Name: **Key**   Period: _____
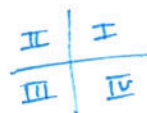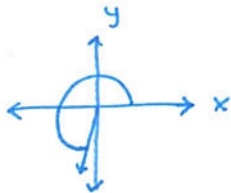
Honors Advanced Algebra: 2nd Semester Review

Target 6.A -- Utilize a unit circle to determine trigonometric values of special angles.
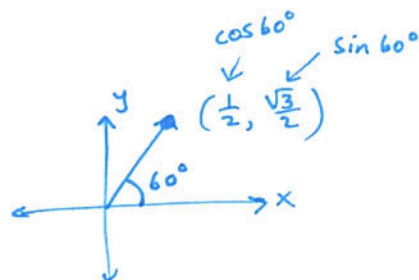
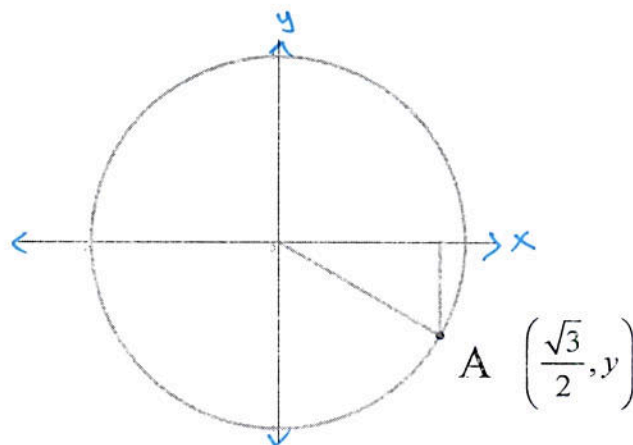1. An angle in standard position with a measure of 265° lies in what quadrant?

    $\boxed{\text{Quadrant } \underline{III}}$

2. Find the exact value of tan 60°. Show work.

$$\tan 60° = \frac{\sin 60°}{\cos 60°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$$

$$= \boxed{\sqrt{3}}$$

$\cos 60° \quad \sin 60°$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$60°$

3. Find y such that the point A is on the unit circle.

$$y = -\frac{1}{2}$$

A $\left(\frac{\sqrt{3}}{2}, y\right)$

Know the unit circle!

4. Change 330° to radians.

$$330° \cdot \frac{\pi \text{ rad}}{180°} = \frac{330\pi \text{ rad}}{180}$$

$$= \boxed{\frac{11\pi}{6} \text{ rad}} \quad \text{or} \quad \boxed{\frac{11\pi}{6}}$$

$$y = a \cos b\theta$$

Target 6.B – Graph, transform, and identify the key features of trigonometric functions.

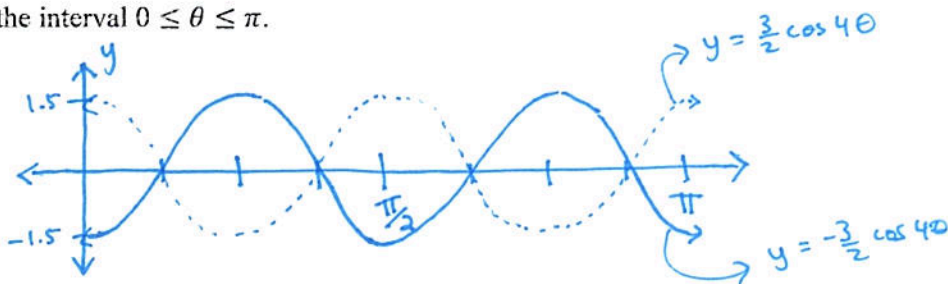5. What is the period of $y = -\frac{3}{2}\cos 4\theta$?

$$\frac{2\pi}{|4|} = \frac{2\pi}{4}$$

$$\text{Period}: \quad \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$$

6. Graph the function $y = -\frac{3}{2}\cos 4\theta$ for the interval $0 \le \theta \le \pi$.

Amplitude: $\left|-\frac{3}{2}\right| = 1.5$

Period: $\frac{\pi}{2}$

$y = \frac{3}{2}\cos 4\theta$

$y = -\frac{3}{2}\cos 4\theta$

7. What is the amplitude of $y = -\frac{3}{2}\cos 4\theta$?

Amplitude: $\left|-\frac{3}{2}\right| = \frac{3}{2}$ or 1.5

Target 6.C – Use the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$, to find $\sin\theta$, $\cos\theta$ or $\tan\theta$, and the quadrant of the angle.

8. Find $\sin\theta$ given that $\tan\theta = -\frac{7}{24}$ in Quadrant IV.

$$\tan\theta = -\frac{7 \text{ opp}}{24 \text{ adj}}$$
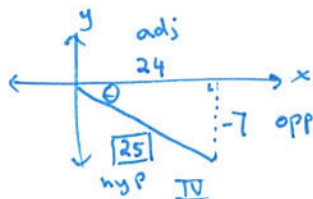
$$24^2 + (-7)^2 = r^2$$

$$\boxed{25 = r}$$

$$\boxed{\therefore \sin\theta = -\frac{7}{25}}$$

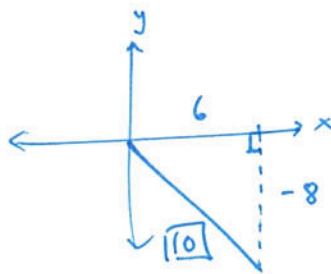| Students	All
Take	Calculus

9. In what quadrant is $\cos\theta$ positive and $\tan\theta$ negative?

$\tan\theta$ "$-$" | $\cos\theta$ "$+$"
$\cos\theta$ "$+$" tanθ "$-$"

ALL STUDENTS TAKE CALCULUS

$$\boxed{\therefore \text{Quadrant IV}}$$

10. Find $\sin\theta$ and $\cos\theta$ for an angle θ, in standard position whose terminal side contains the point (6, -8)?

$$x^2 + y^2 = r^2$$
$$6^2 + (-8)^2 = r^2$$
$$36 + 64 = r^2$$
$$100 = r^2$$
$$\boxed{10 = r}$$

hypotenuse always! positive

$$\therefore \boxed{\sin\theta = \frac{-8}{10} = -\frac{4}{5}}$$

Simplify ratio!

$$\boxed{\cos\theta = \frac{6}{10} = \frac{3}{5}}$$

Target 6.D – Apply trigonometric functions to model real life phenomena.

11. Given the equation $y = 3\sin 2\theta$, identify the amplitude, ~~interval,~~ and the period.
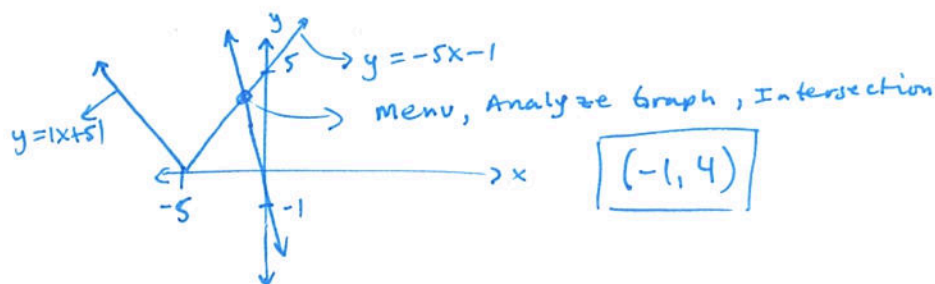
Amplitude : $|a| = |3| = 3$

Period : $\dfrac{2\pi}{|b|} = \dfrac{2\pi}{|2|} = \dfrac{2\pi}{2} = \pi$

Target 7.A – Solve a system of various functions using technology.

12. Find the solution(s) to this system of equations: (show work)

$$y = |x + 5| \text{ and } y = -5x - 1$$

Graph using Nspire!

$y = -5x - 1$

$y = |x+5|$

Menu, Analyze Graph, Intersection

$(-1, 4)$

13. Find the solutions(s) to this system of equations: (show work)

$$y = -(x + 3)^2 + 5$$
$$y = -2x - 4$$

Can graph and find intersection, too!

$-(x+3)^2 + 5 = -2x - 4$

$-[(x+3)(x+3)] + 5 = -2x - 4$

$-(x^2 + 3x + 3x + 9) + 5 = -2x - 4$

$-(x^2 + 6x + 9) + 5 = -2x - 4$

$-x^2 - 6x - 9 + 5 = -2x - 4$

$-x^2 - 6x - 4 = -2x - 4$
$ + 2x + 4 \quad + 2x + 4$

$-x^2 - 4x = 0$

$-x(x + 4) = 0$

$-x = 0 \quad$ or $\quad x + 4 = 0$
$\boxed{x = 0} \qquad\qquad \boxed{x = -4}$

When $x = 0$:
$y = -2(0) - 4$
$y = -4$
$\boxed{\therefore (0, -4)}$

When $x = -4$:
$y = -2(-4) - 4$
$y = 8 - 4 = 4$
$\boxed{\therefore (-4, 4)}$

Target 7.B – Key features of multiple functions can be compared using various representations.

14. Given the equations $y = 2x$, $y = 2^x$, and $y = x^2$, which one grows at a faster rate given $x \geq 4$ and why? Identify each type of function.

$y = 2x$ is a linear function.

$y = 2^x$ is an exponential function.

$y = x^2$ is a Quadratic function.

x	$2x$	2^x	x^2
4	8	16	16
5	10	32	25
6	12	64	36
7	14	128	49

$\downarrow$

grows @ fastest rate

OR $36 = 6^2$, $16 = 4^2$, $4 = 2^2$, $0 = 0^2$, $4 = 2^2$, $16 = 4^2$, $36 = 6^2$

↳ Quadratic

15. Which type of function best models the data below: linear, exponential, or quadratic?

∴ Quadratic

x	-3	-2	-1	0	1	2	3
y	36	16	4	0	4	16	36

-20   -12   -4   4   12   20

8   8   8   8   8   → 2nd difference constant

16. Which type of function best models the data below: linear, exponential, or quadratic?

x	y
-2	216
-1	144
0	96
1	64
2	$\frac{128}{3}$
3	$\frac{256}{9}$

→ -72
→ -48
→ -32
→ $-\frac{64}{3}$
→ $-\frac{128}{9}$

Look like exponential decay ... can verify with graph (plot pts (-2, 216), (-1, 144), ...)

17. Select function(s) that has the greatest y-intercept.

Function A

$y = 10 * 2^x$

$y = 10 \cdot 2^0$
$y = 10 \cdot 1$
$y = 10$

∴ $(0, 10)$
y-intercept

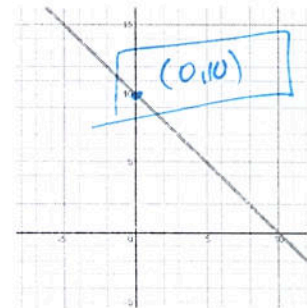Function B

x	-2	-1	0	1	2
y	3	5	8	5	3

∴ y-intercept
$(0, 8)$

∴ Function A and C have greatest y-intercept

Function C

$(0, 10)$

18. Select the choice where the minimum value is larger.

Function A

$y = 2x^2 - 8x + 2$

$= (2, -6)$
min value $= -6$
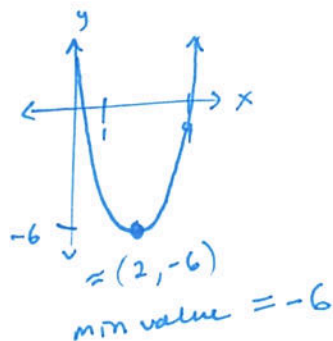
Function B
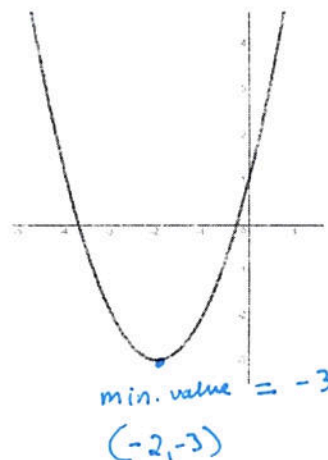
min. value $= -3$
$(-2, -3)$

Function C

X	Y
-3	13
-2	3
-1	-3
0	-5
1	-3
2	3
3	13

min value ∴ -5

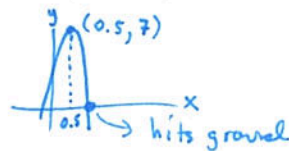∴ Function B has the largest min. value of -3.

Use the information below to answer questions 19, 20, and 21:

Can graph it all

You are on a ledge 3ft high. You throw a ball into the air. Its path is given by the equation: $y = -16t^2 + 16t + 3$

19. What is the maximum height in feet the ball reaches?

@ $t = \dfrac{-b}{2a} = \dfrac{-16}{2(-16)} = \dfrac{1}{2}$ or 0.5 seconds

∴ max height is $y = -16(\frac{1}{2})^2 + 16(\frac{1}{2}) + 3 = -16(\frac{1}{4}) + 8 + 3 = -4 + 8 + 3 = \boxed{7\,ft}$

(0.5, 7) → hits ground

20. At what time does the ball reach its maximum height?

The ball reaches it max height @ $t = \dfrac{-b}{2a} = \dfrac{-16}{2(-16)} = 0.5$ seconds

Using Nspire, find maximum: (time, height)
↓
answer

21. What time does the ball hit the ground?

Ball hits ground when $y = 0$:

Use Nspire.

$0 = -16t^2 + 16t + 3$
$a = -16$
$b = 16$
$c = 3$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2(a)}$

$= \dfrac{-16 \pm \sqrt{16^2 - 4(-16)(3)}}{2(-16)} \approx \boxed{1.16\ sec}$

Target 8A. Identify if two events are independent utilizing probability tests.

22. Give an example of each an independent event and a dependent event. (Answers may vary)

Independent: flipping coin and spinning spinner   Dependent: Selecting cards from deck without replacement

23. The probability that event A occurs is 0.45 and the probability that event B occurs is 0.3. If events A and B are independent events, what is the probability that events A and B occur together?

$P(A \cap B) = P(A) \cdot P(B)$ If events A and B are independent
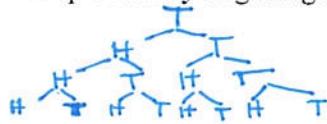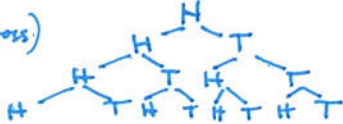$= 0.45 \cdot 0.3$
$= 0.135$ ∴ $\boxed{P(A \cap B) = 0.135}$

Target 8B. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

24. Jack and You and your friends want to play a game flipping a coin 4 times. Determine the sample space of the possible outcomes and then determine the probability of getting a tails on the third toss.

P(tails 3rd toss)
$\frac{8}{16} = \boxed{\frac{1}{2}}$

$S = \{$ HHHH, HHHT, HHTH
HHTT, HTHH, HTHT
HTTH, HTTT, THHH
THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT $\}$

25. When you roll a standard number cube, what is the probability of rolling a number smaller than 5 or an odd number?

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= \frac{4}{6} + \frac{3}{6} - \frac{2}{6}$
$= \boxed{\frac{5}{6}}$

A = roll smaller than 5   (1) 2 (3) 4
B = roll th that's odd   (1) (3) 5

Target 8C. Explain conditional probability and independence using everyday examples of events based on the context of the problem.

$$P(A|B) = P(A)$$

26. The probability of liking apples is 0.72 and the probability of linking bananas is 0.55. The probability of liking apples is independent of the probability of liking bananas. Determine the probability of liking apples *given* that you like bananas.

$$P(\text{like apples} \mid \text{like bananas}) = P(\text{like apples}) = 0.72$$

Since A, B events are independent

27. Of all of the Honors Advanced Algebra students in Morton District 201, 75% have an A in the class, 42% have perfect attendance, and 26% have both an A in Honors Advanced Algebra and an A in the class. What is the conditional probability that a person who has an A in Honors Advanced Algebra also has perfect attendance?

A = A in class
B = perfect attendance

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.26}{0.75} \approx 0.3467 \text{ or } 35\%$$

Target 8D. Compute probabilities of independent, dependent and compound events and use these to interpret data.

28. The tree diagram shows the percent of boys and girls in the 10th grade at a school and whether they joined the student government club. Compete the diagram and then determine the combined probability of the students that joined the club.
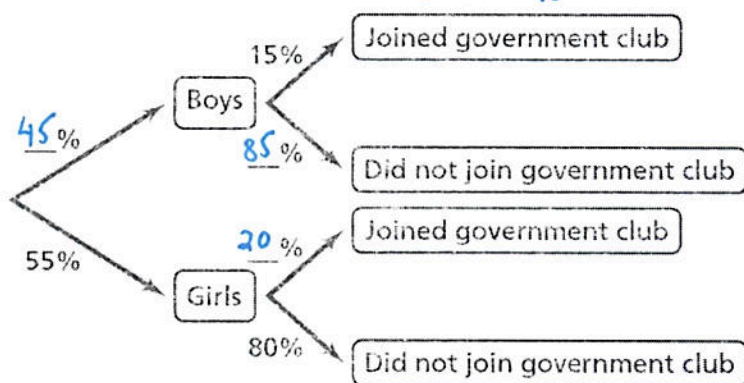
$$P(\text{joined club}) = P(\text{joined} \mid \text{boys}) P(\text{boys}) + P(\text{joined} \mid \text{girls}) P(\text{girls})$$
$$= 0.15 \cdot 0.45 + 0.20 \cdot 0.55 = \boxed{0.1775}$$
$$\text{or } 17.75\%$$

45% → Boys
  15% → Joined government club
  85% → Did not join government club
55% → Girls
  20% → Joined government club
  80% → Did not join government club

29. A survey reported that 63% of students at Morton East high school prefer drinking water, while 37% prefer a sugared drink (juice, soda, etc.). This survey also reported students' snack preference between granola bars and chips. Of the students that prefer water, 59% prefer granola bars. Of the students that prefer a sugared drink, 21% prefer granola bars. Determine the probability that a randomly selected student prefers water *and* prefers granola bars.

63% water
  59% Granola Bars
  41% Chips
37% Sugar Drink
  21% Granola Bars
  79% Chips
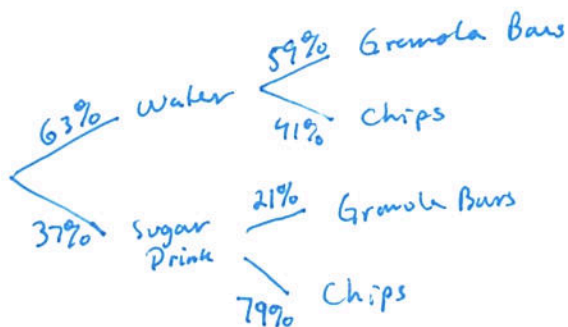
$$P(\text{water AND granola Bars}) = ?$$
$$0.63 \cdot 0.59 = 0.3713$$
$$\text{or} \approx 37\%$$

Target 8E. Construct and interpret a two-way frequency table and test for independence.

30. The two-way frequency table below shows where a random sample of sophomores and juniors prefer to study. Complete the table, and then use then table to determine P(freshman|home).

	Library	Home	Cafe	Totals
Freshman	1	8	1	10
Sophomore	3	4	3	10
Totals	4	12	4	20

$$P(\text{freshman} \mid \text{home})$$
$$= \frac{8}{12} = \boxed{\frac{2}{3}}$$

31. Using the two-way frequency table from question 9, are the events A and B listed below independent events? Show work and explain your reasoning.

Event A: the selected student is a Freshman
Event B: the selected student prefers to study in the Library.

CHECK :
$$P(A \mid B) = P(A) \qquad P(B \mid A) = P(B)$$
$$\frac{1}{4} \neq \frac{10}{20} = \frac{1}{2} \qquad \frac{1}{10} \neq \frac{4}{20} = \frac{1}{5}$$
$$\therefore \text{NOT independent}$$

Target 9A: Use simulation to determine if the data collected is consistent with the selected model or if another model is required.

32. Using the line from the random digit table below, which numbers would you use to choose 4 students from a group of 75 students?

83023 19785 88676 32528 27323

02, 31, 67, 63

33. A bus depot shows that 10% of buses arrive late, 30% of ~~airplanes~~ buses arrive early, and 60% of ~~airplanes~~ buses arrive on time. Out of 50 buses, 15 arrive late, 17 arrive early, and 18 arrive on time, is this model valid? ~~(Assume p = 0.01.)~~

$$\frac{15}{50} = 0.3 \text{ or } 30\% \qquad \frac{17}{50} = 0.34 \text{ or } 34\% \qquad \frac{18}{50} = 0.36 \text{ or } 36\%$$

Invalid

34. In each roll of a game piece, there is a 35% chance of winning 10 points and a 65% chance of losing 4 points. What is the expected value for each roll?

$$0.35(+10) + 0.65(-4) = 3.5 + (-2.6) = \boxed{0.9 \text{ pts}}$$

Target 9B: Use the mean and standard deviation of data to fit into a normal distribution and to estimate population percentages.

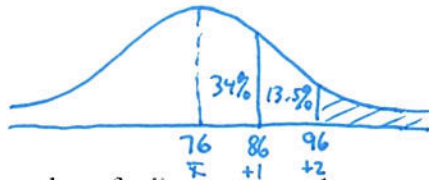35. What is the value of two standard deviations for the normal distribution shown at the below?

496	556	616	676	736	796	856
-3	-2	-1		+1	+2	+3
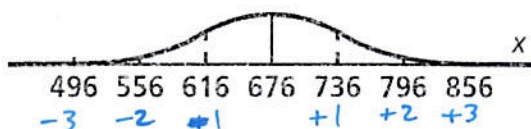
∴ The value of two std. dev.
is 60 + 60 = $\boxed{120}$

$\bar{x} = 676 + 60 = 736$ +1 std dev
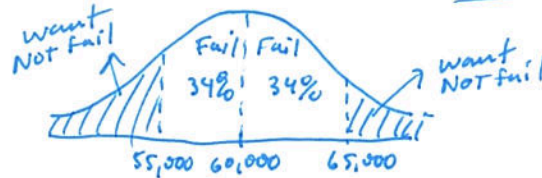736 + 60 = 796   + 2 std dev

36. Scores on an exam are distributed normally with a mean of 76 and a standard deviation of 10. Out of 230 tests, about how many students score above 96?

$$50 - (34.5 + 13.5) = 2.5\%$$

$$0.025 \cdot 230 \approx \boxed{6} \text{ students}$$

34% | 13.5%

76   86   96
$\bar{x}$ +1   +2

37. The number of miles on a car when a certain part fails is normally distributed, with a mean of 60,000 and a standard deviation of 5000. What is the probability that the part will **NOT fail** between 55,000 and 65,000 miles?

want Not Fail · Fail | Fail 34% | 34% · want Not fail

$$100 - (34 + 34)$$
$$100 - 68 = \boxed{32\%}$$

55,000 60,000 65,000

38. The local bakery makes chocolate chip cookies. The number of chips in a cookie is approximately normally distributed with a mean 11.4 and standard deviation 1.3. What is the range of chocolate chips that cover about 95% of the number of chocolate chips?

$$13.5 + 13.5 + 34 + 34 = 95\%$$

13.5 | 34 | 34 | 13.5

8.8) 0.1   11.4   12.7 (14)
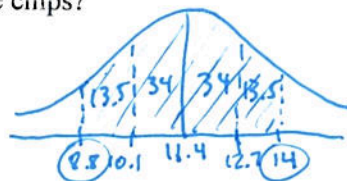
Range 8.8 to 14.0

Target 9C: Estimate a population mean or proportion and utilize it to determine how often the true population mean or proportion is within the margin of error.

39. Which of the following best represents a sample mean?

   A. It's found that on average, 72% of students prefer winter break over spring break.
   B. Radio shows average about 39 minutes of music for every 1 hour of broadcasting.
   C. The average height of a building in downtown Chicago is 625 feet.

40. A company specializing in building robots that clean your house has found that the average amount of time kids are forced to spend cleaning their houses is about 2 hours per week. If their sample was 1000 randomly chose kids and the standard deviation was 0.3 hours, what is the margin of error for a confidence level of 95%?

$$ME = 1.96 \cdot \frac{s}{\sqrt{n}} = 1.96 \cdot \frac{0.3}{\sqrt{1000}} \approx \boxed{0.0186}$$

41. A consumer research group tested the battery life of 36 randomly chose batteries to establish the likely battery life for the population of same type of battery. Use the data below to find the 95% confidence interval for the population mean.

Battery Life (in Hours)			
63.2	84.6	78.4	85.3
62.1	81.8	63.6	64.2
79.4	75.2	54.1	73.4
66.3	74.5	71.6	60.1
61.2	74.5	72.4	81.3
61.4	83.6	75.6	74.1
68.3	82.2	59.3	42.6
86.2	64.3	72.7	71.6
71.4	63.6	59.6	68.1

$\bar{x} = 70.486$

$s = 9.454$

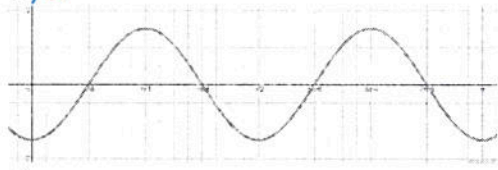$$M.E. = 1.96 \cdot \frac{9.454}{\sqrt{36}} \approx 3.088$$

$$\bar{x} \pm ME$$

$$70.486 + 3.088 = 73.574$$
$$70.486 - 3.088 = 67.398$$

$$[67.398, 73.574]$$

Answer Key

1. III
2. $\sqrt{3}$
3. $-\dfrac{1}{2}$
4. $\dfrac{11\pi}{6}$
5. $\pi/2$
6.
7. $\dfrac{3}{2}$
8. $-\dfrac{7}{25}$
9. IV
10. $\sin\theta = -\dfrac{4}{5}$ $\cos\theta = \dfrac{3}{5}$
11. Amplitude = 3, ~~Intervals = 2~~, Period = π
12. $(-1, 4)$
13. $(-4, 4), (0, -4)$
14. $y = 2x$ linear

    $y = 2^x$ exponential, grows at a faster rate,

    $y = x^2$ quadratic
15. quadratic

16. exponential
17. function A and function C
18. function B
19. 7 feet
20. 0.5 seconds
21. 1.16 seconds
22. (Answers may vary)
23. 0.135
24. S={HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

    P(tails on 3rd toss)=1/2
25. 5/6
26. 0.72
27. About 35% or 0.3467 →correct
28. 45%, 85%, 20% ; 0.1775 or 17.75%
29. 0.3717 or about 37%
30. 2/3
31. The events are NOT independent
32. 02, 31, 67, 63
33. Invalid
34. 0.9
35. 120
36. 6
37. 32%
38. 8.8 to 14.0
39. C
40. 0.018
41. [67.5, 73.5]