiz

41. If something has a general fee and cost per unit, rational equations can be used to determine how many units a person must buy in order for the actual unit price to be a given number. Answers should include the following.
- To solve $\frac{500 + 5x}{x} = 6$, multiply each side of the equation by x to eliminate the rational expression. Then subtract $5x$ from each side. Therefore, $500 = x$. A person would need to make 500 minutes of long distance minutes to make the actual unit price 6¢.
- Since the cost is 5¢ per minute plus $5.00 per month, the actual cost per minute could never be 5¢ or less.

4 Assess

Open-Ended Assessment

Writing Have students write their own real-world problems similar to some they have seen in this lesson, but using their own data. Then solve them.

Tips for New Teachers

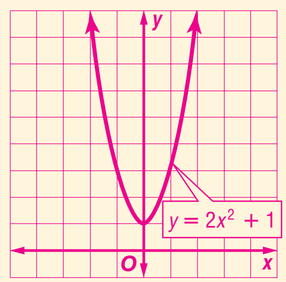
Intervention Make sure every student is clear about why some values must be excluded, even though they appear as solutions in the course of working a problem. Point out that multiplying each side of an equation by the variable may introduce extraneous roots.

Assessment Options

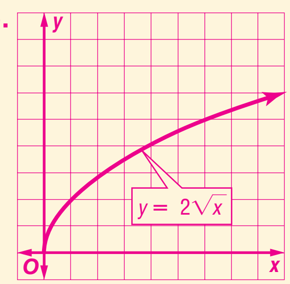
Quiz (Lesson 9-6) is available on p. 568 of the *Chapter 9 Resource Masters*.

Answers

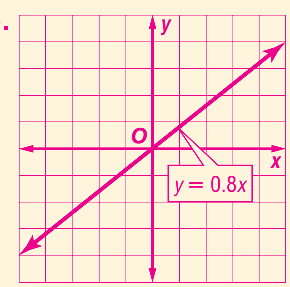
44.



45.



46.



Graphing Calculator Investigation

A Follow-Up of Lesson 9-6

Getting Started

Know Your Calculator To see the vertical asymptote for the graph of y_1, students should check to see that the calculator is in Connected mode.

Using Parentheses When students enter functions on the Y= list, they should use parentheses around any numerator or denominator that is not a single number or variable.

Teach

After they enter the two equations, have students enter the CALC menu and select 5:Intersect. Then have them move the cursor and press [ENTER] to identify each of the graphs. In response to Guess?, move the cursor to an estimated point of intersection and press [ENTER].

Assess

Ask students to describe some ways you can identify excluded values.

Graphing Calculator Investigation

A Follow-Up of Lesson 9-6

Solving Rational Equations by Graphing

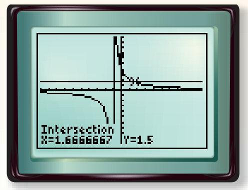
You can use a graphing calculator to solve rational equations. You need to graph both sides of the equation and locate the point(s) of intersection. You can also use a graphing calculator to confirm solutions that you have found algebraically.

Example

Use a graphing calculator to solve $\frac{4}{x+1} = \frac{3}{2}$.

- First, rewrite as two functions, $y_1 = \frac{4}{x+1}$ and $y_2 = \frac{3}{2}$.
- Next, graph the two functions on your calculator.

KEYSTROKES: [Y=] 4 [÷] [(] [X,T,θ,n] [+] 1 [)] [▼] 3 [÷] 2 [ZOOM] 6



[−10, 10] scl: 1 by [−10, 10] scl: 1

Notice that because the calculator is in connected mode, a vertical line is shown connecting the two branches of the hyperbola. This line is not part of the graph.

- Next, locate the point(s) of intersection.

KEYSTROKES: [2nd] [CALC] 5

Select one graph and press [ENTER]. Select the other graph, press [ENTER], and press [ENTER] again. The solution is $1\frac{2}{3}$. Check this solution by substitution.

Exercises

Use a graphing calculator to solve each equation.

1. $\frac{1}{x} + \frac{1}{2} = \frac{2}{x}$ **2**
2. $\frac{1}{x-4} = \frac{2}{x-2}$ **6**
3. $\frac{4}{x} = \frac{6}{x^2}$ **1.5**
4. $\frac{1}{1-x} = 1 - \frac{x}{x-1}$ **all real numbers except 1**
5. $\frac{1}{x+4} = \frac{2}{x^2+3x-4} - \frac{1}{1-x}$ **no real solution**
6. $\frac{1}{x-1} + \frac{1}{x+2} = \frac{1}{2}$ **−1, 4**

Solve each equation algebraically. Then, confirm your solution(s) using a graphing calculator. **11. $\frac{-3 \pm \sqrt{17}}{2}$ or about −3.56 and 0.56**

7. $\frac{3}{x} + \frac{7}{x} = 9$ **$1\frac{1}{9}$**
8. $\frac{1}{x-1} + \frac{2}{x} = 0$ **$\frac{2}{3}$**
9. $1 + \frac{5}{x-1} = \frac{7}{6}$ **31**
10. $\frac{1}{x^2-1} = \frac{2}{x^2+x-2}$ **0**
11. $\frac{6}{x^2+2x} - \frac{x+1}{x+2} = \frac{2}{x}$
12. $\frac{3}{x^2+5x+6} + \frac{x-1}{x+2} = \frac{7}{x+3}$ **7**

www.algebra2.com/other_calculator_keystrokes

Chapter 9

Study Guide and Review

Vocabulary and Concept Check

asymptote (p. 485)	direct variation (p. 492)	rational equation (p. 505)
complex fraction (p. 475)	inverse variation (p. 493)	rational expression (p. 472)
constant of variation (p. 492)	joint variation (p. 493)	rational function (p. 485)
continuity (p. 485)	point discontinuity (p. 485)	rational inequality (p. 508)

State whether each sentence is *true* or *false*. If false, replace the underlined word or number to make a true sentence.

1. The equation $y = \frac{x^2-1}{x+1}$ has a(n) <u>*asymptote*</u> at $x = -1$. **false; point discontinuity**
2. The equation $y = 3x$ is an example of a <u>*direct*</u> variation equation. **true**
3. The equation $y = \frac{x^2}{x+1}$ is a(n) <u>*polynomial*</u> equation. **false; rational**
4. The graph of $y = \frac{4}{x-4}$ has a(n) <u>*variation*</u> at $x = 4$. **false; asymptote**
5. The equation $b = \frac{2}{a}$ is a(n) <u>*inverse*</u> variation equation. **true**
6. On the graph of $y = \frac{x-5}{x+2}$, there is a break in continuity at $x = $ <u>2</u>. **false; −2**

Lesson-by-Lesson Review

9-1 Multiplying and Dividing Rational Expressions

See pages 472–478.

Concept Summary

- Multiplying and dividing rational expressions is similar to multiplying and dividing fractions.

Examples

1 Simplify $\frac{3x}{2y} \cdot \frac{8y^3}{6x^2}$.

$$\frac{3x}{2y} \cdot \frac{8y^3}{6x^2} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{y}} \cdot y \cdot y}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{y}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{x}} \cdot x}$$

$$= \frac{2y^2}{x}$$

2 Simplify $\frac{p^2+7p}{3p} \div \frac{49-p^2}{3p-21}$.

$$\frac{p^2+7p}{3p} \div \frac{49-p^2}{3p-21} = \frac{p^2+7p}{3p} \cdot \frac{3p-21}{49-p^2}$$

$$= \frac{\overset{1}{\cancel{p}}\overset{1}{\cancel{(7+p)}}}{\underset{1}{\cancel{3}}\underset{1}{\cancel{p}}} \cdot \frac{\overset{-1}{\cancel{-3}}\cancel{(7-p)}}{\underset{1}{\cancel{(7+p)}}\underset{1}{\cancel{(7-p)}}}$$

$$= -1$$

Exercises Simplify each expression. *See Examples 4–7 on pages 474 and 475.*

7. $\frac{-4ab}{21c} \cdot \frac{14c^2}{22a^2}$ $-\frac{4bc}{33a}$
8. $\frac{a^2-b^2}{6b} \div \frac{a+b}{36b^2}$ $6b(a-b)$
9. $\frac{y^2-y-12}{y+2} \div \frac{y-4}{y^2-4y-12}$ $(y+3)(y-6)$
10. $\dfrac{\frac{x^2+7x+10}{x+2}}{\frac{x^2+2x-15}{x+2}}$ $\frac{x+2}{x-3}$
11. $\dfrac{\frac{1}{n^2-6n+9}}{\frac{n+3}{2n^2-18}}$ $\frac{2}{n-3}$
12. $\frac{x^2+3x-10}{x^2+8x+15} \cdot \frac{x^2+5x+6}{x^2+4x+4}$

12. $\frac{(x-2)(x+2)}{(x+3)^2}$

www.algebra2.com/vocabulary_review

Chapter 9

Study Guide and Review

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 9 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 9 is available on p. 566 of the *Chapter 9 Resource Masters*.

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

FOLDABLES™ Study Organizer

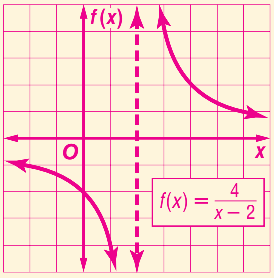
For more information about Foldables, see *Teaching Mathematics with Foldables.*

Suggest that students think of a concept map as a visual organizer that is related to a linear outline, but better shows interrelated ideas. Remind students that different people will organize, remember, and study differently, so they should make a Foldable that works well for them, rather than copying someone else's way of doing notes.

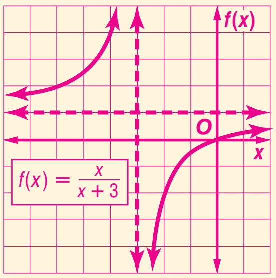
Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Answers

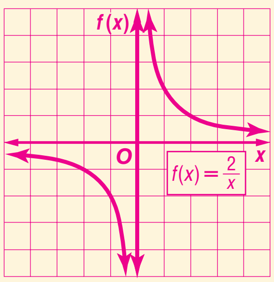
19.



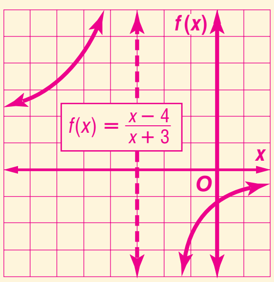
20.



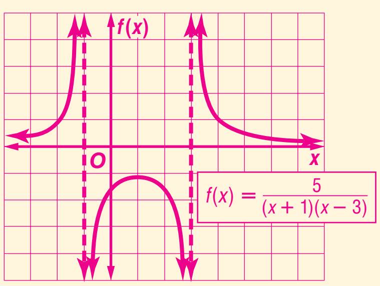
21.



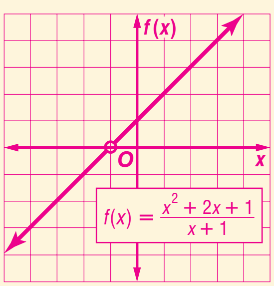
22.



23.



24.



9-2 Adding and Subtracting Rational Expressions

See pages 479–484.

Concept Summary

- To add or subtract rational expressions, find a common denominator.
- To simplify complex fractions, simplify the numerator and the denominator separately, and then simplify the resulting expression.

Example Simplify $\frac{14}{x+y} - \frac{9x}{x^2-y^2}$.

$$\frac{14}{x+y} - \frac{9x}{x^2-y^2} = \frac{14}{x+y} - \frac{9x}{(x+y)(x-y)} \quad \text{Factor the denominators.}$$

$$= \frac{14(x-y)}{(x+y)(x-y)} - \frac{9x}{(x+y)(x-y)} \quad \text{The LCD is } (x+y)(x-y).$$

$$= \frac{14(x-y)-9x}{(x+y)(x-y)} \quad \text{Subtract the numerators.}$$

$$= \frac{14x-14y-9x}{(x+y)(x-y)} \quad \text{Distributive Property}$$

$$= \frac{5x-14y}{(x+y)(x-y)} \quad \text{Simplify.}$$

Exercises Simplify each expression. *See Examples 3 and 4 on page 480.*

13. $\frac{x+2}{x-5} + 6$ **$\frac{7(x-4)}{x-5}$**

14. $\frac{x-1}{x^2-1} + \frac{2}{5x+5}$ **14. $\frac{7}{5(x+1)}$**

15. $\frac{7}{y} - \frac{2}{3y}$ **$\frac{19}{3y}$**

16. $\frac{7}{y-2} - \frac{11}{2-y}$ **$\frac{18}{y-2}$**

17. $\frac{3}{4b} - \frac{2}{5b} - \frac{1}{2b}$ **$-\frac{3}{20b}$**

18. $\frac{m+3}{m^2-6m+9} - \frac{8m-24}{9-m^2}$ **$\frac{3(3m^2-14m+27)}{(m+3)(m-3)^2}$**

9-3 Graphing Rational Functions

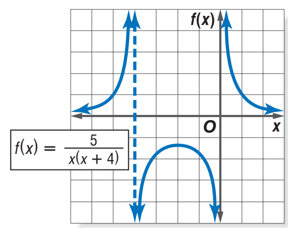
See pages 485–490.

Concept Summary

- Functions are undefined at any x value where the denominator is zero.
- An asymptote is a line that the graph of the function approaches, but never crosses.

Example Graph $f(x) = \frac{5}{x(x+4)}$.

The function is undefined for $x = 0$ and $x = -4$. Since $\frac{5}{x(x+4)}$ is in simplest form, $x = 0$ and $x = -4$ are vertical asymptotes. Draw the two asymptotes and sketch the graph.



19–24. See margin.

Exercises Graph each rational function. *See Examples 2–4 on pages 486–488.*

19. $f(x) = \frac{4}{x-2}$

20. $f(x) = \frac{x}{x+3}$

21. $f(x) = \frac{2}{x}$

22. $f(x) = \frac{x-4}{x+3}$

23. $f(x) = \frac{5}{(x+1)(x-3)}$

24. $f(x) = \frac{x^2+2x+1}{x+1}$

514 Chapter 9 Rational Expressions and Equations

9-4 Direct, Joint, and Inverse Variation

See pages 492–498.

Concept Summary

- Direct Variation: There is a nonzero constant k such that $y = kx$.
- Joint Variation: There is a number k such that $y = kxz$, where $x \neq 0$ and $z \neq 0$.
- Inverse Variation: There is a nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$.

Example **If y varies inversely as x and $x = 14$ when $y = -6$, find x when $y = -11$.**

$\frac{x_1}{y_2} = \frac{x_2}{y_1}$ Inverse variation

$\frac{14}{-11} = \frac{x_2}{-6}$ $x_1 = 14, y_1 = -6, y_2 = -11$

$14(-6) = -11(x_2)$ Cross multiply.

$-84 = -11x_2$ Simplify.

$7\frac{7}{11} = x_2$ When $y = -11$, the value of x is $7\frac{7}{11}$.

Exercises **Find each value.** *See Examples 1–3 on pages 493 and 494.*

25. If y varies directly as x and $y = 21$ when $x = 7$, find x when $y = -5$. $-1\frac{2}{3}$

26. If y varies inversely as x and $y = 9$ when $x = 2.5$, find y when $x = -0.6$. -37.5

27. If y varies inversely as x and $x = 28$ when $y = 18$, find x when $y = 63$. 8

28. If y varies directly as x and $x = 28$ when $y = 18$, find x when $y = 63$. 98

29. If y varies jointly as x and z and $x = 2$ and $z = 4$ when $y = 16$, find y when $x = 5$ and $z = 8$. 80

9-5 Classes of Functions

See pages 499–504.

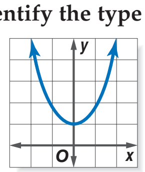
Concept Summary

The following is a list of special functions.

- constant function
- direct variation function
- identity function
- greatest integer function
- absolute value function
- quadratic function
- square root function
- rational function
- inverse variation function

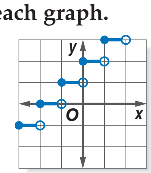
Examples **Identify the type of function represented by each graph.**

1



The graph has a parabolic shape, therefore it is a quadratic function.

2

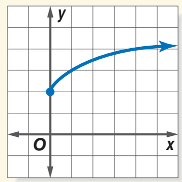


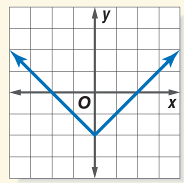
The graph has a stair-step pattern, therefore it is a greatest integer function.

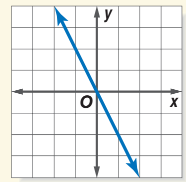
Chapter 9 Study Guide and Review 515

Chapter 9 For More ...
• Extra Practice, see pages 847–849.
• Mixed Problem Solving, see page 870.

Exercises Identify the type of function represented by each graph.
See Example 1 on page 500.

30. 

31. 

32. 

square root — absolute value — direct variation

9-6 Solving Rational Equations and Inequalities

See pages 505–511.

Concept Summary

- Eliminate fractions in rational equations by multiplying each side of the equation by the LCD.
- Possible solutions to a rational equation must exclude values that result in zero in the denominator.
- To solve rational inequalities, find the excluded values, solve the related equation, and use these values to divide a number line into regions. Then test a value in each region to determine which regions satisfy the original inequality.

Example Solve $\frac{1}{x-1} + \frac{2}{x} = 0$.

The LCD is $x(x-1)$.

$$\frac{1}{x-1} + \frac{2}{x} = 0 \qquad \text{Original equation}$$

$$x(x-1)\left(\frac{1}{x-1} + \frac{2}{x}\right) = x(x-1)(0) \qquad \text{Multiply each side by } x(x-1).$$

$$x(x-1)\left(\frac{1}{x-1}\right) + x(x-1)\left(\frac{2}{x}\right) = x(x-1)(0) \qquad \text{Distributive Property}$$

$$1(x) + 2(x-1) = 0 \qquad \text{Simplify.}$$

$$x + 2x - 2 = 0 \qquad \text{Distributive Property}$$

$$3x - 2 = 0 \qquad \text{Simplify.}$$

$$3x = 2 \qquad \text{Add 2 to each side.}$$

$$x = \frac{2}{3} \qquad \text{Divide each side by 3.}$$

The solution is $\frac{2}{3}$.

Exercises Solve each equation or inequality. Check your solutions.
See Examples 1, 2, and 5 on pages 505, 506, 508, and 509.

33. $\frac{3}{y} + \frac{7}{y} = 9$ $1\frac{1}{9}$

34. $1 + \frac{5}{y-1} = \frac{7}{6}$ 31

35. $\frac{3x+2}{4} = \frac{9}{4} - \frac{3-2x}{6}$ 3

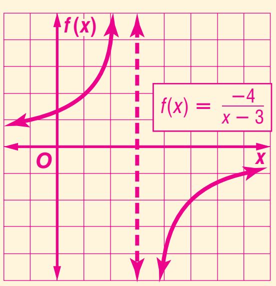
36. $\frac{1}{r^2-1} = \frac{2}{r^2+r-2}$ 0

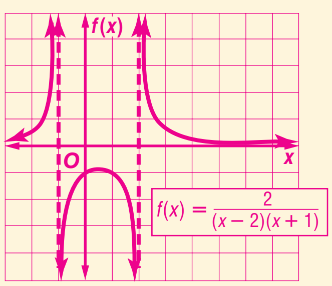
37. $\frac{x}{x^2-1} + \frac{2}{x+1} = 1 + \frac{1}{2x-2}$ $1\frac{1}{2}$

38. $\frac{1}{3b} - \frac{3}{4b} > \frac{1}{6}$ $-2\frac{1}{2} < b < 0$

Answers

Practice Test

12. 

13. 

Chapter 9

Practice Test

Vocabulary and Concepts

Match each example with the correct term.

1. $y = 4xz$ **c**
2. $y = 5x$ **b**
3. $y = \frac{7}{x}$ **a**

a. inverse variation equation
b. direct variation equation
c. joint variation equation

Skills and Applications

Simplify each expression.

4. $\frac{a^2 - ab}{3a} \div \frac{a - b}{15b^2}$ $\mathbf{5b^2}$

5. $\frac{x^2 - y^2}{y^2} \cdot \frac{y^3}{y - x}$ $\mathbf{-y(x + y)}$

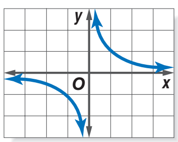
6. $\frac{x^2 - 2x + 1}{y - 5} \div \frac{x - 1}{y^2 - 25}$ $\mathbf{(x - 1)(y + 5)}$

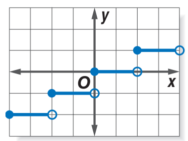
7. $\dfrac{\frac{x^2 - 1}{x^2 - 3x - 10}}{\frac{x^2 + 3x + 2}{x^2 - 12x + 35}}$ $\mathbf{\frac{(x - 1)(x - 7)}{(x + 2)^2}}$

8. $\frac{x - 2}{x - 1} + \frac{6}{7x - 7}$ $\mathbf{\frac{7x - 8}{7(x - 1)}}$

9. $\frac{x}{x^2 - 9} + \frac{1}{2x + 6}$ $\mathbf{\frac{3(x - 1)}{2(x - 3)(x + 3)}}$

Identify the type of function represented by each graph.

10. 
inverse variation

11. 
greatest integer

Graph each rational function. **12–13. See margin.**

12. $f(x) = \frac{-4}{x - 3}$

13. $f(x) = \frac{2}{(x - 2)(x + 1)}$

Solve each equation or inequality.

14. $\frac{2}{x - 1} = 4 - \frac{x}{x - 1}$ **2**

15. $\frac{9}{28} + \frac{3}{z + 2} = \frac{3}{4}$ **5**

16. $5 + \frac{3}{t} > -\frac{2}{t}$ $\mathbf{t < -1 \text{ or } t > 0}$

17. $x + \frac{12}{x} - 8 = 0$ **2, 6**

18. $\frac{5}{6} - \frac{2m}{2m + 3} = \frac{19}{6}$ $\mathbf{-1\frac{1}{20}}$

19. $\frac{x - 3}{2x} = \frac{x - 2}{2x + 1} - \frac{1}{2}$ $\mathbf{\pm\frac{\sqrt{6}}{2}}$

20. If y varies inversely as x and $y = 9$ when $x = -\frac{2}{3}$, find x when $y = -7$. $\mathbf{\frac{6}{7}}$

21. If g varies directly as w and $g = 10$ when $w = -3$, find w when $g = 4$. $\mathbf{-1\frac{1}{5}}$

22. Suppose y varies jointly as x and z. If $x = 10$ when $y = 250$ and $z = 5$, find x when $y = 2.5$ and $z = 4.5$. $\mathbf{\frac{1}{9}}$

23. **AUTO MAINTENANCE** When air is pumped into a tire, the pressure required varies inversely as the volume of the air. If the pressure is 30 pounds per square inch when the volume is 140 cubic inches, find the pressure when the volume is 100 cubic inches. $\mathbf{42\ lb/in^2}$

24. **ELECTRICITY** The current I in a circuit varies inversely with the resistance R.
a. Use the table at the right to write an equation relating the current and the resistance. $\mathbf{I = \frac{6}{R}}$
b. What is the constant of variation? **6**

I	0.5	1.0	1.5	2.0	2.5	3.0	5.0
R	12.0	6.0	4.0	3.0	2.4	2.0	1.2

25. **STANDARDIZED TEST PRACTICE** If $m = \frac{1}{x}$, $n = 7m$, $p = \frac{1}{n}$, $q = 14p$, and $r = \frac{1}{\frac{1}{2}q}$, find x. **D**

(A) r (B) q (C) p (D) $\frac{1}{r}$ (E) $\frac{1}{q}$

www.algebra2.com/chapter_test

Chapter 9 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 9 can be found on p. 566 of the *Chapter 9 Resource Masters*.

Chapter Tests There are six Chapter 9 Tests and an Open-Ended Assessment task available in the *Chapter 9 Resource Masters*.

Chapter 9 Tests			
Form	**Type**	**Level**	**Pages**
1	MC	basic	553–554
2A	MC	average	555–556
2B	MC	average	557–558
2C	FR	average	559–560
2D	FR	average	561–562
3	FR	advanced	563–564

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment
Performance tasks for Chapter 9 can be found on p. 565 of the *Chapter 9 Resource Masters*. A sample scoring rubric for these tasks appears on p. A25.

TestCheck and Worksheet Builder

This **networkable software** has three modules for assessment.

- **Worksheet Builder** to make worksheets and tests.
- **Student Module** to take tests on-screen.
- **Management System** to keep student records.

Portfolio Suggestion

Introduction In this lesson, you have been working with several kinds of variation.

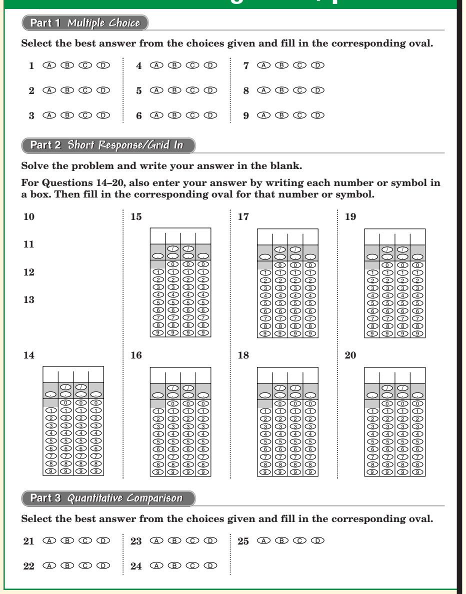
Ask Students Write an application problem for each of the three types of variations (direct, inverse, and joint), and show your steps for each problem. Then describe how your problems are modeled by each of the variations, and how your answers relate to the solutions for the problems.

Chapter 9 Standardized Test Practice

These two pages contain practice questions in the various formats that can be found on the most frequently given standardized tests.

A practice answer sheet for these two pages can be found on p. A1 of the *Chapter 9 Resource Masters*.

Standardized Test Practice Student Recording Sheet, p. A1


Part 1 Multiple Choice

Select the best answer from the choices given and fill in the corresponding oval.

1 Ⓐ Ⓑ Ⓒ Ⓓ 4 Ⓐ Ⓑ Ⓒ Ⓓ 7 Ⓐ Ⓑ Ⓒ Ⓓ

2 Ⓐ Ⓑ Ⓒ Ⓓ 5 Ⓐ Ⓑ Ⓒ Ⓓ 8 Ⓐ Ⓑ Ⓒ Ⓓ

3 Ⓐ Ⓑ Ⓒ Ⓓ 6 Ⓐ Ⓑ Ⓒ Ⓓ 9 Ⓐ Ⓑ Ⓒ Ⓓ

Part 2 Short Response/Grid In

Solve the problem and write your answer in the blank.

For Questions 14–20, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

10 15 17 19

11

12

13

14 16 18 20

Part 3 Quantitative Comparison

Select the best answer from the choices given and fill in the corresponding oval.

21 Ⓐ Ⓑ Ⓒ Ⓓ 23 Ⓐ Ⓑ Ⓒ Ⓓ 25 Ⓐ Ⓑ Ⓒ Ⓓ

22 Ⓐ Ⓑ Ⓒ Ⓓ 24 Ⓐ Ⓑ Ⓒ Ⓓ

Additional Practice

See pp. 571–572 in the *Chapter 9 Resource Masters* for additional standardized test practice.

Chapter 9 Standardized Test Practice

Part 1 Multiple Choice

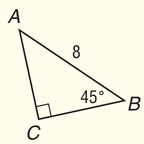
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Best Bikes has 5000 bikes in stock on May 1. By the end of May, 40 percent of the bikes have been sold. By the end of June, 40 percent of the remaining bikes have been sold. How many bikes remain unsold? **C**

Ⓐ 1000 Ⓑ 1200

Ⓒ 1800 Ⓓ 2000

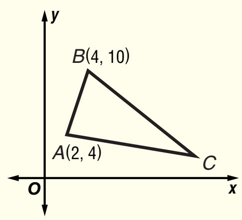
2. In $\triangle ABC$, if AB is equal to 8, then BC is equal to **C**



Ⓐ $\frac{\sqrt{2}}{8}$. Ⓑ 4.

Ⓒ $4\sqrt{2}$. Ⓓ 8.

3. In the figure, the slope of $\overline{AC}$ is $-\frac{1}{3}$ and $m\angle C = 30°$. What is the length of $\overline{BC}$? **D**



Ⓐ $\sqrt{10}$ Ⓑ $2\sqrt{10}$

Ⓒ $3\sqrt{10}$ Ⓓ $4\sqrt{10}$

4. Given that $-|2 - 4k| = -14$, which of the following could be k? **B**

Ⓐ 5 Ⓑ 4

Ⓒ 3 Ⓓ 2

5. In a hardware store, n nails cost c cents. Which of the following expresses the cost of k nails? **B**

Ⓐ nck Ⓑ $\frac{kc}{n}$

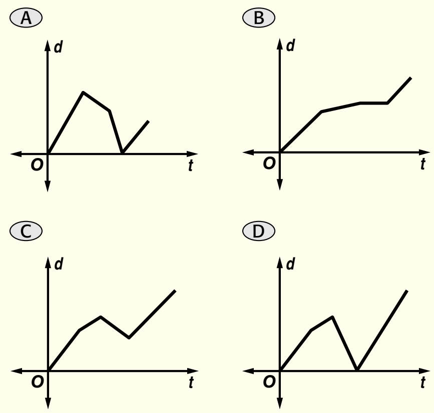
Ⓒ $n + \frac{k}{c}$ Ⓓ $n + \frac{c}{n}$

6. If $5w + 3 \leq w - 9$, then **D**

Ⓐ $w \leq 3$. Ⓑ $w \geq 3$.

Ⓒ $w \leq 12$. Ⓓ $w \leq -3$.

7. The graphs show a driver's distance d from a designated point as a function of time t. The driver passed the designated point at 60 mph and continued at that speed for 2 hours. Then she slowed to 50 mph for 1 hour. She stopped for gas and lunch for 1 hour and then drove at 60 mph for 1 hour. Which graph best represents this trip? **B**



8. Which equation has roots of $-2n$, $2n$, and 2? **D**

Ⓐ $2x^2 - 8n^2 = 0$

Ⓑ $8n^2 - 2x^2 = 0$

Ⓒ $x^3 - 2x^2 - 4n^2x - 8n^2 = 0$

Ⓓ $x^3 - 2x^2 - 4n^2x + 8n^2 = 0$

9. What point is on the graph of $y - x^2 = 2$ and has a y-coordinate of 5? **A**

Ⓐ $(-\sqrt{3}, 5)$ Ⓑ $(\sqrt{7}, 5)$

Ⓒ $(5, \sqrt{3})$ Ⓓ $(3, 5)$

Log On for Test Practice

The Princeton Review offers additional test-taking tips and practice problems at their web site. Visit www.princetonreview.com or www.review.com

TestCheck and Worksheet Builder

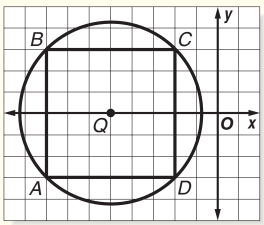
Special banks of standardized test questions similar to those on the SAT, ACT, TIMSS 8, NAEP 8, and Algebra 1 End-of-Course tests can be found on this CD-ROM.

Aligned and verified by The Princeton Review

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. In the figure, what is the equation of the circle Q that is circumscribed around the square $ABCD$? **$(x + 5)^2 + y^2 = 18$**



11. Find one possible value for k such that k is an integer between 20 and 40 that has a remainder of 2 when it is divided by 3 and that has a remainder of 2 when divided by 4. **26 or 38**

12. The coordinates of the vertices of a triangle are $(2, -4)$, $(10, -4)$, and (a, b). If the area of the triangle is 36 square units, what is a possible value for b? **-13 or 5**

13. If $(x + 2)(x - 3) = 6$, what is a possible value of x? **-3 or 4**

14. If the average of five consecutive even integers is 76, what is the greatest of these integers? **80**

15. In May, Hank's Camping Supply Store sold 45 tents. In June, it sold 90 tents. What is the percent increase in the number of tents sold? **100**

16. If $2^{n-4} = 64$, what is the value of n? **10**

17. If $xy = 5$ and $x^2 + y^2 = 20$, what is the value of $(x + y)^2$? **30**

18. If $\frac{2}{a} - \frac{8}{a^2} = \frac{-8}{a^3}$, then what is the value of a? **2**

19. If $\sqrt[x]{80} = 2\sqrt[x]{5}$, what is the value of x? **4**

20. What is the y-intercept of the graph of $3x + 2 = 4y - 6$? **2**

www.algebra2.com/standardized_test

Part 3 Quantitative Comparison

Compare the quantity in Column A and the quantity in Column B. Then determine whether:

- Ⓐ **the quantity in Column A is greater,**
- Ⓑ **the quantity in Column B is greater,**
- Ⓒ **the two quantities are equal, or**
- Ⓓ **the relationship cannot be determined from the information given.**

	Column A	Column B
21. **A**	the number of distinct prime factors of 105	the number of distinct prime factors of 189
22. **D** $\frac{x}{y} = \frac{3}{7}$	x	y
23. **D**	t	$3t$
24. **A** $0 < x < 1$	x^2	x^3
25. **B** $0 < x < 1$	x	$\sqrt{x}$

The Princeton Review Test-Taking Tip

Questions 22–25 In quantitative comparison questions that involve variables, make sure you consider all of the possible values of the variables before you make a comparison. Consider positive and negative integers, positive and negative fractions, and 0.

Chapter 9 Standardized Test Practice 519

Page 477, Lesson 9-1

49. A rational expression can be used to express the fraction of a nut mixture that is peanuts. Answers should include the following.

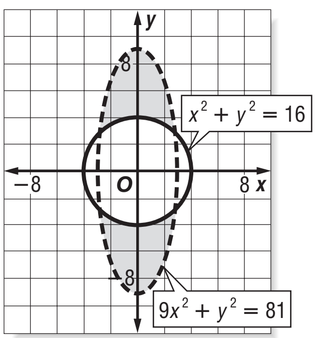
- The rational expression $\frac{8+x}{13+x}$ is in simplest form because the numerator and the denominator have no common factors.
- Sample answer: $\frac{8+x}{13+x+y}$ could be used to represent the fraction that is peanuts if x pounds of peanuts and y pounds of cashews were added to the original mixture.

Page 484, Lesson 9-2

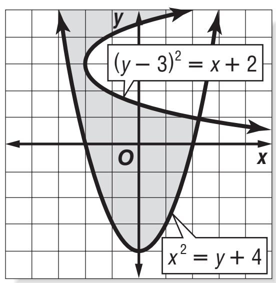
51. Subtraction of rational expressions can be used to determine the distance between the lens and the film if the focal length of the lens and the distance between the lens and the object are known. Answers should include the following.

- To subtract rational expressions, first find a common denominator. Then, write each fraction as an equivalent fraction with the common denominator. Subtract the numerators and place the difference over the common denominator. If possible, reduce the answer.
- $\frac{1}{q} = \frac{1}{10} - \frac{1}{60}$ could be used to determine the distance between the lens and the film if the focal length of the lens is 10 cm and the distance between the lens and the object is 60 cm.

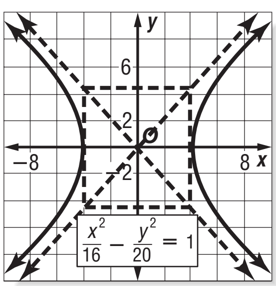
56.



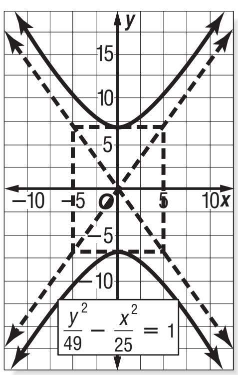
57.



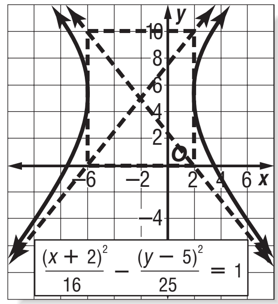
59.



60.

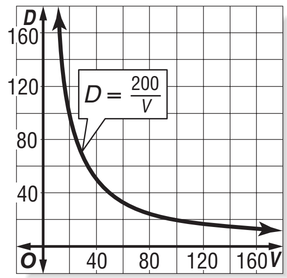


61.



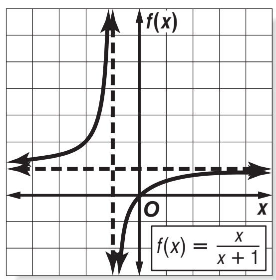
Page 487, Algebra Activity

1.

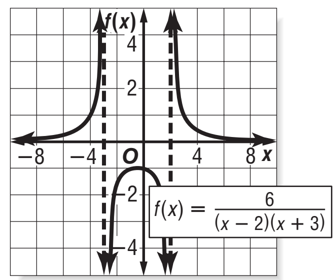


Pages 488–489, Lesson 9-3

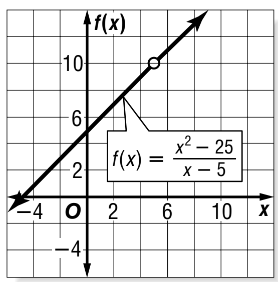
6.



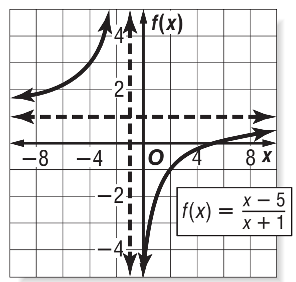
7.



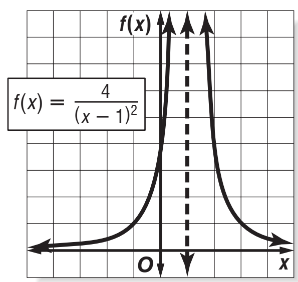
8.



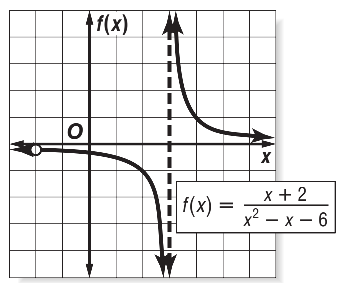
9.



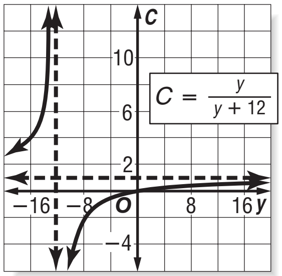
10.



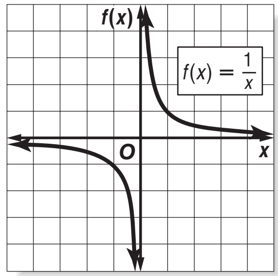
11.



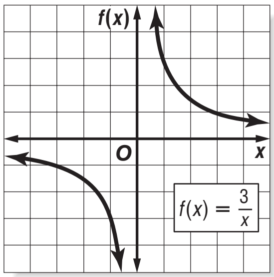
13.



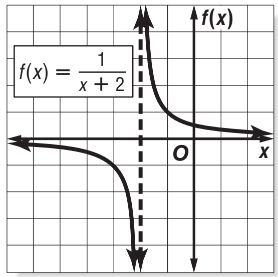
22.



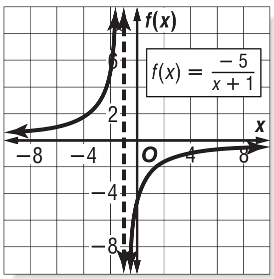
23.



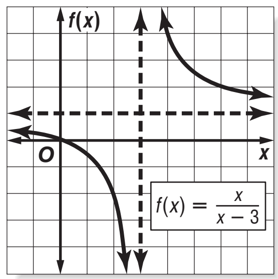
24.



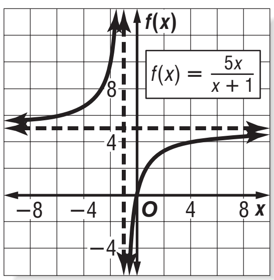
25.



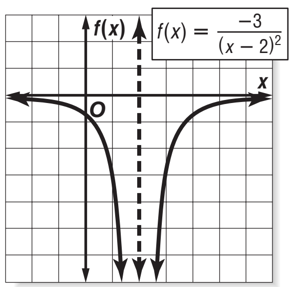
26.



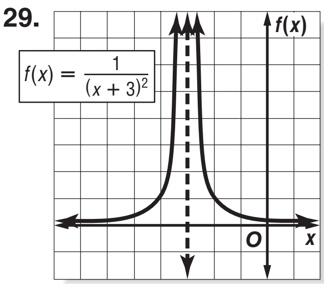
27.



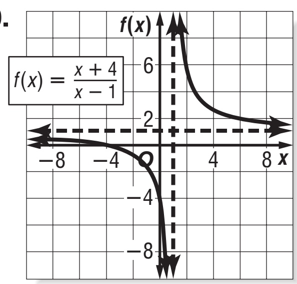
28.



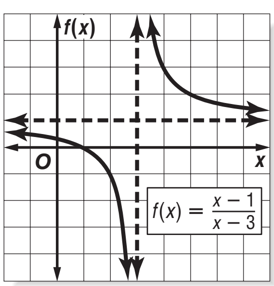
29.



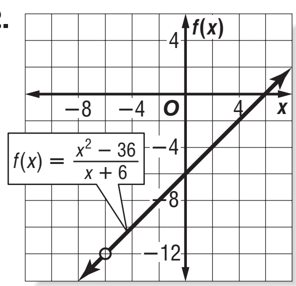
30.



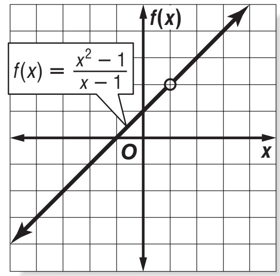
31.



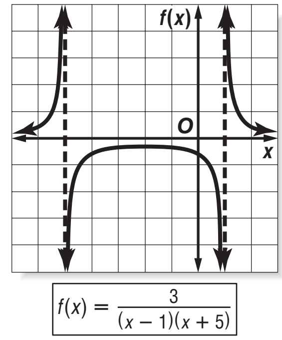
32.



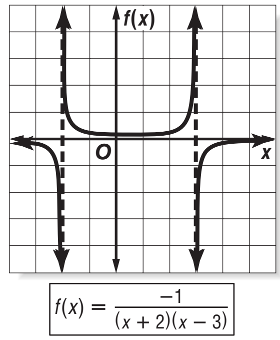
33.



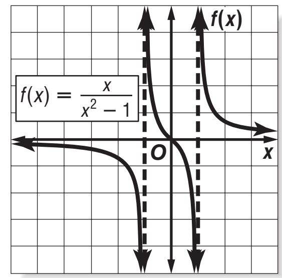
34.



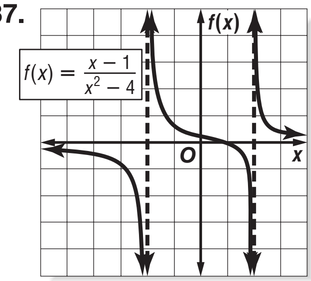
35.



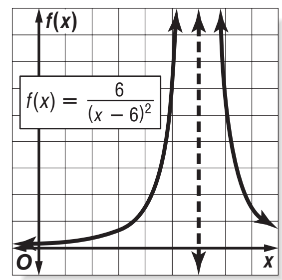
36.



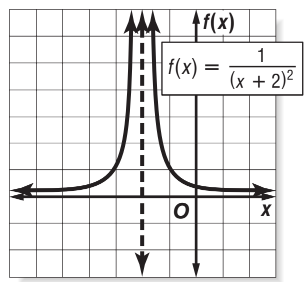
37.



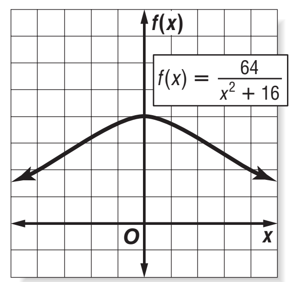
38.



39.

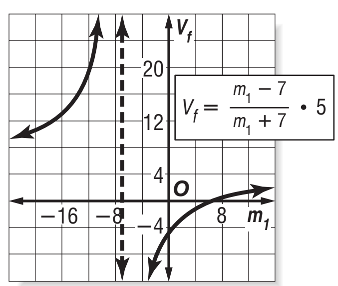


40.



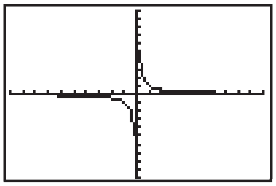
42. Since $\frac{-64}{x^2 + 16} = -\left(\frac{64}{x^2 + 16}\right)$, the graph of $f(x) = \frac{-64}{x^2 + 16}$ would be a reflection of the graph of $f(x) = \frac{64}{x^2 + 16}$ over the x-axis.

43.

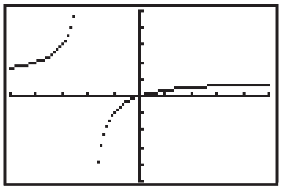


Page 491, Follow-Up of Lesson 9-3 Graphing Calculator Investigation

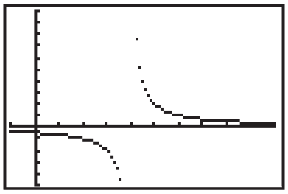
1.



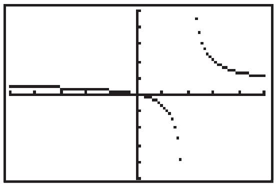
2.



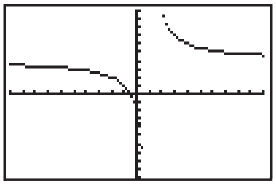
3.



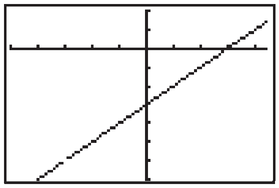
4.



5.

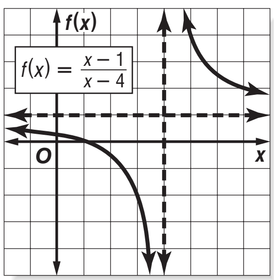


6.

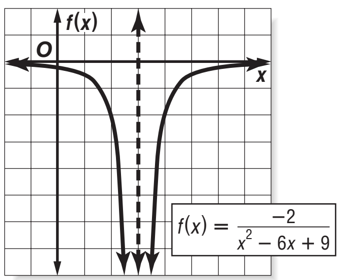


Page 498, Practice Quiz 2

1.

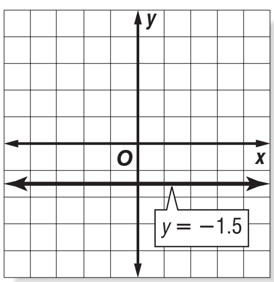


2.

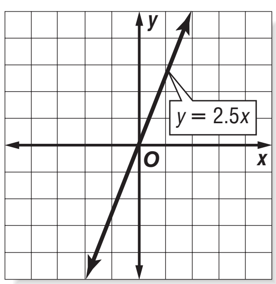


Pages 502–504, Lesson 9-5

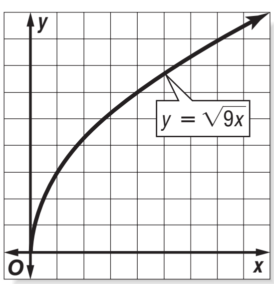
23.



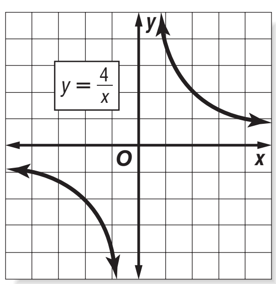
24.



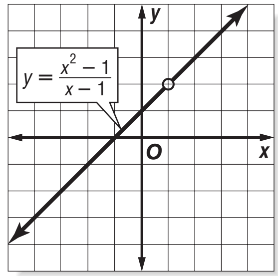
25.



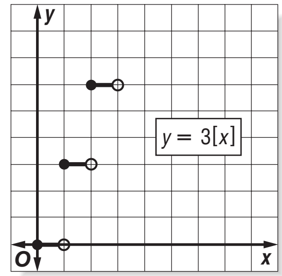
26.



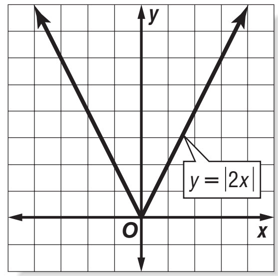
27.



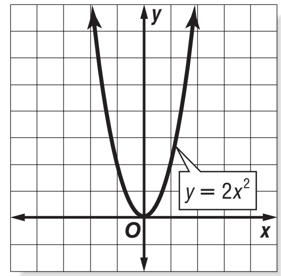
28.



29.

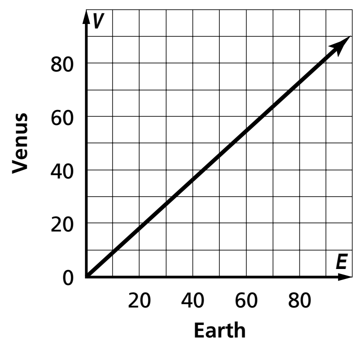


30.

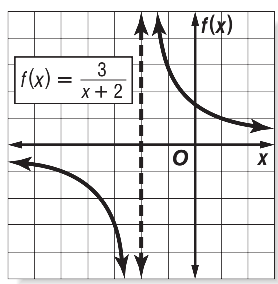


38. A graph of the function that relates a person's weight on Earth with his or her weight on a different planet can be used to determine a person's weight on the other planet by finding the point on the graph that corresponds with the weight on Earth and determining the value on the other planet's axis. Answers should include the following.

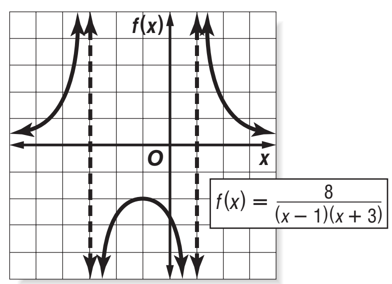
- The graph comparing weight on Earth and Mars represents a direct variation function because it is a straight line passing through the origin and is neither horizontal nor vertical.
- The equation $V = 0.9E$ compares a person's weight on Earth with his or her weight on Venus.



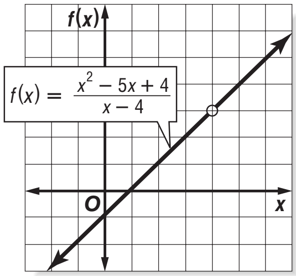
42.



43.



44.



Additional Answers for Chapter 9