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8.5. Honors Geometry

DATE: 3/5/13

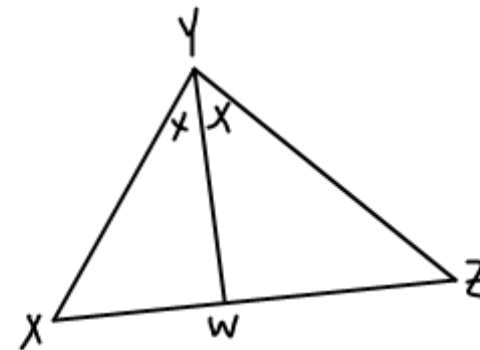
Target 7E. Understand and apply theorems relating proportions in triangles

Angle Bisector Theorem: If a ray bisects an angle of a triangle, then it divides the opposite side into segments that are proportional to the adjacent sides.

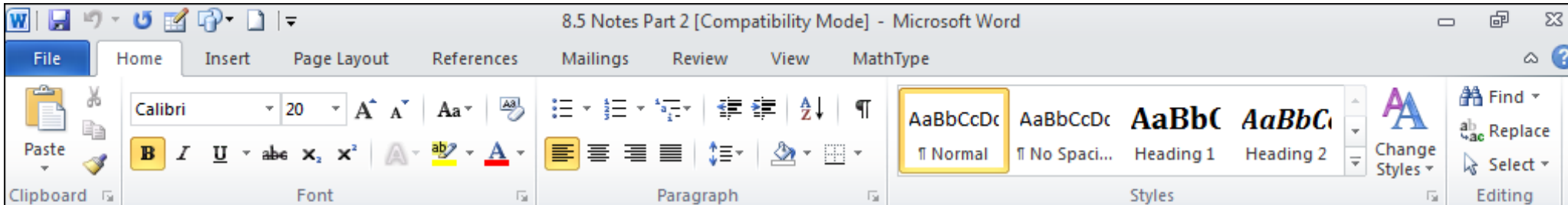
Given: $\triangle XYZ$, \overline{YW} bisects $\angle XYZ$

Diagram:

Prove: $\frac{XW}{WZ} = \frac{XY}{YZ}$



The proof of this theorem was completed in class after a construction using the Nspire calculator. You may wish to read the proof on pg. 352 in your book.



Examples

Find PR . By the \angle bisector theorem, we know

$$\frac{OP}{PR} = \frac{OM}{MR} \Rightarrow \frac{6}{x} = \frac{18}{12}$$

$$\Rightarrow 6(12) = 18x \quad \therefore PR = 4$$

$$\Rightarrow \frac{72}{18} = \frac{18x}{18}$$

$$\Rightarrow 4 = x$$

\overline{BD} bisects $\angle ABC$. Find AC .

Again by \angle bisector thm,

$$\frac{AD}{DC} = \frac{AB}{BC} \Rightarrow \frac{3}{x} = \frac{2x}{24}$$

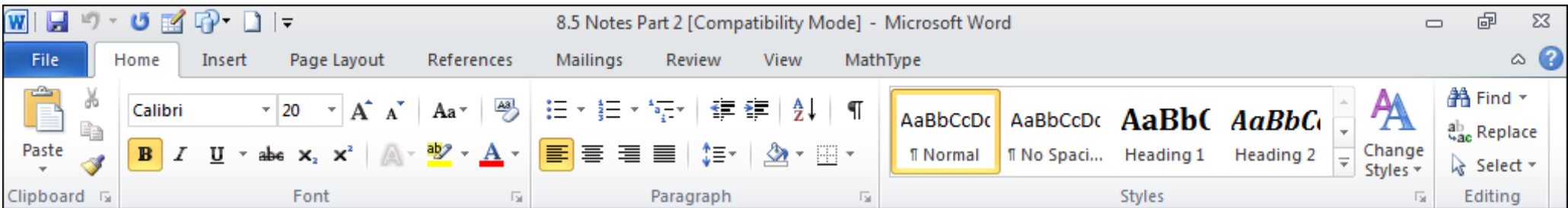
$$\Rightarrow 3(24) = 2x \cdot x$$

$$\Rightarrow \frac{72}{2} = \frac{2x^2}{2}$$

$$\Rightarrow \sqrt{36} = \sqrt{x^2}$$

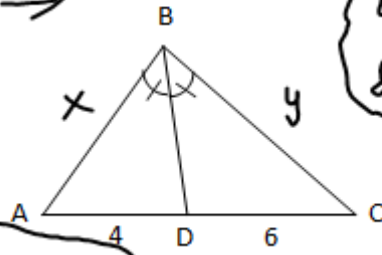
$$\Rightarrow \pm 6 = x$$

But (-6) cannot be the length of \overline{DC} (why?)
 $\therefore DC = 6$, so $AC = 3 + 6 = 9$



The perimeter of $\triangle ABC$ is 25. Find BC. Again, \propto bis.thm \Rightarrow

$$\frac{AD}{DC} = \frac{AB}{BC} \Rightarrow \frac{4}{6} = \frac{x}{y} \Rightarrow 4y = 6x$$



Let $AB = x$
& $BC = y$

Since we don't know length...

Also, the perimeter of $\triangle ABC$ is 25. Since perimeter of $\triangle ABC$ is the sum of $AC + BC + AB$, then

$$\begin{array}{r} 4 + 6 + y + x = 25 \\ 10 + y + x = 25 \\ \hline -10 \quad -10 \\ \hline y + x = 15 \end{array}$$

$$\begin{array}{r} y + x = 15 \\ -x \quad -x \\ \hline y = 15 - x \end{array}$$

By substitution,

$$\begin{array}{r} 4(15 - x) = 6x \\ 60 - 4x = 6x \\ +4x \quad +4x \\ \hline 60 = 10x \end{array}$$

$$\begin{array}{r} \text{So } y = 15 - x \\ = 15 - 6 \\ = 9 \end{array}$$

"choose to solve for x or y." I choose to solve for y.

$$\frac{60}{10} = \frac{10x}{10} \Rightarrow 6 = x$$

BC = 9