Group members____

Pass the Trigonometric Proof

Write the Pythagorean Identities
(Write one and pass it on. The next person checks previous person's work)

1. $\cos^2\theta + \sin^2\theta = 1$

2. $1 + \tan^2\theta = \sec^2\theta$

3. $\cot^2\theta + 1 = \csc^2\theta$

Prove the Identities
(Write one step and pass it on. Then next person checks previous person's work)

1. $\frac{\tan^2 x+1}{1+\cot^2 x} = \tan^2 x$

Pf: $\frac{\tan^2 x + 1}{1+\cot^2 x} = \frac{\sec^2 x}{\csc^2 x}$

$= \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}$

$= \frac{\sin^2 x}{\cos^2 x}$

$= \tan^2 x$

$\therefore$, $\frac{\tan^2 x+1}{1+\cot^2 x} = \tan^2 x$.

2. $\frac{\sin^2\alpha\cot^2\alpha}{1-\sin^2\alpha} = 1$

Pf: $\frac{\sin^2\alpha\cot^2\alpha}{1-\sin^2\alpha} = \frac{\sin^2\alpha \cdot \frac{\cos^2\alpha}{\sin^2\alpha}}{\cos^2\alpha}$

$= \frac{\cos^2\alpha}{\cos^2\alpha}$

$= 1$

$\therefore$, $\frac{\sin^2\alpha\cot^2\alpha}{1-\sin^2\alpha} = 1$.

3. $-\cos^2 x\sin^2 x = \frac{\cos^2 x}{-1-\cot^2 x}$

Pf: $\frac{\cos^2 x}{-1-\cot^2 x} = -\frac{\cos^2 x}{1+\cot^2 x}$

$= -\frac{\cos^2 x}{\csc^2 x}$

$= -\frac{\cos^2 x}{\frac{1}{\sin^2 x}}$

$= -\cos^2 x\sin^2 x$

$\therefore$, $\frac{\cos^2 x}{-1-\cot^2 x} = -\cos^2 x\sin^2 x$.

4. $\sin\theta + \cos\theta = \frac{\tan\theta+1}{\sec\theta}$

Pf: $\frac{\tan\theta + 1}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta} + 1}{\frac{1}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{\frac{\sin\theta + \cos\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{\sin\theta+\cos\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}$

$= \sin\theta + \cos\theta$

$\therefore$, $\frac{\tan\theta+1}{\sec\theta} = \sin\theta + \cos\theta$.

5. $\frac{\tan x - \tan x \sin^2 x}{2 \sin x \cos x} = \frac{1}{2}$

Pf: $\frac{\tan x - \tan x \sin^2 x}{2 \sin x \cos x} = \frac{\tan x (1 - \sin^2 x)}{2 \sin x \cos x}$

$= \frac{\tan x \cdot \cos^2 x}{2 \sin x \cdot \cos x}$

$= \frac{\frac{\sin x}{\cos x} \cdot \cos^2 x}{2 \sin x \cdot \cos x}$

$= \frac{1 \cdot \cancel{\sin x} \cdot \cancel{\cos x}}{2 \cancel{\sin x} \cdot \cancel{\cos x}}$

$= \frac{1}{2}$

$\therefore \frac{\tan x - \tan x \sin^2 x}{2 \sin x \cos x} = \frac{1}{2}$.

6. $\frac{1 + \cot x}{\csc x} = \sin x + \cos x$

Pf: $\frac{1 + \cot x}{\csc x} = \frac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \frac{\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \frac{\frac{\sin x + \cos x}{\sin x}}{\frac{1}{\sin x}}$

$= \frac{\sin x + \cos x}{\sin x} \cdot \sin x$

$= \sin x + \cos x$

$\therefore \frac{1 + \cot x}{\csc x} = \sin x + \cos x$.

CHALLENGE: As a group, write your own identity for another group to prove.

You try it!

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Confirm that $\cosh^2 x - \sinh^2 x = 1$ if $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$.

Pf: $\cosh^2 x - \sinh^2 x = \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$

$= \frac{(e^x + e^{-x})^2}{2^2} - \frac{(e^x - e^{-x})^2}{2^2}$

$= \frac{(e^x + e^{-x})(e^x + e^{-x}) - [(e^x - e^{-x})(e^x - e^{-x})]}{4}$

$= \frac{e^{2x} + e^0 + e^0 + e^{-2x} - [e^{2x} - e^0 - e^0 + e^{-2x}]}{4}$ Distribute "−"

$= \frac{\cancel{e^{2x}} + 1 + 1 + \cancel{e^{-2x}} - \cancel{e^{2x}} + 1 + 1 - \cancel{e^{-2x}}}{4}$

$= \frac{4}{4}$

$= 1$ Confirmed ☺