Factoring Review

Essential Understanding You can write some trinomials of the form $x^2 + bx + c$ as the product of two binomials.

To understand how, consider the product of binomials below.

$$(x + 3)(x + 7) = x^2 + (7 + 3)x + 3 \cdot 7 = x^2 + 10x + 21$$

The coefficient of the trinomial's x^2-term is 1. The coefficient of the trinomial's x-term, 10, is the *sum* of the numbers 3 and 7 in the binomials. The trinomial's constant term, 21, is the *product* of the same numbers, 3 and 7. To factor a trinomial of the form $x^2 + bx + c$ as the product of binomials, you must find two numbers that have a sum of b and a product of c.

Problem 1 Factoring $x^2 + bx + c$ Where $b > 0$, $c > 0$

What is the factored form of $x^2 + 8x + 15$?

List the pairs of factors of 15. Identify the pair that has a sum of 8.

Factors of 15	Sum of Factors
1 and 15	16
3 and 5	8 ✔

$x^2 + 8x + 15 = (x + 3)(x + 5)$

Check $(x + 3)(x + 5) = x^2 + 5x + 3x + 15$

$= x^2 + 8x + 15$ ✔

Problem 2 Factoring $x^2 + bx + c$ Where $b < 0$, $c > 0$

What is the factored form of $x^2 - 11x + 24$?

List the pairs of negative factors of 24. Identify the pair that has a sum of -11.

Factors of 24	Sum of Factors
-1 and -24	-25
-2 and -12	-14
-3 and -8	-11 ✔
-4 and -6	-10

$x^2 - 11x + 24 = (x - 3)(x - 8)$

Check $(x - 3)(x - 8) = x^2 - 8x - 3x + 24$

$= x^2 - 11x + 24$ ✔

Problem 3 Factoring $x^2 + bx + c$ Where $c < 0$

What is the factored form of $x^2 + 2x - 15$?

Identify the pair of factors of -15 that has a sum of 2.

Factors of -15	Sum of Factors
1 and -15	-14
-1 and 15	14
3 and -5	-2
-3 and 5	2 ✔

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Essential Understanding You can write some trinomials of the form $ax^2 + bx + c$ as the product of two binomials.

Consider the trinomial $6x^2 + 23x + 7$. To factor it, think of $23x$ as $2x + 21x$.

$$\begin{aligned} 6x^2 + 23x + 7 &= 6x^2 + 2x + 21x + 7 && \text{Rewrite } 23x \text{ as } 2x + 21x. \\ &= 2x(3x + 1) + 7(3x + 1) && \text{Factor out the GCF of each pair of terms.} \\ &= (2x + 7)(3x + 1) && \text{Distributive Property} \end{aligned}$$

How do you know to rewrite $23x$ as $2x + 21x$? Notice that multiplying 2 and 21 gives 42, which is the product of the x^2-coefficient 6 and the constant term 7. This example suggests that to factor a trinomial of the form $ax^2 + bx + c$ you should look for factors of the product ac that have a sum of b.

Problem 1 Factoring When *ac* Is Positive

What is the factored form of $5x^2 + 11x + 2$?

Step 1 Find factors of ac that have sum b. Since $ac = 10$ and $b = 11$, find positive factors of 10 that have sum 11.

Factors of 10	1, 10	2, 5
Sum of Factors	11 ✔	7

Step 2 To factor the trinomial, use the factors you found to rewrite bx.

$$\begin{aligned} 5x^2 + 11x + 2 &= 5x^2 + 1x + 10x + 2 && \text{Rewrite } bx\text{: } 11x = 1x + 10x. \\ &= x(5x + 1) + 2(5x + 1) && \text{Factor out the GCF of each pair of terms.} \\ &= (x + 2)(5x + 1) && \text{Distributive Property} \end{aligned}$$

Problem 2 Factoring When *ac* Is Negative

What is the factored form of $3x^2 + 4x - 15$?

Step 1 Find factors of ac that have sum b. Since $ac = -45$ and $b = 4$, find factors of -45 that have sum 4.

Factors of −45	1, −45	−1, 45	3, −15	−3, 15	5, −9	−5, 9
Sum of Factors	−44	44	−12	12	−4	4 ✔

Step 2 To factor the trinomial, use the factors you found to rewrite bx.

$$\begin{aligned} 3x^2 + 4x - 15 &= 3x^2 - 5x + 9x - 15 && \text{Rewrite } bx\text{: } 4x = -5x + 9x. \\ &= x(3x - 5) + 3(3x - 5) && \text{Factor out the GCF of each pair of terms.} \\ &= (3x - 5)(x + 3) && \text{Distributive Property} \end{aligned}$$

To factor a polynomial completely, first factor out the GCF of the polynomial's terms. Then factor the remaining polynomial until it is written as the product of polynomials that cannot be factored further.

Problem 4 Factoring Out a Monomial First

What is the factored form of $18x^2 - 33x + 12$?

Think

Factor out the GCF.

Write

$$18x^2 - 33x + 12 = 3(6x^2 - 11x + 4)$$

Factor $6x^2 - 11x + 4$. Since $ac = 24$ and $b = -11$, find negative factors of 24 that have sum -11.

Factors of 24	−1, −24	−2, −12	−3, −8	−4, −6
Sum of Factors	−25	−14	−11 ✔	−10

Rewrite the term bx. Then use the Distributive Property to finish factoring.

$$3(6x^2 - 3x - 8x + 4)$$
$$3[3x(2x - 1) - 4(2x - 1)]$$
$$3(3x - 4)(2x - 1)$$

Problem 5 Factoring Out a Common Factor

What is the factored form of $24g^2 - 6$?

$24g^2 - 6 = 6(4g^2 - 1)$ Factor out the GCF, 6.

$= 6[(2g)^2 - 1^2]$ Write the difference as $a^2 - b^2$.

$= 6(2g + 1)(2g - 1)$ Use the rule for the difference of squares.

take note

Key Concept Factoring a Difference of Two Squares

Algebra For all real numbers a and b:
$a^2 - b^2 = (a + b)(a - b)$

Examples $x^2 - 64 = (x + 8)(x - 8)$
$25x^2 - 36 = (5x + 6)(5x - 6)$

Problem 3 Factoring a Difference of Two Squares

What is the factored form of $z^2 - 9$?

Think	Write
Rewrite 9 as a square.	$z^2 - 9 = z^2 - 3^2$
Factor using the rule for a difference of two squares.	$= (z + 3)(z - 3)$
Check your answer by multiplying the factored form.	$(z + 3)(z - 3) = z^2 - 3z + 3z - 9$ $= z^2 - 9$ ✔

Problem 4 Factoring a Difference of Two Squares

What is the factored form of $16x^2 - 81$?

$16x^2 - 81 = (4x)^2 - 9^2$ Write each term as a square.

$= (4x + 9)(4x - 9)$ Use the rule for the difference of squares.