10.3. Advanced Algebra DATE: 12/9

Properties of Logarithms

Target 4E. Solve exponential and logarithmic equations.
Target 4F. Condense and expand logarithms using logarithmic properties.

Since logarithms are exponents, the properties of logarithms are derived from the properties of exponents.

Product Property of Logarithms:

Use the N-spire to evaluate the logarithmic expressions below:

1a) $\log_2(4 \cdot 16) = 6$ 1b) $\log_2 4 + \log_2 16 = 6$

2a) $\log_3(27 \cdot 9) = 5$ 2b) $\log_3 27 + \log_3 9 = 5$

3a) $\log_5(25 \cdot 15{,}625) = 8$ 3b) $\log_5 25 + \log_5 15{,}625 = 8$

Conclusion: The logarithm of a product is the sum of the logarithms of its factors. In symbols, for all positive numbers m, n, and b, where $b \neq 1$,

$$\log_b(m \cdot n) = \log_b m + \log_b n$$

Expand the following Logarithms:

1. $\log_2 7x$ $\log_2 7 + \log_2 x$

2. $\log_6 2ab$ $\log_6 2 + \log_6 a + \log_6 b$

Condense the following Logarithms:

3. $\log_3 4 + \log_3 m$ $\log_3 4m$

4. $\log_9 3 + \log_9 5 + \log_9 u + \log_9 v$ $\log_9 3 \cdot 5 \cdot u \cdot v = \log_9 15uv$

10-3 Properties of Logarithms - Microsoft Word

Quotient Property of Logarithms:

Use the N-spire to evaluate the logarithmic expressions below:

1a) $\log_2\left(\frac{16}{4}\right) = 2$ 1b) $\log_2 16 - \log_2 4 = 2$

2a) $\log_3\left(\frac{27}{9}\right) = 1$ 2b) $\log_3 27 - \log_3 9 = 1$

3a) $\log_5\left(\frac{15{,}625}{25}\right) = 4$ 3b) $\log_5 15{,}625 - \log_5 25 = 4$

Conclusion: The logarithm of a quotient is the difference of the logarithms of numerator and denominator. In symbols, for all positive numbers m, n, and b, where $b \neq 1$,

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

Expand the following Logarithms:

5. $\log_2 \frac{7}{x}$ $\log_2 7 - \log_2 x$

6. $\log_6 \frac{2a}{b}$ $\log_6 2a - \log_6 b$ $= \log_6 2 + \log_6 a - \log_6 b$

Condense the following Logarithms:

7. $\log_3 4 - \log_3 m$ $\log_3 \frac{4}{m}$

8. $\log_9 15 - \log_9 3$ $\log_9 \frac{15}{3} = \log_9 5$

Power Property of Logarithms:

Use the N-spire to evaluate the logarithmic expressions below:

1a) $\log\ 4^3 =$ 1b) $3 \cdot \log\ 4 =$

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Power Property of Logarithms:

Use the N-spire to evaluate the logarithmic expressions below:

1a) $\log_2 4^3 = 6$ 1b) $3\log_2 4 = 6$

2a) $\log_3 27^{17} = 51$ 2b) $17\log_3 27 = 51$

3a) $\log_5 15{,}625^4 = 24$ 3b) $4\log_5 15{,}625 = 24$

Conclusion: The logarithm of a ___power___ is the ___product___ of the logarithm and the exponent. In symbols, for any real number p and positive numbers m and b, where $b \neq 1$,

$$\log_b m^p = p \cdot \log_b m$$

Expand the following Logarithms:

9. $\log_2 5^x$ $x \cdot \log_2 5$

10. $\log_6 x^5$ $5 \cdot \log_6 x$

Condense the following Logarithms:

11. $2\log_3 7 - \log_3 m$

$\log_3 7^2 - \log_3 m = \log_3 \frac{7^2}{m}$

$= \log_3 \frac{49}{m}$

12. $3\log_9(4x) + \log_9 3$

$\log_9 (4x)^3 + \log_9 3$

$\log_9 4^3x^3 + \log_9 3$

$= \log_9(4^3 \cdot 3 \cdot x^3) = \log_9 192x^3$ ✓

$64 \times 3 = 192$

Solve Equations Using Properties of Logarithms

Solve each equation.

13. $3\log_5 x - \log_5 4 = \log_5 16$

$\log_5 x^3 - \log_5 4 = \log_5 16$

$\cancel{\log_5} \frac{x^3}{4} = \cancel{\log_5} 16$

$\frac{x^3}{4} \times \frac{16}{1} \Rightarrow x^3 = 64$

$\therefore x = 4$

14. $\log_4 x + \log_4(x-6) = 2$

$\log_4 x \cdot (x-6) = 2$

$4^2 = x(x-6)$

$16 = x^2 - 6x$

$0 = x^2 - 6x - 16$

$0 = (x-8)(x+2)$

$0 = x - 8$ or $0 = x + 2$

$8 = x$ $-2 = x$