Trig Extended: Circular Functions Cont'd (Target 6A)

Review of Prior Concepts

(PARCC Sample Question)

1. Angle θ is in Quadrant II, and $\sin\theta = \frac{4}{5}$. $\frac{opp}{hyp}$

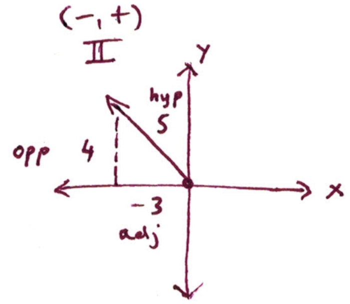
What is the value of $\cos\theta$?

A. $\frac{4}{5}$

B. $\frac{3}{5}$

C. $-\frac{3}{5}$

D. $-\frac{4}{5}$



Like (3, 4, 5)

$\cos\theta = \frac{adj}{hyp} = \frac{-3}{5}$

(ACT Sample Question)

2. In the following triangle, what is the value of $\csc n$?

$\csc n = \frac{hyp}{opp} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

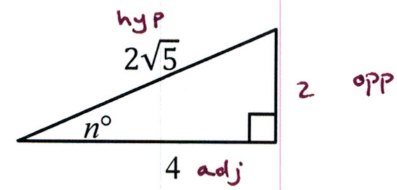
A. $\sqrt{5}$

B. $2\sqrt{5}$

C. $\frac{\sqrt{5}}{2}$

D. $\frac{\sqrt{5}}{5}$

E. $\frac{2\sqrt{5}}{5}$



$a^2 + b^2 = c^2$

$4^2 + b^2 = (2\sqrt{5})^2$

$(2\sqrt{5})^2$
$(2\sqrt{5})(2\sqrt{5})$
$4 \cdot 5$

$16 + b^2 = 4 \cdot 5$

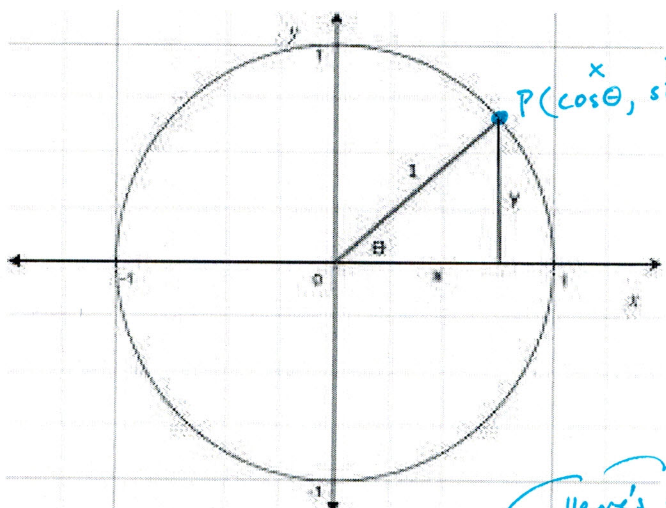
$16 + b^2 = 20$

$-16 \quad -16$

$b^2 = 4$

$b = 2$

Target 6C explained



$P(\cos\theta, \sin\theta)$ (x, y)

$x^2 + y^2 = 1^2$

$(\cos\theta)^2 + (\sin\theta)^2 = 1$

$\boxed{\cos^2\theta + \sin^2\theta = 1}$ ✓

Here's another approach!

Prove $\cos^2\theta + \sin^2\theta = 1$. ☺

Pf: $\cos^2\theta + \sin^2\theta = (\cos\theta)^2 + (\sin\theta)^2$

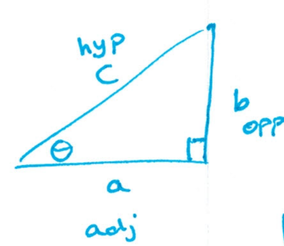
$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$

$= \frac{a^2 + b^2}{c^2}$ $(a^2 + b^2 = c^2)$

$= \frac{c^2}{c^2}$

$= 1$ ✓



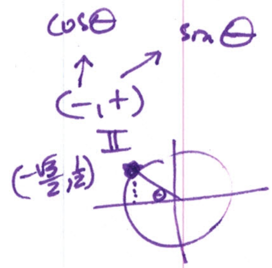
Pythagorean Thm:

$a^2 + b^2 = c^2$

$\boxed{\cos\theta = \frac{a}{c}}$

$\boxed{\sin\theta = \frac{b}{c}}$

Examples



1. Find $\cos\theta$ given that $\sin\theta = \boxed{\frac{1}{2}}$ in Quadrant II.

$\cos^2\theta + \sin^2\theta = 1$ substitute $\frac{1}{2}$ for $\sin\theta$

$\cos^2\theta + (\sin\theta)^2 = 1$

$\cos^2\theta + \left(\frac{1}{2}\right)^2 = 1$

$\cos^2\theta + \frac{1}{4} = 1$

$\quad -\frac{1}{4} \quad -\frac{1}{4}$

$\left(\frac{1}{2}\right)^2 = \frac{1^2}{2^2} = \frac{1}{4}$

$\sqrt{\cos^2\theta} = \pm\sqrt{\frac{3}{4}}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$

Since we are in Q II, $\boxed{\cos\theta = -\frac{\sqrt{3}}{2}}$ ✓

2. Find $\cos\theta$ given that $\sin\theta = -\frac{\sqrt{2}}{2}$ in Quadrant IV. [Note: #1 can be done with unit ⊙]

Same idea as #1.

$\cos^2\theta + \sin^2\theta = 1$

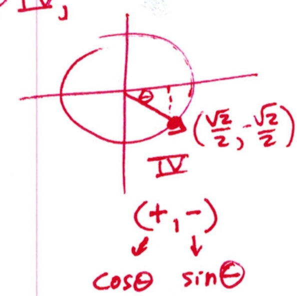
$\cos^2\theta + \left(-\frac{\sqrt{2}}{2}\right)^2 = 1$

$\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{4}}{4} = \frac{2}{4} = \frac{1}{2}$

$\cos^2\theta + \frac{1}{2} = 1$

$\cos^2\theta = \frac{1}{2} \implies \cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

Since we are in Q IV, $\boxed{\cos\theta = \frac{\sqrt{2}}{2}}$ ✓



3. Find $\sin\theta$ given that $\cos\theta = \frac{5}{13}$ in Quadrant I.

Same idea as #1 & #2

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{5}{13}\right)^2 + \sin^2\theta = 1$

$\frac{25}{169} + \sin^2\theta = 1$

$-\frac{25}{169} \quad -\frac{25}{169}$

$\sin^2\theta = 1 - \frac{25}{169}$

$= \frac{169}{169} - \frac{25}{169}$

$= \frac{169-25}{169}$

$= \frac{144}{169}$

$\therefore \sin^2\theta = \frac{144}{169}$

$\sin\theta = \pm\sqrt{\frac{144}{169}}$

$\sin\theta = \pm\frac{12}{13}$

Since we are in Q I, $\boxed{\sin\theta = \frac{12}{13}}$ ✓

4. tan θ is negative. In which quadrant(s) would θ be located if $\sin\theta$ is negative?

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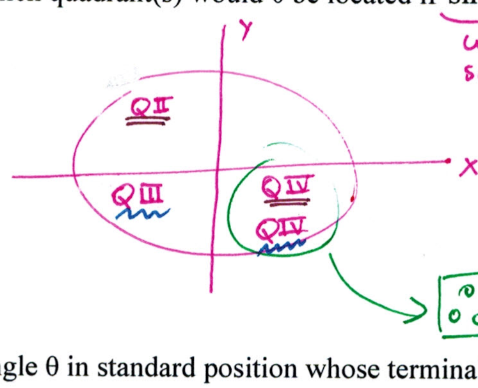
Where is $\tan\theta$ negative?

① Tan θ is negative in Q II & Q IV

Where is $\sin\theta$ negative?

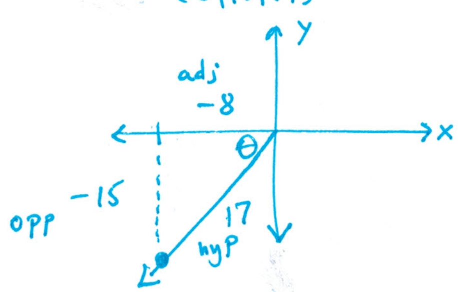
② $\sin\theta$ is negative in Q III & Q IV



$\boxed{\therefore \text{Q IV}}$

5. What is $\cos\theta$ for an angle θ in standard position whose terminal side contains the point $(-8, -15)$?

(8, 15, 17)



$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$= \frac{-8}{17}$

$\boxed{\therefore \cos\theta = \frac{-8}{17}}$