

## 4.4 Graphs of Sinusoidal Functions (Target 5E)

### Review of Prior Concepts

- 1) From the parent function  $f(x) = x^2$ , describe the transformation of  $g(x) = (x - 1)^2 + 3$  and give the domain and range of  $g(x)$ .

vertical shift up 3 units,  
horizontal shift right 1 unit

Domain:  $(-\infty, \infty)$

Range:  $[3, \infty)$

- 2) From the parent function  $h(x) = e^x$ , describe the transformation of  $k(x) = e^{x+1} - 3$  and give the domain and range of  $k(x)$ .

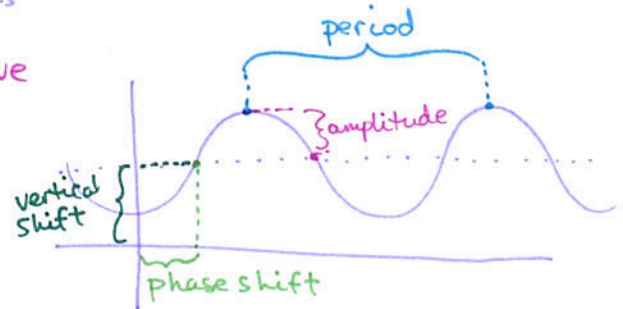
vertical shift down 3 units  
horizontal shift left 1 unit.

Domain:  $(-\infty, \infty)$

Range:  $(-3, \infty)$

### Vocabulary

- Sinusoidal Functions – sine and cosine functions
- Amplitude –  $\frac{1}{2}$  height of the sine/cosine wave
- Period – one cycle of the sine wave
- Phase Shift – horizontal (left/right) shift
- Vertical Shift – up/down shift



$$y = a \sin(bx + c) + d \quad \text{OR} \quad y = a \cos(bx + c) + d$$

Open the TI-Nspire document: *Basic\_Transformations.tns*

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1. Drag the sliders to change the values of  $a$  in the function  $f(x) = a \sin(bx)$ .

- a) How does the value of  $a$  affect the shape of the graph?

vertical shrink or stretch

- b) What happens to the graph if  $a$  is negative?

reflects over the x-axis

- c) How does the value of  $b$  affect the shape of the graph?

horizontal shrink or stretch

### Conclusion:

For  $a \neq 0$  and  $b > 0$ , the graph of  $f(x) = a \sin(bx)$  has an amplitude of  $|a|$  and a period of  $\frac{2\pi}{|b|}$ .

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2. Drag the sliders to change the value of  $d$  in the function of  $f(x) = \sin(x) + d$ .

How does the value of  $d$  affect the shape of the graph?

vertical shift (up/down)

Conclusion:

The graph of  $f(x) = \sin(x) + d$  has a vertical shift of  $d$ .

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3. Drag the sliders to change the value of  $c$  in the function of  $f(x) = \sin(x + c)$ .

How does the value of  $c$  affect the shape of the graph?

horizontal (left/right) shift

Conclusion:

The graph of  $f(x) = \sin(x + c)$  has a phase shift of  $c$ .

$c > 0$ , shift left  
 $c < 0$ , shift right

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4. Drag the sliders to change the value of  $a$ ,  $b$ ,  $c$  and  $d$  in the function  $f(x) = a \sin(bx + c) + d$ .

Which of the four parameters have an impact on the phase shift of the graph?

$c$  and  $b$

$a \sin(b(x + \frac{c}{b})) + d$   
 ↓  
 phase shift

Conclusion:

The graph of  $f(x) = a \sin(bx + c) + d$  has a phase shift of  $\frac{c}{b}$ .



Transformation	General Form $f(x) = a \sin(bx + c) + d$ OR $f(x) = a \cos(bx + c) + d$	Example $f(x) = 3 \sin(2x + \pi) - 4$
Amplitude	$ a $	3
Period	$\frac{2\pi}{ b }$	$\frac{2\pi}{ 2 } = \frac{2\pi}{2} = \pi$
Phase Shift	$\frac{c}{b}$	$\frac{\pi}{2}$ (to the left)
Vertical Shift	$d$	4 (down)

Sketch the graph of:  $f(x) = 3 \sin(2x + \pi) - 4$

① graph parent function

② consider period

$$\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$$

③ consider amplitude

$$|a| = |3| = 3$$

④ consider phase shift

$$\frac{c}{b} = \frac{\pi}{2} \text{ (1/2 left)}$$

⑤ consider vertical shift

$$d = -4 \text{ (4 down)}$$

