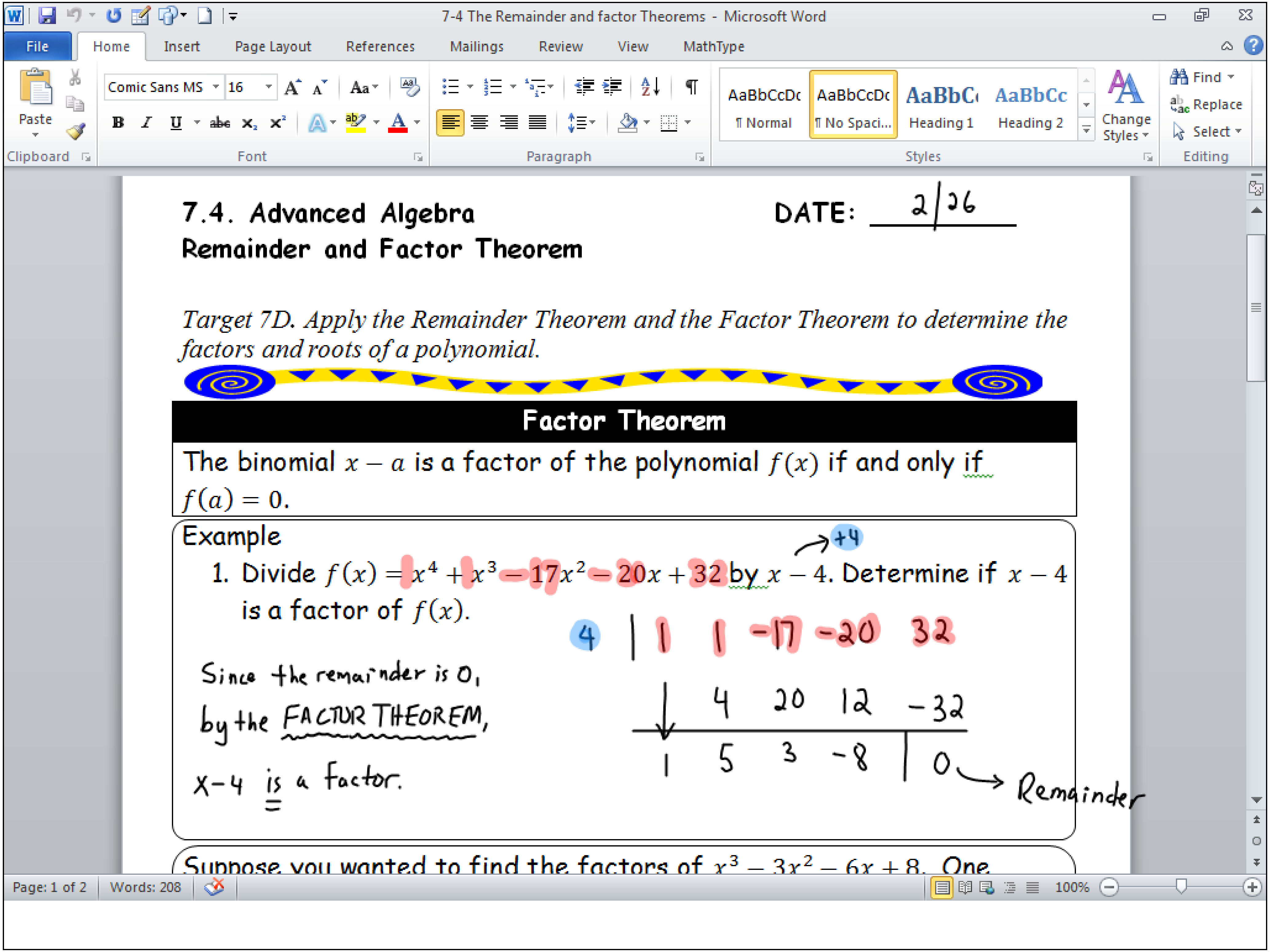


7.4. Advanced Algebra
Remainder and Factor Theorem

DATE: 2/26

Target 7D. Apply the Remainder Theorem and the Factor Theorem to determine the factors and roots of a polynomial.

Factor Theorem

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

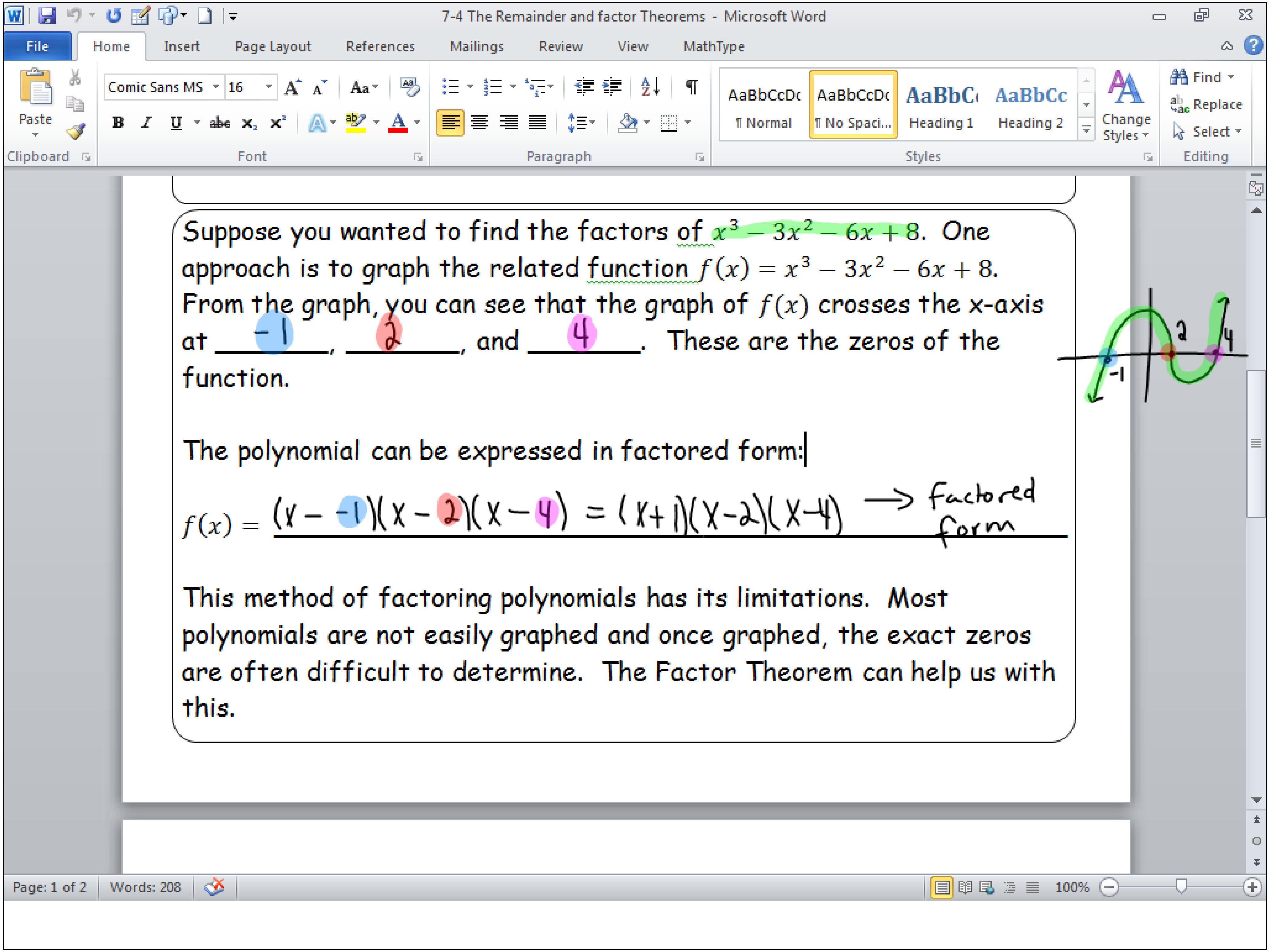
Example

1. Divide $f(x) = x^4 + x^3 - 17x^2 - 20x + 32$ by $x \overset{+4}{-} 4$. Determine if $x - 4$ is a factor of $f(x)$.

$$\begin{array}{r|rrrrr} 4 & 1 & 1 & -17 & -20 & 32 \\ & \downarrow & 4 & 20 & 12 & -32 \\ \hline & 1 & 5 & 3 & -8 & \big| \; 0 \end{array} \xrightarrow{} \text{Remainder}$$

Since the remainder is 0,
by the FACTOR THEOREM,
x-4 is a factor.

Suppose you wanted to find the factors of $x^3 - 3x^2 - 6x + 8$. One

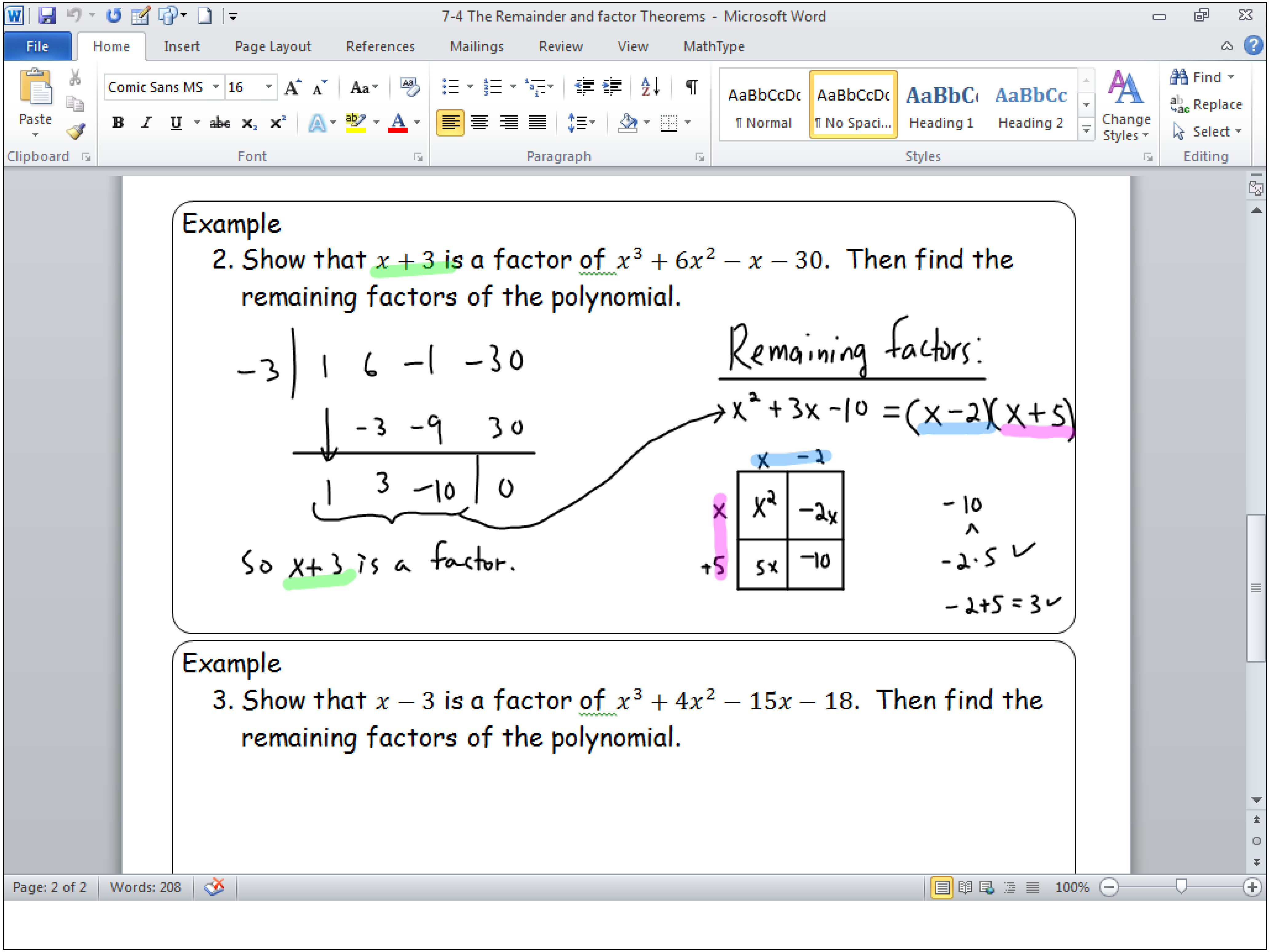


Suppose you wanted to find the factors of $x^3 - 3x^2 - 6x + 8$. One approach is to graph the related function $f(x) = x^3 - 3x^2 - 6x + 8$. From the graph, you can see that the graph of $f(x)$ crosses the x-axis at -1, 2, and 4. These are the zeros of the function.

The polynomial can be expressed in factored form:

$$f(x) = (x - -1)(x - 2)(x - 4) = (x+1)(x-2)(x-4) \longrightarrow \text{Factored form}$$

This method of factoring polynomials has its limitations. Most polynomials are not easily graphed and once graphed, the exact zeros are often difficult to determine. The Factor Theorem can help us with this.



Example

2. Show that $x + 3$ is a factor of $x^3 + 6x^2 - x - 30$. Then find the remaining factors of the polynomial.

$$\begin{array}{r|rrrr} -3 & 1 & 6 & -1 & -30 \\ & \downarrow & -3 & -9 & 30 \\ \hline & 1 & 3 & -10 & 0 \end{array}$$

So $x+3$ is a factor.

Remaining factors:

$x^2 + 3x - 10 = (x-2)(x+5)$

$$\begin{array}{c|c|c} & x & -2 \\ \hline x & x^2 & -2x \\ \hline +5 & 5x & -10 \end{array}$$

-10

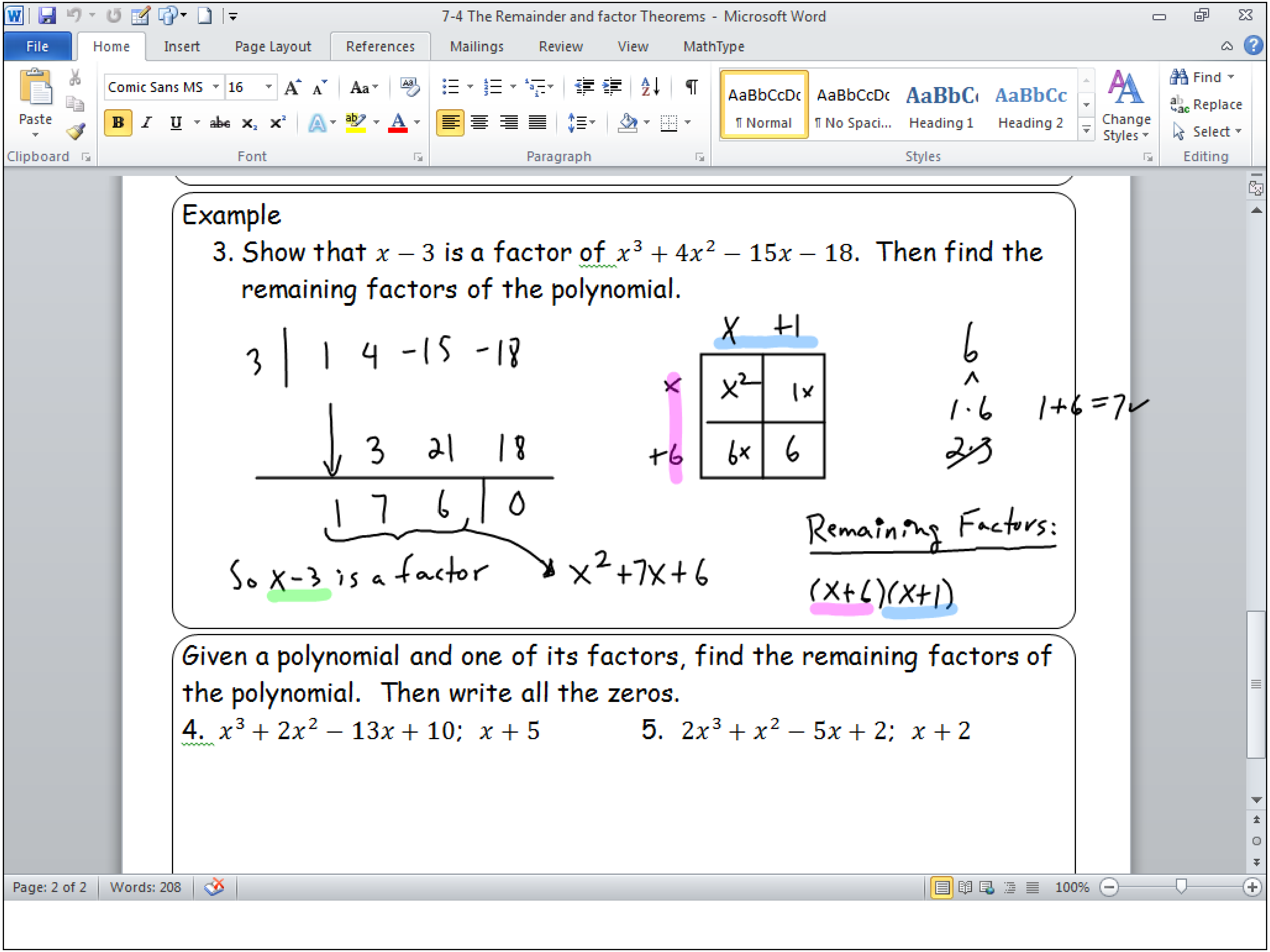
$\wedge$

$-2 \cdot 5$ ✓

$-2+5=3$ ✓

Example

3. Show that $x - 3$ is a factor of $x^3 + 4x^2 - 15x - 18$. Then find the remaining factors of the polynomial.



Example

3. Show that $x - 3$ is a factor of $x^3 + 4x^2 - 15x - 18$. Then find the remaining factors of the polynomial.

$$\begin{array}{c|cccc} 3 & 1 & 4 & -15 & -18 \\ & \downarrow & 3 & 21 & 18 \\ \hline & 1 & 7 & 6 & 0 \end{array}$$

So $x-3$ is a factor → x^2+7x+6

	x	$+1$
x	x^2	$1x$
$+6$	$6x$	6

6

$1 \cdot 6$ $1+6=7$ ✓

~~$2 \cdot 3$~~

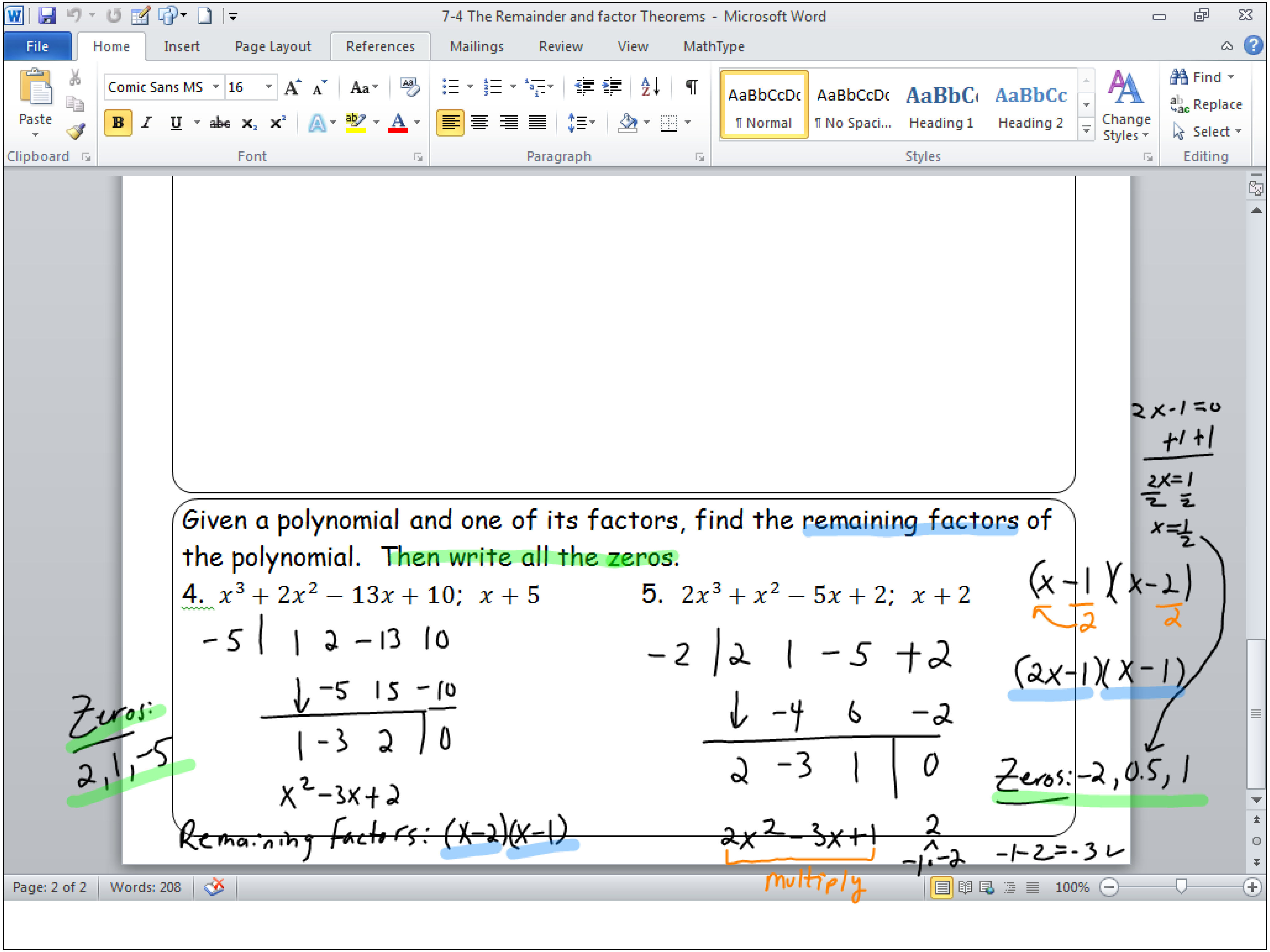
Remaining Factors:

$(x+6)(x+1)$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Then write all the zeros.

4. $x^3 + 2x^2 - 13x + 10;\ x + 5$

5. $2x^3 + x^2 - 5x + 2;\ x + 2$



Given a polynomial and one of its factors, find the remaining factors of the polynomial. Then write all the zeros.

4. $x^3 + 2x^2 - 13x + 10;\ \ x + 5$

$$\begin{array}{r|rrrr} -5 & 1 & 2 & -13 & 10 \\ & \downarrow & -5 & 15 & -10 \\ \hline & 1 & -3 & 2 & 0 \end{array}$$

x^2-3x+2

Remaining factors: $(x-2)(x-1)$

Zeros: 2, 1, -5

5. $2x^3 + x^2 - 5x + 2;\ \ x + 2$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & -5 & +2 \\ & \downarrow & -4 & 6 & -2 \\ \hline & 2 & -3 & 1 & 0 \end{array}$$

$2x^2-3x+1$ multiply

2

$-1 \cdot -2$

$-1-2=-3$ ✓

$(x-1)(x-2)$

$2 \quad 2$

$(2x-1)(x-1)$

$2x-1=0$

$+1 \ +1$

$2x=1$

$\frac{2x}{2}=\frac{1}{2}$

$x=\frac{1}{2}$

Zeros: -2, 0.5, 1