Honors Advanced Algebra DATE:_____

6-5 Solving Radical Equations

Target 4D. Solve an equation involving radicals or rational exponents and identify, if any extraneous solutions

Solving Radical Equations

Solve. Check for extraneous solutions.

1. $\sqrt{x+1}+2=4$

$-2 \quad -2$

$(\sqrt{x+1})^2=(2)^2$ "square both sides"

$x+1=4$

$-1 \quad -1$

$\boxed{x=3}$

CHECK: $\sqrt{x+1}+2=4$

$\sqrt{3+1}+2=4$

$\sqrt{4}+2=4$

$2+2=4$

$4=4$ ✓

Recall we can check to see that $x=3$ is a solution by graphing $y=\sqrt{x+1}+2$ and $y=4$ and finding the intersection of the two graphs.

2. $(\sqrt{x-15})^2=(3-\sqrt{x})^2$ "square both sides"

$x-15=(3-\sqrt{x})(3-\sqrt{x})$

	3	$-\sqrt{x}$
3	9	$-3\sqrt{x}$
$-\sqrt{x}$	$-3\sqrt{x}$	$\sqrt{x^2}=x$

$x-15=9-6\sqrt{x}+x$

$-x \qquad -x$

$-15=9-6\sqrt{x}$

$-9 \quad -9$

$-24=-6\sqrt{x}$

$\frac{-24}{-6}=\frac{-6\sqrt{x}}{-6}$

$(4)^2=(\sqrt{x})^2$

$\boxed{16=x}$

CHECK: $\sqrt{x-15} \stackrel{?}{\neq} 3-\sqrt{x}$

$\sqrt{16-15} \stackrel{?}{\neq} 3-\sqrt{16}$

$\sqrt{1} \stackrel{?}{\neq} 3-4$

$1 \stackrel{?}{\neq} -1$ Doesn't ✓

NO SOLUTION

3. $(\sqrt{x^2-2x})^2=(\sqrt{-6x+21})^2$ "Square both sides"

$x^2-2x=-6x+21$

$+6x \quad +6x$

$x^2+4x=21$

$-21 \quad -21$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x+7=0$ or $x-3=0$

$\boxed{x=-7}$ or $\boxed{x=3}$

CHECK:

$x=-7$

$\sqrt{(-7)^2-2(-7)}=\sqrt{-6(-7)+21}$

$\sqrt{49+14}=\sqrt{42+21}$

$\sqrt{63}=\sqrt{63}$ ✓

$x=3$

$\sqrt{(3)^2-2(3)}=\sqrt{-6(3)+21}$

$\sqrt{9-6}=\sqrt{-18+21}$

$\sqrt{3}=\sqrt{3}$ ✓

4. $5\sqrt[3]{m}+28=8$ "cube both sides"

$-28 \quad -28$

$\frac{5\sqrt[3]{m}}{5}=\frac{-20}{5}$

$(\sqrt[3]{m})^3=(-4)^3$

$\boxed{m=-64}$

CHECK:

$5\sqrt[3]{m}+28=8$

$5\sqrt[3]{-64}+28=8$

$5\sqrt[3]{(-4)^3}+28=8$

$5(-4)+28=8$

$-20+28=8$

$8=8$ ✓

5. $\frac{3}{3}(n-2)^{\frac{3}{4}} = \frac{24}{3}$

$(n-2)^{\frac{3}{4}} = 8$

$\left((n-2)^{\frac{3}{4}}\right)^{\frac{4}{3}} = (8)^{\frac{4}{3}}$ "Raise to Reciprocal power"

$(n-2)^{\frac{12}{12}} = 8^{\frac{4}{3}}$

$n-2 = (2^3)^{\frac{4}{3}}$

$n-2 = 2^4$

$n-2 = 16$

$+2 \quad +2$

$\boxed{n = 18}$

CHECK:

$3(n-2)^{\frac{3}{4}} = 24$

$3(18-2)^{\frac{3}{4}} = 24$

$3(16)^{\frac{3}{4}} = 24$

$3\cdot(2^4)^{\frac{3}{4}} = 24$

$3\cdot 2^3 = 24$

$3\cdot 8 = 24$

$24 = 24$ ✓

6. $(z+1)^{\frac{3}{2}} - 2 = 25$

$+2 \quad +2$

$(z+1)^{\frac{3}{2}} = 27$

$\left((z+1)^{\frac{3}{2}}\right)^{\frac{2}{3}} = (27)^{\frac{2}{3}}$

$z+1 = (3^3)^{\frac{2}{3}}$

$z+1 = 3^2$

$z+1 = 9$

$z = 8$

CHECK:

$(z+1)^{\frac{3}{2}} - 2 = 25$

$(8+1)^{\frac{3}{2}} - 2 = 25$

$(9)^{\frac{3}{2}} - 2 = 25$

$(3^2)^{\frac{3}{2}} - 2 = 25$

$3^3 - 2 = 25$

$27 - 2 = 25$

$25 = 25$ ✓

7. $(x-3)^2 = (\sqrt{4x})^2$

$(x-3)(x-3) = 4x$

$x^2 - 3x - 3x + 9 = 4x$

$x^2 - 6x + 9 = 4x$

$-4x \quad -4x$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$\boxed{x=9}$ or ~~$x=1$~~

↓ The only solution

CHECK:

$(x-3) = \sqrt{4x}$

$9-3 = \sqrt{4(9)}$

$6 = \sqrt{36}$

$6 = 6$ ✓

$(x-3) \neq \sqrt{4x}$

$1-3 \neq \sqrt{4(1)}$

$-2 \neq \sqrt{4}$

$-2 \neq 2$ ✗

8. $3 + \sqrt[5]{5x+7} = 5$

$-3 \quad -3$

$\left(\sqrt[5]{5x+7}\right)^5 = (2)^5$ "Raise both sides to 5th power"

$5x+7 = 32$

$-7 \quad -7$

$5x = 25$

$\boxed{x = 5}$

CHECK!

$3 + \sqrt[5]{5x+7} = 5$

$3 + \sqrt[5]{5(5)+7} = 5$

$3 + \sqrt[5]{32} = 5$

$3 + \sqrt[5]{2^5} = 5$

$3 + 2 = 5$

$5 = 5$ ✓

9. $(\sqrt{3k+1})^2 = (\sqrt{5k}-1)^2$ "Square both sides"

$3k+1 = (\sqrt{5k}-1)(\sqrt{5k}-1)$

	$\sqrt{5k}$	-1
$\sqrt{5k}$	$5k$	$-1\sqrt{5k}$
-1	$-1\sqrt{5k}$	$+1$

$3k+1 = 5k - 2\sqrt{5k} + 1$

$-1 \quad -1$

$3k = 5k - 2\sqrt{5k}$

$-5k \quad -5k$

$-2k = -2\sqrt{5k}$

$\frac{-2k}{-2} = \frac{-2\sqrt{5k}}{-2}$

$(k)^2 = (\sqrt{5k})^2$ "Square both sides again"

$k^2 = 5k$

$-5k \quad -5k$

$k^2 - 5k = 0 \Rightarrow k(k-5) = 0$

$k=0$ or $k-5=0 \Rightarrow \boxed{k=5}$ ← Only solution

CHECK:

$\sqrt{3(0)+1} \neq \sqrt{5(0)} - 1$

$\sqrt{1} \neq \sqrt{0} - 1$

$1 \neq 0 - 1$

$1 \neq -1$ ✗

CHECK:

$\sqrt{3(5)+1} = \sqrt{5(5)} - 1$

$\sqrt{15+1} = \sqrt{25} - 1$

$\sqrt{16} = 5 - 1$

$4 = 4$ ✓